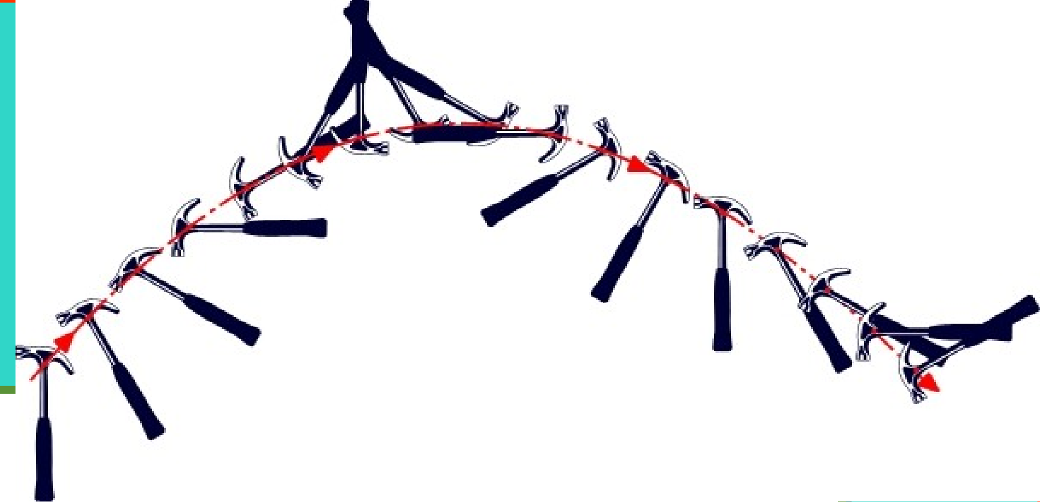
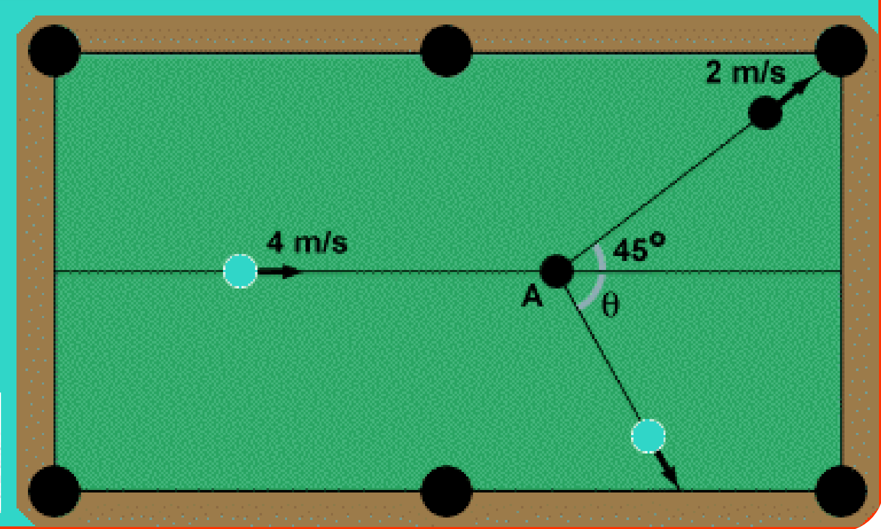


Chapter 9

Center of Mass and Linear Momentum

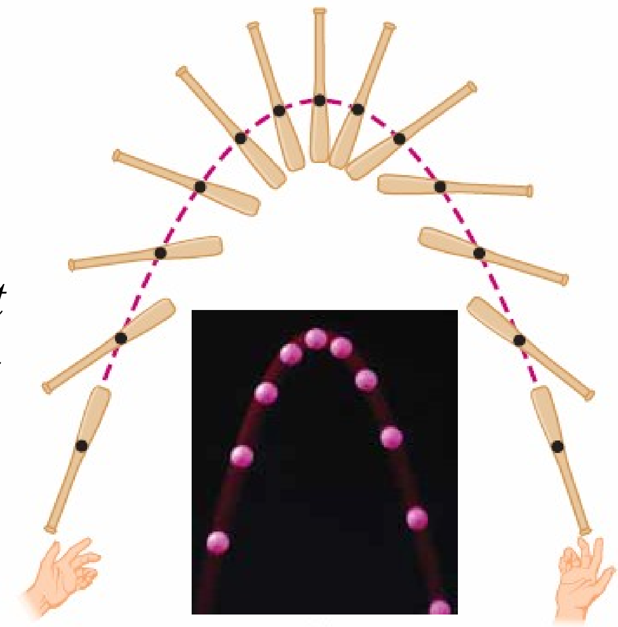


İZMİR KÂTİP ÇELEBİ ÜNİVERSİTESİ

9 CENTER OF MASS AND LINEAR MOMENTUM 201

since mass is conserved → velocities are changed

- The motion of rotating objects can be complicated (imagine flipping a baseball bat into the air).
  - •But there is a special point on the object for which the motion is simple.
  - •*The center of mass (black dot) of a baseball bat flipped into the air follows a parabolic path, but all other points of the bat follow more complicated curved paths.*



- The **center of mass (COM)** of a system of particles is the point that moves as though
1. All of the system's mass were concentrated there.
2. All external forces were applied there.

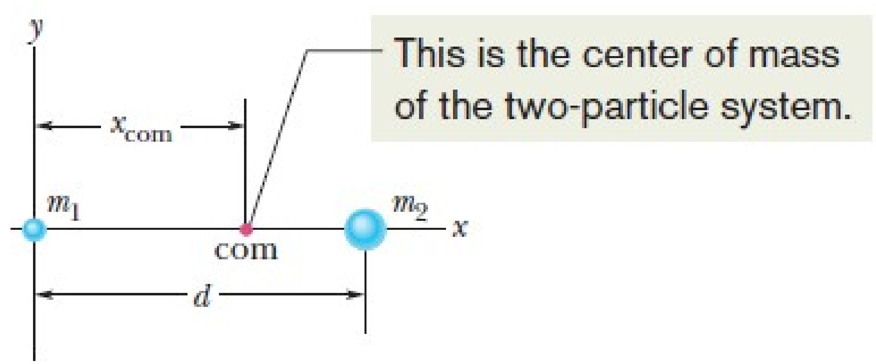
object → a point like particle

F_{ext} → at COM

For two particles separated by a distance *d*, where the origin is chosen at the position of particle 1:

$x_1=0$ & $x_2=d$

$$x_{com} = \frac{m_2}{m_1+m_2}d.$$

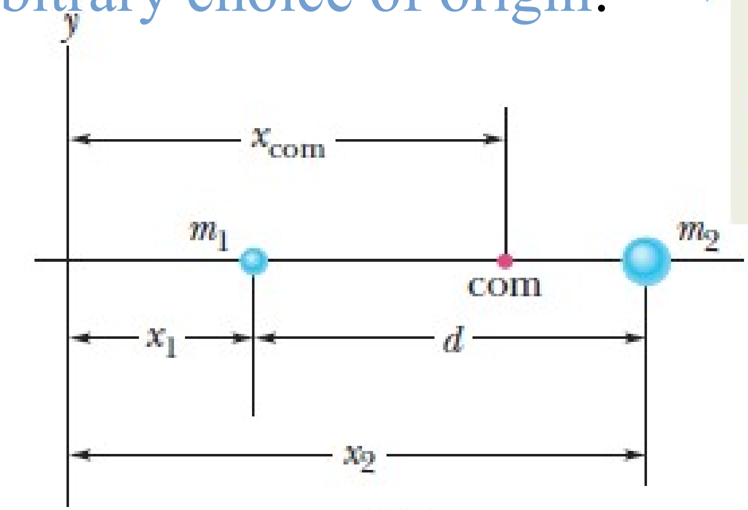


- For two particles, for an arbitrary choice of origin:

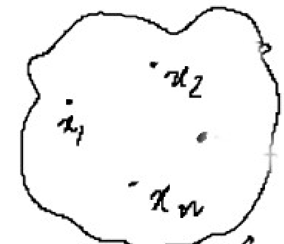
$$x_{com} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}.$$

$$\boxed{x_{com} = \frac{m_1 x_1 + m_2 x_2}{M},}$$

$$M = m_1 + m_2.$$



Shifting the axis does not change the relative position of the com.



system of particles

- For many particles, we can generalize the equation. Consider a situation in which n particles are strung out along the x axis.
  - Let the mass of the particles are m_1, m_2,….m_n, and let them be located at x_1, x_2,…x_n respectively.
- Then if the total mass is $M = m_1+m_2+...+m_n$, then the location of the center of mass, x_{com}, is

$$x_{com} = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3 + \cdots + m_n x_n}{M}$$

$$\boxed{= \frac{1}{M}\sum_{i=1}^{n} m_i x_i.}$$

X_{COM}, Y_{COM}, Z_{COM}

9.2 The Center of Mass: System of Particles

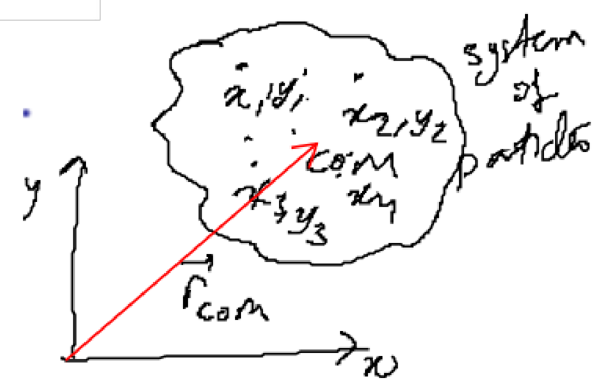
- The center of mass is in the same location regardless of the coordinate system used.
    - It is a property of the particles, not the coordinates.
- In three dimensions, we find the center of mass along each axis separately:

$$x_{\text{com}} = \frac{1}{M}\sum_{i=1}^{n} m_i x_i, \qquad y_{\text{com}} = \frac{1}{M}\sum_{i=1}^{n} m_i y_i, \qquad z_{\text{com}} = \frac{1}{M}\sum_{i=1}^{n} m_i z_i.$$

The position of the center of mass can be expressed in vector notation as:

$$\vec{r}_{\text{com}} = x_{\text{com}}\hat{\text{i}} + y_{\text{com}}\hat{\text{j}} + z_{\text{com}}\hat{\text{k}}.$$

$$\boxed{\vec{r}_{\text{com}} = \frac{1}{M}\sum_{i=1}^{n} m_i \vec{r}_i,}$$



9.2 The Center of Mass: Solid Body

- For solid bodies, we take the limit of an infinite sum of infinitely small particles → integration! (m → Δm → dm)

$$x_{com} = \frac{1}{M}\int x\,dm, \qquad y_{com} = \frac{1}{M}\int y\,dm, \qquad z_{com} = \frac{1}{M}\int z\,dm$$

where M is the mass of the object. If the object has uniform density, ρ, defined as: $\rho = \frac{dm}{dV} = \frac{M}{V}$

$$\left\{ dm = \underbrace{\rho}_{\frac{M}{V}}\, dV \right.$$

$\Sigma \to \int$

$m \to dm$

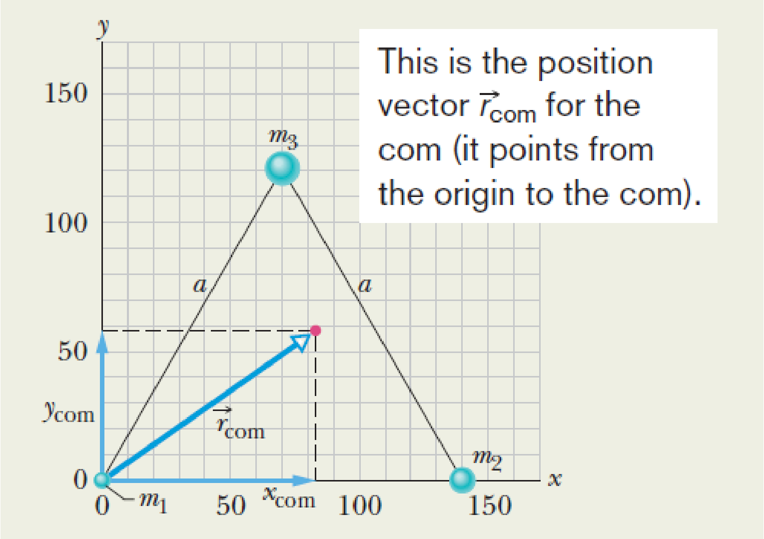
- We limit ourselves to objects of uniform density, ρ, for the sake of simplicity, then

$$x_{com} = \frac{1}{V}\int x\,dV, \qquad y_{com} = \frac{1}{V}\int y\,dV, \qquad z_{com} = \frac{1}{V}\int z\,dV.$$

where V is the volume of the object. You can bypass one or more of these integrals if the object has symmetry.

Sample problem: COM of 3 particles

Three particles of masses $m_1 = 1.2$ kg, $m_2 = 2.5$ kg, and $m_3 = 3.4$ kg form an equilateral triangle of edge length $a = 140$ cm. Where is the center of mass of this system?



We are given the following data:

Particle	Mass (kg)	x (cm)	y (cm)
1	1.2	0	0
2	2.5	140	0
3	3.4	70	120

The total mass M of the system is 7.1 kg.

The coordinates of the center of mass are therefore:

$$x_{com} = \frac{1}{M}\sum_{i=1}^{3} m_i x_i = \frac{m_1x_1 + m_2x_2 + m_3x_3}{M}$$

$$= \frac{(1.2\text{ kg})(0) + (2.5\text{ kg})(140\text{ cm}) + (3.4\text{ kg})(70\text{ cm})}{7.1\text{ kg}}$$

$$= 83\text{ cm} \qquad \text{(Answer)}$$

and $$y_{com} = \frac{1}{M}\sum_{i=1}^{3} m_i y_i = \frac{m_1y_1 + m_2y_2 + m_3y_3}{M}$$

$$= \frac{(1.2\text{ kg})(0) + (2.5\text{ kg})(0) + (3.4\text{ kg})(120\text{ cm})}{7.1\text{ kg}}$$

$$= 58\text{ cm}. \qquad \text{(Answer)}$$

Note that the $z_{com} = 0$.

$\vec{r}_{com} = 83\text{ cm}\,\hat{i} + 58\text{ cm}\,\hat{j}$

- Center of mass motion continues unaffected by forces internal to a system (collisions between billiard balls).

The vector equation that governs the motion of the center of mass of such a system of particles is:

$$\vec{F}_{net} = M\vec{a}_{com} \qquad \text{(system of particles).}$$

$$F_{net,x} = Ma_{com,x} \qquad F_{net,y} = Ma_{com,y} \qquad F_{net,z} = Ma_{com,z}.$$

Note that:

1. F_{net} is the net force of all external forces that act on the system. Forces on one part of the system from another part of the system (internal forces) are not included.
2. M is the total mass of the system. M remains ***constant***, and the system is said to be ***closed***.
3. **$\mathbf{a}_{com}$** is the acceleration of the center of mass of the system.

The internal forces of the explosion cannot change the path of the com.

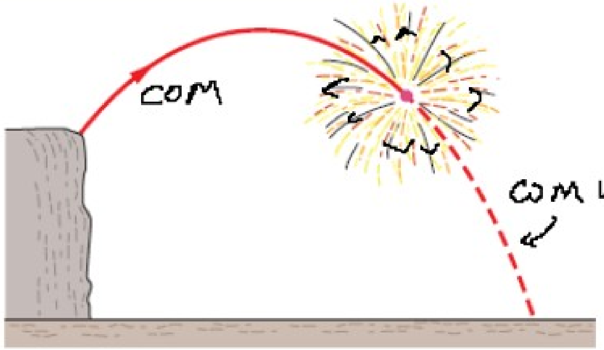


Fig. 9-5 A fireworks rocket explodes in flight. In the absence of air drag, the center of mass of the fragments would continue to follow the original parabolic path, until fragments began to hit the ground.

9.3 Newton's 2nd Law for a System of Particles: Proof of Final Result

- For a system of *n* particles, $M\vec{r}_{com} = m_1\vec{r}_1 + m_2\vec{r}_2 + m_3\vec{r}_3 + \cdots + m_n\vec{r}_n,$

where M is the total mass, and $\mathbf{r}_i$ are the position vectors of the masses m_i.

- Differentiating, $M\vec{v}_{com} = m_1\vec{v}_1 + m_2\vec{v}_2 + m_3\vec{v}_3 + \cdots + m_n\vec{v}_n.$

where the **v** vectors are velocity vectors.

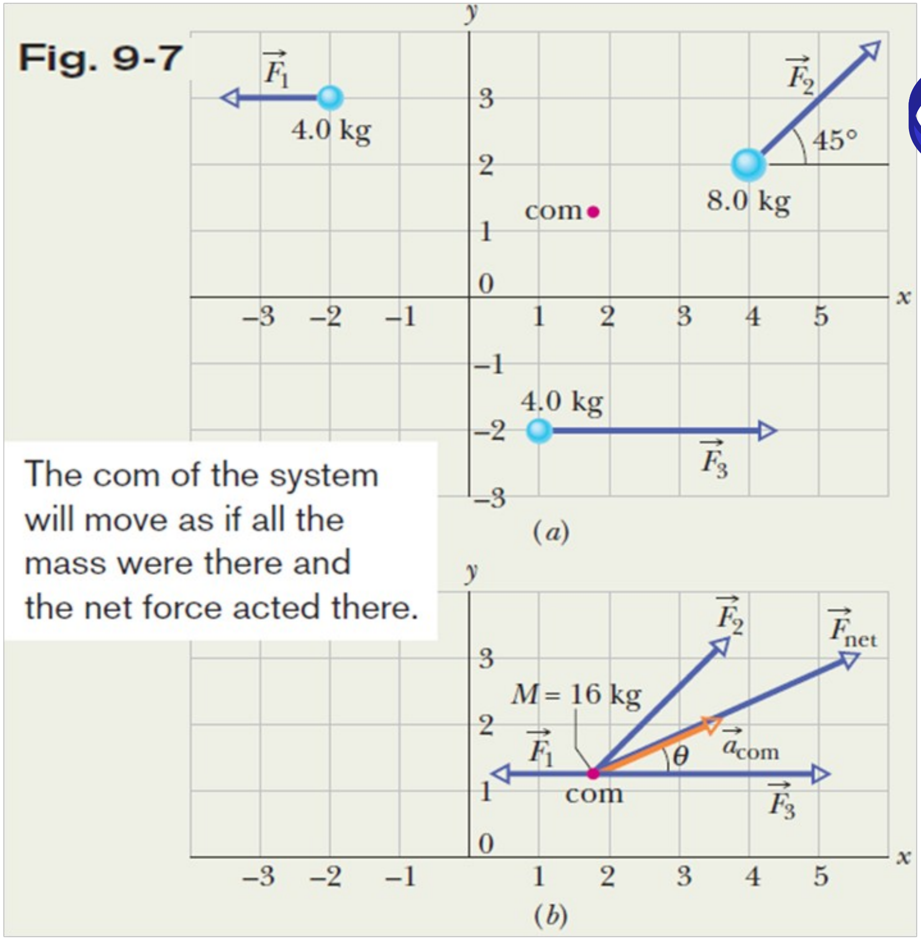
- This leads to $M\vec{a}_{com} = m_1\vec{a}_1 + m_2\vec{a}_2 + m_3\vec{a}_3 + \cdots + m_n\vec{a}_n.$

- Finally, $\boxed{M\vec{a}_{com} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \cdots + \vec{F}_n. \;=\mathbf{F}_{net}}$

What remains on the right hand side is the vector sum of all the external forces that act on the system, while the internal forces cancel out by Newton's 3rd Law.

Sample problem: Motion of the COM of 3 Particles

The three particles in Fig. 9-7*a* are initially at rest. Each experiences an *external* force due to bodies outside the three-particle system. The directions are indicated, and the magnitudes are $F_1 = 6.0$ N, $F_2 = 12$ N, and $F_3 = 14$ N. What is the acceleration of the center of mass of the system, and in what direction does it move?



Calculations: Applying Newton's second law to the center of mass,

$$\vec{F}_{net} = M\vec{a}_{com}$$

$$\vec{F}_1 + \vec{F}_2 + \vec{F}_3 = M\vec{a}_{com}$$

$$\vec{a}_{com} = \frac{\vec{F}_1 + \vec{F}_2 + \vec{F}_3}{M}.$$

$$a_{com,x} = \frac{F_{1x} + F_{2x} + F_{3x}}{M}$$

$$= \frac{-6.0\text{ N} + (12\text{ N})\cos 45° + 14\text{ N}}{16\text{ kg}} = 1.03\text{ m/s}^2.$$

Along the y axis, we have

$$a_{com,y} = \frac{F_{1y} + F_{2y} + F_{3y}}{M}$$

$$= \frac{0 + (12\text{ N})\sin 45° + 0}{16\text{ kg}} = 0.530\text{ m/s}^2.$$

From these components, we find that $\vec{a}_{com}$ has the magnitude

$$a_{com} = \sqrt{(a_{com,x})^2 + (a_{com,y})^2}$$

$$= 1.16\text{ m/s}^2 \approx 1.2\text{ m/s}^2 \qquad \text{(Answer)}$$

and the angle (from the positive direction of the x axis)

$$\theta = \tan^{-1}\frac{a_{com,y}}{a_{com,x}} = 27°. \qquad \text{(Answer)}$$

9.4 Linear Momentum

DEFINITION:

linear: since not rotation

$$\vec{p} = m\vec{v} \qquad \text{(linear momentum of a particle)}$$

in which m is the mass of the particle and **v** is its velocity.

•The time rate of change of the momentum of a particle is equal to the net force acting on the particle and is in the direction of that force.

$$\vec{F}_{\text{net}} = \frac{d\vec{p}}{dt}.$$

Manipulating this equation:

$$\vec{F}_{\text{net}} = \frac{d\vec{p}}{dt} = \frac{d}{dt}(m\vec{v}) = m\frac{d\vec{v}}{dt} = m\vec{a}. \qquad \text{(Newton's } 2^{\text{nd}} \text{ Law)}$$

9.5 Linear Momentum of a System of Particles

- The linear momentum of a system of particles is equal to the product of the total mass M of the system and the velocity of the center of mass.

$$\vec{P} = M\vec{v}_{com} \qquad \text{(linear momentum, system of particles),}$$

The time rate of change of the momentum of a particle is equal to the net force acting on the particle and is in the direction of that force.

$$\vec{F}_{net} = \frac{d\vec{P}}{dt} \qquad \text{(system of particles),}$$

- The net external force on a system changes linear momentum.
- Without a net external force, the *total* linear momentum of a system of particles cannot change.

The collision of a ball with a bat collapses part of the ball. *(Photo by Harold E. Edgerton. ©The Harold and Esther Edgerton Family Trust, courtesy of Palm Press, Inc.)*

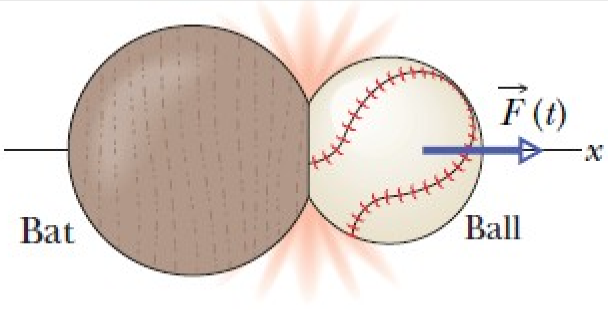


Fig. 9-8 Force $\vec{F}(t)$ acts on a ball as the ball and a bat collide.

- In this case, the collision is brief, and the ball experiences a force that is great enough to slow, stop, or even reverse its motion. Δt
- The figure depicts the collision at one instant.
- The ball experiences a force F(t) that varies during the collision and changes the linear momentum of the ball.

- In a collision, momentum of a particle can change. $\Delta \vec{p} = m \Delta \vec{v}$
- The change in linear momentum is related to the force by Newton's second law written in the form

$$\vec{F} = d\vec{p}/dt. \longrightarrow \int_{t_i}^{t_f} d\vec{p} = \int_{t_i}^{t_f} \vec{F}(t)\, dt.$$

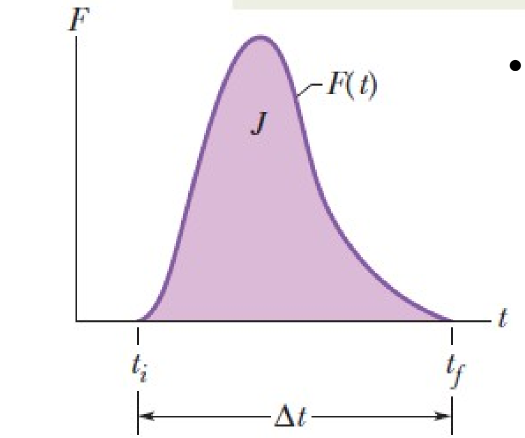
$\int F(t)\,!$

$$\longrightarrow \boxed{\vec{J} = \int_{t_i}^{t_f} \vec{F}(t)\, dt} \qquad \text{(impulse defined).}$$

- The right side of the equation is a measure of both the magnitude and the duration of the collision force, and is called the ***impulse of the collision,* J** (Unit: Ns).
- This means that the applied impulse is equal to the change in momentum of the object during the collision:

$$\boxed{\Delta \vec{p} = \vec{J} \qquad \text{(linear momentum–impulse theorem).}}$$

The impulse in the collision is equal to the area under the curve.

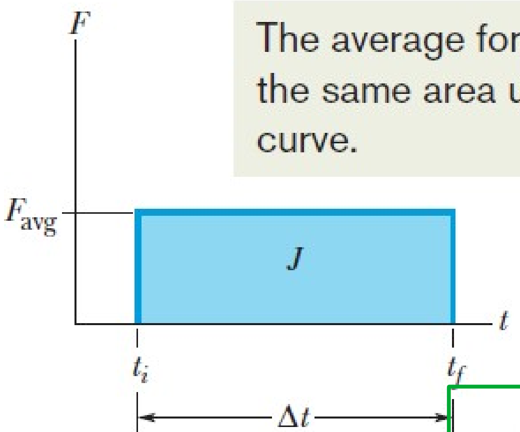


(a)

- We are integrating: we only need to know the area under the force curve!

Fig. 9-9 (*a*) The curve shows the magnitude of the time-varying force $F(t)$ that acts on the ball in the collision of Fig. 9-8. The area under the curve is equal to the magnitude of the impulse $\vec{J}$ on the ball in the collision. (*b*) The height of the rectangle represents the average force F_{avg} acting on the ball over the time interval Δt. The area within the rectangle is equal to the area under the curve in (*a*) and thus is also equal to the magnitude of the impulse $\vec{J}$ in the collision.

The average force gives the same area under the curve.



(b)

$$\boxed{J = F_{avg}\,\Delta t.}$$

- Instead of the ball, one can focus on the bat.
- At any instant, Newton's third law says that the force on the bat has the same magnitude but the opposite direction as the force on the ball.
- That means that the impulse on the bat has the same magnitude but the opposite direction as the impulse on the ball.

$\vec{J}$ $\vec{J}'$

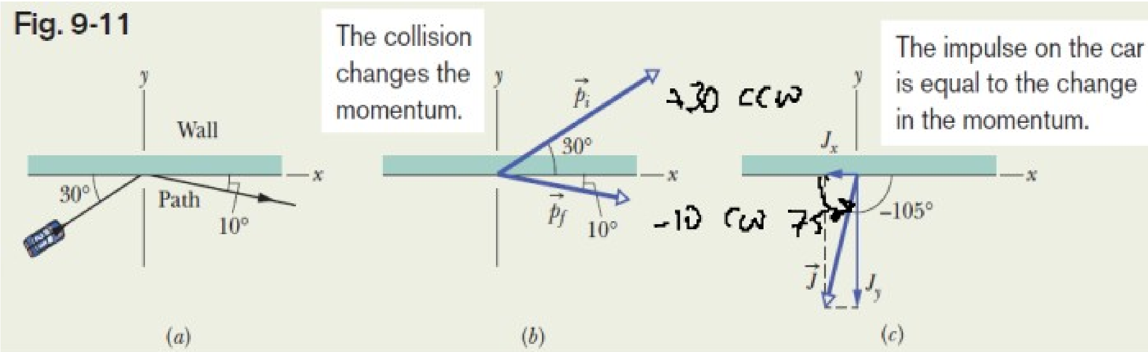
order!

$\vec{J}$ ball on bat

(-) $\vec{J}'$ bat on ball

Sample problem: 2-D Impulse

Race car–wall collision. Figure 9-11a is an overhead view of the path taken by a race car driver as his car collides with the racetrack wall. Just before the collision, he is traveling at speed $v_i = 70$ m/s along a straight line at 30° from the wall. Just after the collision, he is traveling at speed $v_f = 50$ m/s along a straight line at 10° from the wall. His mass m is 80 kg.



(a) What is the impulse $\vec{J}$ on the driver due to the collision?

Calculations: Figure 9-11b shows the driver's momentum $\vec{p}_i$ before the collision (at angle 30° from the positive x direction) and his momentum $\vec{p}_f$ after the collision (at angle −10°).

$$\vec{J} = \vec{p}_f - \vec{p}_i = m\vec{v}_f - m\vec{v}_i = m(\vec{v}_f - \vec{v}_i).$$

x component: Along the x axis we have

$$\begin{aligned} J_x &= m(v_{fx} - v_{ix}) \\ &= (80\text{ kg})[(50\text{ m/s})\cos(-10°) - (70\text{ m/s})\cos 30°] \\ &= -910\text{ kg}\cdot\text{m/s}. \end{aligned}$$

y component: Along the y axis,

$$\begin{aligned} J_y &= m(v_{fy} - v_{iy}) \\ &= (80\text{ kg})[(50\text{ m/s})\sin(-10°) - (70\text{ m/s})\sin 30°] \\ &= -3495\text{ kg}\cdot\text{m/s} \approx -3500\text{ kg}\cdot\text{m/s}. \end{aligned}$$

Impulse: The impulse is then

$$\vec{J} = (-910\hat{\mathrm{i}} - 3500\hat{\mathrm{j}})\text{ kg}\cdot\text{m/s}, \qquad \text{(Answer)}$$

which means the impulse magnitude is

$$J = \sqrt{J_x^2 + J_y^2} = 3616\text{ kg}\cdot\text{m/s} \approx 3600\text{ kg}\cdot\text{m/s}.$$

The angle of $\vec{J}$ is given by

$$\theta = \tan^{-1}\frac{J_y}{J_x}, \qquad \text{(Answer)}$$

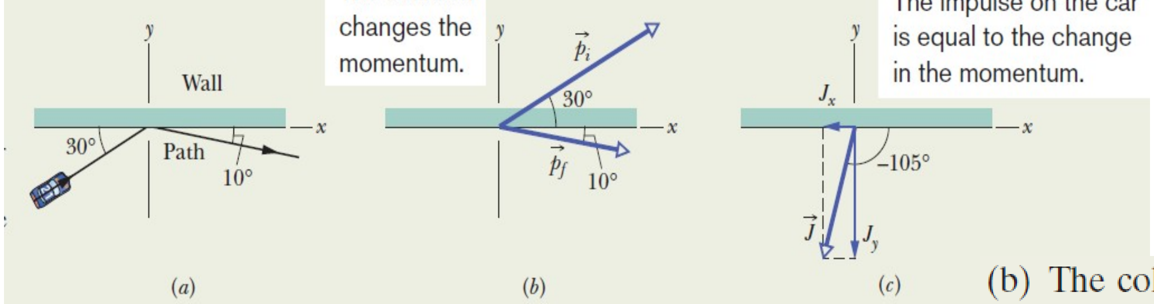
which a calculator evaluates as 75.4°. Recall that the physically correct result of an inverse tangent might be the displayed answer plus 180°. We can tell which is correct here by drawing the components of $\vec{J}$ (Fig. 9-11c). We find that θ is actually 75.4° + 180° = 255.4°, which we can write as

$$\theta = -105°. \qquad \text{(Answer)}$$

Race car–wall collision. Figure 9-11*a* is an overhead view of the path taken by a race car driver as his car collides with the racetrack wall. Just before the collision, he is traveling at speed $v_i = 70$ m/s along a straight line at 30° from the wall. Just after the collision, he is traveling at speed $v_f = 50$ m/s along a straight line at 10° from the wall. His mass m is 80 kg.

Sample problem: 2-D Impulse contd.

Fig. 9-11



(b) The collision lasts for 14 ms. What is the magnitude of the average force on the driver during the collision?

Calculations: We have

$$F_{avg} = \frac{J}{\Delta t} = \frac{3616 \text{ kg}\cdot\text{m/s}}{0.014 \text{ s}}$$
$$= 2.583 \times 10^5 \text{ N} \approx 2.6 \times 10^5 \text{ N}. \qquad \text{(Answer)}$$

Using $F = ma$ with $m = 80$ kg, you can show that the magnitude of the driver's average acceleration during the collision is about 3.22×10^3 m/s^2 = $329g$, which is fatal.

Surviving: Mechanical engineers attempt to reduce the chances of a fatality by designing and building racetrack walls with more "give," so that a collision lasts longer. For example, if the collision here lasted 10 times longer and the other data remained the same, the magnitudes of the average force and average acceleration would be 10 times less and probably survivable.

- If no net external force acts on a system of particles, the total linear momentum, **P**, of the system cannot change. (an impulse of zero)

$$\vec{P} = \text{constant} \qquad \text{(closed, isolated system).}$$

If the component of the net external force on a closed system is zero along an axis, then the component of the linear momentum of the system along that axis cannot change.

$$\boxed{\vec{P}_i = \vec{P}_f \qquad \text{(closed, isolated system).}}$$

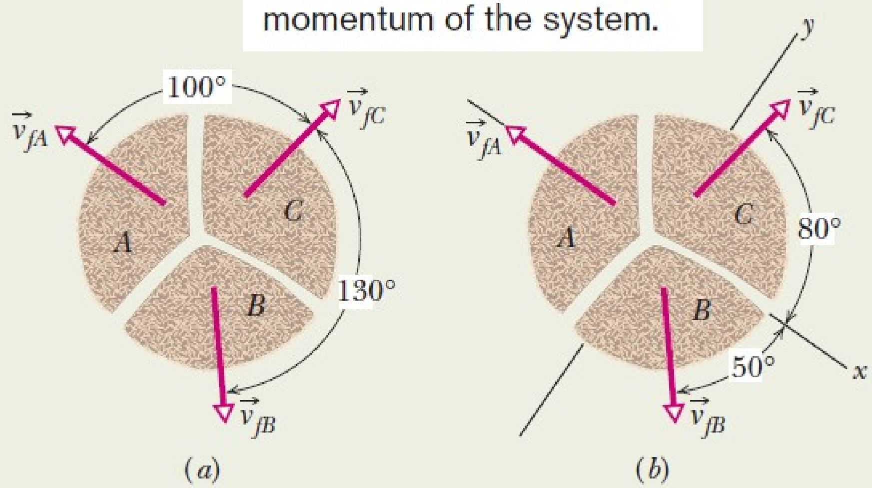
$$\begin{pmatrix}\text{total linear momentum} \\ \text{at some initial time } t_i\end{pmatrix} = \begin{pmatrix}\text{total linear momentum} \\ \text{at some later time } t_f\end{pmatrix}.$$

- This is called the **law of conservation of linear momentum.**
- Internal forces can change momenta of parts of the system, but cannot change the linear momentum of the entire system.

Sample problem: 2-D Explosion

Two-dimensional explosion: A firecracker placed inside a coconut of mass M, initially at rest on a frictionless floor, blows the coconut into three pieces that slide across the floor. An overhead view is shown in Fig. 9-13*a*. Piece C, with mass $0.30M$, has final speed $v_{fC} = 5.0$ m/s.

Fig. 9-13 The explosive separation can change the momentum of the parts but not the momentum of the system.



(a) What is the speed of piece B, with mass $0.20M$?

Calculations: To get started, we superimpose an xy coordinate system as shown in Fig. 9-13*b*, with the negative direction of the x axis coinciding with the direction of $\vec{v}_{fA}$. The x axis is at 80° with the direction of $\vec{v}_{fC}$ and 50° with the direction of $\vec{v}_{fB}$.

$$P_{iy} = P_{fy},$$

where subscript i refers to the initial value (before the explosion), and subscript y refers to the y component of $\vec{P}_i$ or $\vec{P}_f$.

The component P_{iy} of the initial linear momentum is zero, because the coconut is initially at rest. To get an expression for P_{fy}, we find the y component of the final linear momentum of each piece, using the y-component version of Eq. 9-22 ($p_y = mv_y$):

$$p_{fA,y} = 0,$$
$$p_{fB,y} = -0.20Mv_{fB,y} = -0.20Mv_{fB}\sin 50°,$$
$$p_{fC,y} = 0.30Mv_{fC,y} = 0.30Mv_{fC}\sin 80°.$$

(Note that $p_{fA,y} = 0$ because of our choice of axes.) Equation 9-48 can now be written as

$$P_{iy} = P_{fy} = p_{fA,y} + p_{fB,y} + p_{fC,y}.$$

Then, with $v_{fC} = 5.0$ m/s, we have

$$0 = 0 - 0.20Mv_{fB}\sin 50° + (0.30M)(5.0\text{ m/s})\sin 80°,$$

from which we find

$$v_{fB} = 9.64\text{ m/s} \approx 9.6\text{ m/s}. \qquad \text{(Answer)}$$

(b) What is the speed of piece A?

$$p_{fA,x} = -0.50Mv_{fA},$$
$$p_{fB,x} = 0.20Mv_{fB,x} = 0.20Mv_{fB}\cos 50°,$$
$$p_{fC,x} = 0.30Mv_{fC,x} = 0.30Mv_{fC}\cos 80°.$$

$$P_{ix} = P_{fx} = p_{fA,x} + p_{fB,x} + p_{fC,x}.$$

Then, with $v_{fC} = 5.0$ m/s and $v_{fB} = 9.64$ m/s, we have

$$0 = -0.50Mv_{fA} + 0.20M(9.64\text{ m/s})\cos 50° + 0.30M(5.0\text{ m/s})\cos 80°,$$

from which we find

$$v_{fA} = 3.0\text{ m/s}. \qquad \text{(Answer)}$$

1. In a closed and isolated system, if there are two colliding bodies, and total kinetic energy is unchanged (conserved) by the collision.
   - Such a collision is called an ***elastic collision***.
   - A useful approximation for common situations.
   - In real collisions, some energy is always transferred.
2. If during the collision, some energy is always transferred from kinetic energy to other forms of energy, such as thermal energy or energy of sound, then the kinetic energy of the system is not conserved.
   - Such a collision is called an ***inelastic collision***.
   - Some energy is transferred.
3. ***Completely inelastic collisions:***
   - The objects stick together
   - Greatest loss of kinetic energy.

check if KE is conserved or NOT

elastic inelastic

Inelastic collision:

Here is the generic setup for an inelastic collision.

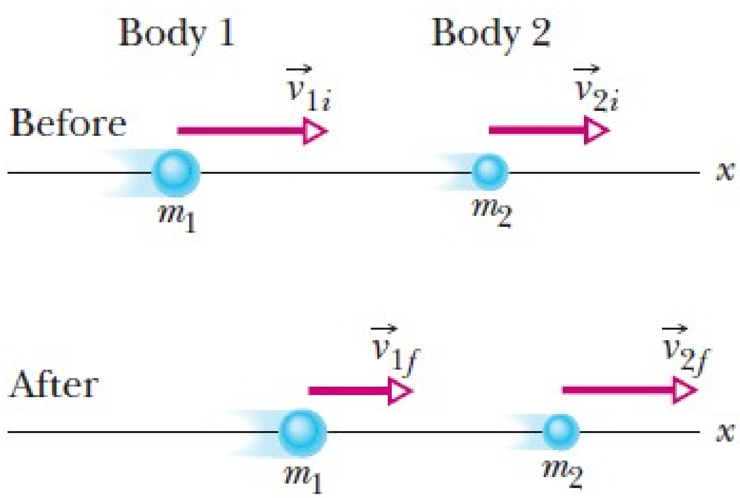


Fig. 9-14 Bodies 1 and 2 move along an x axis, before and after they have an inelastic collision.

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}.$$

in x-direction

Completely inelastic collision, for target at rest:

In a completely inelastic collision, the bodies stick together.

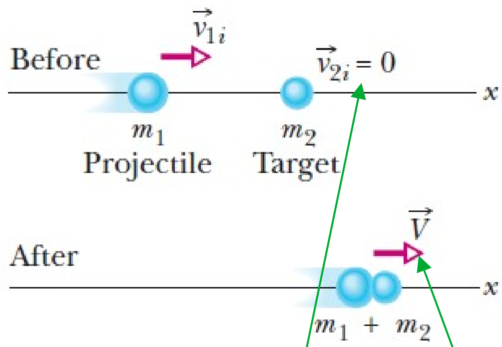


Fig. 9-15 A completely inelastic collision between two bodies. Before the collision, the body with mass m_2 is at rest and the body with mass m_1 moves directly toward it. After the collision, the stuck-together bodies move with the same velocity $\vec{V}$.

$+m_2 v_{2i} = 0$ & $v_{1f} = v_{2f} = V$

$$m_1 v_{1i} = (m_1 + m_2)V$$

$$V = \frac{m_1}{m_1 + m_2} v_{1i}.$$

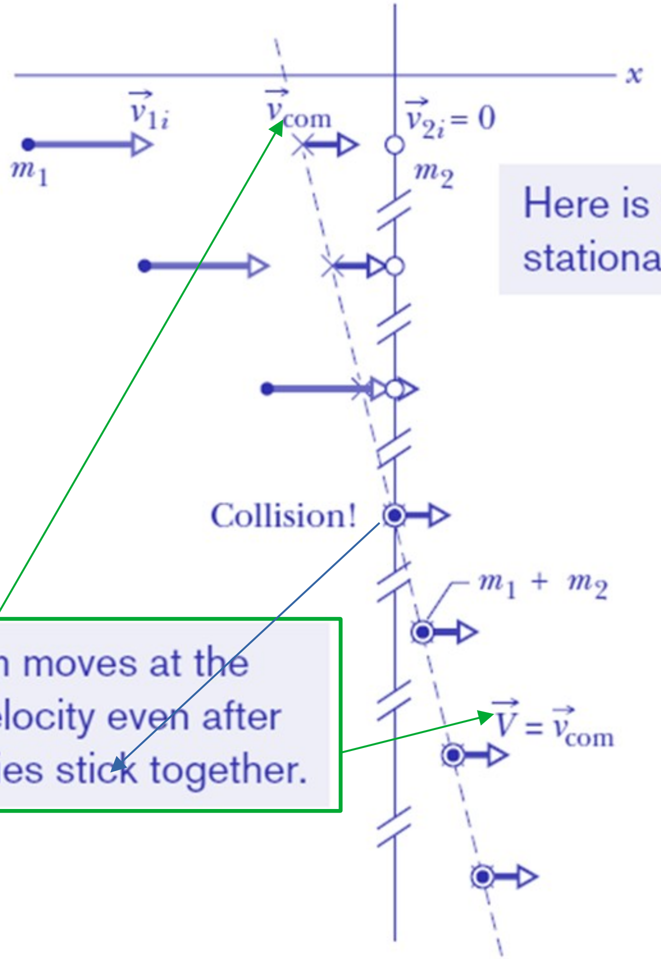
The center of mass velocity remains unchanged:

The com of the two bodies is between them and moves at a constant velocity.

$$\vec{v}_{com} = \frac{\vec{P}}{m_1 + m_2} = \frac{\vec{p}_{1i} + \vec{p}_{2i}}{m_1 + m_2}.$$

Here is the incoming projectile.

Here is the stationary target.



The com moves at the same velocity even after the bodies stick together.

Fig. 9-16 Some freeze frames of a two-body system, which undergoes a completely *inelastic collision*. The system's center of mass is shown in each freeze-frame.

- The velocity v_{com} of the center of mass is unaffected by the collision.
- Because the bodies stick together after the collision, their common velocity V must be equal to v_{com}.

Sample problem: conservation of momentum

The *ballistic pendulum* was used to measure the speeds of bullets before electronic timing devices were developed. The version shown in Fig. 9-17 consists of a large block of wood of mass $M = 5.4$ kg, hanging from two long cords. A bullet of mass $m = 9.5$ g is fired into the block, coming quickly to rest. The *block + bullet* then swing upward, their center of mass rising a vertical distance $h = 6.3$ cm before the pendulum comes momentarily to rest at the end of its arc. What is the speed of the bullet just prior to the collision?

There are two events here. The bullet collides with the block. Then the bullet–block system swings upward by height h.

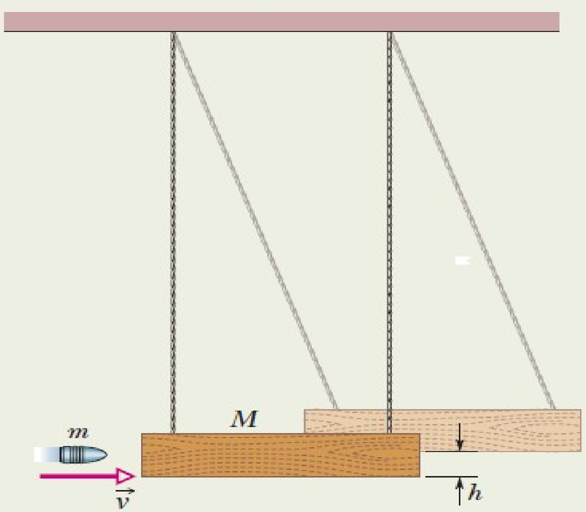


Fig. 9-17 A ballistic pendulum, used to measure the speeds of bullets.

The collision within the bullet– block system is so brief. Therefore:

(1)During the collision, the gravitational force on the block and the force on the block from the cords are still balanced. Thus, during the collision, the net external impulse on the bullet-block system is zero. Therefore, the system is isolated and its total linear momentum is conserved.

(2) The collision is one-dimensional in the sense that the direction of the bullet and block just after the collision is in the bullet's original direction of motion.

$$V = \frac{m}{m + M} v.$$

As the bullet and block now swing up together, the mechanical energy of the bullet–block–Earth system is conserved:

$$\tfrac{1}{2}(m + M)V^2 = (m + M)gh.$$

Combining steps:

$$v = \frac{m + M}{m}\sqrt{2gh}$$

$$= \left(\frac{0.0095 \text{ kg} + 5.4 \text{ kg}}{0.0095 \text{ kg}}\right)\sqrt{(2)(9.8 \text{ m/s}^2)(0.063 \text{ m})}$$

$$= 630 \text{ m/s.} \qquad \text{(Answer)}$$

In an elastic collision, the kinetic energy of each colliding body may change, but the total kinetic energy of the system does not change.

Here is the generic setup for an elastic collision with a stationary target.

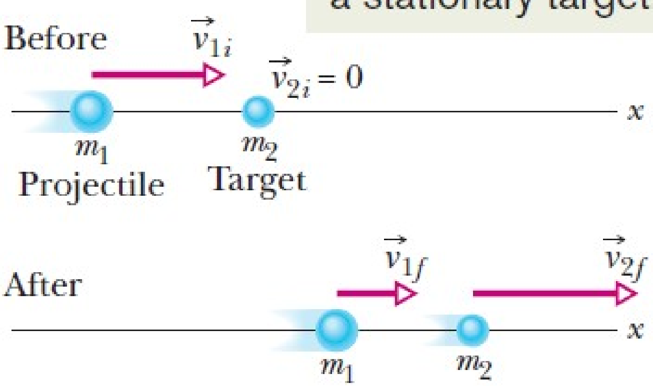


Fig. 9-18 Body 1 moves along an x axis before having an elastic collision with body 2, which is initially at rest. Both bodies move along that axis after the collision.

$$m_1 v_{1i} = m_1 v_{1f} + m_2 v_{2f} \qquad \text{(linear momentum).}$$

$$\tfrac{1}{2} m_1 v_{1i}^2 = \tfrac{1}{2} m_1 v_{1f}^2 + \tfrac{1}{2} m_2 v_{2f}^2 \qquad \text{(kinetic energy).}$$

initially

m_1, m_2 ✓
v_{1i}, v_{2i}

With some algebra we get:

$$\boxed{v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i} \qquad v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i}.}$$

unknowns

knowns

finally

$v_{1f} \sim v_{1i}$
$v_{2f} \sim v_{1i}$
v_{2i}

- Results:

- Equal masses, $v_{1f} = 0$, $v_{2f} = v_{1i}$: the first object stops.
- Massive target, $m_2 >> m_1$: the first object just bounces back, speed mostly unchanged.
- Massive projectile, $v_{1f} \approx v_{1i}$, $v_{2f} \approx 2v_{1i}$: the first object keeps going, the target flies forward at about twice its speed.

9.10 Elastic collisions in 1-D: Moving Target

For a target that is also moving,

Here is the generic setup for an elastic collision with a moving target.

Fig. 9-19 Two bodies headed for a one-dimensional elastic collision.

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f},$$

$$\tfrac{1}{2} m_1 v_{1i}^2 + \tfrac{1}{2} m_2 v_{2i}^2 = \tfrac{1}{2} m_1 v_{1f}^2 + \tfrac{1}{2} m_2 v_{2f}^2.$$

With some algebra we get:

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i} + \frac{2m_2}{m_1 + m_2} v_{2i}$$

unknowns

knowns

$$v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i} + \frac{m_2 - m_1}{m_1 + m_2} v_{2i}.$$

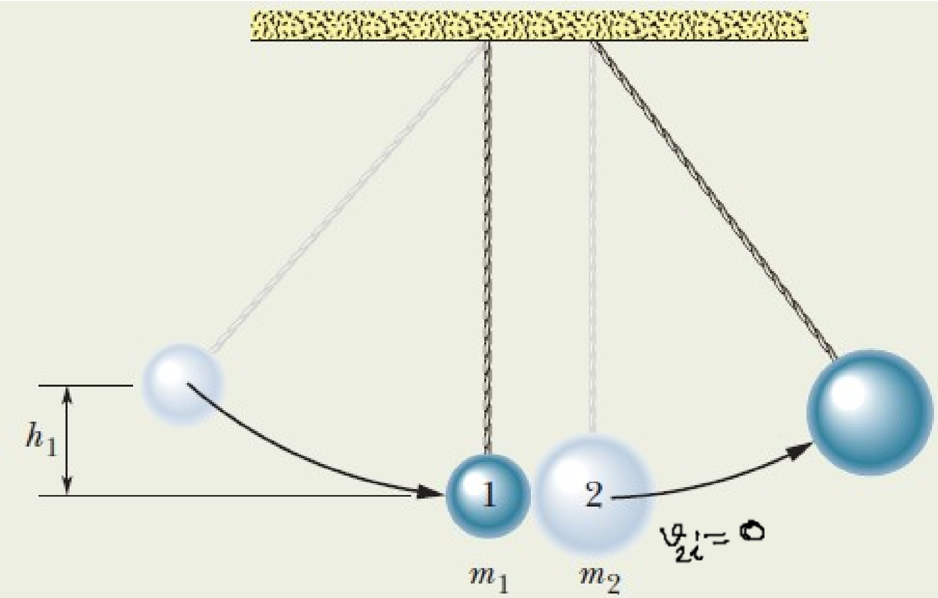
$v_{1f} \sim v_{1i}$ & v_{2i}

$v_{2f} \sim v_{1i}$ & v_{2i}

İZMİR KÂTİP ÇELEBİ ÜNİVERSİTESİ

Sample problem: Two Pendulums

Two metal spheres, suspended by vertical cords, initially just touch, as shown in Fig. 9-20. Sphere 1, with mass $m_1 = 30$ g, is pulled to the left to height $h_1 = 8.0$ cm, and then released from rest. After swinging down, it undergoes an elastic collision with sphere 2, whose mass $m_2 = 75$ g. What is the velocity v_{1f} of sphere 1 just after the collision?



Step 1: As sphere 1 swings down, the mechanical energy of the sphere–Earth system is conserved. (The mechanical energy is not changed by the force of the cord on sphere 1 because that force is always directed perpendicular to the sphere's direction of travel.)

Calculation: Let's take the lowest level as our reference level of zero gravitational potential energy. Then the kinetic energy of sphere 1 at the lowest level must equal the gravitational potential energy of the system when sphere 1 is at height h_1. Thus,

$$\tfrac{1}{2}m_1 v_{1i}^2 = m_1 g h_1,$$

which we solve for the speed v_{1i} of sphere 1 just before the collision:

$$v_{1i} = \sqrt{2gh_1} = \sqrt{(2)(9.8\ \text{m/s}^2)(0.080\ \text{m})} = 1.252\ \text{m/s}.$$

Step 2: Here we can make two assumptions in addition to the assumption that the collision is elastic. First, we can assume that the collision is one-dimensional because the motions of the spheres are approximately horizontal from just before the collision to just after it. Second, because the collision is so

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i} = \frac{0.030\ \text{kg} - 0.075\ \text{kg}}{0.030\ \text{kg} + 0.075\ \text{kg}} (1.252\ \text{m/s}) = -0.537\ \text{m/s} \approx -0.54\ \text{m/s}. \quad \text{(Answer)}$$

The minus sign tells us that sphere 1 moves to the left just after the collision.

$$v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i} \quad \checkmark$$

$$\vec{P}_{1i} + \vec{P}_{2i} = \vec{P}_{1f} + \vec{P}_{2f}\,.$$

A glancing collision that conserves both momentum and kinetic energy.

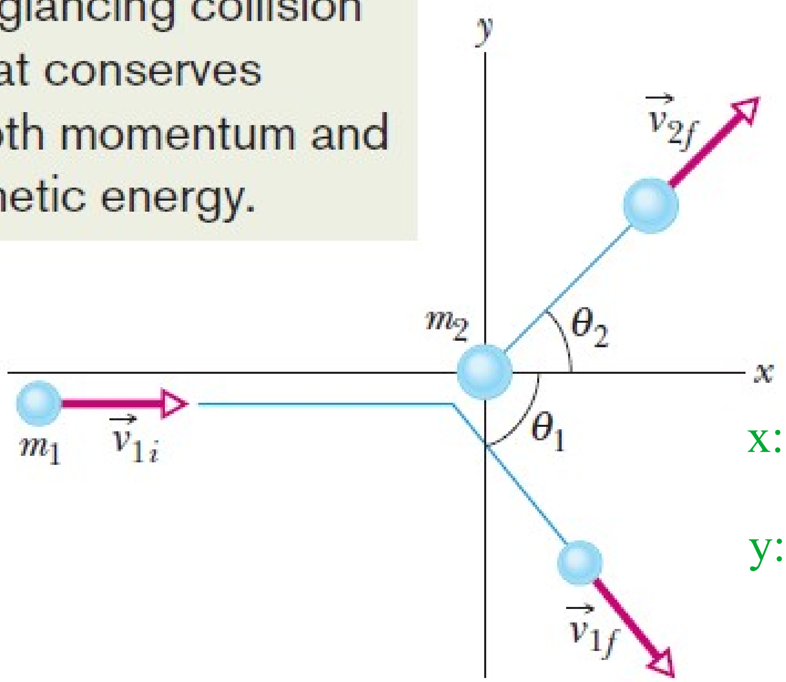


Fig. 9-21 An elastic collision between two bodies in which the collision is not head-on. The body with mass m_2 (the target) is initially at rest.

If elastic,

$$K_{1i} + K_{2i} = K_{1f} + K_{2f}.$$

Apply the conservation of momentum along each axis:

x: $$m_1 v_{1i} = m_1 v_{1f} \cos\theta_1 + m_2 v_{2f} \cos\theta_2,$$

y: $$0 = -m_1 v_{1f} \sin\theta_1 + m_2 v_{2f} \sin\theta_2.$$

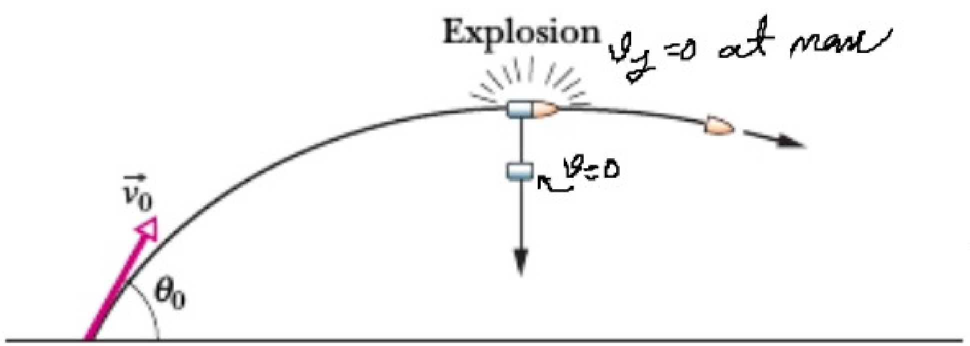
Also, apply conservation of energy for elastic collisions:

$$\tfrac{1}{2} m_1 v_{1i}^2 = \tfrac{1}{2} m_1 v_{1f}^2 + \tfrac{1}{2} m_2 v_{2f}^2$$

known(s) $\rightarrow$ unknown(s) !

1. A shell is shot with an initial velocity of $\mathbf{v_0}$=20 m/s, at an angle of θ=60° with the horizontal. At the top of the trajectory, the shell explodes into two fragments of equal mass (see Figure). One fragment, whose speed immediately after the explosion is zero, falls vertically. How far from the gun does the other fragment land, assuming that the terrain is level and that air drag is negligible?

as max height



$v_y = 0$ at max

$x = x_0 + v_{0x} t$

$y = y_0 + v_{0y} t - \frac{1}{2} g t^2$

$v_x = v_{0x}$

$v_y = v_{0y} - g t$

$v_{0x} = v_0 \cos\theta$

$v_{0y} = v_0 \sin\theta$

1st step: find time to reach max height

2nd step: find coordinates at explosion

3rd step: Reanalyze the question: find new velocity for half-mass by using conservation of linear momentum

3)(13) $\vec{v}_0 = 20\,m/s$, $\theta = 60°$

Shell explodes at top of the trajectory

Two fragments of equal mass

① $v=0$ falls vertically

② $x=?$

where the shell explodes? $v_2 = ?$

$v = v_{0y} - gt = 0$ at top of trajectory

Step 1

$\rightarrow t = \dfrac{v_0 \sin\theta}{g}$: time of explosion

$\rightarrow x = v_{0x}t = v_0 t \cos\theta = \dfrac{v_0^2}{g}\sin\theta\cos\theta = \dfrac{(20\,m/s)^2}{9.8\,m/s^2}\sin 60 \cos 60° = 17.7\,m$

$y = v_{0y}t - \frac{1}{2}gt^2 = v_0 \sin\theta \dfrac{v_0 \sin\theta}{g} - \frac{1}{2} g \dfrac{v_0^2 \sin^2\theta}{g^2} = \frac{1}{2}\dfrac{v_0^2 \sin^2\theta}{g}$

$= \frac{1}{2}\dfrac{(20\,m/s)^2 \sin^2 60}{9.8\,m/s^2} = 15.3\,m$

Step 2

$\Rightarrow$ Coordinates of explosion is $(17.7\,m, 15.3\,m)$

No external forces acting. Momentum is conserved. $\vec{P}_i = \vec{P}_f$

v_{com} does not change

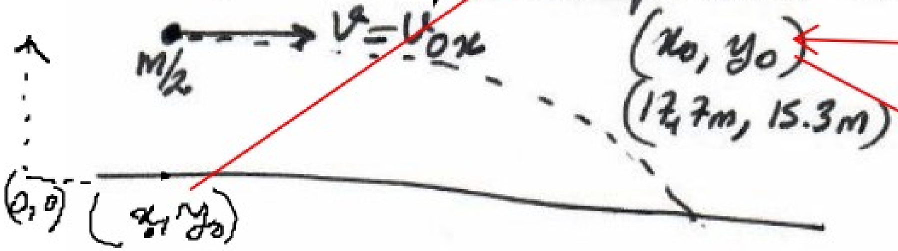
total mass

$m v_{0x} = 0 + \frac{m}{2} v$

Step 3

$\Rightarrow v = 2 v_0 \cos 60 = 20\,m/s$

Now, we have a new picture as



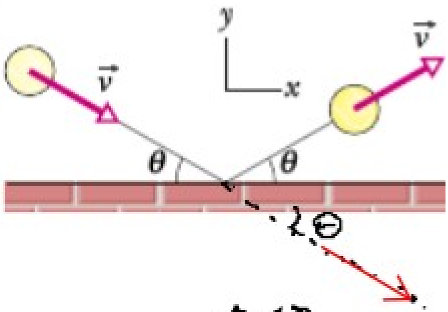
$y = y_0 - \frac{1}{2}gt^2 \Rightarrow 0 - 15.3\,m = -\frac{1}{2} 9.8\,m/s^2\, t^2$

$\Rightarrow t = \sqrt{\dfrac{30.6\,m}{9.8\,m/s^2}}$

$x = x_0 + v'_{0x}t = 17.7\,m + 20\,m/s\sqrt{\dfrac{30.6\,m}{9.8\,m/s^2}}$

$\boxed{x = 53\,m}$

2. In the overhead view of Figure, a 300 g ball with a speed v of 6.0 m/s strikes a wall at an angle θ of 30° and then rebounds with the same speed and angle. It is in contact with the wall for 10 ms. In unit-vector notation, what are (a) the impulse on the ball from the wall and (b) the average force on the wall from the ball? F_{wb}



38) $m = 300\,g = 0.3\,kg$

$v = 6.0\,m/s$

$\theta_i = 30°$

rebounds with same speed and angle

impact time = 10 ms

i) in unit vector notation $\vec{J} = ?$

$\vec{v}_i = v\cos\theta\,\hat{i} - v\sin\theta\,\hat{j} = 6.9\,\hat{i} - 4.0\,\hat{j}$

$\vec{v}_f = v\cos\theta\,\hat{i} + v\sin\theta\,\hat{j} = 6.9\,\hat{i} + 4.0\,\hat{j}$

$\vec{J} = \Delta\vec{p} = m(\vec{v}_f - \vec{v}_i) = 0.3\,kg\,(6.9\hat{i} + 4.0\hat{j} - (6.9\hat{i} - 4.0\hat{j}))$ Ns

$= 2.4\,Ns\,\hat{j}$

ii) Average force on the ~~ball~~ wall from the ball?

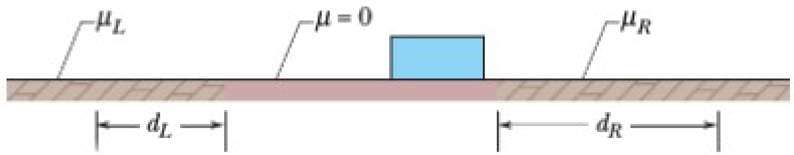
$\vec{F}_{avg} = \frac{\vec{J}}{\Delta t} = \frac{2.4\,N.s\,\hat{j}}{10\times10^{-3}s} = (240\,N)\hat{j}$ ← force on the ball by the wall

Newton's 3rd law → $(-240\,N\,\hat{j})$

$\uparrow F_{bw} \sim +240\,N\,\hat{j}$

$\downarrow F_{wb} \sim -240\,N\,\hat{j}$

3. In Figure, a stationary block explodes into two pieces *L* and *R* that slide across a frictionless floor and then into regions with friction, where they stop. Piece *L*, with a mass of 2.0 kg, encounters a coefficient of kinetic friction $\mu_L = 0.40$ and slides to a stop in distance $d_L = 0.15$ m. Piece *R* encounters a coefficient of kinetic friction $\mu_R = 0.50$ and slides to a stop in distance $d_R = 0.25$ m. What was the mass of the block?



$\Delta K + \cancel{\Delta U} + \Delta E_{th} = W = 0$

$\cancel{K_f} - K_i = -f_k d$

8) (44)

$M_L + m_R = M = ?$

$M_L = 2.0$ kg

$\mu_{k_L} = 0.40$

stops at distance $d_L = 0.15$ m

$\mu_{k_R} = 0.50$ " " " $d_R = 0.25$ m

KE is converting into thermal energy

$\Rightarrow \frac{1}{2}\cancel{m_L} v_L^2 = 0.40\, \cancel{m_L} (9.8\, m/s^2)(0.15\, m)$

$\frac{1}{2}\cancel{m_R} v_R^2 = 0.50\, \cancel{m_R} (9.8\, m/s^2)(0.25\, m)$

$\vec{P_i} = \vec{P_f}$

$0 = m_L v_L (-\hat{i}) + m_R v_R \hat{i}$

$\left(\sqrt{2 \times 0.40 (9.8\, m/s^2) 0.15\, m}\right) 2.0\, kg = \sqrt{2 \times 0.50 (9.8\, m/s^2)(0.25\, m)}\; m_R$

$(0.06)\, 4.0\, kg^2 = 0.125\, m_R^2 \rightarrow m_R = 1.39\, kg \Rightarrow \boxed{M = 3.39\, kg}$

Total Mass

4. A 5.20 g bullet moving at 672 m/s strikes a 700 g wooden block at rest on a frictionless surface. The bullet emerges, traveling in the same direction with its speed reduced to 428 m/s. (a) What is the resulting speed of the block? (b) What is the speed of the bullet–block center of mass?

v_i

v_f

So) $m = 0.0052$ kg

$v_i = 672$ m/s

strikes to

$M = 0.7$ kg at rest

$v_f = 428$ m/s

y, x

i) Resulting speed of the block, M?

$m v_{mi} + M v_{Mi} = m v_{mf} + M v_{Mf}$

Conservation of linear momentum

v_f

v_i

$(0.0052\,kg)\,672\,m/s + (0.7\,kg)(0) = (0.0052\,kg)\,428\,m/s + (0.7\,kg)\,v_f$

$\Rightarrow v_{Mf} = 1.82\,m/s$

ii) speed of the bullet-block com?

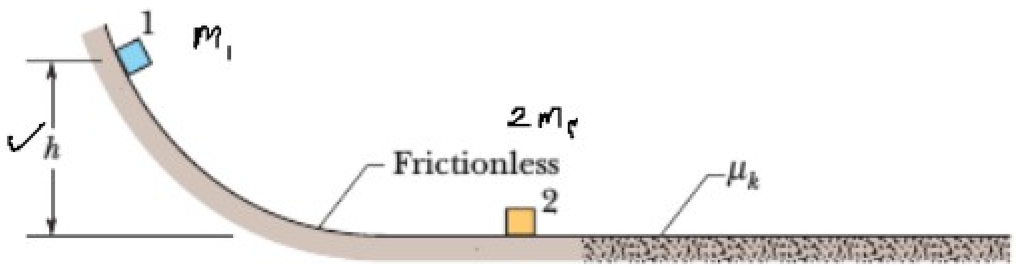
velocity of com is not changed $\Leftrightarrow$ Cons. of mom.

~~exact~~ calculate before and after velocity.

Say before $\vec{v}_{com} = \dfrac{m v_{mi} + M v_{Mi}}{m + M} = \dfrac{(0.052\,kg)(672\,m/s)}{(0.0052 + 0.7)\,kg} = 4.96\,m/s$

check for after one!

5. A In Figure, block 1 of mass m_1 slides from rest along a frictionless ramp from height h=2.50 m and then collides with stationary block 2, which has mass m_2=2m_1. After the collision, block 2 slides into a region where the coefficient of kinetic friction is μ_k=0.500 and comes to a stop in distance d within that region. What is the value of distance d if the collision is (a) elastic and (b) completely inelastic?



· need the initial velocity of m_1 before collision: Conservation of energy $\Delta K + \Delta U = 0$

$K_f + \cancel{U_f} = \cancel{K_i} + U_i$

· apply the eqns for elastic collisions / inelastic (completely) to get v_{2f}

10) (68)

m_1 slides from rest
$v_1 = 0$
frictionless ramp
$h = 2.50\ m$
collides with a stationary block
$2m_1 = m_2$
$v_2 = 0$
$\mu_k = 0.50$
stops at distance d.
d = ? if the collision is elastic & inelastic

$\Delta K = -\Delta U \rightarrow K_2 - K_1 = -U_2 + U_1 \rightarrow K_2 + U_2 = K_1 + U_1$

$K_1 = 0$, $U_2 = 0$ $\Big|$ $K_2 = m_1 g h \rightarrow \frac{1}{2} m_1 v_1^2 = m_1 g h \rightarrow v_1 = \sqrt{2gh}$

Just before impact (v_{1i})

i) elastic collision

$v_{2f} = \frac{2m_1}{m_1+m_2} v_{1i} = \frac{2m_1}{3m_1}\sqrt{2gh} = \frac{2}{3}\sqrt{2gh}$

$K_f - K_i = -\mu_k m g d$

$KE = \Delta E_{the} = f_s d = \mu_k m_2 g d$

$\frac{1}{2}\cancel{m_2} v_2^2 = \mu_k \cancel{m_2} g d \rightarrow d = \frac{1}{2}\frac{4}{9}\frac{2gh}{\mu_k g} = \frac{4}{9}\frac{(2.50m)}{(0.5)} = \frac{20}{9} m$

$d = 2.22\ m$?

ii) $v_{2f} = \frac{m_1}{m_1+m_2} v_{1i} = \frac{1}{3}\sqrt{2gh} \rightarrow d = \frac{1}{2}\frac{1}{9}\frac{2gh}{\mu_k g} = \frac{1}{4}\left(\frac{20}{9}\right) m$

$d = 0.555$?

6. Two 2.0 kg bodies, *A* and *B*, collide. The velocities before the collision are $\mathbf{v}_A=(15\mathbf{i}+30\mathbf{j})$ m/s and $\mathbf{v}_B=(-10\mathbf{i}+5.0\mathbf{j})$ m/s. After the collision, $\mathbf{v}'_A=(-5.0\mathbf{i}+20\mathbf{j})$ m/s. What are (a) the final velocity of *B* and (b) the change in the total kinetic energy (including sign)?

$m_A = m_B = 2\,kg$

7(4) $m_A = m_B = 2\,kg$

$\vec{v}_{A_i} = (15\hat{i}+30\hat{j})\,m/s$

$\vec{v}_{B_i} = (-10\hat{i}+5\hat{j})\,m/s$

$\vec{v}_{A_f} = (-5.0\hat{i}+20\hat{j})\,m/s$

i) $\vec{v}_{B_f} = ?$ Cons. of lin. mom.

$\cancel{m_A}\vec{v}_{A_i} + \cancel{m_B}\vec{v}_{B_i} = \cancel{m_A}\vec{v}_{A_f} + \cancel{m_B}\vec{v}_{B_f}$

$(15-10+5)\hat{i} + (30+5-20)\hat{j} = \vec{v}_{B_f}$

$\vec{v}_{B_f} = (10\hat{i}+15\hat{j})\,m/s$

ii) $\Delta KE = ?$ $K_f - K_i = \frac{1}{2}m_A v_{A_f}^2 + \frac{1}{2}m_B v_{B_f}^2 - \left(\frac{1}{2}m_A v_{A_i}^2 + \frac{1}{2}m_B v_{B_i}^2\right)$

$= \frac{1}{2}\,2\,kg\left[-\underbrace{\left((15^2+30^2)+((-10)^2+5^2)\right)}_{\text{initial } 1.3\times10^2\,J} + \underbrace{\left(((-5)^2+(20^2))+(10^2+15^2)\right)}_{\text{final } 8\times10^2\,J}\right]\,m^2/s^2$

$\Delta KE = -5\times10^2\,J$

initial KE is lost (500 J)

$(K_{B_f}+K_{A_f}) - (K_{B_i}+K_{A_i})$

$K_f = K_i$
$\Delta KE = 0$

elastic collision

Linear Momentum & Newton's 2nd Law

- Linear momentum defined as:

$$\vec{P} = M\vec{v}_{com}$$ Eq. (9-25)

- Write Newton's 2nd law:

$$\vec{F}_{net} = \frac{d\vec{P}}{dt}$$ Eq. (9-27)

Conservation of Linear Momentum

$$\vec{P} = \text{constant} \quad \text{(closed, isolated system).}$$ Eq. (9-42)

Collision and Impulse

- Defined as $\vec{J} = \int_{t_i}^{t_f} \vec{F}(t)\, dt$ Eq. (9-30)
- Impulse causes changes in linear momentum

Inelastic Collision in 1D

- Momentum conserved along that dimension

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}.$$ Eq. (9-51)

Motion of the Center of Mass

- Unaffected by collisions/internal forces

Collisions in Two Dimensions

- Apply conservation of momentum along each axis individually
- Conserve K if elastic

Elastic Collisions in One Dimension

- K is also conserved

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i}$$ Eq. (9-67)

$$v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i}.$$ Eq. (9-68)

Variable-Mass Systems

$$Rv_{rel} = Ma \quad \text{(first rocket equation).}$$ Eq. (9-87)

$$v_f - v_i = v_{rel} \ln \frac{M_i}{M_f} \quad \text{(second rocket equation)}$$ Eq. (9-88)

Additional Materials

Sample problem: COM

Figure 9-3*a* shows a uniform metal plate *P* of radius 2*R* from which a disk of radius *R* has been stamped out (removed) in an assembly line. The disk is shown in Fig. 9-3b. Using the *xy* coordinate system shown, locate the center of mass com_P of the remaining plate.

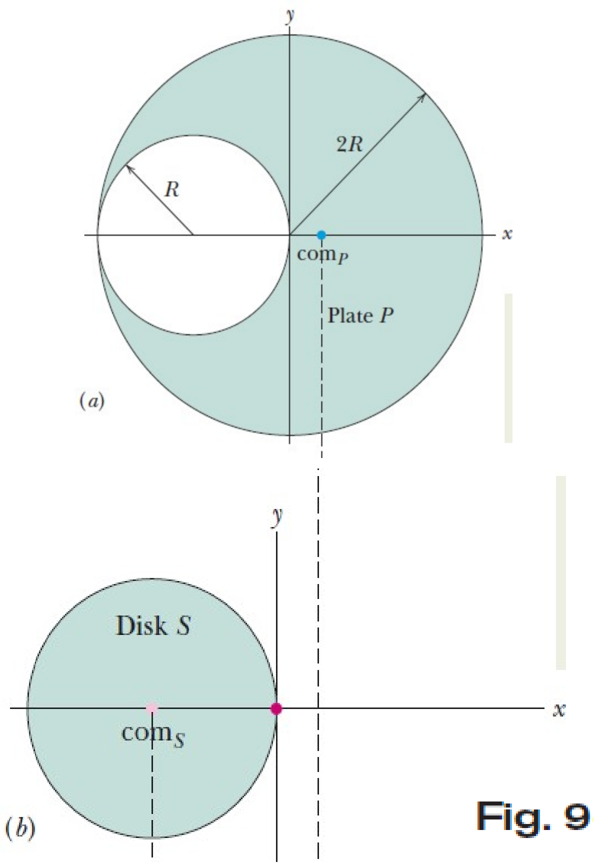


Fig. 9-3

Calculations:

- First, put the stamped-out disk (call it disk S) back into place to form the original composite plate (call it plate C).
- Because of its circular symmetry, the center of mass com_S for disk S is at the center of S, at $x = -R$.
- Similarly, the center of mass com_C for composite plate C is at the center of C, at the origin.
- Assume that mass m_S of disk S is concentrated in a particle at $x_S = -R$, and mass m_P is concentrated in a particle at x_P.
- Next treat these two particles as a two particle system, and find their center of mass x_{S+P}

$$x_{S+P} = \frac{m_S x_S + m_P x_P}{m_S + m_P}.$$

- Next note that the combination of disk S and plate P is composite plate C. Thus, the position x_{S+P} of com_{S+P} must coincide with the position x_C of com_C, which is at the origin; so $x_{S+P} = x_C = 0$.

$$x_P = -x_S \frac{m_S}{m_P}.$$

$$\frac{m_S}{m_P} = \frac{\text{density}_S}{\text{density}_P} \times \frac{\text{thickness}_S}{\text{thickness}_P} \times \frac{\text{area}_S}{\text{area}_P}$$

But,

$$\frac{m_S}{m_P} = \frac{\text{area}_S}{\text{area}_P} = \frac{\text{area}_S}{\text{area}_C - \text{area}_S} = \frac{\pi R^2}{\pi (2R)^2 - \pi R^2} = \frac{1}{3}.$$

and $x_S = -R$ $\Rightarrow$ $x_P = \frac{1}{3}R.$

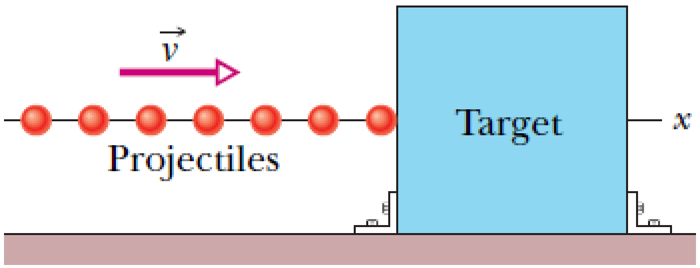


Fig. 9-10 A steady stream of projectiles, with identical linear momenta, collides with a target, which is fixed in place. The average force F_{avg} on the target is to the right and has a magnitude that depends on the rate at which the projectiles collide with the target or, equivalently, the rate at which mass collides with the target.

- Let n be the number of projectiles that collide in a time interval Δt.
- Each projectile has initial momentum mv and undergoes a change Δ p in linear momentum because of the collision.
- The total change in linear momentum for n projectiles during interval Δ t is n Δ p.
- The resulting impulse on the target during Δ t is along the x axis and has the same magnitude of nΔp but is in the opposite direction.

$$\boxed{J = -n\,\Delta p,}$$

$$\boxed{F_{avg} = \frac{J}{\Delta t} = -\frac{n}{\Delta t}\Delta p = -\frac{n}{\Delta t} m\,\Delta v.}$$

- In time interval Δt, an amount of mass Δ m =nm collides with the target.

$$\boxed{F_{avg} = -\frac{\Delta m}{\Delta t}\Delta v.}$$

- If the particles stop:

$$\Delta v = v_f - v_i = 0 - v = -v,$$

- If the particles bounce back with equal speed

$$\Delta v = v_f - v_i = -v - v = -2v.$$

- Rocket and exhaust products form an isolated system.
- Conserve momentum.

The ejection of mass from the rocket's rear increases the rocket's speed.

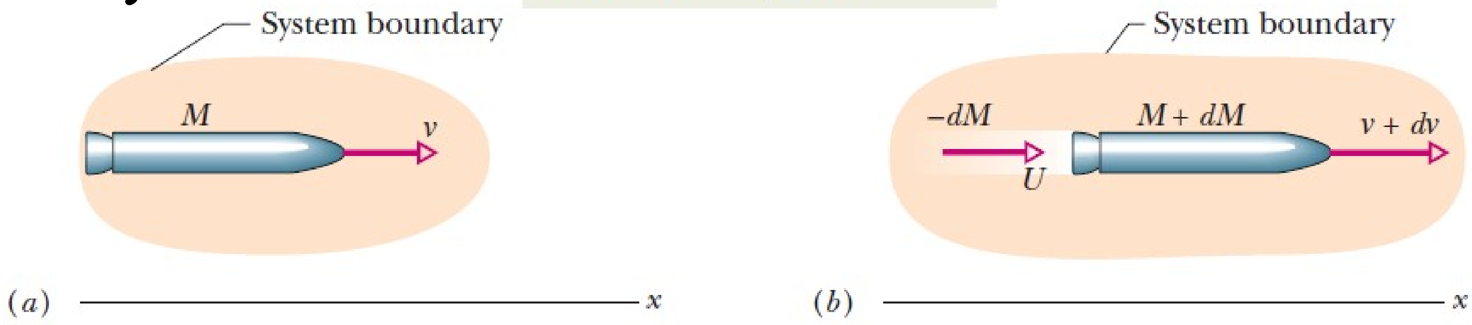


Fig. 9-22 (*a*) An accelerating rocket of mass M at time t, as seen from an inertial reference frame. (*b*) The same but at time $t + dt$. The exhaust products released during interval dt are shown.

- The system here consists of the rocket and the exhaust products released during interval dt. The system is closed and isolated, so the linear momentum of the system must be conserved during dt, where the subscripts i and f indicate the values at the beginning and end of time interval dt.

$$P_i = P_f, \quad \Longrightarrow \quad Mv = -dM\,U + (M + dM)(v + dv)$$

$$\begin{pmatrix}\text{velocity of rocket}\\ \text{relative to frame}\end{pmatrix} = \begin{pmatrix}\text{velocity of rocket}\\ \text{relative to products}\end{pmatrix} + \begin{pmatrix}\text{velocity of products}\\ \text{relative to frame}\end{pmatrix}$$

$$(v + dv) = v_{\text{rel}} + U, \qquad U = v + dv - v_{\text{rel}}. \quad \Longrightarrow \quad -\frac{dM}{dt} v_{\text{rel}} = M\frac{dv}{dt}. \quad \Longrightarrow \quad Rv_{\text{rel}} = Ma$$

$$dv = -v_{\text{rel}} \frac{dM}{M}.$$

$$\int_{v_i}^{v_f} dv = -v_{\text{rel}} \int_{M_i}^{M_f} \frac{dM}{M},$$

in which M_i is the initial mass of the rocket and M_f its final mass. Evaluating the integrals then gives

$$v_f - v_i = v_{\text{rel}} \ln \frac{M_i}{M_f}$$

for the increase in the speed of the rocket during the change in mass from M_i to M_f .

Sample problem: Rocket Engine, Thrust, Acceleration

A rocket whose initial mass M_i is 850 kg consumes fuel at the rate $R = 2.3$ kg/s. The speed v_{rel} of the exhaust gases relative to the rocket engine is 2800 m/s. What thrust does the rocket engine provide?

KEY IDEA

Thrust T is equal to the product of the fuel consumption rate R and the relative speed v_{rel} at which exhaust gases are expelled, as given by Eq. 9-87.

Calculation: Here we find

$$T = Rv_{rel} = (2.3 \text{ kg/s})(2800 \text{ m/s}) = 6440 \text{ N} \approx 6400 \text{ N}. \qquad \text{(Answer)}$$

(b) What is the initial acceleration of the rocket?

KEY IDEA

We can relate the thrust T of a rocket to the magnitude a of the resulting acceleration with $T = Ma$, where M is the rocket's mass. However, M decreases and a increases as fuel is consumed. Because we want the initial value of a here, we must use the intial value M_i of the mass.

Calculation: We find

$$a = \frac{T}{M_i} = \frac{6440 \text{ N}}{850 \text{ kg}} = 7.6 \text{ m/s}^2. \qquad \text{(Answer)}$$

To be launched from Earth's surface, a rocket must have an initial acceleration greater than $g = 9.8$ m/s². That is, it must be greater than the gravitational acceleration at the surface. Put another way, the thrust T of the rocket engine must exceed the initial gravitational force on the rocket, which here has the magnitude $M_i g$, which gives us

$$(850 \text{ kg})(9.8 \text{ m/s}^2) = 8330 \text{ N}.$$

Because the acceleration or thrust requirement is not met (here $T = 6400$ N), our rocket could not be launched from Earth's surface by itself; it would require another, more powerful, rocket.