Chapter 1- ARITHMETIC SEQUENCES

Number Pattern

Number pattern is a pattern or sequence in a series of numbers.

Number sequences

A set of numbers written as first, second, third so on according to particular rule is called number sequence.

Eg: a) 1,2,3,4,5,........................ b) 2,4,6,8,......................3) 1,2,4,8,16,32,64,...............

problems from Text.

(1) Make the following number sequences, from the sequence of equilateral triangles, squares, regular pentagons and so on, of regular polygons:

Number of sides 3, 4, 5, ...

Sum of inner angles

Sum of outer angles

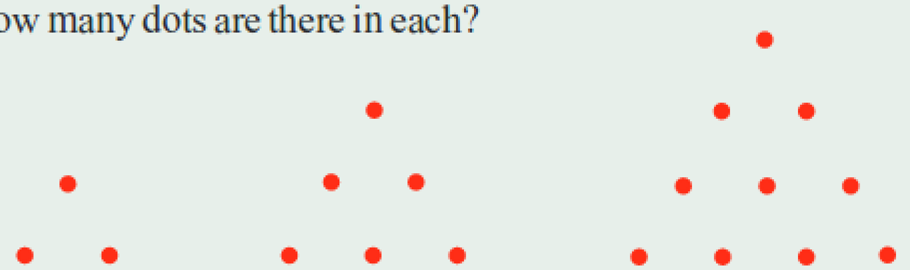
One inner angle

One outer angle

Answer :
Sum of inner angles : 180, 360,540,720,900,....
Sum of Outer angles : 360,360,360,........
One Inner angle : 60,90,108,120,128.57,.....
One outer angles : 120,90,72,51.43

(2)

(1) Look at these triangles made with dots.
How many dots are there in each?



Compute the number of dots needed to make the next two triangles.

Answer :

Number of dots in each picture 3, 6, 10,

Number of dots needed to make next two triangles are 15, 21

(2) Make the following number sequences, from the sequence of equilateral triangles, squares, regular pentagons and so on, of regular polygons:

Number of sides 3, 4, 5, ...

Sum of interior angles

Sum of exterior angles

One interior angle

One exterior angle

Answer:

Sequence of regular polygons with sides 3,4,5,.......

Sum of interior angles	180, 360, 540, ...
Sum of exterior angles	360,360,360, ...
One Interior Angle	60,90,108, ...
One Exterior angle	120,90,72,..........

(3) Write down the sequence of natural numbers leaving remainder 1 on division by 3 and the sequence of natural numbers leaving remainder 2 on division by 3.

Answer:

Sequence of natural Numbers leaving remainder 1 on division by 3 is
1,4,7,10,.........

Sequence of natural Numbers leaving remainder 2 on division by 3 is

2,5,8, ...

(4) Write down the sequence of natural numbers ending in 1 or 6 and describe it in two other ways.

Answer: :
Sequence of natural numbers ending in 1 or 6

1,6,11,16,21........

Sequence can be described in two ways :
a) Natural Numbers starting from 1 with difference 5
b) Numbers leaves remainder 1 when divided by 5

(5) One cubic centimetre of iron weighs 7.8 grams. Write as sequences, the volumes of weights of iron cubes of sides 1 centimetre, 2 centimetres and so on.

Answer:
Volume of cube = side X side X side = a^3.
Volumes of the iron cubes with sides 1cm, 2cm, 3cm .....
$1^3, 2^3, 3^3$, ...
= 1, 8, 27, ...
Weights of iron cubes

1x7.8, 8x7.8, 27x7.8...........
7.8, 62,4, 210.6 ....
(6)

A tank contains 1000 litres of water and it flows out at the rate of 5 litres per second.

How much water is there in the tank after each second? Write their numbers as a sequence.

Answer:

Seconds :	1	2	3	4	5	6	7	
Litters of water in the tank :	995	990	985	980	975	970	965	....

Algebra of Sequences

In the sequence 4,8,12,16,......

each number is a ***Term*** *in the sequence. We can decide the position of each term of this sequence*

1st	2nd	3rd	4th	5th
4	8	12	16	......

What is the 20^{th} term of the sequence ?
For solving this we are using algebra

x_1	x_2	x_3	x_4	......	20^{th}
4	8	12	16	......	?

x_1=4x1, x_2= 4x 2, x_3 = 4x3 so 20th term = 4x 20 =80

So algebraic expression of this sequence is 4n

Write the algebraic expression of the following sequence .
1, 4, 9 16, ... Answer : Algebraic expression =n^2

(1) Write the algebraic expression for each of the sequences below:

i) Sequence of odd numbers

ii) Sequence of natural numbers which leave remainder 1 on division by 3.

iii) The sequence of natural numbers ending in 1.

iv) The sequence of natural numbers ending in 1 or 6.

Answer :

i) Sequence odd numbers 1, 3, 5, 7, ...
position 1, 2, 3, 4, ...

1= 2 x 1 -1 3= 2x2-1 5= 2 x3 -1 7=2x4-1 ....x_n = 2xn-1 =2n-1

So algebraic form of odd sequences = **2n-1**

ii) Sequence of natural numbers which leave remainder 1 when division by 3 are

1, 4, 7, 10, ... and position 1,2,3,4, ...

1 =3x1-2, 4 = 3x2-2 7=3x3-2 10= 3x4-2 ....x_n = 3xn-2
so algebraic expression of this sequence = **x_n = 3n-2**

iii) Sequence of natural numbers ending in 1 are 1, 11, 21, 31, ...
position 1, 2, 3, 4, ...
1= 10x1-9, 11=10x2-9, 21=10x3-9, 31=10x4-9, ...
x_n= 10xn-9 =**10n-9**
iv) Sequence of natural numbers ending in 1 or 6

1, 6,11,16,21, ... with position numbers 1,2,3,4,........

algebraic expression of this sequence **x_n =x_n 5n-4.**

(2) For the sequence of regular polygons starting with an equilateral triangle, write the algebraic expressions for the sequence of the sums of interior angles, the sums of the exterior angles, the measures of an interior angle, and the measures of an exterior angle.

Answer:

Sequence of sums of interior angles 180,360, 540, 720,..........

positions 1 2 3 4 ....

180= 1x 180, 360= 2x180 540= 3x180 720=4x180..........

x_n = nx180. **x_n = 180n** Which is the algebraic expression.

Sequence of sum of exterior angles 360, 360,360,......

Algebraic expression **x_n = 360**

Sequence of measures of interior angles $\frac{180}{3}, \frac{360}{4}, \frac{540}{5}, \frac{720}{6}$...........

Positions 1 2 3 4 ....

Algebraic expression = **X_n** $= \frac{180n}{(n+2)}$

Sequence of measures of exterior angles $\frac{360}{3}, \frac{360}{4}, \frac{360}{5}, \frac{360}{6}$.....

Algebraic expression = **X_n** $= \frac{360}{(n+2)}$

(3) Look at these pictures:

The second picture is obtained by removing the small triangle formed by joining the midpoints of the first triangle. The third picture is got by removing such a middle triangle from each of the red triangles of the second picture.

i) How many red triangles are there in each picture?

ii) Taking the area of the first triangle as 1, compute the area of a small triangle in each picture.

iii) What is the total area of all the red triangles in each picture?

iv) Write the algebraic expressions for these three sequences obtained by continuing this process.

Answer :

1) Number of red triangles in each picture 1, 3, 9, ...

2) Area of small triangles 1, 1/4, 1/16, ...

3) Total area of red triangles 1, 3/4, 9/16 ......

4) Algebraic expression of these sequences

1,3,9,......... $\mathbf{x_n = 3^{(n-1)}}$

1, 1/4, 1/16, ... $x_n = (1/4)^{(n-1)}$

1, 3/4, 9/16 ...... $xn = (3/4)^{(n-1)}$

Arithmetic Sequence

An arithmetic sequence is a sequence in which we get the same number on subtracting from any term, the term immediately preceding it.

This constant difference got by subtracting from any term the just previous term, is called the common difference of an arithmetic sequence.

(1) Check whether each of the sequences given below is an arithmetic sequence. Give reasons. For the arithmetic sequences, write the common difference also.

i) Sequence of odd numbers

ii) Sequence of even numbers

iii) Sequence of fractions got as half the odd numbers

iv) Sequence of powers of 2

v) Sequence of reciprocals of natural numbers

Answer :
i) Sequence of odd numbers : 1,3,5,7,9,.......Arithmetic sequence with common difference 2

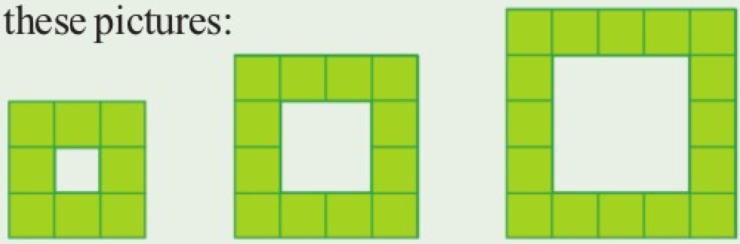
ii) Sequences of even numbers : 2,4,6,8........Arithmetic sequence with common difference 2

iii) Sequence of fractions got as half the odd numbers
1/2, 3/2,5/2, 7/2 ....Arithmetic sequence with common difference 1

iv) Sequence of powers of 2 = 2,4,8,16, No common difference so this sequence is not an Arithmetic sequence.
v) Sequence of reciprocals of natural numbers

1, 1/2, 1/3, 1/4, ... Which does not have a common difference and not an Arithmetic sequence.

(2) Look at these pictures:



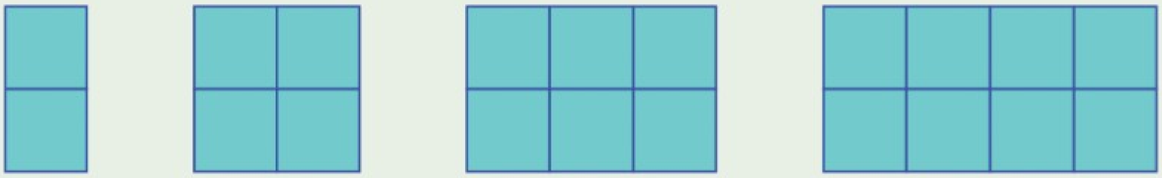
If the pattern is continued, do the numbers of coloured squares form an arithmetic sequence? Give reasons.

Answer :

numbers of coloured squares = 8, 12, 16, ...

This is an arithmetic sequence with common difference 4

(3) See the pictures below:



i) How many small squares are there in each rectangle?

ii) How many large squares?

iii) How many squares in all?

Continuing this pattern, is each such sequence of numbers, an arithmetic sequence?

Answer :

i) Small squares in each rectangle : 2, 4, 6, 8, ...

ii) Number of large squares : 0, 1, 2, 3,

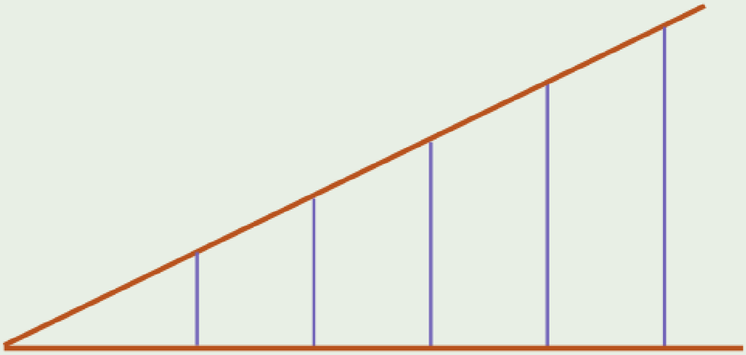
iii) All squares in each picture : 2, 5, 8, 11, ...

First sequence is an arithmetic sequence with common difference 2

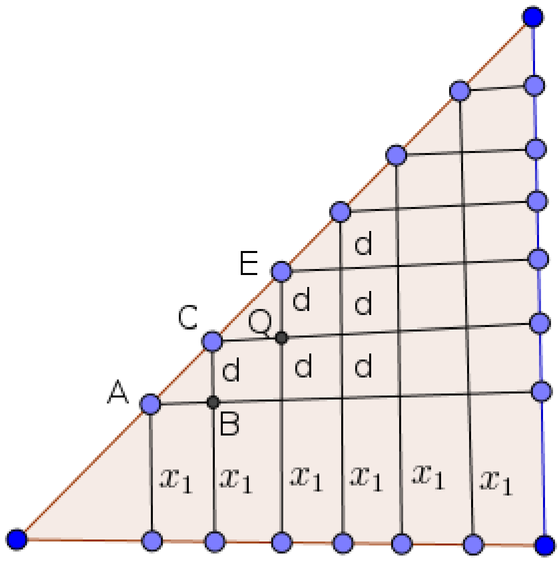
Second sequence is an arithmetic sequence with common difference 1

Third sequence is an arithmetic sequence with common difference 3

(4) In this picture, the perpendiculars to the bottom line are equally spaced. Prove that, continuing like this, the lengths of perpendiculars form an arithmetic sequence.



Answer : In the figure < BAC = < QCE -Corresponding angles Δ



<B and <Q are right angles
AB =CQ
there fore these triangles are equal. (ASA)

Lengths of perpendicular lines.
$x_1, x_1 + d, x_1 + 2d,$ ....

Which is an arithmetic sequence with common difference d.

(5) The algebraic expression of a sequence is

$$x_n = n^3 - 6n^2 + 13n - 7$$

Is it an arithmetic sequence?

Answer :

Algebraic expression of the sequence is $x_n = n^3 - 6n^2 + 13n - 7$

First term = $x_1 = 1 - 6 + 13 - 7$ = 1

Second term = $x_2 = 2^3 - 6X2^2 + 13X2 - 7$ = 8 -24 +26 -7 = 3

Third term = $x_3 = 3^3 - 6X3^2 + 13X3 - 7$

= 27 -54+39 -7

= 5

Fourth term = $x_4 = 4^3 - 6X4^2 + 13X4 - 7$

= 64- 96 + 52 -7

= 116- 103

= 13

Now the sequence 1, 3, 5, 13, ... is not an arithmetic sequence because there is no common difference.

Position and Term

A) Can you make an arithmetic sequence with 1 and 11 as the first and second terms?

Answer:

1 ,11, 21, 31,....

B)Can you make an arithmetic sequence with 1 and 11 as the first and third terms?

Answer :

1 ,6, 11, 16, 21,.......

C) Find an arithmetic sequence with the 3rd term 37 and 7th term 72?

Number of times the common difference added = 7-3 =4 times

4 Times common difference = 36

4 X d =36

$d = \frac{36}{4}$ = 9

First term = 3rd trem – 2d = 37 -18 =19
Second term = 19+9 = 28,
Arithmetic Sequence = 19, 28, 37, 46, 55, 64, 73, ...

The difference between any two terms of an arithmetic sequence is the product of the difference of positions and the common difference.

we get a formula from this idea. d $= \frac{x_n - x_m}{n - m}$

We can put it like this also:

In an arithmetic sequence, term difference is proportional to position difference; and the constant of proportionality is the common difference.

We can use this to check whether a given number is a term of a given arithmetic sequence.

D) Check whether 1000 a term of the sequence 19, 28, 37, ...

Answer :
Common difference = d =9

Difference of last term and first term =1000-19 = 981 and $\frac{981}{9}$ = 109

Now we get from $\frac{x_n - x_m}{d} = n - m$

109 = n – 1 ,
n= 10 9 + 1

n = 110, means 1000 is 110th term of the sequence 19, 28, 37, ...

E) Is every power of 10 from 100 onwards, a term of the arithmetic sequence 19, 28, 37, ...?

Answer : First we will check 100

We have $\frac{100-19}{9}=n-1$

$\frac{81}{9}=n-1$

9 = n-1

n=10

100 is 10th term. Similarly 1000 is 110th term, 10000 is 1110th term and so on. Means every power of 10 is a term in the sequence.

(1) In each of the arithmetic sequences below, some terms are missing and their positions are marked with ◯. Find them.

i) 24, 42, ◯, ◯, ...

ii) ◯, 24, 42 , ◯, ...

iii) ◯, ◯, 24, 42, ...

iv) 24, ◯, 42, ◯, ...

v) ◯, 24, ◯, 42, ...

vi) 24, ◯, ◯, 42, ...

Answer :

i) 24, 42, 60, 78, ...

ii) 6, 24, 42, 60, ...

iii) -12, 6, 24, 42, ...

iv) 24, 33, 42, 51, ...

v) 15, 24, 33, 42, 51, ...

vi) 24 +3d =42 , 3d = 42-24 = 18

d= $\frac{18}{3}$ =6,

sequence = 24, 30, 36, 42, ...

(2) The terms in two positions of some arithmetic sequences are given below. Write the first five terms of each:

i) 3^{rd} term 34
6^{th} term 67

ii) 3^{rd} term 43
6^{th} term 76

iii) 3^{rd} term 2
5^{th} term 3

iv) 4^{th} term 2
7^{th} term 3

v) 2^{nd} term 5
5^{th} term 2

Answer :

i) 12, 23, 34, 45, ...

ii) 21, 32, 43, 54,......

iii) $d = \frac{term \quad difference}{position difference}$ = $\frac{3-2}{5-3}$ =$\frac{1}{2}$

Sequence is

1, $1\frac{1}{2}$, 2, $2\frac{1}{2}$, ...

iv) $d = \frac{term \quad difference}{position difference}$ = $\frac{3-2}{7-4}$ =$\frac{1}{3}$

sequence is 1, $\frac{4}{3}$, $\frac{5}{3}$, 2, $\frac{7}{3}$, ...

v)

$d = \frac{term \quad difference}{position difference}$= $\frac{2-5}{5-2}$ =-1

Sequence 6, 5, 4, 3, ...

(3) The 5^{th} term of an arithmetic sequence is 38 and the 9^{th} term is 66. What is its 25^{th} term?

$$d = \frac{term \quad difference}{position difference} = \frac{66-38}{9-5} = \frac{28}{4} = 7$$

25th term = 9th term + (25-9)d
= 66+16 X 7
= 66+112
= 178
====

(4) Is 101 a term of the arithmetic sequence 13, 24, 35, ...? What about 1001?

Answer :

Common Difference =11

101-13 = 88 and $\frac{88}{11}$ = 8 so 88 is a multiple of 11 , 101 is a term.

1001-13 = 988 and $\frac{988}{11}$ =89.81 not a multiple of 11 , 1001 is not a term

(5) How many three-digit numbers are there, which leave a remainder 3 on division by 7?

Answer :

$\frac{100}{7}$ gives remainder 2 . so First three digit number which gives remainder 3 when divided by 7 is 101.

$\frac{1000}{7}$ gives remainder 6, so last three digit number which gives remainder 3 when divided by 7 is 997.

ie sequence = 101, 108, 115, ..., 997
now 101+7n = 997

$$n = \frac{997-101}{7} = 128,$$

so 997 is 128+1 = 129th term

ie there are 129 terms in the sequence.

(6) Fill up the empty cells of the square below such that the numbers in each row and column form arithmetic sequences:

1			4
7			28

What if we use other numbers instead of 1, 4, 28 and 7?

Answer:

1	2	3	4
3	6	9	12
5	10	15	20
7	14	21	28

Another table for the same question

5			14
17			26

Answer:

5	8	11	14
9	12	15	18
13	16	19	22
17	20	23	26

Try another table using other four numbers

(7) In the table below, some arithmetic sequences are given with two numbers against each. Check whether each belongs to the sequence or not.

Sequence	Numbers	Yes/No
11, 22, 33, ...	123	
	132	
12, 23, 34, ...	100	
	1000	
21, 32, 43, ...	100	
	1000	
$\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, \ldots$	3	
	4	
$\frac{3}{4}, 1\frac{1}{2}, 2\frac{1}{4}, \ldots$	3	
	4	

Answer:

11,22,33,......................
d= 11 when divided by 11 gives reminder 0.
Now,

$\frac{123}{'11}$ gives reminder 1 so 123 is not a term in the sequence.

Algebra of arithmetic sequences

A)
Prove that sum of any three consecutive natural numbers is three times the middle number
Answer :
Let the number are (n-1), n, (n+1)
Sum = n-1 + n + n+1 = 3n =Three times middle number.

For any arithmetic sequence, the sum of three consecutive terms is thrice the middle one.

We can say it in another way.

In any three consecutive terms of an arithmetic sequence, the middle one is half the sum of the first and the last.

If x, y, z are three consecutive terms of an arithmetic sequence, then	
x + y + z = 3 y	$y = \frac{x+z}{2}$

B) Prove that sum any 5 consecutive terms of an arithmetic sequence is five times its middle term.
Answer : Let the numbers are x-2d, x-d, x, x+d, x+2d
sum = x-2d + x+d +x + x+d +x+2d = 5x = Five times middle term

C) Prove that sum any 7 consecutive terms of an arithmetic sequence is 7 times its middle term.
Answer :
x-3d, x-2d + x+d +x + x+d +x+2d , x+3d
sum = x-3d+x-2d + x+d +x + x+d +x+2d +x+3d= 7x = Seven times middle term.

Now algebraic expression

Positions :	**1**	**2**	**3**	**4**	**5**	**.................. n**
Terms :	f	f+d	f+2d	f+3d	f+4d	.................f+(n-1)d

Taking the first term of an arithmetic sequence as f and the common difference as d, the n^{th} term is f + (n − 1) d = dn + (f − d)

That is n^{th} term = common difference X position + a fixed number.

So

Any arithmetic sequence is of the form

$$x_n = an + b$$

where *a* and *b* are fixed numbers; conversely, any sequence of this form is an arithmetic sequence.

Here a is the common difference and a+b is the first term.

D) Prove that this sequence $x_n = \dfrac{2n+1}{6}$ contains no natural numbers.
Answer :
Numerator is odd and denominator is even, there is no natural numbers in this series.

(1) Write three arithmetic sequences with 30 as the sum of the first five terms.

Answer :

Sum of 5 consecutive terms = 5 x middle term =30
middle term = 6
Arithmetic sequences with sum 30
4,5,6,7,8
2,4,6,8,10
0,3,6,9,12

(2) The first term of an arithmetic sequence is 1 and the sum of the first four terms is 100. Find the first four terms.

Answer : f= 1, given 1+1+d+1+2d+1+3d =100
4 +6d =100, 6d=96

$d = \frac{96}{6}$ = 16

The arithmetic sequence = 1, 17, 33, 49, ...

(3) Prove that for any four consecutive terms of an arithmetic sequence, the sum of the two terms on the two ends and the sum of the two terms in the middle are the same.

Answer:
Let four consecutive terms are f, f+d, f+2d, f+3d

Sum of end terms = f + f +3d =2f+3d
Sum of middles =f+d+f+2d= 2f+3d
both are same

(4) Write four arithmetic sequences with 100 as the sum of the first four terms.

Answer :
Let four terms f, f+d, f+2d, f+3d
given f, f+d, f+2d, f+3d =100
4f+6d= 100
2f+3d =50
When f=1
3d=48, d =16
Sequence is 1, 17, 33,49 sum 1+17+33+49 =100

When f =4, then d =14
4,18,32,46
When f=7 , d =12
7, 19, 32, 43

(5) The 8^{th} term of an arithmetic sequence is 12 and its 12^{th} term is 8. What is the algebraic expression for this sequence?

Answer :
8^{th} term = f+7d =12
12^{th} term =f+11d =8
solving common difference = -1
then f= 19
Algebraic expression =x_n =an+b
a =-1
f= a+b, 19 =-1+b
b=20
algebraic expression x_n =-n +20 =20-n

(6) The Bird problem in Class 8 (The lesson, **Equations**) can be slightly changed as follows.

One bird said:

"We and we again, together with half of us and half of that, and one more is a natural number"

Write the possible number of birds in order. For each of these, write the sum told by the bird also.

Find the algebraic expression for these two sequences.

Answer :
Let number of birds =x

given $x + x + \frac{x}{2} + \frac{x}{4}$ +1=n

$\frac{11x}{4}$ +1 = n
possible values of x , which is number of birds
4, 8, 12, ... algebraic expression =4n

possible values of n
12, 23, 34,..... algebraic expression 11n+1
**

(7) Prove that the arithmetic sequence with first term $\frac{1}{3}$ and common difference $\frac{1}{6}$ contains all natural numbers.

Answer:
Sequence is $\frac{6}{18}, \frac{9}{18}, \frac{12}{18}, ...$
Algebraic expression $x_n = dn + f - d$

$x_n = \frac{n}{6} + [\frac{1}{3} - \frac{1}{6}]$ = $\frac{n}{6} + \frac{1}{6}$ = $\frac{n+1}{6}$
Giving values 5, 11, 17, 23, ... to n we get natural number 1,2,3, ...
so this sequence contains all natural numbers.

(8) Prove that the arithmetic sequence with first term $\frac{1}{3}$ and common difference $\frac{2}{3}$ contains all odd numbers, but no even number.

Answer :

algebraic expression of this sequence = x_n = dn+f-d

= $\frac{2n}{3} + \frac{2}{3} - \frac{1}{3}$
= $\frac{2n}{3} - \frac{1}{3}$
= $\frac{2n-1}{3}$ an odd number divided by an odd number gives an odd number.

By putting values 2, 5, 8, 11, ... we get all odd numbers but no even numbers.

(9) Prove that the squares of all the terms of the arithmetic sequence 4, 7, 10, ... belong to the sequence.

Answer :

Algebraic expression = x_n = dn+f-d

= 3n +4-3

= 3n +1

Square of this term = $(3n+1)^2$ = $9n^2+6n+1$

To prove $x_n - x_1$ is a multiple of common difference 3

$9n^2+6n+1$ -4 = $9n^2+6n-3$ = 3($3n^2+3n-1$)

which is a multiple of 3 , so all squares of n^{th} terms in this sequence.

(10) Prove that the arithmetic sequence 5, 8, 11, ... contains no perfect squares.

Answer :

Algebraic expression = x_n = dn+f-d

= 3n + 5-3

= 3n+2

Square of n^{th} term = $(3n+2)^2$ if this term contained in the sequence

$x_n - x_1$ should be a multiple of 3

ie $(3n+2)^2$ -5 = $9n^2+12n-1$ which not a multiple of common difference 3 , so no perfect square in this sequence.

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(11) The angles of a pentagon are in arithmetic sequence. Prove that its smallest angle is greater than 36º.

Answer :

Let smallest angle =36

so angles of this pentagon are 36, 36+d, 36+2d, 36+3d, 36+4d

we have 36 +36+d+ 36+2d+ 36+3d+ 36+4d = 540

ie 180 + 10d =540
ie d = 36
ie angles are 36, 72, 108, 144, 180, but we cannot construct pentagon with one angle 180,
that first angle, the smallest angle should be greater than 36

(12) Write the whole numbers in the arithmetic sequence $\frac{11}{8}, \frac{14}{8}, \frac{17}{8}, ...$

Do they form an arithmetic sequence?

Answer :
Algebraic expression = x_n = dn+f-d

$$= \frac{3}{8}n + \frac{11}{8} - \frac{3}{8}$$

$= \frac{3n}{8} + 1$, this term becomes a whole number when n is a multiple of 8, that is values of n are 8, 16, 24 .....

when n = 8 , $\frac{3n}{8} + 1$ = 4,
when n= 16, $\frac{3n}{8} + 1$ = 7,
when n = 24, $\frac{3n}{8} + 1$ = 10
....
....
so sequence of whole numbers in the above sequence are
4,7,10, ...
This is an arithmetic sequence with common difference 3.

13.

The 8th term of an arithmetic sequence is 12 and its 12th term is 8. What is the algebraic expression for this sequence?

Answer:

Given that $x_8 = 12$ and $x_{12} = 8$

We know $x_n = an+b$

ie $8a+b=12$----------------(1)

ie $12a+b=8$----------------(2)

Solving, (1) -(2)

$-4a = 4$

$a = -1$

common difference is -1

put this value in (1)

$-8 + b = 12$

$b = 20$

algebraic expression is $x_n = -n+20 = 20-n$

If m^{th} term is n and n^{th} term is m then common difference is always -1

Sums

The sum of any number of consecutive natural numbers, starting with one, is half the product of the last number and the next natural number.

In the language of algebra,

$$1 + 2 + 3 + \cdots + n = \frac{1}{2}\, n\,(n+1)$$

A) Find the sum of natural numbers up to 100

Answer :

1+2+3+.......+ 100 =

B) Find the sum of terms 2,4,6, ..., 100

Answer :

2+4+6+ .... +100 = 2(1+2+3+.......+50)

= 2 x $\frac{50 \times 51}{2}$

= 50 x 51

= 2550

**

C) Find the sum first n odd Numbers.

Answer :

1+3+5+.................. n terms

n^{th} term = dn +f-d = 2n+1-2 = 2n-1

1 term = 2 X1-1 =1

2 term= 2 x2-1 =3

3 rd term 2x3-1 = 5

sum =1+3+5+...........+2n-1 = 2x1-1 + 2x2-1 + 2x3-1 +....+2xn-1

ie = 2(1+2+3+.......+n) -1-1-1-1 (n times)

= 2(1+2+3+....... +n) – (1+1+1+ n times)

= 2 x $\frac{n \times (n+1)}{2}$ - n

= n(n+1)-n

= $n^2 + n - n$

= n^2

Sum of odd number up to n = n^2

D) Calculate the sum of any arithmetic sequence.

Answer :

Let the term are $x_1, x_2, x_3, \ldots\ldots\ldots\ldots, x_n$

Sum = $x_1 + x_2 + x_3 + \ldots\ldots\ldots\ldots + x_n$

here $x_n =$ an +b

first term = a x 1 +b, = 1a+b

Second term = a x 2 +b = 2a+b

third term = a x 3 +b = 3a+b

....

.... = na+b

Sum = a+b+2a+b+3a+b+...........+na+b

= a+2a+3a+..............+na + b +b+b+....(n times)

=a(1+2+3+,,,,,,,,,,+n) + nb

$$= a \times \frac{n \times (n+1)}{2} + nb$$

$$= \frac{1}{2} an(n+1) + nb$$

For the arithmetic sequence

$$x_n = an + b$$

the sum of the first n terms is

$$x_1 + x_2 + \dots + x_n = \frac{1}{2} an(n+1) + nb$$

Now try this formula in another fashion,

$$\begin{aligned} \frac{1}{2} an(n+1) + nb &= \frac{1}{2} n(a(n+1) + 2b) \\ &= \frac{1}{2} n((an+b) + (a+b)) \\ &= \frac{1}{2} n[x_n + x_1] \end{aligned}$$

$$\boxed{x_1 + x_2 + \dots + x_n = \frac{1}{2} n(x_n + x_1)}$$

(1) Find the sum of the first 25 terms of each of the arithmetic sequences below.

i) 11, 22, 33, ... ii) 12, 23, 34, ... iii) 21, 32, 43, ...
iv) 19, 28, 37, ... v) 1, 6, 11, ...

Answer :

Sum of 25 terms = =$\frac{1}{2}$ an (n+1) +nb

a= 11, n =25, b =f-d =0

Sum of 25 terms = S_{25} =$\frac{1}{2}$ x11x 25 x 26 +0 = 3575

ii) Answer =3600

iii) Answer = 3825

iv)Answer =3175

v)Answer =1525

(2) What is the difference between the sum of the first 20 terms and the next 20 terms of the arithmetic sequence 6, 10, 14, ...?

Answer :

Given sequence 6,10,14, ...

$x_n =$ dn+f-d

20th term = x_{20} = 4 x20 + 6-4

= 80 + 2

= 82

Sum of n terms = S_n = $\frac{1}{2}n[x_n + x_1]$

Sum of 20 terms = $\frac{20}{2}[6 + 82]$

= 10 x 88

= 880

To find sum of next 20 terms.

21st term = 82+4 =86

x_{40} = 4 x20 +86-4

= 80 + 82

=162

S_{20} = $\frac{20}{2}[86 + 162]$

= 10 x 248

= 2480

Difference between these sums = 2480- 880 = 1600

=======

(3) Calculate the difference between the sum of the first 20 terms of the arithmetic sequences 6, 10, 14, ... and 15, 19, 23, ...

Sum of 20 terms of first sequence

x_{20} =dn+f-d = 4 x 20 +6-4 = 82

Sum of first 20 terms = S_{20} =$\frac{20}{2}[6+82]$ = <u>880</u>

Sum of 20 terms of second sequence

x_{20} =dn+f-d = 4 x 20 +15-4 = 80 +11 =91

Sum of first 20 terms = S_{20} =$\frac{20}{2}[15+91]$ = <u>1060</u>

Difference between the sums =1060- 880 =180

===

(4) Find the sum of all three digit numbers, which are multiples of 9.

Answer :

First three digit number a multip$\frac{20}{2}[15+91]$le of 9 =108

Last three digit number a multiple of 9 =999

Number of terms = $n = \frac{x_n - x_1}{d} + 1$

$$n = \frac{999-108}{9} + 1$$

$$n = \frac{891}{9} + 1$$

= 99+1

= 100

Sum of 100 terms

$$S_{100} = \frac{100}{2}[999+108]$$

= 50 x 1107

= 55350

====

(5) Find n in the equation $5^2 \times 5^4 \times 5^6 \times ... \times 5^{2n} = (0.008)^{-30}$.

Answer :

$5^2 \times 5^4 \times 5^6 \times .... \times 5^{2n} = (0.008)^{-30}$

$5^{2+4+6+......+2n}$ = $\frac{8}{1000}^{-3^0}$

$5^{2(1+2+3+...+n)}$ = $\frac{1}{125}^{-3^0}$

$5^{n(n+1)}$ = $(5^{-3})^{-30}$

$5^{n(n+1)}$ = 5^{90}

ie n(n+1) = 90

9 x 10 = 90

so n = 9

==

(6) The expressions for the sum to n terms of some arithmetic sequences are given below. Find the expression for the n^{th} term of each:

i) $n^2 + 2n$ ii) $2n^2 + n$ iii) $n^2 - 2n$

iv) $2n^2 - n$ v) $n^2 - n$

Answer :

Sum of n terms = $n^2 + 2n$

sum of 1 term = $1^2 + 2$ = 3

Sum of 2 terms = $2^2 + 2 \times 2$ =8
Second term = 8-3 =5
Sequence is 3, 5, 7, ...
$x_n =$ dn+f-d = 2n +3-2 = 2n+1

ii) 4n-1
iii)2n-3
iv)4n-3
v)2n-2

(7) Calculate in head, the sums of the following arithmetic sequences.

i) 51 + 52 + 53 + ... + 70

ii) $1\frac{1}{2} + 2\frac{1}{2} + ... + 12\frac{1}{2}$

iii) $\frac{1}{2} + 1 + 1\frac{1}{2} + 2 + 2\frac{1}{2} + ... + 12\frac{1}{2}$

Answer :
i) 1210
ii)84
iii)162.5
**

(8) The sum of the first 10 terms of an arithmetic sequence is 350 and the sum of the first 5 terms is 100. Write the algebraic expression for the sequence.

Answer :
Sum of 10 terms = $\frac{10}{2}[x_1 + x_{10}]$
= 5 $[x_1 + x_1 + 9d]$
=5[2x_1 +9d]
given
5[2x_1 +9d] = 350

$[2x_1 + 9d]$ = 70 -------------(1)

Sum 5 terms $= \frac{5}{2}[x_1 + x_5][2x_1 + 9d]$ = 70

$= \frac{5}{2}[x_1 + x_1 + 4d]$

$= \frac{5}{2}[2x_1 + 4d]$

given

$\frac{5}{2}[2x_1 + 4d]$ = 100

$5(2x_1 + 4d)$ = 200

$2x_1 + 4d$ = 40 ----------------------(2)

Solving (1) & (2)

$2x_1 + 9d$ = 70 ----------------------(1)

$2x_1 + 4d$ = 40 ----------------------(2)

5d =30

d =6

$2x_1 + 24 = 40$

$2x_1 = 16$

$x_1 = 8$

First term = 8 , common difference = 6

Sequence is 8,14,20,.........

n^{th} term = dn+f-d =6n+8-6= 6n+2

====

(9) Prove that the sum of any number of terms of the arithmetic sequence 16, 24, 32, ... starting from the first, added to 9 gives a perfect square.

Answer :

Sequence is 16,24,32, ...

n^{th} term = dn+f-d = 8n +16-8 = 8n +8

Sum of first n terms $= \frac{n}{2}[x_1 + x_n]$

$= \frac{n}{2}[16 + 8n + 8]$

$= \frac{n}{2}[8n + 24]$

$= 4n^2 + 12n$

when 9 is added to sum of n terms

$4n^2 + 12n + 9 = (2n+3)^2$ which is a perfect square

(10) 4

7 10

13 16 19

22 25 28 31

....

....

....

Write the next two lines of the pattern above. Calculate the first and last terms of the 20th line.

Answer :

34,37,40,43,46

49,52,55,58,61,64

Number of terms up to last number of 19th line

$= 1+2+3+......+19 = \frac{19 \times 20}{2}$

= 190 terms

190th term =dn+f-d =3 x 190 +1

= 571

first number in 20th line = 571 +3 =574

there are 20 terms in 20th line.

so Last number in 20th line = 3 x20 + 571

= 631

====

Extra Questions:

1. Algebra of a number sequence is n^2+n.

(a) Write this sequence.

(b) Is this an arithmetic sequence? Why?

Answer :

1 (a) :

Given algebra of Number sequence = $n^2 + n$

Sequence is $1^2 + 1, 2^2 + 2, 3^2 +3, 4^2 + 4,$ ....

= 2, 6, 12, 20, ...

1 (b):

No common difference. So this is not an Arithmetic Sequence.