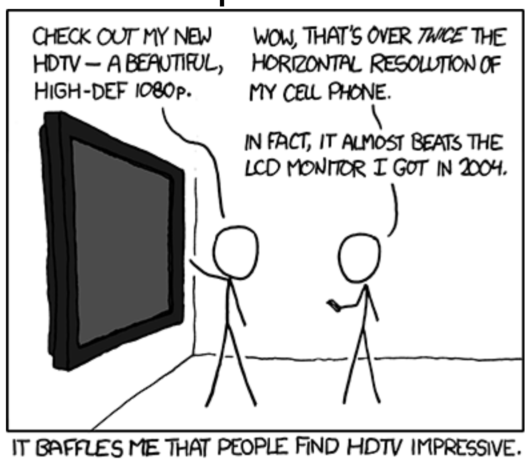
Centroids

Method of Composite Areas

A small boy swallowed some coins and was taken to a hospital. When his grandmother telephoned to ask how he was a nurse said 'No change yet'.



Centroids

- Previously, we developed a general formulation for finding the centroid for a series of n areas

$$\bar{x} = \frac{\sum_{i=1}^{n} x_i A_i}{\sum_{i=1}^{n} A_i}$$

Centroids

- x_i was the distance from the y-axis to the local centroid of the area A_i

$$\bar{x} = \frac{\sum_{i=1}^{n} x_i A_i}{\sum_{i=1}^{n} A_i}$$

Centroids

- If we can break up a shape into a series of smaller shapes that have predefined local centroid locations, we can use this formula to locate the centroid of the composite shape

$$\bar{x} = \frac{\sum_{i=1}^{n} x_i A_i}{\sum_{i=1}^{n} A_i}$$

Centroid by Composite Bodies

- There is a table in the back cover of your book that gives you the location of local centroids for a select group of shapes
- The point labeled C is the location of the centroid of that shape.

Centroid by Composite Bodies

- Please note that these are local centroids, they are given in reference to the x and y axes as shown in the table.

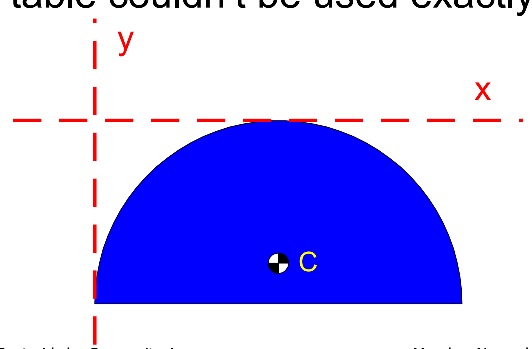
Centroid by Composite Bodies

- For example, the centroid location of the semicircular area has the y-axis through the center of the area and the x-axis at the bottom of the area
- The x-centroid would be located at 0 and the y-centroid would be located at

$$\frac{4r}{3\pi}$$

Centroid by Composite Bodies

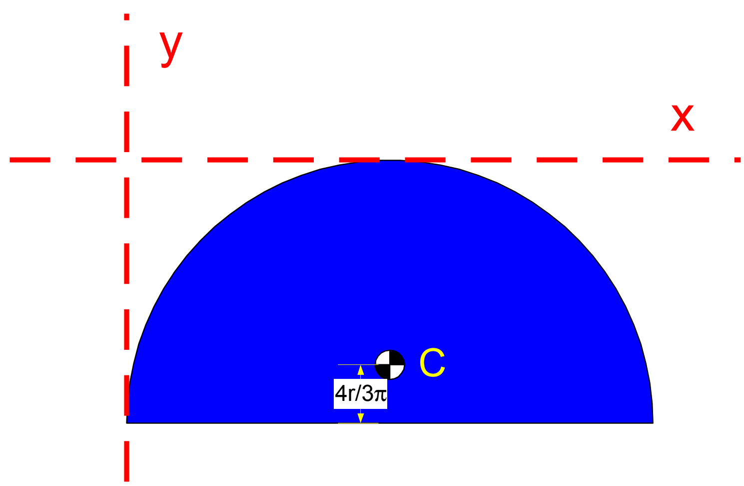
- If we wanted the centroid with respect to another axis, say along the top of the semicircle and along the left edge, the values in the table couldn't be used exactly



Centroid by Composite Bodies

- The table would give you the distance of C above the base of the semicircle, but that isn't the distance from the centroid to the x-axis

Centroid by Composite Bodies

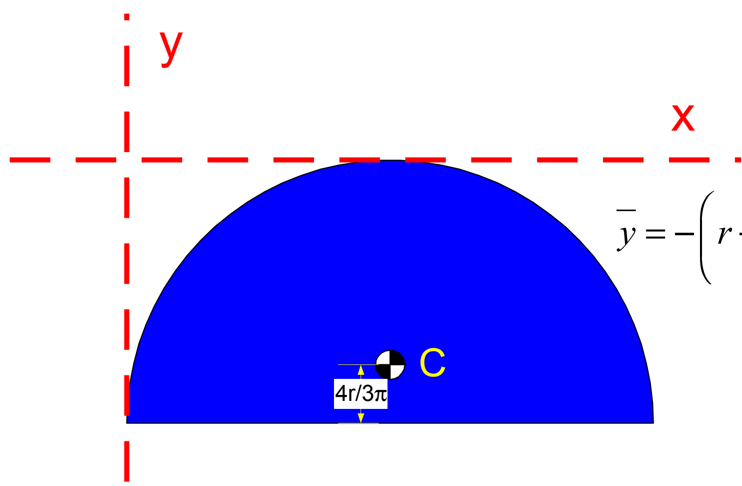


Centroid by Composite Bodies

- Since the radius of the semicircle, in this case the distance to the y-centroid would be

$$\bar{y} = -\left(r - \frac{4r}{3\pi}\right)$$

Centroid by Composite Bodies



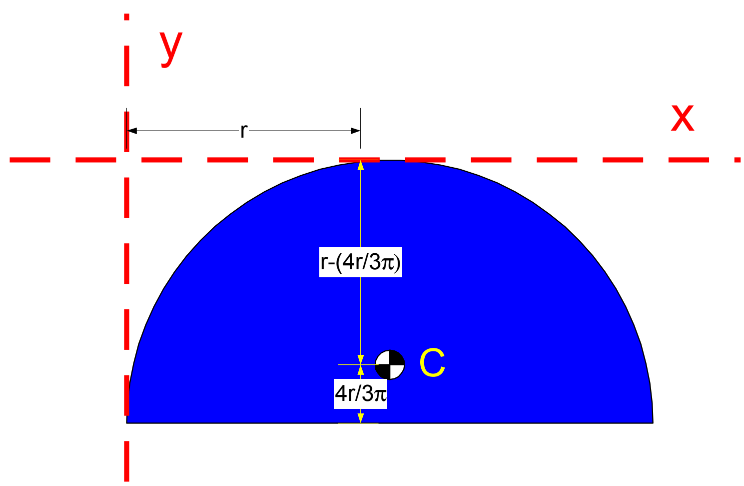
$$\bar{y} = -\left(r - \frac{4r}{3\pi}\right)$$

Centroid by Composite Bodies

- By the same logic, the distance to the x-centroid would be

$$\bar{x} = r$$

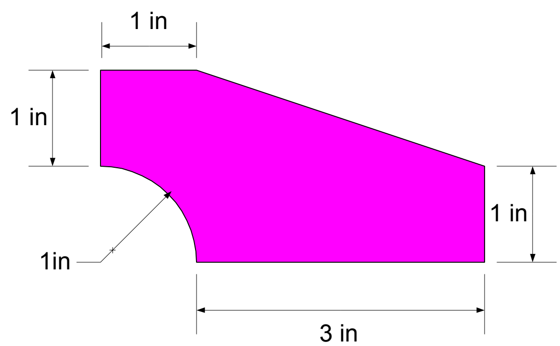
Centroid by Composite Bodies



An Example

- Lets start with an example problem and see how this develops

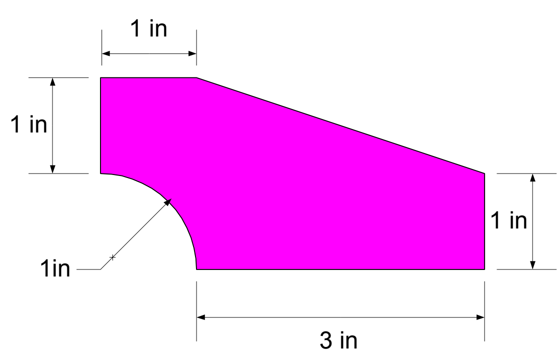
$$\bar{x} = \frac{\sum_{i=1}^{n} x_i A_i}{\sum_{i=1}^{n} A_i}$$



An Example

- We want to locate both the x and y centroids

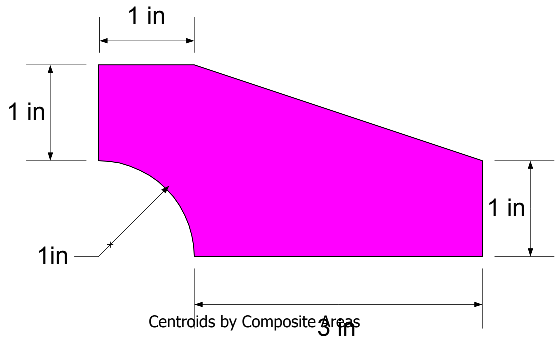
$$\bar{x} = \frac{\sum_{i=1}^{n} x_i A_i}{\sum_{i=1}^{n} A_i}$$



An Example

- There isn't much of a chance of developing a function that is easy to integrate in this case

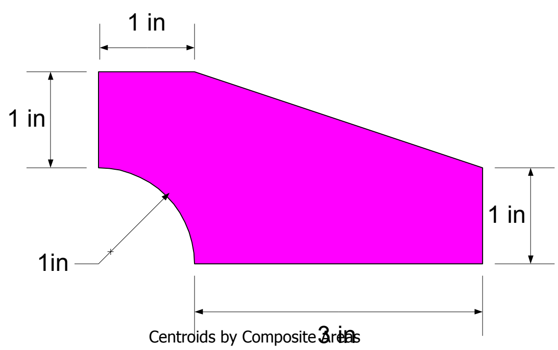
$$\bar{x} = \frac{\sum_{i=1}^{n} x_i A_i}{\sum_{i=1}^{n} A_i}$$



An Example

- We can break this figure up into a series of shapes and find the location of the local centroid of each

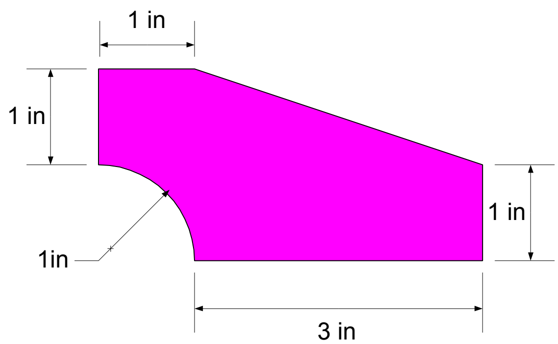
$$\bar{x} = \frac{\sum_{i=1}^{n} x_i A_i}{\sum_{i=1}^{n} A_i}$$



An Example

- There are multiple ways to do this as long as you are consistent

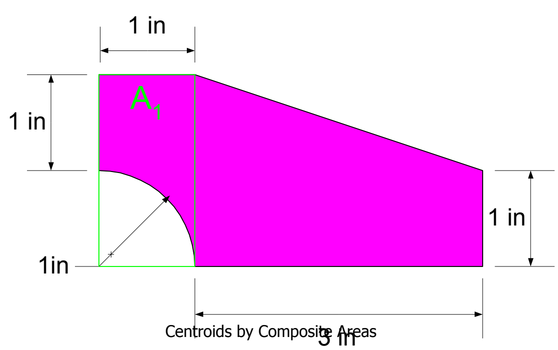
$$\bar{x} = \frac{\sum_{i=1}^{n} x_i A_i}{\sum_{i=1}^{n} A_i}$$



An Example

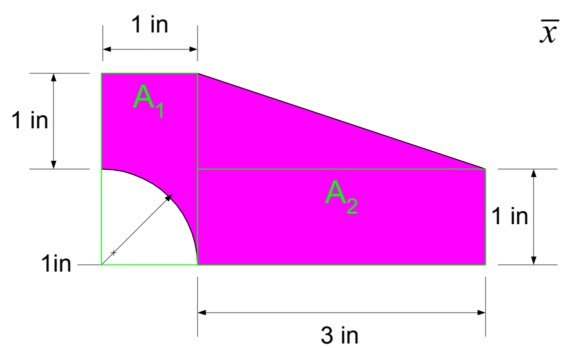
- First, we can develop a rectangle on the left side of the diagram, we will label that as area 1, A_1

$$\bar{x} = \frac{\sum_{i=1}^{n} x_i A_i}{\sum_{i=1}^{n} A_i}$$



An Example

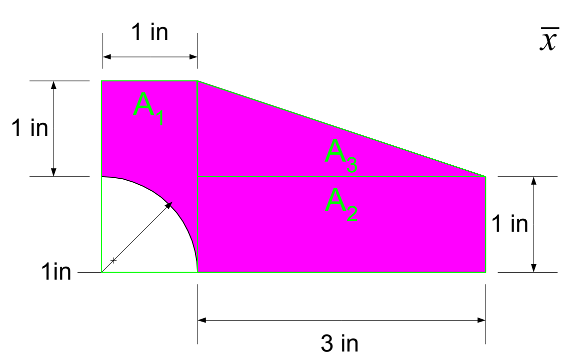
- A second rectangle will be placed in the bottom of the figure, we will label it A_2



$$\bar{x} = \frac{\sum_{i=1}^{n} x_i A_i}{\sum_{i=1}^{n} A_i}$$

An Example

- A right triangle will complete the upper right side of the figure, label it A_3

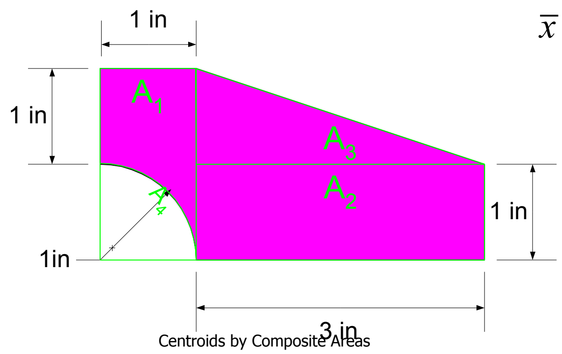


$$\bar{x} = \frac{\sum_{i=1}^{n} x_i A_i}{\sum_{i=1}^{n} A_i}$$

An Example

- Finally, we will develop a negative area to remove the quarter circle in the lower left hand corner, label it A_4

$$\bar{x} = \frac{\sum_{i=1}^{n} x_i A_i}{\sum_{i=1}^{n} A_i}$$

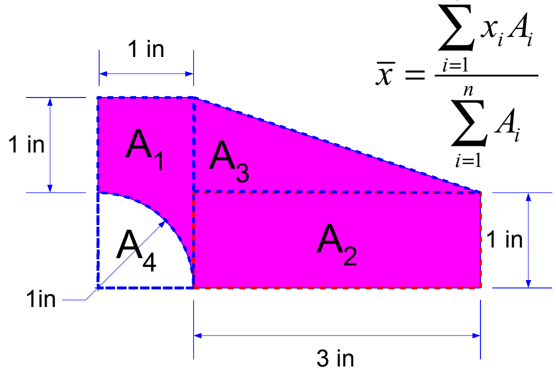


An Example

- We will begin to build a table so that keeping up with things will be easier
- The first column will be the areas

$$\bar{x} = \frac{\sum_{i=1}^{n} x_i A_i}{\sum_{i=1}^{n} A_i}$$

ID	Area
	(in^2)
A_1	2
A_2	3
A_3	1.5
A_4	-0.7854

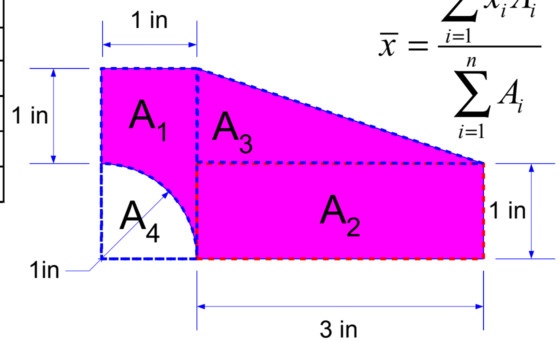


An Example

- Now we will calculate the distance to the local centroids from the y-axis (we are calculating an x-centroid)

ID	Area	x_i
	(in^2)	(in)
A_1	2	0.5
A_2	3	2.5
A_3	1.5	2
A_4	-0.7854	0.42441

$$\bar{x} = \frac{\sum_{i=1}^{n} x_i A_i}{\sum_{i=1}^{n} A_i}$$

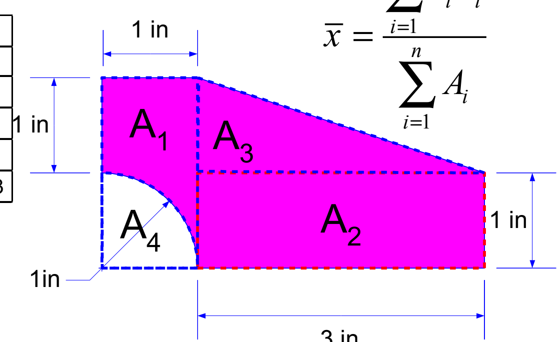


An Example

- To calculate the top term in the expression we need to multiply the entries in the last two columns by one another

ID	Area	x_i	x_i*Area
	(in^2)	(in)	(in^3)
A_1	2	0.5	1
A_2	3	2.5	7.5
A_3	1.5	2	3
A_4	-0.7854	0.42441	-0.33333

$$\bar{x} = \frac{\sum_{i=1}^{n} x_i A_i}{\sum_{i=1}^{n} A_i}$$

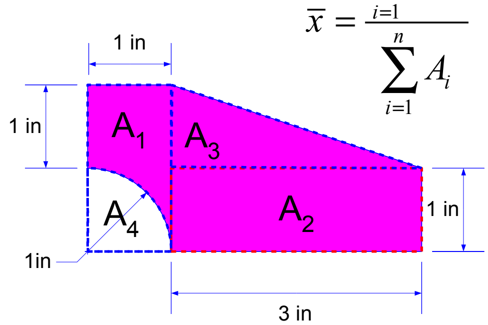


An Example

- If we sum the second column, we have the bottom term in the division, the total area

ID	Area	x_i	x_i*Area
	(in^2)	(in)	(in^3)
A_1	2	0.5	1
A_2	3	2.5	7.5
A_3	1.5	2	3
A_4	-0.7854	0.42441	-0.33333
	5.714602		

$$\bar{x} = \frac{\sum_{i=1}^{n} x_i A_i}{\sum_{i=1}^{n} A_i}$$

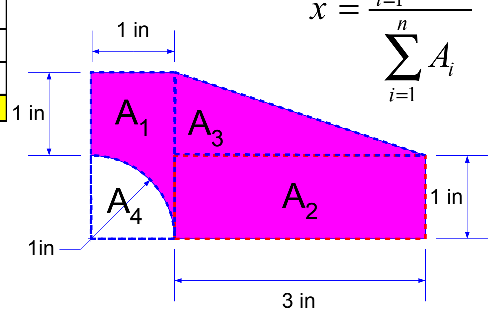


An Example

- And if we sum the fourth column, we have the top term, the area moment

ID	Area	x_i	x_i*Area
	(in^2)	(in)	(in^3)
A_1	2	0.5	1
A_2	3	2.5	7.5
A_3	1.5	2	3
A_4	-0.7854	0.42441	-0.33333
	5.714602		11.16667

$$\bar{x} = \frac{\sum_{i=1}^{n} x_i A_i}{\sum_{i=1}^{n} A_i}$$

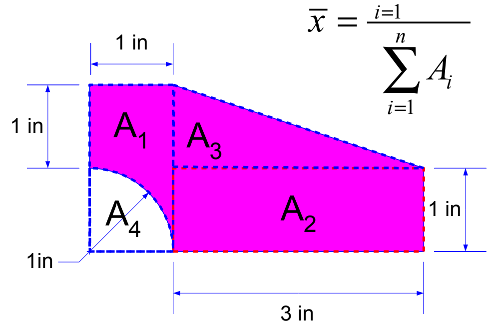


An Example

- Dividing the sum of the area moments by the total area we calculate the x-centroid

ID	Area	x_i	x_i*Area
	(in^2)	(in)	(in^3)
A_1	2	0.5	1
A_2	3	2.5	7.5
A_3	1.5	2	3
A_4	-0.7854	0.42441	-0.33333
	5.714602		11.16667
	x_{bar}	1.9541	

$$\bar{x} = \frac{\sum_{i=1}^{n} x_i A_i}{\sum_{i=1}^{n} A_i}$$

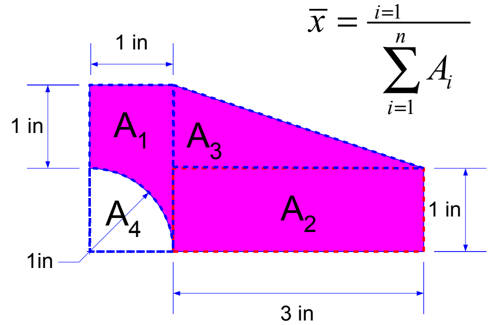


An Example

- You can always remember which to divide by if you look at the final units, remember that a centroid is a distance

ID	Area	x_i	x_i*Area
	(in^2)	(in)	(in^3)
A_1	2	0.5	1
A_2	3	2.5	7.5
A_3	1.5	2	3
A_4	-0.7854	0.42441	-0.33333
	5.714602		11.16667
	x_{bar}	1.9541	

$$\bar{x} = \frac{\sum_{i=1}^{n} x_i A_i}{\sum_{i=1}^{n} A_i}$$

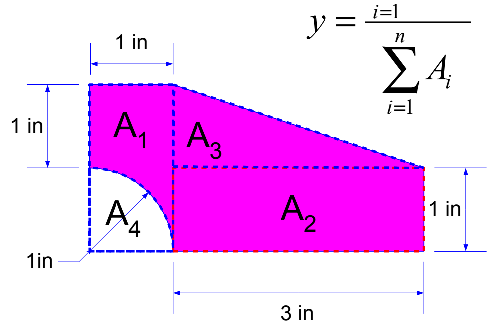


An Example

- We can do the same process with the y centroid

ID	Area	x_i	x_i*Area
	(in^2)	(in)	(in^3)
A_1	2	0.5	1
A_2	3	2.5	7.5
A_3	1.5	2	3
A_4	-0.7854	0.42441	-0.33333
	5.714602		11.16667
	x_{bar}	1.9541	

$$\bar{y} = \frac{\sum_{i=1}^{n} y_i A_i}{\sum_{i=1}^{n} A_i}$$

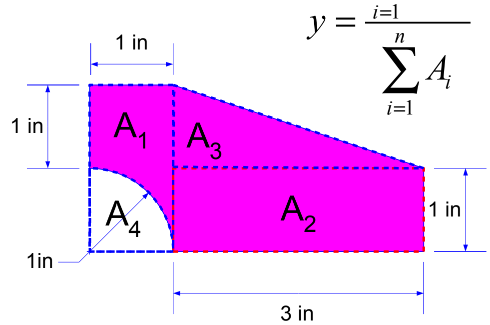


An Example

- Notice that the bottom term doesn't change, the area of the figure hasn't changed

ID	Area	x_i	x_i*Area
	(in^2)	(in)	(in^3)
A_1	2	0.5	1
A_2	3	2.5	7.5
A_3	1.5	2	3
A_4	-0.7854	0.42441	-0.33333
	5.714602		11.16667
	x_{bar}	1.9541	

$$\bar{y} = \frac{\sum_{i=1}^{n} y_i A_i}{\sum_{i=1}^{n} A_i}$$

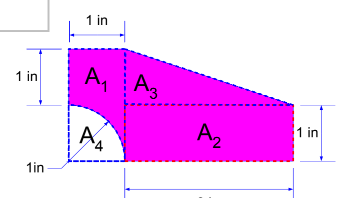


An Example

- We only need to add a column of y_i's

ID	Area	x_i	x_i*Area	y_i
	(in^2)	(in)	(in^3)	(in)
A_1	2	0.5	1	1
A_2	3	2.5	7.5	0.5
A_3	1.5	2	3	1.333333
A_4	-0.7854	0.42441	-0.33333	0.42441
	5.714602		11.16667	
	x_{bar}	1.9541		

$$\bar{y} = \frac{\sum_{i=1}^{n} y_i A_i}{\sum_{i=1}^{n} A_i}$$

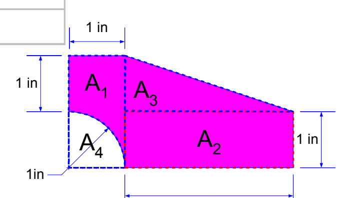


An Example

- Calculate the area moments about the x-axis

ID	Area	x_i	x_i*Area	y_i	y_i*Area
	(in^2)	(in)	(in^3)	(in)	(in^3)
A_1	2	0.5	1	1	2
A_2	3	2.5	7.5	0.5	1.5
A_3	1.5	2	3	1.333333	2
A_4	-0.7854	0.42441	-0.33333	0.42441	-0.33333
	5.714602		11.16667		
	x_{bar}	1.9541			

$$\bar{y} = \frac{\sum_{i=1}^{n} y_i A_i}{\sum_{i=1}^{n} A_i}$$

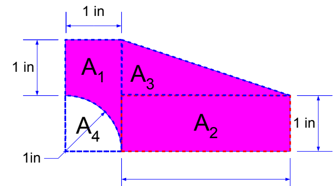


An Example

- Sum the area moments

ID	Area	x_i	x_i*Area	y_i	y_i*Area
	(in^2)	(in)	(in^3)	(in)	(in^3)
A_1	2	0.5	1	1	2
A_2	3	2.5	7.5	0.5	1.5
A_3	1.5	2	3	1.333333	2
A_4	-0.7854	0.42441	-0.33333	0.42441	-0.33333
	5.714602		11.16667		5.166667
	x_{bar}	1.9541			

$$\bar{y} = \frac{\sum_{i=1}^{n} y_i A_i}{\sum_{i=1}^{n} A_i}$$

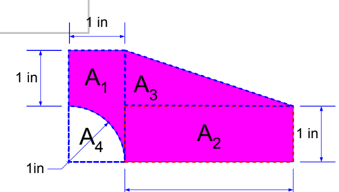


An Example

- And make the division of the area moments by the total area

ID	Area	x_i	x_i*Area	y_i	y_i*Area
	(in^2)	(in)	(in^3)	(in)	(in^3)
A_1	2	0.5	1	1	2
A_2	3	2.5	7.5	0.5	1.5
A_3	1.5	2	3	1.333333	2
A_4	-0.7854	0.42441	-0.33333	0.42441	-0.33333
	5.714602		11.16667		5.166667
	x_{bar}	1.9541	y_{bar}	0.904117	

$$\bar{y} = \frac{\sum_{i=1}^{n} y_i A_i}{\sum_{i=1}^{n} A_i}$$



Example Problem 9-60

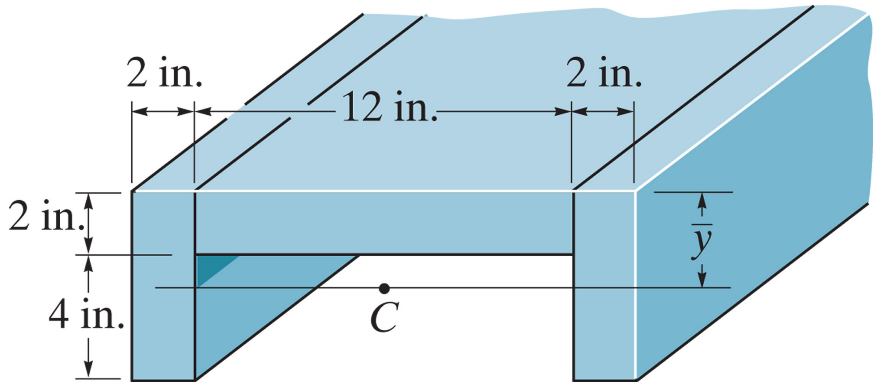


Figure: 09_P060

Example Problem 9-61

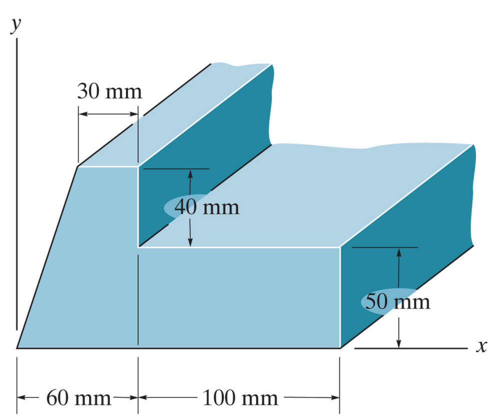


Figure: 09_P061

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Department of Civil Engineering

Homework

- Problem 9-64
- Problem 9-65
- Problem 9-71