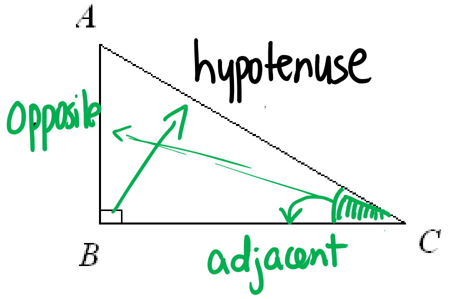
**First Name: ________________________ Last Name: ________________________ Block: ______**

# Ch. 2 – Trigonometry Notes

# 2.1 – The Tangent Ratio

**Definitions**:

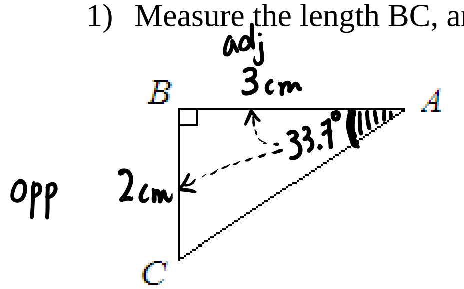



**Hypotenuse**: The side opposite the right angle in a right triangle.
**Opposite** side: side opposite the angle of interest
**Adjacent** side: side that is adjacent to the angle of interest (that is not the hypotenuse)

| From angle, | Opposite Side | Adjacent Side | Hypotenuse |
|---|---|---|---|
| ∠A | BC | AB | AC |
| ∠C | AB | BC | AC |

**Exercise**:

1) Measure the length BC, and AB to determine the ratio, $\frac{BC}{AB}$ to the nearest 5 decimal places.




$\overline{AB} \doteq 3$ cm

$\overline{BC} \doteq 2$ cm

$\frac{BC}{AB} = \frac{2\text{ cm}}{3\text{ cm}} = \frac{2}{3} \doteq 0.66667$

a) If you measure the angle A to the nearest 8 decimal places, it will be $\angle A \approx 33.69006753°$

b) Using your calculator, calculate tan(A) to the nearest 5 decimal places. $\tan(33.69006753°) \doteq 0.66667$

c) What do you notice? $\tan A = \frac{opp}{adj}$

**Conclusions**:
The ratio you found in the above exercise is called ***tangent ratio***.

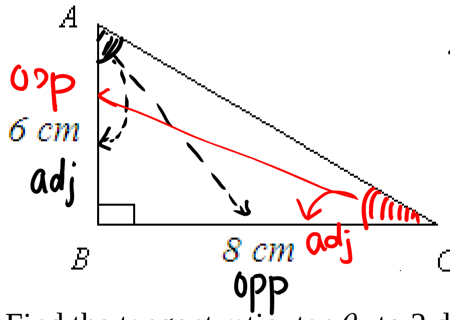
The tangent ratio for an angle can be determined with the (TAN) key on your scientific calculator.

$$\tan\ \theta = \frac{\text{Oppposite side length}}{\text{Adjacent side length}}$$

$$\boxed{\tan\theta = \frac{opp}{adj}}$$

**Examples**: ratio

1. Determine tan(A), and tan(C).

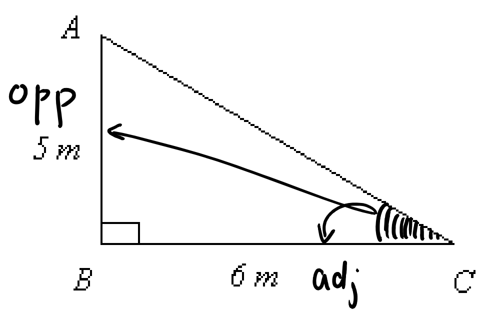



$\tan A = \frac{8}{6}$ $\tan C = \frac{6}{8}$

2. Find the tangent ratio, $\tan\theta$, to 2 decimal places, for each angle, $\theta$
   a. $\theta = 50°$  $\tan 50° \doteq 1.19$

   b. $\theta = 39°$  $\tan 39° \doteq 0.81$

3. Find tan C. ① Calculate ② $\angle C$ to the nearest degree.

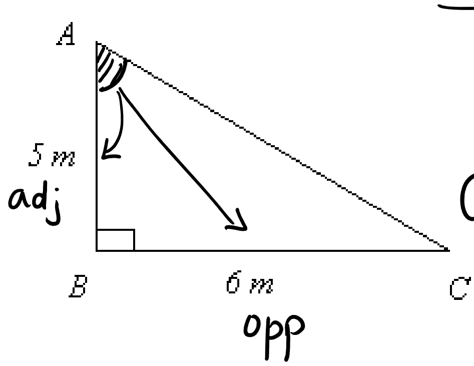



$\tan C = \frac{opp}{adj}$

① $\tan C = \frac{5}{6}$

② $\angle C = \tan^{-1}\left(\frac{5}{6}\right) \doteq 39.81° \doteq \boxed{40°}$

4. Find tan A. ① Calculate ② $\angle A$ to the nearest degree.




$\tan A = \frac{opp}{adj}$

① $\tan A = \frac{6}{5}$

② $\angle A = \tan^{-1}\left(\frac{6}{5}\right) \doteq 50°$

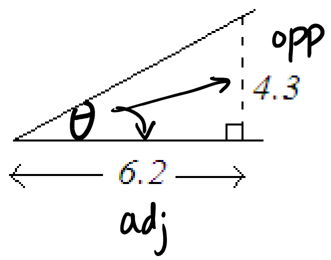
5. Find each angle, measure the angle to the nearest degree, for each tangent ratio
   a. $\tan\theta = 3.892$

   $$\theta = \tan^{-1}(3.892) \doteq 76^\circ$$

   b. $\tan\theta = 1.891$

   $$\theta = \tan^{-1}(1.891) \doteq 62^\circ$$

6. Determine the angle of inclination of each line to the nearest tenth of a degree.



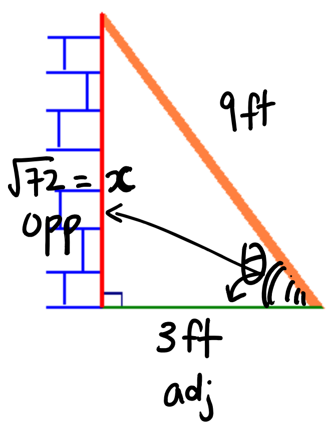

$$\tan\theta = \frac{opp}{adj}$$

$$\tan\theta = \frac{4.3}{6.2}$$

$$\theta = \tan^{-1}\left(\frac{4.3}{6.2}\right)$$

$$\theta \doteq 34.7^\circ$$

7. A 9-ft. ladder leans against the side of a building with its base 3 ft. from the wall. What angle, to the nearest degree, does the ladder make with the ground?




$$3^2 + x^2 = 9^2$$

$$9 + x^2 = 81$$

$$-9 \qquad -9$$

$$x^2 = 72$$

$$x = \sqrt{72}$$

$$\tan\theta = \frac{\sqrt{72}}{3}$$

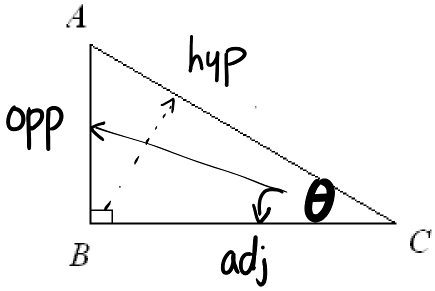
$$\theta = \tan^{-1}\left(\frac{\sqrt{72}}{3}\right)$$

$$\theta \doteq 71^\circ$$

***Ch. 2.1 HW: p. 75 #3 – 16, 19***

# 2.2 – Using the Tangent Ratio to Calculate Lengths

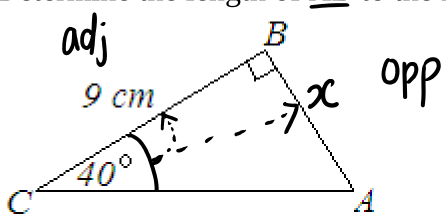
**Recap**:




$\tan\theta = \frac{opp}{adj}$

**Examples: Determining the Length of a Side Opposite a Given Angle**

1) Determine the length of AB to the nearest tenth of a centimeter.

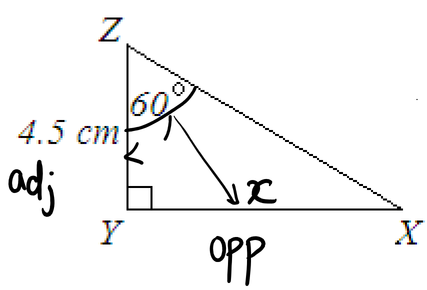



$\tan\theta = \frac{opp}{adj}$

$9 \cdot \tan 40° = \frac{x}{9} \cdot 9$

$x = 9 \cdot \tan 40° \doteq 7.6$ cm

2) Determine the length of XY to the nearest tenth of a centimeter.



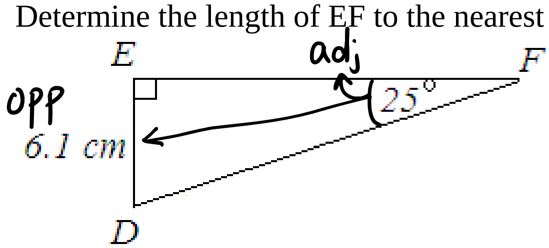

$\tan\theta = \frac{opp}{adj}$

$\tan 60° = \frac{x}{4.5}$

$x = 4.5 \cdot \tan 60° \doteq 7.8$ cm

**Examples: Determining the Length of a Side Adjacent a Given Angle**

3) Determine the length of EF to the nearest tenth of a centimeter.

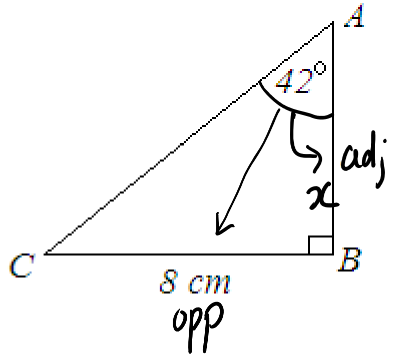



$\tan\theta = \frac{opp}{adj}$

$\tan 25° = \frac{6.1}{x}$

$x = \frac{6.1}{\tan 25°} \doteq 13.1$ cm

4) Determine the length of AB to the nearest tenth of a centimeter.



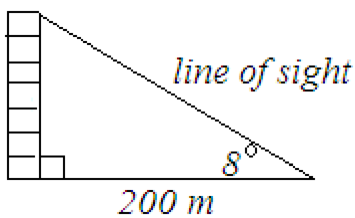

$$\tan\theta = \frac{opp}{adj}$$

$$\tan 42^\circ = \frac{8}{x}$$

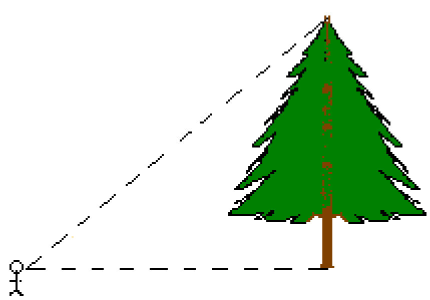
$$x = \frac{8}{\tan 42^\circ} \doteq 8.9 \text{ cm}$$

**Examples: Using Tangent to Solve an Indirect Measurement Problem**

5) At a horizontal distance of 200 m from the base of an observation tower, the angle between the ground and the line of sight to the top of the tower is $8°$. How high is the tower to the nearest metre? The diagram is not drawn to scale.



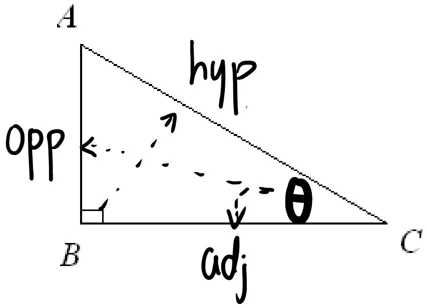

6) One of Canada's tallest trees is a Douglas fir on Vancouver Island. The angle of elevation measured by an observer from 78 m from the base of the tree is $50°$. How tall is this tree, to the nearest metre?



*Ch. 2.2 HW: p. 82 # 3 – 5 (a, c), #6 – 14*

# 2.4 – The Sine and Cosine Ratios

**Recap**:




$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

Similar to the tangent ratio,

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

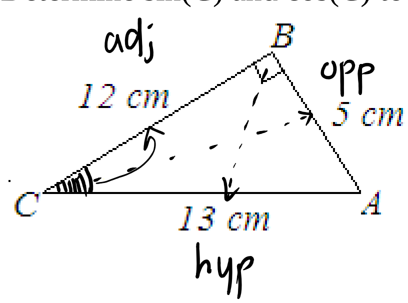
$$\cos\theta = \frac{\textit{adjacent}}{\textit{hypotenuse}}$$

sine opp hyp cosine adj hyp tan opp adj

You can remember this better with ' SOH CAH TOA '

**Examples: Determining the Sine and Cosine of an Angle**

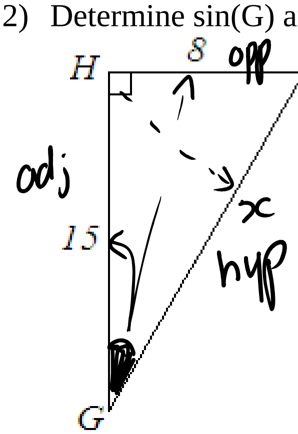
1) Determine sin(C) and cos(C) to the nearest hundredth.




$\sin C = \frac{opp}{hyp}$ $\cos C = \frac{adj}{hyp}$

$\sin C = \frac{5}{13}$ $\cos C = \frac{12}{13}$

2) Determine sin(G) and cos(G) to the nearest hundredth.




$8^2 + 15^2 = x^2$

$64 + 225 = x^2$

$289 = x^2$

$x = \sqrt{289}$

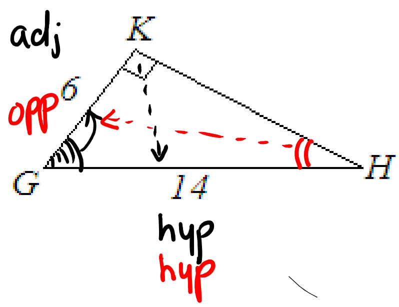
$x = 17$

$\sin G = \frac{opp}{hyp}$ $\cos G = \frac{adj}{hyp}$

$\sin G = \frac{8}{17}$ $\cos G = \frac{15}{17}$

**Examples: Using Sine or Cosine to Determine the Measure of an Angle**

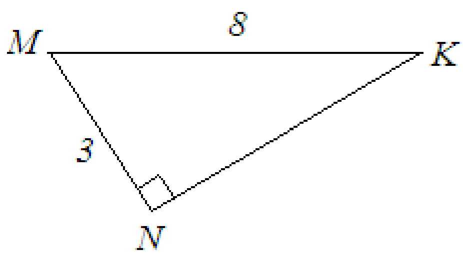
3) Determine the measures of $\angle G$ and $\angle H$ to the nearest tenth of a degree.




$$\cos G = \frac{adj}{hyp}$$
$$\cos G = \frac{6}{14}$$
$$\angle G = \cos^{-1}\left(\frac{6}{14}\right)$$
$$\angle G \doteq 64.6^\circ$$

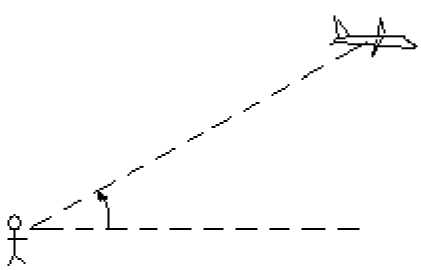
$$\sin H = \frac{opp}{hyp}$$
$$\sin H = \frac{6}{14}$$
$$\angle H = \sin^{-1}\left(\frac{6}{14}\right)$$
$$\angle H \doteq 25.4^\circ$$

4) Determine the measures of $\angle K$ and $\angle M$ to the nearest tenth of a degree.




**Examples: Using Sine or cosine to Solve a Problem**

5) An observer is sitting on a dock watching a float plane in Vancouver harbor. At a certain time, the plane is 300 m above the water and 430 m from the observer. Determine the angle of elevation of the plane measured from the observer, to the nearest degree.



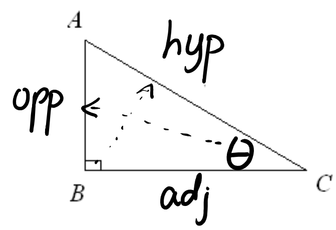
***CH. 2.4 HW: p. 95 #4 – 13***

# 2.5 – Using the Sine and Cosine Ratios to Calculate Lengths

**Recap**:

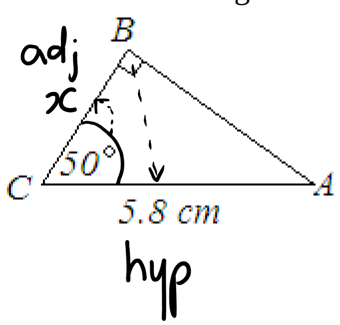
$\sin\theta = \dfrac{opp}{hyp}$

$\cos\theta = \dfrac{adj}{hyp}$




**Examples: Using the Sine or Cosine Ratio to Determine the Lengths of a Leg**

1) Determine the length of BC to the nearest tenth of a centimeter.



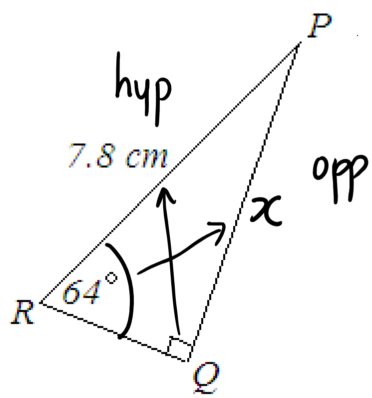

$\cos\theta = \dfrac{adj}{hyp}$

$5.8 \cdot \cos 50° = \dfrac{x}{\cancel{5.8}} \cdot \cancel{5.8}$

$x = 5.8\cos 50° \doteq 3.7$ cm

$\therefore \overline{BC} \doteq 3.7$ cm

2) Determine the length of PQ to the nearest tenth of a centimeter.



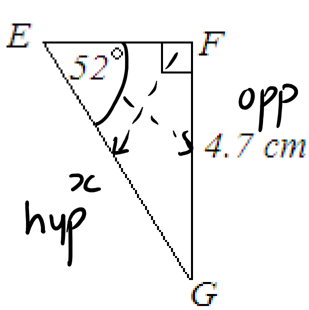

$\sin\theta = \dfrac{opp}{hyp}$

$\sin 64° = \dfrac{x}{7.8}$

$x = 7.8\sin 64° \doteq 7.0$ cm

**Examples: Using Sine or Cosine to Determine the Length of the Hypotenuse**

3) Determine the length of EG to the nearest tenth.



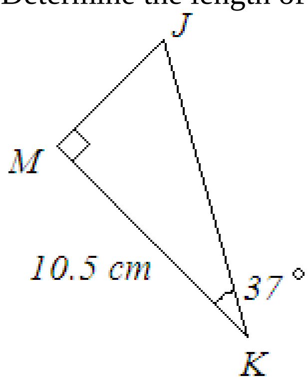

$\sin\theta = \dfrac{opp}{hyp}$

$\dfrac{\sin 52°}{1} = \dfrac{4.7}{x}$

$4.7 \cdot \dfrac{1}{\sin 52°} = \dfrac{x}{\cancel{4.7}} \cdot \cancel{4.7}$

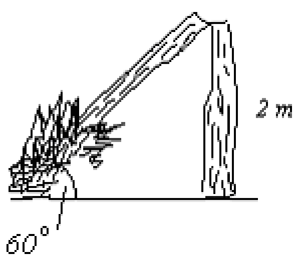
$x = \dfrac{4.7}{\sin 52°} \doteq 6.0$ cm

4) Determine the length of JK to the nearest tenth of a centimeter.

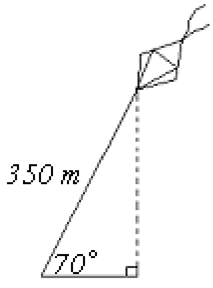



**Examples: Solving an Indirect Measurement Problem**

5) A tree is splintered by lightning 2 m up its trunk, so that the top part of the tree touches the ground. The angle the top of the tree forms with the ground is $60°$. Approximately, how tall is the tree, to the nearest tenth of a meter?




6) A kite string is 350 m long. The angle the string makes with the ground is $70°$. How far from the person holding the string is a person standing directly under the kite? Round to the nearest metre.




***Ch. 2.5 HW: p. 101 #3 – 5 (a, c), #6 – 12***

# 2.6 – Applying the Trigonometric Ratios

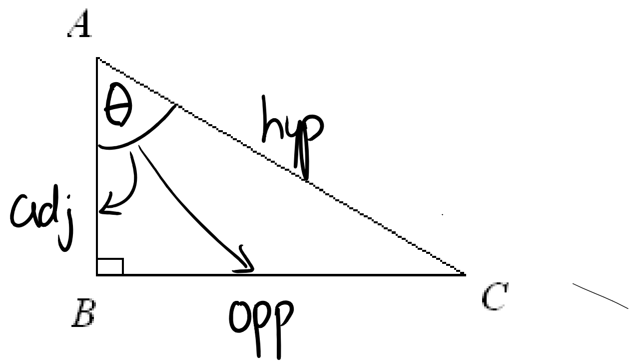
SOH CAH TOA

**Recap**:

$\sin\theta = \frac{opp}{hyp}$

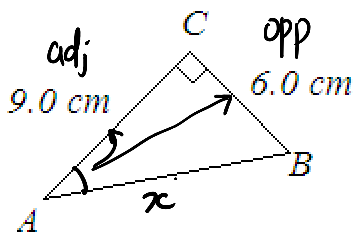
$\cos\theta = \frac{adj}{hyp}$

$\tan\theta = \frac{opp}{adj}$

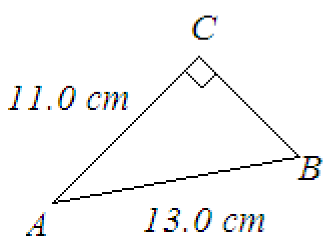



**Examples: Solving a Right Triangle Given Two Sides**

1) Solve $\Delta$ ABC. Give the measures to the nearest tenth

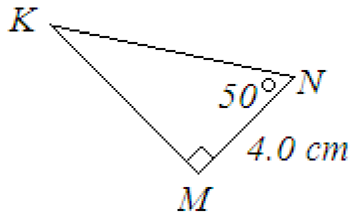
| | |
|---|---|
|   | Solve for AB:<br>$6^2+9^2=x^2$<br>$36+81=x^2$<br>$117=x^2$<br>$x=\sqrt{117}$<br>$\therefore \overline{AB} \doteq 10.8$ cm |
| Solve for $\angle$A:<br>$\tan A = \frac{6}{9}$<br>$\angle A = \tan^{-1}\left(\frac{6}{9}\right) \doteq 33.7^\circ$ | Solve for $\angle$B:<br>$\angle B \doteq 180^\circ - 90^\circ - 33.7^\circ = 56.3^\circ$ |

2) Solve $\Delta$ABC. Give the measures to the nearest tenth.

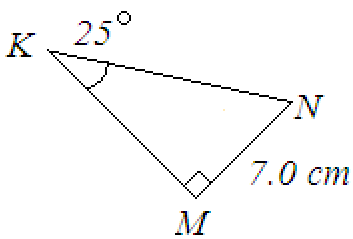
| | |
|---|---|
|   | Solve for BC:<br>6.9 cm |
| Solve for $\angle$A:<br>$32.2^\circ$ | Solve for $\angle$B:<br>$57.8^\circ$ |

## Examples: Solving a Right Triangle Given Two Sides

3) Solve $\Delta$KMN. Give the measures to the nearest tenth. (Given an angle and one side)

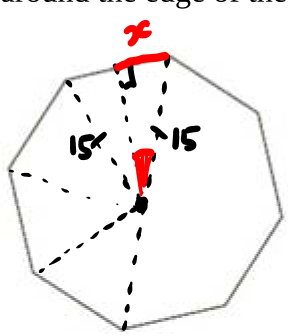
| | |
|---|---|
|  | Solve for KM:<br><br>4.8 cm |
| Solve for KN:<br><br>6.2 cm | Solve for $\angle$K:<br><br>40° |

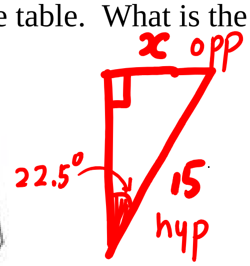
4) Solve $\Delta$KMN. Give the measures to the nearest tenth. (Given an angle and one side)

| | |
|---|---|
|  | Solve for KM: |
| Solve for KN: | Solve for $\angle$N: |

## Examples: Solving a Problem Using the Trigonometric Ratios

A small table has the shape of a regular octagon. The distance from one vertex to the opposite vertex, measured through the centre of the table, is approximately 30cm. There is a strip of wood veneer around the edge of the table. What is the length of this veneer to the nearest centimeter?




$\sin 22.5° = \frac{x}{15}$

$x = 15 \cdot \sin 22.5° \doteq 5.74$ cm

$\therefore$ length of the veneer $\doteq 16 \times 5.74$ cm $= 91.84$ cm

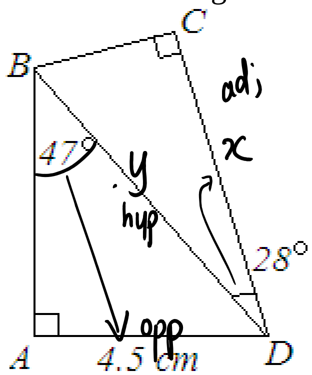
$\doteq 92$ cm

*CH. 2. 6 HW:p. 111 #3 – 6 (a, c), # 7, 8, 11, 12b*

# 2.7 – Solving Problems Involving More than One Right Triangle

**Examples: Calculating Side Length Using More than One Triangle**

1) Calculate the length of CD to the nearest tenth of a centimeter.




$\sin 47° = \frac{4.5}{y}$

$y = \frac{4.5}{\sin 47°}$

$y \doteq 6.15$ cm

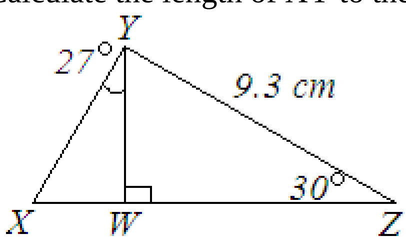
$\cos 28° \doteq \frac{x}{6.15}$

$x \doteq 6.15 \cdot \cos 28°$

$x \doteq 5.4$

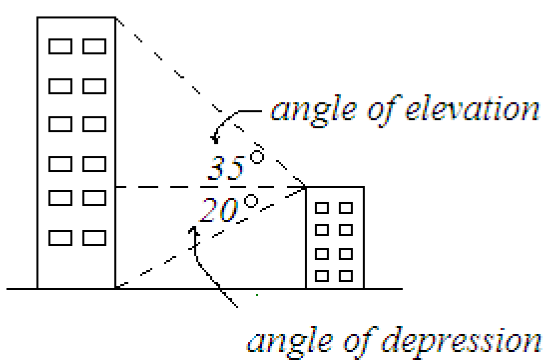
$\therefore \boxed{CD \doteq 5.4 \text{ cm}}$

2) Calculate the length of XY to the nearest tenth of a centimeter.

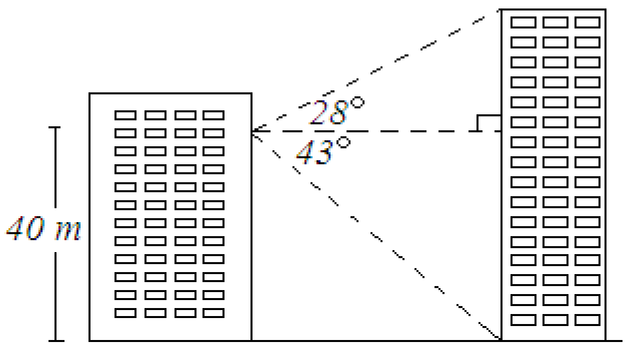



**Examples: Solving a Problem with Triangles in the Same Plane**

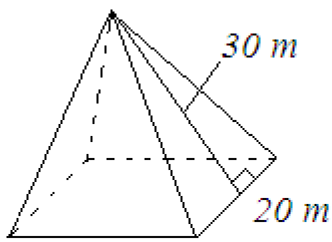
3) From the top of a 20-m high building, a surveyor measured the angle of elevation of the top of another building and the angle of depression of the base of that building. The surveyor sketched this plan of her measurements. Determine the height of the taller building to the nearest tenth of a metre.



4) A surveyor stands at a window on the 11th floor of an office tower. He uses a clinometers to measure the angles of elevation and depression of the top and the base of a taller building. The surveyor sketches this plan of his measurements. Determine the height of the taller building to the nearest tenth of a metre.




5) Given a pyramid with 4 congruent triangular faces, determine the measure of each of the three angles in the triangular face.

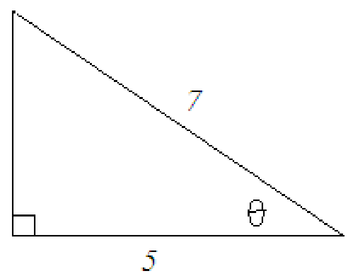



***Ch. 2.7 HW: p. 118 #3 – 5 (a, c), #6 – 14***

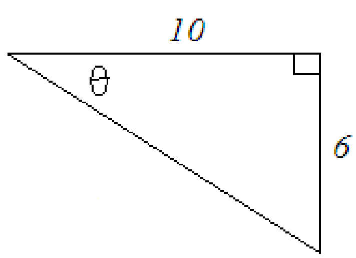
# Ch. 2 - Review

1. Determine each ratio. (Write the ratio in fraction and in decimal. Round the decimal value to the nearest hundredths): [3 marks]

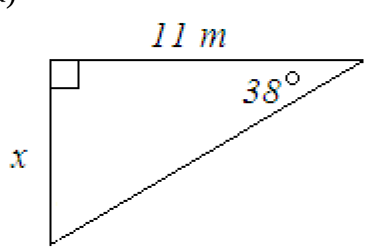
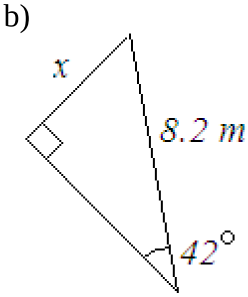
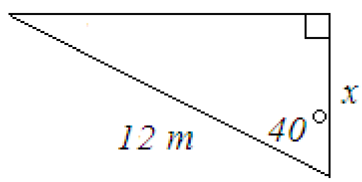
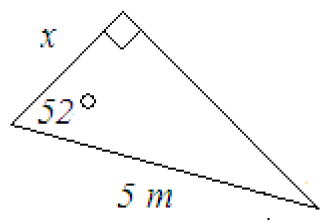
a)

| | $\tan\ \theta$ | $\cos\ \theta$ | $\sin\ \theta$ |
|---|---|---|---|
|  | | | |

b)

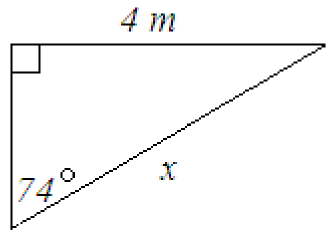
| | $\tan\ \theta$ | $\cos\ \theta$ | $\sin\ \theta$ |
|---|---|---|---|
|  | | | |

2. Calculate $x$ to the nearest tenth of a meter.

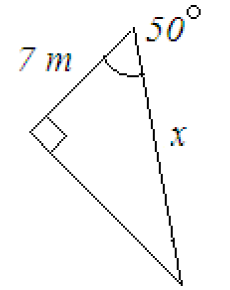
| | |
|---|---|
| a)  | b)  |
| c)  | d)  |

3. Calculate $x$ to the nearest tenth of a meter.

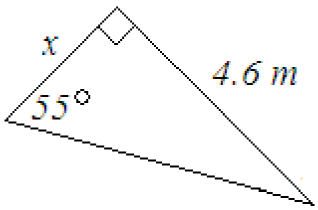
a)



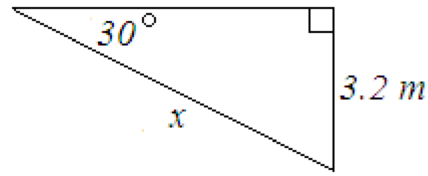

b)




c)

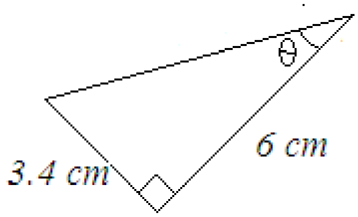



d)

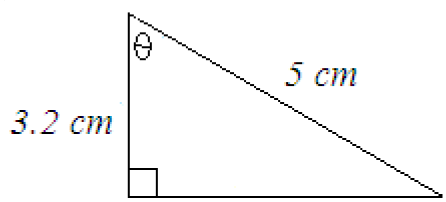



4. Calculate $\theta$ to the nearest **degree**.

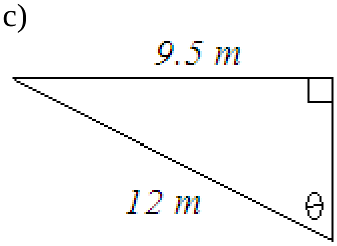
a)



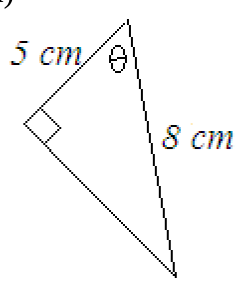

b)




c)

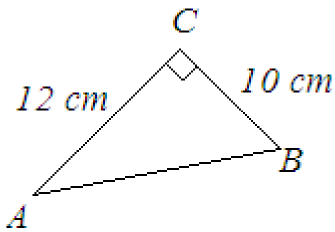



d)

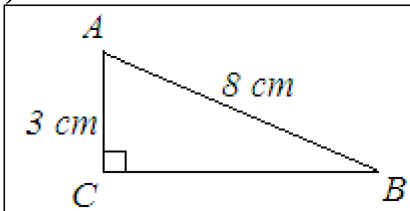


5. Solve $\Delta ABC$. Give the measures to the nearest tenth. (Given 2 sides)

a)

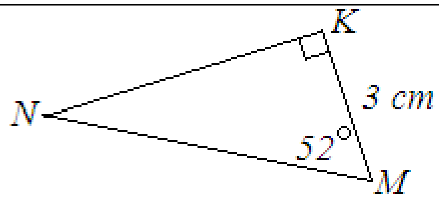
| | |
|---|---|
|   | Solve for AB: |
| Solve for $\angle A$: | Solve for $\angle B$: |

b)

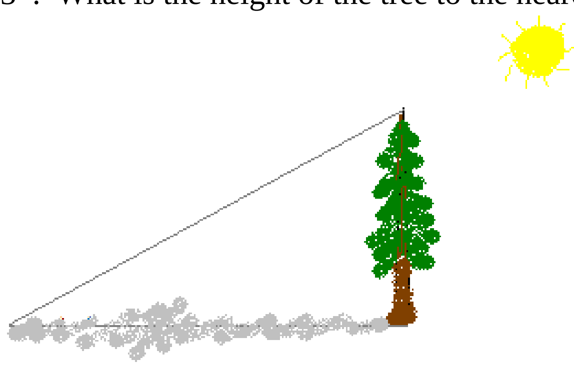
| | |
|---|---|
|   | Solve for BC: |
| Solve for $\angle A$: | Solve for $\angle B$: |

6. Solve $\Delta KMN$. Give the measures to the nearest tenth. (Given an angle and one side)

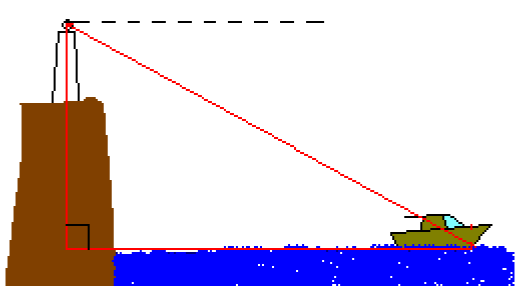
a)

| | |
|---|---|
|   | Solve for KN: |
| Solve for MN: | Solve for $\angle N$: |

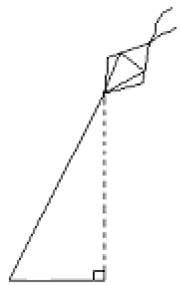
7. A tree casts a shadow that is 10.5 m long when the angle between the sun's rays and the ground is $23°$. What is the height of the tree to the nearest tenth of a metre?



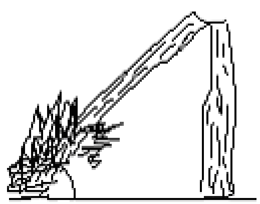
8. A lighthouse sits at the top of a sheer cliff. The top of the lighthouse is 33 m above sea level. The **angle of depression to sight** a small fishing boat at sea is $24°$. How far from the base of the cliff is the fishing boat, to the nearest metre?



9. A kite string is 250 m long. The angle the string makes with the ground is $25°$. How far from the person holding the string is a person standing directly under the kite? Round the answer to the nearest tenth of a metre.



10. A tree is splintered by lightning 2.3 m up its trunk, so that the top part of the tree touches the ground. The angle the top of the tree forms with the ground is $55°$. Approximately, how tall is the tree, to the nearest tenth of a meter?



11. A surveyor stands at a window on the $11^{th}$ floor of an office tower. He uses a clinometers to measure the angles of elevation and depression of the top and the base of a taller building to be to be $20°$ and $39°$ respectively. The surveyor sketches this plan of his measurements. Determine the height of the taller building to the nearest tenth of a metre.