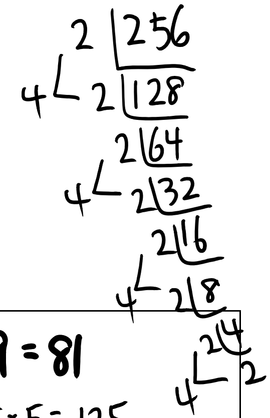
First Name: ________________________ Last Name: ________________________ Block: ______

Ch. 4 – Roots and Powers Notes

Ch. 4.1 – Estimating Roots



$$\sqrt[n]{X}$$

Recap:	**You try:**
$\sqrt{4} = 2$ since $2 \times 2 = 4$	$\sqrt{81} = 9$ since $9 \times 9 = 81$
$\sqrt[3]{27} = 3$ since $3 \times 3 \times 3 = 27$	$\sqrt[3]{125} = 5$ since $5 \times 5 \times 5 = 125$
$\sqrt[4]{81} = 3$ since $3 \times 3 \times 3 \times 3 = 81$	$\sqrt[4]{256} = 4$ since $4 \times 4 \times 4 \times 4 = 256$

What if the radicand is a negative value?
Evaluate each root:

1) $\sqrt{-25}$ = undefined (not real #)

2) $\sqrt[3]{-8} = -2$ since $(-2)(-2)(-2) = -8$

3) $\sqrt[3]{-27} = -3$ " $(-3)(-3)(-3) = -27$

4) $\sqrt[4]{-16}$ = undefined (not real #)
because there is no real # when repeated 4 times to produce a negative #.

What if the radicand is not an integer?
Evaluate each root:

5) $\sqrt{\dfrac{4}{25}} = \dfrac{2}{5}$ since $\left(\dfrac{2}{5}\right)\left(\dfrac{2}{5}\right) = \dfrac{4}{25}$

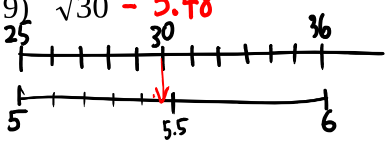
6) $\sqrt[3]{\dfrac{27}{64}} = \dfrac{3}{4}$ since $\left(\dfrac{3}{4}\right)\left(\dfrac{3}{4}\right)\left(\dfrac{3}{4}\right) = \dfrac{27}{64}$

7) $\sqrt{1.44} = \sqrt{\dfrac{144}{100}} = \dfrac{12}{10} = 1.2$

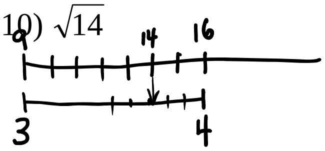
8) $\sqrt[3]{0.027} = \sqrt[3]{\dfrac{27}{1000}} = \dfrac{3}{10} = 0.3$

What if we can not evaluate the root? We can at least estimate to the nearest ~~tenth~~. hundredths.

9) $\sqrt{30} \doteq 5.48$

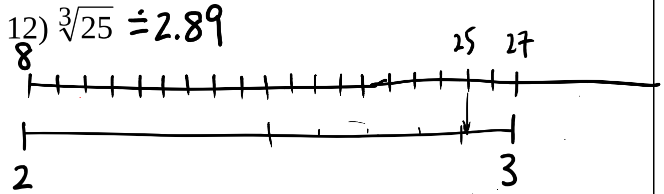


10) $\sqrt{14}$ $\quad \sqrt{14} \doteq 3.71$



11) $\sqrt{3.4} \doteq 1.83$

12) $\sqrt[3]{25} \doteq 2.89$



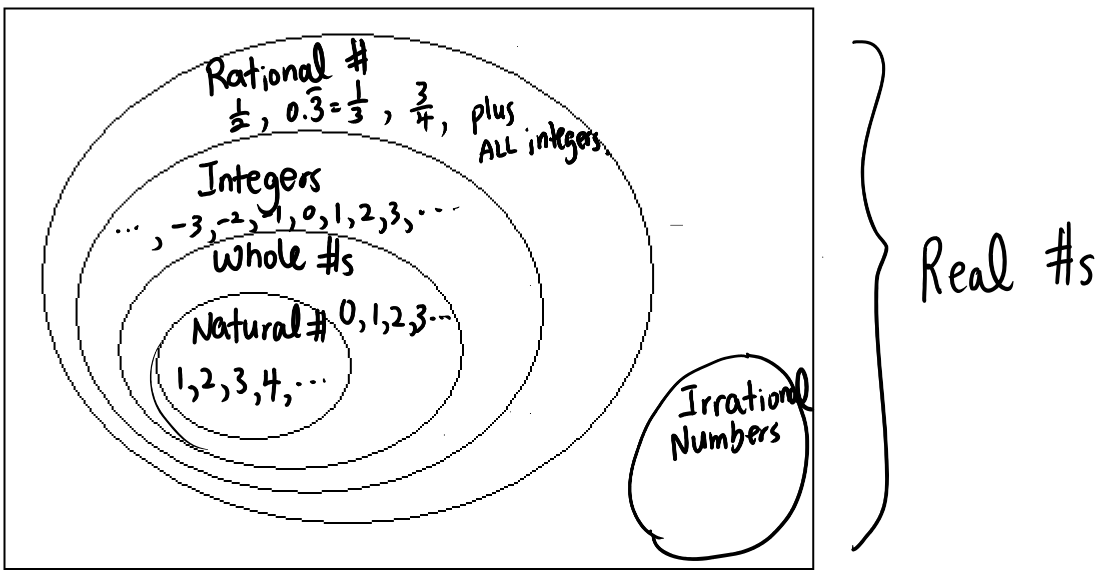
13) $\sqrt[4]{17}$

Ch. 4.1 HW: p. 206 #2 – 5

Ch. 4.2 – Irrational Numbers

Definitions:

- **Natural Numbers**: "counting numbers" (eg: 1, 2, 3, 4, 5, 6, 7, …)
- **Whole Numbers**: 0 + Natural Numbers (eg: 0, 1, 2, 3, 4, 5, 6, 7, ….)
- **Integers**: positive or negative of whole numbers (eg: …, -3, -2, -1, 0, 1, 2, 3, …)
- **Rational Numbers**: Numbers that can be written in the form, $\frac{a}{b}$, where a and b are integers, $b \neq 0$. Ex: $\frac{3}{4}$, $\frac{1}{2}$, $5 = \frac{5}{1}$, $-7.1 = \frac{-71}{10}$, $0.\bar{3} = \frac{1}{3}$, $1.\bar{4} = \frac{13}{9}$
- **Irrational Numbers**: Numbers that can NOT be written in the form, $\frac{a}{b}$, where a and b are integers, $b \neq 0$. Ex: π, $\sqrt{2}$, if the decimals do <u>not</u> terminate & do <u>not</u> repeat.
- **Real Numbers**: All numbers that include natural numbers, whole numbers, integers, rational numbers and irrational numbers.



Examples:

1) Identify whether each number is rational or irrational:

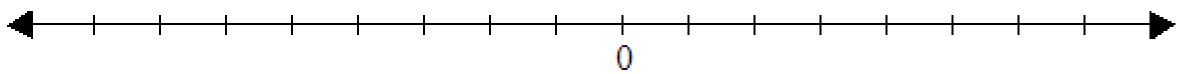
Real Numbers	Rational or Irrational?
$\sqrt{100} = 10 = \frac{10}{1}$	Rational #
$\sqrt[3]{\frac{-8}{27}} = \frac{-2}{3}$	Rational #.
$1.\bar{4} = \frac{13}{9}$	Rational #. Note: Repeating decimals can be converted to fraction.
$\sqrt{5} = 2.23606\ldots$	Irrational # Note: Decimals do <u>not</u> terminate nor repeat.
$\sqrt[3]{10}$	Irrational #
$2.1358912 = \frac{21358912}{10,000,000}$	Rational #

2) True or False

Statements	**True or False?**	**If false, give a counter example**
All natural numbers are integers	True	
All Integers are natural numbers	False	-2 is an integer but not a natural #.
All irrational numbers are roots	False	π is an irrational # but a root.
All roots are irrational numbers	False	$\sqrt{4} = 2$ is a root but a rational #.
All whole numbers are rational numbers	True	

3) Number each number on a number line:

$\sqrt{30}$, $\sqrt[3]{-8}$, $\frac{2}{3}$, $\sqrt[4]{256}$, π, $\sqrt[5]{32}$



4) Write a number that is:

a) a rational number but not a natural number

b) a irrational number but not a root

c) a whole number but not a natural number

Ch. 4.2 HW: p. 211 #3 – 17

4.3 – Mixed and Entire Radicals

Pattern Recognition:

$\sqrt{36}$	$= 6$	Notice: $36 = 4 \cdot 9$	$\sqrt{4} \cdot \sqrt{9}$	$= 2 \cdot 3 = 6$
$\sqrt{400}$	$= 20$	$400 = 100 \cdot 4$	$\sqrt{100} \cdot \sqrt{4}$	$= 10 \cdot 2 = 20$
$\sqrt[3]{216}$	$= 6$	$216 = 8 \cdot 27$	$\sqrt[3]{8} \cdot \sqrt[3]{27}$	$= 2 \cdot 3 = 6$
$\sqrt[3]{512}$	$= 8$	$512 = 8 \cdot 8 \cdot 8$	$\cancel{\sqrt[3]{8} \cdot \sqrt[3]{64}}$ $\sqrt[3]{8} \cdot \sqrt[3]{8} \cdot \sqrt[3]{8}$	$= 2 \cdot 2 \cdot 2 = 8$
$\sqrt[4]{1296}$			$\sqrt[4]{16} \cdot \sqrt[4]{81}$	

What do you notice?

Multiplication Property of Radicals:

$$\boxed{\sqrt[n]{a \cdot b} = \sqrt[n]{a} \cdot \sqrt[n]{b}}$$

where n is a natural number, and a and b are real numbers.

Examples: Simplify Radicals Using Prime Factorization

1) Simplify each radical (Write each entire radical to mixed radical).

a) $\sqrt{63} = \sqrt{3 \cdot 3 \cdot 7} = \sqrt{9 \cdot 7} = \sqrt{9} \cdot \sqrt{7} = 3\sqrt{7}$

b) $\sqrt[3]{108} = \sqrt[3]{3 \cdot 3 \cdot 3 \cdot 2 \cdot 2} = \sqrt[3]{27 \cdot 4} = \sqrt[3]{27} \cdot \sqrt[3]{4} = 3\sqrt[3]{4}$

c) $\sqrt[4]{128} = 2\sqrt[4]{8}$

2|128
2|64
2|32
2|16
2|8
2|4
2

2|30
3|15
5

2) Write each radical in simplest form, if possible

a) $\sqrt{30}$ can not be simplified. $= \sqrt{2 \cdot 3 \cdot 5}$	d) $\sqrt[3]{32} = 2\sqrt[3]{4}$
b) $\sqrt{80} = \sqrt{2 \cdot 2 \cdot 2 \cdot 2 \cdot 5} = \sqrt{4} \cdot \sqrt{4} \cdot \sqrt{5} = 2 \cdot 2 \cdot \sqrt{5} = 4\sqrt{5}$	e) $\sqrt[3]{108} = 3\sqrt[3]{4}$
c) $\sqrt{180} = 6\sqrt{5}$	f) $\sqrt[4]{48} = 2\sqrt[4]{3}$

3) Writing mixed radicals as Entire Radicals

a) $4\sqrt{3} = \sqrt{4 \cdot 4} \cdot \sqrt{3} = \sqrt{16} \cdot \sqrt{3} = \sqrt{16 \cdot 3} = \sqrt{48}$	d) $2\sqrt[3]{4} = \sqrt[3]{2 \cdot 2 \cdot 2} \cdot \sqrt[3]{4} = \sqrt[3]{32}$
b) $7\sqrt{3} = \sqrt{7 \cdot 7} \cdot \sqrt{3} = \sqrt{7 \cdot 7 \cdot 3} = \sqrt{147}$	e) $3\sqrt[3]{5} = \sqrt[3]{3 \cdot 3 \cdot 3} \cdot \sqrt[3]{5} = \sqrt[3]{135}$
c) $-3\sqrt{5} = -\sqrt{3 \cdot 3} \cdot \sqrt{5} = -\sqrt{3 \cdot 3 \cdot 5} = -\sqrt{45}$	f) $2\sqrt[4]{3} = \sqrt[4]{2 \cdot 2 \cdot 2 \cdot 2} \cdot \sqrt[4]{3} = \sqrt[4]{48}$

Ch. 4.3 HW: p. 218 #3 – 7, #9, #10 – 12 odd letters, #13 – 18, 20

4.4 – Fraction Exponents and Radicals

Radicals as Powers:

$\sqrt[n]{x}$ } Radical vs $x^{1/n}$ } power

We can represent radicals as a power:

Radicals	**Written as a Power**
$\sqrt{3}$	$= 3^{\frac{1}{2}}$
$\sqrt[3]{5}$	$= 5^{\frac{1}{3}}$
$\sqrt[4]{8}$	$= 8^{\frac{1}{4}}$
…	
$\sqrt[n]{x}$	$= x^{\frac{1}{n}}$

Recap: Exponent Laws

Power	**Meaning**	~~**Simplified Form**~~ Shortcut
$(5^2)^3$	$(5^2)(5^2)(5^2) = (5\cdot5)(5\cdot5)(5\cdot5) = 5^6$	$5^{2\times3} = 5^6$
$(2^3)^4$	$(2^3)(2^3)(2^3)(2^3) = 2^{12}$	$2^{3\times4} = 2^{12}$
$(7^5)^2$	$(7^5)(7^5) = 7^{10}$	$7^{5\times2} = 7^{10}$
…	…	…
$(x^m)^n$ $= x^{m\cdot n}$		

Extend Exponent Laws:

Powers in Radical and Exponent Form	**Simplified Form**
$(\sqrt{5})^3$	$= (5^{\frac{1}{2}})^3 = 5^{\frac{1}{2}\times3} = 5^{\frac{3}{2}}$
$(\sqrt[3]{5})^5$	$= (5^{\frac{1}{3}})^5 = 5^{\frac{1}{3}\times5} = 5^{\frac{5}{3}}$
$(\sqrt[4]{6})^3$	$= (6^{\frac{1}{4}})^3 = 6^{\frac{1}{4}\times3} = 6^{\frac{3}{4}}$
In general $(\sqrt[n]{x})^m = \left(x^{\frac{1}{n}}\right)^m = x^{\frac{m}{n}}$	

$\sqrt[4]{3^3}$	$= \left(3^3\right)^{\frac{1}{4}} = 3^{3\times\frac{1}{4}} = 3^{\frac{3}{4}}$
$\sqrt[3]{6^2}$	$= \left(6^2\right)^{\frac{1}{3}} = 6^{2\times\frac{1}{3}} = 6^{\frac{2}{3}}$
$\sqrt{2^5}$	$= \left(2^5\right)^{\frac{1}{2}} = 2^{5\times\frac{1}{2}} = 2^{\frac{5}{2}}$
In general, $\sqrt[n]{x^m} = \left(x^m\right)^{\frac{1}{n}} = x^{m\times\frac{1}{n}}$	$= x^{\frac{m}{n}}$

Examples: Rewriting Powers in Radical and Exponent Form

1) Write $30^{\frac{3}{4}}$ in radical form in 2 ways. $30^{3/4} = \left(30^3\right)^{1/4} = \sqrt[4]{30^3}$ OR $30^{3/4} = \left(30^{\frac{1}{4}}\right)^3 = \left(\sqrt[4]{30}\right)^3$

2) Write $\sqrt{4^5}$ as a power.

3) Write $\sqrt[3]{5^2}$ as a power.

4) Write $\left(\sqrt[4]{70}\right)^3$ as a power.

Examples: Evaluating Powers of the Form $a^{\frac{1}{n}}$ without a calculator

5) $1000^{\frac{1}{3}} = \sqrt[3]{1000} = 10$

6) $0.25^{\frac{1}{2}} = \sqrt{0.25} = \sqrt{\frac{25}{100}} = \frac{5}{10} = \frac{1}{2} = 0.5$

7) $(-8)^{\frac{1}{3}} = \sqrt[3]{-8} = -2$

8) $\left(\frac{16}{81}\right)^{\frac{1}{4}} = \sqrt[4]{\frac{16}{81}} = \frac{2}{3}$

Examples: Evaluating Powers with Rational Exponents and Rational Bases

9) Evaluate $0.01^{\frac{3}{2}}$ without a calculator. $= \left(\sqrt{0.01}\right)^3 = \left(\sqrt{\frac{1}{100}}\right)^3 = \left(\frac{1}{10}\right)^3 = \frac{1}{1000} = 0.001$

10) Evaluate $(-27)^{\frac{4}{3}}$ without a calculator. $= \left(\sqrt[3]{-27}\right)^4 = (-3)^4 = \underbrace{(-3)(-3)}_{9}\times\underbrace{(-3)(-3)}_{9} = 81$

11) Evaluate $81^{\frac{3}{4}}$ without a calculator.

12) Evaluate $0.75^{1.2}$ using a calculator

Ch. 4.4 HW: p. 227 #3 – 7 odd letters, 8 – 14, 17, 19

4.5 – Negative Exponents and Reciprocals

Recap: Exponent Laws:

	Use repeated multiplication to simplify	What's the shortcut?
$\frac{3^6}{3^2}$	$=\frac{\cancel{3}\cdot\cancel{3}\cdot 3\cdot 3\cdot 3\cdot 3}{\cancel{3}\cdot\cancel{3}}=\frac{3^4}{1}=3^4$	$3^{6-2}=3^4$
$\frac{x^5}{x^3}$	$=\frac{\cancel{x}\cdot\cancel{x}\cdot\cancel{x}\cdot x\cdot x}{\cancel{x}\cdot\cancel{x}\cdot\cancel{x}}=\frac{x^2}{1}=x^2$	$x^{5-3}=x^2$
$\frac{x^4}{x^3}$	$=\frac{\cancel{x}\cdot\cancel{x}\cdot\cancel{x}\cdot x}{\cancel{x}\cdot\cancel{x}\cdot\cancel{x}}=\frac{x}{1}=x$	$x^{4-3}=x^1=x$

In general:

$$\boxed{\frac{x^m}{x^n}=x^{m-n}}$$

What does it mean to have a negative exponent?

	Use repeated multiplication to simplify	Use the Exponent Law:	Conclusion
$\frac{x^2}{x^3}$	$=\frac{\cancel{x}\cdot\cancel{x}}{\cancel{x}\cdot\cancel{x}\cdot x}=\frac{1}{x}$	$x^{2-3}=x^{-1}$	$x^{-1}=\frac{1}{x}$
$\frac{x^3}{x^5}$	$=\frac{\cancel{x}\cdot\cancel{x}\cdot\cancel{x}}{\cancel{x}\cdot\cancel{x}\cdot\cancel{x}\cdot x\cdot x}=\frac{1}{x^2}$	$x^{3-5}=x^{-2}$	$x^{-2}=\frac{1}{x^2}$
$\frac{x^4}{x^7}$	$=\frac{\cancel{x}\cdot\cancel{x}\cdot\cancel{x}\cdot\cancel{x}}{\cancel{x}\cdot\cancel{x}\cdot\cancel{x}\cdot\cancel{x}\cdot x\cdot x\cdot x}=\frac{1}{x^3}$	$x^{4-7}=x^{-3}$	$x^{-3}=\frac{1}{x^3}$

In general,

$$\boxed{x^{-n}=\left(\frac{1}{x}\right)^n=\frac{1}{x^n}}$$

Examples:

1) Evaluate each power:

a) $3^{-2}=\frac{1}{3^2}=\frac{1}{9}$

b) $4^{-1}=\frac{1}{4^1}=\frac{1}{4}$

c) $2^{-3}=\frac{1}{2^3}=\frac{1}{8}$

d) $\frac{1}{3^{-2}}=\frac{1}{\frac{1}{3^2}}=1\div\frac{1}{3^2}=1\times\frac{3^2}{1}=3^2=9$

can you take a shortcut?

e) $\frac{1}{2^{-3}} = \frac{1}{\frac{1}{2^3}} = 1 \div \frac{1}{2^3} = 1 \times \frac{2^3}{1} = 2^3 = 8$

What if the base is a rational number of the form, $\frac{x}{y}$?

	Use repeated multiplication to simplify	Use the Exponent Law:	Conclusion
$\frac{\left(\frac{1}{2}\right)^2}{\left(\frac{1}{2}\right)^3}$	$= \frac{\cancel{\left(\frac{1}{2}\right)}\cancel{\left(\frac{1}{2}\right)}}{\cancel{\left(\frac{1}{2}\right)}\cancel{\left(\frac{1}{2}\right)}\left(\frac{1}{2}\right)} = \frac{1}{\frac{1}{2}} = 1 \div \frac{1}{2} = 1 \times \frac{2}{1} = \frac{2}{1}$	$\left(\frac{1}{2}\right)^{2-3} = \left(\frac{1}{2}\right)^{-1}$	$\left(\frac{1}{2}\right)^{-1} = \left(\frac{2}{1}\right)^{1}$
$\frac{\left(\frac{2}{3}\right)^2}{\left(\frac{2}{3}\right)^4}$	$\frac{\cancel{\left(\frac{2}{3}\right)}\cancel{\left(\frac{2}{3}\right)}}{\cancel{\left(\frac{2}{3}\right)}\cancel{\left(\frac{2}{3}\right)}\left(\frac{2}{3}\right)\left(\frac{2}{3}\right)} = \frac{1}{\left(\frac{2}{3}\right)^2} = \frac{1}{\frac{4}{9}} = \frac{9}{4} = \left(\frac{3}{2}\right)^2$	$\left(\frac{2}{3}\right)^{2-4} = \left(\frac{2}{3}\right)^{-2}$	$\left(\frac{2}{3}\right)^{-2} = \left(\frac{3}{2}\right)^{2}$

In general,

$$\boxed{\left(\frac{x}{y}\right)^{-n} = \left(\frac{y}{x}\right)^{n}}$$

Examples:

2) Evaluate each power:

a) $\left(-\frac{3}{4}\right)^{-2} = \left(-\frac{4}{3}\right)^{2} = \left(-\frac{4}{3}\right)\left(-\frac{4}{3}\right) = \frac{16}{9}$

b) $\left(\frac{3}{10}\right)^{-1} = \left(\frac{10}{3}\right)^{1} = \frac{10}{3}$

c) $\left(\frac{5}{2}\right)^{-3} = \left(\frac{2}{5}\right)^{3} = \left(\frac{2}{5}\right)\left(\frac{2}{5}\right)\left(\frac{2}{5}\right) = \frac{8}{125}$

Note: 8^{-2} can be thought of as $\left(\frac{8}{1}\right)^{-2}$

Examples: Evaluating Powers with Negative Rational exponents

3) Evaluate each power without using a calculator

a) $8^{-\frac{2}{3}}$

b) $\left(\frac{9}{16}\right)^{-\frac{3}{2}}$

c) $\left(\frac{25}{36}\right)^{-\frac{1}{2}}$

d) $16^{-\frac{5}{4}}$

Ch. 4.5 HW: p. 233 #1 – 10, 13, 19, 20

4.6 – Applying the Exponent Laws (Part I)

Recap: Exponent Laws

Product of Powers

$3^2 \cdot 3^5$	$(3\cdot3)\cdot(3\cdot3\cdot3\cdot3\cdot3) = 3^7$	$3^{2+5} = 3^7$
$(-2)^3(-2)^2$	$(-2)(-2)(-2)\cdot(-2)(-2) = (-2)^5$	$(-2)^{3+2} = (-2)^5$
In general $a^m \cdot a^n =$		a^{m+n}

Quotient of Powers

$\frac{3^6}{3^4}$	$= \frac{\cancel{3}\cdot\cancel{3}\cdot\cancel{3}\cdot\cancel{3}\cdot3\cdot3}{\cancel{3}\cdot\cancel{3}\cdot\cancel{3}\cdot\cancel{3}} = 3^2$	$3^{6-4} = 3^2$
$2^5 \div 2^4 = \frac{2^5}{2^4}$	$= \frac{\cancel{2}\cdot\cancel{2}\cdot\cancel{2}\cdot\cancel{2}\cdot2}{\cancel{2}\cdot\cancel{2}\cdot\cancel{2}\cdot\cancel{2}} = \frac{2}{1} = 2$	$2^{5-4} = 2^1 = 2$
In general $a^m \div a^n =$		a^{m-n}

Power of a Power

$(3^2)^4$	$= (3^2)(3^2)(3^2)(3^2) = 3^8$	$3^{2\times4} = 3^8$
In general $(a^m)^n$		$a^{m\cdot n}$

Power of a Product

$(2\cdot3)^4$	$= (2\cdot3)(2\cdot3)(2\cdot3)(2\cdot3) = 2^4\cdot3^4$	
In general $(ab)^m$	$= a^m\cdot b^m$	

Power of a Quotient

$\left(\frac{3}{4}\right)^2$	$= \left(\frac{3}{4}\right)\left(\frac{3}{4}\right) = \frac{3\cdot3}{4\cdot4} = \frac{3^2}{4^2}$	
$\left(\frac{2}{5}\right)^3$	$= \left(\frac{2}{5}\right)\left(\frac{2}{5}\right)\left(\frac{2}{5}\right) = \frac{2\cdot2\cdot2}{5\cdot5\cdot5} = \frac{2^3}{5^3}$	
In general $\left(\frac{a}{b}\right)^m$	$= \frac{a^m}{b^m}$	

Zero Exponent

	Write in repeated multiplication and simplify	Use Exponent Law to simplify	Conclusion
$\frac{3^2}{3^2}$	$=\frac{1\cancel{3}\cdot\cancel{3}}{1\cancel{3}\cdot\cancel{3}}=\frac{1}{1}=1$	$3^{2-2}=3^0$	$3^0=1$
$\frac{(-2)^3}{(-2)^3}$	$=\frac{1\cancel{(-2)}\cancel{(-2)}\cancel{(-2)}}{1\cancel{(-2)}\cancel{(-2)}\cancel{(-2)}}=\frac{1}{1}=1$	$(-2)^{3-3}=(-2)^0$	$(-2)^0=1$
In general x^0			

Applying the Exponent Laws

1. Simplify.
   a) $a^2 \cdot a^6 = a^{2+6} = a^8$
   b) $a^1 \cdot a^{10} = a^{1+10} = a^{11}$
   c) $a^3 \cdot a^{-4} = a^{3+(-4)} = a^{-1}$ OR $a^{-1} = \frac{1}{a}$
   d) $x^3 \cdot x^2 \cdot x^{-1} = x^{3+2+(-1)} = x^4$
   e) $\frac{x^3}{x^5} = x^{3-5} = x^{-2}$ OR $x^{-2} = \frac{1}{x^2}$
   f) $\frac{x^3}{x^{-5}} = x^{3-(-5)} = x^{3+5} = x^8$
   g) $\frac{b^{-2}}{b^{-3}} = b^{-2-(-3)} = b^{-2+3} = b^1 = b$
   h) $a^{-1} \div a^3 = a^{-1-3} = a^{-4}$ OR $a^{-4} = \frac{1}{a^4}$

2. Simplify.
   a) $(n^2)^3 = n^6$
   b) $(a^3)^4 = a^{12}$
   c) $(x^{-1})^4 = x^{-4}$ OR $\frac{1}{x^4}$
   d) $(a^{-3})^{-2} = a^6$
   e) $(x \cdot y)^2 = x^2y^2$
   f) $(ab)^4 = a^4b^4$
   g) $(a^2b)^3 = a^6b^3$
   h) $(x^3y^{-1})^{-2} = x^{-6}y^2$ OR $\frac{y^2}{x^6}$

not necessary

3. Write as a single power.

a) $0.2^3 \cdot 0.2^5 = 0.2^8$

b) $\dfrac{3.5^2}{3.5^{-5}} = 3.5^{2-(-5)} = 3.5^{2+5} = 3.5^7$

c) $\left[\left(-\frac{2}{3}\right)^{-3}\right]^2 \left[\left(-\frac{2}{3}\right)^2\right]^4 = \left(-\frac{2}{3}\right)^{-6} \cdot \left(-\frac{2}{3}\right)^{8} = \left(-\frac{2}{3}\right)^{2}$

d) $\dfrac{(1.5^2)(1.5)^3}{1.5^{-3}} = \dfrac{1.5^2 \cdot 1.5^3}{1.5^{-3}} = \dfrac{1.5^5}{1.5^{-3}} = 1.5^{5-(-3)}$
$= 1.5^{5+3} = 1.5^8$

e) $\left(\dfrac{7^{\frac{2}{3}}}{7^{\frac{1}{3}} \cdot 7^{\frac{5}{3}}}\right)^6 = \left(\dfrac{7^{\frac{2}{3}}}{7^{\frac{6}{3}}}\right)^6 = \left(7^{-\frac{4}{3}}\right)^6 = 7^{-8}$ or $\dfrac{1}{7^8}$

4. Simplify.

a) $(x^3y^2)(x^3y^{-2}) = x^6y^0 = x^6 \cdot 1 = x^6$

b) $(m^4n^{-3})(m^2n^1) = \boxed{m^6n^{-2}} = m^6 \cdot \dfrac{1}{n^2} = \boxed{\dfrac{m^6}{n^2}}$

c) $(3a^2)^3(4a^3)^0 = 3^3a^6 \cdot 1 = 3^3a^6 = \boxed{27a^6}$

d) $\dfrac{(a^3b)^2}{a^{-3}} = \dfrac{a^6b^2}{a^{-3}} = \boxed{a^9 \cdot b^2}$

e) $\dfrac{6a^4b^3}{3a^2b} = 2a^2b^2$

$3-(-2) = 3+2 = 5$

f) $\left(\dfrac{a^3b^2}{a^{-2}}\right)^2 = (a^5 \cdot b^2)^2 = a^{10} \cdot b^4$

g) $\dfrac{-10x^{-2}y^3}{5x^3y^{-1}} = -2x^{-5}y^4$ or $\dfrac{-2}{1} \cdot \dfrac{1}{x^5} \cdot \dfrac{y^4}{1} = \dfrac{-2y^4}{x^5}$

$-2-3 = -5$

$3-(-1) = 4$

h) $\dfrac{3x^4y^{-2}}{6xy^{-3}} = \dfrac{1}{2} \cdot \dfrac{x^3}{1} \cdot \dfrac{y^1}{1} = \dfrac{x^3y}{2}$

$-2-(-3) = -2+3 = 1$

$-10-4=-14$

i) $\dfrac{(3a^5)^{-2}}{a^4} = \dfrac{3^{-2}a^{-10}}{a^4} = \dfrac{3^{-2}}{1}\cdot\dfrac{a^{-10}}{a^4} = \dfrac{1}{3^2}\cdot\dfrac{a^{-14}}{1} = \dfrac{1}{3^2}\cdot\dfrac{1}{a^{14}} = \boxed{\dfrac{1}{9a^{14}}}$

j) $\left(\dfrac{a^{-2}b}{b^{-2}}\right)^{-4} = \left(a^{-2}\cdot b^{3}\right)^{-4} = \boxed{a^8b^{-12}}$ or $= \boxed{\dfrac{a^8}{b^{12}}}$

$1-(-2)=1+2=3$

skip.

k) $\left(\dfrac{25x^a}{125x^3}\right)^3$

$\underbrace{2\cdot2}_{4}\cdot\underbrace{2\cdot2}_{4}$ $\underbrace{3\cdot3}_{9}\cdot\underbrace{3\cdot3}_{9}$

5. Simplify

a) $\left(x^{-1}y^{-2}\right)^{-4} = x^4\cdot y^8$

b) $\left(\dfrac{3}{2}m^{-2}n^{-3}\right)^{-4} = \left(\dfrac{3}{2}\right)^{-4}\left(m^{-2}\right)^{-4}\cdot\left(n^{-3}\right)^{-4} = \left(\dfrac{2}{3}\right)^4\cdot m^8\cdot n^{12} = \dfrac{16m^8n^{12}}{81}$

6. Simplify and then evaluate

a) $\left(\dfrac{3}{4}\right)^{3/4}\cdot\left(\dfrac{3}{4}\right)^{5/4} = \left(\dfrac{3}{4}\right)^{\frac{3}{4}+\frac{5}{4}} = \left(\dfrac{3}{4}\right)^{\frac{8}{4}} = \underbrace{\left(\dfrac{3}{4}\right)^{2}}_{\text{simplified}} = \left(\dfrac{3}{4}\right)\left(\dfrac{3}{4}\right) = \underbrace{\dfrac{9}{16}}_{\text{evaluated.}}$

b) $\dfrac{(0.027)^{\frac{5}{3}}}{(0.027)^{\frac{4}{3}}} = (0.027)^{\frac{5}{3}-\frac{4}{3}} = \boxed{0.027^{\frac{1}{3}}} = \sqrt[3]{0.027} = \sqrt[3]{\dfrac{27}{1000}} = \dfrac{3}{10} = \boxed{0.3}$

simplified evaluated.

Examples: Solving Problems Using the Exponent Laws

7. A cone with equal height and radius has volume 1234 cm^3. What is the height of the cone to the nearest tenth of a centimeter?

Ch. 4. 6 (part I) HW: p. 241 # 3 – 11 odd letters, 12 – 15

4.6 – Applying the Exponent Laws (Part II)

Examples: Simplifying Expression with Rational Exponents

1. Simplify

c) $\left(x^{\frac{3}{2}}y^2\right)\left(x^{\frac{1}{2}}y^{-1}\right) = x^2y$

d) $\dfrac{12x^{-5}y^{\frac{5}{2}}}{3x^{\frac{1}{2}}y^{-\frac{1}{2}}} = 4x^{-5-\frac{1}{2}}y^{\frac{5}{2}-\left(-\frac{1}{2}\right)} = 4x^{\frac{-11}{2}}y^{3} = \dfrac{4y^3}{x^{11/2}}$

$-\frac{5}{1}-\frac{1}{2} = \frac{-10}{2} - \frac{1}{2} = \frac{-11}{2}$

$\frac{5}{2}-\left(\frac{-1}{2}\right) = \frac{5}{2}+\frac{1}{2} = \frac{6}{2}$
$= 3$

e) $\left(\dfrac{50x^2y^4}{2x^4y^7}\right)^{\frac{1}{2}} = \left(25x^{-2}y^{-3}\right)^{\frac{1}{2}} = 25^{\frac{1}{2}}x^{-1}y^{\frac{-3}{2}} = 5 \cdot \frac{1}{x} \cdot \frac{1}{y^{3/2}} = \dfrac{5}{xy^{3/2}}$

$2-4 = -2$

$4-7 = -3$

$\frac{-2}{1} \times \frac{1}{2} = \frac{-2}{2} = -1$

Examples: Simplifying Expressions with Radicals

2. Simplify

a) $\sqrt{x^3} \div \sqrt[3]{x^4} = x^{3/2} \div x^{\frac{4}{3}} = x^{\frac{3}{2}-\frac{4}{3}} = x^{\frac{9}{6}-\frac{8}{6}} = x^{\frac{1}{6}} = \sqrt[6]{x}$

b) $\dfrac{\sqrt[3]{x}\cdot\sqrt[2]{x}}{x} = \dfrac{x^{\frac{1}{3}}\cdot x^{\frac{1}{2}}}{x^1} = \dfrac{x^{\frac{5}{6}}}{x^1} = x^{-1/6} = \dfrac{1}{x^{1/6}} = \dfrac{1}{\sqrt[6]{x}}$

c) $\dfrac{\left(\sqrt[3]{a}\right)^2\left(\sqrt[3]{a}\right)}{\sqrt[3]{a^4}} = \dfrac{\left(a^{\frac{1}{3}}\right)^2\cdot a^{\frac{1}{3}}}{\left(a^4\right)^{1/3}} = \dfrac{a^{\frac{2}{3}}\cdot a^{\frac{1}{3}}}{a^{4/3}} = \dfrac{a^{\frac{3}{3}}}{a^{4/3}} = a^{\frac{-1}{3}} = \dfrac{1}{a^{1/3}} = \dfrac{1}{\sqrt[3]{a}}$

$\frac{1}{3}+\frac{1}{2} = \frac{2}{6}+\frac{3}{6} = \frac{5}{6}$

$\frac{5}{6}-\frac{6}{6} = \frac{-1}{6}$

$\frac{3}{3}-\frac{4}{3} = \frac{-1}{3}$

Examples: Simplifying Algebraic Expressions to Simplest Radical Form

3. Convert to simplest radical form.

a) $\left(-c^2\right)^{-\frac{1}{3}} = \dfrac{1}{\left(-c^2\right)^{1/3}} = \dfrac{1}{\sqrt[3]{-c^2}}$ or $\dfrac{1}{-\sqrt[3]{c^2}}$ since $\dfrac{1}{\sqrt[3]{-c^2}} = \dfrac{1}{\sqrt[3]{(-1)(-1)(-1)c^2}}$

-1 can come out!

b) $\dfrac{\left(a^3\right)^{\frac{1}{2}}}{\left(a^2\right)^{-\frac{1}{3}}} = \dfrac{a^{\frac{3}{2}}}{a^{-2/3}} = a^{\frac{3}{2}-\left(-\frac{2}{3}\right)} = a^{\frac{3}{2}+\frac{2}{3}} = a^{\frac{9}{6}+\frac{4}{6}} = a^{\frac{13}{6}} = \sqrt[6]{a^{13}}$

Ch. 4. 6 (part II) HW: p. 242 # 16, 17, 19, 21

Ch. 4 Review – Roots and Powers

First Name: ________________________ Last Name: _________________________ Block: _____

Part I: Calculators are not allowed for this test.

1. Which of the following statements are true?

I. $\sqrt{0.81} = 0.9$ since $0.9 \times 0.9 = 0.81$ II. $\sqrt{0.12} = 0.6$ since $0.6 \times 0.6 = 0.12$ III. $\sqrt[4]{16} = 2$ since $2 \times 2 \times 2 \times 2 = 16$ IV. $\sqrt[5]{100000} = 10$ since $10^5 = 100000$	a) I and II only b) I and III only c) I, III and IV only d) I, II, III, and IV

2. Which of the following statements are true?

I. $\sqrt{-25} = -5$ II. $\sqrt[3]{-8} = -2$ III. $\sqrt{\frac{64}{49}} = \frac{8}{7}$ IV. $\sqrt[4]{-10000}$ = undefined	a) I and IV only b) II and III only c) II, III, and IV only d) I, II, III and IV

3. Which of the following statements are true?

I. $\sqrt{-4}$ is a real number II. $\sqrt{25}$ is a rational number III. $\sqrt{25}$ is an integer IV. $\sqrt{25}$ is a whole number	e) II and IV only f) III and IV only g) II, III, and IV only h) I, II, III and IV

4. Which of the following statements are true?

I. All natural numbers are integers II. All integers are rational numbers III. All rational numbers have terminating decimals IV. All irrational numbers are radicals	i) I and II only j) I, II, and III only k) I, III, and IV only l) I, II, III and IV

5. Estimate $\sqrt{27}$

a) 4.9	b) 5.2	c) 5.4	d) 5.7

6. Express $3\sqrt{2}$ as an entire radical

a) $\sqrt{6}$	b) $\sqrt{18}$	c) $\sqrt{11}$	d) $\sqrt{36}$

7. Order the numbers from the smallest value to the largest value.

I. $-2\sqrt{2}$ II. $\sqrt{25}$ III. $2\sqrt{3}$ IV. $-3\sqrt{3}$

a) I, IV, II, III	b) I, IV, III, II	c) IV, I, II, III	d) IV, I, III, II

8. Evaluate $\left(\frac{1}{2}\right)^{-3}$

a) 6	b) 8	c) $-\frac{1}{8}$	d) $-\frac{1}{6}$

9. Evaluate $(-27)^{\frac{2}{3}}$

a) 9	b) -9	c) 3	d) 6

10. Write $\left(\sqrt[4]{70}\right)^3$ as a power

a) $3^{\frac{1}{4}}$	b) 70^3	c) $70^{\frac{4}{3}}$	d) $70^{\frac{3}{4}}$

11. Evaluate 9^{-1}

a) -9	b) $\frac{1}{3}$	c) $\frac{1}{9}$	d) 0

12. Evaluate $\left(\frac{2}{3}\right)^{-2}$

a) $\left(\frac{3}{2}\right)^2 = \frac{9}{4}$	b) $\left(\frac{3}{2}\right)^2 = \frac{6}{4}$	c) $-\left(\frac{2}{3}\right)^2 = -\frac{4}{9}$	d) $-\left(\frac{2}{3}\right)^2 = -\frac{4}{6}$

13. Evaluate $\frac{1}{2^{-3}}$

a) 8	b) -8	c) $\frac{1}{-6}$	d) $\frac{1}{8}$

Written Response of Part I: No calculator is allowed.

14. Estimate $\sqrt{52.3}$

15. Evaluate $\sqrt[3]{0.027}$

16. Simplify $\sqrt[3]{128}$

17. Write as an entire radical: $2\sqrt{7}$

18. Evaluate $\left(\frac{4}{25}\right)^{\frac{3}{2}}$

19. Evaluate $\left(\sqrt{\frac{1}{9^{-1}}}\right)$

Part II: You may use your calculator. Show **ALL** your work for full marks

20. Simplify
$(2x^3)^3 \cdot 3x^4$

a) $24x^{36}$	b) $24x^{13}$	c) $6x^{13}$	d) $18x^{36}$

21. Simplify $\sqrt[3]{1080}$

a) $2\sqrt[3]{135}$	b) $3\sqrt[3]{40}$	c) $6\sqrt[3]{5}$	d) $6\sqrt[3]{30}$

22. Simplify $\left(3a^2\right)^3\left(4a^2\right)^0$

a) $9a^6$	b) $36a^8$	c) $27a^6$	d) $108a^8$

23. Which expression is equivalent to $\left(-a^2\right)^{-\frac{1}{5}}$

a) $\sqrt[5]{-a^2}$	b) $\dfrac{1}{\sqrt{a^5}}$	c) $\dfrac{1}{\sqrt[5]{a^2}}$	d) $\dfrac{1}{\sqrt[5]{-a^2}}$

24. Simplify $\sqrt{x^3} \div \sqrt[3]{x^5}$

a) $\dfrac{1}{\sqrt[6]{x}}$	b) $\sqrt[5]{x^8}$	c) $\dfrac{1}{\sqrt{x^2}}$	d) $\sqrt[3]{x^8}$

25. Simplify $\dfrac{-9a^4b^2}{3a^2b^3}$

a) $\dfrac{3a^2}{b}$	b) $\dfrac{-3a^2}{b}$	c) $\dfrac{-a}{3b}$	d) $-3a^2b$

26. Simplify $\dfrac{5x^{-2}y^2}{10x^2y}$

a) $\dfrac{y^2}{2x^5}$	b) $\dfrac{y}{2x}$	c) $2y$	d) $\dfrac{y}{2x^4}$

Written Response:

27. Simplify.

$$\left(\frac{a^{-3}b}{b^3c}\right)^2$$

28. Write $\left(\sqrt[3]{-1.5}\right)^2$ as a power

29. Write $5^{\frac{2}{3}}$ as a radical in two ways

30. Evaluate $\sqrt{\dfrac{1}{4^{-2}}}$

31. Simplify $\dfrac{\left(a^{2}b^{-2}\right)^{-1}}{\left(2a^{-2}b\right)^{2}}$

32. Simplify $\left(\dfrac{x^{2}}{y}\right)^{-2}$

33. Simplify $\left(ab^{2}\right)^{3}\left(-2a^{-1}b\right)^{2}$