Mechanics of materials

Chapter 1+2

Mechanical properties of materials

By

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Mechanics of materials

Mechanical properties of materials

Stress and strain

- If a body is subjected to external loads, internal loads represented as distributed load will be generated due to the interrelation between the material crystals (or particles). This type of force is **resistance** to the effect of the external loading.
- The internal resistance for external loads is called the **strength of the material** and the distribute load is called **stress.**
- The effects of external loads are not the stress but there is **the strain** effect. When the material is subjected to external loads, it will be **deformed (i.e. change in its shape and/or size).** The percentage of deformation is called strain

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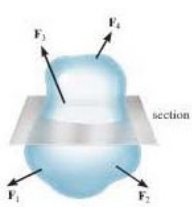
Mechanical properties of materials

Stress

- To understand the concept of stress, we can start with a random body subjected to system of external loads as shown in fig.a.



(a)

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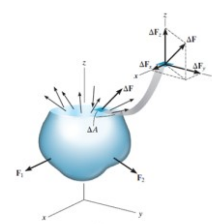
Mechanical properties of materials

Stress

- Make a section as shown in the fig.b. as you can see, we can assume that the stress is a distributed load. However, the direction of the stress is in every way. This assumption is because the infinite number of crystals in the material and the its random distribution.



(b)

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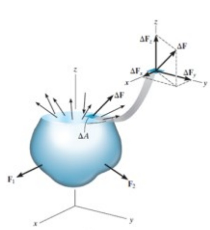
Stress

- If we take an infinitesimal area element as shown with ΔF force acting on it. The force element ΔF can be resolved to its rectangular components: ΔF_x, ΔF_y and ΔF_z.
- Later on we will discuss the equilibrium condition.



(b)

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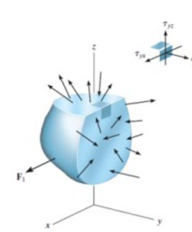
Stress

- Apply other section as shown in the figure. At this stage, we can divide the internal forces into normal forces (tensile) and tangential forces (shear).
- In engineering, we give the normal stress the symbol (σ) and the tangential stress the symbol (τ). Further on, we will call the normal stress as **tensile** or **compression** stress according to the direction and the tangential stress the **shear** stress.



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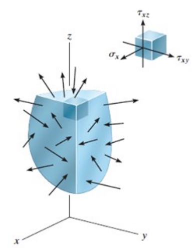
Mechanical properties of materials

Stress

Mechanics of materials

❑A final section can be performed as shown in the figure. A three dimensional section is the best way to examine the stress concept.

❑The normal stresses can be in the three dimensions (x, y and z) and the tangential (or shear) stresses are distributed on the surface of the cubic element shown in the figure (two components for each surface)



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Stress

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❑In general, if the loads are in the three dimensions, the internal loads will be: 3 normal components (σ) and 6 tangential (τ).

❑To obtain an average numerical value for these stresses, the limits of the load over the area is taken. Mathematically:

$$\sigma_x = \lim_{\Delta A \to 0} \frac{\Delta F_x}{\Delta A} \qquad \tau_{xy} = \lim_{\Delta A \to 0} \frac{\Delta F_y}{\Delta A} \qquad \tau_{xz} = \lim_{\Delta A \to 0} \frac{\Delta F_z}{\Delta A}$$

$$\sigma_y = \lim_{\Delta A \to 0} \frac{\Delta F_y}{\Delta A} \qquad \tau_{yx} = \lim_{\Delta A \to 0} \frac{\Delta F_x}{\Delta A} \qquad \tau_{yz} = \lim_{\Delta A \to 0} \frac{\Delta F_z}{\Delta A}$$

$$\sigma_z = \lim_{\Delta A \to 0} \frac{\Delta F_z}{\Delta A} \qquad \tau_{zx} = \lim_{\Delta A \to 0} \frac{\Delta F_x}{\Delta A} \qquad \tau_{zy} = \lim_{\Delta A \to 0} \frac{\Delta F_y}{\Delta A}$$

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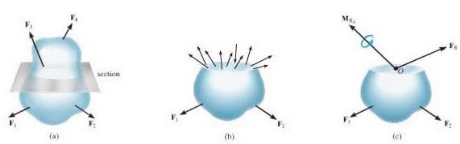
Mechanical properties of materials

Stress

Mechanics of materials

❑The stress analysis can be reduced to single force – single moment system (as you learn in Static course) at the centroid of the section as shown in the figure.



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Stress

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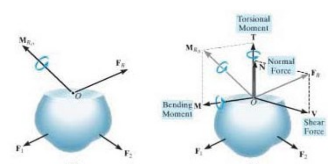
❑Resolve F_R and $M_{R,o}$ to its rectangular components:



❑As you can see there are two force components : **normal** and **shear**

❑Also you can see there are two moment components : **torsion** and **bending**

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Stress

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❑In general, the internal loads (or stresses) generated inside a under external load body are:

- ➢Normal stresses: tensile or compression
- ➢Shear stresses
- ➢Torsion stresses
- ➢Bending stresses

❑In the next chapters, we will discuses each one alone (i.e. pure stress).

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Stress

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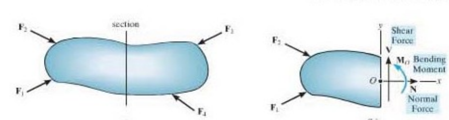
❑If the external loads are **coplaner** , the internal stresses generated will be: **normal, shear** and **bending** as shown in the figure



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EXAMPLE 1.1

Determine the resultant internal loadings acting on the cross section at C of the cantilevered beam shown in Fig. 1–4a.

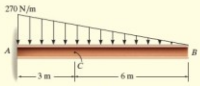
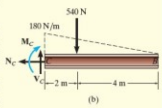


Fig. 1–4

SOLUTION

Support Reactions. The support reactions at A do not have to be determined if segment CB is considered.

Free-Body Diagram. The free-body diagram of segment CB is shown in Fig. 1–4b. It is important to keep the distributed loading on the segment until *after* the section is made. Only then should this loading be replaced by a single resultant force. Notice that the intensity of the distributed loading at C is found by proportion, i.e., from Fig. 1–4a, $w/6\text{ m} = (270\text{ N/m})/9\text{ m}$, $w = 180\text{ N/m}$. The magnitude of the resultant of the distributed load is equal to the area under the loading curve (triangle) and acts through the centroid of this area. Thus, $F = \frac{1}{2}(180\text{ N/m})(6\text{ m}) = 540\text{ N}$, which acts $\frac{1}{3}(6\text{ m}) = 2\text{ m}$ from C as shown in Fig. 1–4b.



EXAMPLE 1.1 CONTINUED

Equations of Equilibrium. Applying the equations of equilibrium we have

$\xrightarrow{+}\Sigma F_x = 0;\quad -N_C = 0$

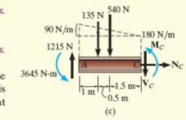
$N_C = 0$ *Ans.*

$+\uparrow\Sigma F_y = 0;\quad V_C - 540\text{ N} = 0$

$V_C = 540\text{ N}$ *Ans.*

$\circlearrowleft + \Sigma M_C = 0;\quad -M_C - 540\text{ N}(2\text{ m}) = 0$

$M_C = -1080\text{ N}\cdot\text{m}$ *Ans.*



NOTE: The negative sign indicates that $\mathbf{M}_C$ acts in the opposite direction to that shown on the free-body diagram. Try solving this problem using segment AC, by first obtaining the support reactions at A, which are given in Fig. 1–4c.

EXAMPLE 1.2

Determine the resultant internal loadings acting on the cross section at C of the machine shaft shown in Fig. 1–5a. The shaft is supported by journal bearings at A and B, which only exert vertical forces on the shaft.

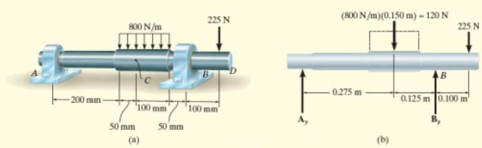


Fig. 1–5

EXAMPLE 1.2 CONTINUED

SOLUTION

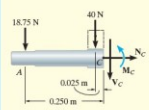
We will solve this problem using segment AC of the shaft.

Support Reactions. The free-body diagram of the entire shaft is shown in Fig. 1–5b. Since segment AC is to be considered, only the reaction at A has to be determined. Why?

$\circlearrowleft + \Sigma M_B = 0;\ -A_y(0.400\text{ m}) + 120\text{ N}(0.125\text{ m}) - 225\text{ N}(0.100\text{ m}) = 0$

$A_y = -18.75\text{ N}$

The negative sign indicates that $\mathbf{A}_y$ acts in the *opposite sense* to that shown on the free-body diagram.



Free-Body Diagram. The free-body diagram of segment AC is shown in Fig. 1–5c.

Equations of Equilibrium.

$\xrightarrow{+}\Sigma F_x = 0;\quad N_C = 0$ *Ans.*

$+\uparrow\Sigma F_y = 0;\quad -18.75\text{ N} - 40\text{ N} - V_C = 0$

$V_C = -58.8\text{ N}$ *Ans.*

$\circlearrowleft + \Sigma M_C = 0;\quad M_C + 40\text{ N}(0.025\text{ m}) + 18.75\text{ N}(0.250\text{ m}) = 0$

$M_C = -5.69\text{ N}\cdot\text{m}$ *Ans.*

NOTE: The negative signs for V_C and M_C indicate they act in the opposite directions on the free-body diagram. As an exercise, calculate the reaction at B and try to obtain the same results using segment CBD of the shaft.

EXAMPLE 1.3

The 500-kg engine is suspended from the crane boom in Fig. 1–6a. Determine the resultant internal loadings acting on the cross section of the boom at point E.

SOLUTION

Support Reactions. We will consider segment AE of the boom so we must first determine the pin reactions at A. Notice that member CD is a two-force member. The free-body diagram of the boom is shown in Fig. 1–6b. Applying the equations of equilibrium,

$\circlearrowleft + \Sigma M_A = 0;\quad F_{CD}(\frac{3}{5})(2\text{ m}) - [500(9.81)\text{ N}](3\text{ m}) = 0$

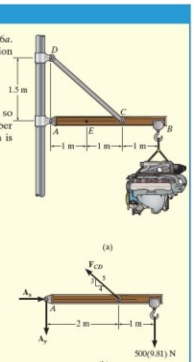
$F_{CD} = 12\,262.5\text{ N}$

$\xrightarrow{+}\Sigma F_x = 0;\quad A_x - (12\,262.5\text{ N})(\frac{4}{5}) = 0$

$A_x = 9810\text{ N}$

$+\uparrow\Sigma F_y = 0;\quad -A_y + (12\,262.5\text{ N})(\frac{3}{5}) - 500(9.81)\text{ N} = 0$

$A_y = 2452.5\text{ N}$



EXAMPLE 1.3 CONTINUED

Free-Body Diagram. The free-body diagram of segment AE is shown in Fig. 1–6c.

Equations of Equilibrium.

$\xrightarrow{+}\Sigma F_x = 0;\quad N_E + 9810\text{ N} = 0$

$N_E = -9810\text{ N} = -9.81\text{ kN}$ *Ans.*

$+\uparrow\Sigma F_y = 0;\quad -V_E - 2452.5\text{ N} = 0$

$V_E = -2452.5\text{ N} = -2.45\text{ kN}$ *Ans.*

$\circlearrowleft + \Sigma M_E = 0;\quad M_E + (2452.5\text{ N})(1\text{ m}) = 0$

$M_E = -2452.5\text{ N}\cdot\text{m} = -2.45\text{ kN}\cdot\text{m}$ *Ans.*

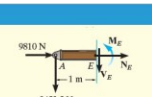


Fig. 1–6

EXAMPLE 1.4

Determine the resultant internal loadings acting on the cross section at G of the beam shown in Fig. 1–7a. Each joint is pin connected.

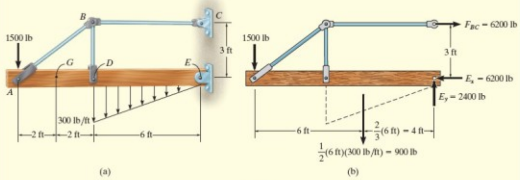
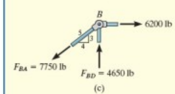


Fig. 1–7

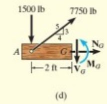
EXAMPLE 1.4 CONTINUED



SOLUTION

Support Reactions. Here we will consider segment AG. The free-body diagram of the *entire* structure is shown in Fig. 1–7b. Verify the calculated reactions at E and C. In particular, note that BC is a *two-force member* since only two forces act on it. For this reason the force at C must act along BC, which is horizontal as shown.

Since BA and BD are also two-force members, the free-body diagram of joint B is shown in Fig. 1–7c. Again, verify the magnitudes of forces $\mathbf{F}_{BA}$ and $\mathbf{F}_{BD}$.



Free-Body Diagram. Using the result for $\mathbf{F}_{BA}$, the free-body diagram of segment AG is shown in Fig. 1–7d.

Equations of Equilibrium.

$$\xrightarrow{+} \Sigma F_x = 0; \quad 7750\text{ lb}\left(\tfrac{4}{5}\right) + N_G = 0 \qquad N_G = -6200\text{ lb} \qquad \textit{Ans.}$$

$$+\uparrow \Sigma F_y = 0; \quad -1500\text{ lb} + 7750\text{ lb}\left(\tfrac{3}{5}\right) - V_G = 0$$

$$V_G = 3150\text{ lb} \qquad \textit{Ans.}$$

$$\circlearrowleft + \Sigma M_G = 0; \quad M_G - (7750\text{ lb})\left(\tfrac{3}{5}\right)(2\text{ ft}) + 1500\text{ lb}(2\text{ ft}) = 0$$

$$M_G = 6300\text{ lb}\cdot\text{ft} \qquad \textit{Ans.}$$

EXAMPLE 1.5

Determine the resultant internal loadings acting on the cross section at B of the pipe shown in Fig. 1–8a. The pipe has a mass of 2 kg/m and is subjected to both a vertical force of 50 N and a couple moment of 70 N · m at its end A. It is fixed to the wall at C.

SOLUTION

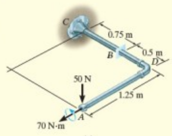
The problem can be solved by considering segment AB, so we do not need to calculate the support reactions at C.

Free-Body Diagram. The x, y, z axes are established at B and the free-body diagram of segment AB is shown in Fig. 1–8b. The resultant force and moment components at the section are assumed to act in the positive coordinate directions and to pass through the *centroid* of the cross-sectional area at B. The weight of each segment of pipe is calculated as follows:

$$W_{BD} = (2\text{ kg/m})(0.5\text{ m})(9.81\text{ N/kg}) = 9.81\text{ N}$$

$$W_{AD} = (2\text{ kg/m})(1.25\text{ m})(9.81\text{ N/kg}) = 24.525\text{ N}$$

These forces act through the center of gravity of each segment.



EXAMPLE 1.5 CONTINUED

Equations of Equilibrium. Applying the six scalar equations of equilibrium, we have*

$$\Sigma F_x = 0; \qquad (F_B)_x = 0 \qquad \textit{Ans.}$$

$$\Sigma F_y = 0; \qquad (F_B)_y = 0 \qquad \textit{Ans.}$$

$$\Sigma F_z = 0; \quad (F_B)_z - 9.81\text{ N} - 24.525\text{ N} - 50\text{ N} = 0$$

$$(F_B)_z = 84.3\text{ N} \qquad \textit{Ans.}$$

$$\Sigma (M_B)_x = 0; \quad (M_B)_x + 70\text{ N}\cdot\text{m} - 50\text{ N}(0.5\text{ m}) - 24.525\text{ N}(0.5\text{ m}) - 9.81\text{ N}(0.25\text{ m}) = 0$$

$$(M_B)_x = -30.3\text{ N}\cdot\text{m} \qquad \textit{Ans.}$$

$$\Sigma (M_B)_y = 0; \quad (M_B)_y + 24.525\text{ N}(0.625\text{ m}) + 50\text{ N}(1.25\text{ m}) = 0$$

$$(M_B)_y = -77.8\text{ N}\cdot\text{m} \qquad \textit{Ans.}$$

$$\Sigma (M_B)_z = 0; \qquad (M_B)_z = 0 \qquad \textit{Ans.}$$

NOTE: What do the negative signs for $(M_B)_x$ and $(M_B)_y$ indicate? Note that the normal force $N_B = (F_B)_y = 0$, whereas the shear force is $V_B = \sqrt{(0)^2 + (84.3)^2} = 84.3$ N. Also, the torsional moment is $T_B = (M_B)_y = 77.8$ N · m and the bending moment is $M_B = \sqrt{(30.3)^2 + (0)^2} = 30.3$ N · m.

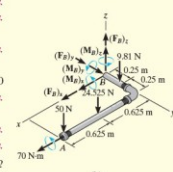


Fig. 1–8

*The *magnitude* of each moment about an axis is equal to the magnitude of each force times the perpendicular distance from the axis to the line of action of the force. The *direction* of each moment is determined using the right-hand rule, with positive moments (thumb) directed along the positive coordinate axes.

Mechanical properties of materials

Mechanics of materials

Stress

❑If we desire to find average stress (normal or shear), we reduce the limits presented previously as shown below

$$\sigma_x = \lim_{\Delta A \to 0} \frac{\Delta F_x}{\Delta A} = \frac{F_x}{A} \qquad \tau_{xy} = \frac{F_y}{A_y} \quad \tau_{xz} = \frac{F_z}{A_z}$$

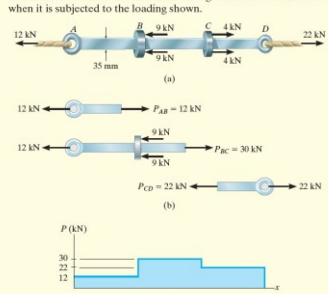
$$\sigma_y = \lim_{\Delta A \to 0} \frac{\Delta F_y}{\Delta A} = \frac{F_y}{A} \qquad \tau_{yx} = \frac{F_x}{A_x} \quad \tau_{yz} = \frac{F_z}{A_z}$$

$$\sigma_z = \lim_{\Delta A \to 0} \frac{\Delta F_z}{\Delta A} = \frac{F_z}{A} \qquad \tau_{zx} = \frac{F_x}{A_x} \quad \tau_{zy} = \frac{F_y}{A_y}$$

❑In general: **average stress = force / area: σ=P/A and τ = V/A**

EXAMPLE 1.6

The bar in Fig. 1–16a has a constant width of 35 mm and a thickness of 10 mm. Determine the maximum average normal stress in the bar when it is subjected to the loading shown.



EXAMPLE 1.6 CONTINUED

SOLUTION

Internal Loading. By inspection, the internal axial forces in regions *AB*, *BC*, and *CD* are all constant yet have different magnitudes. Using the method of sections, these loadings are determined in Fig. 1–16*b*; and the normal force diagram which represents these results graphically is shown in Fig. 1–16*c*. The largest loading is in region *BC*, where $P_{BC} = 30$ kN. Since the cross-sectional area of the bar is *constant*, the largest average normal stress also occurs within this region of the bar.

Average Normal Stress. Applying Eq. 1–6, we have

$$\sigma_{BC} = \frac{P_{BC}}{A} = \frac{30(10^3)\text{ N}}{(0.035\text{ m})(0.010\text{ m})} = 85.7\text{ MPa} \qquad \textit{Ans.}$$

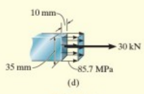


Fig. 1–16

NOTE: The stress distribution acting on an arbitrary cross section of the bar within region *BC* is shown in Fig. 1–16*d*. Graphically the *volume* (or "block") represented by this distribution of stress is equivalent to the load of 30 kN; that is, 30 kN = (85.7 MPa)(35 mm)(10 mm).

EXAMPLE 1.7

The 80-kg lamp is supported by two rods *AB* and *BC* as shown in Fig. 1–17*a*. If *AB* has a diameter of 10 mm and *BC* has a diameter of 8 mm, determine the average normal stress in each rod.

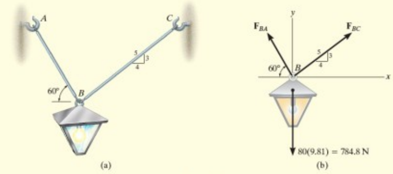


Fig. 1–17

EXAMPLE 1.7 CONTINUED

SOLUTION

Internal Loading. We must first determine the axial force in each rod. A free-body diagram of the lamp is shown in Fig. 1–17*b*. Applying the equations of force equilibrium,

$$\xrightarrow{+} \Sigma F_x = 0; \quad F_{BC}\left(\tfrac{4}{5}\right) - F_{BA}\cos 60^\circ = 0$$

$$+\uparrow \Sigma F_y = 0; \quad F_{BC}\left(\tfrac{3}{5}\right) + F_{BA}\sin 60^\circ - 784.8\text{ N} = 0$$

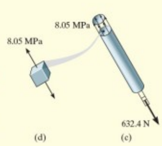
$$F_{BC} = 395.2\text{ N}, \qquad F_{BA} = 632.4\text{ N}$$

By Newton's third law of action, equal but opposite reaction, these forces subject the rods to tension throughout their length.

Average Normal Stress. Applying Eq. 1–6,

$$\sigma_{BC} = \frac{F_{BC}}{A_{BC}} = \frac{395.2\text{ N}}{\pi(0.004\text{ m})^2} = 7.86\text{ MPa} \qquad \textit{Ans.}$$

$$\sigma_{BA} = \frac{F_{BA}}{A_{BA}} = \frac{632.4\text{ N}}{\pi(0.005\text{ m})^2} = 8.05\text{ MPa} \qquad \textit{Ans.}$$



NOTE: The average normal stress distribution acting over a cross section of rod *AB* is shown in Fig. 1–17*c*, and at a point on this cross section, an element of material is stressed as shown in Fig. 1–17*d*.

EXAMPLE 1.8

The casting shown in Fig. 1–18*a* is made of steel having a specific weight of $\gamma_{st} = 490$ lb/ft³. Determine the average compressive stress acting at points *A* and *B*.

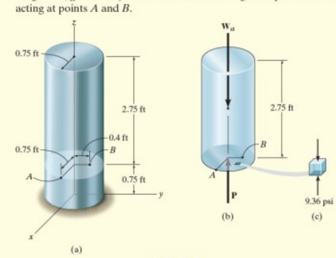


Fig. 1–18

EXAMPLE 1.8 CONTINUED

SOLUTION

Internal Loading. A free-body diagram of the top segment of the casting where the section passes through points *A* and *B* is shown in Fig. 1–18*b*. The weight of this segment is determined from $W_{st} = \gamma_{st} V_{st}$. Thus the internal axial force *P* at the section is

$$+\uparrow \Sigma F_z = 0; \qquad P - W_{st} = 0$$

$$P - (490\text{ lb/ft}^3)(2.75\text{ ft})[\pi(0.75\text{ ft})^2] = 0$$

$$P = 2381\text{ lb}$$

Average Compressive Stress. The cross-sectional area at the section is $A = \pi(0.75\text{ ft})^2$, and so the average compressive stress becomes

$$\sigma = \frac{P}{A} = \frac{2381\text{ lb}}{\pi(0.75\text{ ft})^2} = 1347.5\text{ lb/ft}^2$$

$$\sigma = 1347.5\text{ lb/ft}^2\,(1\text{ ft}^2/144\text{ in}^2) = 9.36\text{ psi} \qquad \textit{Ans.}$$

NOTE: The stress shown on the volume element of material in Fig. 1–18*c* is representative of the conditions at either point *A* or *B*. Notice that this stress acts *upward* on the bottom or shaded face of the element since this face forms part of the bottom surface area of the section, and on this surface, the resultant internal force **P** is pushing upward.

EXAMPLE 1.9

Member *AC* shown in Fig. 1–19*a* is subjected to a vertical force of 3 kN. Determine the position *x* of this force so that the average compressive stress at the smooth support *C* is equal to the average tensile stress in the tie rod *AB*. The rod has a cross-sectional area of 400 mm² and the contact area at *C* is 650 mm².

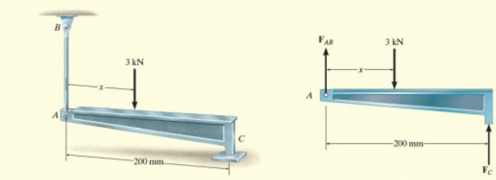


Fig. 1–19

EXAMPLE 1.9 CONTINUED

SOLUTION

Internal Loading. The forces at A and C can be related by considering the free-body diagram for member AC, Fig. 1–19b. There are three unknowns, namely, F_{AB}, F_C, and x. To solve this problem we will work in units of newtons and millimeters.

$$+\uparrow \Sigma F_y = 0; \qquad F_{AB} + F_C - 3000\ \text{N} = 0 \qquad (1)$$

$$\circlearrowleft + \Sigma M_A = 0; \qquad -3000\ \text{N}(x) + F_C(200\ \text{mm}) = 0 \qquad (2)$$

Average Normal Stress. A necessary third equation can be written that requires the tensile stress in the bar AB and the compressive stress at C to be equivalent, i.e.,

$$\sigma = \frac{F_{AB}}{400\ \text{mm}^2} = \frac{F_C}{650\ \text{mm}^2}$$

$$F_C = 1.625F_{AB}$$

Substituting this into Eq. 1, solving for F_{AB}, then solving for F_C, we obtain

$$F_{AB} = 1143\ \text{N}$$

$$F_C = 1857\ \text{N}$$

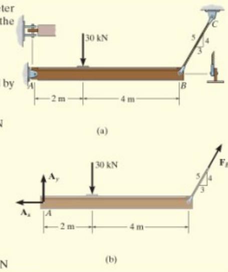
The position of the applied load is determined from Eq. 2,

$$x = 124\ \text{mm} \qquad \textit{Ans.}$$

NOTE: $0 < x < 200$ mm, as required.

EXAMPLE 1.10

Determine the average shear stress in the 20-mm-diameter pin at A and the 30-mm-diameter pin at B that support the beam in Fig. 1–22a.



SOLUTION

Internal Loadings. The forces on the pins can be obtained by considering the equilibrium of the beam, Fig. 1–22b.

$$\circlearrowleft + \Sigma M_A = 0; \quad F_B\left(\frac{4}{5}\right)(6\ \text{m}) - 30\ \text{kN}(2\ \text{m}) = 0 \quad F_B = 12.5\ \text{kN}$$

$$\xrightarrow{+} \Sigma F_x = 0; \quad (12.5\ \text{kN})\left(\frac{3}{5}\right) - A_x = 0 \quad A_x = 7.50\ \text{kN}$$

$$+\uparrow \Sigma F_y = 0; \quad A_y + (12.5\ \text{kN})\left(\frac{4}{5}\right) - 30\ \text{kN} = 0$$

$$A_y = 20\ \text{kN}$$

Thus, the resultant force acting on pin A is

$$F_A = \sqrt{A_x^2 + A_y^2} = \sqrt{(7.50\ \text{kN})^2 + (20\ \text{kN})^2} = 21.36\ \text{kN}$$

EXAMPLE 1.10 CONTINUED

The pin at A is supported by two fixed "leaves" and so the free-body diagram of the center segment of the pin shown in Fig. 1–22c has *two* shearing surfaces between the beam and each leaf. The force of the beam (21.36 kN) acting on the pin is therefore supported by shear force on each of these surfaces. This case is called *double shear*. Thus,

$$V_A = \frac{F_A}{2} = \frac{21.36\ \text{kN}}{2} = 10.68\ \text{kN}$$

In Fig. 1–22a, note that pin B is subjected to *single shear*, which occurs on the section between the cable and beam, Fig. 1–22d. For this pin segment,

$$V_B = F_B = 12.5\ \text{kN}$$

Average Shear Stress.

$$(\tau_A)_{avg} = \frac{V_A}{A_A} = \frac{10.68(10^3)\ \text{N}}{\frac{\pi}{4}(0.02\ \text{m})^2} = 34.0\ \text{MPa} \qquad \textit{Ans.}$$

$$(\tau_B)_{avg} = \frac{V_B}{A_B} = \frac{12.5(10^3)\ \text{N}}{\frac{\pi}{4}(0.03\ \text{m})^2} = 17.7\ \text{MPa} \qquad \textit{Ans.}$$

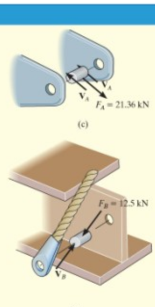


Fig. 1–22

EXAMPLE 1.11

If the wood joint in Fig. 1–23a has a width of 150 mm, determine the average shear stress developed along shear planes a–a and b–b. For each plane, represent the state of stress on an element of the material.

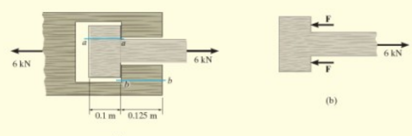


Fig. 1–23

EXAMPLE 1.11 CONTINUED

SOLUTION

Internal Loadings. Referring to the free-body diagram of the member, Fig. 1–23b,

$$\xrightarrow{+} \Sigma F_x = 0; \quad 6\ \text{kN} - F - F = 0 \qquad F = 3\ \text{kN}$$

Now consider the equilibrium of segments cut across shear planes a–a and b–b, shown in Figs. 1–23c and 1–23d.

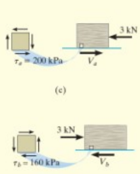
$$\xrightarrow{+} \Sigma F_x = 0; \quad V_a - 3\ \text{kN} = 0 \qquad V_a = 3\ \text{kN}$$

$$\xrightarrow{+} \Sigma F_x = 0; \quad 3\ \text{kN} - V_b = 0 \qquad V_b = 3\ \text{kN}$$

Average Shear Stress.

$$(\tau_a)_{avg} = \frac{V_a}{A_a} = \frac{3(10^3)\ \text{N}}{(0.1\ \text{m})(0.15\ \text{m})} = 200\ \text{kPa} \qquad \textit{Ans.}$$

$$(\tau_b)_{avg} = \frac{V_b}{A_b} = \frac{3(10^3)\ \text{N}}{(0.125\ \text{m})(0.15\ \text{m})} = 160\ \text{kPa} \qquad \textit{Ans.}$$



The state of stress on elements located on sections a–a and b–b is shown in Figs. 1–23c and 1–23d, respectively.

EXAMPLE 1.12

The inclined member in Fig. 1–24a is subjected to a compressive force of 600 lb. Determine the average compressive stress along the smooth areas of contact defined by AB and BC, and the average shear stress along the horizontal plane defined by DB.

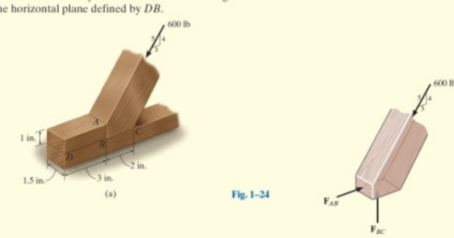


Fig. 1–24

EXAMPLE 1.12 CONTINUED

SOLUTION

Internal Loadings. The free-body diagram of the inclined member is shown in Fig. 1–24*b*. The compressive forces acting on the areas of contact are

$\xrightarrow{+} \Sigma F_x = 0;\quad F_{AB} - 600\text{ lb}\left(\tfrac{3}{5}\right) = 0 \quad F_{AB} = 360\text{ lb}$

$+\uparrow \Sigma F_y = 0;\quad F_{BC} - 600\text{ lb}\left(\tfrac{4}{5}\right) = 0 \quad F_{BC} = 480\text{ lb}$

Also, from the free-body diagram of the top segment *ABD* of the bottom member, Fig. 1–24*c*, the shear force acting on the sectioned horizontal plane *DB* is

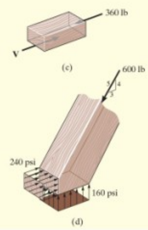
$\xrightarrow{+} \Sigma F_x = 0;\quad V = 360\text{ lb}$

Average Stress. The average compressive stresses along the horizontal and vertical planes of the inclined member are

$$\sigma_{AB} = \frac{F_{AB}}{A_{AB}} = \frac{360\text{ lb}}{(1\text{ in.})(1.5\text{ in.})} = 240\text{ psi} \qquad \textit{Ans.}$$

$$\sigma_{BC} = \frac{F_{BC}}{A_{BC}} = \frac{480\text{ lb}}{(2\text{ in.})(1.5\text{ in.})} = 160\text{ psi} \qquad \textit{Ans.}$$

These stress distributions are shown in Fig. 1–24*d*.

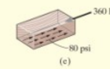


EXAMPLE 1.12 CONTINUED

The average shear stress acting on the horizontal plane defined by *DB* is

$$\tau_{avg} = \frac{360\text{ lb}}{(3\text{ in.})(1.5\text{ in.})} = 80\text{ psi} \qquad \textit{Ans.}$$

This stress is shown uniformly distributed over the sectioned area in Fig. 1–24*e*.



Mechanics of materials

Mechanical properties of materials

Stress

- Factor of safety is a ratio between the failure load over the allowable stress.
- The allowable stress is the stress theta you can not exceed through the design process.
- Because stress depends on **load** and **area** and the load is something enforced on us the area is the variable that we can use to design a member can handle the applied load. Mathematically:

$$F.S = \frac{F_{Fail}}{F_{Allow}} \qquad A_{normal} = \frac{P}{\sigma_{allow}} \qquad A_{shear} = \frac{V}{\tau_{allow}}$$

EXAMPLE 1.13

The control arm is subjected to the loading shown in Fig. 1–26*a*. Determine to the nearest $\frac{1}{4}$ in. the required diameter of the steel pin at *C* if the allowable shear stress for the steel is $\tau_{allow} = 8$ ksi.

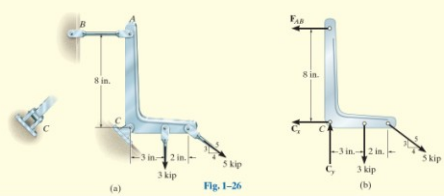


Fig. 1–26

EXAMPLE 1.13 CONTINUED

SOLUTION

Internal Shear Force. A free-body diagram of the arm is shown in Fig. 1–26*b*. For equilibrium we have

$\circlearrowleft + \Sigma M_C = 0;\quad F_{AB}(8\text{ in.}) - 3\text{ kip}\,(3\text{ in.}) - 5\text{ kip}\left(\tfrac{3}{5}\right)(5\text{ in.}) = 0$

$F_{AB} = 3\text{ kip}$

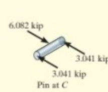
$\xrightarrow{+} \Sigma F_x = 0;\quad -3\text{ kip} - C_x + 5\text{ kip}\left(\tfrac{4}{5}\right) = 0 \quad C_x = 1\text{ kip}$

$+\uparrow \Sigma F_y = 0;\quad C_y - 3\text{ kip} - 5\text{ kip}\left(\tfrac{3}{5}\right) = 0 \quad C_y = 6\text{ kip}$

The pin at *C* resists the resultant force at *C*, which is

$$F_C = \sqrt{(1\text{ kip})^2 + (6\text{ kip})^2} = 6.082\text{ kip}$$

Since the pin is subjected to double shear, a shear force of 3.041 kip acts over its cross-sectional area *between* the arm and each supporting leaf for the pin, Fig. 1–26*c*.



EXAMPLE 1.13 CONTINUED

Required Area. We have

$$A = \frac{V}{\tau_{allow}} = \frac{3.041\text{ kip}}{8\text{ kip/in}^2} = 0.3802\text{ in}^2$$

$$\pi\left(\frac{d}{2}\right)^2 = 0.3802\text{ in}^2$$

$$d = 0.696\text{ in.}$$

Use a pin having a diameter of

$$d = \tfrac{3}{4}\text{ in.} = 0.750\text{ in.} \qquad \textit{Ans.}$$

EXAMPLE 1.14

The suspender rod is supported at its end by a fixed-connected circular disk as shown in Fig. 1–27*a*. If the rod passes through a 40-mm-diameter hole, determine the minimum required diameter of the rod and the minimum thickness of the disk needed to support the 20-kN load. The allowable normal stress for the rod is $\sigma_{allow} = 60$ MPa, and the allowable shear stress for the disk is $\tau_{allow} = 35$ MPa.

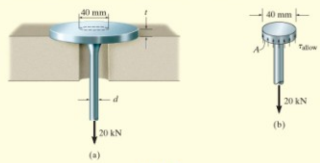


Fig. 1–27

EXAMPLE 1.14 CONTINUED

SOLUTION

Diameter of Rod. By inspection, the axial force in the rod is 20 kN. Thus the required cross-sectional area of the rod is

$$A = \frac{P}{\sigma_{allow}}; \qquad \frac{\pi}{4}d^2 = \frac{20(10^3)\text{ N}}{60(10^6)\text{ N/m}^2}$$

so that

$$d = 0.0206\text{ m} = 20.6\text{ mm} \qquad \textit{Ans.}$$

Thickness of Disk. As shown on the free-body diagram in Fig. 1–27*b*, the material at the sectioned area of the disk must resist *shear stress* to prevent movement of the disk through the hole. If this shear stress is *assumed* to be uniformly distributed over the sectioned area, then, since $V = 20$ kN, we have

$$A = \frac{V}{\tau_{allow}}; \qquad 2\pi(0.02\text{ m})(t) = \frac{20(10^3)\text{ N}}{35(10^6)\text{ N/m}^2}$$

$$t = 4.55(10^{-3})\text{ m} = 4.55\text{ mm} \qquad \textit{Ans.}$$

EXAMPLE 1.15

The shaft shown in Fig. 1–28*a* is supported by the collar at *C*, which is attached to the shaft and located on the right side of the bearing at *B*. Determine the largest value of *P* for the axial forces at *E* and *F* so that the bearing stress on the collar does not exceed an allowable stress of $(\sigma_b)_{allow} = 75$ MPa and the average normal stress in the shaft does not exceed an allowable stress of $(\sigma_t)_{allow} = 55$ MPa.

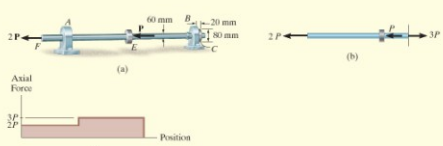


Fig. 1–28

EXAMPLE 1.15 CONTINUED

SOLUTION

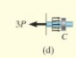
To solve the problem we will determine *P* for each possible failure condition. Then we will choose the *smallest* value. Why?

Normal Stress. Using the method of sections, the axial load within region *FE* of the shaft is 2*P*, whereas the *largest* axial force, 3*P*, occurs within region *EC*, Fig. 1–28*b*. The variation of the internal loading is clearly shown on the normal-force diagram, Fig. 1–28*c*. Since the cross-sectional area of the entire shaft is constant, region *EC* is subjected to the maximum average normal stress. Applying Eq. 1–11, we have

$$A = \frac{P}{\sigma_{allow}}; \qquad \pi(0.03\text{ m})^2 = \frac{3P}{55(10^6)\text{ N/m}^2}$$

$$P = 51.8\text{ kN} \qquad \textit{Ans.}$$

Bearing Stress. As shown on the free-body diagram in Fig. 1–28*d*, the collar at *C* must resist the load of 3*P*, which acts over a bearing area of $A_b = [\pi(0.04\text{ m})^2 - \pi(0.03\text{ m})^2] = 2.199(10^{-3})\text{ m}^2$. Thus,



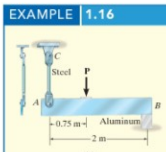
$$A = \frac{P}{\sigma_{allow}}; \qquad 2.199(10^{-3})\text{ m}^2 = \frac{3P}{75(10^6)\text{ N/m}^2}$$

$$P = 55.0\text{ kN}$$

By comparison, the largest load that can be applied to the shaft is $P = 51.8$ kN, since any load larger than this will cause the allowable normal stress in the shaft to be exceeded.

NOTE: Here we have not considered a possible shear failure of the collar as in Example 1.14.

EXAMPLE 1.16



The rigid bar *AB* shown in Fig. 1–29*a* is supported by a steel rod *AC* having a diameter of 20 mm and an aluminum block having a cross-sectional area of 1800 mm². The 18-mm-diameter pins at *A* and *C* are subjected to *single shear*. If the failure stress for the steel and aluminum is $(\sigma_{st})_{fail} = 680$ MPa and $(\sigma_{al})_{fail} = 70$ MPa, respectively, and the failure shear stress for each pin is $\tau_{fail} = 900$ MPa, determine the largest load *P* that can be applied to the bar. Apply a factor of safety of F.S. = 2.

SOLUTION

Using Eqs. 1–9 and 1–10, the allowable stresses are

$$(\sigma_{st})_{allow} = \frac{(\sigma_{st})_{fail}}{\text{F.S.}} = \frac{680\text{ MPa}}{2} = 340\text{ MPa}$$

$$(\sigma_{al})_{allow} = \frac{(\sigma_{al})_{fail}}{\text{F.S.}} = \frac{70\text{ MPa}}{2} = 35\text{ MPa}$$

$$\tau_{allow} = \frac{\tau_{fail}}{\text{F.S.}} = \frac{900\text{ MPa}}{2} = 450\text{ MPa}$$

EXAMPLE 1.16 CONTINUED

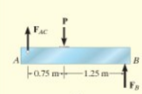


Fig. 1–29

The free-body diagram of the bar is shown in Fig. 1–29*b*. There are three unknowns. Here we will apply the moment equations of equilibrium in order to express F_{AC} and F_B in terms of the applied load *P*. We have

$$\circlearrowleft+\Sigma M_B = 0; \qquad P(1.25\text{ m}) - F_{AC}(2\text{ m}) = 0 \qquad (1)$$

$$\circlearrowleft+\Sigma M_A = 0; \qquad F_B(2\text{ m}) - P(0.75\text{ m}) = 0 \qquad (2)$$

We will now determine each value of *P* that creates the allowable stress in the rod, block, and pins, respectively.

Rod *AC*. This requires

$$F_{AC} = (\sigma_{st})_{allow}(A_{AC}) = 340(10^6)\text{ N/m}^2\,[\pi(0.01\text{ m})^2] = 106.8\text{ kN}$$

Using Eq. 1,

$$P = \frac{(106.8\text{ kN})(2\text{ m})}{1.25\text{ m}} = 171\text{ kN}$$

Block *B*. In this case,

$$F_B = (\sigma_{al})_{allow}A_B = 35(10^6)\text{ N/m}^2\,[1800\text{ mm}^2\,(10^{-6})\text{ m}^2/\text{mm}^2] = 63.0\text{ kN}$$

Using Eq. 2,

$$P = \frac{(63.0\text{ kN})(2\text{ m})}{0.75\text{ m}} = 168\text{ kN}$$

EXAMPLE 1.16 CONTINUED

Pin A or C. Due to single shear,

$$F_{AC} = V = \tau_{allow} A = 450(10^6)\ \text{N/m}^2\,[\pi(0.009\ \text{m})^2] = 114.5\ \text{kN}$$

From Eq. 1,

$$P = \frac{114.5\ \text{kN}\,(2\ \text{m})}{1.25\ \text{m}} = 183\ \text{kN}$$

By comparison, as P reaches its *smallest value* (168 kN), the allowable normal stress will first be developed in the aluminum block. Hence,

$$P = 168\ \text{kN} \qquad \textit{Ans.}$$

Mechanical properties of materials

Mechanics of materials

Strain

- Strain is the ratio between the change in the dimension (normal or shear) and the original dimension.
- Is the body is subjected to pure normal load, the strain **(ε) is calculated as**

$$\varepsilon = \frac{L - L_o}{L_o}$$

- **Where L is the instantaneous length and Lo is the initial length**
- if the body was under pure shear, the shear strain (γ) is the found as

$$\gamma_{nt} = \frac{\pi}{2} - \lim_{\substack{B \to A \text{ along } n \\ C \to A \text{ along } t}} \theta'$$

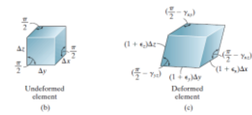


Fig. 2–3

Mechanical properties of materials

Mechanics of materials

Strain

- In most of engineering examples, the strain is in two dimensional (coplanar forces system) and so ε is found using the previous equation and γ is calculated using the geometry of the problem.
- ε is unit less however in many references it is given **mm/mm or m/m** or it is represented as percentage (%)
- In the next chapter we are going to study the relation between stress – strain and the classification of materials according to something called ***stress – strain diagram.*** For this moment, the stress is P/A or V/A and the strain is ε =(L-Lo)/Lo and γ is found from the geometry of the problem
- See the next examples

EXAMPLE 2.1

The slender rod shown in Fig. 2–4 is subjected to an increase of temperature along its axis, which creates a normal strain in the rod of $\epsilon_z = 40(10^{-3})z^{1/2}$, where z is measured in meters. Determine (a) the displacement of the end B of the rod due to the temperature increase, and (b) the average normal strain in the rod.

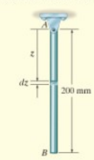


Fig. 2–4

EXAMPLE 2.1 CONTINUED

SOLUTION

Part (a). Since the normal strain is reported at each point along the rod, a differential segment dz, located at position z, Fig. 2–4, has a deformed length that can be determined from Eq. 2–1; that is,

$$dz' = dz + \epsilon_z\, dz$$

$$dz' = \left[1 + 40(10^{-3})z^{1/2}\right] dz$$

The sum of these segments along the axis yields the *deformed length* of the rod, i.e.,

$$z' = \int_0^{0.2\ \text{m}} \left[1 + 40(10^{-3})z^{1/2}\right] dz$$

$$= \left[z + 40(10^{-3})\tfrac{2}{3}z^{3/2}\right]\Big|_0^{0.2\ \text{m}}$$

$$= 0.20239\ \text{m}$$

The displacement of the end of the rod is therefore

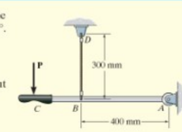
$$\Delta_B = 0.20239\ \text{m} - 0.2\ \text{m} = 0.00239\ \text{m} = 2.39\ \text{mm} \downarrow \qquad \textit{Ans.}$$

Part (b). The average normal strain in the rod is determined from Eq. 2–1, which assumes that the rod or "line segment" has an original length of 200 mm and a change in length of 2.39 mm. Hence,

$$\epsilon_{avg} = \frac{\Delta s' - \Delta s}{\Delta s} = \frac{2.39\ \text{mm}}{200\ \text{mm}} = 0.0119\ \text{mm/mm} \qquad \textit{Ans.}$$

EXAMPLE 2.2

When force **P** is applied to the rigid lever arm ABC in Fig. 2–5a, the arm rotates counterclockwise about pin A through an angle of 0.05°. Determine the normal strain developed in wire BD.



(a)

SOLUTION I

Geometry. The orientation of the lever arm after it rotates about point A is shown in Fig. 2–5b. From the geometry of this figure,

$$\alpha = \tan^{-1}\left(\frac{400\ \text{mm}}{300\ \text{mm}}\right) = 53.1301^\circ$$

Then

$$\phi = 90^\circ - \alpha + 0.05^\circ = 90^\circ - 53.1301^\circ + 0.05^\circ = 36.92^\circ$$

For triangle ABD the Pythagorean theorem gives

$$L_{AD} = \sqrt{(300\ \text{mm})^2 + (400\ \text{mm})^2} = 500\ \text{mm}$$

EXAMPLE 2.2 CONTINUED

Using this result and applying the law of cosines to triangle $AB'D$,

$$L_{B'D} = \sqrt{L_{AD}^2 + L_{AB'}^2 - 2(L_{AD})(L_{AB'})\cos\phi}$$
$$= \sqrt{(500\text{ mm})^2 + (400\text{ mm})^2 - 2(500\text{ mm})(400\text{ mm})\cos 36.92^\circ}$$
$$= 300.3491\text{ mm}$$

Normal Strain.

$$\epsilon_{BD} = \frac{L_{B'D} - L_{BD}}{L_{BD}} = \frac{300.3491\text{ mm} - 300\text{ mm}}{300\text{ mm}} = 0.00116\text{ mm/mm} \quad \textit{Ans.}$$

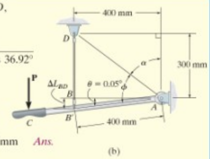


Fig. 2–5

EXAMPLE 2.2 CONTINUED

SOLUTION II

Since the strain is small, this same result can be obtained by approximating the elongation of wire BD as ΔL_{BD}, shown in Fig. 2–5b. Here,

$$\Delta L_{BD} = \theta L_{AB} = \left[\left(\frac{0.05^\circ}{180^\circ}\right)(\pi\text{ rad})\right](400\text{ mm}) = 0.3491\text{ mm}$$

Therefore,

$$\epsilon_{BD} = \frac{\Delta L_{BD}}{L_{BD}} = \frac{0.3491\text{ mm}}{300\text{ mm}} = 0.00116\text{ mm/mm} \quad \textit{Ans.}$$

EXAMPLE 2.3

Due to a loading, the plate is deformed into the dashed shape shown in Fig. 2–6a. Determine (a) the average normal strain along the side AB, and (b) the average shear strain in the plate at A relative to the x and y axes.

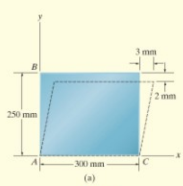
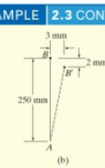
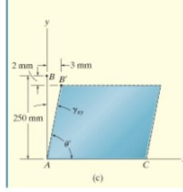


Fig. 2–6

EXAMPLE 2.3 CONTINUED





SOLUTION

Part (a). Line AB, coincident with the y axis, becomes line AB' after deformation, as shown in Fig. 2–6b. The length of AB' is

$$AB' = \sqrt{(250\text{ mm} - 2\text{ mm})^2 + (3\text{ mm})^2} = 248.018\text{ mm}$$

The average normal strain for AB is therefore

$$(\epsilon_{AB})_{avg} = \frac{AB' - AB}{AB} = \frac{248.018\text{ mm} - 250\text{ mm}}{250\text{ mm}}$$
$$= -7.93(10^{-3})\text{ mm/mm} \quad \textit{Ans.}$$

The negative sign indicates the strain causes a contraction of AB.

Part (b). As noted in Fig. 2–6c, the once 90° angle BAC between the sides of the plate at A changes to θ' due to the displacement of B to B'. Since $\gamma_{xy} = \pi/2 - \theta'$, then γ_{xy} is the angle shown in the figure. Thus,

$$\gamma_{xy} = \tan^{-1}\left(\frac{3\text{ mm}}{250\text{ mm} - 2\text{ mm}}\right) = 0.0121\text{ rad} \quad \textit{Ans.}$$

EXAMPLE 2.4

The plate shown in Fig. 2–7a is fixed connected along AB and held in the horizontal guides at its top and bottom, AD and BC. If its right side CD is given a uniform horizontal displacement of 2 mm, determine (a) the average normal strain along the diagonal AC, and (b) the shear strain at E relative to the x, y axes.

SOLUTION

Part (a). When the plate is deformed, the diagonal AC becomes AC', Fig. 2–7b. The length of diagonals AC and AC' can be found from the Pythagorean theorem. We have

$$AC = \sqrt{(0.150\text{ m})^2 + (0.150\text{ m})^2} = 0.21213\text{ m}$$
$$AC' = \sqrt{(0.150\text{ m})^2 + (0.152\text{ m})^2} = 0.21355\text{ m}$$

Therefore the average normal strain along the diagonal is

$$(\epsilon_{AC})_{avg} = \frac{AC' - AC}{AC} = \frac{0.21355\text{ m} - 0.21213\text{ m}}{0.21213\text{ m}}$$
$$= 0.00669\text{ mm/mm} \quad \textit{Ans.}$$

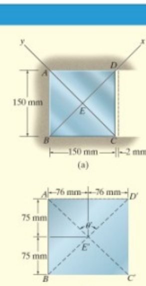


Fig. 2–7

EXAMPLE 2.4 CONTINUED

Part (b). To find the shear strain at E relative to the x and y axes, it is first necessary to find the angle θ' after deformation, Fig. 2–7b. We have

$$\tan\left(\frac{\theta'}{2}\right) = \frac{76\text{ mm}}{75\text{ mm}}$$
$$\theta' = 90.759^\circ = \left(\frac{\pi}{180^\circ}\right)(90.759^\circ) = 1.58404\text{ rad}$$

Applying Eq. 2–3, the shear strain at E is therefore

$$\gamma_{xy} = \frac{\pi}{2} - 1.58404\text{ rad} = -0.0132\text{ rad} \quad \textit{Ans.}$$

The *negative sign* indicates that the angle θ' is *greater than* 90°.

NOTE: If the x and y axes were horizontal and vertical at point E, then the 90° angle between these axes would not change due to the deformation, and so $\gamma_{xy} = 0$ at point E.