CH 27

Circuits

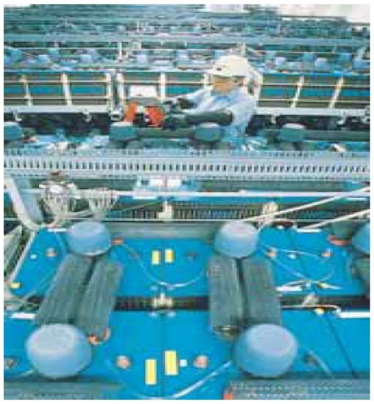
I. "Pumping" Charges:

A. In order to produce a steady flow of charge through a resistor, one needs a "charge pump," a device that—by doing work on the charge carriers—maintains a potential difference between a pair of terminals.

B. Such a device is called an *emf*, or ________________________________.

C. A common *emf* device is the *battery,* used to power a wide variety of machines from wristwatches to submarines. The emf device that most influences our daily lives is the *electric generator,* which, by means of electrical connections (wires) from a generating plant, creates a potential difference in our homes and workplaces.

D. Some other emf devices known are *solar cells, fuel cells.* An emf device does not have to be an instrument—living systems, ranging from electric eels and human beings to plants, have physiological emf devices.



The world's largest battery energy storage plant (dismantled in 1996) connected over 8000 large lead-acid batteries in 8 strings at 1000 V each with a capability of 10 MW of power for 4 hours. Charged up at night, the batteries were then put to use during peak power demands on the electrical system. *(Courtesy Southern California Edison Company)*

II. Work, Energy, and Emf:

A. Simple Circuit:

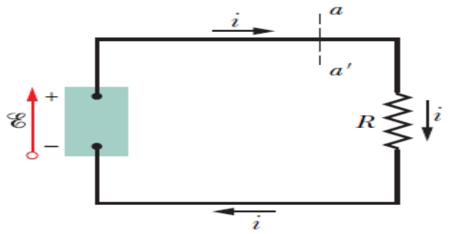


Fig. 27-1 A simple electric circuit, in which a device of emf ℰ does work on the charge carriers and maintains a steady current *i* in a resistor of resistance *R*.

1. In any time interval *dt,* a charge *dq* passes through any cross section of the circuit shown, such as *aa′.* This same amount of charge must enter the emf device at its low-potential end and leave at its high-potential end.

2. The emf device must do an amount of work *dW* on the charge *dq* to force it to move in this way.

3. We define the emf of the emf device in terms of this work:

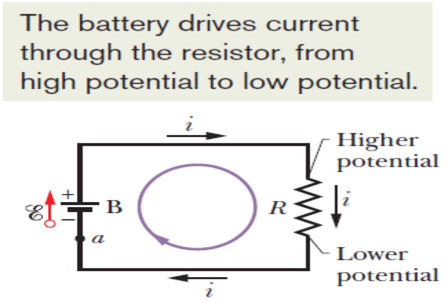
(definition of ℰ).

B. An <u>ideal emf device</u> is one that has no internal resistance to the internal movement of charge from terminal to terminal. The potential difference between the terminals of an ideal emf device is exactly equal to the emf of the device.

C. A <u>real emf device</u>, such as any real battery, has internal resistance to the internal movement of charge. When a real emf device is not connected to a circuit, and thus does not have current through it, the potential difference between its terminals is equal to its emf. However, when that device has current through it, the potential difference between its terminals differs from its emf.

III. Calculating the Current in a Single-Loop Circuit:

A. Circuit:



The battery drives current through the resistor, from high potential to low potential.

Fig. 27-3 A single-loop circuit in which a resistance R is connected across an ideal battery B with emf $\mathscr{E}$. The resulting current i is the same throughout the circuit.

B. The equation $P = i^2R$ tells us that in a time interval dt an amount of energy given by $i^2R\,dt$ will appear in the resistor, as shown in the figure, as thermal energy.

C. During the same interval, a charge $dq = i\,dt$ will have moved through battery B, and the work that the battery will have done on this charge, is

$$dW =$$

D. From the principle of conservation of energy, the work done by the (ideal) battery must equal the thermal energy that appears in the resistor:

E. Therefore, the energy per unit charge transferred to the moving charges is equal to the energy per unit charge transferred from them.

IV. Calculating the Current in a Single-Loop Circuit, Potential Method:

A. Loop Rule (Kirchoff's voltage law): The algebraic sum of the changes in potential encountered in a complete traversal of any loop of a circuit must be ZERO.

B. Circuit:

The battery drives current through the resistor, from high potential to low potential.

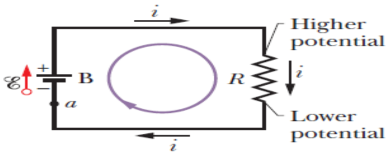


Fig. 27-3 A single-loop circuit in which a resistance R is connected across an ideal battery B with emf $\mathscr{E}$. The resulting current i is the same throughout the circuit.

1. In the figure, let us start at point *a,* whose potential is V_a, and mentally go clockwise around the circuit until we are back at *a,* keeping track of potential changes as we move.

2. Our starting point is at the low-potential terminal of the battery. Since the battery is ideal, the potential difference between its terminals is equal to $\mathscr{E}$.

3. As we go along the top wire to the top end of the resistor, there is no potential change because the wire has negligible resistance.

4. When we pass through the resistor, however, the potential decreases by *iR.*

5. We return to point *a* by moving along the bottom wire. At point *a,* the potential is again V_a. The initial potential, as modified for potential changes along the way, must be equal to our final potential; that is

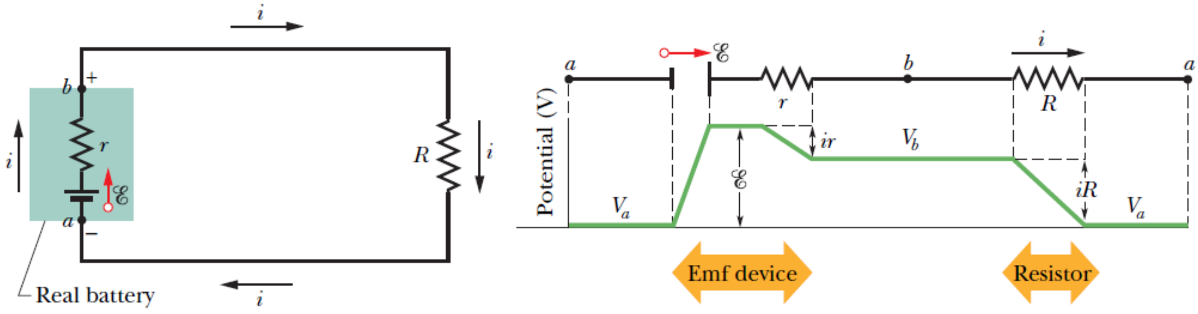
C. For circuits that are more complex than that of the previous figure, two basic rules are usually followed for finding potential differences as we move around a loop:

1. **Resistance Rule: For a move through a resistance in the direction of the current, the change in potential is –iR; in the opposite direction it is +iR.**

2. **EMF Rule: For a move through an ideal emf device in the direction of the emf arrow, the change in potential is + $\mathscr{E}$; in the opposite direction it is – $\mathscr{E}$.**

V. Other Single-Loop Circuits, Internal Resistance:

A. Figure



(a) (b)

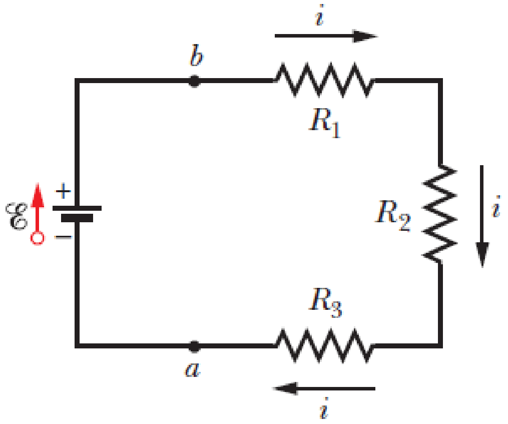
Fig. 27-4 (a) A single-loop circuit containing a real battery having internal resistance r and emf $\mathscr{E}$. (b) The same circuit, now spread out in a line. The potentials encountered in traversing the circuit clockwise from a are also shown. The potential V_a is arbitrarily assigned a value of zero, and other potentials in the circuit are graphed relative to V_a.

1. **The figure above shows a real battery, with internal resistance *r,* wired to an external resistor of resistance *R.* According to the potential rule,**

VI. Other Single-Loop Circuits, Resistances in Series:

A. Kirchoff's Voltage law: The algebraic sum of the changes in potential encountered in a complete traversal of any loop of a circuit must be ZERO.

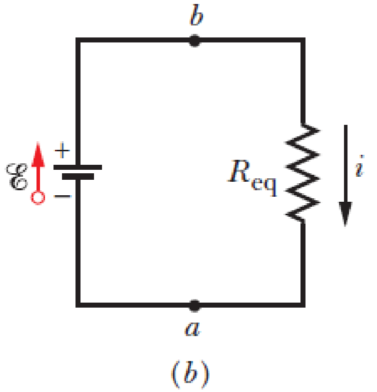
1. In other words: When a potential difference V is applied across resistances connected in series, the resistances have identical currents i. The sum of the potential differences across the resistances is equal to the applied potential difference V.



2.

3. Thus,

B. Resistances connected in series can be replaced with an equivalent resistance R_{eq} that has the same current i and the same *total* potential difference V as the actual resistances.



Series resistors and their equivalent have the same current ("ser-i").

VII. Potential between two points:

A. To find the potential between any two points in a circuit, start at one point and traverse the circuit to the other point, follow any path, and add algebraically the changes in potential you encounter.

B. The circuit:

The internal resistance reduces the potential difference between the terminals.

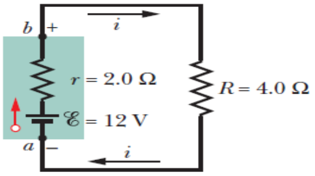


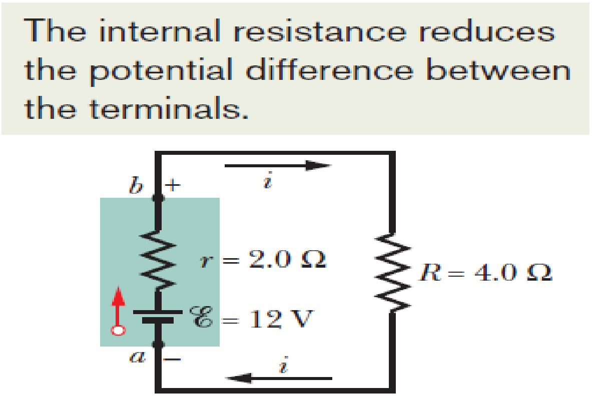
Fig. 27-6 Points *a* and *b*, which are at the terminals of a real battery, differ in potential.

1. **Going clockwise from a:**

2. **Going counterclockwise from a:**

VIII. Potential across a real battery:

A. Circuit:



The internal resistance reduces the potential difference between the terminals.

Fig. 27-6 Points a and b, which are at the terminals of a real battery, differ in potential.

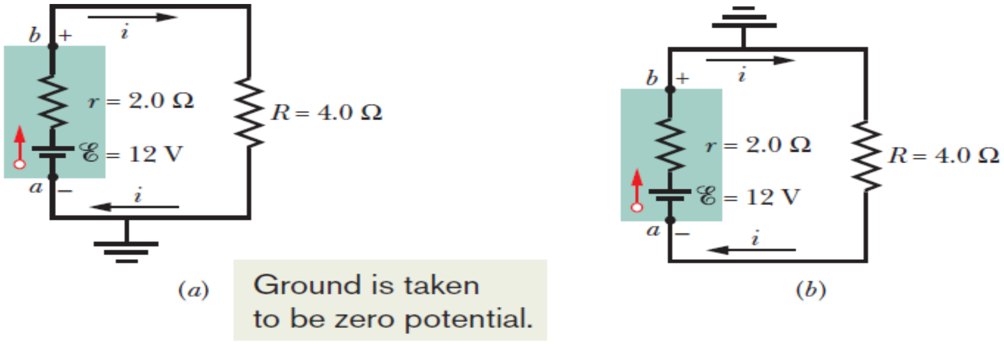
1. If the internal resistance r of the battery in the previous case were zero, V would be equal to the emf $\mathscr{E}$ of the battery—namely, *12 V.*

2. However, since $r = 2.0\Omega$, V is less than $\mathscr{E}$.

3. The result depends on the value of the current through the battery. If the same battery were in a different circuit and had a different current through it, V would have some other value.

IX. Grounding a Circuit:

A. Circuit:



(a) (b)

Ground is taken to be zero potential.

Fig. 27-7 (*a*) Point *a* is directly connected to ground. (*b*) Point *b* is directly connected to ground.

1. This is the same example as in the previous slide, except that battery terminal *a* is grounded in Fig. 27-7*a*. *Grounding a circuit* usually means connecting the circuit to a conducting path to Earth's surface, and such a connection means that the potential is defined to be zero at the grounding point in the circuit.

2. In Fig. 27-7*a*, the potential at *a* is defined to be $V_a = 0$. Therefore, the potential at *b* is $V_b = 8.0$ V.

X. Power, Potential, and Emf:

A. The net rate *P* of energy transfer from the emf device to the charge carriers is given by:

where *V* is the potential across the terminals of the emf device.

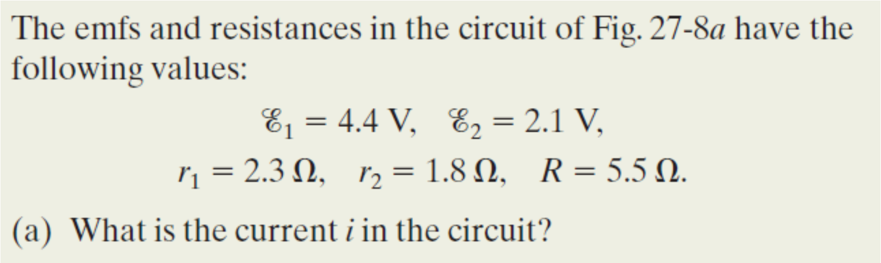
B. But , therefore

1. But P_r is the rate of energy transfer to thermal energy within the emf device:

C. Therefore the term $i\mathcal{E}$ *must* be the rate P_{emf} at which the emf device transfers energy both to the charge carriers and to internal thermal energy.

XI. Example, Single loop circuit with two real batteries:

A.



The emfs and resistances in the circuit of Fig. 27-8*a* have the following values:

$$\mathscr{E}_1 = 4.4\ \text{V},\quad \mathscr{E}_2 = 2.1\ \text{V},$$
$$r_1 = 2.3\ \Omega,\quad r_2 = 1.8\ \Omega,\quad R = 5.5\ \Omega.$$

(a) What is the current i in the circuit?

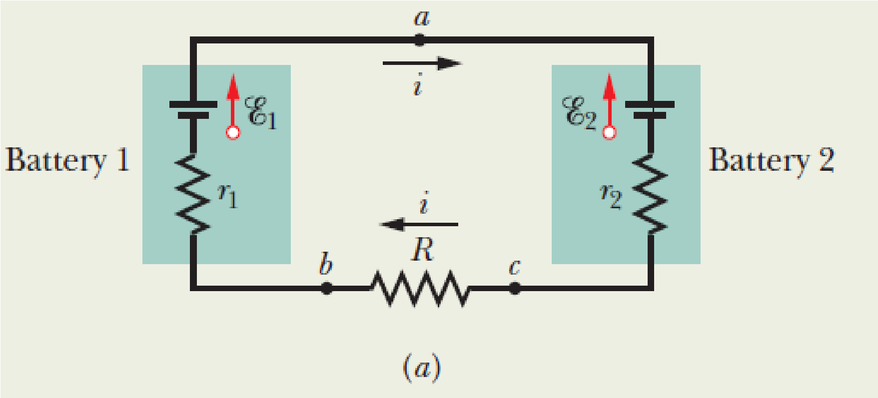
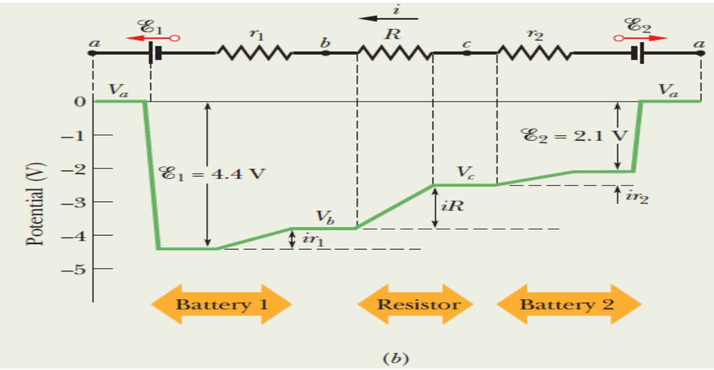


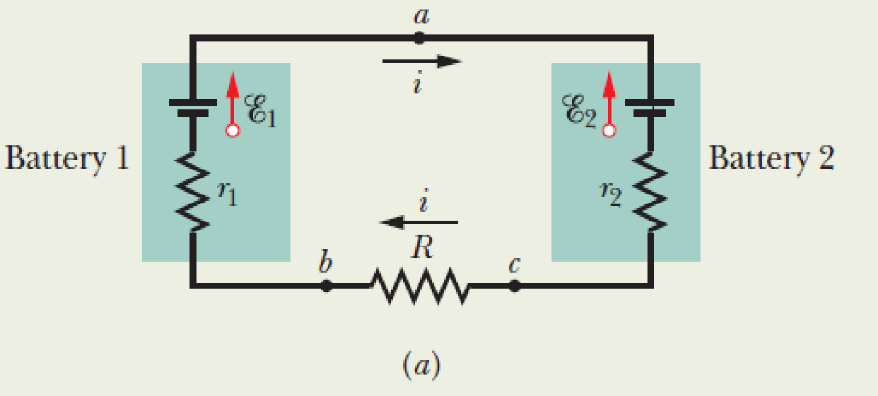
Fig. 27-8

1. **Solution:**

B. Graph:



C. (b) What is the potential difference between the terminals of battery 1 in Fig. 27-8*a*?

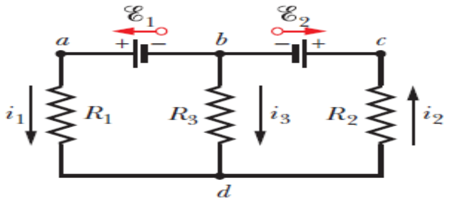


1. **Solution:**

XII. Multi-loop Circuits:

A. <u>Junction/Node Rule</u> [Kirchhoff's current law]: The algebraic sum of the currents entering any junction, or "Node", must be equal to the algebraic sum of the currents leaving that junction/Node

B. Circuit:



The current into the junction must equal the current out (charge is conserved).

Fig. 27-9 A multiloop circuit consisting of three branches: left-hand branch *bad*, right-hand branch *bcd*, and central branch *bd*. The circuit also consists of three loops: left-hand loop *badb*, right-hand loop *bcdb*, and big loop *badcb*.

1. Consider junction *d* in the circuit. Incoming currents i_1 and i_3, and it leaves via outgoing current i_2. Since there is no variation in the charge at the junction, the total incoming current must equal the total outgoing current:

2. This rule is often called Kirchhoff's junction rule (or Kirchhoff's current law).

3. Notice there are multiple Kirchoff's Voltage Law loops possible:

a) *For the left-hand loop,*

b) *For the right-hand loop,*

c) *And for the entire loop(or aka outer loop),*

XIII. Multi-loop Circuits, Resistors in Parallel:

A. When a potential difference V is applied across resistance connected in parallel, the resistances all have that same potential difference V.

B. Resistances connected in parallel can be replaced with an equivalent R_{eq} that has the same potential difference V and the same *<u>total</u>* current i as the actual resistances.

C. Circuit:

Parallel resistors and their equivalent have the same potential difference ("par-V").

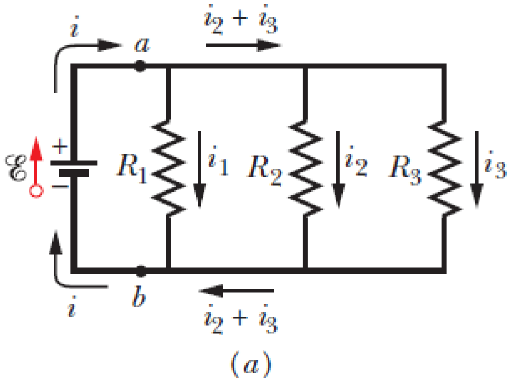


Fig. 27-10 (*a*) Three resistors connected in parallel across points *a* and *b*. (*b*) An equivalent circuit, with the three resistors replaced with their equivalent resistance R_{eq}.

XIV. Multi-loop Circuits:

A. Summary Table

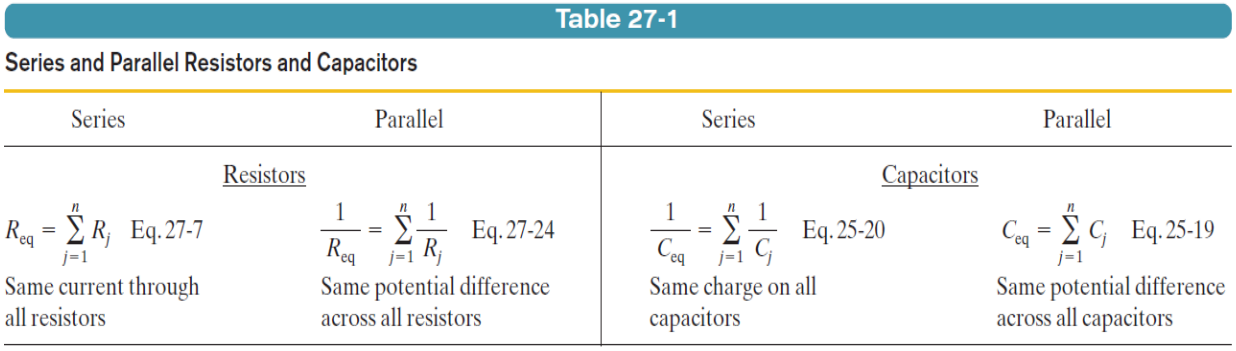


Table 27-1

Series and Parallel Resistors and Capacitors

Series	Parallel	Series	Parallel
Resistors		Capacitors	
$R_{eq} = \sum_{j=1}^{n} R_j$ Eq. 27-7	$\frac{1}{R_{eq}} = \sum_{j=1}^{n} \frac{1}{R_j}$ Eq. 27-24	$\frac{1}{C_{eq}} = \sum_{j=1}^{n} \frac{1}{C_j}$ Eq. 25-20	$C_{eq} = \sum_{j=1}^{n} C_j$ Eq. 25-19
Same current through all resistors	Same potential difference across all resistors	Same charge on all capacitors	Same potential difference across all capacitors

XV. Example, Resistors in Parallel and in Series:

A. Circuit

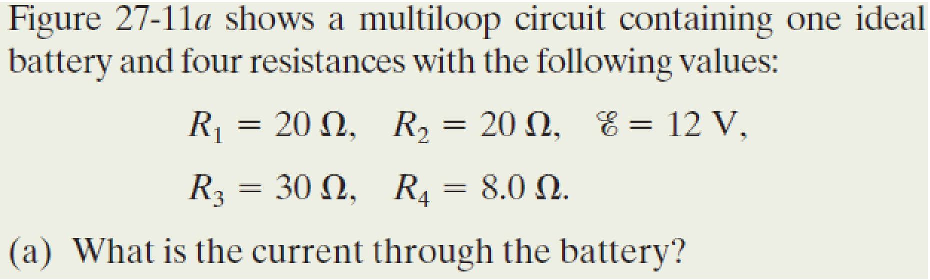
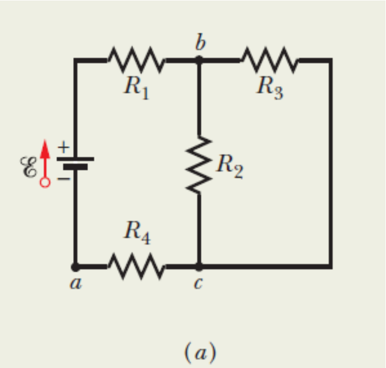


Figure 27-11*a* shows a multiloop circuit containing one ideal battery and four resistances with the following values:

$$R_1 = 20\ \Omega, \quad R_2 = 20\ \Omega, \quad \mathscr{E} = 12\ \text{V},$$

$$R_3 = 30\ \Omega, \quad R_4 = 8.0\ \Omega.$$

(a) What is the current through the battery?



(*a*)

1. Solution in steps:

(b) What is the current i_2 through R_2?

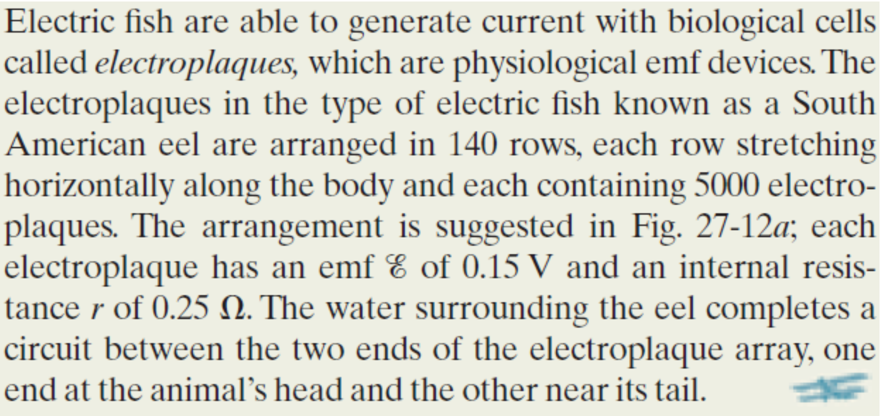
2. **Solution:**

(c) What is the current i_3 through R_3?

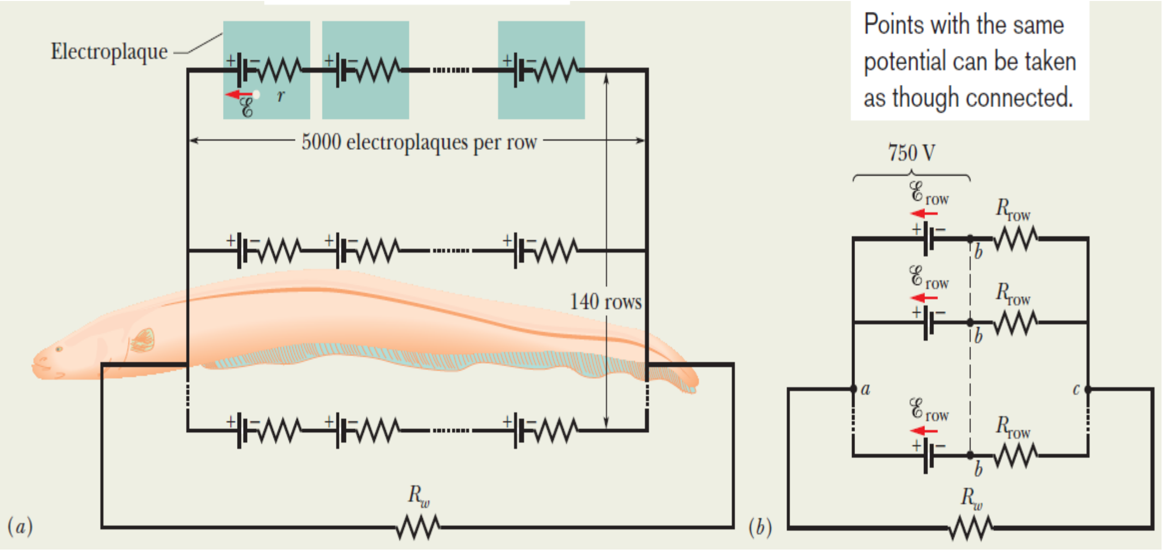
3. **Solution:**

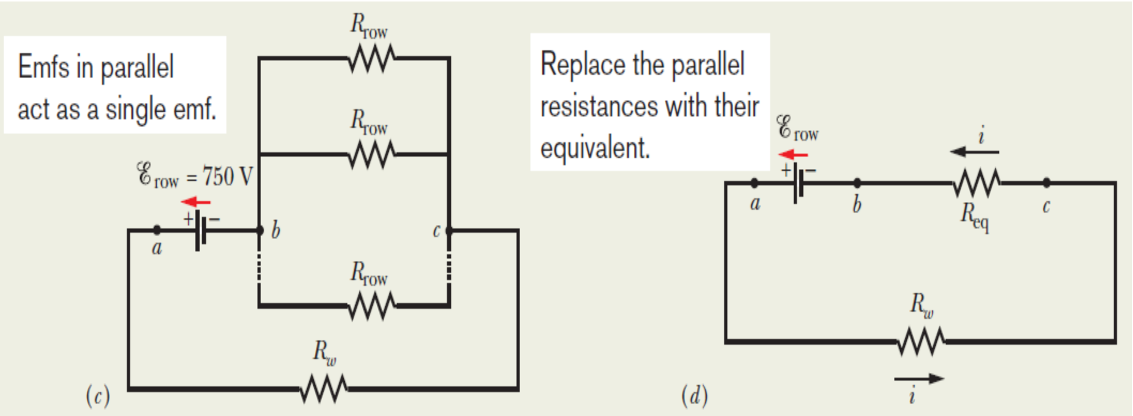
XVI. Example, Real batteries in series and parallel:

A. Electric Fish for example

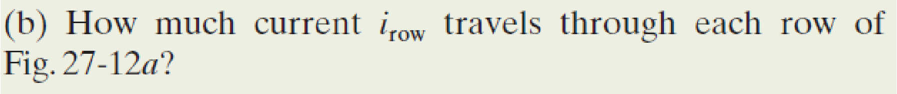


Electric fish are able to generate current with biological cells called *electroplaques*, which are physiological emf devices. The electroplaques in the type of electric fish known as a South American eel are arranged in 140 rows, each row stretching horizontally along the body and each containing 5000 electroplaques. The arrangement is suggested in Fig. 27-12*a*; each electroplaque has an emf $\mathscr{E}$ of 0.15 V and an internal resistance r of 0.25 Ω. The water surrounding the eel completes a circuit between the two ends of the electroplaque array, one end at the animal's head and the other near its tail.





1. **(a) If the water surrounding the eel has resistance $R_w = 800\ \Omega$, how much current can the eel produce in the water?**

2. 

(b) How much current i_{row} travels through each row of Fig. 27-12*a*?

a) Solution:

XVII. Multi-loop circuit and simultaneous loop equations:

A. Example

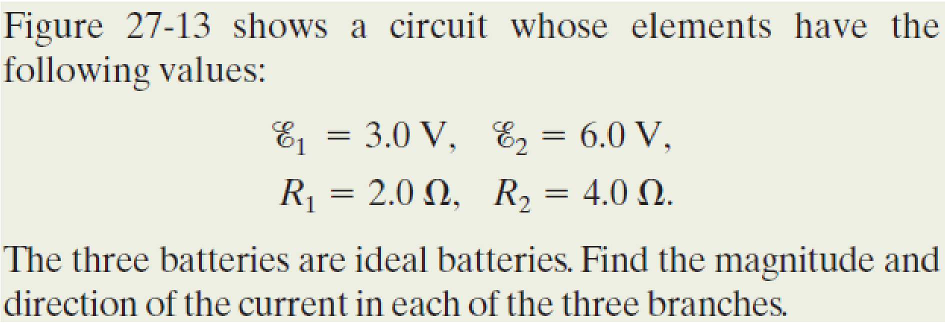


Figure 27-13 shows a circuit whose elements have the following values:

$$\mathscr{E}_1 = 3.0\ \text{V}, \quad \mathscr{E}_2 = 6.0\ \text{V},$$
$$R_1 = 2.0\ \Omega, \quad R_2 = 4.0\ \Omega.$$

The three batteries are ideal batteries. Find the magnitude and direction of the current in each of the three branches.

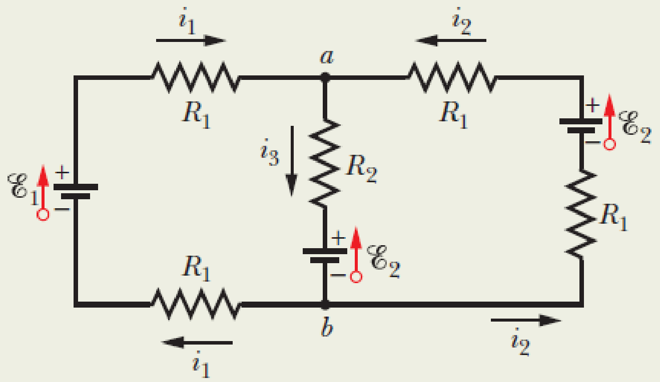


Fig. 27-13 A multiloop circuit with three ideal batteries and five resistances.

1. **Solution:**

2. Solution continued:

XVIII. Ammeter and Voltmeter:

A. An instrument used to measure currents is called an *ammeter.* It is essential that the resistance R_A of the ammeter be very much smaller than other resistances in the circuit.

B. A meter used to measure potential differences is called a *voltmeter.* It is essential that the resistance R_V of a voltmeter be very much larger than the resistance of any circuit element across which the voltmeter is connected.

C. Diagram of how to connect ammeters and voltage meters.

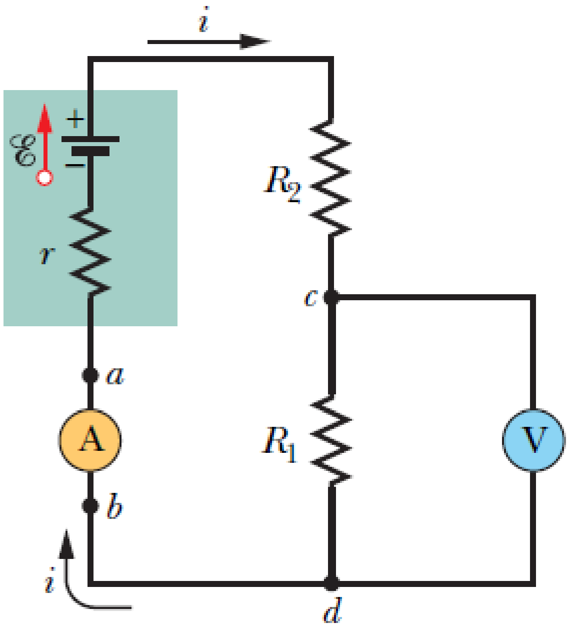


Fig. 27-14 A single-loop circuit, showing how to connect an ammeter (A) and a voltmeter (V).

✦ **1. Ammeters are connected in SERIES so that they measure the current you are interested in.**

✦ **2. Voltage meters are connected in parallel so that they measure the voltage you are interested in.**

XIX. RC Circuits, Charging a Capacitor:

A. Circuit:

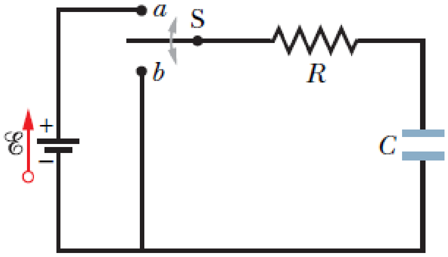


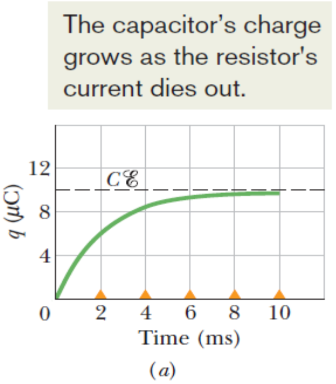
Fig. 27-15 When switch S is closed on *a*, the capacitor is *charged* through the resistor. When the switch is afterward closed on *b*, the capacitor *discharges* through the resistor.

1. **It turns out that:**

2. **We know that:**

3.

4.



5. **Graph**

6. A capacitor that is being charged initially acts like an ordinary connecting wire relative to the charging current. A long time later, it acts like a broken wire.

B. RC Circuits, Time Constant:

1. The product *RC* is called the capacitive time constant of the circuit and is represented with the symbol τ:

2. At time $t = \tau = (RC)$, the charge on the initially uncharged capacitor increases from zero to:

3. During the first time constant τ the charge has increased from zero to 63% of its final value $C\mathscr{E}$.

4. i=

XX. RC Circuits, Discharging a Capacitor:

A. Assume that the capacitor of the figure is fully charged to a potential V_0 equal to the emf of the battery $\mathscr{E}$.

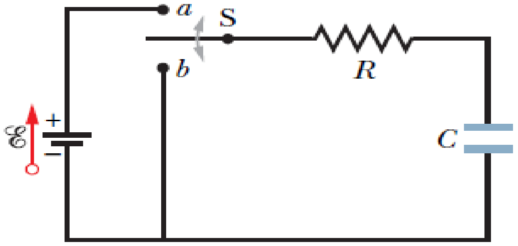


Fig. 27-15 When switch S is closed on *a*, the capacitor is *charged* through the resistor. When the switch is afterward closed on *b*, the capacitor *discharges* through the resistor.

B. At a new time $t = 0$, switch *S* is thrown from *a* to *b* so that the capacitor can discharge through resistance *R*.

1.

2.

3. **Graph**

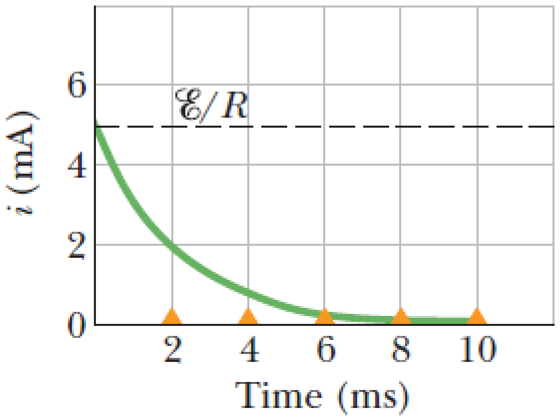
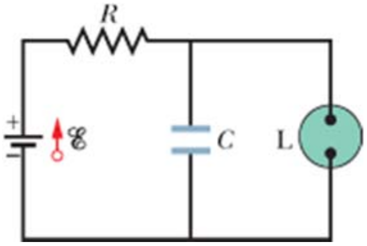


Figure: This shows the decline of the charging current in the circuit. The curves are plotted for *R =2000 Ω, C =1 μF, and* $\mathscr{E}$ *=10 V;* the small triangles represent successive intervals of one time constant τ.

4. **i=**

C. Sample Problem:

1. **Figure below shows the circuit of a flashing lamp, like those attached to barrels at highway construction sites. The fluorescent lamp L (of negligible capacitance) is connected in parallel across the capacitor *C* of an *RC* circuit. There is a current through the lamp only when the potential difference across it reaches the breakdown voltage V_L; then the capacitor discharges completely through the lamp and the lamp flashes briefly. For a lamp with breakdown voltage V_L = 72.0 V, wired to a 95.0 V ideal battery and a 0.150 μF capacitor, what resistance *R* is needed for two flashes per second?**

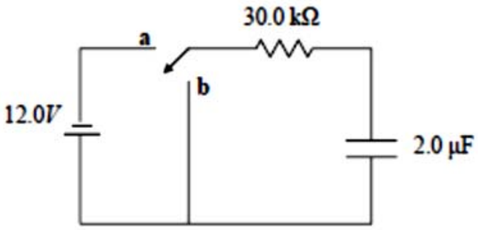


2. **Solution:**

D. Sample Problem#2 &3: both problems deal with the *RC* circuit shown below. The capacitor is initially uncharged.

1. At time *t* = 0, the switch is placed in position a. The time at which the capacitor reaches 70% of its maximum charge is closest to

A. 0.023 s.
B. 1.02 s.
C. 0.051 s.
D. 1.07 s.
E. 0.072 s.



2. Once the capacitor is fully charged, the switch is placed in position b. At *t* = 0.020 s after the switch is placed in position b, the magnitude of the voltage across the RESISTOR is closest to

A. 0.
B. 8.6 V.
C. 6.2 V.
D. 12.0 V.
E. 3.4 V.