01 Mathematical Logic

Subtopics

1.1 Statement

1.2 Logical Connectives, Compound Statements and Truth Tables

1.3 Statement Pattern and Logical Equivalence Tautology, Contradiction and Contingency

1.4 Quantifiers and Quantified Statements

1.5 Duality

1.6 Negation of Compound Statement

1.7 Algebra of Statements (Some Standard equivalent Statements)

1.8 Application of Logic to Switching Circuits

Type of Problems	Exercise	Q. Nos.
Identify the statements and write down their Truth Value	1.1	Q.1
	Miscellaneous	Q.1
Express the statements in Symbolic Form/Write the statement in Symbolic Form	1.2	Q.1
	1.4	Q.1, 2
	Miscellaneous	Q.5
Write the Truth values of Statements	1.2	Q.2
	1.4	Q.3, 5
	1.6	Q.1
	Miscellaneous	Q.2, 3, 9
Write the Negation of Statements/Using the Rules of Negation write the Negation of Statements	1.3	Q.1
	1.8	Q.1, 2, 4
	Miscellaneous	Q.4, 11, 22
Write the Verbal statement for the given Symbolic Statement	1.4	Q.6
	Miscellaneous	Q.6
Converse, Inverse and Contrapositive of the statement	1.4	Q.4
	Miscellaneous	Q.19, 21
Using Quantifiers Convert Open sentences into True statement	1.6	Q.2
Prepare the Truth Table/Find Truth Values of p and q for given cases	1.4	Q.7
	1.5	Q.1
	Miscellaneous	Q. 12, 15
Examine the statement Patterns (Tautology, Contradiction, Contingency)	1.5	Q.3
	Miscellaneous	Q.13, 14, 16
Using Truth Table, Verify Logical Equivalence	1.5	Q.2
	Miscellaneous	Q.7, 18
Write Dual of the statement	1.7	Q.1, 2, 3, 4
Algebra of statements (without using Truth Table verify the Logical Equivalence)/Rewrite the statement without using the conditional form	1.8	Q.3
	Miscellaneous	Q.8, 17, 20
Change the statements in the form if then	Miscellaneous	Q.10
Applications of logic to switching circuits	1.9	Q.1 to 5
	Miscellaneous	Q.23 to 29

Introduction

Mathematics is an exact science. Every mathematical statement must be precise. Hence, there has to be proper reasoning in every mathematical proof.

Proper reasoning involves logic. The study of logic helps in increasing one's ability of systematic and logical reasoning. It also helps to develop the skills of understanding various statements and their validity.

Logic has a wide scale application in circuit designing, computer programming etc. Hence, the study of logic becomes essential.

Statement and its truth value

There are various means of communication viz., verbal, written etc. Most of the communication involves the use of language whereby, the ideas are conveyed through sentences.

There are various types of sentences such as:

i. Declarative (Assertive)
ii. Imperative (A command or a request)
iii. Exclamatory (Emotions, excitement)
iv. Interrogative (Question)

Statement

A statement is a declarative sentence which is either true or false but not both simultaneously. Statements are denoted by the letters p, q, r….

For example:

i. 3 is an odd number.
ii. 5 is a perfect square.
iii. Sun rises in the east.
iv. $x + 3 = 6$, when $x = 3$.

Truth Value

A statement is either True or False. The Truth value of a 'true' statement is defined to be T (TRUE) and that of a 'false' statement is defined to be F (FALSE).

Note: 0 and 1 can also be used for T and F respectively.

Consider the following statements:

i. There is no prime number between 23 and 29.
ii. The Sun rises in the west.
iii. The square of a real number is negative.
iv. The sum of the angles of a plane triangle is 180°.

Here, the truth value of statement i. and iv. is T and that of ii. and iii. is F.

Note:
The sentences like exclamatory, interrogative, imperative etc., are not considered as statements as the truth value for these statements cannot be determined.

Open sentence

An open sentence is a sentence whose truth can vary according to some conditions, which are not stated in the sentence.

Note:
Open sentence is not considered as statement in logic.

For example:

i. $x \times 5 = 20$
This is an open sentence as its truth depends on value of x (if $x = 4$, it is true and if $x \neq 4$, it is false).

ii. Chinese food is very tasty.
This is an open sentence as its truth varies from individual to individual.

Exercise 1.1

State which of the following sentences are statements. Justify your answer. In case of the statements, write down the truth value.

i. The Sun is a star.
ii. May God bless you!
iii. The sum of interior angles of a triangle is 180°.
iv. Every real number is a complex number.
v. Why are you upset?
vi. Every quadratic equation has two real roots.
vii. $\sqrt{-9}$ is a rational number.
viii. $x^2 - 3x + 2 = 0$, implies that $x = -1$ or $x = -2$.
ix. The sum of cube roots of unity is one.
x. Please get me a glass of water.
xi. He is a good person.
xii. Two is the only even prime number.
xiii. $\sin 2\theta = 2\sin\theta\cos\theta$ for all $\theta \in R$.
xiv. What a horrible sight it was!
xv. Do not disturb.
xvi. $x^2 - 3x - 4 = 0, x = -1$.
xvii. Can you speak in French?
xviii. The square of every real number is positive.
xix. It is red in colour.
xx. Every parallelogram is a rhombus.

Solution:

i. It is a statement which is true, hence its truth value is 'T'.
ii. It is an exclamatory sentence, hence, it is not a statement.
iii. It is a statement which is true, hence its truth value is 'T'.
iv. It is a statement which is true, hence its truth value is 'T'.
v. It is an interrogative sentence, hence it is not a statement.
vi. It is a statement which is false, hence its truth value is 'F'.
vii. It is a statement which is false, hence its truth value is 'F'.
viii. It is a statement which is false, hence its truth value is 'F'.
ix. It is a statement which is false, hence its truth value is 'F'.
x. It is an imperative sentence, hence it is not a statement.
xi. It is an open sentence, hence it is not a statement.
xii. It is a statement which is true, hence its truth value is 'T'.
xiii. It is a statement which is true, hence its truth value is 'T'.
xiv. It is an exclamatory sentence, hence it is not a statement.
xv. It is an imperative sentence, hence it is not a statement.
xvi. It is a statement which is true, hence its truth value is 'T'.
xvii. It is an interrogative sentence, hence, it is not a statement.
xviii. It is a statement which is false, hence its truth value is 'F'. (Since, 0 is a real number and square of 0 is 0 which is neither positive nor negative).
xix. It is an open sentence, hence it is not a statement. (The truth of this sentence depends upon the reference for the pronoun 'It'.)
xx. It is a statement which is false, hence its truth value is 'F'.

Logical Connectives, Compound Statements and Truth Tables

Logical Connectives:
The words or group of words such as "and, or, if …. then, if and only if, not" are used to join or connect two or more simple sentences. These connecting words are called logical connectives.

Compound Statements:
The new statement that is formed by combining two or more simple statements by using logical connectives are called compound statements.

Component Statements:
The simple statements that are joined using logical connectives are called component statements.

For example:
Consider the following simple statements,
i. e is a vowel
ii. b is a consonant

These two component statements can be joined by using the logical connective 'or' as shown below:
'e is a vowel or b is a consonant'
The above statement is called **compound statement** formed by using logical connective **'or'.**

Truth Table

A table that shows the relationship between truth values of simple statements and the truth values of compounds statements formed by using these simple statements is called truth table.

Note:
The truth value of a compoud statement depends upon the truth values of its component statements.

Logical Connectives

A. AND [$\wedge$] (Conjunction):
If p and q are any two statements connected by the word 'and', then the resulting compound statement 'p and q' is called conjunction of p and q which is written in the symbolic form as '$p \wedge q$'.

For example:
p: Today is a pleasant day.
q: I want to go for shopping.
The conjunction of above two statements is '$p \wedge q$' i.e. 'Today is a pleasant day and I want to go for shopping'.
A conjunction is true if and only if both p and q are true.

Truth table for conjunction of p and q is as shown below:

p	q	$p \wedge q$
T	**T**	**T**
T	F	F
F	T	F
F	F	F

Note:
The words such as but, yet, still, inspite, though, moreover are also used to connect the simple statements.
These words are generally used by replacing 'and'.

B. OR [$\vee$] (Disjunction):
If p and q are any two statements connected by the word 'or', then the resulting compound statement 'p or q' is called disjunction of p and q which is written in the symbolic form as '$p \vee q$'.
The word 'or' is used in English language in two distinct senses, exclusive and inclusive.
For example:
i. Rahul will pass or fail in the exam.
ii. Candidate must be graduate or post-graduate.

In eg. (i), 'or' indicates that only one of the two possibilities exists but not both which is called exclusive sense of 'or'. In eg. (ii), 'or' indicates that first or second or both the possibilities may exist which is called inclusive sense of 'or'.
A disjunction is false only when both p and q are false.

Truth table for disjunction of p and q is as shown below:

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

Exercise 1.2

1. Express the following statements in symbolic form:
i. Mango is a fruit but potato is a vegetable.
ii. Either we play football or go for cycling.
iii. Milk is white or grass is green.
iv. Inspite of physical disability, Rahul stood first in the class.
v. Jagdish stays at home while Shrijeet and Shalmali go for a movie.

***Solution*:**
i. Let p : Mango is a fruit, q : Potato is a vegetable.
∴ The symbolic form of the given statement is $p \wedge q$.

ii. Let p : We play football, q : We go for cycling.
∴ The symbolic form of the given statement is $p \vee q$.

iii. Let p : Milk is white, q : Grass is green.
∴ The symbolic form of the given statement is $p \vee q$.

iv. Let p : Rahul has physical disability,
q : Rahul stood first in the class.
The given statement can be considered as 'Rahul has physical disability and he stood first in the class.'
∴ The symbolic form of the given statement is $p \wedge q$.

v. Let p : Jagdish stays at home,
q : Shrijeet and Shalmali go for a movie.
The given statement can be considered as 'Jagdish stays at home and Shrijeet and Shalmali go for a movie.'
∴ The symbolic form of the given statement is $p \wedge q$.

2. Write the truth values of following statements.
i. $\sqrt{3}$ is a rational number or 3 + i is a complex number.
ii. Jupiter is a planet and Mars is a star.
iii. $2 + 3 \neq 5$ or $2 \times 3 < 5$
iv. $2 \times 0 = 2$ and $2 + 0 = 2$
v. 9 is a perfect square but 11 is a prime number.
vi. Moscow is in Russia or London is in France.

***Solution*:**
i. Let p : $\sqrt{3}$ is a rational number,
q : 3 + i is a complex number.
∴ The symbolic form of the given statement is $p \vee q$.
Since, truth value of p is F and that of q is T.
∴ truth value of $p \vee q$ is T

ii. Let p : Jupiter is a planet,
q : Mars is a star.
∴ The symbolic form of the given statement is $p \wedge q$.
Since, truth value of p is T and that of q is F.
∴ truth value of $p \wedge q$ is F

iii. Let $p : 2 + 3 \neq 5$,
$q : 2 \times 3 < 5$.

$\therefore$ The symbolic form of the given statement is $p \vee q$.
Since, truth value of both p and q is F.

$\therefore$ truth value of $p \vee q$ is F

iv. Let $p : 2 \times 0 = 2$,
$q : 2 + 0 = 2$.

$\therefore$ The symbolic form of the given statement is $p \wedge q$.
Since, truth value of p is F and that of q is T.

$\therefore$ truth value of $p \wedge q$ is F

v. Let p : 9 is a perfect square,
q : 11 is a prime number.

$\therefore$ The symbolic form of the given statement is $p \wedge q$.
Since, truth value of both p and q is T.

$\therefore$ truth value of $p \wedge q$ is T

vi. Let p : Moscow is in Russia,
q : London is in France.

$\therefore$ The symbolic form of the given statement is $p \vee q$.
Since, truth value of p is T and that of q is F.

$\therefore$ truth value of $p \vee q$ is T

C. Not [~] (Negation):

If p is any statement then negation of p i.e., 'not p' is denoted by ~p. Negation of any simple statement p can also be formed by writing 'It is not true that' or 'It is false that', before p.

For example:

p : Mango is a fruit.
~p : Mango is not a fruit.

Truth table for negation is as shown below:

p	~p
T	F
F	T

Note: If a statement is true its negation is false and vice-versa.

Exercise 1.3

Write negations of the following statements:

i. **Rome is in Italy.**
ii. **5 + 5 = 10**
iii. **3 is greater than 4.**
iv. **John is good in river rafting.**
v. **π is an irrational number.**
vi. **The square of a real number is positive.**
vii. **Zero is not a complex number.**
viii. **Re (z) ≤ | z |.**
ix. **The sun sets in the East.**
x. **It is not true that the mangoes are inexpensive.**

***Solution*:**

i. Rome is not in Italy.
ii. $5 + 5 \neq 10$
iii. 3 is not greater than 4.
iv. John is not good in river rafting.
v. π is not an irrational number.
vi. The square of a real number is not positive.
vii. Zero is a complex number.
viii. Re (z) > |z|.
ix. The sun does not set in the East.
x. It is true that the mangoes are inexpensive.

D. If….then (Implication, ⟶) (Conditional):

If p and q are any two simple statements, then the compound statement, 'if p then q', meaning "statement p implies statement q or statement q is implied by statement p", is called a conditional statement and is denoted by $p \rightarrow q$ or $p \Rightarrow q$.

Here p is called the antecedent (hypothesis) and q is called the consequent (conclusion).

For example:

Let p: I travel by train.
q: My journey will be cheaper.

Here the conditional statement is

'$p \rightarrow q$: If I travel by train then my journey will be cheaper.'

Conditional statement is false if and only if antecedent is true and consequent is false.

Truth table for conditional is as shown below:

p	q	$p \rightarrow q$
T	T	T
T	**F**	**F**
F	T	T
F	F	T

Note: Equivalent forms of the conditional statement

$p \rightarrow q$:

a. p is sufficient for q.
b. q is necessary for p.
c. p implies q.
d. p only if q.
e. q follows from p.

E. Converse, Inverse and Contrapositive statements:

If $p \rightarrow q$ is given, then its

converse is $q \rightarrow p$

inverse is $\sim p \rightarrow \sim q$

contrapositive is $\sim q \rightarrow \sim p$

For example:

Let p : Smita is intelligent.

q : Smita will join Medical.

i. $q \rightarrow p$: If Smita joins Medical then she is intelligent.

ii. $\sim p \rightarrow \sim q$: If Smita is not intelligent then she will not join Medical.

iii. $\sim q \rightarrow \sim p$: If Smita does not join Medical then she is not intelligent.

Consider, the following truth table:

p	q	$p\rightarrow q$	$\sim p$	$\sim q$	$q\rightarrow p$	$\sim q\rightarrow\sim p$	$\sim p\rightarrow\sim q$
T	T	T	F	F	T	T	T
T	F	F	F	T	T	F	T
F	T	T	T	F	F	T	F
F	F	T	T	T	T	T	T

From the above table, we conclude that

i. a conditional statement and its contrapositive are always equivalent.

ii. converse and inverse of the conditional statement are always equivalent.

F. If and only if (Double Implication, $\leftrightarrow$) (Biconditional):

If p and q are any two statements, then 'p if and only if q' or 'p iff q' is called the biconditional statement and is denoted by $p \leftrightarrow q$. Here, both p and q are called implicants.

For example:

Let p : price increases

q : demand falls

Here the Biconditional statement is

'$p \leftrightarrow q$: Price increases if and only if demand falls'.

A biconditional statement is true if and only if both the implicants have same truth value.

Truth table for biconditional is as shown below:

p	q	$p \leftrightarrow q$
T	**T**	**T**
T	F	F
F	T	F
F	**F**	**T**

Exercise 1.4

1. Express the following in symbolic form.

i. I like playing but not singing.

ii. Anand neither likes cricket nor tennis.

iii. Rekha and Rama are twins.

iv. It is not true that 'i' is a real number.

v. Either 25 is a perfect square or 41 is divisible by 7.

vi. Rani never works hard yet she gets good marks.

vii. Eventhough it is not cloudy, it is still raining.

***Solution*:**

i. Let p: I like playing, q: I like singing,

∴ The symbolic form of the given statement is $p \wedge \sim q$.

ii. Let p: Anand likes cricket, q: Anand likes tennis.

∴ The symbolic form of the given statement is $\sim p \wedge \sim q$.

iii. In this statement 'and' is combining two nouns and not two simple statements.
Hence, it is not used as a connective, so given statement is a simple statement which can be symbolically expressed as p itself.

iv. Let, p : 'i' is a real number.

∴ The symbolic form of the given statement is $\sim p$.

v. Let p : 25 is a perfect square,

q : 41 is divisible by 7.

∴ The symbolic form of the given statement is $p \vee q$.

vi. Let p : Rani works hard, q : Rani gets good marks.

∴ The symbolic form of the given statement is $\sim p \wedge q$.

vii. Let p : It is cloudy, q : It is still raining.

∴ The symbolic form of the given statement is $\sim p \wedge q$.

---

2. If p: girls are happy, q: girls are playing, express the following sentences in symbolic form.

i. Either the girls are happy or they are not playing.

ii. Girls are unhappy but they are playing.

iii. It is not true that the girls are not playing but they are happy.

Solution:

i. $p \vee \sim q$ ii. $\sim p \wedge q$

iii. $\sim(\sim q \wedge p)$

3. Find the truth value of the following statements.

i. 14 is a composite number or 15 is a prime number.

ii. Neither 21 is a prime number nor it is divisible by 3.

iii. It is not true that 4+3i is a real number.

iv. 2 is the only even prime number and 5 divides 26.

v. Either 64 is a perfect square or 46 is a prime number.

vi. 3 + 5 > 7 if and only if 4 + 6 < 10.

Solution:

i. Let p : 14 is a composite number,
q : 15 is a prime number.

∴ The symbolic form of the given statement is $p \vee q$.
Since, truth value of p is T and that of q is F.

∴ truth value of $p \vee q$ is T.

ii. Let p: 21 is a prime number,
q: 21 is divisible by 3.

∴ The symbolic form of the given statement is $\sim p \wedge \sim q$.
Since, truth value of p is F and that of q is T

∴ truth value of $\sim p \wedge \sim q$ is F.

iii. Let p: 4 + 3i is a real number.

∴ The symbolic form of the given statement is $\sim p$.
Since, truth value of p is F.

∴ truth value of $\sim p$ is T.

iv. Let p: 2 is the only even prime number,
q: 5 divides 26.

∴ The symbolic form of the given statement is $p \wedge q$.
Since, truth value of p is T and that of q is F

∴ truth value of $p \wedge q$ is F.

v. Let p: 64 is a perfect square,
q: 46 is a prime number.

∴ The symbolic form of the given statement is $p \vee q$.
Since, truth value of p is T and that of q is F.

∴ truth value of $p \vee q$ is T

vi. Let p: $3 + 5 > 7$, q: $4 + 6 < 10$

∴ The symbolic form of the given statement is $p \leftrightarrow q$.
Since, truth value of p is T and that of q is F.

∴ truth value of $p \leftrightarrow q$ is F

4. State the converse, inverse and contrapositive of the following conditional statements:

i. If it rains then the match will be cancelled.

ii. If a function is differentiable then it is continuous.

iii. If surface area decreases then the pressure increases.

iv. If a sequence is bounded then it is convergent.

Solution:

i. Let p : It rains, q : the match will be cancelled.

∴ The symbolic form of the given statement is $p \rightarrow q$.
Converse: $q \rightarrow p$
i.e., If the match is cancelled then it rains.
Inverse: $\sim p \rightarrow \sim q$
i.e., If it does not rain then the match will not be cancelled.
Contrapositive: $\sim q \rightarrow \sim p$
i.e. If the match is not cancelled then it does not rain.

ii. Let p: A function is differentiable,
q: It is continuous.

∴ The symbolic form of the given statement is $p \rightarrow q$.
Converse: $q \rightarrow p$
i.e. If a function is continuous then it is differentiable.
Inverse: $\sim p \rightarrow \sim q$
i.e. If a function is not differentiable then it is not continuous.
Contrapositive: $\sim q \rightarrow \sim p$
i.e. If a function is not continuous then it is not differentiable.

iii. Let p: Surface area decreases,
q: The pressure increases.

∴ The symbolic form of the given statement is $p \rightarrow q$
Converse: $q \rightarrow p$
i.e. If the pressure increases then the surface area decreases.

Inverse: $\sim p \to \sim q$

i.e. If the surface area does not decrease then the pressure does not increase.

Contrapositive: $\sim q \to \sim p$

i.e. If the pressure does not increase then the surface area does not decrease.

iv. Let p: A sequence is bounded,

q: It is convergent.

$\therefore$ The symbolic form of the given statement is $p \to q$

Converse: $q \to p$

i.e. If a sequence is convergent then it is bounded.

Inverse: $\sim p \to \sim q$

i.e. If a sequence is not bounded then it is not convergent.

Contrapositive: $\sim q \to \sim p$

i.e. If a sequence is not convergent then it is not bounded.

5. If p and q are true and r and s are false statements, find the truth value of the following statements:

i. $(p \wedge q) \vee r$

ii. $p \wedge (r \to s)$

iii. $(p \vee s) \leftrightarrow (q \wedge r)$

iv. $\sim (p \wedge \sim r) \vee (\sim q \vee s)$

***Solution*:**

Given that p and q are T and r and s are F.

i. $(p \wedge q) \vee r$

$\equiv (T \wedge T) \vee F$

$\equiv T \vee F$

$\equiv T$

$\therefore$ truth value of the given statement is true.

ii. $p \wedge (r \to s)$

$\equiv T \wedge (F \to F)$

$\equiv T \wedge T$

$\equiv T$

$\therefore$ truth value of the given statement is true.

iii. $(p \vee s) \leftrightarrow (q \wedge r)$

$\equiv (T \vee F) \leftrightarrow (T \wedge F)$

$\equiv T \leftrightarrow F$

$\equiv F$

$\therefore$ truth value of the given statement is false.

iv. $\sim(p \wedge \sim r) \vee (\sim q \vee s)$

$\equiv \sim (T \wedge T) \vee (F \vee F)$

$\equiv \sim (T) \vee F$

$\equiv F \vee F$

$\equiv F$

$\therefore$ truth value of the given statement is false.

6. If p: It is daytime, q: It is warm
Give the compound statements in verbal form denoted by

i. $p \wedge \sim q$ [Oct 14]

ii. $p \vee q$

iii. $p \to q$

iv. $q \leftrightarrow p$ [Oct 14]

***Solution*:**

i. It is daytime but it is not warm. ***[1 Mark]***

ii. It is daytime or it is warm.

iii. If it is daytime then it is warm.

iv. It is warm if and only if it is daytime. ***[1 Mark]***

7. Prepare the truth tables for the following:

i. $\sim p \wedge q$ ii. $p \to (p \vee q)$

iii. $\sim p \leftrightarrow q$

***Solution*:**

i. $\sim p \wedge q$

p	q	$\sim p$	$\sim p \wedge q$
T	T	F	F
T	F	F	F
F	T	T	T
F	F	T	F

ii. $p \to (p \vee q)$

p	q	$p \vee q$	$p \to (p \vee q)$
T	T	T	T
T	F	T	T
F	T	T	T
F	F	F	T

iii. $\sim p \leftrightarrow q$

p	q	$\sim p$	$\sim p \leftrightarrow q$
T	T	F	F
T	F	F	T
F	T	T	T
F	F	T	F

Statement Pattern and Logical Equivalence

A. Statement Pattern

Let, p, q, r,… be simple statements. A compound statement obtained from these simple statements and by using one or more connectives $\wedge$, $\vee$, $\sim$, $\rightarrow$, $\leftrightarrow$ is called a statement pattern.

Following points must be noted while preparing truth tables of the statement patterns:

i. Parentheses must be introduced wherever necessary.

For example:

$\sim(p \wedge q)$ and $\sim p \wedge q$ are not the same.

ii. If a statement pattern consists of 'n' statements and 'm' connectives, then truth table consists of 2^n rows and $(m + n)$ columns.

B. Logical equivalence

Two logical statements are said to be equivalent if and only if the truth values in their respective columns in the joint truth table are identical.

If S_1 and S_2 are logically equivalent statement patterns, we write

$S_1 \equiv S_2$.

For example:

To prove: $p \wedge q \equiv \sim(p \rightarrow \sim q)$ **[Mar 08]**

p	q	$p \wedge q$	$\sim q$	$(p \rightarrow \sim q)$	$\sim(p \rightarrow \sim q)$
T	T	T	F	F	T
T	F	F	T	T	F
F	T	F	F	T	F
F	F	F	T	T	F

In the above truth table, all the entries in the columns of $p \wedge q$ and $\sim(p \rightarrow \sim q)$ are identical.

$\therefore$ **$p \wedge q \equiv \sim(p \rightarrow \sim q)$.**

Note:

i. $\sim(p \vee q) \equiv \sim p \wedge \sim q$ (De-Morgan's 1st Law) **[Mar 96]**

identical

p	q	$\sim p$	$\sim q$	$p \vee q$	$\sim(p \vee q)$	$\sim p \wedge \sim q$
T	T	F	F	T	F	F
T	F	F	T	T	F	F
F	T	T	F	T	F	F
F	F	T	T	F	T	T

ii. $\sim(p \wedge q) \equiv \sim p \vee \sim q$ (De-Morgan's 2nd Law)

identical

p	q	$\sim p$	$\sim q$	$p \wedge q$	$\sim(p \wedge q)$	$\sim p \vee \sim q$
T	T	F	F	T	F	F
T	F	F	T	F	T	T
F	T	T	F	F	T	T
F	F	T	T	F	T	T

iii. $p \rightarrow q \equiv (\sim p) \vee q$

identical

p	q	$\sim p$	$p \rightarrow q$	$\sim p \vee q$
T	T	F	T	T
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

iv. $p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$ **[Mar 98, Oct 00, 01,04]**

identical

p	q	$p \rightarrow q$	$q \rightarrow p$	$p \leftrightarrow q$	$(p \rightarrow q) \wedge (q \rightarrow p)$
T	T	T	T	T	T
T	F	F	T	F	F
F	T	T	F	F	F
F	F	T	T	T	T

Tautology, Contradiction and Contingency

Tautology

A statement pattern having truth value always T, irrespective of the truth values of its component statement is called Tautology.

For example, consider $(p \leftrightarrow q) \leftrightarrow (q \leftrightarrow p)$

p	q	$p \leftrightarrow q$	$q \leftrightarrow p$	$(p \leftrightarrow q) \leftrightarrow (q \leftrightarrow p)$
T	T	T	T	T
T	F	F	F	T
F	T	F	F	T
F	F	T	T	T

In the above truth table, all the entries in the last column are T.

$\therefore$ The given statement pattern is a tautology.

Contradiction

A statement pattern having truth value always F, irrespective of the truth values of its component statement is called a Contradiction.

For example, consider p ∧ ~ p

p	~ p	p ∧ ~ p
T	F	F
F	T	F

In the above truth table, all the entries in the last column are F.

∴ The given statement pattern is a contradiction.

Contingency

A statement pattern which is neither a tautology nor a contradiction is called Contingency.

For example, consider (p ↔ q) ∧ ~(p → ~ q)

p	q	p ↔ q	~q	p →~q	~(p → ~q)	(p ↔ q) ∧ ~(p → ~q)
T	T	T	F	F	T	T
T	F	F	T	T	F	F
F	T	F	F	T	F	F
F	F	T	T	T	F	F

In the above truth table, the entries in the last column are a combination of T and F.

∴ The given statement pattern is neither a tautology nor a contradiction, it is a contingency.

Exercise 1.5

1. **Prepare the truth table of the following statement patterns:**
   i. **[(p → q) ∧ q] → p**
   ii. **(p ∧ q) → (~p)**
   iii. **(p → q) ↔ (~p ∨ q)**
   iv. **(p ↔ r) ∧ (q ↔ p)**
   v. **(p ∨ ~q) → (r ∧ p)**

***Solution*:**

i. [(p → q) ∧ q] → p

p	q	p → q	(p → q) ∧ q	[(p → q) ∧ q] → p
T	T	T	T	T
T	F	F	F	T
F	T	T	T	F
F	F	T	F	T

Note:
Here, the statement pattern consists of 2 statements and 3 connectives. Therefore, the truth table has,
Number of rows = $2^2 = 4$
and number of columns = (2+3) = 5.

ii. (p ∧ q) → (~p)

p	q	p ∧ q	~p	(p ∧ q) → (~p)
T	T	T	F	F
T	F	F	F	T
F	T	F	T	T
F	F	F	T	T

iii. (p → q) ↔ (~p ∨ q)

p	q	p → q	~p	~p ∨ q	(p → q) ↔ (~p ∨ q)
T	T	T	F	T	T
T	F	F	F	F	T
F	T	T	T	T	T
F	F	T	T	T	T

iv. (p ↔ r) ∧ (q ↔ p)

p	q	r	p ↔ r	q ↔ p	(p ↔ r) ∧ (q ↔ p)
T	T	T	T	T	T
T	T	F	F	T	F
T	F	T	T	F	F
T	F	F	F	F	F
F	T	T	F	F	F
F	T	F	T	F	F
F	F	T	F	T	F
F	F	F	T	T	T

v. (p ∨ ~q) → (r ∧ p)

p	q	r	~q	p ∨ ~ q	r ∧ p	(p ∨ ~q) → (r ∧ p)
T	T	T	F	T	T	T
T	T	F	F	T	F	F
T	F	T	T	T	T	T
T	F	F	T	T	F	F
F	T	T	F	F	F	T
F	T	F	F	F	F	T
F	F	T	T	T	F	F
F	F	F	T	T	F	F

2. **Using truth tables, prove the following logical equivalences:**
   i. **(p ∧ q) ≡ ~ (p → ~ q)**
   ii. **p ↔ q ≡ (p ∧ q) ∨ (~p ∧ ~q)** **[Mar 13; Oct 15]**
   iii. **(p ∧ q) → r ≡ p → (q → r)** **[Oct 14]**
   iv. **p ∨ (q ∧ r) ≡ (p ∨ q) ∧ (p ∨ r)**

***Solution*:**

i.

1	2	3	4	5	6
p	q	$p \wedge q$	~q	$p \to$ ~q	~ (p $\to$ ~q)
T	T	T	F	F	T
T	F	F	T	T	F
F	T	F	F	T	F
F	F	F	T	T	F

The entries in the columns 3 and 6 are identical.

$\therefore$ $(p \wedge q) \equiv \sim (p \to \sim q)$

ii.

1	2	3	4	5	6	7	8
p	q	$p \leftrightarrow q$	~p	~q	$p \wedge q$	$\sim p \wedge \sim q$	$(p \wedge q) \vee (\sim p \wedge \sim q)$
T	T	T	F	F	T	F	T
T	F	F	F	T	F	F	F
F	T	F	T	F	F	F	F
F	F	T	T	T	F	T	T

The entries in columns 3 and 8 are identical.

$\therefore$ $p \leftrightarrow q \equiv (p \wedge q) \vee (\sim p \wedge \sim q)$ ***[1 Mark]***

[1 mark each for column 3 and column 8]

iii.

1	2	3	4	5	6	7
p	q	r	$p \wedge q$	$q \to r$	$(p \wedge q) \to r$	$p \to (q \to r)$
T	T	T	T	T	T	T
T	T	F	T	F	F	F
T	F	T	F	T	T	T
T	F	F	F	T	T	T
F	T	T	F	T	T	T
F	T	F	F	F	T	T
F	F	T	F	T	T	T
F	F	F	F	T	T	T

The entries in the columns 6 and 7 are identical.

$\therefore$ $(p \wedge q) \to r \equiv p \to (q \to r)$ ***[1 Mark]***

[1 mark each for column 6 and column 7]

iv.

1	2	3	4	5	6	7	8
p	q	r	$q \wedge r$	$p \vee (q \wedge r)$	$p \vee q$	$p \vee r$	$(p \vee q) \wedge (p \vee r)$
T	T	T	T	T	T	T	T
T	T	F	F	T	T	T	T
T	F	T	F	T	T	T	T
T	F	F	F	T	T	T	T
F	T	T	T	T	T	T	T
F	T	F	F	F	T	F	F
F	F	T	F	F	F	T	F
F	F	F	F	F	F	F	F

The entries in the columns 5 and 8 are identical

$\therefore$ $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$

---

3. Using truth tables examine whether the following statement patterns are tautology, contradiction or contingency.

i. $(p \wedge \sim q) \leftrightarrow (p \to q)$ [Mar 13]

ii. $(\sim p \wedge q) \wedge (q \to p)$

iii. $(p \wedge q) \vee (p \wedge r)$

iv. $[(p \vee q) \vee r] \leftrightarrow [p \vee (q \vee r)]$

v. $(p \vee q) \wedge (p \vee r)$

***Solution*:**

i.

1	2	3	4	5	6
p	q	~q	$p \wedge \sim q$	$p \to q$	$(p \wedge \sim q) \leftrightarrow (p \to q)$
T	T	F	F	T	F
T	F	T	T	F	F
F	T	F	F	T	F
F	F	T	F	T	F

In the above truth table, all the entries in the last column are F.

$\therefore$ $(p \wedge \sim q) \leftrightarrow (p \to q)$ is a contradiction. ***[1 Mark]***

[1 mark each for column 5 and column 6]

ii.

p	q	~p	$\sim p \wedge q$	$q \to p$	$(\sim p \wedge q) \wedge (q \to p)$
T	T	F	F	T	F
T	F	F	F	T	F
F	T	T	T	F	F
F	F	T	F	T	F

In the above truth table, all the entries in the last column are F.

$\therefore$ $(\sim p \wedge q) \wedge (q \to p)$ is a contradiction.

iii.

p	q	r	$p \wedge q$	$p \wedge r$	$(p \wedge q) \vee (p \wedge r)$
T	T	T	T	T	T
T	T	F	T	F	T
T	F	T	F	T	T
T	F	F	F	F	F
F	T	T	F	F	F
F	T	F	F	F	F
F	F	T	F	F	F
F	F	F	F	F	F

In the above truth table, the entries in the last column are a combination of T and F.

$\therefore$ $(p \wedge q) \vee (p \wedge r)$ is a contingency.

iv.

p	q	r	$p \vee q$	$q \vee r$	$(p \vee q) \vee r$	$p \vee (q \vee r)$	$[(p\vee q)\vee r] \leftrightarrow [p\vee(q \vee r)]$
T	T	T	T	T	T	T	T
T	T	F	T	T	T	T	T
T	F	T	T	T	T	T	T
T	F	F	T	F	T	T	T
F	T	T	T	T	T	T	T
F	T	F	T	T	T	T	T
F	F	T	F	T	T	T	T
F	F	F	F	F	F	F	T

In the above truth table, all the entries in the last column are T.

$\therefore$ $[(p \vee q) \vee r] \leftrightarrow [p \vee (q \vee r)]$ is a tautology.

v.

p	q	r	$p \vee q$	$p \vee r$	$(p \vee q) \wedge (p \vee r)$
T	T	T	T	T	T
T	T	F	T	T	T
T	F	T	T	T	T
T	F	F	T	T	T
F	T	T	T	T	T
F	T	F	T	F	F
F	F	T	F	T	F
F	F	F	F	F	F

In the above truth table, the entries in the last column are a combination of T and F.

$\therefore$ $(p \vee q) \wedge (p \vee r)$ is a contingency.

Quantifiers and Quantified Statements

Quantifiers

Quantifiers are the symbols used to denote a group of words or a phrase. Generally, two types of quantifiers are used. They are as follows:

i. Universal Quantifier:

The symbol '$\forall$' stands for "all values of" and is known as universal quantifier.

For example, Consider A = {1, 2, 3}

Let p: $\forall\, x \in A, x < 4$

Here, the statement p uses the quantifier 'for all'($\forall$).

This statement is true if and only if each and every element of set A satisfies the condition '$x < 4$' and is false otherwise.

Here, the given statement is true for all the elements of set A, as 1, 2, 3 satisfy the condition, '$x \in A, x < 4$'.

ii. Existential Quantifier:

The symbol '$\exists$' stands for 'there exists' and is known as existential quantifier.

For example, Consider A = {4, 14, 66, 70}.

Let p: $\exists\, x \in A$ such that x is an odd number.

Here, the statement p uses the quantifier 'there exists' ($\exists$).

This statement is true if atleast one element of set A satisfies the condition 'x is an odd number' and is false otherwise.

Here, the given statement is false as none of the elements of set A satisfy the condition,

'$x \in A$ such that x is an odd number'.

Quantified statement

The statement containing quantifiers is known as quantified statement. Generally, an open sentence with a quantifier becomes a statement and is called quantified statement.

For example:

Use quantifiers to convert open sentence $x + 2 < 4$ into a statement.

***Solution*:**

$\exists\ x \in N$ such that $x + 2 < 4$, is a true statement, since $x = 1 \in N$ satisfies $x + 2 < 4$.

Exercise 1.6

1. If A = {3, 4, 6, 8}, determine the truth value of each of the following:

i. $\exists\, x \in A$, **such that** $x + 4 = 7$

ii. $\forall\, x \in A, x + 4 < 10$.

iii. $\forall\, x \in A, x + 5 \geq 13$.

iv. $\exists\, x \in A$, **such that** x **is odd.**

v. $\exists\, x \in A$, **such that** $(x - 3) \in N$

Solution:

i. Since $x = 3 \in$ A, satisfies $x + 4 = 7$
$\therefore$ the given statement is true.
$\therefore$ Its truth value is 'T'.

ii. Since $x = 6, 8 \in$ A, do not satisfy $x + 4 < 10$,
$\therefore$ the given statement is false.
$\therefore$ Its truth value is 'F'

iii. Since $x = 3, 4, 6 \in$ A, do not satisfy $x + 5 \geq 13$,
$\therefore$ the given statement is false.
$\therefore$ Its truth value is 'F'.

iv. Since $x = 3 \in$ A, satisfies the given statement,
$\therefore$ the given statement is true.
$\therefore$ Its truth value is 'T'.

v. Since $x = 4, 6, 8 \in$ A, satisfy $(x - 3) \in$ N,
$\therefore$ the given statement is true.
$\therefore$ Its truth value is 'T'.

---

2. Use quantifiers to convert each of the following open sentences defined on N, into a true statement:

i. $x^2 = 25$ **ii.** $2x + 3 < 15$
iii. $x - 3 = 11$ **iv.** $x^2 + 1 \leq 5$
v. $x^2 - 3x + 2 = 0$

Solution:

i. $\exists\, x \in$ N, such that $x^2 = 25$.
This is a true statement since $x = 5 \in$ N satisfies $x^2 = 25$.

ii. $\exists\, x \in$ N, such that $2x + 3 < 15$.
This is a true statement since $x = 1, 2, 3, 4, 5 \in$ N satisfy $2x + 3 < 15$.

iii. $\exists\, x \in$ N such that $x - 3 = 11$.
This is a true statement since $x = 14 \in$ N satisfies $x - 3 = 11$.

iv. $\exists\, x \in$ N, such that $x^2 + 1 \leq 5$.
This is a true statement since $x = 1, 2 \in$ N satisfy $x^2 + 1 \leq 5$.

v. $\exists\, x \in$ N such that $x^2 - 3x + 2 = 0$.
This is a true statement since $x = 1, 2 \in$ N satisfy $x^2 - 3x + 2 = 0$.

Duality

Two compound statements S_1 and S_2 are said to be duals of each other, if one can be obtained from the other by interchanging '$\wedge$' and '$\vee$' and vice-versa. The connectives '$\wedge$' and '$\vee$' are duals of each other. Also, a dual is obtained by replacing t by c and c by t, where 't' denotes tautology and 'c' denotes contradiction.

Remarks:

i. The symbol '~' is not changed while finding the dual.
ii. Dual of a dual is the statement itself.
iii. The special statements 't' (tautology) and 'c' (contradiction) are duals of each other.
iv. T is changed to F and vice-versa.

Principle of Duality:

If a compound statement S_1 contains only ~, $\wedge$ and $\vee$ and statement S_2 arises from S_1 by replacing $\wedge$ by $\vee$ and $\vee$ by $\wedge$, then S_1 is a tautology if and only if S_2 is a contradiction.

Exercise 1.7

1. Write the duals of the following statements:

i. $(p \wedge q) \vee r$
ii. $T \vee (p \vee q)$
iii. $p \wedge [\sim q \vee (p \wedge q) \vee \sim r]$

Solution:

i. $(p \vee q) \wedge r$
ii. $F \wedge (p \wedge q)$
iii. $p \vee [\sim q \wedge (p \vee q) \wedge \sim r]$

---

2. Write the dual statement of each of the following compound statements:

i. Vijay and Vinay cannot speak Hindi.
ii. Ravi or Avinash went to Chennai.
iii. Madhuri has curly hair and brown eyes. [Mar 14]

Solution:

i. Vijay or Vinay cannot speak Hindi.
ii. Ravi and Avinash went to Chennai.
iii. Madhuri has curly hair or brown eyes. ***[1 Mark]***

---

3. Write the duals of the following statements:

i. $(p \vee q) \vee r \equiv p \vee (q \vee r)$
ii. $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$

Solution:

i. $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$
ii. $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$

---

4. Write duals of each of the following statements where t is a tautology and c is a contradiction.

i. $p \wedge q \wedge c$ **ii.** $\sim p \wedge (q \vee c)$
iii. $(p \wedge t) \vee (c \wedge \sim q)$

Solution:

i. $p \vee q \vee t$ ii. $\sim p \vee (q \wedge t)$
iii. $(p \vee c) \wedge (t \vee \sim q)$

Negation of compound statement

i. **Negation of conjunction:**
Negation of the conjunction of two simple statements is the disjunction of their negations.
i.e. $\sim(p \wedge q) \equiv \sim p \vee \sim q$

ii. **Negation of disjunction:**
Negation of the disjunction of two simple statements is the conjunction of their negations.
i.e. $\sim(p \vee q) \equiv \sim p \wedge \sim q$

iii. **Negation of negation:**
The negation of negation of a simple statement is the statement itself.
i.e. If p is a simple statement then $\sim(\sim p) \equiv p$

iv. **Negation of conditional (implication) statement:**
The negation of a conditional statement $p \to q$ is p but not q.
i.e. $\sim(p \to q) \equiv p \wedge \sim q$

v. **Negation of biconditional (double implication) statement:**
The negation of a biconditional statement $p \leftrightarrow q$ is the negation of $p \to q$ or $q \to p$.
i.e. $\sim(p \leftrightarrow q) \equiv (p \wedge \sim q) \vee (q \wedge \sim p)$

Note:
Negation of statement pattern involving one or more simple statements p, q, r, …. and one or more connectives $\sim$, $\wedge$ or $\vee$ is obtained by replacing $\vee$ by $\wedge$, $\wedge$ by $\vee$. Statements p, q, r are replaced by their negations $\sim p$, $\sim q$, $\sim r$ etc. and vice-versa.

vi. **Negation of a quantified statement:**
While finding the negations of quantified statements, the word 'all' is replaced by 'some' and 'for every' is replaced by 'there exists' and vice-versa.

Algebra of Statements

Some standard equivalent statements:

a. Idempotent Law:
  i. $p \vee p \equiv p$
  ii. $p \wedge p \equiv p$

b. Commutative Law:
  i. $p \vee q \equiv q \vee p$
  ii. $p \wedge q \equiv q \wedge p$

c. Associative Law:
  i. $(p \vee q) \vee r \equiv p \vee (q \vee r) \equiv p \vee q \vee r$
  ii. $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r) \equiv p \wedge q \wedge r$

d. Distributive Law:
  i. $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$
  ii. $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$

e. Identity Law:
  i. $p \vee F \equiv p$
  ii. $p \wedge F \equiv F$
  iii. $p \vee T \equiv T$
  iv. $p \wedge T \equiv p$

f. Complement Law:
  i. $p \vee \sim p \equiv T$
  ii. $p \wedge \sim p \equiv F$

g. Involution Law:
  i. $\sim T \equiv F$
  ii. $\sim F \equiv T$
  iii. $\sim(\sim p) \equiv p$

h. DeMorgan's Law:
  i. $\sim(p \vee q) \equiv \sim p \wedge \sim q$
  ii. $\sim(p \wedge q) \equiv \sim p \vee \sim q$

i. Absorption Law:
  i. $p \vee (p \wedge q) \equiv p$
  ii. $p \wedge (p \vee q) \equiv p$

j. Conditional Law:
  i. $p \to q \equiv \sim p \vee q$
  ii. $p \leftrightarrow q \equiv (\sim p \vee q) \wedge (\sim q \vee p)$

Exercise 1.8

1. Write the negations of following statements.

i. All equilateral triangles are isosceles.
ii. Some real numbers are not complex numbers.
iii. Every student has paid the fees.
iv. $\forall$ **n** $\in$ **N, n + 1 > 2.**
v. $\forall x \in$ **N,** $x^2 + x$ **is even number.**
vi. $\exists$ **n** $\in$ **N, such that** $n^2 = n$.
vii. $\exists x \in$ **R, such that** $x^2 < x$.
viii. All students of this college live in the hostel.
ix. Some continuous functions are differentiable.
x. Democracy survives if the leaders are not corrupt.

xi. The necessary and sufficient condition for a person to be successful is to be honest.

xii. Some quadratic equations have unequal roots.

***Solution*:**

i. Some equilateral triangles are not isosceles.

ii. All real numbers are complex numbers.

iii. Some students have not paid the fees.

iv. $\exists$ n $\in$ N, such that n + 1 $\le$ 2.

v. $\exists\ x \in$ N, such that $x^2 + x$ is not an even number.

vi. $\forall$ n $\in$ N, $n^2 \neq n$.

vii. $\forall\ x \in$ R, $x^2 \ge x$.

viii. Some students of this college do not live in the hostel.

ix. All continuous functions are not differentiable.

x. Let, p: The leaders are not corrupt.
q: Democracy survives.

$\therefore$ The given statement is of the form, p $\to$ q
Its negation is of the form p $\wedge$ ~ q.
i.e. 'the leaders are not corrupt, but democracy does not survive.'

xi. Let, p: A person is successful.
q: A person is honest.

$\therefore$ The given statement is of the form, p $\leftrightarrow$ q
Its negation is of the form,
(p $\wedge$ ~ q) $\vee$ (q $\wedge$ ~ p)
i.e. 'a person is successful, but he is not honest or a person is honest, but he is not successful.'

xii. All quadratic equations have equal roots.

---

2. Using the rules of negation, write the negations of the following.

i. p $\wedge$ (q $\to$ r) ii. (~p $\vee$ q) $\wedge$ r

iii. (~p $\wedge$ ~q) $\vee$ (p $\wedge$ ~q)

***Solution*:**

i. ~[p $\wedge$ (q $\to$ r)]
$\equiv$ ~p $\vee$ ~(q $\to$ r) ….(Negation of conjunction)
$\equiv$ ~p $\vee$ (q $\wedge$ ~r) ….(Negation of implication)

ii. ~[(~p $\vee$ q) $\wedge$ r]
$\equiv$ ~(~p $\vee$ q) $\vee$ ~r ....(Negation of conjunction)
$\equiv$ [~(~p) $\wedge$ ~q] $\vee$ ~r
.... (Negation of disjunction)
$\equiv$ (p$\wedge$ ~q) $\vee$ ~r ….(Negation of negation)

iii. ~[(~p $\wedge$ ~q) $\vee$ (p $\wedge$ ~q)]
$\equiv$ ~(~p $\wedge$ ~q) $\wedge$ ~(p $\wedge$ ~q)
…(Negation of disjunction)
$\equiv$ [~(~p) $\vee$~ (~q)] $\wedge$ [~p $\vee$ ~(~q)]
....(Negation of conjunction)
$\equiv$ (p $\vee$ q) $\wedge$ (~p $\vee$ q) ….(Negation of negation)

---

3. Without using truth tables, show that

i. p $\wedge$ (q $\vee$ ~p) $\equiv$ p $\wedge$ q

ii. (p $\wedge$ q) $\vee$ (~p $\wedge$ q) $\vee$ (~q $\wedge$ r) $\equiv$ q $\vee$ r

***Solution*:**

i. L.H.S. = p $\wedge$ (q $\vee$ ~p)
$\equiv$ (p $\wedge$ q) $\vee$ (p $\wedge$ ~p)
…(Distributive law)
$\equiv$ (p $\wedge$ q) $\vee$ F ….(Complement law)
$\equiv$ p $\wedge$ q ….(Identity law)

$\therefore$ L.H.S. = R.H.S.

$\therefore$ p $\wedge$ (q $\vee$ ~p) $\equiv$ p $\wedge$ q

ii. L.H.S. = (p $\wedge$ q) $\vee$ (~p $\wedge$ q) $\vee$ (~q $\wedge$ r)
$\equiv$ [(p $\vee$ ~p) $\wedge$ q] $\vee$ (~q $\wedge$ r)
… (Associative and distributive law)
$\equiv$ (T $\wedge$ q) $\vee$ (~q $\wedge$ r)
....(Complement law)
$\equiv$ q $\vee$ (~q $\wedge$ r)
....(Identity law)
$\equiv$ (q $\vee$ ~q) $\wedge$ (q $\vee$ r)
....(Distributive law)
$\equiv$ T $\wedge$ (q $\vee$ r) ....(Complement law)
$\equiv$ q $\vee$ r ....(Identity law)

$\therefore$ L.H.S. = R.H.S.

$\therefore$ (p $\wedge$ q) $\vee$ (~p $\wedge$ q) $\vee$ (~q $\wedge$ r) $\equiv$ q $\vee$ r

---

4. Form the negations of the following statements by giving justification.

i. (p $\wedge$ q) $\to$ (~p $\vee$ r)

ii. (q $\vee$ ~r) $\wedge$ (p $\vee$ q)

Solution:

i. ~[(p $\wedge$ q) $\to$ (~p $\vee$ r)]
$\equiv$ (p $\wedge$ q) $\wedge$ ~(~p $\vee$ r)
…. (Negation of implication)
$\equiv$ (p $\wedge$ q) $\wedge$ [~(~p) $\wedge$ ~r]
…. (Negation of disjunction)
$\equiv$ (p $\wedge$ q) $\wedge$ (p $\wedge$ ~r)
…. (Negation of negation)

ii. ~[(q $\vee$ ~r) $\wedge$ (p $\vee$ q)]
$\equiv$ ~(q $\vee$ ~r) $\vee$ ~(p $\vee$ q)
….(Negation of conjunction)
$\equiv$ [~q $\wedge$ ~(~r)] $\vee$ (~p $\wedge$ ~q)
…. (Negation of disjunction)

$\equiv (\sim q \wedge r) \vee (\sim p \wedge \sim q)$ …. (Negation of negation)

$\equiv (\sim q \wedge r) \vee (\sim q \wedge \sim p)$ ….(Commutative law)

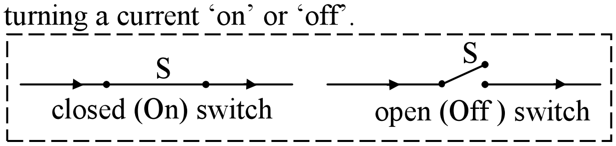
$\equiv \sim q \wedge (r \vee \sim p)$ ….(Distributive law)

Application of Logic to Switching Circuits

The working of an electric switch is similar to a logical statement which has exactly two outcomes, namely, T or F. A switch also has two outcomes or results (current flows and current does not flow) depending upon the status of the switch i.e. (ON or OFF). This analogy is very useful in solving problems of circuit design with the help of logic.

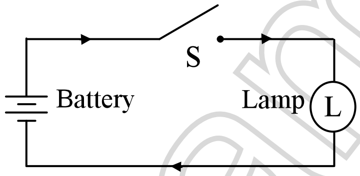
Switch

An electric switch is a two state device used for turning a current 'on' or 'off'.



As shown in the above figures, if the switch is on i.e., circuit is closed, current passes through the circuit and vice-versa.

Consider a simple circuit having a switch 'S', a battery and a lamp 'L'. When the switch 'S' is closed (i.e., ON, current is flowing through the circuit), the lamp glows (is on). Similarly, when the swtich is open (i.e., OFF, current is not flowing through the circuit), the lamp does not glow (is off).



Thus, if p is a statement 'the switch is closed' and if l is a statement 'the lamp glows' then p is equivalent to l i.e $p \equiv l$.

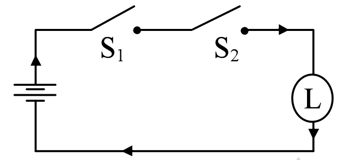
Note:

i. $\sim p$ means 'the switch is open'. In this case, the lamp will not glow and thus $\sim p \equiv \sim l$.

ii. If a switch is 'ON' then its truth value is T or 1 and if the switch is 'OFF', its truth value is F or 0.

If there are two switches, then they can be connected in the following ways:

i. **Switches S_1 and S_2 connected in series:**



Let p: the switch S_1 is closed
q: the switch S_2 is closed
l: the lamp glows

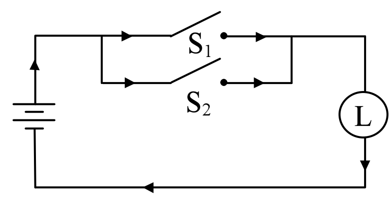
In this case, the lamp glows, if and only if both the switches are closed.

Thus we have, $p \wedge q \equiv l$.

Input – Output (switching) table:

p	q	$p \wedge q$
T (1)	T (1)	T (1)
T (1)	F (0)	F (0)
F (0)	T (1)	F (0)
F (0)	F (0)	F (0)

ii. **Switches S_1 and S_2 are in parallel**



Let p : the switch S_1 is closed
q : the switch S_2 is closed
l : the lamp glows

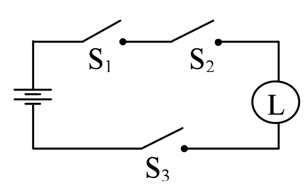
In this case, the lamp glows, if at least one of the switches is closed.

Thus, we have $p \vee q \equiv l$,

Input – output (switching) table:

p	q	$p \vee q$
T (1)	T (1)	T (1)
T (1)	F (0)	T (1)
F (0)	T (1)	T (1)
F (0)	F (0)	F (0)

The above two networks can be combined to form a complicated network as shown below:



Let p : The switch S_1 is closed

q : The switch S_2 is closed

r : The switch S_3 is closed

l : The lamp glows

In this case, the lamp glows, if S_1 and S_2 both are closed or if S_3 is closed.

Thus we have, $(p \wedge q) \vee r \equiv l$.

Note: i. If two or more switches in a circuit are open or closed simultaneously, then they are denoted by same letter and are called **'equivalent switches.'**

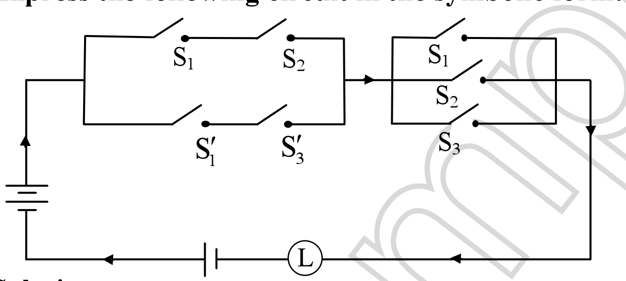
ii. Any two switches in a circuit having opposite states are called **complementary switches.**
For example, if S_1 and S_2 are the two switches such that when S_1 is closed, S_2 is open and vice-versa, then the switches S_1 and S_2 are called complementary switches and S_2 is denoted as S_1'. In such a situation, one of them is considered as p and the other as ~p or p′.

iii. Two circuits are called **equivalent** if output of the two circuits is always same.

iv. A circuit is called **simpler** if it contains lesser number of switches.

Example :

Express the following circuit in the symbolic form:



Solution:

Let p : The switch S_1 is closed.

q : The switch S_2 is closed.

r : The switch S_3 is closed.

$\sim p$: The switch S_1' is closed.

$\sim r$: The switch S_3' is closed.

l : Lamp L is 'on'.

The lamp L is 'on' if and only if –

i. Switch S_1 and Switch S_2 are closed.
($\because$ S_1 and S_2 are in series.)

or ii. Switch S_1' and S_3' are closed.
($\because$ S_1' and S_3' are in series)

and iii. Switch S_1 or S_2 or S_3 are closed.
($\because$ S_1, S_2, S_3 are in parallel)

$\therefore$ Symbolic form of the given circuit is,

$[(p \wedge q) \vee (\sim p \wedge \sim r)] \wedge (p \vee q \vee r) \equiv l$

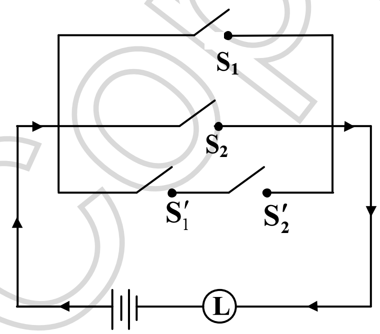
Generally l is not written and therefore the symbolic form is

$[(p \wedge q) \vee (\sim p \wedge \sim r)] \wedge (p \vee q \vee r)$

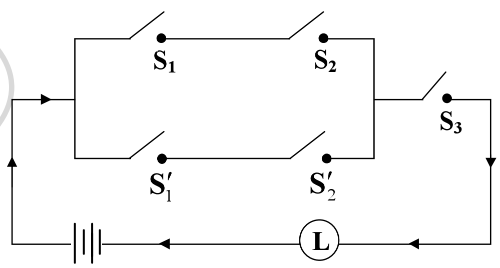
Exercise 1.9

1. Represent the following circuits symbolically and write the input-output or switching table.

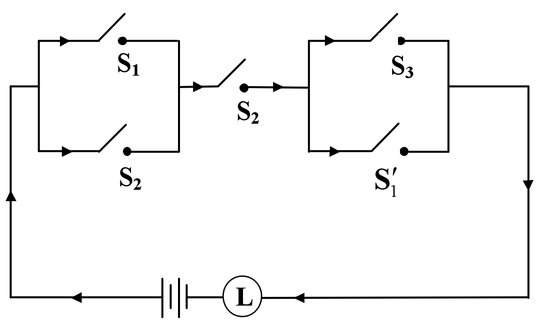
i.



ii.



iii.



***Solution*:**

i. Let p: The switch S_1 is closed.

q: The switch S_2 is closed.

$\sim p$: The switch S_1' is closed or the switch S_1 is open.

$\sim q$: The switch S_2' is closed or the switch S_2 is open.

$\therefore$ The symbolic form of the given circuit is

$(p \vee q) \vee (\sim p \wedge \sim q)$

Input-output Table:

p	q	~p	~q	$p \vee q$	$\sim p \wedge \sim q$	$(p \vee q) \vee (\sim p \wedge \sim q)$
1	1	0	0	1	0	1
1	0	0	1	1	0	1
0	1	1	0	1	0	1
0	0	1	1	0	1	1

Switching Table:

p	q	~p	~q	$p \vee q$	$\sim p \wedge \sim q$	$(p \vee q) \vee (\sim p \wedge \sim q)$
T	T	F	F	T	F	T
T	F	F	T	T	F	T
F	T	T	F	T	F	T
F	F	T	T	F	T	T

ii. Let p: The switch S_1 is closed.
q: The switch S_2 is closed.
r: The switch S_3 is closed.
~p: The switch S_1' is closed or the switch S_1 is open.
~q: The switch S_2' is closed or the switch S_2 is open.

∴ The symbolic form of the given circuit is $[(p \wedge q) \vee (\sim p \wedge \sim q)] \wedge r$.
Let $a \equiv (p \wedge q)$, $b \equiv (\sim p \wedge \sim q)$

Input-output Table:

p	q	r	~p	~q	a	b	$a \vee b$	$(a \vee b) \wedge r$
1	1	1	0	0	1	0	1	1
1	1	0	0	0	1	0	1	0
1	0	1	0	1	0	0	0	0
1	0	0	0	1	0	0	0	0
0	1	1	1	0	0	0	0	0
0	1	0	1	0	0	0	0	0
0	0	1	1	1	0	1	1	1
0	0	0	1	1	0	1	1	0

Switching table:

p	q	r	~p	~q	a	b	$a \vee b$	$(a \vee b) \wedge r$
T	T	T	F	F	T	F	T	T
T	T	F	F	F	T	F	T	F
T	F	T	F	T	F	F	F	F
T	F	F	F	T	F	F	F	F
F	T	T	T	F	F	F	F	F
F	T	F	T	F	F	F	F	F
F	F	T	T	T	F	T	T	T
F	F	F	T	T	F	T	T	F

iii. Let p: The switch S_1 is closed
q: The switch S_2 is closed
r: The switch S_3 is closed
~p: The switch S_1' is closed or the switch S_1 is open.

∴ The symbolic form of the given circuit is $(p \vee q) \wedge q \wedge (r \vee \sim p)$

Input- output Table:

p	q	r	~p	$p \vee q$	$(p \vee q) \wedge q$	$(r \vee \sim p)$	$(p \vee q) \wedge q \wedge (r \vee \sim p)$
1	1	1	0	1	1	1	1
1	1	0	0	1	1	0	0
1	0	1	0	1	0	1	0
1	0	0	0	1	0	0	0
0	1	1	1	1	1	1	1
0	1	0	1	1	1	1	1
0	0	1	1	0	0	1	0
0	0	0	1	0	0	1	0

Switching table:

p	q	r	~p	$p \vee q$	$(p \vee q) \wedge q$	$r \vee \sim p$	$(p \vee q) \wedge q \wedge (r \vee \sim p)$
T	T	T	F	T	T	T	T
T	T	F	F	T	T	F	F
T	F	T	F	T	F	T	F
T	F	F	F	T	F	F	F
F	T	T	T	T	T	T	T
F	T	F	T	T	T	T	T
F	F	T	T	F	F	T	F
F	F	F	T	F	F	T	F

2. Construct the switching circuits of the following statements.

i. $[p \vee (\sim p \wedge q)] \vee [(\sim q \wedge r) \vee \sim p]$ **[Mar 15]**

ii. $(p \wedge q \wedge r) \vee [\sim p \vee (q \wedge \sim r)]$

iii. $[(p \wedge r) \vee (\sim q \wedge \sim r)] \vee (\sim p \wedge \sim r)$

***Solution*:**

i. Let p: The switch S_1 is closed
q: The switch S_2 is closed.
r: The switch S_3 is closed.
~p: The switch S_1' is closed or the switch S_1 is open.
~q: The switch S_2' is closed or the switch S_2 is open.
~r : The switch S_3' is closed or the switch S_3 is open. ***[1 Mark]***

Consider the given statement,

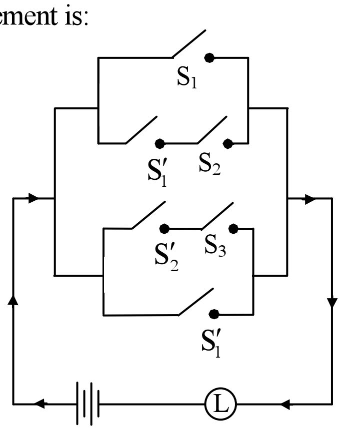
$[p \vee (\sim p \wedge q)] \vee [(\sim q \wedge r) \vee \sim p]$.

$p \vee (\sim p \wedge q)$: represents that switch S_1 is connected in parallel with the series combination of S_1' and S_2.

$(\sim q \wedge r) \vee \sim p$: represents that switch S_1' is connected in parallel with the series combination of S_2' and S_3.

Therefore, $[p \vee (\sim p \wedge q)] \vee [(\sim q \wedge r) \vee \sim p]$ represents that the circuits corresponding to $[p \vee (\sim p \wedge q)]$ and $[(\sim q \wedge r) \vee \sim p]$ are connected in parallel with each other. ***[1 Mark]***

∴ Switching Circuit corresponding to the given statement is:



[1 Mark]

ii. Let p : The switch S_1 is closed.

q : The switch S_2 is closed.

r : The switch S_3 is closed.

~p : The switch S_1' is closed or the switch S_1 is open.

~r : The switch S_3' is closed or the switch S_3 is open.

Consider the given statement,

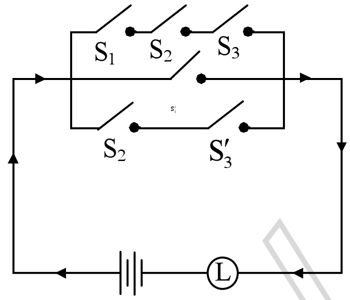
$(p \wedge q \wedge r) \vee [\sim p \vee (q \wedge \sim r)]$

$p \wedge q \wedge r$: represents that switches S_1, S_2, S_3 are connected in series.

$\sim p \vee (q \wedge \sim r)$: represents that the series combination of S_2 and S_3' is connected in parallel with S_1'.

Therefore, $(p \wedge q \wedge r) \vee [\sim p \vee (q \wedge \sim r)$ represents that the circuits corresponding to $(p \wedge q \wedge r)$ and $[\sim p \vee (q \wedge \sim r)]$ are connected in parallel with each other.

∴ Switching circuit corresponding to the given statement is:



iii. Let p: The switch S_1 is closed

q: The switch S_2 is closed.

r: The switch S_3 is closed.

~p: The switch S_1' is closed or the switch S_1 is open.

~q: The switch S_2' is closed or the switch S_2 is open.

~r : The switch S_3' is closed or the switch S_3 is open.

Consider the given statement,

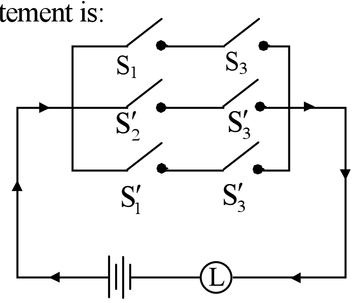
$[(p \wedge r) \vee (\sim q \wedge \sim r)] \vee (\sim p \wedge \sim r)$

$(p \wedge r) \vee (\sim q \wedge \sim r)$: represents that the series combination of S_1 and S_3 and series combination of S_2' and S_3' are connected in parallel with each other.

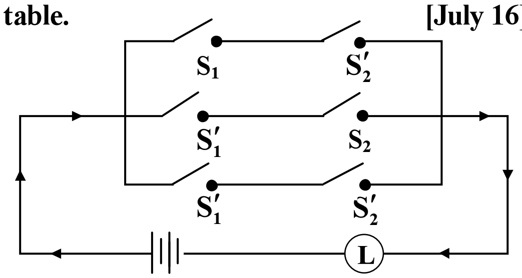
$\sim p \wedge \sim r$: represents that switches S_1' and S_3' are connected in series.

Therefore, $[(p \wedge r) \vee (\sim q \wedge \sim r)] \vee (\sim p \wedge \sim r)$ represents that the circuits corresponding to $[(p \wedge r) \vee (\sim q \wedge \sim r)]$ and $(\sim p \wedge \sim r)$ are connected in parallel with each other.

∴ Switching Circuit corresponding to the given statement is:



3. Give an alternative arrangement for the following circuit, so that the new circuit has two switches only. Also write the switching table. [July 16]



***Solution*:**

Let p: The switch S_1 is closed.
q: The switch S_2 is closed.
~p: The switch S_1' is closed or the switch S_1 is open.
~q: The switch S_2' is closed or the switch S_2 is open.

$\therefore$ symbolic form of the given circuit is

$(p \wedge \sim q) \vee (\sim p \wedge q) \vee (\sim p \wedge \sim q)$ ***[1 Mark]***

$\equiv (p \wedge \sim q) \vee [\sim p \wedge (q \vee \sim q)]$
…(Associative and Distributive law)

$\equiv (p \wedge \sim q) \vee (\sim p \wedge T)$ ….(Complement law)

$\equiv (p \wedge \sim q) \vee \sim p$ ....(Identity law)

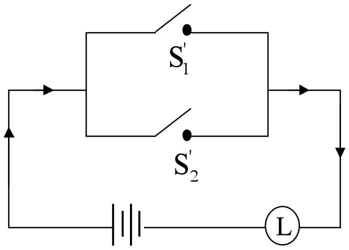
$\equiv (p \vee \sim p) \wedge (\sim q \vee \sim p)$ ....(Distributive law)
[1 Mark]

$\equiv (p \vee \sim p) \wedge (\sim p \wedge \sim q)$ ….(Commutative law)

$\equiv T \wedge (\sim p \vee \sim q)$ ….(Complement law)

$\equiv \sim p \vee \sim q$ ….(Identity law)
[1 Mark]

The alternative arrangement for given circuit is:

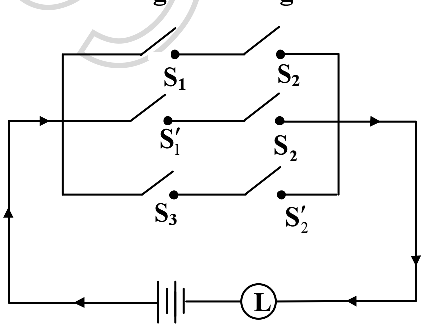


[1 Mark]

Switching Table:

p	q	~p	~q	$p \wedge \sim q$	$\sim p \wedge q$	$\sim p \wedge \sim q$	$(p \wedge \sim q) \vee (\sim p \wedge q) \vee (\sim p \wedge \sim q)$
T	T	F	F	F	F	F	F
T	F	F	T	T	F	F	T
F	T	T	F	F	T	F	T
F	F	T	T	F	F	T	T

4. Find
i. symbolic form
ii. switching table and
iii. draw simplified switching circuit for the following switching circuit.



***Solution*:**

i. Let p: The switch S_1 is closed.
q: The switch S_2 is closed.
r: The switch S_3 is closed.
~p: The switch S_1' is closed or the switch S_1 is open.
~q: The switch S_2' is closed or the switch S_2 is open.

$\therefore$ The symbolic form of the given circuit is

$(p \wedge q) \vee (\sim p \wedge q) \vee (r \wedge \sim q)$

ii. **Switching table:**

Let $a \equiv (p \wedge q) \vee (\sim p \wedge q) \vee (r \wedge \sim q)$

p	q	r	$p \wedge q$	~p	~q	$\sim p \wedge q$	$r \wedge \sim q$	a
T	T	T	T	F	F	F	F	T
T	T	F	T	F	F	F	F	T
T	F	T	F	F	T	F	T	T
T	F	F	F	F	T	F	F	F
F	T	T	F	T	F	T	F	T
F	T	F	F	T	F	T	F	T
F	F	T	F	T	T	F	T	T
F	F	F	F	T	T	F	F	F

iii. For simplified switching circuit,

Consider $(p \wedge q) \vee (\sim p \wedge q) \vee (r \wedge \sim q)$

$\equiv [(p \vee \sim p) \wedge q] \vee (r \wedge \sim q)$
....(Associative and Distributive law)

$\equiv (T \wedge q) \vee (r \wedge \sim q)$....(Complement law)

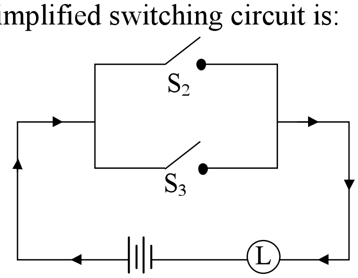
$\equiv q \vee (r \wedge \sim q)$ ....(Identity law)

$\equiv (q \vee r) \wedge (q \vee \sim q)$ ....(Distributive law)

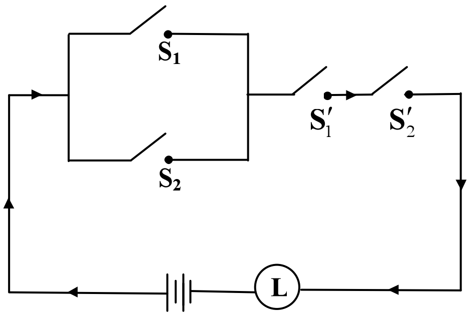
$\equiv (q \vee r) \wedge T$ ....(Complement law)

$\equiv q \vee r$ ....(Identity law)

$\therefore$ Simplified switching circuit is:



5. **Find the symbolic form of the following switching circuit, construct its switching table and interpret your result.**



***Solution*:**

Let p : The Switch S_1 is closed.
q : The Switch S_2 is closed.
~p : The Switch S'_1 is closed or the switch S_1 is open.
~q : The switch S'_2 is closed or the switch S_2 is open.

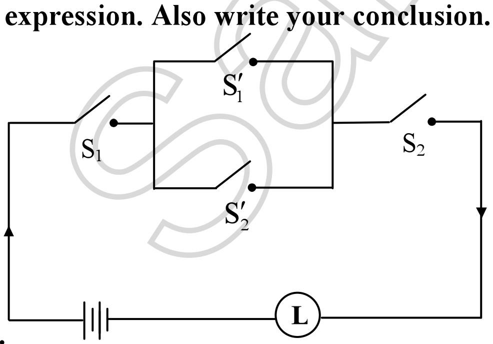
∴ The symbolic form of the given circuit is:
$(p \vee q) \wedge (\sim p) \wedge (\sim q)$

Switching table:

p	q	~p	~q	p∨q	(p∨q) ∧ (~p) ∧ (~q)
T	T	F	F	T	F
T	F	F	T	T	F
F	T	T	F	T	F
F	F	T	T	F	F

In the above truth table, all the entries in the last column are 'F',
∴ the given circuit represents a contradiction.
∴ Irrespective of whether the switches S_1 and S_2 are open or closed, the given circuit will always be open (i.e. off).

6. **Simplify the given circuit by writing its logical expression. Also write your conclusion.**



***Solution*:**

Let, p : The switch S_1 is closed
q : The switch S_2 is closed
~p : The switch S'_1 is closed or the switch S_1 is open.
~q : The switch S'_2 is closed or the switch S_2 is open.

∴ The symbolic form of the given circuit is

$[p \wedge (\sim p \vee \sim q)] \wedge q$

$\equiv [(p \wedge \sim p) \vee (p \wedge \sim q)] \wedge q$ ….[Associative and Distributive law]

$\equiv [F \vee (p \wedge \sim q)] \wedge q$ ….[Complement law]

$\equiv (p \wedge \sim q) \wedge q$ ….[Identity law]

$\equiv (p \wedge q) \wedge (\sim q \wedge q)$ ….[Distributive law]

$\equiv (p \wedge q) \wedge F$ ….[Complement law]

$\equiv F$ ….[Identity law]

∴ Irrespective of the status of the switches, the current will not flow in the circuit, that is, the circuit will always be open.

Miscellaneous Exercise - 1

1. **Which of the following sentences are statements in logic? Justify your answer.**
   - **i π is a real number.**
   - **ii. 5! = 120**
   - **iii. Himalaya is an ocean and Ganga is a river.**
   - **iv. Please get me a cup of tea.**
   - **v. Bring me a notebook.**
   - **vi. Alas! We lost the match**
   - **vii. $\cos 2\theta = \cos^2\theta - \sin^2\theta$, for all $\theta \in R$.**
   - **viii. If x is a real number then $x^2 \geq 0$.**

Solution:

i. It is a statement.
ii. It is a statement.
iii. It is a statement.
iv. It is an imperative sentence, hence it is not a statement.
v. It is an imperative sentence, hence it is not a statement.
vi. It is an exclamatory sentence, hence it is not a statement.
vii. It is a statement.
viii. It is a statement.

2. **Write the truth values of the following statements:**
   - **i. The square of any odd number is even or the cube of any even number is even.**
   - **ii. $\sqrt{5}$ is irrational but $3 + \sqrt{5}$ is a complex number. [Oct 14]**
   - **iii. $\exists$ n $\in$ N, such that n + 5 > 10. [Oct 14]**
   - **iv. $\forall$ n $\in$ N, n + 3 > 5.**

v. **If ABC is a triangle and all its sides are equal then each angle has measure 30°.**

vi. **$\forall$ n $\in$ N, n^2 + n is an even number while n^2 – n is an odd number.**

***Solution*:**

i. Let p: The square of any odd number is even.
q: The cube of any even number is even.

$\therefore$ The symbolic form of the given statement is $p \vee q$.
Since the truth value of p is F and that of q is T,

$\therefore$ truth value of $p \vee q$ is T

ii. Let p: $\sqrt{5}$ is irrational.
q: $3 + \sqrt{5}$ is a complex number.

$\therefore$ The symbolic form of the given statement is $p \wedge q$
Since the truth value of p is T and that of q is F,

$\therefore$ truth value of $p \wedge q$ is F. *[1 Mark]*

iii. Consider the statement, $\exists$ n $\in$ N, n + 5 > 10
Clearly n $\geq$ 6, n$\in$N satisfy n + 5 > 10.

$\therefore$ its truth value is T. *[1 Mark]*

iv. Consider the statement, $\forall$ n $\in$ N, n + 3 > 5

$\because$ n = 1 and n = 2 $\in$ N do not satisfy n + 3 > 5

$\therefore$ truth value of p is F.

v. Let p: ABC is a triangle and all its sides are equal.
q: Each angle has measure 30°.

$\therefore$ The symbolic form of the given statement is $p \to q$.
Since the truth value of p is T and that of q is F,

$\therefore$ truth value of $p \to q$ is F

vi. Let p: $\forall$ n $\in$ N, n^2 + n is an even number.
q: $\forall$n$\in$N, n^2 – n is an odd number.

$\therefore$ The symbolic form of the given statement is $p \wedge q$.
Since, the truth value of p is T and q is F,

$\therefore$ truth value of $p \wedge q$ is F

---

3. If A = {4, 5, 7, 9}, determine the truth value of each of the following quantified statements.

i. $\exists$ $x \in$ A, such that $x + 2 = 7$.

ii. $\forall$ $x \in$ A, $x + 3 < 10$.

iii. $\exists$ $x \in$ A, such that $x + 5 \geq 9$.

iv. $\exists$ $x \in$ A, such that x is even.

v. $\forall$ $x \in$ A, $2x \leq 17$.

***Solution*:**

i. Since $x = 5 \in$ A, satisfies $x + 2 = 7$.

$\therefore$ the given statement is true.

$\therefore$ Its truth value is 'T'.

ii. Since, $x = 7, 9 \in$ A, do not satisfy $x + 3 < 10$.

$\therefore$ the given statement is false.

$\therefore$ Its truth value is 'F'.

iii. Since, $x = 4, 5, 7, 9 \in$ A, satisfy $x + 5 \geq 9$.

$\therefore$ the given statement is true.

$\therefore$ Its truth value is 'T'.

iv. Since, $x = 4 \in$ A, satisfies 'x is even'.

$\therefore$ the given statement is true.
Its truth value is 'T'.

v. Since $x = 9 \in$ A does not satisfy $2x \leq 17$.

$\therefore$ the given statement is false.

$\therefore$ Its truth value is 'F'.

---

4. Write negations of the following statements

i. Some buildings in this area are multistoried.

ii. All parents care for their children.

iii. $\forall$n $\in$ N, n + 7 > 6.

iv. $\exists$ $x \in$ A, such that $x + 5 > 8$.

***Solution*:**

i. All buildings in this area are not multistoried.

ii. Some parents do not care for their children.

iii. $\exists$ n $\in$ N, such that n + 7 $\leq$ 6.

iv. $\forall$ $x \in$ A, $x + 5 \leq 8$.

---

5. Write the following statements in symbolic form:

i. Ramesh is cruel or strict.

ii. I am brave is necessary and sufficient condition to climb the Mount Everest.

iii. I can travel by train provided I get my ticket reserved.

iv. Sandeep neither likes tea nor coffee but enjoys a soft-drink.

v. ABC is a triangle only if AB + BC > AC.

vi. Rajesh is studious but does not get good marks.

***Solution*:**

i. Let p: Ramesh is cruel,
q: Ramesh is strict.

$\therefore$ The symbolic form of the given statement is $p \vee q$.

ii. Let p: I am brave.
q: I can climb the Mount Everest.

$\therefore$ The symbolic form of the given statement is $p \leftrightarrow q$.

iii. Let p: I can travel by train,
q: I get my ticket reserved.

∴ The symbolic form of the given statement is $q \to p$.

iv. Let p: Sandeep likes tea,
q: Sandeep likes coffee.
r: Sandeep enjoys a soft-drink.

∴ The symbolic form of the given statement is $(\sim p \wedge \sim q) \wedge r$.

v. Let p: ABC is a triangle,
q: AB + BC > AC.

∴ The symbolic form of the given statement is $p \to q$.

vi. Let p: Rajesh is studious,
q: Rajesh gets good marks.

∴ The symbolic form of the given statement is $p \wedge \sim q$.

6. If p : The examinations are approaching, q : Students study hard, give a verbal statement for each of the following:

i. $p \wedge \sim q$ **ii.** $p \leftrightarrow q$
iii. $\sim p \to q$ **iv.** $p \vee q$
v. $\sim q \to \sim p$

***Solution*:**

i. The examinations are approaching but the students do not study hard.
ii. The examinations are approaching if and only if the students study hard.
iii. If the examinations are not approaching, then the students study hard.
iv. The examinations are approaching or the students study hard.
v. If the students do not study hard then the examinations are not approaching.

7. If p: It is raining, q: The weather is humid, which of the following statements are logically equivalent? Justify!

i. If it is not raining then the weather is not humid.
ii. It is raining if and only if the weather is humid.
iii. It is not true that it is not raining or the weather is humid.
iv. It is raining but the weather is not humid.
v. The weather is humid only if it is raining.

***Solution*:**

The symbolic forms of the given statements are:

i. $\sim p \to \sim q$ ii. $p \leftrightarrow q$
iii. $\sim(\sim p \vee q)$ iv. $p \wedge \sim q$
v. $q \to p$

Truth table for all the above statements:

p	q	~p	~q	(~p∨q)	(i)	(ii)	(iii)	(iv)	(v)
T	T	F	F	T	T	T	F	F	T
T	F	F	T	F	T	F	T	T	T
F	T	T	F	T	F	F	F	F	F
F	F	T	T	T	T	T	F	F	T

Note: In the above table, the numbers (i), (ii), (iii), (iv) and (v) represent corresponding statements.

In the above table, the columns of statements (i) and (v) are same.

∴ They are logically equivalent.

Similarly, the columns of statements (iii) and (iv) are same.

∴ They are also logically equivalent.

8. Rewrite the following statements without using the conditional form:

i. If prices increase then the wages rise.
ii. If it is cold, we wear woolen clothes.
iii. I can catch cold if I take cold water bath.

***Solution*:**

All these statements are of the form $p \to q \equiv \sim p \vee q$.

∴ The statements without using conditional form will be,

i. Prices do not increase or the wages rise.
ii. It is not cold or we wear woolen clothes.
iii. I do not take cold water bath or I catch cold.

9. If p, q, r are statements with truth values T, F, T respectively, determine the truth values of the following:

i. $q \to (p \vee \sim r)$
ii. $(\sim r \wedge p) \vee \sim q$
iii. $(p \to q) \vee r$
iv. $(r \wedge q) \leftrightarrow \sim p$ **[July 16]**
v. $(p \vee q) \to (q \vee r)$

***Solution*:**

Truth value of p, q and r are T, F and T respectively.

i. $q \to (p \vee \sim r)$
$\equiv F \to (T \vee \sim T)$
$\equiv F \to (T \vee F)$
$\equiv F \to T$
$\equiv T$
Hence, the truth value is 'T'.

ii. $(\sim r \wedge p) \vee \sim q$
$\equiv (\sim T \wedge T) \vee \sim F$
$\equiv (F \wedge T) \vee T$

$\equiv F \vee T$

$\equiv T$

Hence, the truth value is 'T'.

iii. $(p \rightarrow q) \vee r$

$\equiv (T \rightarrow F) \vee T$

$\equiv F \vee T$

$\equiv T$

Hence, the truth value is 'T'.

iv. $(r \wedge q) \leftrightarrow \sim p$

$\equiv (T \wedge F) \leftrightarrow \sim T$

$\equiv (T \wedge F) \leftrightarrow F$ ***[1 Mark]***

$\equiv F \leftrightarrow F$

$\equiv T$

Hence, the truth value is 'T' ***[1 Mark]***

v. $(p \vee q) \rightarrow (q \vee r)$

$\equiv (T \vee F) \rightarrow (F \vee T)$

$\equiv T \rightarrow T$

$\equiv T$

Hence, the truth value is 'T'.

---

10. Change each of the following statement in the form if… then…

i. I shall come provided I finish my work.

ii. Rights follow from performing the duties sincerely.

iii. $x = 1$ only if $x^2 = x$.

iv. The sufficient condition for being rich is to be rational.

v. Getting bonus is necessary condition for me to purchase a car.

***Solution*:**

i. If I finish my work then I shall come.

ii. If the duties are performed sincerely then the rights follow.

iii. If $x = 1$ then $x^2 = x$.

iv. If a man is rational, then he is rich.

v. If I purchase a car then I get bonus.

---

11. Write negations of the following statements:

i. 6 is an even number or 36 is a perfect square.

ii. If diagonals of a parallelogram are perpendicular then it is a rhombus.

iii. If 10 > 5 and 5 < 8 then 8 < 7.

iv. A person is rich if and only if he is a software engineer.

v. Mangoes are delicious but expensive.

vi. It is false that the sky is not blue.

vii. If the weather is fine then my friends will come and we go for a picnic.

***Solution*:**

i. Let, p: 6 is an even number.

q: 36 is a perfect square.

$\therefore$ The given statement is of the form $p \vee q$

Its Negation is $\sim p \wedge \sim q$

i.e. 6 is not an even number and 36 is not a perfect square.

ii. Let, p: Diagonals of a parallelogram are perpendicular.

q : It is a rhombus.

$\therefore$ The given statement is of the form $p \rightarrow q$.

Its negation is $p \wedge \sim q$.

i.e. diagonals of a parallelogram are perpendicular but it is not a rhombus.

iii. Let, p : $10 > 5$ and $5 < 8$

q : $8 < 7$

$\therefore$ The given statement is of the form $p \rightarrow q$

Its negation is $p \wedge \sim q$

i.e, $10 > 5$ and $5 < 8$ but $8 \geq 7$.

iv. Let, p: A person is rich.

q: He is a software engineer.

$\therefore$ The given statement is of the form $p \leftrightarrow q$.

Its negation is $(p \wedge \sim q) \vee (q \wedge \sim p)$

i.e., a person is rich and he is not a software engineer or a person is software engineer and he is not rich.

v. Let, p: Mangoes are delicious.

q: Mangoes are expensive.

$\therefore$ The given statement is of the form $p \wedge q$

Its negation is, $\sim p \vee \sim q$.

i.e., mangoes are not delicious or they are not expensive.

vi. Let, p: Sky is not blue.

$\therefore$ The given statement is of the form $(\sim p)$

Its negation will be, $\sim(\sim p) \equiv p$

i.e., sky is not blue.

vii. Let, p: The weather is fine.

q: My friends are coming.

r: We go for a picnic.

$\therefore$ The given statement is of the form $p \rightarrow (q \wedge r)$

Its negation is, $p \wedge \sim(q \wedge r) \equiv p \wedge (\sim q \vee \sim r)$

i.e., the weather is fine but my friends are not coming or we are not going for a picnic.

---

12. Construct the truth table for each of the following statement patterns:

i. $p \rightarrow (q \rightarrow p)$

ii. $(\sim p \vee \sim q) \leftrightarrow [\sim (p \wedge q)]$

iii. $\sim(\sim p \wedge \sim q) \vee q$

iv. $[(p \wedge q) \vee r] \wedge [\sim r \vee (p \wedge q)]$

v. $[(\sim p \vee q) \wedge (q \to r)] \to (p \to r)$

***Solution*:**

i. $p \to (q \to p)$

p	q	$q \to p$	$p \to (q \to p)$
T	T	T	T
T	F	T	T
F	T	F	T
F	F	T	T

ii. $(\sim p \vee \sim q) \leftrightarrow [\sim (p \wedge q)]$

p	q	~p	~q	$\sim p \vee \sim q$	$p \wedge q$	$\sim(p \wedge q)$	$(\sim p \vee \sim q) \leftrightarrow [\sim(p \wedge q)]$
T	T	F	F	F	T	F	T
T	F	F	T	T	F	T	T
F	T	T	F	T	F	T	T
F	F	T	T	T	F	T	T

iii. $\sim(\sim p \wedge \sim q) \vee q$

p	q	~p	~q	$\sim p \wedge \sim q$	$\sim(\sim p \wedge \sim q)$	$\sim(\sim p \wedge \sim q) \vee q$
T	T	F	F	F	T	T
T	F	F	T	F	T	T
F	T	T	F	F	T	T
F	F	T	T	T	F	F

iv. $[(p \wedge q) \vee r] \wedge [\sim r \vee (p \wedge q)]$

p	q	r	$p \wedge q$	$[(p \wedge q) \vee r]$	~r	$[\sim r \vee (p \wedge q)]$	$[(p \wedge q) \vee r] \wedge [\sim r \vee (p \wedge q)]$
T	T	T	T	T	F	T	T
T	T	F	T	T	T	T	T
T	F	T	F	T	F	F	F
T	F	F	F	F	T	T	F
F	T	T	F	T	F	F	F
F	T	F	F	F	T	T	F
F	F	T	F	T	F	F	F
F	F	F	F	F	T	T	F

v. $[(\sim p \vee q) \wedge (q \to r)] \to (p \to r)$

p	q	r	~p	$\sim p \vee q$	$q \to r$	$(\sim p \vee q) \wedge (q \to r)$	$p \to r$	$[(\sim p \vee q) \wedge (q \to r)] \to (p \to r)$
T	T	T	F	T	T	T	T	T
T	T	F	F	T	F	F	F	T
T	F	T	F	F	T	F	T	T
T	F	F	F	F	T	F	F	T
F	T	T	T	T	T	T	T	T
F	T	F	T	T	F	F	T	T
F	F	T	T	T	T	T	T	T
F	F	F	T	T	T	T	T	T

13. Using truth tables show that following statement patterns are tautologies.

i. $[(p \to q) \wedge \sim q] \to (\sim p)$

ii. $(p \to q) \vee (q \to p)$

iii. $[p \to (q \to r)] \leftrightarrow [(p \wedge q) \to r]$

***Solution*:**

i.

p	q	~p	~q	$(p \to q)$	$[(p \to q) \wedge \sim q]$	$[(p \to q) \wedge \sim q] \to (\sim p)$
T	T	F	F	T	F	T
T	F	F	T	F	F	T
F	T	T	F	T	F	T
F	F	T	T	T	T	T

In the above truth table, all the entries in the last column are T.

∴ The given statement pattern is a tautology.

ii.

p	q	$p \to q$	$q \to p$	$(p \to q) \vee (q \to p)$
T	T	T	T	T
T	F	F	T	T
F	T	T	F	T
F	F	T	T	T

In the above truth table, all the entries in the last column are T.

∴ The given statement pattern is a tautology.

iii.

p	q	r	$q \to r$	$[p \to (q \to r)]$	$(p \wedge q)$	$[(p \wedge q) \to r]$	$[p \to (q \to r)] \leftrightarrow [(p \wedge q) \to r]$
T	T	T	T	T	T	T	T
T	T	F	F	F	T	F	T
T	F	T	T	T	F	T	T
T	F	F	T	T	F	T	T
F	T	T	T	T	F	T	T
F	T	F	F	T	F	T	T
F	F	T	T	T	F	T	T
F	F	F	T	T	F	T	T

In the above truth table, all the entries in the last column are T.

∴ The given statement pattern is a tautology.

14. Using truth tables show that following statement patterns are contradictions.

i. $[(p \vee q) \wedge \sim p] \wedge (\sim q)$

ii. $(p \wedge q) \wedge (\sim p \vee \sim q)$

***Solution*:**

i.

p	q	~p	~q	$p \vee q$	$[(p \vee q) \wedge \sim p]$	$[(p \vee q) \wedge \sim p] \wedge (\sim q)$
T	T	F	F	T	F	F
T	F	F	T	T	F	F
F	T	T	F	T	T	F
F	F	T	T	F	F	F

In the above truth table, all the entries in the last column are F.

∴ The given statement pattern is a contradiction.

ii.

p	q	~p	~q	$p \wedge q$	$(\sim p \vee \sim q)$	$(p \wedge q) \wedge (\sim p \vee \sim q)$
T	T	F	F	T	F	F
T	F	F	T	F	T	F
F	T	T	F	F	T	F
F	F	T	T	F	T	F

In the above truth table, all the entries in the last column are F.

∴ The given statement pattern is a contradiction.

15. Find truth values of p and q in the following cases:

i. $(p \vee q)$ is T and $(p \wedge q)$ is T.

ii. $(p \vee q)$ is T and $(p \vee q) \rightarrow q$ is F.

iii. $(p \wedge q)$ is F and $(p \wedge q) \rightarrow q$ is T.

***Solution*:**

i. $(p \vee q)$ is T and $(p \wedge q)$ is T.

Consider the following truth table:

p	q	$p \vee q$	$p \wedge q$
T	T	T	T
T	F	T	F
F	T	T	F
F	F	F	F

∴ If $(p \vee q)$ is T and $(p \wedge q)$ is T, then both p and q have to be true i.e. their truth value must be T.

ii. $(p \vee q)$ is T and $(p \vee q) \rightarrow q$ is F.

Consider the following truth table:

p	q	$p \vee q$	$(p \vee q) \rightarrow q$
T	T	T	T
T	F	T	F
F	T	T	T
F	F	F	T

∴ If $(p \vee q)$ is T and $(p \vee q) \rightarrow q$ is F, then p is true and q is false i.e., truth value of p is T and that of q is F.

iii. $(p \wedge q)$ is F and $(p \wedge q) \rightarrow q$ is T

Consider the following truth table:

p	q	$p \wedge q$	$(p \wedge q) \rightarrow q$
T	T	T	T
T	F	F	T
F	T	F	T
F	F	F	T

∴ If $(p \wedge q)$ is F and $(p \wedge q) \rightarrow q$ is T, then there are three possibilities for the truth value of p and q.

Either, p is T and q is F or
p is F and q is T or
both p and q are F.

16. Determine whether the following statement patterns are tautologies, contradictions or contingencies.

i. $(p \rightarrow q) \wedge (p \wedge \sim q)$

ii. $(p \wedge q) \vee (\sim p \wedge q) \vee (p \wedge \sim q) \vee (\sim p \wedge \sim q)$

iii. $[p \wedge (p \rightarrow q)] \rightarrow q$

iv. $[(p \vee \sim q) \vee (\sim p \wedge q)] \wedge r$

Solution:

i.

p	q	$p \rightarrow q$	~q	$p \wedge \sim q$	$(p \rightarrow q) \wedge (p \wedge \sim q)$
T	T	T	F	F	F
T	F	F	T	T	F
F	T	T	F	F	F
F	F	T	T	F	F

In the above truth table, all the entries in the last column are F.

$\therefore$ $(p \to q) \wedge (p \wedge \sim q)$ is a contradiction.

ii.

p	q	~p	~q	$p \wedge q$	$\sim p \wedge q$	$p \wedge \sim q$	$\sim p \wedge \sim q$	$(p \wedge q) \vee (\sim p \wedge q) \vee (p \wedge \sim q) \vee (\sim p \wedge \sim q)$
T	T	F	F	T	F	F	F	T
T	F	F	T	F	F	T	F	T
F	T	T	F	F	T	F	F	T
F	F	T	T	F	F	F	T	T

In the above truth table, all the entries in the last column are T.

$\therefore$ $(p \wedge q) \vee (\sim p \wedge q) \vee (p \wedge \sim q) \vee (\sim p \wedge \sim q)$ is a tautology.

iii.

p	q	$p \to q$	$p \wedge (p \to q)$	$[p \wedge (p \to q)] \to q$
T	T	T	T	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T

In the above truth table, all the entries in the last column are T.

$\therefore$ $[p \wedge (p \to q)] \to q$ is a tautology.

iv.

p	q	r	~p	~q	$p \vee \sim q$	$\sim p \wedge q$	$(p \vee \sim q) \vee (\sim p \wedge q)$	$[(p \vee \sim q) \vee (\sim p \wedge q)] \wedge r$
T	T	T	F	F	T	F	T	T
T	T	F	F	F	T	F	T	F
T	F	T	F	T	T	F	T	T
T	F	F	F	T	T	F	T	F
F	T	T	T	F	F	T	T	T
F	T	F	T	F	F	T	T	F
F	F	T	T	T	T	F	T	T
F	F	F	T	T	T	F	T	F

In the above truth table, the entries in the last column are a combination of T and F.

$\therefore$ $[(p \vee \sim q) \vee (\sim p \wedge q)] \wedge r$ is a contingency.

17. Using the rules of logic, prove the following logical equivalences.

i. $p \leftrightarrow q \equiv \sim(p \wedge \sim q) \wedge \sim(q \wedge \sim p)$

ii. $\sim(p \vee q) \vee (\sim p \wedge q) \equiv \sim p$ [Mar 16]

iii. $\sim p \wedge q \equiv (p \vee q) \wedge \sim p$

Solution:

i. Consider, RHS $= \sim(p \wedge \sim q) \wedge \sim(q \wedge \sim p)$

$\equiv \sim[(p \wedge \sim q) \vee (q \wedge \sim p)]$ ….[Negation of disjunction]

$\equiv \sim[\sim(p \leftrightarrow q)]$ ….[Negation of double implication]

$\equiv p \leftrightarrow q$ ….[Negation of negation]

$=$ LHS.

$\therefore$ $p \leftrightarrow q \equiv \sim(p \wedge \sim q) \wedge \sim(q \wedge \sim p)$

ii. Consider,

LHS $= \sim(p \vee q) \vee (\sim p \wedge q)$

$\equiv (\sim p \wedge \sim q) \vee (\sim p \wedge q)$ …. (Negation of disjunction) *[1 Mark]*

$\equiv \sim p \wedge (\sim q \vee q)$ ….(Distributive law) *[1 Mark]*

$\equiv \sim p \wedge T$ ....(Complement law)

$\equiv \sim p$ ....(Identity law) *[1 Mark]*

$=$ RHS.

$\therefore$ $\sim(p \vee q) \vee (\sim p \wedge q) \equiv \sim p$

iii. Consider, RHS $= (p \vee q) \wedge \sim p$

$\equiv (p \wedge \sim p) \vee (q \wedge \sim p)$ …. [Distributive law]

$\equiv (p \wedge \sim p) \vee (\sim p \wedge q)$ ….[Commutative law]

$\equiv F \vee (\sim p \wedge q)$ …. [Complement law]

$\equiv \sim p \wedge q$ …. [Identity law]

$=$ LHS

$\therefore$ $\sim p \wedge q \equiv (p \vee q) \wedge \sim p$

18. Using truth tables prove the following logical equivalences:

i. $p \leftrightarrow q \equiv (p \wedge q) \vee (\sim p \wedge \sim q)$

ii. $(p \wedge q) \to r \equiv p \to (q \to r)$

***Solution*:**

i. *Ref Exercise 1.5 Q2 (ii)*

ii. *Ref Exercise 1.5 Q2 (iii)*

19. Write converse, inverse and contrapositive of the following conditional statements:

i. If an angle is a right angle then its measure is 90°.

ii. If two triangles are congruent then their areas are equal. [Mar 15]

iii. If f(2) = 0 then f(x) is divisible by (x – 2).

***Solution*:**

i. Let p : An angle is a right angle,
q : Its measure is 90°.

∴ The symbolic form of the given statement is $p \to q$.

Converse: $q \to p$

i.e. If the measure of an angle is 90°, then it is a right angle.

Inverse: $\sim p \to \sim q$

i.e. If an angle is not a right angle, then its measure is not 90°.

Contrapositive: $\sim q \to \sim p$

i.e. If the measure of an angle is not 90° then it is not a right angle.

ii. Let p : Two triangles are congruent,
q : Their areas are equal.

∴ The symbolic form of the given statement is $p \to q$.

Converse: $q \to p$

i.e. If areas of two triangles are equal then they are congruent. ***[1 Mark]***

Inverse: $\sim p \to \sim q$

i.e. If two triangles are not congruent then their areas are not equal.

Contrapositive: $\sim q \to \sim p$

i.e. If areas of two triangles are not equal then they are not congruent. ***[1 Mark]***

iii. Let p: f(2) = 0,
q: f(x) is divisible by (x – 2)

∴ The symbolic form of the given statement is $p \to q$.

Converse: $q \to p$

i.e. If f(x) is divisible by (x – 2), then f(2) = 0.

Inverse: $\sim p \to \sim q$

i.e. If f(2) ≠ 0, then f(x) is not divisible by (x – 2).

Contrapositive: $\sim q \to \sim p$

i.e. If f(x) is not divisible by (x–2) then f(2) ≠ 0.

---

20. Without using truth table, prove that $[(p \vee q) \wedge \sim p] \to q$ is a tautology.

***Solution*:**

$[(p \vee q) \wedge \sim p] \to q$

$\equiv [(p \wedge \sim p) \vee (q \wedge \sim p)] \to q$ ….[Distributive law]

$\equiv [F \vee (q \wedge \sim p)] \to q$ ….[Complement law]

$\equiv (q \wedge \sim p) \to q$ ….[Identity law]

$\equiv \sim(q \wedge \sim p) \vee q$ …[Conditional law]

$\equiv [(\sim q \vee \sim(\sim p)] \vee q$ ....[Negation of conjunction]

$\equiv (\sim q \vee p) \vee q$ ….[Negation of negation]

$\equiv (p \vee \sim q) \vee q$ ....[Commutative law]

$\equiv p \vee (\sim q \vee q)$ ....[Associative law]

$\equiv p \vee T$ ....[Complement law]

$\equiv T$ ....[Identity law]

Since, the truth value of the given statement pattern is T, therefore, it is a tautology.

---

21. Consider following statements.

i. If a person is social then he is happy.

ii. If a person is not social then he is not happy.

iii. If a person is unhappy then he is not social.

iv. If a person is happy then he is social.

Identify the pairs of statements having same meaning.

***Solution*:**

Let p: A person is social, q: He is happy.

The symbolic forms of the given statements are:

i. $p \to q$

ii. $\sim p \to \sim q$

iii. $\sim q \to \sim p$

iv. $q \to p$

Statements (i) and (iii) have same meaning.

Since, a statement and its contrapositive are equivalent.

Also, statements (ii) and (iv) have the same meaning. Since, converse and inverse of a compound statement are also equivalent.

---

22. Using the rules of logic, write the negations of the following statements:

i. $(p \vee q) \wedge (q \vee \sim r)$

ii. $(\sim p \wedge q) \vee (p \wedge \sim q)$

iii. $p \wedge (q \vee r)$

iv. $(p \to q) \wedge r$

***Solution*:**

i. $\sim[(p \vee q) \wedge (q \vee \sim r)]$

$\equiv \sim(p \vee q) \vee \sim(q \vee \sim r)$
….[Negation of conjunction]

$\equiv (\sim p \wedge \sim q) \vee (\sim q \wedge \sim(\sim r))$
….[Negation of disjunction]

$\equiv (\sim p \wedge \sim q) \vee (\sim q \wedge r)$
….[Negation of negation]

$\equiv (\sim p \wedge \sim q) \vee (r \wedge \sim q)$
....[Commutative law]

$\equiv (\sim p \vee r) \wedge \sim q$
….[Distributive law]

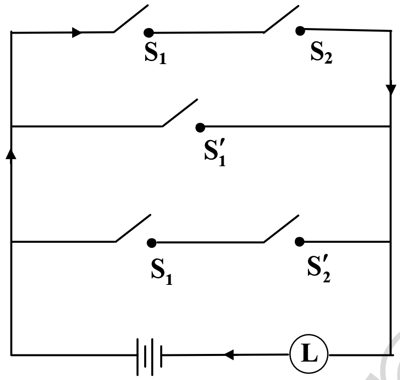
ii. $\sim[(\sim p \wedge q) \vee (p \wedge \sim q)]$
$\equiv \sim(\sim p \wedge q) \wedge \sim (p \wedge \sim q)$ ….[Negation of disjunction]
$\equiv [\sim(\sim p) \vee \sim q] \wedge [\sim p \vee \sim(\sim q)]$ ….[Negation of conjunction]
$\equiv (p \vee \sim q) \wedge [\sim p \vee q]$ ….[Negation of negation]

iii. $\sim[p \wedge (q \vee r)] \equiv \sim p \vee \sim(q \vee r)$ ….[Negation of conjunction]
$\equiv \sim p \vee (\sim q \wedge \sim r)$ ….[Negation of disjunction]

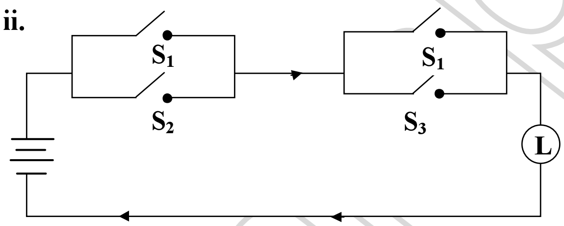
iv. $\sim[(p \to q) \wedge r]$
$\equiv \sim(p \to q) \vee \sim r$ ....[Negation of conjunction]
$\equiv (p \wedge \sim q) \vee \sim r$ ....[Negation of implication]

23. Express the given circuits in symbolic form.

i.



ii.



***Solution*:**

i. Let p: the switch S_1 is closed.
q: the switch S_2 is closed.
~p: the switch S_1' is closed or the switch S_1 is open.
~q: the switch S_2' is closed or the switch S_2 is open.

$\therefore$ The symbolic form of the given circuit is
$(p \wedge q) \vee (\sim p) \vee (p \wedge \sim q)$

ii. Let p: the switch S_1 is closed
q: the switch S_2 is closed
r: the switch S_3 is closed

$\therefore$ The symbolic form of the given circuit is
$(p \vee q) \wedge (p \vee r)$

24. Construct the switching circuits of the following statements:

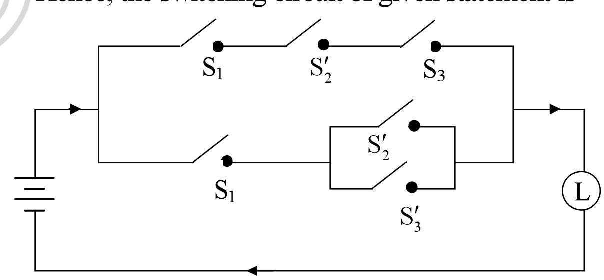
i. $\mathbf{(p \wedge \sim q \wedge r) \vee [p \wedge (\sim q \vee \sim r)]}$

ii. $\mathbf{[(p \wedge r) \vee (\sim q \wedge \sim r)] \wedge (\sim p \wedge \sim r)}$

***Solution*:**

i. Let p: the switch S_1 is closed.
q: the switch S_2 is closed.
r: the switch S_3 is closed.
~q: the switch S_2' is closed or the switch S_2 is open.
~r: the switch S_3' is closed or the switch S_3 is open.

Consider the given statement,
$(p \wedge \sim q \wedge r) \vee [p \wedge (\sim q \vee \sim r)]$
$p \wedge \sim q \vee r$: represents that the switches S_1, S_2' and S_3 are connected in series.
$p \wedge (\sim q \vee \sim r)$: represents that parallel combination of S_2' and S_3' is connected in series with S_1.
Therefore, $(p \wedge \sim q \wedge r) \vee [p \wedge (\sim q \vee \sim r)]$ represents that the circuits corresponding to $[(p \wedge \sim q \wedge r)$ and $[p \wedge (\sim q \vee \sim r)]$ are connected in parallel with each other.
Hence, the switching circuit of given statement is

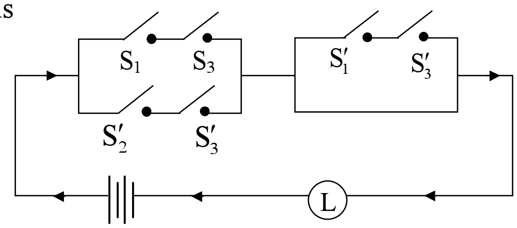


ii. Let p: the switch S_1 is closed
q: the switch S_2 is closed
r: the switch S_3 is closed
~p: the switch S_1' is closed or the switch S_1 is open.
~q: the switch S_2' is closed or the switch S_2 is open.
~r: the switch S_3' is closed or the switch S_3 is open.

Consider the given statement,
$[(p \wedge r) \vee (\sim q \wedge \sim r)] \wedge (\sim p \wedge \sim r)$
$(p \wedge r) \vee (\sim q \wedge \sim r)$: represents that series combination of S_1 and S_3 and series combination of S_2' and S_3' are connected in parallel.
$\sim p \wedge \sim r$: represents that S_1' and S_3' are connected in series.

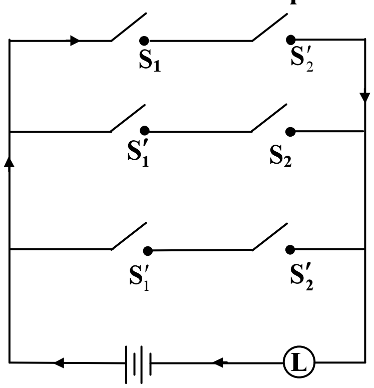
Therefore, $[(p \wedge r) \vee (\sim q \wedge \sim r)] \wedge (\sim p \wedge \sim r)$ represents that the circuits corresponding to $[(p \wedge r) \vee (\sim q \wedge \sim r)]$ and $(\sim p \wedge \sim r)$ are connected in series.

Hence, switching circuit of the given statement is

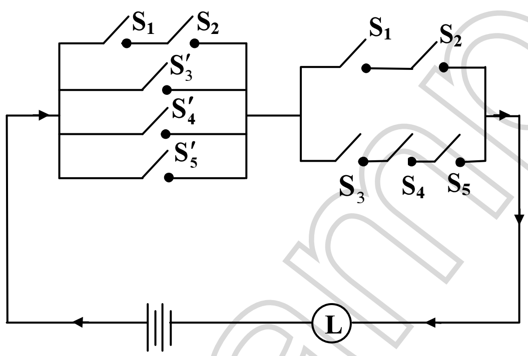


25. Simplify the following circuits so that the new circuit has minimum number of switches. Also draw the simplified circuit.

i.



ii.



***Solution*:**

i. *Refer to Exercise 1.9 Q.3.*

ii. Let, p: The switch S_1 is closed.

q: The switch S_2 closed.

r: The switch S_3 is closed.

s: The switch S_4 is closed.

t: The switch S_5 is closed.

$\sim r$: The switch S_3' is closed or the switch S_3 is open.

$\sim s$: The switch S_4' is closed or the switch S_4 is open.

$\sim t$: The switch S_5' is closed or the switch S_5 is open.

The symbolic form of the given circuit is,

$[(p \wedge q) \vee (\sim r \vee \sim s \vee \sim t)] \wedge [(p \wedge q) \vee (r \wedge s \wedge t)]$

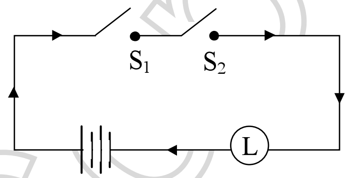
$\equiv [(p \wedge q) \vee \sim(r \wedge s \wedge t)] \wedge [(p \wedge q) \vee (r \wedge s \wedge t)]$ ….[De-Morgan's law]

$\equiv (p \wedge q) \vee [\sim(r \wedge s \wedge t) \wedge (r \wedge s \wedge t)]$ ….[Distributive law]

$\equiv (p \wedge q) \vee F$ ….[Complement law]

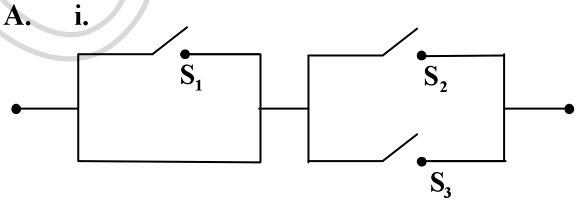
$\equiv p \wedge q$ ….[Identity law]

Hence, the simplified circuit is

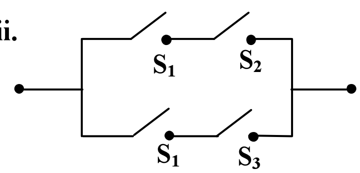


26. Check whether the following switching circuits are logically equivalent. Justify!

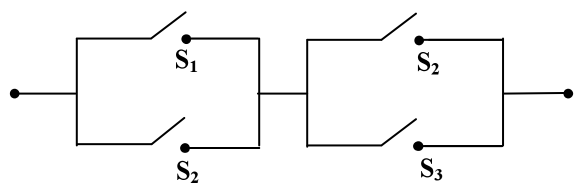
A. i.



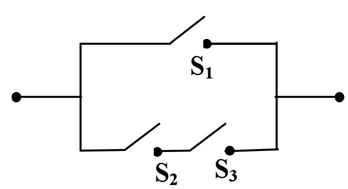
ii.



B. i.



ii.



***Solution*:**

Let,

p : The switch S_1 is closed.

q : The switch S_2 is closed.

r : The switch S_3 is closed.

A. i. The symbolic form of given circuit is,

$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$ ….(i)

….[distributive law]

ii. The symbolic form of given circuit is,

$(p \wedge q) \vee (p \wedge r)$ …. (ii)

$\therefore$ The given circuits are logically equivalent ....[From (i) and (ii)]

B. i. The symbolic form of the given circuit is

$(p \vee q) \wedge (q \vee r)$ ....(i)

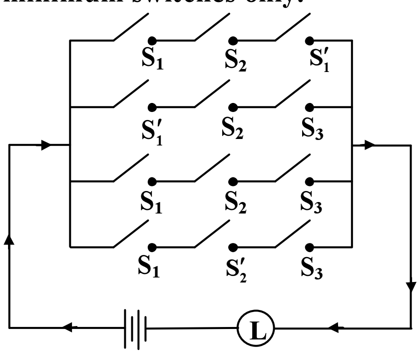
ii. The symbolic form of the given circuit is,

$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$

….[distributive law] ....(ii)

$\therefore$ The given circuits are not logically equivalent ....[From (i) and (ii)]

27. Give alternative arrangement of the following circuit, so that the new circuit has minimum switches only.



***Solution*:**

Let, p : The switch S_1 is closed.

q : The switch S_2 is closed.

r : The switch S_3 is closed.

~p: The switch S_1' is closed or the switch S_1 is open.

~q: The switch S_2' is closed or the switch S_2 is open.

The symbolic form of given circuit is,

$(p \wedge q \wedge \sim p) \vee (\sim p \wedge q \wedge r) \vee (p \wedge q \wedge r) \vee (p \wedge \sim q \wedge r)$

$\equiv (p \wedge \sim p \wedge q) \vee (\sim p \wedge q \wedge r) \vee (p \wedge q \wedge r) \vee (p \wedge \sim q \wedge r)$ ….[Commutative law]

$\equiv (F \wedge q) \vee (\sim p \wedge q \wedge r) \vee (p \wedge q \wedge r) \vee (p \wedge \sim q \wedge r)$ ….[Complement law]

$\equiv F \vee (\sim p \wedge q \wedge r) \vee (p \wedge q \wedge r) \vee (p \wedge \sim q \wedge r)$ ….[Identity law]

$\equiv (\sim p \wedge q \wedge r) \vee (p \wedge q \wedge r) \vee (p \wedge \sim q \wedge r)$ ….[Identity Law]

$\equiv [(\sim p \vee p) \wedge (q \wedge r)] \vee (p \wedge \sim q \wedge r)$ ….[Distributive law]

$\equiv [T \wedge (q \wedge r)] \vee (p \wedge \sim q \wedge r)$ ….[Complement law]

$\equiv (q \wedge r) \vee (p \wedge \sim q \wedge r)$ ….[Identity law]

$\equiv [q \vee (p \wedge \sim q)] \wedge r$ ….[Distributive law]

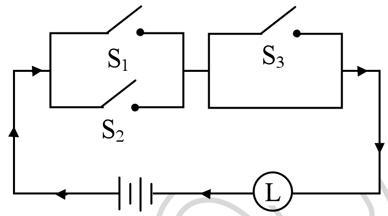
$\equiv [(q \vee p) \wedge (q \vee \sim q)] \wedge r$ ….[Distributive law]

$\equiv [(q \vee p) \wedge T] \wedge r$ ….[Complement law]

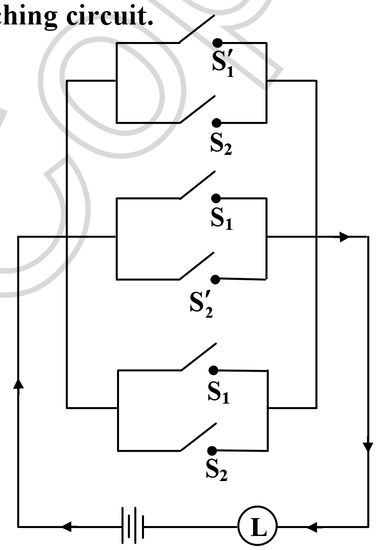
$\equiv (q \vee p) \wedge r$ ….[Identity law]

$\equiv (p \vee q) \wedge r$ ….[Commutative law]

$\therefore$ The switching circuit corresponding to the given statement is:



28. Draw the simplified circuit of the following switching circuit.



***Solution*:**

Let p : The switch S_1 is closed.

q : The switch S_2 is closed.

~p : The switch S_1' is closed or the switch S_1 is open.

~q : The switch S_2' is closed or the switch S_2 is open.

The symbolic form of the given circuit is

$(\sim p \vee q) \vee (p \vee \sim q) \vee (p \vee q)$

$\equiv (\sim p \vee q) \vee [p \vee (q \vee \sim q)]$ ….[Commutative and Distributive law]

$\equiv (\sim p \vee q) \vee (p \vee T)$ ….[Complement law]

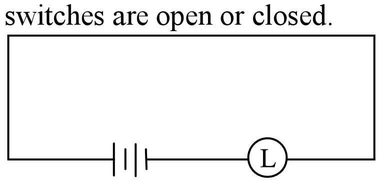
$\equiv (\sim p \vee q) \vee T$ ….[Identity law]

$\equiv \sim p \vee (q \vee T)$ ….[Commutative law]

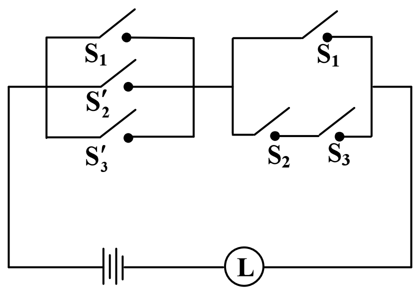
$\equiv \sim p \vee T$ ….[Identity law]

$\equiv T$ ….[Identity law]

$\therefore$ The symbolic form of the given circuit is a tautology. Hence, the current will always flow through the circuit irrespective of whether the switches are open or closed.



29. Represent the following switching circuit in symbolic form and construct its switching table. Write your conclusion from the switching table.



***Solution*:**

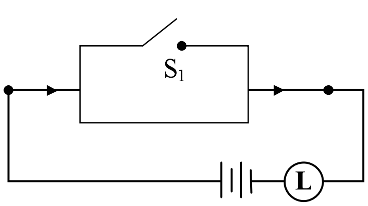
Let p : The switch S_1 is closed.
q : The switch S_2 is closed.
r : The switch S_3 is closed.
~p : The switch S_1' is closed or the switch S_1 is open
~q : The switch S_2' is closed or the switch S_2 is open
~r : The switch S_3' is closed or the switch S_3 is open.

The symbolic form of the given circuit is
$(p \vee \sim q \vee \sim r) \wedge [p \vee (q \wedge r)]$

$\therefore$ The switching table corresponding to the given statements is :

p	q	r	~q	~r	$p \vee (\sim q) \vee (\sim r)$	$q \wedge r$	$p \vee (q \wedge r)$	$[p \vee (\sim q) \vee (\sim r)] \wedge [p \vee (q \wedge r)]$
T	T	T	F	F	T	T	T	T
T	T	F	F	T	T	F	T	T
T	F	T	T	F	T	F	T	T
T	F	F	T	T	T	F	T	T
F	T	T	F	F	F	T	T	F
F	T	F	F	T	T	F	F	F
F	F	T	T	F	T	F	F	F
F	F	F	T	T	T	F	F	F

The final column of the above table is equivalent to the column of 'p' i.e. column corresponding to switch S_1. Hence, the given circuit is equivalent to the circuit where only switch S_1 is present.
Hence, switching circuit is as follows:



Multiple Choice Questions

1. Which of the following is a statement?
   (A) Stand up!
   (B) Will you help me?
   (C) Do you like social studies?
   (D) 27 is a perfect cube.

2. Which of the following is not a statement?
   (A) Please do me a favour.
   (B) 2 is an even integer.
   (C) 2 + 1 = 3.
   (D) The number 17 is prime.

3. Which of the following is an open statement?
   (A) x is a natural number.
   (B) Give me a glass of water.
   (C) Wish you best of luck.
   (D) Good morning to all.

4. Which of the following is not a proposition in logic.
   (A) $\sqrt{3}$ is a prime.
   (B) $\sqrt{2}$ is a irrational.
   (C) Mathematics is interesting.
   (D) 5 is an even integer.

5. If p: The sun has set
   q: The moon has risen,
   then the statement 'The sun has not set or the moon has not risen' in symbolic form is written as
   (A) $\sim p \vee \sim q$ (B) $\sim p \wedge q$
   (C) $p \wedge \sim q$ (D) $p \vee \sim q$

6. Assuming p: She is beautiful, q: She is clever, the verbal form of $p \wedge (\sim q)$ is
   (A) She is beautiful but not clever.
   (B) She is beautiful and clever.
   (C) She is not beautiful and not clever.
   (D) She is beautiful or not clever.

7. Let p: 'It is hot' and q: 'It is raining'. The verbal statement for $(p \wedge \sim q) \rightarrow p$ is
   (A) If it is hot and not raining, then it is hot.
   (B) If it is hot and raining, then it is hot.
   (C) If it is hot or raining, then it is not hot.
   (D) If it is hot and raining, then it is not hot.

8. Using the statements
   p: Kiran passed the examination,
   s : Kiran is sad.
   the statement 'It is not true that Kiran passes therefore he is sad' in symbolic form is
   (A) $\sim p \rightarrow s$ (B) $\sim (p \rightarrow \sim s)$
   (C) $\sim p \rightarrow \sim s$ (D) $\sim (p \rightarrow s)$

9. Assuming p: She is beautiful, q: She is clever, the verbal form of $\sim p \wedge (\sim q)$ is
(A) She is beautiful but not clever.
(B) She is beautiful and clever.
(C) She is not beautiful and not clever.
(D) She is beautiful or not clever.

10. The converse of the statement 'If it is raining then it is cool' is
(A) If it is cool then it is raining.
(B) If it is not cool then it is raining.
(C) If it is not cool then it is not raining.
(D) If it is not raining then it is not cool.

11. If p and q are simple propositions, then $p \wedge q$ is true when
(A) p is true and q is false.
(B) p is false and q is true.
(C) p is true and q is true.
(D) p is false q is false.

12. Which of the following is logically equivalent to $\sim[\sim p \rightarrow q]$
(A) $p \vee \sim q$ (B) $\sim p \wedge q$
(C) $\sim p \wedge q$ (D) $\sim p \wedge \sim q$

13. The logically equivalent statement of $p \rightarrow q$ is
(A) $\sim p \vee q$ (B) $q \rightarrow \sim p$
(C) $\sim q \vee p$ (D) $\sim q \vee \sim p$

14. The logically equivalent statement of $\sim p \vee \sim q$ is
(A) $\sim p \wedge \sim q$ (B) $\sim(p \wedge q)$
(C) $\sim(p \vee q)$ (D) $p \wedge q$

15. The contrapositive of $(p \vee q) \rightarrow r$ is
(A) $\sim r \rightarrow \sim p \wedge \sim q$ (B) $\sim r \rightarrow (p \vee q)$
(C) $r \rightarrow (p \vee q)$ (D) $p \rightarrow (q \vee r)$

16. Which of the following propositions is true?
(A) $p \rightarrow q \equiv \sim p \rightarrow \sim q$
(B) $\sim(p \rightarrow \sim q) \equiv \sim p \wedge q$
(C) $\sim(p \leftrightarrow q) \equiv [\sim (p \rightarrow q) \wedge \sim (q \rightarrow p)]$
(D) $\sim(\sim p \rightarrow \sim q) \equiv \sim p \wedge q$

17. When two statements are connected by the connective 'if and only if' then the compound statement is called
(A) conjunction of the statements.
(B) disjunction of the statements.
(C) biconditional statement.
(D) conditional statement.

18. If p and q be two statements then the conjunction of the statements, $p \wedge q$ is false when
(A) both p and q are true.
(B) either p or q are true
(C) either p or q or both are false.
(D) both p and q are false.

19. The negation of the statement, "The question paper is not easy and we shall not pass" is
(A) The question paper is not easy or we shall not pass.
(B) The question paper is not easy implies we shall not pass.
(C) The question paper is easy or we shall pass.
(D) We shall pass implies the question paper is not easy.

20. The statement $(p \wedge q) \wedge (\sim p \vee \sim q)$ is
(A) a contradiction.
(B) a tautology.
(C) neither a contradiction nor a tautology.
(D) equivalent to $p \vee q$.

21. The proposition $p \wedge \sim p$ is a
(A) tautology and contradiction.
(B) contingency.
(C) tautology.
(D) contradiction.

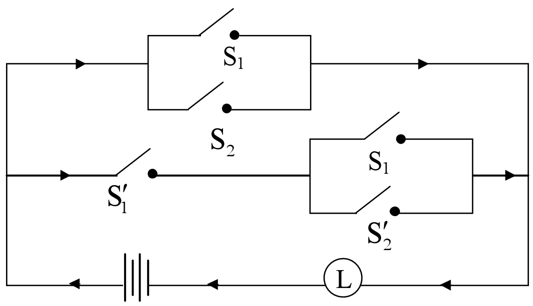
22. The proposition $p \rightarrow \sim (p \wedge q)$ is a
(A) tautology
(B) contradiction
(C) contingency
(D) either (A) or (B)

23. The false statement in the following is
(A) $p \wedge (\sim p)$ is a contradiction.
(B) $(p \rightarrow q) \rightarrow (\sim q \rightarrow \sim p)$ is a contradiction.
(C) $\sim(\sim p) \rightarrow p$ is a tautology.
(D) $p \vee (\sim p)$ is a tautology.

24. Negation of $\sim(p \vee q)$ is
(A) $\sim p \vee \sim q$
(B) $\sim p \wedge \sim q$
(C) $p \wedge \sim q$
(D) $p \vee \sim q$

25. The dual of $\sim(p\vee q) \vee [p\vee(q\wedge\sim r)]$is,
(A) $\sim(p\wedge q) \wedge [p \vee (q\wedge\sim r)]$
(B) $(p\wedge q) \wedge [p \wedge (q\vee \sim r)]$
(C) $\sim(p\wedge q) \wedge [p \wedge (q\wedge r)]$
(D) $\sim(p\wedge q) \wedge [p \wedge (q \vee \sim r)]$

26. The symbolic form of the following circuit, where p: switch S_1 is closed.and q: switch S_2 is closed, is-
(A) $(p \vee q) \wedge [\sim p \vee (p \wedge \sim q)]$
(B) $(\sim p \wedge q) \vee [\sim p \vee (p \wedge \sim q)]$
(C) $(p \vee q) \vee [\sim p \wedge (p \vee \sim q)]$
(D) $(p \wedge q) \vee [\sim p \wedge (p \wedge \sim q)]$



27. If A = {2, 3, 4, 5, 6}, then which of the following is not true? **[Oct 13]**
(A) $\exists\, x \in$ A such that $x + 3 = 8$
(B) $\exists\, x \in$ A such that $x + 2 < 5$
(C) $\exists\, x \in$ A such that $x + 2 < 9$
(D) $\forall\, x \in$ A such that $x + 6 \geq 9$

28. If $p \wedge q = F$, $p \to q = F$, then the truth value of p and q is : **[Oct 15]**
(A) T, T (B) T, F
(C) F, T (D) F, F

29. The negation of $p \wedge (q \to r)$ is **[Mar 16]**
(A) $p \vee (\sim q \vee r)$
(B) $\sim p \wedge (q \to r)$
(C) $\sim p \wedge (\sim q \to \sim r)$
(D) $\sim p \vee (q \wedge \sim r)$

30. Inverse of the statement pattern $(p \vee q) \to (p \wedge q)$ is **[July 16]**
(A) $(p \wedge q) \to (p \vee q)$
(B) $\sim (p \vee q) \to (p \wedge q)$
(C) $(\sim p \vee \sim q) \to (\sim p \wedge \sim q)$
(D) $(\sim p \wedge \sim q) \to (\sim p \vee \sim q)$

Answers to Multiple Choice Questions

1.	(D)	2.	(A)	3.	(A)	4.	(C)
5.	(A)	6.	(A)	7.	(A)	8.	(D)
9.	(C)	10.	(A)	11.	(C)	12.	(D)
13.	(A)	14.	(B)	15.	(A)	16.	(D)
17.	(C)	18.	(C)	19.	(C)	20.	(A)
21.	(D)	22.	(C)	23.	(B)	24.	(B)
25.	(D)	26.	(C)	27.	(D)	28.	(B)
29.	(D)	30.	(D)				