First Name: $\qquad$ Last Name: $\qquad$ Block: $\qquad$

## Ch. 1 - Sequences and Series Notes

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## 1.1 - Arithmetic Sequences

Definition:
Arithmetic Sequence - An ordered list of terms in which the difference between consecutive terms is constant. This constant is called the common difference.


Deriving the formula to calculate the $\mathrm{n}^{\text {th }}$ term, given an arithmetic sequence:


Examples:

1) Given the following arithmetic sequence:
$10,15,20,25, \ldots$
a) What is the value of the first term, $t_{1} ? 10$
b) What is the common difference? $d=5$
c) What is the value of the $100^{\text {th }}$ term?

$$
t_{100}=10+(100-1)(5)=10+99(5)=505
$$

2) An arithmetic sequence does not have to increase. Here's an example of a decreasing arithmetic sequence.

$$
\begin{aligned}
& t_{1}=6 \\
& d=-2
\end{aligned}
$$

$6,4,2,0,-2, \ldots$

$$
t_{n}=t_{1}+(n-1) d
$$

a) What is the value of $t_{10}$ ? $\quad t_{10}=6+(10-1)(-2)=6+(9)(-2)=6-18=-12$
b) What is the value of $t_{100} ? t_{100}=6+(100-1)(-2)=6+99(-2)=-192$
3) For the arithmetic sequence: $-3,2,7,12, \ldots$

Which term in the sequence has the value 212 ?

$$
\begin{aligned}
t_{n} & =t_{1}+(n-1) d \\
212 & =-3+(n-1)(5) \\
212 & =-3+5 n-5 \\
212 & =5 n-8 \\
220 & =5 n
\end{aligned} \quad \therefore \quad \begin{aligned}
& n=44 \\
& t_{44}=212
\end{aligned}
$$

4) Determine the third term of an arithmetic sequence if the first term is 3 and the sixth term is 18 .

$$
\begin{aligned}
& t_{n}=t_{1}+(n-1) d \\
& t_{6}=3+(6-1) d \\
& 18=3+5 d \\
& -3 \\
& \frac{15}{5}=\frac{5 d}{5} \\
& 3=d
\end{aligned}
$$

$$
\begin{aligned}
t_{3} & =3+(3-1) 3 \\
& =3+2(3) \\
& =3+6 \\
t_{3} & =9
\end{aligned}
$$

5) Determine the first term of the arithmetic sequence in which the $20^{\text {th }}$ term is 112 and the common difference is 6 .

$$
t_{n}=\frac{1}{?}+(n-1) d
$$

$$
t_{20}^{\uparrow}=112
$$

$$
\hat{d=6}
$$

$$
\begin{aligned}
& 112=t_{1}+(20-1)(6) \\
& 112=t_{1}+(19)(6) \\
& 112=t_{1}+114 \\
& -114 \\
& \text {-114 } 1,4-7,9,12 \text { (odd letters) }
\end{aligned} \rightarrow t_{1}=-2
$$

1.2 - Arithmetic Series

Carl Friedrich Gauss, mathematician born in 1977:
When Gauss was 10 , his math teacher challenged the class to find the sum of the numbers from 1 to 100 , thinking it will take some time. However, Gauss found the answer, 5050, within minutes. What did he do?

Hint: Let $\underline{S}=$ the sum of the numbers from 1 to 100 . Write the sum in ascending order. Write the sum in descending order. Add the first equation and $2^{\text {nd }}$ equation together and solve for $S$.

$$
\begin{aligned}
S & =1+2+3+4+\cdots \\
+\quad S & =100+99+98+97+\cdots \cdots+100 \\
\hline 2 S & =101+101+101+\cdots \cdot 1 \\
2 S & =100(101) \\
2 S & =10100 \\
& =101+101 \\
& =150
\end{aligned}
$$

Gauss had discovered the underlying principles of an arithmetic series!!
Definition:
Arithmetic Series: A sum of terms that form an arithmetic sequence.
Ex:
2, 5, 8, 11, 14, $17 \leftarrow$ Arithmetic sequence $2+5+8+11+14+17 \leftarrow$ Arithmetic series

Deriving the formula to calculate the sum of n terms, given an arithmetic series:


In general,

$$
S_{n}=\frac{n}{2}\left(t_{1}+t_{n}\right)
$$

Examples:

1) Determine the sum of each arithmetic series.
a) $-3+1+5+9+\ldots+29$
$\overrightarrow{+4}+4+4$
Determine common difference. $\quad d=4$
Determine how many terms. using

$$
\begin{aligned}
t_{n} & =t_{1}+(n-1) d \\
29 & =-3+(n-15(4) \\
29 & =-3+4 n-4 \\
29 & =-7+4 n \\
+7 & +7 \\
36 & =4 n \\
n & =9
\end{aligned}
$$

b) $\frac{1}{2}+\frac{3}{4}+1+\frac{5}{4}+\frac{6}{4}+\ldots+4$

Determine common difference. $\frac{3}{4}-\frac{1}{2}=\frac{3}{4}-\frac{2}{4}=\frac{1}{4}$
Determine how many terms.

$$
\left.\begin{array}{rlrl}
t_{n} & =t_{1}+(n-1) d \\
4 & =\frac{1}{2}+(n-1) \frac{1}{4} \\
4 & =\frac{1}{2}+\frac{1}{4} n-\frac{1}{4} \\
4 & =\frac{1}{4}+\frac{1}{4} n \\
-\frac{1}{4} & -\frac{1}{4} \\
4 \cdot \frac{15}{4} & =\frac{n}{4} \cdot 4 \\
15 & =n
\end{array} \quad \therefore S_{15}=\frac{15}{2}\left[\frac{1}{2}+4\right] \frac{9}{2}\right]=\frac{135}{4}
$$

$$
d=-11-(-4)=-11+4=-7
$$

Chapter 1 - Sequences and Series
2) For each following arithmetic series, determine the indicated sum.
a) $-4-11-18-25-\ldots, S_{28}$
(1) find $t_{28}=-4+(28-1)(-7)=-193$
(2) $S_{28}=\frac{28}{2}[-4+(-193)]=\frac{28}{2}(-197)=-2758$

$$
\begin{aligned}
& \text { b) } 1+3.5+6+8.5+\ldots, \quad S_{42} \\
& t_{42}=1+(41)(2.5)=103.5 \\
& S_{42}=\frac{42}{2}(1+103.5)=2194.5
\end{aligned}
$$

3) Determine the sum, $S_{n}$, for the arithmetic sequence described.

$$
\begin{array}{rr}
t_{1}=-5 & n=8 \\
S_{8}=\frac{8}{2}(-5+72)=268 & t_{8}=72
\end{array}
$$

4) Determine the value of the first term.

$$
\begin{aligned}
d=4 & S_{n}=1380 \\
S_{n} & =\frac{n}{2}\left[t_{1}+t_{1}+(n-1) d\right] \\
1380 & =\frac{20}{2}\left[2 t_{1}+19(4)\right] \\
\frac{1380}{10} & =\frac{10\left[2 t_{1}+76\right]}{10}
\end{aligned}
$$

$n=20$

$$
\begin{aligned}
& 138=2 t_{1}+76 \\
& -76 \\
& \frac{62}{2}=\frac{2 t_{1}}{2} \\
& 31=t_{1}
\end{aligned}
$$

5) For the arithmetic series, determine the value $n$.
$t_{1}=2$

$$
t_{n}=-66
$$

$$
S_{n}=\frac{n}{2}\left[t_{1}+t_{n}\right]
$$

$$
-576=\frac{n}{2}[2+(-66)]
$$

$$
-576=\frac{n}{2}(-64)
$$



$$
18=n
$$

Ch. 1.2 HW: p. 27 \# 1-7, 9, 11, 14 (odd letters)

## 1.3 - Geometric Sequence

What's the pattern?


The ratio of $\frac{t_{n+1}}{t_{n}}$ is 3 throughout.
This is an example of geometric sequence, with a common ratio of $\qquad$ -

## Definition:

Geometric Sequence - An ordered list of terms in which the ratio, $\frac{t_{n+1}}{t_{n}}$, between consecutive terms is constant. This constant is called the common ratio.

Deriving the formula to calculate the $\mathrm{n}^{\text {th }}$ term, given a geometric sequence:


In general,
$t_{n}=t_{1} \cdot r^{n-1}$

## Examples:

1) Determine the first five terms of the geometric sequence where
a) $t_{1}=3, \quad r=4$

b) $t_{1}=-1, \quad r=-\frac{2}{3}$

2) Determine the first five terms of the geometric sequence where
a) 2.3, $\qquad$ , $\qquad$ , $\qquad$ , 186.3


if $r=3$
if $r=-3.6, \underline{60.7}, \underline{62.1}$, 186.3

$$
r^{4}=81
$$

$$
\text { if } r=-3
$$

$$
r= \pm 3
$$

b) $1,5,25,125,625,3125$
$\qquad$
$\qquad$

$$
2.3,-6.9,20.7,-62.1,186.3
$$

 $t_{3}$

$$
t_{6}=t_{3} \cdot r^{3}
$$

$$
\frac{3125}{25}=\frac{25 \cdot r^{3}}{25}
$$

$$
r^{3}=125
$$

$$
r=5
$$

3) Determine a formula for the $\mathrm{n}^{\text {th }}$ term.
a) $200,50,12.5,3.125, \ldots$

$$
r=\frac{5 \sigma}{20 \phi}=\frac{1}{4}
$$

$$
t_{n}=200\left(\frac{1}{4}\right)^{n-1}
$$

b) $t_{5}=-162, \quad t_{10}=39366$
(1)

$$
\begin{array}{rlrl}
t_{10} & =t_{5} \cdot r^{5} & \text { (2) solve for } t_{1} & \text { (3) } t_{n}=-2(-3)^{n-1} \\
39366 & =-162 \cdot r^{5} & t_{n} & =t_{1} r^{10} 1 \\
r^{5} & =-243 & t_{10} & 39 \\
r & =5 \sqrt{-243} & 39366 & =t_{1}(-3)^{9} \\
r & =-3 & t_{1} & =-2
\end{array}
$$

$1,3,9,27 \leftarrow$ Geometric Sequence $1+3+9+27 \longleftarrow$ Geometric Series

Deriving the formula to calculate the sum of $n$ terms, given a geometric series:
Idea: (1)

$$
\begin{align*}
S_{4}=1+3+9+27 \xrightarrow{x r=3} & 3 S_{4}=3(1+3+9+27) \\
3 S_{4} & =3+9+27+81 \tag{2}
\end{align*}
$$

(2)-(1):

$$
\begin{aligned}
3 S_{4} & =3+9+27+81 \\
-\quad S_{4} & =1+3+9+27 \\
\hline 2 S_{4} & =-1 \\
2 S_{4} & =80 \\
S_{4} & =40
\end{aligned}
$$


(1) $S_{n}=t_{1}+t_{1} r+t_{1} r^{2}+\cdots$

$$
\left.\begin{array}{l}
+t_{1} r^{n-1} \\
+t_{1} r^{n}
\end{array}\right) \times r
$$

(2) $r S_{n}=t_{1} r+t_{1} r^{2}+t_{1} r^{3}+\cdots+t_{1} r^{n}$
(2)

$$
\begin{aligned}
& r S_{n}=t_{1} r+t_{1} r^{2}+t_{1} r^{3}+\cdots+t_{1} r^{n-1}+t_{1} r^{n} \\
& \frac{-S_{n}=t_{1}+t_{1} r+t_{1} r^{2}+\cdots \quad t_{1} r^{n-1}}{(r-1) S_{n}=-t_{1}} \quad+t_{1} r^{n} \\
& (r-1) S_{n}=t_{1} r^{n}-t_{1} \\
& S_{n}=\frac{t_{1}\left(r^{n}-1\right)}{r-1}
\end{aligned}
$$

In general,

$$
S_{n}=\frac{t_{1}\left(r^{n}-1\right)}{r-1}
$$

Examples:

1) Given $3+4.5+6.75+\ldots$, determine $S_{10}$ to the nearest hundredth.

$$
n=10
$$

$$
t_{1}=3
$$

$$
t_{1}=3
$$

$$
S_{10}=\frac{3\left(1.5^{10}-1\right)}{1.5-1} \doteq 339.99
$$

2) What is $S_{n}$ expressed in fraction if $t_{1}=\frac{1}{1024}, \quad r=-4, \quad n=10$.

$$
\begin{aligned}
& S_{10}=\frac{\frac{1}{1024}\left[(-4)^{10}-1\right]}{-4-1}=\frac{\frac{1}{1024}(1048576-1)}{-5}=\frac{\frac{1}{1024}(1048575)}{-5} \\
& =\left(\frac{1048575}{1024}\right)\left(-\frac{1}{5}\right)_{\text {geometric series. }}^{1024}
\end{aligned}
$$

3) Determine the sum of each geometric series.

$$
\text { a) } \frac{1}{27}+\frac{1}{9}+\frac{1}{3}+\ldots+729 \quad r=\frac{1}{9} \div \frac{1}{27}=\frac{1}{9} \times \frac{27}{1}=3
$$

(1) use $t_{n}=t_{1} r^{n-1}$ to solve for $n$

$$
r=\frac{4.5}{3}=1.5
$$

$$
t_{1}=3
$$

## 1.5 - Infinite Geometric Series

## Investigation:



1) Use the square provided above to follow the steps below.

Step 1: Divide the square in half and shade (with the color of your choice). What is the area of the shaded region?
Step 2: Divide the unshaded region in half and shade (with the color of your choice).
Repeat steps 1 to 2 until you can no longer shade.
2) Record the areas obtained from each step:

3) Is the sequence arithmetic, geometric, $\delta$ or neither? Justify your answer.

4) Ignoring the physical limitations, could this sequence continue indefinitely? In other words, would this be an infinite sequence?

Sum of the areas:
$S_{7}=8+4+2+1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}=15.875$
$\mathrm{S}_{8}=\cdots \quad \cdots \quad \ldots \quad+\frac{1}{16}=15.9375$
$S_{9}=$ " " " $"+\frac{1}{32}=15.96875$

What do you think $\mathrm{S}_{\mathrm{n}}$ should be as n gets larger and larger?

## Convergent Series:

Consider the series $8+4+2+1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\frac{1}{32}+\frac{1}{64}+\cdots$
$\mathrm{S}_{\mathrm{n}}=\frac{t_{1}\left(r^{n}-1\right)}{r-1}=\frac{8\left(0.5^{n}-1\right)}{-0.5}$
on gr.cale.
$y_{1}=\frac{8\left(0.5^{x}-1\right)}{-0.5}$
Use your graphing calculator to quickly calculate the following sum:
$\mathrm{S}_{11}=15.992$
$\mathrm{S}_{12}=15.996$
$\mathrm{S}_{13} \doteq 15.998$
$\mathrm{S}_{14} \doteqdot 15.999$
As the number of terms increases, the sequence of partial sums approaches a fixed value of 16 .
This series is said to be a convergent series. (able to determine the sum

## Divergent Series:

 of infinitely manyConsider the series $8+16+32+64+128+\cdots$
As the number of terms increases, the sum of the series continues to grow. The sequence of partial sums does not approach a fixed value. Therefore, the sum of this series cannot be calculated. This series is said to be a $\qquad$ series. series. (can't determine the sum
of infinitely many terms) series. (can't determine the sum
of infinitely many terms)

## Infinite Geometric Series:

The formula for the sum of a geometric series is
$\mathrm{S}_{\mathrm{n}}=\frac{t_{1}\left(r^{n}-1\right)}{r-1}=\frac{t_{1}\left(1-r^{n}\right)}{1-r}$


As $n->\infty$ ( $\boldsymbol{t}_{1}$ 百) gets very large), the value of the $r^{n}$ approaches $\qquad$ .

Thereford, the sum of an infinite geometric series is:

$$
\mathrm{S}_{\infty}=\frac{t_{1}}{1-r} \text { where }-1<r<1
$$

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Chapter 1 - Sequences and Series
Examples:

1) State whether each infinite geometric series is convergent or divergent.
a) $t_{1}=3$
$r=\frac{5}{3}$
divergent since $r>1$

$$
\square_{n 0} S_{\infty}
$$

b) $5-2.5+1.25-0.625$

$$
r=\frac{-2.5}{5}=\frac{-1}{2} \quad-1<\frac{-1}{2}<1 \quad \therefore \text { convergent }
$$

2) Determine the sum of each infinite geometric series, if it exists.
a) $\mathrm{t}_{1}=81 \quad \mathrm{r}=-\frac{1}{3}$
$\rightarrow$ convergent $-1<\frac{-1}{3}<1$

$$
\begin{aligned}
S_{\infty} & =\frac{t_{1}}{1-r}=\frac{81}{1-\left(-\frac{1}{3}\right)}=\frac{81}{1+\frac{1}{3}} \\
& =\frac{81}{\frac{4}{3}}=\frac{81}{1} \times \frac{3}{4}=\frac{243}{4}=60.75
\end{aligned}
$$

b) $3+\frac{9}{2}+\frac{27}{4}+\frac{81}{8} \quad \therefore=\frac{9}{2} \div 3=\frac{9}{2} \cdot \frac{1}{3}=\frac{3}{2} \quad \therefore r>1$
$\therefore$ divergent series
$\therefore$ can't calculate $S_{\infty}$ ( $S_{\infty}=$ D.N.E)
3. The sum of an infinite geometric series is 9 and its common ratio is $\frac{1}{3}$. What is the value of the first term? Write the first three terms of the series.

$$
\begin{aligned}
S_{\infty} & =\frac{t_{1}}{1-r} \\
q & =\frac{t_{1}}{1-\frac{1}{3}} \\
\frac{2}{3}(9) & =\frac{t_{1}}{\frac{1}{3}} \cdot\left(\frac{2}{3}\right)
\end{aligned} \quad \begin{aligned}
& t_{1}=9 \cdot \frac{2}{3} \\
& t_{1}=6 \\
& \underbrace{6+2+\frac{2}{3}+\cdots}_{x \frac{1}{3}} \underbrace{}_{x \frac{1}{3}}
\end{aligned}
$$

4. The sum of an infinite series is three times its first term. Determine the value of the common ratio.

$$
\begin{aligned}
& S_{\infty}=\frac{t_{1}}{1-r} \\
&(1-r)\left(3 t_{1}\right)=\left(\frac{t_{1}}{1-r}\right)(1-r) \quad \frac{1}{3}-\frac{3}{3}=\frac{-2}{3} \\
& \frac{(1-r)\left(3 t_{1}\right)}{3 t_{1}}=\frac{t_{1}}{3 t_{1}} \\
& 1-r=\frac{1}{3} \\
&-1 \\
&-r=\frac{-2}{3} \\
& r=\frac{2}{3}
\end{aligned}
$$

Ch. 1.5 HW: p. 63 \#1-5 odd letters, \#6-10

## Ch. 1 - Review

## Multiple Choice Questions:

1. Which of the following is an arithmetic sequence?

| a) $2,4,8,16,32, \ldots$ | b) $-2,0,2,4,6,8, \ldots$ |
| :--- | :--- |
| c) $-1,5,10,-50,100, \ldots$ | d) None |

2. Which of the following is a geometric sequence?

| a) $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \ldots$ | b) $1,-2,4,-8,16, \ldots$ |
| :--- | :--- |
| c) $-1,0,1,2,3, \ldots$ | d) Both a and b |

3. Which of the following is a convergent infinite geometric series?

| a) $0.5+1+1.5+2+2.5+\ldots$ | b) $1-2+4-8+16-\ldots$ |
| :--- | :--- |
| c) $\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\ldots$ | d) $5+4+3+2+1+\ldots$ |

## Written Response Questions:

4. Determine the indicated term of the arithmetic sequence
$6,11,16, \ldots, t_{7}$
5. Two terms of an arithmetic sequence are given. Determine the indicated term.
$t_{4}=24$,
$t_{10}=66$,
determine $t_{1}$
6. In this arithmetic sequence: $3,8,13,18, \ldots$; which term has the value 123 ?
7. In this arithmetic sequence, $k$ is a natural number: $k, \frac{2 k}{3}, \frac{k}{3}, 0, \ldots$
a) Determine $t_{6}$ in terms of $k$.
b) Write an expression for $t_{n}$ in terms of $k$.
c) Suppose $t_{20}=-16$; determine the value of $k$.
8. Determine the sum of the first 20 terms of the arithmetic series:
$-21-15.5-10-4.5-\ldots$
9. Use the given data about the arithmetic series to determine the indicated value.
a) $S_{20}=-850, t_{20}=-90$; determine $t_{1}$
b) $S_{15}=322.5, t_{1}=4$; determine $d$
c) $S_{n}=-126, t_{1}=-1, t_{n}=-20$; determine $n$
d) $t_{1}=1.5, t_{20}=58.5 ;$ determine $S_{15}$
10. Determine the sum of this arithmetic series:

$$
-2+3+8+13+\ldots+158
$$

11. In a finite geometric sequence, $t_{1}=5$ and $t_{5}=1280$
a) Determine $t_{2}$ and $t_{6}$
b) The last term of the sequence is 20480 . How many terms are there?
12. Write the first 4 terms of the geometric sequence if $t_{1}=\frac{1}{2}, r=\frac{2}{3}$. Leave the terms in reduced fraction.
13. Use the given data about the geometric sequence to determine the indicated value.
$r=-3, t_{7}=135 ;$ determine $t_{1}$
14. Determine the sum of the first 15 terms of the geometric series: Round to the nearest hundredth. $40-20+10-5+2.5-\ldots$
15. On the first swing, a pendulum swings through an arc of 40 cm . On each successive swing, the length of the arc is 0.98 times the previous length. In the first 20 swings, what is the total distance that the lower end of the pendulum swings, to the nearest hundredth of a centimeter?
16. Calculate the sum of the geometric series: $1+6+36+\ldots+279936$
17. What is $S_{n}$ for the geometric series if $t_{1}=2, \quad r=-2, \quad \mathrm{n}=12$
18. Write the repeating decimal as an infinite series.
$2.1 \overline{35}=2.13535353535 \ldots$

After the term 2.1, the series forms an infinite geometric with the common ratio $=$ $\qquad$ . Is the series convergent or divergent? $\qquad$ since $\qquad$ $<\mathrm{r}<$ $\qquad$ .
19. Determine the sum of the infinite geometric series, if it exists.
a) $t_{1}=-1, r=\frac{3}{4}$
b) $t_{1}=-4, \quad r=\frac{4}{3}$
20. The first term of an infinite geometric series is -18 , and its sum is -45 . Write the first four terms of the series.

