CFA Level 1 Notes 2017

Ethical conduct improves outcomes for stakeholders, by balancing self interest with impact on others

Code of ethics rules and standards that require minimum level of ethical behaviour

Professional code of ethics way for profession to communicate to public that its members will use knowledge/skill to serve clients in honest and ethical manner → increase public confidence and trust that members will act ethically

Challenges to ethical behaviour:

- Overestimating own ethical character → **overconfidence bias**
- Considering only **near term consequences**
- Letting **situational** (external) influences such as peer pressure, loyalty unduly affect decisions and behaviour
  - Overate ethical quality of own behaviour and overemphasize importance of own personal traits → situational influences are more important than personal/internal

Unethical behaviour (such as providing misleading info) could affect allocation of capital raised → negative consequences on all stakeholders in economy

Ethical principles often set higher standard of behaviour than laws and regulations. New laws result from instances of unethical behaviour (e.g. Securities Act 1933, Glass-Stegal Act, Securities Act 1934, Sarbanes-Oxley laws (Enron and Worldcom), Dodd-Frank Act (GFC). Ethical decisions require more judgment and consideration of impact of behaviour on stakeholders than legal.

Ethical decision making framework:

- *Identify:* facts, stakeholders, duties, ethical principles, conflicts of interest
- *Consider:* situational influences, additional guidance (e.g. friends, compliance department), alternative actions (short and long term effects)
- *Decide and act*
- *Reflect:* evaluate outcomes of actions - was outcome as anticipated?

CFA Institute Professional Conduct Program (PCP)

- PCP in conjunction with DRC are responsible for enforcement of the Code and Standards
- Covered by CFA Institute Bylaws and Rules of Procedure for Proceeding Related to Professional Conduct
- Based on principles of fairness of process to members and maintaining confidentiality of proceedings
- All CFA members and candidates enrolled in CFA are required to comply with code and standards
- ***Disciplinary Review Committee (DRC)*** has responsibility for program and enforcement of Code and Standards volunteer committee
- **INQUIRY:** CFA Institute Professional Conduct staff conducts inquires related to professional conduct → can be prompted by 4 things:
  - <u>Self-disclosure</u> by members on Professional Conduct Statements of involvement in civil litigation or criminal investigation
  - <u>Written complains</u> to PCP about member's conduct
  - Evidence of misconduct my member received through <u>public sources</u>
  - Report by <u>CFA exam proctor</u> of possible violation
  - Analysis of exam material and monitoring social media by CFA Institute
- **INVESTIGATION:** Once inquiry begins, staff may request (in writing) explanation from member and may interview the member, interview complainant/3 [d] party and collect documents/records relevant to investigation

- o No disciplinary sanctions appropriate
- o Issue cautionary letter
- o Discipline member → member must accept or reject sanction. If rejected, matter referred to DRC of Members.
  - Sanctions include *public censure, membership suspension, revocation of charter*

6 Code of ethics
1. Act with **integrity**, **competence**, diligence and respect
2. Place integrity of profession and clients **above personal interests**
3. **Reasonable care** and exercise **independent** professional judgment when making investment recommendations
4. Practice and encourage others to **practice in ethical manner**
5. **Promote** integrity and viability of **global capital markets**
6. Maintain and improve **professional competence**

7 standards of professional conduct
1. **PROFESSIONALISM**
   A. *Knowledge of the law* (including code of ethics and standards of professional conduct)   in the event of a conflict, the <u>stricter law</u>, rule or regulation applies.
   B. *Independence and objectivity*   not offer or accept gift or compensation that would compromise independence/objectivity
   C. *Misrepresentation*   analysis, recommendations or actions
      - Crediting source <u>not required</u> when using statistics, tables and projections from recognised financial and statistical reporting services
   D. *Misconduct*   not engage in conduct involving dishonesty, fraud, deceit
2. **INTEGRITY OF CAPITAL MARKETS**
   A. *Material nonpublic info*   that could affect value of investment
      - Public once it is announced to the marketplace
      - Mosaic theory = reaching investment conclusion through analysis of public info + non-material nonpublic info
      - Members should make effort to achieve public dissemination by the firm of information they possess. Firms should review employee trades and maintain watch lists.
   B. *Market manipulation*   not distort prices or artificially inflate trading volume → only if there is INTENT to mislead.
3. **DUTIES TO CLIENTS**
   A. *Loyalty, Prudence and Care*   act in benefit of client, place clients interest before employer's/own interest
      - Submit at least <u>quarterly statements</u> showing securities in custody and all debits, credit and transactions. Not vote on all proxies.
   B. *Fair Dealing*   dealing with clients when making analysis, recommendations, engagement
      - E.g. do not take shares of an oversubscribe IPO
   C. *Suitability*   <u>risk and return</u> objectives, suitable investments, consistent with objectives and constraints of portfolio
      - Members gather info at beginning of relationship in the form of an investment policy statement (IPS)
      - If trade has material impact, you can update IPS so client accepts changed risk profile that would permit the trade. If client wont

accept the changed IPS, manager may follow firm policy which may allow client-directed account

   D. *Performance presentation*   fair, accurate and complete
   - Member must not state or imply ability to achieve rate of return similar to that achieved in the past
   - Member should present performance of weighted composite of similar portfolios rather than single account
   - Include terminated accounts and state when terminated

   E. *Preservation of confidentiality*   keep info about clients (current and past) confidential unless 3 exceptions: illegal activities, disclosure required by law, client permits disclosure

4. **DUTIES TO EMPLOYERS**
   A. *Loyalty*   act for benefit of employer and not divulge confidential info
   - No requirement to put employer interests ahead of family and personal obligations
   - When leaving an employer, member must continue to act until resignation is effective
   - Violations include misappropriation of trade secrets and client lists, misuse of confidential info, soliciting employer's clients, self-dealing.

   B. *Additional Compensation Arrangements*   not accept gifts, benefits that might create conflict of interest <u>unless obtain written consent</u> from all parties involved
   - If client offers bonus depending on <u>future performance</u>, this is an <u>compensation arrangement</u> → requires written consent in advance
   - If client offers bonus depending on <u>past performance</u>, this is a <u>gift</u> → requires disclosure to employer to comply with Standard I(B) Independence and Objectivity

   C. *Responsibilities of Supervisors*   make sure people comply with laws, regulation and Code and Standards

5. **INVESTMENT ANALYSIS, RECOMMENDATIONS AND ACTIONS**
   A. *Diligence and Reasonable Basis*   reasonable basis supported by research and investigation for analysis, recommendation
   - Application depends on investment philosophy adhered to, members' roles in investment decision making process, and resources and support provided by employer
   - Considerations include economic conditions, firms financial results/operating history, fees and historical results, limitations of quant models, peer group comparisons for valuation are appropriate
   - Members should encourage firm to adopt policy for periodic internal review of quality of 3 [d] party research

   B. *Communication with Clients*   disclose basic principles of investment process and construct portfolios and any changes that might materially affect processes, significant limitations and risks, identifying important factors and communicate them, distinguish between fact and opinion.
   - Expectations based on modeling/analysis are not facts
   - Communicate gains/losses in terms of total returns
   - Explain limitations of model/assumptions used and of the investment itself   e.g. liquidity and capacity

   C. *Record Retention*   develop and maintain records to support analysis and recommendation with clients (e.g. documenting details of convo)

- Member who changes firms must re-create analysis documentation supporting recommendation and must not rely on material created at previous firm
- If no regulatory standards/firm policies in place, recommends <u>7-year minimum holding period</u>

6. **CONFLICT OF INTEREST**
   A. *Disclosure of Conflicts* — matters that could impair independence and objectivity or interfere with duty to clients and employer
      - E.g. ownership of stock in company that recommending
      - E.g. compensation/bonus/commissions
   B. *Priority of Transactions* — <u>clients/employers priority over own</u>
      - Limitations on employee participation in equity IPO, private placement
      - Blackout period — no personal purchase/sale of security in advance of client/employer
   C. *Referral Fees* — compensation received or paid to others for recommendation of products/services

7. **RESPONSIBLE AS A CFA INSTITUTE MEMBER/CANDIDATE**
   1. *Conduct as Participants in CFA Institute Programs* — not compromise reputation or integrity of CFA
      - e.g. exam cheating, improperly using designation, not reveal confidential info regarding CFA, misrepresenting info on Professional Conduct Statement (PCS)
   2. *Reference to CFA Institute, Designation and Program* — not misrepresent or exaggerate meaning/implications
      - Members must sign the PCS annually, and pay CFA membership dues annually → if fail to do this, person will no longer be an active member

<mark>READ STANDARDS OF PRACTICE HANDBOOK 11 EDITION</mark> — 15% of questions based from this book and 2 short readings on GIPS

INTRODUCTION TO THE GLOBAL INVESTMENT PERFORMANCE STANDARDS (GIPS)

GIPS are a set of ethical principles based on standardised, industry-wide approach
- Investment firms can <u>voluntarily</u> follow GIPS in presentation of historical results to clients → Standards seek to avoid misrepresentation of performance
- Only inv mgmt firms that actually manage assets can claim compliance
- Created to make it easier to compare different investment management firms → GIPS allow clients to <u>more easily compare performance</u>

A **composite** is an aggregation of one or more individual discretionary portfolios representing a similar investment strategy or objective — e.g. large cap growth stocks.
- Reporting on composites gives clients info about firm's success in managing various types of securities or results for various investment styles
- Must include all <u>actual fee paying discretionary</u> portfolios (past and present)
  - Actual = no dummy portfolio. Fee paying = no charity. Discretionary = control over stock selection

- Firm should identify composite <u>before</u> performance is known → prevents firm from choosing to include composite in order to create composite with superior returns
  - Must be determined on <u>ex-ante basis</u> (i.e. before period which composite's performance will be calculated) → pre-established criteria

Once firm claims GIPS compliance, they must <u>voluntarily</u> hire a <u>3rd party</u> to perform **verification**
- Verification of GIPS compliance is <u>optional</u>, but if issued by 3 ^d party then report must be issued with respect to <u>whole firm (firm-wide basis)</u>
- Must attest 2 things:
  - Whether firm has complied with all <u>composite construction requirements</u>
  - Firms <u>processes and procedures</u> are established to present performance according with calculation methodology, data requirements and format required by GIPS

<u>**GIPS Objectives**</u> → Goals of the GIPS Executive Committee:
- Obtain global acceptance of calculation and presentation standards in fair, comparable format with full disclosure
- Ensure consistent, accurate performance data in areas of reporting, records, marketing and presentations
- Promote fair competition among inv mgmt firms in all markets without unnecessary entry barriers for new firms
- Promote global "self regulation"

<u>**Key Characteristics of GIPS**</u>
- To claim compliance, firms must define its "firm" → reflect the distinct business entity that is held out to clients and prospects as the firm
- Ethical standards for performance presentation which ensure fair representation
- Include all actual fee-paying, discretionary portfolios in composites for **min 5 years** or since firm/composite inception.
  - After 5 years of compliant data, firm must add annual performance each year going forward up to **min 10 years.**
- Firms required to use certain calculation and presentation standards and make specific disclosures
- GIPS contain both *required* and *recommended* provisions → firms encouraged to adopt recommended provisions
- Encouraged to present all pertinent additional and supplemental info
- Follow local laws when in conflict with GIPS, but disclose the conflict
- Supplemental private equity and real estate provisions contained in GIPS are to be applied to those asset classes

	Requirement	Recommendation
Definition of the firm	• Apply GIPS on firm-wide basis • Firm defined as distinct business unit • Total firm assets include total mkt value of discretionary and non-discretionary assets, including fee-paying and non-fee-paying accounts • Include asset performance of sub-advisors, as long as firm has discretion over sub-advisor selection	• Include broadest definition of the firm, including all geographical offices marketed under same brand name

	• If firm changes its organisation, historical composite results cannot be changed	
Document policies and procedures	• Document, in writing, policies and procedures firm uses to comply with GIPS	
Claim of compliance	• Compliance statement "ABC has prepared …in compliance with GIPS" • No such thing as partial compliance • No statements referring to calculation used in composite presentation or performance of individual client as being "in accordance with GIPS" (unless complaint firm is reporting results directly to client)	
Firm fundamental responsibilities	• Compliant presentation to all prospects (within previous 12 months) • Composite list and description to all prospects that make a request. List discontinued composites for **min 5 years** • If jointly marketing with another firm, clearly define firm separation if noncompliant	

Firm must present min 5 years compliant performance unless firm or composite has been in existence less than 5 years → "since inception" must be presented.
- After initial compliant presentation, 1 year of compliant performance must be added each year to a required minimum history of 10 years.
- Firms may present periods of noncompliant performance as long as no noncompliant performance if presented for any period after 1^s January 2000

Firms that previously presented performance in compliance with particular Country Version of GIPS (CVG) may claim GIPS compliance prior to 1 Jan 2006.
- Must continue to include data until min 10 years compliant performance presented
- Where country specific regulations conflict with GIPS, firms must follow **country-specific regulations** but must also disclosure nature of conflict with GIPS

9 major sections of the GIPS standards

0. *Fundamental of Compliance*
   o Definition of firm
     ▪ Distinct business entity held to clients claiming GIPS compliance
     ▪ Total firm assets = fair value of all assets including discretionary, nondiscretionary, fee-paying and non-fee-paying accounts
   o Documentation of firm policies & procedures with respect to GIPS compliance
     ▪ E.g. make complete list of composites to any prospective client → list must include current and any terminated composites within last 5 years
   o Complying with GIPS updates
   o Claiming compliance in appropriate manner
     ▪ Not claim **partial compliance**.
     ▪ Not state **specific calculation** is GIPS compliant

- o Appropriate verification statement when 3 [d] party verifier is employed
1. *Input data*
   - o Consistent in order to establish full and comparable performance
2. *Calculation methodology*
   - o Uniformity required so results are comparable
   - o Must not use **estimated** trading expenses
3. *Composite construction*
   - o Meaningful, asset-weighted composite with performance based on performance of 1 or more portfolio with same investment strategy
4. *Disclosure*
   - o E.g. claim of compliance, currency, fee schedule, benchmark description, composite creation date
5. *Presentation and reporting*
   - o Investment performance according to GIPS. Other firm-specific info not specifically required by GIPS should be included when appropriate
6. *Real estate*
   - o Provisions apply to ALL real estate investment (land, buildings) regardless of level of control firm has over it, or whether asset is producing revenue or leverage involved
   - o REITS, Commercial mortgage backed securities (CMBS) and private debt instruments are not considered real estate and must follow sections 0-5
7. *Private equity*
   - o PE Investments must be valued according to Private Equity Valuations Principles unless investment is open-ended or evergreen fund → follow GIPS
   - o Not publicly traded regardless of stage of business development (e.g. venture capital, ownership of previously public company, mezzanine financing, limited partnership shares, and fund-of-funds investments
8. *Wrap fee/SMA portfolio*
   - o Section 0 to 5 are supplemented or replaced by requirements specific in this section

Time Value of Money

- The value of the investment's cash flows must be measured at some common point in time → at end of investment horizon (FV) or beginning of investment horizon (PV)

Interest rates are a measure of the TVM.
- Equilibrium interest rates are the **required rate of return** for a particular investment.
- Interest rates are the **opportunity cost** of current consumption
- **Discount rate** for calculating PV of future cash flows
- **Real risk-free rate** is a theoretical rate on a single-period loan when there is no expectation of inflation → investors increase in purchasing power (after adjusting for inflation)
   - o T-bill rates are *nominal risk-free rates* because they contain an inflation premium
   - o Nominal risk-free rate = real risk-free rate + expected inflation rate

Types of security risks:

- **Default risk** risk that borrower will <u>not make promised payments</u> in timely manner
- **Liquidity risk** risk of receiving less than fair value for investment if it must be sold for cash quickly
- **Maturity risk** long-term bonds are more volatile than short-term bonds and hence have maturity risk requiring a maturity risk premium

Each risk factor is associated with risk premium that is added to nominal risk free rate to adjust for greater default risk, less liquidity and longer maturity relative to very liquid, short-term, default risk free rate such as that on T-bills
- Required interest rate = nominal risk free rate + default risk premium + liquidity premium + maturity risk premium
    - Note: nom na r sk free rate = rea r sk free rate + nf at on prem um

Stated annual int rate – quoted int rate that does <u>not</u> account for compounding
Effective annual rate – annual rate of return earned after adjustments made for different compounding periods.
- The rate of interest that investor realizes as result of compounding. (i.e. 8% savings rate compounded quarterly as supposed to 2% per quarter)
- EAR = $(1 + \text{periodic rate})^m - 1$
    - M = number of compounding periods per year
    - Periodic rate = stated annual rate/m
- <u>EAR increases at a decreasing rate as compounding frequency increases</u>
- It is necessary when comparing investment that have different compounding periods → apples-to-apples rate comparison
- **Continuous compounding** – if number of compounding periods become infinite
    - $FV_N = PV(e^{rn})$
    - $e^r - 1 = EAR$

Future value amount which current deposit will grow over time when placed in account paying compound interest → *compound value*
- FV = $PV(1+I/Y)^N$
    - PV = money invested today
    - I/Y = rate of return per compounding period
    - N = total number of compounding periods
    - $(1+I/Y)^N$ referred to as *future value factor*

Present value – today's value of a cash flow that is to be received at some point in the future → amount invested today at a given rate to end up with specific FV
- discounting future cash flows back to the present
- interest rate = *discount rate, opp cost, rate of return, cost of capital*
- PV = $FV/(1+I/Y)^N$
    - Known as present value factor

Annuity stream of equal cash flows that occur at equal intervals over a given period
- *Ordinary annuity* cash flows occurring at <u>end</u> of each compounding period
- *Annuity date* cash flows occurring at <u>beginning</u> of each compounding period

$$FV\ Annuity = PMT \left[\frac{(1 + r)^N - 1}{r} \right]$$

$$PV\ Annuity = PMT \left[\frac{1 - (1 + r)^{-N}}{r} \right]$$

- Annuity due → 1 less discounting period since 1^s CF is at t=0 and hence already its PV
  - *PV Annuity Due = PV Ordinary Annuity * (1+r)*
    - PV of annuity due > PV ordinary annuity

Perpetuity financial instrument that pays fixed amount of money at set intervals over an infinite period of time.
- i.e. perpetual annuity (i.e. set of never-ending sequential cashflows)
- e.g. British Consol bonds or most preferred stock

$$PV\ Perpetuity = \left[\frac{PMT}{r}\right]$$

Increase in compounding frequency → increase effective int rate → increase FV → decrease PV

Loan amortisation process of paying off a loan with a series of period loan payments, whereby a portion of outstanding loan is paid off, or amoritsed, with each payment.

e.g. Amortisation schedule for 10k loan at 10% for 5 years

Period	Beg Bal	PMT	Interest (beg bal * int rate)	Principal (PMT interest)	End Bal (beg bal principal)
1	10,000	$2637.97	1000	1637.97	8362.03
2	8362.03	$2637.97	836.203	1801.767	6560.263
3	6560.263	$2637.97	656.03	1981.94	4578.32
4	4578.32	$2637.97	457.83	2180.14	2398.18
5	2398.18	$2637.97	239.82	2398.18	0

$$Compound\ Annual\ Growth\ Rate\ (CAGR) = \left[\frac{Current}{Previous}\right]^{1/n} - 1$$

Cash flow additivity principle PV of any stream of cash flows equals sum of PV of the cash flows

Discounted Cash Flow Applications

Net Present Value (NPV) of an investment project is the PV of expected cash inflows associated with project less PV of projects expected outflows, discounted at the appropriate cost of capital

Internal Rate of Return (IRR) – discount rate for which NPV of investment is 0
- i.e. rate of return that equates the PV of an investment's expected benefits (inflows) with the PV of its costs (outflows)
- Gives you sense of return on project - 'internal' because depends on investment CF
- Discount rate < IRR = +ve NPV
- Discount rate > IRR = -ve NPV
- If Discount rate (i.e. hurdle rate) = IRR → 0 NPV

NPV Decision Rule if project has positive NPV, this amount goes to firm's shareholders
- Accept projects with positive NPV → increase shareholder wealth

- Reject projects with negative NPV → decrease shareholder wealth
- When 2 projects are <u>mutually exclusive</u>, project <u>with higher positive NPV</u> should be accepted

IRR decision rule   provides analyst with analysing an investment result in terms of rate of return. IRR uses cost of capital as the hurdle rate→ is the return of the project higher than the cost of capital?
- Accept projects with IRR > firm's required rate of return
- Reject projects with IRR < firm's required rate of return

IRR and NPV rules give **same** accept/reject decision when projects are <u>independent</u>
- If <u>mutually exclusive</u> (i.e. conflict) → use which one gives higher NPV

Holding period return (HPR) - % change of value of an investment over the period it is held

$$HPR = \frac{ending\ value + CF\ received}{beginning\ value} - 1$$

2 types of portfolio return measurement tools:
1. **Money Weighted Return (=IRR) -** internal rate of return on a portfolio taking into account all cash inflows and outflows → $PV_{flows} = PV_{outflows}$
   - Inflows = beginning value of account and all deposits into account
   - Outflows = ending value and all withdrawals
   - Returns depends on timing and amount of CF.
   - Appropriate if PM has control of timing and amount of investment
2. **Time Weighted Return** → measures compound rate of growth of $1 initially invested in the portfolio a specified performance horizon
   - Process of averaging a set of values overtime
   - Preferred method of performance because it is <u>not affected by the timing of cash inflows and outflows</u>
       - If funds contributed to portfolio at favourable time → money weighted return > time-weighted rate of return
           - Time weighted removes this distortion
   - Returns do not depend on timing and amount of cash flow.
   - Appropriate if PM <u>does not</u> control timing and amount of investment
   - $[(1+r_{t1}) \times (1+r_{t2}) \times ... (1+r_{tn})] -1$

Annualized Time weighted return = $[(1+HPR_1)(1+HPR_2)]^{/} - 1$
- N = number of periods → *only do 1/n if annualised*

How can we annualize a return when the holding period return is more than one year? For example, how do we annualize an 18-month holding period return? Because one year contains two-thirds of 18-month periods, $c = 2/3$ in the above equation. An 18-month return of 20 percent can be annualized, as shown:

$$r_{annual} = (1 + r_{18month})^{2/3} - 1 = (1 + 0.20)^{2/3} - 1 = 0.1292 = 12.92\%$$

Money market is the market for short-term debt instruments.
- **T-bills (Treasury Bills)** are <u>pure discount instruments</u> and quoted on a <u>bank discount basis</u>

5 different yield measures   Bank Discount Yield, Holding Period Yield, Money Market Yield, Effective Annual Yield, Bond Equivalent Yield.

Bank Discount Yield (BDY)   expresses the dollar amount from the face value as a fraction of face value, not the market price.
- Annualizes using simple interest and ignores effects of compound interest
- Based on 360-day year rather than 365

$$BDY = \frac{Face\ Value - purchase\ price}{Face\ Value} \times \frac{360}{days\ till\ maturity}$$

D scou = V u c ase ce

Holding Period Yield (HPY) total return an investor earns between purchase date and the sale or maturity date (i.e. return for holding security of a period)

- Aka holding period return → actual return investor receives if held to maturity

$$HPY = \frac{price\ at\ maturity - initial + interest\ payment}{initial\ value}$$

Effective Annual Yield = $(1+HPY)^{365/}$ 1

- Annualized HPY on basis of 365 days incorporating effects of compounding

Money market yield (aka CD equivalent yield) is the annualized HPY, assuming a 360 day year → makes yield on T-bill comparable to yield quotes for interest-bearing money market instruments that pay interest on a 360 day basis

- o Annualized yield based on price of 360 day year that does not account for compounding → assumes simple interest
- o MMY > BDY

$$MMY = HPY \times \frac{360}{\#\ days\ to\ maturity}$$

$$r_{MM} = \frac{360 \times r_{BD}}{360 - (t \times r_{BD})}$$

Bond equivalent yield refers to 2 x the semi annual yield
- = 2 * [(1+yield on annual bond)$^{0.5}$) -1]
- Used because Bonds pay coupons semi annually
- Note: EAY > BEY

Statistical Concepts and Market Returns

Central tendency - Provide an indication of investment's expected return
- Arithmetic mean, geometric mean, weighted mean, median and mode.

Dispersion – indicate the riskiness of an investment
- Range, absolute deviation, variance

Lack of symmetry (skewness) and which distribution is peaked (kurtosis)

Descriptive statistics summarise important characteristics of large data
Inferential statistics procedures to make forecasts, estimates and judgments about a large set of data on basis of statistical characteristics of a smaller set (sample)
Population set of all possible members of a stated group.
Sample subset of the population of interest

4 types of measurement scales

- **Nominal scales** — data put into categories that have <u>no particular order</u> → only names used (e.g. bond 1, bond 2 etc…)
- **Ordinal scales** — data put into categories that can be <u>ordered with respect to some characteristics</u> (e.g. ranking 1000 small cap growth stocks by performance)
- **Interval scales** — provides relative ranking plus the assurance that differences between scale values are equal (e.g. temperature — degrees)
- **Ratio scales** — provides ranking and equal differences between scale values and also a true zero point as the origin (e.g. measure of money in $)
    - Absolute zero

Parameter — measure used to describe characteristic of a <u>population</u> (e.g. mean return and SD of return

Sample statistic — measure a characteristic of a sample

Frequency distribution — summarise data by assigning specific groups or intervals and may be measured using any type of measurement scale
- *Absolute frequency = actual number of observations that fall within an interval*
- *Modal interval = interval with the greatest frequency*

$$Relative\ Frequency = \frac{Absolute\ frequency\ of\ each\ return\ interval}{Number\ of\ observations}$$

Histogram — presentation of absolute frequency distribution
- Allows us to quickly see where observations are concentrated

Arithmetic mean is the only measure of central tendency for which sum of deviations from the mean is 0 → **arithmetic mean** = sum of observations/# observations
- Used to estimate avg return over <u>1-period time horizon</u>

Weighted mean recognises different observations may have disproportionate influence of the mean.
- *Return of portfolio is weighted average of returns of individual assets in the portfolio*

<u>Median may be a better measure of central tendency because it is not affected by extreme values</u>

Unimodal – when distribution has 1 value that appears more frequently

Geometric mean – used when calculating avg investment returns over <u>multiple periods</u> or when measuring *compound growth rates*

$$R_G = [(1 + R_1)(1 + R_2)(1 + R_n)]^{1/n} - 1$$

- Geometric mean always less or equal to arithmetic mean → difference increases as dispersion of observation increases
- Since annual returns are compounded each period, GM is appropriate measure of past performance → gives *average annual compound growth*
    - GM best to estimate <u>multi-year returns</u>

Harmonic mean – can be used to find <u>average purchase price</u>

$$Harmonic\ mean = \frac{N}{\frac{1}{x_i} + \frac{1}{x_2} + \frac{1}{x_n}}$$

If returns are variable → Arithmetic mean > Geometric Mean > Harmonic Mean

- Basis for claimed benefit of dollar cost averaging
- If constant, AM = GM = HM
- The greater the variability → the more AM will exceed GM

Quantile   value at or below which a stated proportion of data in a distribution line
- Quartile (quarters), Quintile (5ths), Decile (10ths), Percentile (hundredths)
- Quantiles and measure of central tendency = measure of <u>location</u>
- $L_y \quad (n+1)\frac{y}{100}$

  n   observation y   percentile

Dispersion – variability around the central tendency
Mean absolute deviation (MAD) – avg of <u>*absolute values*</u> of deviations of individual observations from arithmetic mean

$$MAD = \frac{\sum_{i=1}^{n} |X_i - mean|}{n}$$

Standard deviation > MAD

Standard deviation = $\sigma = \sqrt{\dfrac{\sum_{i=1}^{N}(X_i - \mu)^2}{N}}$ = (return – mean)^2

Chebyshev's inequality – % of observations that lie within *k* standard deviations of the mean is at least $1 - 1/k^2$ for all k>1
- Basis for normal distribution (i.e. 75% of observations lie within +/ 2 SD's from mean)
- Used to measure max amount of dispersion <u>regardless of distribution shape</u>

Coefficient of variation (CV)   amount of <u>dispersion</u> in a distribution relative to it's mean→ relative dispersion

$$Coefficient\ of\ Variation = \frac{SD\ x}{Avg\ x}$$

- Enables us to make direct comparison of dispersion across sets of data → used to measure <u>risk</u> (variability) per unit of expected return (mean)
- Higher CV = higher risk

Sharpe ratio   measures *<u>excess return per unit of risk</u>*

$$Sharpe\ ratio = \frac{r_p - r_f}{\sigma_p}$$

r_p = portfo o return = mean return
r_f = r sk-free return
σ_p = SD of portfo o

- Return on portfolio   risk-free return = **excess return** → measures extra reward investor receives for exposing themselves to risk
- Sharpe ratio of 0.14 = investment earned 0.14% of excess return per unit of risk
- <u>2 limitations of sharpe ratio</u>
  - If 2 portfolios have -ve Sharpe ratio, it is not necessarily true that higher Sharpe ratio implies superior risk-adjusted returns

o   Investments with option characteristics have asymmetric return distributions reflecting large probability of small gains with small probability of large loss → SD may underestimate risk and produce higher Sharpe ratio

**Skewness**   extent to which distribution is not symmetrical

- *Positively skewed*   outliers in upper region (right tail) → skewed to right
  o   Mean > Median > Mode
- *Negatively skewed*   outliers in lower region (left tail) → skewed to left
  o   Mode > Median > Mean
  o   Indicates returns below mean are more extreme → <u>more risky</u>
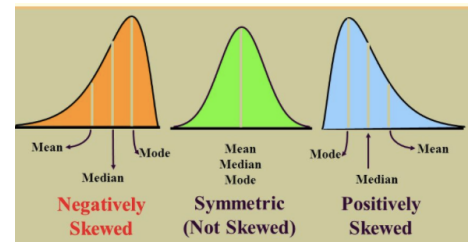
i.e. mean is "pulled" in direction of the skew

Kurtosis   measure of degree to which distribution or more or less "peaked" than normal distribution → probability of extreme outcomes (thickness of tail)

- **Leptokurtic**   distribution that is <u>more</u> peaked than normal distribution
  o   K > 3
  o   More returns clustered around mean and more returns with large deviations (i.e. more peaked with fatter tails   more extreme outliers)
    ▪   *e.g. equity distributions are leptokurtic*
- **Platykurtic**   distribution that is <u>less</u> peaked (i.e. flatter) than normal distribution → <u>better for risk adverse clients</u>
  o   K < 3
- **Mesokurtic**   distribution that is <u>same</u> kurtosis as a normal distribution
  o   K = 3
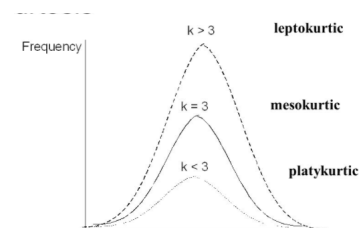
Excess kurtosis = if distribution has more/less kurtosis than normal distribution

- Greater positive kurtosis and negative skew in returns = more risk
- +ve = leptokurtic (more peaked, flat tail) → probability of extreme outcomes greater
- -ve = platykurtic (less peaked, thin tails)
- excess kurtosis > 1 is considered significant

Sample skewness = (observation   mean)^3/SD^3

- Right skewed distribution → +ve sample skewness
- Left-skewed distribution → -ve sample skeweness

Sample kurtosis = (observation   mean)^4/SD^4

- Excess kurtosis = kurtosis   3

Arithmetic mean appropriate for forecasting single period returns in future, while geometric mean appropriate for forecasting future compound returns over multiple periods.

- Geometric mean appropriate for <u>past performance</u>
- Arithmetic mean appropriate for <u>forward-looking context</u>

Probability Concepts

Mutually exclusive event   cannot both happen at the same time
Exhaustive event   include all possible outcomes