Chapter 1:

Analytic Trigonometry

Chapter 1 Overview

Trigonometry is, literally, the study of triangle measures. Geometry investigated the special significance of the relationships between the angles and sides of a triangle, especially in a right triangle. But this is fairly limited, because the angles must be between 0° and 180° in a triangle, due to the definition of an angle as a figure formed by two rays with a common endpoint.

The creation of the Cartesian coordinate system provided a tool to analyze the trigonometry of angles that are not limited in size. By redefining an angle as the rotation of a ray from one position to another, angles greater than 180° (indeed greater than 360°) and negative angles will be explored. This chapter will review the geometric information about trigonometry and expand into the analytic view.

1-1: Analytic Trigonometry

Recall from Geometry the following facts about trigonometry:

Pythagorean Theorem: $a^2 + b^2 = c^2$, where c is the hypotenuse

SOHCAHTOA:

$$\sin A = \frac{\text{opposite}}{\text{hypotenuse}} \qquad \cos A = \frac{\text{adjacent}}{\text{hypotenuse}} \qquad \tan A = \frac{\text{opposite}}{\text{adjacent}}$$

Of course, these definitions and formulas, which were memorized, assume some key details.

1. Angles and their opposite sides in a triangle are labeled with the same letter– upper case for the angle and lower case for the side.

2. Implied in the Pythagorean theorem is that a and b are lengths of the legs of a right triangle and c is the length of the hypotenuse.

3. SOHCAHTOA is a mnemonic for the relations of the sides of a right triangle with an angle. Sin, cos, and tan mean **NOTHING** without an angle for reference. They should be "$\sin A$, $\cos A$, $\tan A$."

4. Most importantly, $\sin A$ does not mean "sine times A." Sine is the operation and there is no multiplication implied. Therefore, do not isolate A by dividing by sin. Sin is a verb, not a noun.

The difference between the trigonometry learned in Geometry and our current focus–namely, analytic trigonometry—is that the figures are placed on and analyzed in the Cartesian coordinate system. The advantage to this is that angles are no longer restricted to measurements between 0° and 360°. In fact, they are no longer restricted to degrees either.

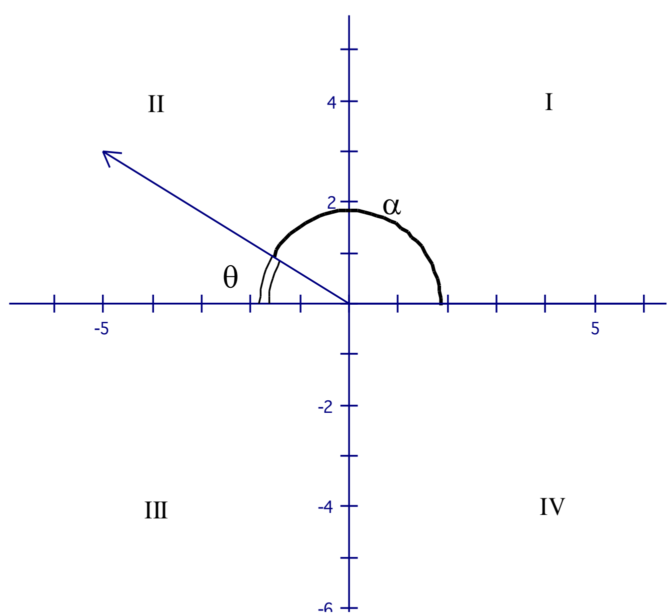
LEARNING OUTCOMES

Draw angles that are negative or are larger than 180°.
Find quadrant and reference angles of a given angle.
Given a point on the terminal side of an angle, find the six exact trigonometric values.
Given a trigonometric value and the quadrant of an angle, find the other five exact trigonometric values.

Vocabulary:

1. ***Angle*** – the rotation of a ray from one position to another. Furthermore, counterclockwise is defined as the positive direction.
2. ***Initial side*** – the starting position of the ray that forms the angle
3. ***Terminal side*** – the ending position of the ray that forms the angle
4. ***Standard position*** – the endpoint of each ray is at the origin and the initial ray is on the positive part of the *x*-axis

Here is an angle α in standard position.
θ represents the reference angle of α.



5. ***Reference angle*** – the angle formed (geometrically) by the terminal side and the *x*-axis. This forms a right triangle that "refers" the analytic angle to a geometric figure.
6. ***Coterminal angle*** – angles with the same initial and terminal sides

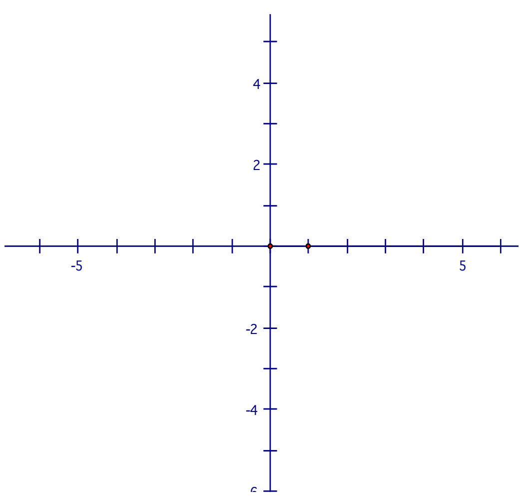
If the rotation that defined the angle was not marked and had only drawn the initial and terminal sides, several distinct angles would be possible. For instance, the angle α in the figure above measures about 150°. But if the arc is not marked, it could be 510° or 870° or even –210°. All the possible angles with the same terminal side would all be separated by a multiple of 360° and are called coterminal angles. This can be generalized as

$$\alpha \pm 360\text{n}$$

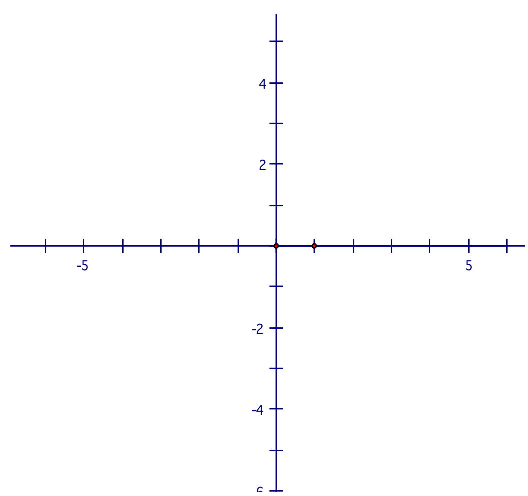
where *n* is a "dummy variable" that represents the number of full rotations about the origin. Note the quadrant numbers (I, II, III, and IV) and the reference angle θ .

EX 1 Draw and find the quadrant and reference angle of

a) 1847°



b) –145°



To find the reference angles, first eliminate the rotations about the origin by subtracting out multiples of 360°.

$1847° - 360(5) = 47°$

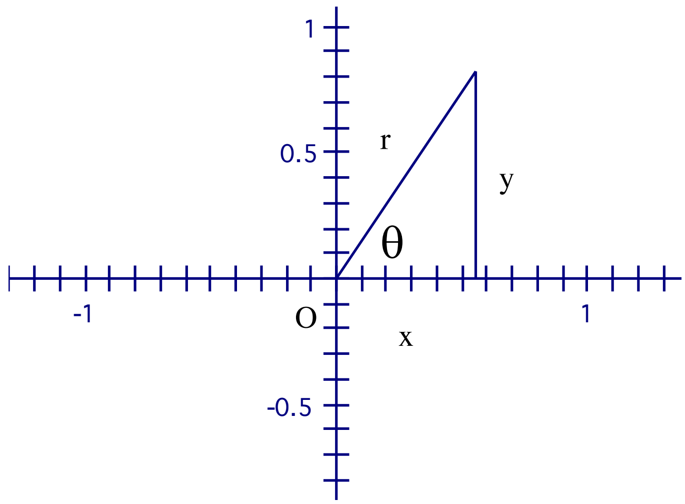
$-145° + 360 = 215°$, which is in QIII.

In QI, $\theta_{\text{ref}} = 47°$

The reference angle θ_{ref} is

$215° - 180° = 35°$

As defined above, the reference angle connects geometric trigonometry with new definitions in analytic trigonometry. With this diagram,



the Pythagorean theorem becomes

$$x^2 + y^2 = r^2$$

and SOHCAHTOA becomes

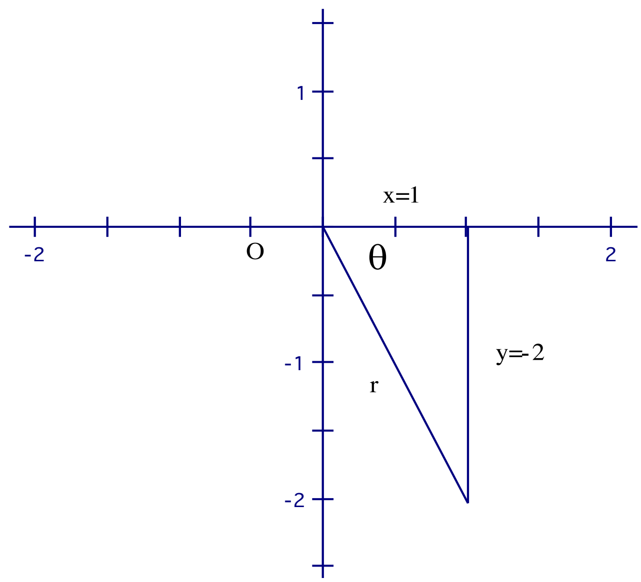
$$\sin\theta = \frac{y}{r} \qquad \cos\theta = \frac{x}{r} \qquad \tan\theta = \frac{y}{x}$$

Expand on that to define three reciprocal trigonometry functions:

$$\csc\theta = \frac{r}{y} \qquad \sec\theta = \frac{r}{x} \qquad \cot\theta = \frac{x}{y}$$

These functions (cosecant, secant, and cotangent) are the reciprocals of the first three.

EX 2 Find the six exact trigonometric values of θ if $(1,\ -2)$ is on the terminal side.



$$1^2 + (-2)^2 = r^2$$
$$r = \sqrt{5}$$

$$\sin\theta = -\frac{2}{\sqrt{5}} \qquad \cos\theta = \frac{1}{\sqrt{5}} \qquad \tan\theta = -2$$
$$\csc\theta = -\frac{\sqrt{5}}{2} \qquad \sec\theta = \sqrt{5} \qquad \cot\theta = -\frac{1}{2}$$

Note the fractions for sine and cosine were not rationalized. While some teachers––as well as the SATs and the answer keys in most textbooks—may require it, the book is not going to emphasize this. That was done in previous classes to help with practice simplifying radicals, but is not a focus of this course.

In problems where one trigonometric function is given, the question arises as to which quadrant the terminal side lies. The answer to this question determines the sign of x and y because the solution to the Pythagorean theorem could be either positive or negative, due to the need to square root a number to solve for x or y. **(This is never a problem with *r*, because, for now, *r* is the <u>length</u> of the ray and must therefore be positive.)** This will be a key issue later when looking at trigonometric inverse operations in general.

EX 3 Find the other five exact trigonometric values of β if $\sin\beta = \frac{5}{13}$ in Quad II.

$$x^2 + 5^2 = 13^2$$
$$x^2 = 144$$
$$x = \pm 12$$

In Quadrant II, x is negative. So $x = -12$.

$$\cos\beta = -\frac{12}{13} \qquad \tan\beta = -\frac{5}{12}$$

$$\csc\beta = \frac{13}{5} \qquad \sec\beta = -\frac{13}{12} \qquad \cot\beta = -\frac{12}{5}$$

EX 4 Find the other five exact trigonometric values of δ if $\cot\delta = \frac{4}{5}$ in Quad III.

$\cot\delta = \frac{x}{y}$, but both x and y are negative in QIII, so

$$4^2 + 5^2 = r^2$$
$$r^2 = 41$$
$$r = \sqrt{41}$$

$$\sin\delta = -\frac{5}{\sqrt{41}} \qquad \cos\delta = -\frac{4}{\sqrt{41}} \qquad \tan\delta = \frac{5}{4}$$

$$\csc\delta = -\frac{\sqrt{41}}{5} \qquad \sec\delta = -\frac{\sqrt{41}}{4}$$

1-1 Free Response Homework

Draw the following angles, find their reference angles, and state the quadrant of the terminal side.

1. 730°

2. –285°

3. 4876°

4. –942°

Find the six exact trigonometric values if the given point is on the terminal side of α.

5. $(6,\ 8)$

6. $(-1,\ -4)$

7. $(2,\ -3)$

8. $(-5,\ 5)$

Find the other five exact trigonometric values for the given angle.

9. $\sin A = \frac{2}{3}$ in Quad I

10. $\cos C = -\frac{4}{9}$ in Quad III

11. $\tan B = \frac{25}{24}$ in Quad III

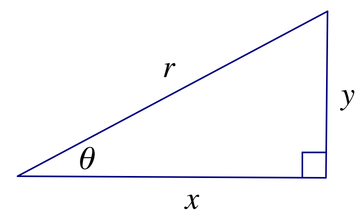
12. $\csc\phi = -\frac{6}{5}$ in Quad IV

13. $\sec\varphi = -\frac{\sqrt{37}}{3}$ in Quad II

14. $\cot\omega = -\frac{11}{60}$ in Quad II

1-1 Multiple Choice Homework

1. In the figure to the right, $\sin\theta\tan\theta =$



a) $\frac{x}{r}$ b) $\frac{y}{r}$ c) $\frac{y^2}{rx}$ d) $\frac{x^2}{ry}$ e) $\frac{xy}{r^2}$

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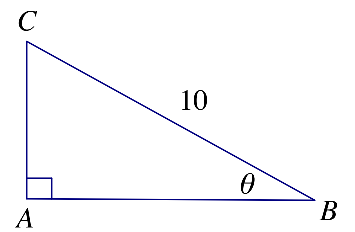
2. From the information given in the table and in the figure shown, which of the following best approximates the length of *AB*?



$\sin\theta$	$\cos\theta$	$\tan\theta$
0.643	0.766	0.839

a) 6.4
b) 7.7
c) 8.0
d) 8.4
e) 13.1

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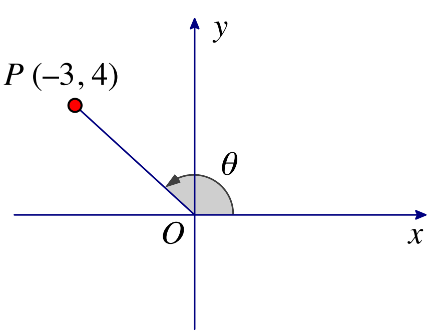
3. In the figure shown $\tan\theta =$,



a) $-\frac{4}{3}$

b) $-\frac{3}{4}$

c) $-\frac{3}{5}$

d) $\frac{3}{4}$

e) $\frac{4}{3}$

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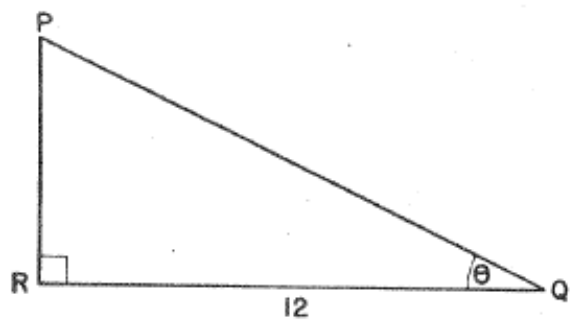
4. In $\triangle PQR$, $RQ = 12$ and $\tan\theta = \frac{1}{3}$. What is the area of $\triangle PQR$?



(a) 18 (b) 24 (c) 36 (d) 48 (e) 216

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5. If the terminal side of α passes through $(-9, 5)$, then $\tan\alpha =$

(a) $-\frac{9}{5}$ (b) $-\frac{5}{9}$ (c) $-\frac{9}{\sqrt{106}}$ (d) $\frac{5}{9}$ (e) $\frac{9}{5}$

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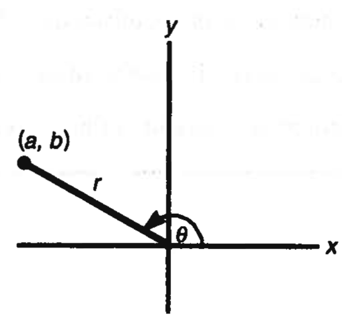
6. In the figure $r\sin\theta$ equals



a) a
b) b
c) $-a$
d) $-b$
e) $a+b$

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7. At a distance of 100 feet, the angle of elevation from the horizontal ground to the top of a building is 42°. The height of the building is

a) 67 feet b) 74 feet c) 90 feet

d) 110 feet e) 229 feet

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8. What is the value of $\cos x$ if $\sin x = \frac{1}{4}$ and x is an acute angle?

a) $\frac{1}{\sqrt{17}}$ b) $\frac{1}{\sqrt{15}}$ c) $\frac{3}{4}$ d) $\frac{\sqrt{15}}{4}$ e) $\frac{\sqrt{17}}{4}$

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9. Simplify the expression $\sin\left(\cos^{-1} 6x\right)$.

a) $\sqrt{1-6x^2}$ b) $\sqrt{1+6x^2}$ c) $\sqrt{36x^2-1}$

d) $\sqrt{1-36x^2}$ e) $\sqrt{1+36x^2}$

1-2: The Unit Circle and Special Triangles

Vocabulary:
1. ***Unit Circle*** – circle on the Cartesian coordinate system with center at the origin and radius of one unit
2. ***Radian measure of an angle*** – the measure of the arc length on the unit circle

The relationship between radian and degree measures is how radian measures are defined. Since a full circle is 360° and the perimeter of a circle of radius 1 is 2π, the proportion of degrees to radians in the unit circle is $\frac{2\pi}{360} = \frac{\pi}{180}$. Since radians are a measure of length, trigonometric functions transcend the limitation of being applied to an angle (in degrees) and allow the purely mathematical situation of taking the trigonometric value of a number not related to an angle. This is why radians are important and this course will almost always be in that mode.

LEARNING OUTCOMES

Convert between radians and degrees.
Use exact values from the special triangles to simplify trigonometric expressions.

Conversions: Degrees $\bullet \frac{\pi}{180°}$ = Radians;
Radians $\bullet \frac{180°}{\pi}$ = Degrees

EX 1 Convert the following angle measures: a) 50° and b) $\frac{\pi}{6}$

a) $50° \bullet \frac{\pi}{180°} = \frac{5\pi}{18}$

b) $\frac{\pi}{6} \bullet \frac{180°}{\pi} = 30°$

Exact trigonometric values can be found for several specific angles. Recall the ratios of the sides in the special triangles—$1: 1: \sqrt{2}$ in the 45-45-90 triangle and $1: \sqrt{3}: 2$ in the 30-60-90 triangle. Also, recall that points on the *x*-axis have a *y*-coordinate = 0 and points on the *y*-axis have an *x*-coordinate = 0. So for angles in QI:

$\cos 0° = 1$ and $\sin 0° = 0$

$\cos 30° = \frac{\sqrt{3}}{2}$ and $\sin 30° = \frac{1}{2}$

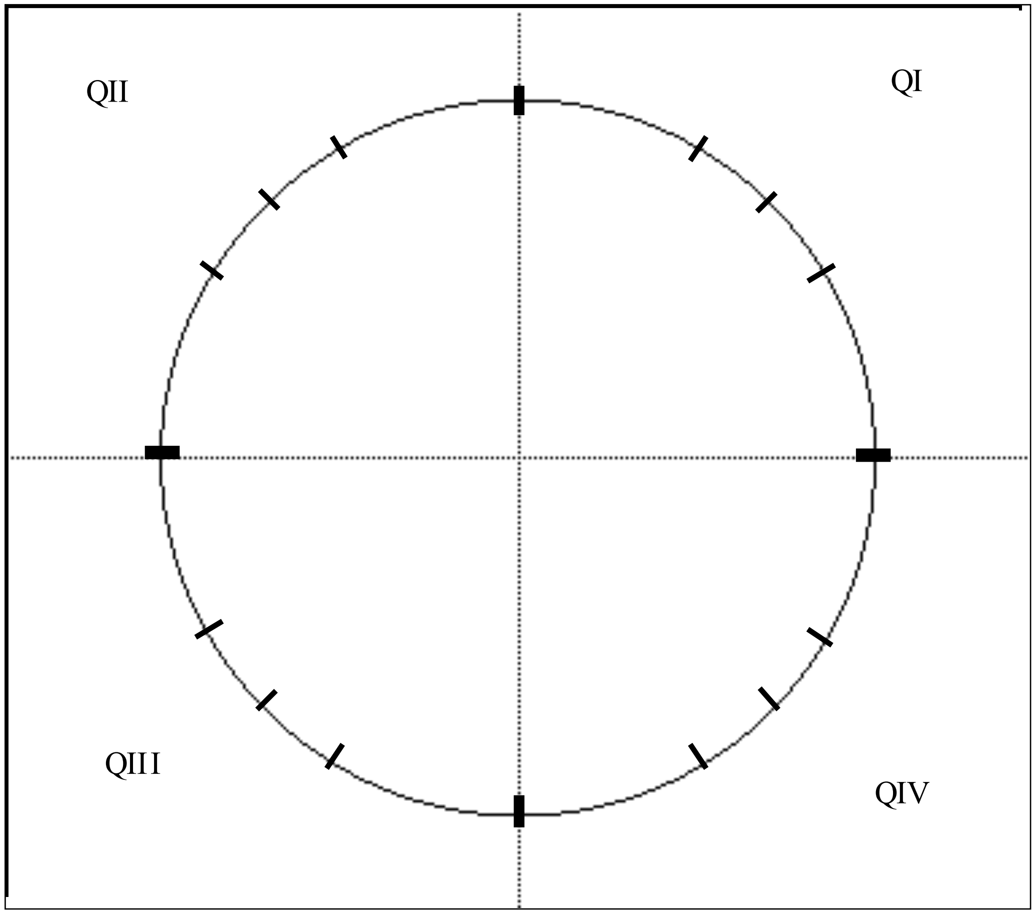
$\cos 45° = \frac{1}{\sqrt{2}}$ and $\sin 45° = \frac{1}{\sqrt{2}}$

$\cos 60° = \frac{1}{2}$ and $\sin 60° = \frac{\sqrt{3}}{2}$

$\cos 90° = 0$ and $\sin 90° = 1$

[Notice that the sine of some of these angles is the cosine of others. That is because the "co" on cosine is short for "complements." The cosine of an angle is the sine of its complement.] Reference angles can be used to develop a table of common exact trigonometric values.

The Unit Circle



The Table

Rads	Deg	$\cos\theta$	$\sin\theta$	$\tan\theta$
	0°			
	30°			
	45°			
	60°			
	90°			
	120°			
	135°			
	150°			
	180°			
	210°			
	225°			
	240°			
	270°			
	300°			
	315°			
	330°			
	360°			

EX 2 Simplify these trigonometric expressions:

a) $\tan^2 \frac{\pi}{6} + \sin^2 \frac{\pi}{6}$

$$\begin{aligned} \tan^2 \frac{\pi}{6} + \sin^2 \frac{\pi}{6} &= \left(\frac{1}{\sqrt{3}}\right)^2 + \left(\frac{1}{2}\right)^2 \\ &= \frac{1}{3} + \frac{1}{4} \\ &= \frac{7}{12} \end{aligned}$$

*Note that $\tan^2 \frac{\pi}{6}$ means $\left(\tan \frac{\pi}{6}\right)^2$

b) $\cos^2 120° - 1$

$$\begin{aligned} \cos^2 120° - 1 &= \left(-\frac{1}{2}\right)^2 - 1 \\ &= \frac{1}{4} - 1 \\ &= -\frac{3}{4} \end{aligned}$$

1-2 Free Response Homework

Convert these degree measures to radians

1. $45°$ 2. $80°$ 3. $-246°$ 4. $540°$

Convert these radian measures to degrees.

5. $\frac{\pi}{10}$ 6. $\frac{3\pi}{5}$ 7. $\frac{9\pi}{2}$ 8. $-\frac{7\pi}{6}$

9. Fill in the table and the circle in this section.

Simplify these trigonometric expressions without using a calculator.

10. $\sin^2\frac{\pi}{3}-\cos^2\frac{\pi}{3}$

11. $\sin\frac{\pi}{6}\cos\frac{\pi}{3}+\sin\frac{\pi}{3}\cos\frac{\pi}{6}$

12. $\tan^2\frac{5\pi}{6}-\cot^2\frac{4\pi}{3}$

13. $\sec^2\frac{5\pi}{3}-\tan^2\frac{2\pi}{3}$

14. $\sin^2 60°+\cos^2 120°$

15. $\csc 135°\tan 45°\cos 315°$

16. $\cot 30°\sec 330°+\csc 210°$

17. $\frac{1}{\csc 45°+\cot 225°}+\frac{1}{\csc 135°-\cot 45°}$

1-2 Multiple Choice Homework

1. If $x \in [0°,\ 90°]$ and $\sin x - 0.5 = 0$, then $x =$

a) $0°$ b) $\frac{1}{2}^{\circ}$ c) $30°$ d) $45°$ e) $60°$

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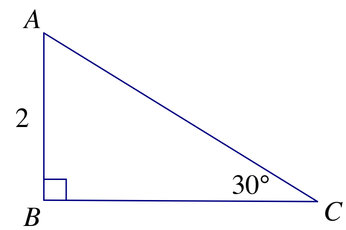
2. What is the area of $\triangle ABC$ to the right?



a) 2
b) $\sqrt{3}$
c) $2\sqrt{3}$
d) $4\sqrt{2}$
e) $4\sqrt{3}$

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3. $\sin 120° =$

(a) $-\frac{\sqrt{3}}{2}$ (b) $-\frac{1}{2}$ (c) $\frac{1}{2}$ (d) $-\frac{\sqrt{3}}{3}$ (e) $\frac{\sqrt{3}}{2}$

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4. Consider the functions $f(x) = \sin x,\ g(x) = 5 - \sqrt{x}$. Find $f \circ g$.

(a) $(f \circ g)(x) = 5 - \sqrt{\sin x}$
(b) $(f \circ g)(x) = \sin(\sin x)$
(c) $(f \circ g)(x) = \sin\left(5 - \sqrt{x}\right)$
(d) $(f \circ g)(x) = 5 - \sqrt{5 - \sqrt{x}}$
(e) $(f \circ g)(x) = \sin x - \sqrt{\sin\sqrt{x} - 5}$

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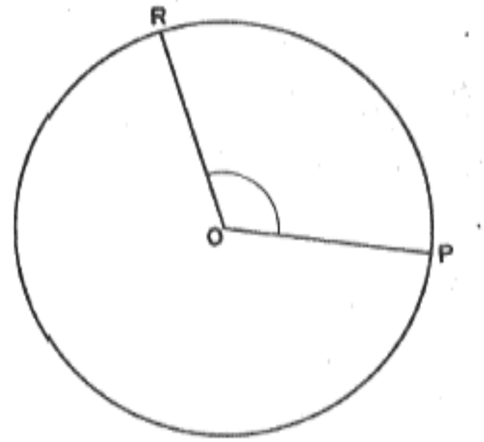
5. In the figure below, Circle O has radius 2 and $\overparen{PR}$ has length 4. What is the radian measure of $\angle POR$?



a. 1 b. 2 c. 4 d. $\frac{1}{\pi}$ e. π

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6. $\sec\frac{2\pi}{3}\tan\frac{7\pi}{6}+\cot\frac{11\pi}{6}\csc\frac{5\pi}{3}=$

a. $2\sqrt{3}+2$ b. $\frac{2}{\sqrt{3}}-2$ c. $\frac{2\sqrt{3}+2}{\sqrt{3}}$

d. $\frac{2\sqrt{3}-2}{\sqrt{3}}$ e. 0

1-3: Calculator Use

Much of trigonometry will require the use of a calculator, since only certain exact values are known. There are a few things to keep in mind with calculator problems:

1. The MODE that the calculator is in makes a HUGE difference. The calculator needs to know if the numbers being entered are degrees or radians. $\sin 3°$ is very different from $\sin 3$.

2. Calculators only have three trigonometric keys (sin, cos, and tan), not six. The other three are reciprocals of the first three.

3. Knowing a trigonometric value alone is not sufficient to determine the measure of the angle. Knowing one trigonometric value only narrows the answers down to the reference angle in two possible quadrants.

4. Stay consistent and round all answers to **three** decimals.

LEARNING OUTCOMES

Use a calculator to find approximate trigonometric values for a given angle.
Use a calculator to find approximate angle values for a given trigonometric value.

EX 1 Find $\sin 4°$ vs. $\sin 4$.

$\sin 4° = 0.070$ $\qquad\qquad$ $\sin 4 = -0.757$

Note that while $4°$ is in QI, 4 radians is equal to $229.183°$, which is in QIII.

EX 2 Find the six approximate trigonometric values of $165°$.

$$\sin 165° = 0.259$$
$$\cos 165° = -0.966$$
$$\tan 165° = -0.268$$
$$\csc 165° = \frac{1}{\sin 165°} = 3.864$$
$$\sec 165° = \frac{1}{\cos 165°} = -1.035$$
$$\cot 165° = \frac{1}{\tan 165°} = -3.732$$

When finding angles by calculator, realize that when a positive sine value is entered, the calculator does not know if the angle in QI or QII was intended. When a positive cosine value is entered, the calculator does not know if the angle in QI or QIV was intended. When a positive tangent value is entered, the calculator does not know if the angle in QI or QIII was intended. Inverses in a calculator are programmed as functions, not operations; therefore they will only give one answer. (We will discuss the range issues of trigonometric inverse functions later.) There are two answers to each inverse problem, but the calculator will only state one of them. In addition, a calculator will not tell about all the coterminal angles. The following is a way to get the second set of answers and list all the coterminal angles.

$$\cos^{-1}\frac{x}{r} = \begin{Bmatrix} \text{calculator} \pm 360°n \\ -\text{calculator} \pm 360°n \end{Bmatrix}$$

$$\sin^{-1}\frac{y}{r} = \begin{Bmatrix} \text{calculator} \pm 360°n \\ 180° - \text{calculator} \pm 360°n \end{Bmatrix}$$

$$\tan^{-1}\frac{y}{x} = \begin{Bmatrix} \text{calculator} \pm 360°n \\ 180 + \text{calculator} \pm 360°n \end{Bmatrix} = \text{calculator} \pm 180°n$$

$$\cos^{-1}\frac{x}{r}=\begin{Bmatrix}\text{calculator}\pm 2\pi n\\ -\text{calculator}\pm 2\pi n\end{Bmatrix}$$

$$\sin^{-1}\frac{y}{r}=\begin{Bmatrix}\text{calculator}\pm 2\pi n\\ \pi-\text{calculator}\pm 2\pi n\end{Bmatrix}$$

$$\tan^{-1}\frac{y}{x}=\begin{Bmatrix}\text{calculator}\pm 2\pi n\\ \pi+\text{calculator}\pm 2\pi n\end{Bmatrix}=\text{calculator}\pm \pi n$$

NB. This book will use the symbols $\cos^{-1}x$, $\sin^{-1}x$, and $\tan^{-1}x$ almost exclusively. But "inverse" is often replaced by "arc"—as in $\arccos x$, $\arcsin x$, and $\arctan x$—which might be seen on the SAT.

EX 3 Find the following approximate angle values in degrees.

a) $\sin\alpha=0.351\rightarrow\alpha=\sin^{-1}(0.351)=\begin{Bmatrix}20.548°\pm 360°n\\ 159.452°\pm 360°n\end{Bmatrix}$

b) $\tan\alpha=1.4\rightarrow\alpha=\arctan(1.4)=54.462°\pm 180°n$

c) $\sec\alpha=-1.6\rightarrow\alpha=\sec^{-1}(-1.6)=\cos^{-1}\left(\frac{1}{-1.6}\right)=\pm 128.682°\pm 360°n$

d) $\csc\alpha=0.654\rightarrow\alpha=\csc^{-1}(0.654)=\arcsin\left(\frac{1}{0.654}\right)=\text{error}$

What would EX 3 look like if asked to approximate the angle values in radians?

EX 3 (again) Find the following approximate angle values in radians.

a) $\sin\alpha = 0.351 \rightarrow \alpha = \sin^{-1}(0.351) = \begin{Bmatrix} 0.359 \pm 2\pi n \\ 2.783 \pm 2\pi n \end{Bmatrix}$

b) $\tan\alpha = 1.4 \rightarrow \alpha = \arctan(1.4) = 0.951 \pm \pi n$

c) $\sec\alpha = -1.6 \rightarrow \alpha = \sec^{-1}(-1.6) = \cos^{-1}\left(\frac{1}{-1.6}\right) = \pm 2.246 \pm 2\pi n$

d) $\csc\alpha = 0.654 \rightarrow \alpha = \csc^{-1}(0.654) = \arcsin\left(\frac{1}{0.654}\right) = \text{error}$

Note it is not allowed to $\sec^{-1}$ or $\csc^{-1}$ a number whose absolute value is less than 1, because secant and cosecant are defined with the numerator as the hypotenuse, which is always has a length greater than or equal to that of the legs. Similarly, trying to $\sin^{-1}$ or $\cos^{-1}$ a number whose absolute value is more than 1 results in an error.

Also note that α was not isolated by division. The inverse (or arc) function was applied. While –1 as an exponent does mean reciprocal in other instances, it does not mean that here. Actually, the –1 means the inverse operation, the operation that cancels the one being referred to. Reciprocals cancel when multiplied, but there is no multiplication here. The sine and cosine are the operations. So,

$$\sin^{-1}\theta \neq \csc\theta \qquad \cos^{-1}\theta \neq \sec\theta \qquad \tan^{-1}\theta \neq \cot\theta$$

In fact, note in EX 3 that the angle value α is not the object of the inverse operations. **The angle α is equal to the inverse of a number**, that number being the trigonometric ratio. If the angle is in radians, the inverse is often referred to as an arcsine or arccosine because they equal the arc length on the unit circle.

1-3 Free Response Homework

Approximate the following trigonometric values to 3 decimal places.

1. $\sin 14°$

2. $\cos 5$

3. $\tan 140°$

4. $\cot 14$

5. $\sec(-6)$

6. $\csc(-195°)$

Approximate the following angle values to 3 decimal places in degrees.

7. $\sin^{-1} 0.652$

8. $\arctan 1.432$

9. $\sec^{-1} 1.781$

Approximate the following angle values to 3 decimal places in radians.

10. $\cos^{-1}(-0.521)$

11. $\text{arccot}(-0.652)$

12. $\text{arccsc}\, 0.395$

Approximate the angle values, in degrees, for the following problems:

13. $\sin A = \frac{2}{3}$ in Quad I

14. $\tan B = \frac{25}{24}$ in Quad III

15. $\sec \varphi = -\frac{\sqrt{37}}{3}$ in Quad II

Approximate the angle values, in radians, for the following problems:

16. $\cos C = -\frac{4}{9}$ in Quad III

17. $\csc\phi = -\frac{6}{5}$ in Quad IV

18. $\cot\omega = -\frac{11}{60}$ in Quad II

1-3 Multiple Choice Homework

1. Find the exact value of $\sin\left(\sin^{-1} 0.6\right)$

a) 1 b) 0.6 c) -0.6 d) -1 e) 0

---

2. If $\sec 1.4 = x$, find the value of $\csc\left(2\,\text{Arctan}\,x\right)$.

a) 0.33 b) 3.03 c) 1 d) 1.06 e) 0.87

---

3. If $\sin A = 0.8364$ and $\tan A = -1.5258$, $\angle A =$

a) 0.99 b) -0.99 c) 5.80 d) 2.151 e) 5.873

---

4. If $0° \le x \le 90°$ and $\tan x = \sin x$, then $x =$

a) 0° b) 30° c) 1

d) 60° e) 90°

---

5. The binary operation * is defined over the set of real numbers to be

$$a*b=\begin{cases} a\sin\frac{b}{a} & \text{if } a>b \\ b\cos\frac{a}{b} & \text{if } b>a \end{cases}$$

. What is the value of $2*(5*3)$?

a) 4.01 b) 3.65 c) 1.84 d) 2.79 e) 2.14

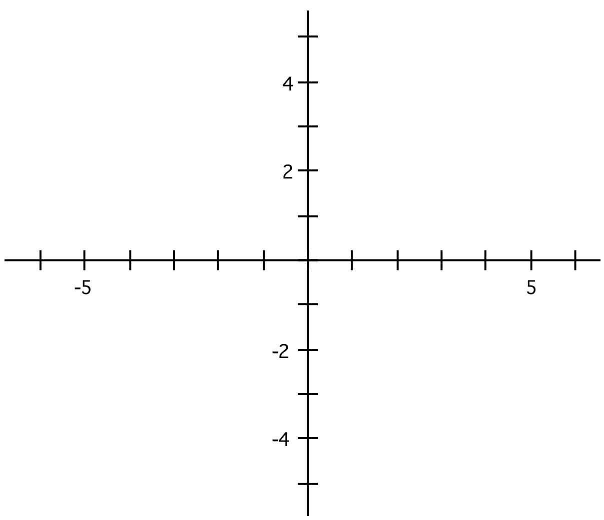
---

6. $\sin^{-1}(\cos 100°)=$

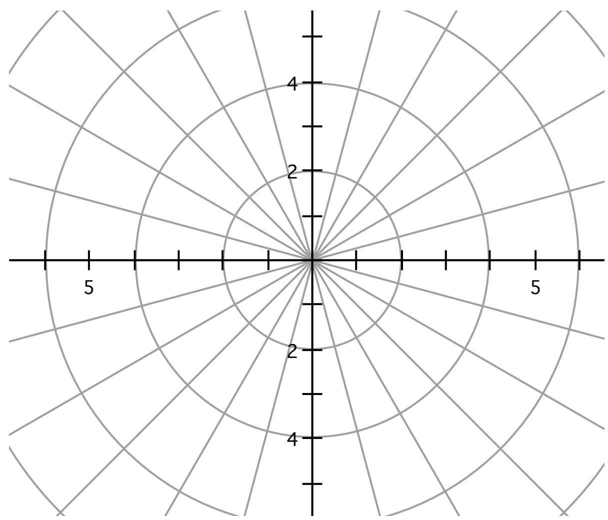
a) 1.0 b) 1.4 c) 0.2 d) –1.4 e) –0.2

1-4: Vectors

There are several applications of basic right triangle trigonometry. Of particular interest are vectors and complex numbers. There are two ways to look at a point on the Cartesian coordinate system. The most common in Algebra is as $(x,\ y)$ coordinates that define how far over and how far up from the origin a point is. This is the well-known Cartesian coordinate system. An alternative is the polar coordinate system, which views the points as $(r,\ \theta)$, r being how far away in a straight line the point is from the origin and θ being the angle at which the direction is measured.



Cartesian Coordinates



Polar Coordinates

This r is the radius and θ is the standard position angle. The same conversions that are used to find the radius and the standard position angle will be used to get $(r,\ \theta)$ from $(x,\ y)$ and vice versa.

Vocabulary:

1. ***Vector*** – a directed line segment. As such, it can be used to represent anything that has magnitude and direction (e.g., force, velocity, etc.).
2. ***Magnitude*** – the length/size of a vector
3. ***Direction of a Vector*** – the standard position angle when a vector is placed with its tail at the origin
4. ***Resultant Vector*** – Defn: sum of two vectors
   Means: the vector forming the third side of a triangle where the first two sides are the vectors laid head to tail.
5. ***Unit Vector*** – a vector with magnitude 1. $\vec{i}$ is a unit vector on the x-axis, and $\vec{j}$ is a unit vector on the y-axis.
6. ***Component Form of a Vector*** (a.k.a. rectangular form) – the form $x\vec{i}+y\vec{j}$, where $(x\ ,\ y)$ is the head and the origin is the tail of the vector

Any vector can be represented by a sum of $\vec{i}$ s and $\vec{j}$ s. This section will use $x\vec{i}+y\vec{j}$ (component form) but there are others. Some texts—and the AP answer keys—use $\langle x, y\rangle$ instead of $x\vec{i}+y\vec{j}$. The SAT uses (x, y), which can get confusing because this is the same notation used to represent a point and to represent an interval. This book will not use (x, y) for vectors, but when the topic of motion is discussed later, $\langle x, y\rangle$ will be used.

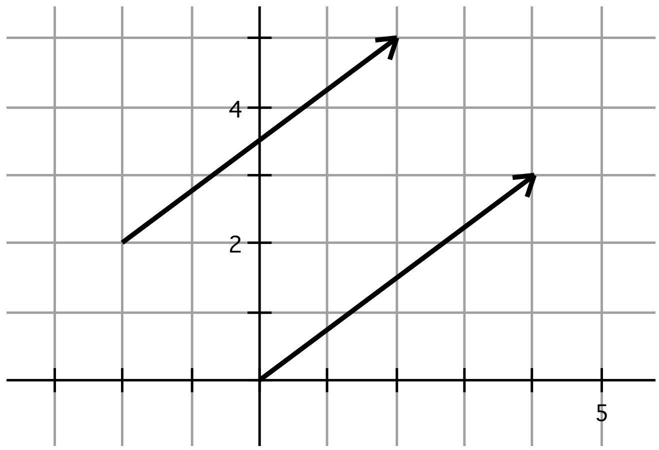
LEARNING OUTCOMES

Find a vector from one point to another.
Find a unit vector in the direction of another vector.
Find a resultant vector.
Convert between the component form and polar form of a vector.

Notice that the definition of a vector has nothing to do with where it is, just how long it is and where it is pointed. But the component form has the vector start at the origin. What if the vector does not start at the origin?

EX 1 Find the component form of a vector from $(-2,\ 2)$ to $(2,\ 5)$.

Since placement does not matter, the same vector can be dragged to have its tail at the origin. Now see that the vector is $4\vec{i}+3\vec{j}$.



Algebraically, it can be understood that the horizontal length is the difference of the *x*-coordinates of the two points and the vertical length is the difference of the *y*-coordinates.

$$2-(-2)=4 \text{ and } 5-2=3,$$

$$\text{so the vector is } 4\vec{i}+3\vec{j}$$

Suppose the magnitude and direction of this vector needs to be known. Some conversion equations will be needed, but they are actually already available: the Pythagorean theorem and SOHCAHTOA. They might need to be stated slightly differently in this context.

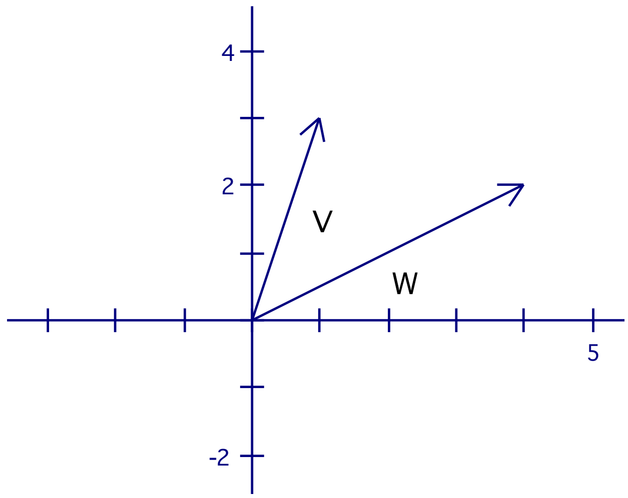
Conversion from Component Form to Polar Form:

$$r=|\vec{v}|=\sqrt{x^2+y^2} \text{ and } \theta=\pm\cos^{-1}\left(\frac{x}{|\vec{v}|}\right),$$

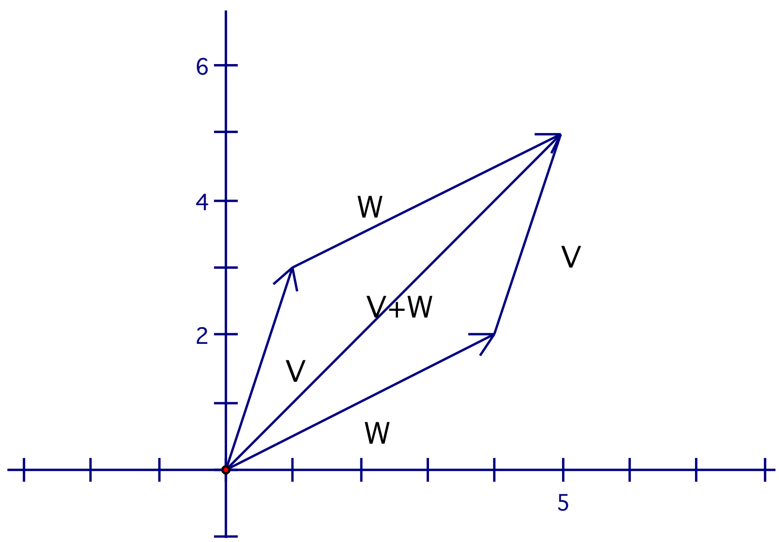
where the sign is the same as *y*.

A resultant vector is the sum of two vectors. To find this in component form, simply add like terms.

EX 2 Given vectors $\vec{V} = 1\vec{i} + 3\vec{j}$ and $\vec{W} = 4\vec{i} + 2\vec{j}$ shown below, find the resultant vector $\overrightarrow{V+W}$.



According to the definition of vector addition above, no matter which vector has its tail on the other's head $\overrightarrow{V+W}$ would look like this:



The component form of $\overrightarrow{V+W}$ is $5\vec{i} + 5\vec{j}$. Really all that is being done mathematically is adding like terms.

$$1\vec{i} + 3\vec{j} + 4\vec{i} + 2\vec{j} = 5\vec{i} + 5\vec{j}$$

Vectors can also be stretched and/or shrunken by multiplying by a factor. A common case here is to find a unit vector in the direction of a given vector.

EX 3 Find the unit vector in the direction of $\overrightarrow{V+W}$ in EX 2.

To find a unit vector, divide the vector by its length (magnitude).

$$\left|\overrightarrow{V+W}\right| = \sqrt{5^2+5^2} = 5\sqrt{2}$$

and

$$\frac{V+W}{\left|\overrightarrow{V+W}\right|} = \frac{5\vec{i}+5\vec{j}}{5\sqrt{2}} = \frac{5}{5\sqrt{2}}\vec{i} + \frac{5}{5\sqrt{2}}\vec{j} = \frac{1}{\sqrt{2}}\vec{i} + \frac{1}{\sqrt{2}}\vec{j}$$

New vectors can be formed by combining these stretched vectors.

EX 4 Given $\vec{V} = 1\vec{i}+3\vec{j}$ and $\vec{W} = 4\vec{i}+2\vec{j}$, find $\overrightarrow{3W-7V}$.

This is a fairly common SAT 2 question which is really just an arithmetic problem:

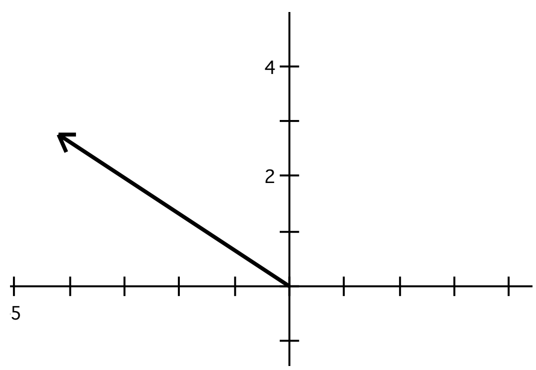
$$\begin{aligned} 3\left(4\vec{i}+2\vec{j}\right) &= 12\vec{i}+6\vec{j} \\ \underline{-7\left(1\vec{i}+3\vec{j}\right)} &= \underline{-7\vec{i}-21\vec{j}} \\ \overrightarrow{3W-7V} &= 5\vec{i}-15\vec{j} \end{aligned}$$

So what if the vectors are not in component form in order to add like terms? There are conversion formulas to change into component form. The conversions from magnitude $\left(r=\left|\vec{v}\right|\right)$ and direction $\theta = \cos^{-1}\left(\frac{x}{\left|\vec{v}\right|}\right)$ to component form are the same as finding r and θ, and, later for converting from polar to rectangular form.

Conversion from Polar to Component Form:

$x = r\cos\theta$ and $y = r\sin\theta$ and $r = \sqrt{x^2 + y^2}$

EX 5 $\vec{v} = 5$ units at $147°$. Find the component form of $\vec{v}$.



$$\begin{aligned} \vec{v} &= 5 \text{ units at } 147° \\ &= 5\cos 147° \vec{i} + 5\sin 147° \vec{j} \\ &= -4.193\vec{i} + 2.723\vec{j} \end{aligned}$$

EX 6 Find the magnitude and direction of the vector from $(-2,\ 2)$ to $(2,\ 5)$.

From EX 1 $\vec{v} = 4\vec{i} + 3\vec{j}$ or $\langle 4,\ 3 \rangle$.

$$\left|\vec{v}\right| = \sqrt{4^2 + 3^2} = 5$$

and

$$\theta = \cos^{-1}\frac{4}{5} = 36.869°$$

EX 7 Find the polar form of the resultant vector $5\vec{i}+5\vec{j}$ in EX 4.

$$\left|\overrightarrow{V+W}\right| = \sqrt{5^2+5^2} = 5\sqrt{2}$$

and

$$\theta = \cos^{-1}\frac{5}{5\sqrt{2}} = 45°$$

Ultimately, these conversions and the finding of a resultant vector come together in one problem. The most common application of the sum of vectors is in navigation and aviation. Speed or distance is the magnitude and the bearing is the direction.

EX 8 A plane flies 500 mph on a bearing of 80°. The wind blows 60 mph on a bearing of 300°. Find the magnitude and direction of the resultant vector.

$$\begin{aligned} p &= (500\ \cos\ 80)\vec{i} + (500\ \sin\ 80)\vec{j} = 86.824\vec{i} + 492.404\vec{j} \\ w &= (60\ \cos\ 300)\vec{i} + (60\ \sin\ 300)\vec{j} = 30\vec{i} - 51.962\vec{j} \\ \hline \overrightarrow{p+w} &= \qquad\qquad = 116.824\vec{i} + 440.442\vec{j} \end{aligned}$$

$$\left|\overrightarrow{p+w}\right| = \sqrt{116.824^2 + 440.442^2} = 455.672 \text{ mph}$$

$$\theta = \cos^{-1}\left(\frac{116.824}{455.672}\right) = 75.145°$$

EX 9 An object moves 90 meters along a bearing of 180°, then turns and moves 130 more meters along a bearing of 250°. Find the polar form of the resultant vector.

$$a = (90\cos 180°)\vec{i} + (90\sin 180°)\vec{j}$$
$$b = (130\cos 250°)\vec{i} + (130\sin 250°)\vec{j}$$
$$\overrightarrow{a+b} = -134.463\vec{i} - 122.160\vec{j}$$

$$\left|\overrightarrow{a+b}\right| = \sqrt{134.463^2 + 122.160^2} = 181.668$$
$$\theta = -\cos^{-1}\left(\frac{-134.463}{181.668}\right) = -137.745°$$

Note that the angle is negative because the *y*-value of the sum was negative.

so $\overrightarrow{a+b} = 181.668$ units at $-137.745°$ (or $222.255°$)

1-4 Free Response Homework

For problems 1-12, assume $\vec{r} = 60\vec{i} + 11\vec{j}$, $\vec{s} = -2\vec{i} + 9\vec{j}$, $\vec{t} = -4\vec{i} + 7\vec{j}$, and $\vec{u} = 15\vec{i} + 8\vec{j}$. Simplify each of the stated expressions.

1. $3\vec{s} - 2\vec{r}$
2. $2\vec{t} + \vec{r}$
3. $3\vec{u} - 7\vec{s}$
4. $-\vec{t} + 4\vec{s}$
5. $|3\vec{s} + 2\vec{r}|$
6. $|-\vec{t} + 4\vec{s}|$
7. $|\vec{u} - 3\vec{s}|$
8. $|2\vec{t} + 3\vec{r}|$
9. The unit vector in the direction $\vec{r}$.
10. The unit vector in the direction $\vec{u} + \vec{r}$.
11. The unit vector in the direction $-3\vec{t} + 4\vec{s}$.
12. The unit vector in the direction $3\vec{s} - 2\vec{r}$.

For problems 13-16, find the magnitude and direction of the resultant vector.

13. $\vec{v} = 6$ units at $30°$ and $\vec{w} = 5$ units at $60°$
14. $\vec{v} = 7$ units at $237°$ and $\vec{w} = 4$ units at $0°$
15. $\vec{v} = 10$ units at $98°$ and $\vec{w} = 13$ units at $260°$
16. $\vec{v} = 30$ units at $190°$ and $\vec{w} = 50$ units at $200°$

17. A plane flies 400 mph on a bearing of 280°. The wind blows 10 mph on a bearing of 190°. Find the actual speed and bearing of the plane.

18. A ship sails 40 miles on a bearing of 30°, then it turns and sails 60 miles on a bearing of 100°. How far away from its starting point and on what bearing is it?

19. A plane flies 500 mph on a bearing of 80°. The wind blows 60 mph on a bearing of 300°. Find the magnitude and direction of the resultant vector.

20. A boat is crossing a river at 5 mph and is traveling perpendicular to the current, which is flowing at 2 mph. The boat is traveling east and the river is flowing south. What is the actual velocity of the boat?

21. A ship in space is acted upon by the gravity of the nearest and largest bodies. At a particular time, the sun pulls 50 lbs at 142°, the moon pulls 20 lbs at 243° and the Earth pulls 120 lbs at 27°. To the nearest pound and degree, what is the sum of the resultant vector?

22. In order to avoid an iceberg, the good ship Valdez is given directions to a Russian port in three segments:

First: 100 miles at 210°
Second: 60 miles at 120°
Third: 30 miles at 70°

Before leaving dock, Captain Hazelwood must give directions to a plane to fly directly to the port. What are the magnitude and direction to the Russian port?

23. A person in prison is trying to tunnel out and must tunnel 100 yards at 0° to escape. If he starts by tunneling 27 yds at 7°, then turns to head –45° and tunnels 5 yards to avoid a rock, and finally turns to face 33° and tunnels 75 yards, find the distance and direction he ends up going, and tell whether he got far enough to get out.

<u>1-4 Multiple Choice Homework</u>

1. If vector $\vec{v} = \left(1,\ \sqrt{3}\right)$ and vector $\vec{u} = \left(3,\ -2\right)$, find the value of $\left|3\vec{v} - \vec{u}\right|$.

a) 52

b) $2 + 3\sqrt{3}$

c) 6

d) $0.2 + 3\sqrt{3}$

e) 7

2. A unit vector parallel to vector $\vec{v} = (2,\ -3,\ 6)$ is vector

a) $(-2,\ 3,\ -6)$

b) $(6,\ -3,\ 2)$

c) $(-0.29,\ 0.43,\ -0.86)$

d) $(0.29,\ 0.43,\ -0.86)$

e) $(-0.36,\ -0.54,\ 1.08)$

---

3. The magnitude of $\vec{v} = 7\vec{i} - \sqrt{6}\vec{j}$ is

a. $\sqrt{55}$ b. 1 c. $\sqrt{43}$ d. 55 e. $\sqrt{85}$

---

4. Which of the following is a unit vector?

a. $0\vec{i} + 0\vec{j}$ b. $\vec{i} - \vec{j}$ c. $\vec{i} + \vec{j}$

d. $\vec{v} = \frac{1}{3}\vec{i} - \frac{2}{3}\vec{j}$ e. $\frac{4}{5}\vec{i} + \frac{3}{5}\vec{j}$

---

5. A boat sails 48 mph at a bearing of 303°. The current flows 8 mph at 34°. The magnitude and bearing of the resultant vector are

a. 32.775 mph at 35.789° b. 32.775 mph at -35.789°

c. 48.524 mph at -35.789° d. 48.524 mph at 47.612°

e. 48.524 mph at -47.612°

Analytic Trigonometry Practice Test
Part 1: CALCULATOR REQUIRED

1. $\operatorname{arcsec} 1.8 + \operatorname{arccsc} 1.8 =$

(a) 74°
(b) 16°
(c) 90°
(d) 0°
(e) 39°

2. The magnitude of vector $\vec{V} = 3\vec{i} - \sqrt{2}\vec{j}$ is

(a) 4.24
(b) 2.45
(c) 3.61
(d) 3.32
(e) 1.59

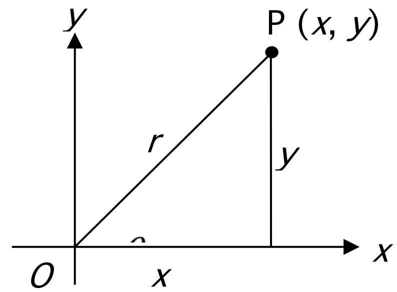
3. If $\sin A = 0.4321$, in Q1, what is the tangent of the supplement of $\angle A$?

(a) 0.3965
(b) –0.4791
(c) 2.0987
(d) –0.3965
(e) 0.4791

4. If $\cos 67° = \tan x°$, then $x =$

(a) 0.4
(b) 6.8
(c) 21
(d) 29.3
(e) 7.8

5. In the figure to the right, $r\cos\theta =$



(a) x
(b) y
(c) r
(d) $x+y$
(e) $r+y$

6. If the terminal side of α passes through $(-9,\ 5)$ then $\tan\alpha =$

(a) $-\frac{9}{5}$ (b) $-\frac{9}{\sqrt{106}}$ (c) $-\frac{5}{9}$ (d) $\frac{5}{9}$ (e) $\frac{9}{5}$

7. If $\arccos(\cos x) = 0$ and $0 \le x \le \frac{\pi}{2}$, then x could equal

(a) 0 (b) $\frac{\pi}{6}$ (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{3}$ (e) $\frac{\pi}{2}$

Part 2: CALCULATOR Allowed

Round to 3 decimal places. Show all work.

1. $(-5,\ 7)$ is on the terminal side of A. Find the six exact trigonometric values:

$\sin A =$ $\cos A =$ $\tan A =$

$\csc A =$ $\sec A =$ $\cot A =$

2. If $\cot B = \frac{7}{23}$ in QIII, find the other five exact trigonometric values:

$\sin B =$ $\cos B =$ $\tan B =$

$\csc B =$ $\sec B =$ $\cot B =$

3. What are the approximate values, in degrees of A and B (from #1and #2)?

$A =$ ____________________ $B =$ ____________________

4. Find the approximate values of:

a) $\csc(-118°) =$ $\cos 136° =$ $\tan 32 =$

b) Find the approximate values (in degrees) of:

$\cos^{-1}(-0.215) =$ $\cot^{-1}(1.48) =$

$\csc^{-1}(0.982) =$ $\sin^{-1}(0.982) =$

5. A plane flies 508 mph at a bearing of 182°. The wind blows at a bearing of 60 mph at 275°. Find the magnitude and bearing of the resultant vector.

6. Identify the quadrant and reference angle of:

a) 1785° Q_________ $\theta_{ref} =$___________

b) –1382° Q_________ $\theta_{ref} =$___________

c) 434° Q_________ $\theta_{ref} =$___________

d) –31° Q_________ $\theta_{ref} =$___________

7. Find the exact value (NO DECIMALS!!!) of the following:

a) $\sin\frac{4\pi}{3}\tan^2\frac{7\pi}{4}$

b) $\cot\frac{7\pi}{6}\sec\frac{5\pi}{4}+\sin\frac{4\pi}{3}$

1-1 Free Response Homework

1. QI, $\theta_{ref} = 10°$ 2. QI, $\theta_{ref} = 75°$ 3. QIII, $\theta_{ref} = 16°$

4. QII, $\theta_{ref} = 42°$

5. $\sin\alpha = \frac{4}{5}; \cos\alpha = \frac{3}{5}$ $\tan\alpha = \frac{4}{3}; \cot\alpha = \frac{3}{4}$ $\csc\alpha = \frac{5}{4}; \sec\alpha = \frac{5}{3}$

6. $\sin\alpha = \frac{-4}{\sqrt{17}}; \cos\alpha = \frac{-1}{\sqrt{17}};$ $\tan\alpha = 4; \cot\alpha = \frac{1}{4};$ $\csc\alpha = \frac{-\sqrt{17}}{4}; \sec\alpha = -\sqrt{17}$

7. $\sin\alpha = \frac{-3}{\sqrt{13}}; \cos\alpha = \frac{2}{\sqrt{13}};$ $\tan\alpha = \frac{-3}{2}; \cot\alpha = -\frac{2}{3};$ $\csc\alpha = \frac{-\sqrt{13}}{3}; \sec\alpha = \frac{\sqrt{13}}{2}$

8. $\sin\alpha = \frac{1}{\sqrt{2}}; \cos\alpha = \frac{-1}{\sqrt{2}};$ $\tan\alpha = -1; \cot\alpha = -1;$ $\csc\alpha = \sqrt{2}; \sec\alpha = -\sqrt{2}$

9. $\sin\alpha = \frac{2}{3}; \cos\alpha = \frac{\sqrt{5}}{3};$ $\tan\alpha = \frac{2}{\sqrt{5}}; \cot\alpha = \frac{\sqrt{5}}{2};$ $\csc\alpha = \frac{3}{2}; \sec\alpha = \frac{3}{\sqrt{5}}$

10. $\sin\alpha = \frac{-\sqrt{65}}{9}; \cos\alpha = \frac{-4}{9};$ $\tan\alpha = \frac{\sqrt{65}}{4}; \cot\alpha = \frac{4}{\sqrt{65}};$ $\csc\alpha = \frac{-9}{\sqrt{65}}; \sec\alpha = \frac{-9}{4}$

11. $\sin\alpha = \frac{-25}{\sqrt{1201}}; \cos\alpha = \frac{-24}{\sqrt{1201}};$ $\tan\alpha = \frac{25}{24}; \cot\alpha = \frac{24}{25};$ $\csc\alpha = \frac{-\sqrt{1201}}{25};$
$\sec\alpha = \frac{-\sqrt{1201}}{24}$

12. $\sin\alpha = \frac{-5}{6}; \cos\alpha = \frac{\sqrt{11}}{6};$ $\tan\alpha = \frac{-5}{\sqrt{11}}; \cot\alpha = \frac{-\sqrt{11}}{5};$ $\csc\alpha = \frac{-6}{5}; \sec\alpha = \frac{6}{\sqrt{11}}$

13. $\sin\alpha = \frac{2\sqrt{7}}{\sqrt{37}}; \cos\alpha = \frac{-3}{\sqrt{37}}; \quad \tan\alpha = \frac{-2\sqrt{7}}{3}; \cot\alpha = \frac{-3}{2\sqrt{7}}; \ \csc\alpha = \frac{\sqrt{37}}{2\sqrt{7}};$

$\sec\alpha = \frac{-\sqrt{37}}{3}$

14. $\sin\alpha = \frac{60}{61}; \cos\alpha = \frac{-11}{61}; \quad \tan\alpha = \frac{-60}{11}; \cot\alpha = \frac{-11}{60}; \ \csc\alpha = \frac{61}{60}; \sec\alpha = \frac{-61}{11}$

<u>1-1 Multiple Choice Homework</u>

1.	C	2.	B	3.	A	4.	C	5.	B
6.	B	7.	C	8.	D	9.	E		

<u>1-2 Free Response Homework</u>

1.	$\frac{\pi}{4}$	2.	$\frac{4\pi}{9}$	3.	$\frac{-41\pi}{30}$	4.	3π	5.	18°
6.	108°	7.	810°	8.	–210°	9.	See notes		
10.	$\frac{1}{2}$	11.	1	12.	0	13.	1		
14.	1	15.	1	16.	0	17.	$2\sqrt{2}$		

<u>1-2 Multiple Choice Homework</u>

1.	C	2.	C	3.	E	4.	C	5.	B	6.	D

<u>1-3 Free Response Homework</u>

1.	0.242	2.	0.284	3.	–0.839	4.	0.138

5. 1.041

6. 3.864

7. $\begin{cases} 40.693° \pm 360° n \\ 139.307° \pm 360° n \end{cases}$

8. $\begin{cases} 55.072° \pm 360° n \\ 235.072° \pm 360° n \end{cases}$ or $55.072° \pm 180° n$

9. $\pm 55.842° \pm 360° n$

10. $\pm 2.119 \pm 2\pi n$

11. $\begin{cases} -0.993 \pm 2\pi n \\ 2.149 \pm 2\pi n \end{cases}$ or $-0.993 \pm \pi n$

12. No solution

13. $41.810° \pm 360° n$

14. $226.169° \pm 360° n$

15. $119.551° \pm 360° n$

16. $-2.031 \pm 2\pi n$

17. $-0.985 \pm 2\pi n$

18. $1.752 \pm 2\pi n$

1-3 Multiple Choice Homework

1. B 2. B 3. D 4. A 5. E 6. E

1-4 Free Response Homework

1. $-126\vec{i} + 5\vec{j}$

2. $52\vec{i} + 25\vec{j}$

3. $59\vec{i} - 39\vec{j}$

4. $-4\vec{i} + 29\vec{j}$

5. $\sqrt{15397} \approx 124.085$

6. $\sqrt{857} \approx 29.275$

7. $\sqrt{802} \approx 28.320$

8. $\sqrt{31793} \approx 178.306$

9. $\frac{60}{61}\vec{i} + \frac{11}{61}\vec{j}$

10. $\frac{75}{\sqrt{5986}}\vec{i} + \frac{19}{\sqrt{5986}}\vec{j}$

11. $\frac{4}{\sqrt{241}}\vec{i} + \frac{15}{\sqrt{241}}\vec{j}$

12. $-\frac{126}{\sqrt{15901}}\vec{i} + \frac{5}{\sqrt{15901}}\vec{j}$

13. 10.628 units at 43.605°

14. 5.874 units at –88.170° (or 271.830°)

15. 4.661 units at –141.527° (or 218.473°)

16. 79.715 units at –163.747 (or 196.253°)

17. 400.125 mph at –81.432° (or 278.568°)

18. 82.715 miles at 72.972°

19. 455.672 mph at 75.145°

20. $\sqrt{29}$ mph

21. 89 lbs at 49°

22. 110.534 miles at 164.184°

23. 101.692 yards at 23.532°; 93.235 yards at 0° no, he does not get out because he only went

<u>1-4 Multiple Choice Homework</u>

1. B 2. C 3. A 4. E 5. E

<u>Analytic Trigonometry Practice Test Answer Key</u>

<u>Multiple Choice</u>

1. C 2. D 3. B 4. C 5. A

<u>Free Response</u>

1. $\frac{7}{\sqrt{74}},-\frac{5}{\sqrt{74}},-\frac{7}{5},\ \frac{\sqrt{74}}{7},-\frac{\sqrt{74}}{5},-\frac{5}{7}$

2. $-\frac{23}{\sqrt{578}},-\frac{7}{\sqrt{578}},\ \frac{23}{7},-\frac{\sqrt{578}}{23},-\frac{\sqrt{578}}{7},\ \frac{7}{23}$

3. $A=125.538°\pm 360°n,\ B=-106.928°\pm 360°n\ \left(\text{or } 253.072°\pm 360°n\right)$

4a. $-1.133, -0.719,\ 0.661$

b. $\pm 102.416° \pm 360° n,$ $34.046° \pm 180° n,$ DNE, $79.113 \pm 360° n,$ and $100.887° \pm 360° n$

5. 508.403mph at $-171.232°$ (or $188.768°$)

6a. QIV, $\theta_{ref} = 15°$

b. QI, $\theta_{ref} = 58°$

c. QI, $\theta_{ref} = 74°$

d. QIV, $\theta_{ref} = 31°$

7a. $-\frac{\sqrt{3}}{2}$ b. $\frac{-2\sqrt{6} - \sqrt{3}}{2}$