CFD Modeling of a Diaphragm Stirling Cryocooler

A. Caughley[1], M Sellier[2], A. Tucker[2], M Gschwendtner[3]

[1]Industrial Research Ltd, Christchurch, New Zealand
[2]University of Canterbury, Christchurch, New Zealand
[3]TS-dot Engineering, Auckland, New Zealand

ABSTRACT

Industrial Research Ltd has developed a unique diaphragm-based pressure wave generator technology for employment in pulse tube and Stirling cryocoolers. The system uses a pair of metal diaphragms to separate the clean cryocooler gas circuit from a conventionally lubricated mechanical driver, thus producing a clean pressure wave with a long-life drive. The same diaphragm concept has been extended to support and seal the displacer in a free piston Stirling expander. Diaphragms allow displacer movement without rubbing or clearance gap seals, hence allowing the development of cost-effective, long-life and efficient Stirling cryocoolers. A proof-of-concept prototype has achieved cryogenic temperatures.

The diaphragm's large diameter and short stroke produces a significant radial component to the oscillating flow fields inside the cryocooler which are not modelled in one-dimensional analysis tools such as Sage. Compared with standard pistons, the gas-to-wall heat transfer is increased due to the higher velocities and smaller hydraulic diameters. This paper presents the results of Computational Fluid Dynamics (CFD) analysis used to model the flow and gas-to-wall heat transfer inside the cryocooler, including experimental validation of the CFD to produce a robust analysis.

INTRODUCTION

Metallic diaphragm pressure wave generators (PWG) developed by Industrial Research Ltd[1] have matured to provide a useful alternative to linear motor or standard piston-based PWGs for Stirling and pulse tube cryocoolers. The metallic diaphragm PWG employs two opposed diaphragms that balance each other's average gas forces. A conventionally lubricated, ambient pressure mechanism sits between the two diaphragms to reciprocate the diaphragms.

The metallic diaphragm PWG's concept has the potential to produce a free piston Stirling cold head without rubbing or high precision clearance seal parts. Figure 1 shows the concept that has been explored. Diaphragms have the potential to support a large displacer without appendix gap leakage; they are tolerant of alignment errors; and their short stroke minimizes vibration. As with the PWG, the diaphragms seal in the cryocooler's working gas. Equally-sized diaphragms balance the large compressive forces on the displacer, leaving the displacer free to move. The stiffness of the diaphragms provides a spring to centre the displacer. The diaphragm's high radial stiffness limits the displacer's movement to the axial direction, and their large area achieves the

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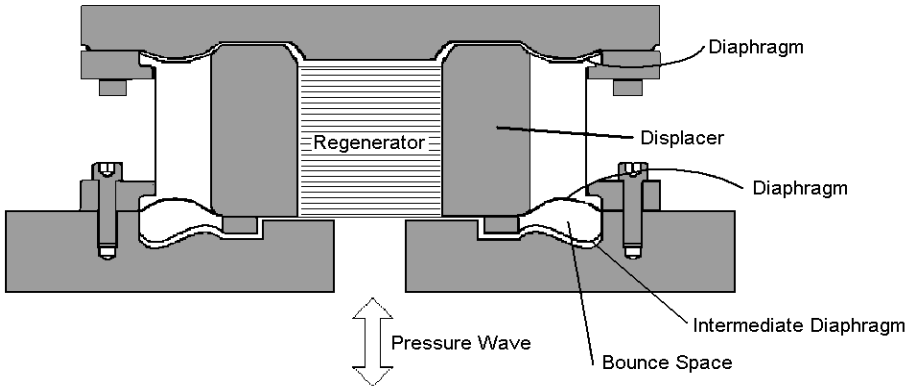

Figure 1. Conceptual layout of a diaphragm free piston Stirling cold head

required swept volume with a small linear motion. The regenerator is housed inside the displacer, thereby adding to its mass and lowering its resonant frequency and improving stroke and phase angle. The displacer can be pneumatically actuated, as in a conventional free piston machine[2], by dividing its warm end into two regions: one that experiences the pressure wave, and one that remains at the average pressure. The area division is provided by a third diaphragm separating an outer annular ring for a bounce space. A slow leak to the bounce space ensures it remains at the average system pressure. Alternatively a central 'shaft' can be used to provide a piston/dashpot effect. The end result is a displacer with no rubbing parts and no fine tolerance clearance gap seals. The large surface areas and radial flows of the diaphragms offer the opportunity for heat transfer without expensive heat exchangers. This possibility was indicated previously by the author who successfully ran a pulse tube with diaphragm PWG using the diaphragm as the after-cooler[1]. In this case the PWG area plus the transfer holes to the cold head's compression space provided the after-cooling function. The cold heat exchanger is the expansion diaphragm's large area.

A proof of concept prototype diaphragm free piston Stirling cryocooler was constructed and tested by the author[3]. It achieved a low temperature of 100 K. Improvements to the design of the proof-of-concept prototype were recommended, including using a smaller diaphragm for the displacer, and use of a dashpot instead of a third diaphragm to achieve timing. A second prototype was constructed from the recommendations of the previous exercise.

This paper describes a Computational Fluid Dynamics (CFD) approach to analysing the gas flows, thermodynamics and heat transfers inside a Diaphragm Free Piston Stirling Cryocooler (DFPSC). CFD technique involves breaking a complicated fluid chamber up into many elements, each with simpler solvable geometry. All the elements' constituent equations are solved simultaneously to produce a representation of the flow field and associated heat transfer. Transient conditions can be modelled by time stepping. CFD allows a very detailed study but, due to its large computational requirements, is not as good for mapping or optimisation as one-dimensional analysis tools such as Sage[4]. CFD requires validation against experimental or well known analytical solutions to ensure that the field equations are appropriate and solved correctly. A common technique is to model a simple situation, with key features of the final study, which can be experimentally measured and/or has an analytical solution. Once the CFD has been validated on a simple geometry, the model's geometry is changed to the final study and the analysis results can be interpreted with confidence.

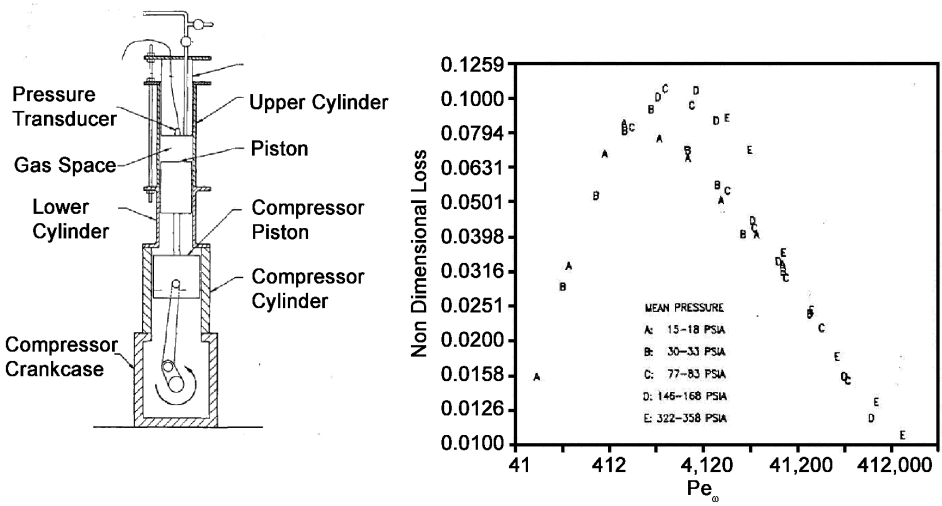

Figure 2. Kornhauser and Smith's experimental rig and results[5]. Note that Kornhauser originally plotted against a Peclet number based on the average piston velocity. The x-axis of the above graph has been rescaled for Pe_ω which is based on the rotational speed to maintain consistency with the rest of this paper.

MODEL VALIDATION

Three methods of validating the CFD model are considered in this work.

The first validation method is the industry standard modelling tool Sage which is a one-dimensional frequency domain modeller that provides fast sinusoidal solutions to oscillating thermodynamic systems such as those existing inside Stirling machines. Sage is an excellent tool for mapping and optimisation but, stemming from its history, is tuned for accuracy for the piston-based models that it is typically applied to, so is good for checking such cases.

The second validation method is a set of experiments by Kornhauser and Smith[5] who measured gas spring hysteresis losses to compare with analytical solutions. Kornhauser's experiments provide a carefully measured situation spanning a large range of conditions from near-isothermal to near-adiabatic. These experiments were used by Huang[6] to verify his CFD analysis of a diaphragm pressure wave generator. Kornhauser's experiments simulated isothermal wall conditions by running for a very short time using the wall heat capacity as a heat sink. This approximation worked well as the heat capacity and thermal diffusivity of the walls was significantly greater than of the adjacent gas. Kornhauser's experiments traversed a range of conditions from near-isothermal to near-adiabatic compression. Running conditions were normalised by using the Peclet number, as defined by

$$Pe_\omega = \frac{\omega D}{\alpha_o} \tag{1}$$

where ω is the oscillation speed [rad/s], D is the hydraulic diameter [m] and α_o is the thermal diffusivity [m^2/s]. A low Pe_ω, less than 100, indicates near-isothermal conditions where the gas has time to transfer heat – resulting from temperature oscillations – to the walls. A high Pe_ω, greater than 100,000 indicates near adiabatic conditions where there is insufficient time to transfer heat from temperature oscillations to the walls.

Gas spring performance was normalised by dividing the work done in a cycle by the adiabatic work done during compression of that cycle

$$L_{nd} = \frac{\oint p dV}{W_{adiab}} \tag{2}$$

Kornhauser's experiment had a 2 inch (50.8mm) diameter piston with a volumetric compression ratio of 2:1. It was found that the gas spring's hysteresis loss, as expressed by the work done by the piston, was low for both low and high Pe_ω but there was a significant increase at intermediate values of Pe_ω where transfer of the heat of compression to the walls was significant, requiring significant temperature gradients and therefore entropy generation. Figure 2 shows Kornhauserss apparatus and results.

The third validation method is an experiment with radial flows and geometry typical of the DFPSC prototype. Again gas spring behaviour is useful for comparison between the experiment and model. An alternative measurement of thermodynamic losses is the polytropic index, defined by

$$Pr = Vr^n, \tag{3}$$

where Pr = Pressure ratio, Vr=Volume ratio and n = polytropic index.

The polytropic index is used as a way of determining the conditions during compression and is used as a way of normalising the pressure ratio results indicating where the compression is on the isothermal to adiabatic scale. Isothermal compression has a polytropic index of 1, and for adiabatic compression it is the ratio of specific heats, γ, which is 1.67 for helium. The polytropic index measurement is easily carried out, requiring only a pressure reading and a volume ratio, importantly it does not require accurate timing for the pressure/volume phase shift essential for Kornhauser's non-dimensional loss.

In this work, Ansys CFX was used as the CFD code. Ansys Release 12 was used for the initial modelling and verification, and Release 13 for the consequent full prototype modelling.

Gas spring verification of the CFD modelling technique

A CFD model of Kornhauser's gas spring experimental gas space was created in Ansys CFX[7]. The cylinder was modelled as a 10 degree-wedge with symmetry conditions on the wedge faces, creating a pseudo 2-D axi-symmetric model. Isothermal boundary conditions with heat transfer were applied to the outer and end walls. One end wall was moved in accordance with Kornhauser's experimental conditions. The mesh was refined with smaller elements next to the cylinder wall to give better resolution where thermal gradients were high. Transient analysis runs were conducted with 400 steps per cycle. The fluid was helium, assumed to behave as an ideal gas.

Figure 3 shows Kornhauser's experimental results overlaid by the results of the Ansys CFX model and a Sage model of the same situation. A good agreement was achieved over a large range

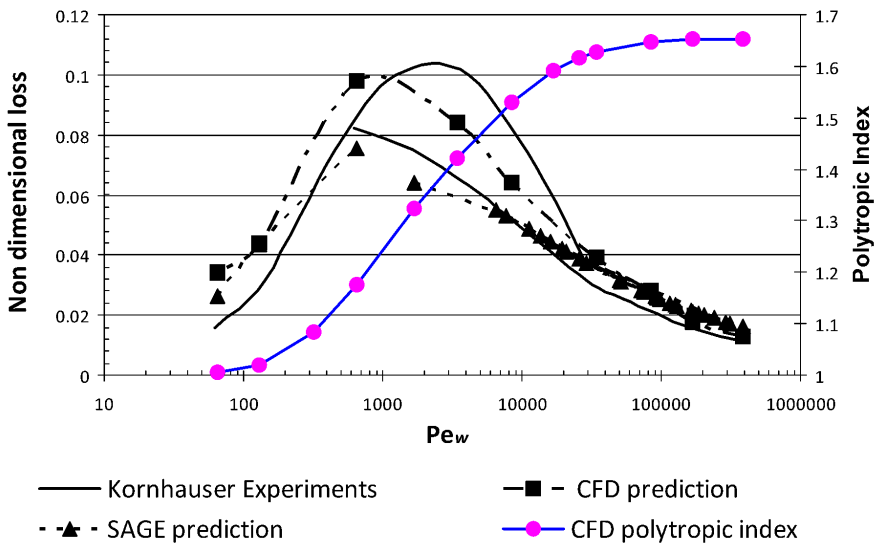

Figure 3. CFD model of a segment of Kornhauser's experiment.

of conditions, indicating that Ansys CFX modelled the heat transfer and entropy generation in the model with sufficient accuracy. Sage predicted high and low Pe_ω cases well but did not converge for high hysteresis loss cases, hence the gap in data and slope discontinuity.

The corresponding polytropic index plot for Kornhauser's experiments shows that the region of high loss is between a Pe_ω of ~100 to ~100,000, the region where the polytropic index changes from 1.1 to 1.6.

Radial Flow Verification Experiment

An experiment was conducted to verify the CFX model with radial flows and geometries typical of the diaphragm Stirling cryocooler. The experiment used the DFPSC prototype without a regenerator or moving displacer to allow validation without the complexities of a matrix or refrigeration cycle. A radial compression space equivalent to the cryocooler's expansion space was incorporated at the cold end diaphragm. The overall volume and compression ratio of the prototype was maintained. The CFD model, Figure 4, was segmented like the model of Kornhauser's experiment. The bottom surface, equivalent to the pressure wave generator diaphragm, was moved to produce the pressure wave. Walls were set to isothermal at 300 K with heat transfer between the gas and walls as in the model of Kornhauser's gas springs.

Experiments were conducted with helium at a pressure range from 1 to 5 bar (absolute) and speeds from 5 to 50 Hz, covering a Pe_ω range of 700 to 20,000. A National Instruments Rio data acquisition system was used to measure piston position, pressure, and average temperatures.

Four gas volume geometries were measured as shown in Figure 5. Case (a) was the standard geometry with a radial section equivalent to the prototype's cold space, Case (b) has an additional annular volume to create more radial flow, Case (c) has the radial section removed, and Case(d) has additional cylindrical length to equal (a)'s volume. These different cases were achieved by changing packing plates; Figure 4, left. They represent different radial flow conditions. Still referring to Figure 5, the hypothesis was: Cases (a) and (b) had radial flow sections not present in (c) and (d) and would therefore exhibit more isothermal gas spring behaviour. Moreover Case (b) would have more radial flow and thus allow more heat transfer than (a), although the volume sections exhibit larger hydraulic diameter and could counter this. In all cases the CFD analysis should be able to predict the differences experienced in the experimental work.

A CFD analysis was completed for each case at 3 bar (absolute) gas pressure over the range of frequencies from 5 to 50 Hz. The other pressures were not modelled as the CFD's computational time was 12 hours per data point. The CFD was run at higher and lower speeds for Case (a), to get very high and low Peclet numbers that correspond to near-adiabatic and near-isothermal behaviour

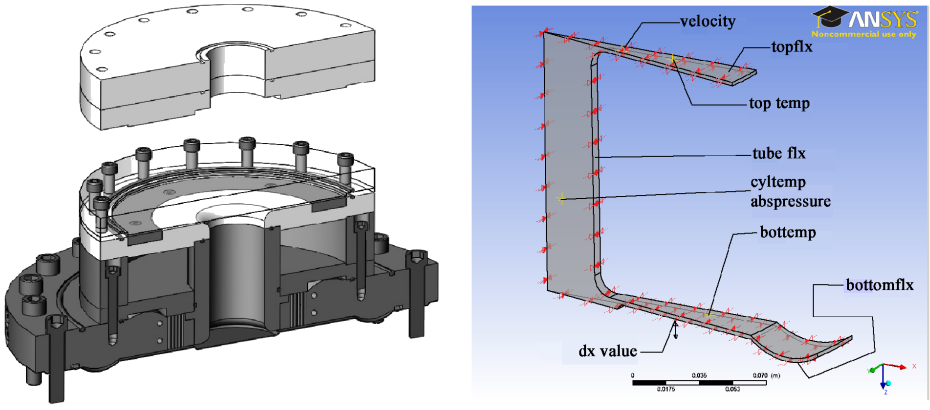

Figure 4. Radial flow experiment apparatus (left) and CFD model (right)

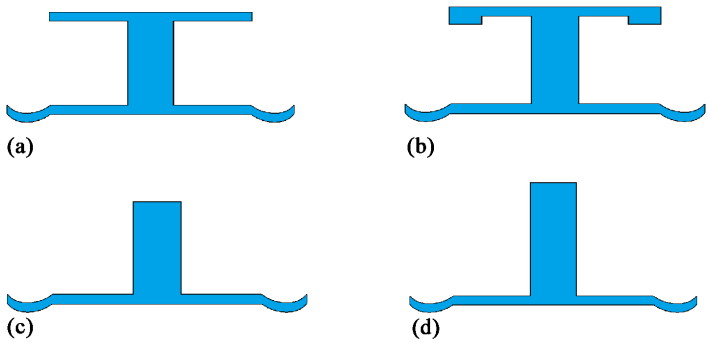

Figure 5. The four volume cases. (a) the 'standard' case representing the expansion space equivalent volume of the Stirling prototype. (b) the standard case with additional volume as an annular ring to enhance radial flow. (c) the standard case without the radial section, capped off so it has a smaller volume. (d) as per (c) with extra cylindrical length to achieve the same volume as (a).

respectively. It was also possible with the CFD to impose pure isothermal or adiabatic behaviour on the model at a mid-point to test the model.

Figure 6 shows the CFD model's prediction of the polytropic index for the case (a) experiment. Each experimental line represents a frequency sweep at a given charge pressure. Sage, which is a 1-D modelling tool, does not predict the experiment well because of the presence of significant radial flows. The CFD model also allowed investigation of higher Pe_ω runs than were physically possible on the experiment, (numbers 13,15,17 and 19) and lower Pe_ω runs than were possible with the experimental apparatus (14 & 16). Run 18 was with isothermal gas conditions – agreeing with a polytropic index of 1, and Run 20 was with adiabatic walls, producing a polytropic index, *n*, of ~1.7 as expected. The fast runs showed a tendency for *n* to approach 1.7 (adiabatic) but then at very high Pe_ω they rose sharply (runs 17,19) which was probably due to fluid flow pressure losses and friction

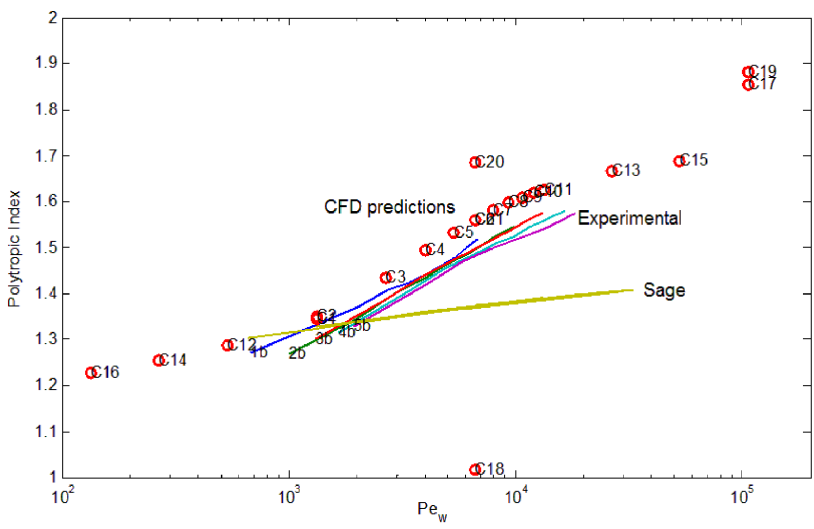

Figure 6. Polytropic compression experiment compared with CFD results for Case (a). The numbered circles each represent a CFD analysis.

producing a pressure ratio higher than for adiabatic compression. The polytropic index has the same typical S-shaped curve as Kornhauser's gas spring experiments, in a similar Pe_ω range.

The CFD analysis for the remaining Cases, (b), (c) and (d) produced similar results to Case (a). Predictions of the polytropic index were close to experimental for the cases without the radial portion ((c) and (d)). For Case (b), with the extra annular ring, the CFD slightly overestimated *n*, but had the same slope over the frequency range.

This verification experiment raises an interesting question of how to describe a compression volume that has a number of different hydraulic diameters. In this case the diaphragm piston and compression spaces had hydraulic diameters in the order of 4mm whereas the central tube's hydraulic diameter was 64 mm. It was clear from post-processing plots that there were runs where the diaphragm spaces were near isothermal while the central tube was clearly near adiabatic. Mass transferring from chamber to chamber adds a further complication. Other possible approaches might be: use a mass-averaged hydraulic diameter or Peclet number; or that the Peclet number might not be able to describe a whole machine and that one should only describe individual chambers in a machine with a "local" Peclet number indicating individual chamber behaviour.

Describing the experiment in terms of the Peclet number for the radial spaces gives similar results to Kornhauser's experiments, if the Pe_ω was based on the 64 mm tube section then the results would be shifted up in Pe_ω by an order of magnitude. The correlation with Kornhauser's experiments then suggests that heat transfer in the radial diaphragm spaces dominates this situation. This is consistent with the trend for cases (c) and (d) having results shifted to the left on the graph from Cases (a) and (b). Cases (c) and (d) are missing the top radial sections and thus the large diameter tube increases the system's average hydraulic diameter, shifting the system's average behaviour towards adiabatic, and resulting in a higher polytropic index.

FULL MODEL OF THE PROTOTYPE

The CFD model of the prototype added a regenerator and moving displacer to the model of the verification experiment, as shown in Figure 8. At the time of writing, the full prototype CFD model was still running so only preliminary results are discussed here.

The regenerator in the prototype was a stack of Stainless Steel (SS) 400 mesh discs with 30 micron wires. Ansys CFX modelled the regenerator with the following parameters: porosity, interfacial area density, resistance loss coefficient, permeability, and heat transfer coefficient.

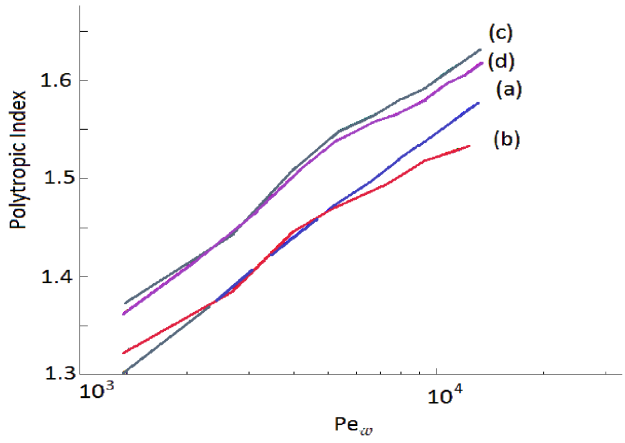

Figure 7. Experimental polytropic index vs Pe_ω for the four cases at 3 bar charge pressure.

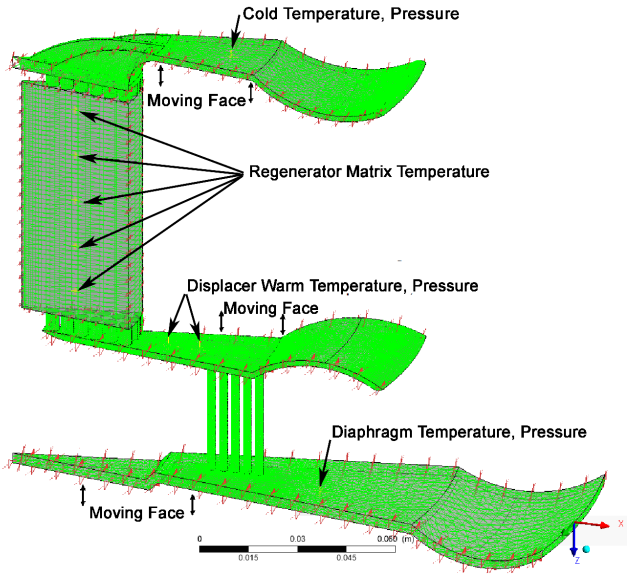

Figure 8. Full CFD model of prototype – needs positions of temp sensors etc.

The porosity and interfacial area density were directly calculable from the physical geometry of the SS mesh discs. The resistance loss coefficient, permeability and heat transfer coefficients were not so apparent. Sage has a good model for mesh regenerators based on a combination of theory and empirical experience. A factorial experiment was carried out with a very simple model: piston compressing of a cylinder, through a regenerator stack, and into a volume. It was hypothesised that if the CFD model could predict the same behaviour as the Sage model for a case where Sage was well verified then we would have reasonable parameter values to use for this radial flow model. The objective of this exercise was to study heat transfer in oscillating radial flows, not to develop a robust regenerator model. Starting values for the factorial experiment were derived from experimental measurements of regenerator properties by Cha[8] for use in a Fluent[9] CFD analysis. Fluent's regenerator model had equivalent properties with different nomenclature to CFX. The factorial experiment identified that the dominant parameter was the resistance loss coefficient, whereas the heat transfer coefficient could be varied between 100 and 1000 (typical values for gas-to-wall heat transfer) with little effect. The final values chosen to get a good correlation were close to Cha's measurements. The values that produced the best correlation were: a porosity of 0.7; interfacial area density of $4x10^8$ m^{-1}; heat transfer coefficient of 100 W m^{-2} K^{-1}; permeability of $4x10^{-11}$; and a resistance coefficient of $7x10^3$ m^{-1}.

The CFD model was constructed as a wedge (Figure 8), the sides having symmetry conditions. The wedge angle was defined by the prototype's hole pattern for transferring gas from the diaphragm compression space to the warm side of the displacer. The dashpot was not modelled. The walls of the cold side of the displacer were set to an isothermal 200 K and the walls of the warm side to 300 K. A linear temperature gradient was set as an initial condition for the gas and matrix in the regenerator.

Prototype Characterization

A characterisation experiment was conducted on the prototype to determine the phase and amplitude of the displacer movement for the CFD analysis. Running conditions were 20 bar average pressure with frequencies ranging from 30 to 55 Hz. Measurements were taken with the cold end temperature at approximately 200 K, a temperature the cooler could maintain under a variety of conditions. Frequency sweeps were conducted as the machine cooled down. No

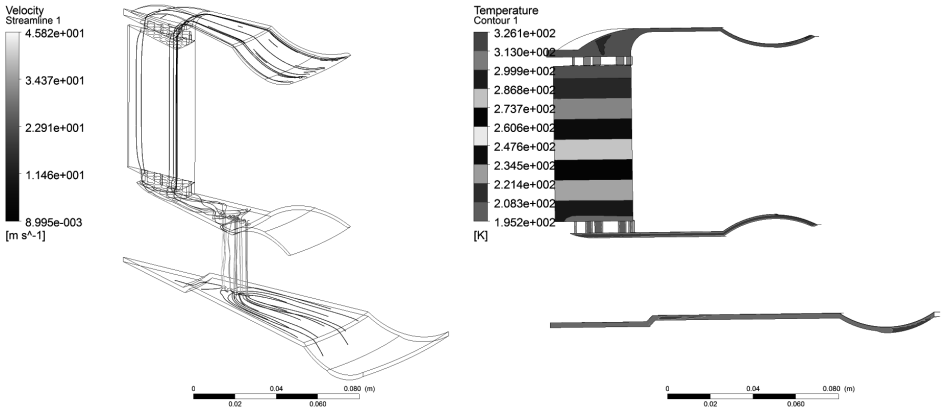

Figure 9. Left, streamlines showing flow in the model towards the end of the compression stroke. Right, gas temperature model towards the end of the compression stroke

significant sensitivity to temperature was apparent. The displacer amplitude at 40 Hz was measured to be 0.32 mm at an angle of 50 degrees in advance of the compression piston. These conditions were used in the CFD analysis.

CFD Model results

Preliminary CFD results are shown and discussed in this paper. Figure 9 shows streamlines on the left and temperatures on the right. The CFD analysis predicted high gas velocities in the transfer holes between the diaphragm compression space and the warm side of the displacer. While the highest heat flux was in the region of the transfer holes, the area represented by these holes was small, so the greatest amount of heat transfer came from the surfaces adjacent to the warm side of the displacer as a consequence of the large area and medium gas velocity. Cold end velocities were considerably lower, highest in the centre and reducing to the stagnant edges, which did not participate significantly in the overall cycle.

The model had not converged to a steady state solution after the 24 cycles completed at the time of writing. Figure 10 shows a temperature trace during a cycle show the effect of heated gas leaving the regenerator on the expansion stroke, where the gas on the warm side of the displacer initially starts to cool with expansion, then gas warmed by the regenerator passes the measurement point. The diaphragm and cold space temperatures oscillate sinusoidally.

DISCUSSION

The agreement between the CFD, Sage and experimental results gives good confidence in the CFD model's ability to predict the heat flow and transfer inside the diaphragm Stirling cryocooler. From this confidence it will be possible to analyse in detail: where significant flow and heat transfer effects take place; identify areas for improvement; and test proposed improvements before committing to prototype changes.

Pe_ω The validation experiments show that high gas spring losses occur under conditions where , is between 100 and 100000. This is a design condition to be avoided in areas such as the periphery of the diaphragm spaces. In the prototypes (which ran at 20 bar and 50 Hz and a typical hydraulic diameter of 4 mm), Pe_ω is 1,300,000 which is clearly adiabatic and will contribute little gas spring loss. This does not mean that heat transfer will not occur; rather it means that the oscillating temperature fluctuations of the bulk fluid will not be damped by the walls. Any heat transfer to the walls will be from the average gas-to-wall temperature difference. Typical heat exchange components such as the regenerator or slotted heat exchangers have Pe_ω

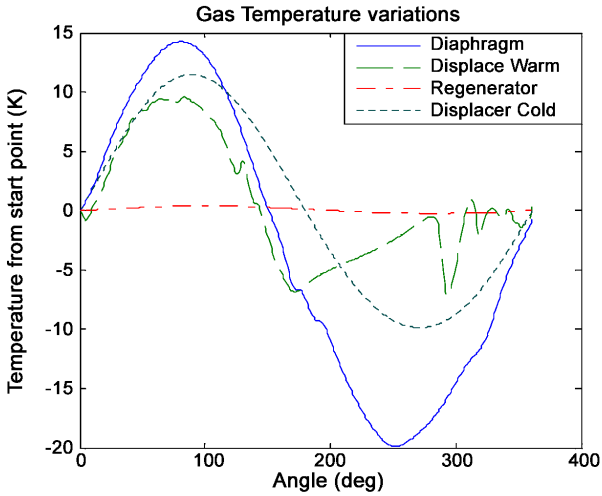

Figure 10. Temperature oscillations in the CFD model during cycle 24. The Diaphragm plot is on a 300 K average, the Displacer Warm lines on a 305 K average, the Regenerator gas on a 250 K average (mid regenerator) and the cold on a 200 K average.

values of approximately 1000 and 10000 respectively, indicating that they are designed for high heat transfer, but are subject to hysteresis losses as they damp pressure oscillations.

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