

# Chapter 1

## BASIC RADIATION PHYSICS

E.B. PODGORSK

Department of Medical Physics,  
McGill University Health Centre,  
Montreal, Quebec, Canada

### 1.1. INTRODUCTION

#### 1.1.1. Fundamental physical constants (rounded off to four significant figures)

- Avogadro's number:  $N_A = 6.022 \times 10^{23}$  atoms/g-atom.
- Avogadro's number:  $N_A = 6.022 \times 10^{23}$  molecules/g-mole.
- Speed of light in vacuum:  $c = 299\,792\,458$  m/s ( $\approx 3 \times 10^8$  m/s).
- Electron charge:  $e = 1.602 \times 10^{-19}$  C.
- Electron rest mass:  $m_{e^-} = 0.5110$  MeV/ $c^2$ .
- Positron rest mass:  $m_{e^+} = 0.5110$  MeV/ $c^2$ .
- Proton rest mass:  $m_p = 938.3$  MeV/ $c^2$ .
- Neutron rest mass:  $m_n = 939.6$  MeV/ $c^2$ .
- Atomic mass unit:  $u = 931.5$  MeV/ $c^2$ .
- Planck's constant:  $h = 6.626 \times 10^{-34}$  J·s.
- Permittivity of vacuum:  $\epsilon_0 = 8.854 \times 10^{-12}$  C/(V·m).
- Permeability of vacuum:  $\mu_0 = 4\pi \times 10^{-7}$  (V·s)/(A·m).
- Newtonian gravitation constant:  $G = 6.672 \times 10^{-11}$  m<sup>3</sup>·kg<sup>-1</sup>·s<sup>-2</sup>.
- Proton mass/electron mass:  $m_p/m_e = 1836.0$ .
- Specific charge of electron:  $e/m_e = 1.758 \times 10^{11}$  C/kg.

#### 1.1.2. Important derived physical constants and relationships

- Speed of light in a vacuum:

$$c = \frac{1}{\sqrt{\epsilon_0 \mu_0}} \approx 3 \times 10^8 \text{ m/s} \quad (1.1)$$

## CHAPTER 1

- Reduced Planck's constant  $\times$  speed of light in a vacuum:

$$\hbar c = \frac{h}{2\pi} c = 197.3 \text{ MeV} \cdot \text{fm} \approx 200 \text{ MeV} \cdot \text{fm} \quad (1.2)$$

- Fine structure constant:

$$\alpha = \frac{e^2}{4\pi\epsilon_0} \frac{1}{\hbar c} = \frac{1}{137} \quad (1.3)$$

- Bohr radius:

$$a_0 = \frac{\hbar c}{\alpha m_e c^2} = \frac{4\pi\epsilon_0 (\hbar c)^2}{e^2 m_e c^2} = 0.5292 \text{ \AA} \quad (1.4)$$

- Rydberg energy:

$$E_R = \frac{1}{2} m_e c^2 \alpha^2 = \frac{1}{2} \left( \frac{e^2}{4\pi\epsilon_0} \right)^2 \frac{m_e c^2}{(\hbar c)^2} = 13.61 \text{ eV} \quad (1.5)$$

- Rydberg constant:

$$R_\infty = \frac{E_R}{2\pi\hbar c} = \frac{m_e c^2 \alpha^2}{4\pi\hbar c} = \frac{1}{4\pi} \left( \frac{e^2}{4\pi\epsilon_0} \right)^2 \frac{m_e c^2}{(\hbar c)^3} = 109\,737 \text{ cm}^{-1} \quad (1.6)$$

- Classical electron radius:

$$r_e = \frac{e^2}{4\pi\epsilon_0 m_e c^2} = 2.818 \text{ fm} \quad (1.7)$$

- Compton wavelength of the electron:

$$\lambda_C = \frac{h}{m_e c} = 0.0243 \text{ \AA} \quad (1.8)$$

## BASIC RADIATION PHYSICS

### 1.1.3. Physical quantities and units

- Physical quantities are characterized by their numerical value (magnitude) and associated unit.
- Symbols for physical quantities are set in italic type, while symbols for units are set in roman type (e.g.  $m = 21$  kg;  $E = 15$  MeV).
- The numerical value and the unit of a physical quantity must be separated by a space (e.g. 21 kg and not 21kg; 15 MeV and not 15MeV).
- The currently used metric system of units is known as the *Système international d'unités* (International System of Units), with the international abbreviation SI. The system is founded on base units for seven basic physical quantities:

Length  $l$ : metre (m).

Mass  $m$ : kilogram (kg).

Time  $t$ : second (s).

Electric current  $I$ : ampere (A).

Temperature  $T$ : kelvin (K).

Amount of substance: mole (mol).

Luminous intensity: candela (cd).

All other quantities and units are derived from the seven base quantities and units (see Table 1.1).

TABLE 1.1. THE BASIC AND SEVERAL DERIVED PHYSICAL QUANTITIES AND THEIR UNITS IN THE INTERNATIONAL SYSTEM OF UNITS AND IN RADIATION PHYSICS

Physical quantity	Symbol	Unit in SI	Units used in radiation physics	Conversion
Length	$l$	m	nm, Å, fm	$1 \text{ m} = 10^9 \text{ nm} = 10^{10} \text{ Å} = 10^{15} \text{ fm}$
Mass	$m$	kg	MeV/ $c^2$	$1 \text{ MeV}/c^2 = 1.78 \times 10^{-30} \text{ kg}$
Time	$t$	s	ms, $\mu$ s, ns, ps	$1 \text{ s} = 10^3 \text{ ms} = 10^6 \mu\text{s} = 10^9 \text{ ns} = 10^{12} \text{ ps}$
Current	$I$	A	mA, $\mu$ A, nA, pA	$1 \text{ A} = 10^3 \text{ mA} = 10^6 \mu\text{A} = 10^9 \text{ nA}$
Charge	$Q$	C	$e$	$1 e = 1.602 \times 10^{-19} \text{ C}$
Force	$F$	N		$1 \text{ N} = 1 \text{ kg}\cdot\text{m}\cdot\text{s}^{-2}$
Momentum	$p$	N·s		$1 \text{ N}\cdot\text{s} = 1 \text{ kg}\cdot\text{m}\cdot\text{s}^{-1}$
Energy	$E$	J	eV, keV, MeV	$1 \text{ eV} = 1.602 \times 10^{-19} \text{ J} = 10^{-3} \text{ keV}$

**1.1.4. Classification of forces in nature**

There are four distinct forces observed in the interaction between various types of particle (see Table 1.2). These forces, listed in decreasing order of strength, are the strong force, electromagnetic (EM) force, weak force and gravitational force, with relative strengths of 1, 1/137,  $10^{-6}$  and  $10^{-39}$ , respectively.

- The ranges of the EM and gravitational forces are infinite ( $1/r^2$  dependence, where  $r$  is the separation between two interacting particles);
- The ranges of the strong and weak forces are extremely short (of the order of a few femtometres).

Each force results from a particular intrinsic property of the particles, such as:

- Strong charge for the strong force transmitted by massless particles called gluons;
- Electric charge for the EM force transmitted by photons;
- Weak charge for the weak force transmitted by particles called W and  $Z^0$ ;
- Energy for the gravitational force transmitted by hypothetical particles called gravitons.

**1.1.5. Classification of fundamental particles**

Two classes of fundamental particle are known: quarks and leptons.

- Quarks are particles that exhibit strong interactions. They are constituents of hadrons (protons and neutrons) with a fractional electric charge ( $2/3$  or  $-1/3$ ) and are characterized by one of three types of strong charge called colour: red, blue and green. There are six known quarks: up, down, strange, charm, top and bottom.

TABLE 1.2. THE FOUR FUNDAMENTAL FORCES IN NATURE

Force	Source	Transmitted particle	Relative strength
Strong	Strong charge	Gluon	1
EM	Electric charge	Photon	1/137
Weak	Weak charge	W and $Z^0$	$10^{-6}$
Gravitational	Energy	Graviton	$10^{-39}$

## BASIC RADIATION PHYSICS

- Leptons are particles that do not interact strongly. Electrons ( $e$ ), muons ( $\mu$ ), taus ( $\tau$ ) and their corresponding neutrinos ( $\nu_e, \nu_\mu, \nu_\tau$ ) are in this category.

### 1.1.6. Classification of radiation

As shown in Fig. 1.1, radiation is classified into two main categories, non-ionizing and ionizing, depending on its ability to ionize matter. The ionization potential of atoms (i.e. the minimum energy required to ionize an atom) ranges from a few electronvolts for alkali elements to 24.5 eV for helium (noble gas).

- Non-ionizing radiation (cannot ionize matter).
- Ionizing radiation (can ionize matter either directly or indirectly):
  - Directly ionizing radiation (charged particles): electrons, protons,  $\alpha$  particles and heavy ions.
  - Indirectly ionizing radiation (neutral particles): photons (X rays and  $\gamma$  rays), neutrons.

Directly ionizing radiation deposits energy in the medium through direct Coulomb interactions between the directly ionizing charged particle and orbital electrons of atoms in the medium.

Indirectly ionizing radiation (photons or neutrons) deposits energy in the medium through a two step process:

- In the first step a charged particle is released in the medium (photons release electrons or positrons, neutrons release protons or heavier ions);
- In the second step the released charged particles deposit energy to the medium through direct Coulomb interactions with orbital electrons of the atoms in the medium.

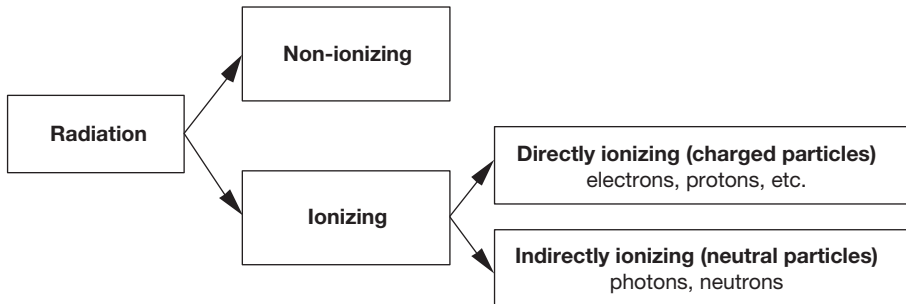


FIG. 1.1. Classification of radiation.

## CHAPTER 1

Both directly and indirectly ionizing radiations are used in the treatment of disease, mainly but not exclusively for malignant disease. The branch of medicine that uses radiation in the treatment of disease is called radiotherapy, therapeutic radiology or radiation oncology. Diagnostic radiology and nuclear medicine are branches of medicine that use ionizing radiation in the diagnosis of disease.

### 1.1.7. Classification of ionizing photon radiation

- Characteristic X rays: resulting from electron transitions between atomic shells.
- Bremsstrahlung: resulting from electron–nucleus Coulomb interactions.
- $\gamma$  rays: resulting from nuclear transitions.
- Annihilation quanta: resulting from positron–electron annihilation.

### 1.1.8. Einstein's relativistic mass, energy and momentum relationships

$$m(v) = \frac{m_0}{\sqrt{1 - \left(\frac{v}{c}\right)^2}} = \frac{m_0}{\sqrt{1 - \beta^2}} = \gamma m_0 \quad (1.9)$$

$$E = m(v)c^2 \quad (1.10)$$

$$E_0 = m_0c^2 \quad (1.11)$$

$$E_K = E - E_0 = (\gamma - 1)E_0 \quad (1.12)$$

$$E^2 = E_0^2 + p^2c^2 \quad (1.13)$$

where

- $v$  is the particle velocity;  
 $c$  is the speed of light in a vacuum;  
 $\beta$  is the normalized particle velocity (i.e.  $\beta = v/c$ );  
 $m(v)$  is the particle mass at velocity  $v$ ;  
 $m_0$  is the particle rest mass (at velocity  $v = 0$ );  
 $E$  is the total energy of the particle;  
 $E_0$  is the rest energy of the particle;  
 $E_K$  is the kinetic energy of the particle;  
 $p$  is the momentum of the particle.

## BASIC RADIATION PHYSICS

- For photons,  $E = h\nu$  and  $E_0 = 0$ ; thus using Eq. (1.13) we obtain  $p = h\nu/c = h/\lambda$ , where  $\nu$  and  $\lambda$  are the photon frequency and wavelength, respectively.

### 1.1.9. Radiation quantities and units

The most important radiation quantities and their units are listed in Table 1.3. Also listed are the definitions of the various quantities and the relationships between the old and the SI units for these quantities.

## 1.2. ATOMIC AND NUCLEAR STRUCTURE

### 1.2.1. Basic definitions for atomic structure

The constituent particles forming an atom are protons, neutrons and electrons. Protons and neutrons are known as nucleons and form the nucleus of the atom.

- Atomic number  $Z$ : number of protons and number of electrons in an atom.
- Atomic mass number  $A$ : number of nucleons in an atom (i.e. number of protons  $Z$  plus number of neutrons  $N$  in an atom:  $A = Z + N$ ).
- There is no basic relation between  $A$  and  $Z$ , but the empirical relationship

$$Z = \frac{A}{1.98 + 0.0155A^{2/3}} \quad (1.14)$$

furnishes a good approximation for stable nuclei.

- Atomic mass  $M$ : expressed in atomic mass units  $u$ , where  $1 u$  is equal to  $1/12$  of the mass of the  $^{12}\text{C}$  atom or  $931.5 \text{ MeV}/c^2$ . The atomic mass  $M$  is smaller than the sum of the individual masses of constituent particles because of the intrinsic energy associated with binding the particles (nucleons) within the nucleus.
- Atomic g-atom (gram-atom): number of grams that correspond to  $N_A$  atoms of an element, where  $N_A = 6.022 \times 10^{23}$  atoms/g-atom (Avogadro's number). The atomic mass numbers of all elements are defined such that  $A$  grams of every element contain exactly  $N_A$  atoms. For example: 1 g-atom of  $^{60}\text{Co}$  is 60 g of  $^{60}\text{Co}$ . In 60 g of  $^{60}\text{Co}$  (1 g-atom) there is Avogadro's number of  $^{60}\text{Co}$  atoms.

**CHAPTER 1**

**TABLE 1.3. RADIATION QUANTITIES, UNITS AND CONVERSION BETWEEN OLD AND SI UNITS**

Quantity	Definition	SI unit	Old unit	Conversion
Exposure ( $X$ )	$X = \frac{\Delta Q}{\Delta m_{\text{air}}}$	$2.58 \times \frac{10^{-4} \text{C}}{\text{kg air}}$	$R = \frac{1 \text{ esu}}{\text{cm}^3 \text{ air}_{\text{STP}}}$	$1 R = 2.58 \times \frac{10^{-4} \text{C}}{\text{kg air}}$
Dose ( $D$ )	$D = \frac{\Delta E_{\text{ab}}}{\Delta m}$	$1 \text{ Gy} = 1 \frac{\text{J}}{\text{kg}}$	$1 \text{ rad} = 100 \frac{\text{erg}}{\text{g}}$	$1 \text{ Gy} = 100 \text{ rad}$
Equivalent dose ( $H$ )	$H = D w_{\text{R}}$	$1 \text{ Sv}$	$1 \text{ rem}$	$1 \text{ Sv} = 100 \text{ rem}$
Activity ( $\mathcal{A}$ )	$\mathcal{A} = \lambda N$	$1 \text{ Bq} = 1 \text{ s}^{-1}$	$1 \text{ Ci} = 3.7 \times 10^{10} \text{ s}^{-1}$	$1 \text{ Bq} = \frac{1 \text{ Ci}}{3.7 \times 10^{10}}$

$\Delta Q$  is the charge of either sign collected;

$\Delta m_{\text{air}}$  is the mass of air;

$\Delta E_{\text{ab}}$  is the absorbed energy;

$\Delta m$  is the mass of medium;

$w_{\text{R}}$  is the radiation weighing factor;

$\lambda$  is the decay constant;

$N$  is the number of radioactive atoms;

R stands for roentgen;

Gy stands for gray;

Sv stands for sievert;

Bq stands for becquerel;

Ci stands for curie;

STP stands for standard temperature (273.2 K) and standard pressure (101.3 kPa).

- Number of atoms  $N_{\text{a}}$  per mass of an element:

$$\frac{N_{\text{a}}}{m} = \frac{N_{\text{A}}}{A}$$

- Number of electrons per volume of an element:

$$Z \frac{N_{\text{a}}}{V} = \rho Z \frac{N_{\text{a}}}{m} = \rho Z \frac{N_{\text{A}}}{A}$$



## BASIC RADIATION PHYSICS

- Number of electrons per mass of an element:

$$Z \frac{N_a}{m} = \frac{Z}{A} N_A$$

Note that  $(Z/A) \approx 0.5$  for all elements, with the one notable exception of hydrogen, for which  $(Z/A) = 1$ . Actually,  $(Z/A)$  slowly decreases from 0.5 for low  $Z$  elements to 0.4 for high  $Z$  elements.

- In nuclear physics the convention is to designate a nucleus  $X$  as  ${}^A_ZX$ , where  $A$  is the atomic mass number and  $Z$  is the atomic number; for example, the  ${}^{60}\text{Co}$  nucleus is identified as  ${}^{60}_{27}\text{Co}$ , the  ${}^{226}\text{Ra}$  nucleus as  ${}^{226}_{88}\text{Ra}$ .
- In ion physics the convention is to designate ions with + or – superscripts. For example,  ${}^4_2\text{He}^+$  stands for a singly ionized  ${}^4\text{He}$  atom and  ${}^4_2\text{He}^{2+}$  stands for a doubly ionized  ${}^4\text{He}$  atom, which is the  $\alpha$  particle.
- If we assume that the mass of a molecule is equal to the sum of the masses of the atoms that make up the molecule, then for any molecular compound there are  $N_A$  molecules per g-mole of the compound, where the g-mole (gram-mole or mole) in grams is defined as the sum of the atomic mass numbers of the atoms making up the molecule; for example, a g-mole of water is 18 g of water and a g-mole of  $\text{CO}_2$  is 44 g of  $\text{CO}_2$ . Thus 18 g of water or 44 g of carbon dioxide contain exactly  $N_A$  molecules (or  $3N_A$  atoms, since each molecule of water and carbon dioxide contains three atoms).

### 1.2.2. Rutherford's model of the atom

The model is based on the results of an experiment carried out by Geiger and Marsden in 1909 with  $\alpha$  particles scattered on thin gold foils. The experiment tested the validity of the Thomson atomic model, which postulated that the positive charges and negative electrons were uniformly distributed over the spherical atomic volume, the radius of which was of the order of a few ångström. Theoretical calculations predict that the probability for an  $\alpha$  particle to be scattered on such an atom with a scattering angle exceeding  $90^\circ$  is of the order of  $10^{-3500}$ , while the Geiger–Marsden experiment showed that approximately 1 in  $10^4$   $\alpha$  particles was scattered with a scattering angle  $\theta > 90^\circ$  (probability  $10^{-4}$ ).

From the findings of the Geiger–Marsden experiment, Rutherford in 1911 concluded that the positive charge and most of the mass of the atom are concentrated in the atomic nucleus (diameter a few femtometres) and negative electrons are smeared over on the periphery of the atom (diameter a few ångströms).

In  $\alpha$  particle scattering the positively charged  $\alpha$  particle has a repulsive Coulomb interaction with the more massive and positively charged nucleus.

## CHAPTER 1

The interaction produces a hyperbolic trajectory of the  $\alpha$  particle, and the scattering angle  $\theta$  is a function of the impact parameter  $b$ . The limiting case is a direct hit with  $b = 0$  and  $\theta = \pi$  (backscattering) that, assuming conservation of energy, determines the distance of closest approach  $D_{\alpha-N}$  in the backscattering interaction:

$$E_K(\alpha) = \frac{z_\alpha Z_N e^2}{4\pi\epsilon_0 D_{\alpha-N}} \quad \Rightarrow \quad D_{\alpha-N} = \frac{z_\alpha Z_N e^2}{4\pi\epsilon_0 E_K(\alpha)} \quad (1.15)$$

where

$z_\alpha$  is the atomic number of the  $\alpha$  particle;  
 $Z_N$  is the atomic number of the scattering material;  
 $E_K(\alpha)$  is the initial kinetic energy of the  $\alpha$  particle.

The repulsive Coulomb force between the  $\alpha$  particle (charge  $+2e$ ) and the nucleus (charge  $+Ze$ ) is governed by  $1/r^2$  as follows:

$$F_{\text{Coul}} = \frac{2Ze^2}{4\pi\epsilon_0 r^2} \quad (1.16)$$

resulting in the following  $b$  versus  $\theta$  relationship:

$$b = \frac{1}{2} D_{\alpha-N} \cot \frac{\theta}{2} \quad (1.17)$$

The differential Rutherford scattering cross-section is then expressed as follows:

$$\left( \frac{d\sigma}{d\Omega} \right)_R = \left( \frac{D_{\alpha-N}}{4} \right)^2 \frac{1}{\sin^4(\theta/2)} \quad (1.18)$$

### 1.2.3. Bohr's model of the hydrogen atom

Bohr expanded Rutherford's atomic model in 1913 and based it on four postulates that combine classical, non-relativistic mechanics with the concept of angular momentum quantization. Bohr's model successfully deals with one-electron entities such as the hydrogen atom, singly ionized helium atom, doubly ionized lithium atom, etc.

## BASIC RADIATION PHYSICS

The four Bohr postulates are as follows:

- Postulate 1: Electrons revolve about the Rutherford nucleus in well defined, allowed orbits (shells). The Coulomb force of attraction  $F_{\text{Coul}} = Ze^2/(4\pi\epsilon_0 r^2)$  between the negative electrons and the positively charged nucleus is balanced by the centrifugal force  $F_{\text{cent}} = m_e v^2/r$ , where  $Z$  is the number of protons in the nucleus (atomic number),  $r$  is the radius of the orbit,  $m_e$  is the electron mass and  $v$  is the velocity of the electron in the orbit.
- Postulate 2: While in orbit, the electron does not lose any energy despite being constantly accelerated (this postulate is in contravention of the basic law of nature, which is that an accelerated charged particle will lose part of its energy in the form of radiation).
- Postulate 3: The angular momentum  $L = m_e v r$  of the electron in an allowed orbit is quantized and given as  $L = n\hbar$ , where  $n$  is an integer referred to as the principal quantum number and  $\hbar = h/(2\pi)$ , where  $h$  is Planck's constant. The simple quantization of angular momentum stipulates that the angular momentum can have only integral multiples of a basic value ( $\hbar$ ).
- Postulate 4: An atom or ion emits radiation when an electron makes a transition from an initial orbit with quantum number  $n_i$  to a final orbit with quantum number  $n_f$  for  $n_i > n_f$ .

The radius  $r_n$  of a one-electron Bohr atom is given by:

$$r_n = a_0 \left( \frac{n^2}{Z} \right) = 0.529 \text{ \AA} \left( \frac{n^2}{Z} \right) \quad (1.19)$$

where  $a_0$  is the Bohr radius ( $a_0 = 0.529 \text{ \AA}$ ).

The velocity  $v_n$  of the electron in a one-electron Bohr atom is:

$$v_n = \alpha c \left( \frac{Z}{n} \right) = \frac{c}{137} \left( \frac{Z}{n} \right) \quad (1.20)$$

where  $\alpha$  is the fine structure constant ( $\alpha = 1/137$ ).

The energy levels for orbital electron shells in mono-electronic atoms (e.g. hydrogen, singly ionized helium and doubly ionized lithium) are given by:

$$E_n = -E_R \left( \frac{Z}{n} \right)^2 = -13.6 \text{ eV} \left( \frac{Z}{n} \right)^2 \quad (1.21)$$

## CHAPTER 1

where

- $E_R$  is the Rydberg energy (13.61 eV);  
 $n$  is the principal quantum number ( $n = 1$ , ground state;  $n > 1$ , excited state);  
 $Z$  is the atomic number ( $Z = 1$  for a hydrogen atom,  $Z = 2$  for singly ionized helium,  $Z = 3$  for doubly ionized lithium, etc.).

The wave number  $k$  of the emitted photon is given by:

$$k = \frac{1}{\lambda} = R_\infty Z^2 \left( \frac{1}{n_f^2} - \frac{1}{n_i^2} \right) = 109\,737 \text{ cm}^{-1} Z^2 \left( \frac{1}{n_f^2} - \frac{1}{n_i^2} \right) \quad (1.22)$$

where  $R_\infty$  is the Rydberg constant.

Bohr's model results in the energy level diagram for the hydrogen atom shown in Fig. 1.2.

### 1.2.4. Multielectron atoms

For multielectron atoms the fundamental concepts of the Bohr atomic theory provide qualitative data for orbital electron binding energies and electron transitions resulting in emission of photons. Electrons occupy allowed shells, but the number of electrons per shell is limited to  $2n^2$ , where  $n$  is the shell number (the principal quantum number).

- The K shell binding energies  $E_B(\text{K})$  for atoms with  $Z > 20$  may be estimated with the following relationship:

$$E_B(\text{K}) = E_R Z_{\text{eff}}^2 = E_R (Z - s)^2 = E_R (Z - 2)^2 \quad (1.23)$$

where  $Z_{\text{eff}}$ , the effective atomic number, is given by  $Z_{\text{eff}} = Z - s$ , where  $s$  is the screening constant equal to 2 for K shell electrons.

- Excitation of an atom occurs when an electron is moved from a given shell to a higher  $n$  shell that is either empty or does not contain a full complement of electrons.
- Ionization of an atom occurs when an electron is removed from the atom (i.e. the electron is supplied with enough energy to overcome its binding energy in a shell).
- Excitation and ionization processes occur in an atom through various possible interactions in which orbital electrons are supplied with a given amount of energy. Some of these interactions are: (i) Coulomb

## BASIC RADIATION PHYSICS

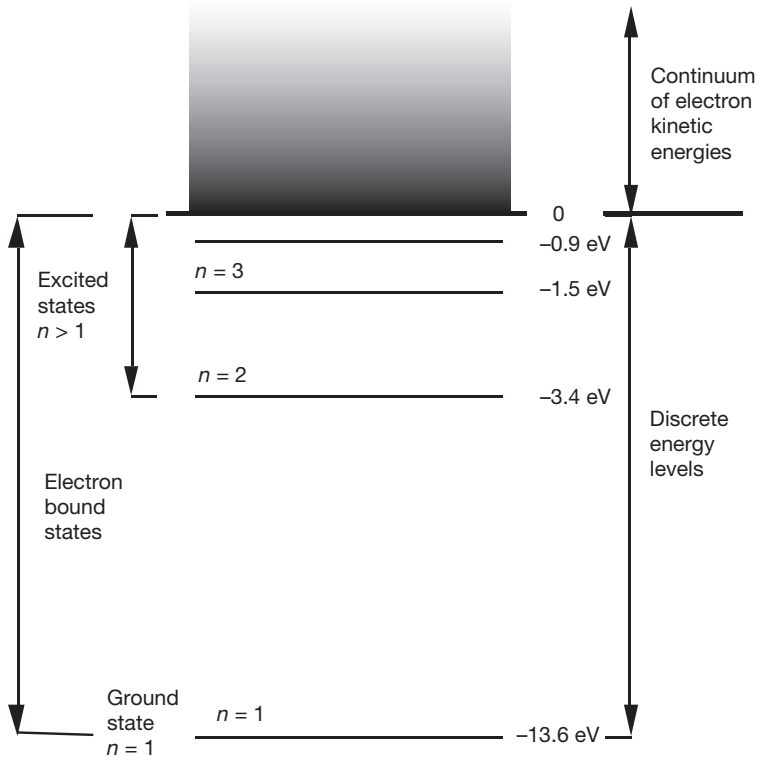


FIG. 1.2. Energy level diagram for a hydrogen atom (ground state:  $n = 1$ , excited states:  $n > 1$ ).

interaction with a charged particle; (ii) the photoelectric effect; (iii) the Compton effect; (iv) triplet production; (v) internal conversion; (vi) electron capture; (vii) the Auger effect; and (viii) positron annihilation.

- An orbital electron from a higher  $n$  shell will fill an electron vacancy in a lower  $n$  atomic shell. The energy difference between the two shells will be either emitted in the form of a characteristic photon or it will be transferred to a higher  $n$  shell electron, which will be ejected from the atom as an Auger electron.
- Energy level diagrams of multielectron atoms resemble those of one-electron structures, except that inner shell electrons are bound with much larger energies, as shown for a lead atom in Fig. 1.3.
- The number of characteristic photons (sometimes called fluorescent photons) emitted per orbital electron shell vacancy is referred to as fluorescent yield  $\omega$ , while the number of Auger electrons emitted per orbital

## CHAPTER 1

electron vacancy is equal to  $(1 - \omega)$ . The fluorescent yield depends on the atomic number  $Z$  of the atom and on the principal quantum number of a shell. For atoms with  $Z < 10$  the fluorescent yield  $\omega_K = 0$ ; for  $Z \approx 30$  the fluorescent yield  $\omega_K \approx 0.5$ ; and for high atomic number atoms  $\omega_K = 0.96$ , where  $\omega_K$  refers to the fluorescent yield for the K shell (see Fig. 1.9).

### 1.2.5. Nuclear structure

Most of the atomic mass is concentrated in the atomic nucleus consisting of  $Z$  protons and  $(A - Z)$  neutrons, where  $Z$  is the atomic number and  $A$  is the atomic mass number of a given nucleus.

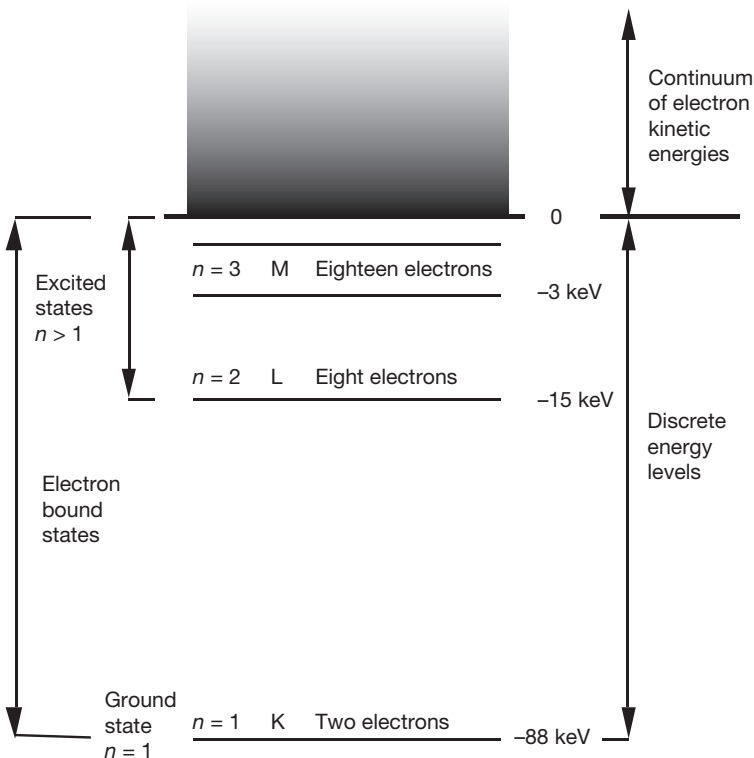


FIG. 1.3. Energy level diagram for a multielectron atom (lead). The  $n = 1, 2, 3, 4 \dots$  shells are referred to as the K, L, M, O... shells, respectively. Electronic transitions that end in low  $n$  shells are referred to as X ray transitions because the resulting photons are in the X ray energy range. Electronic transitions that end in high  $n$  shells are referred to as optical transitions because they result in ultraviolet, visible or infrared photons.

## BASIC RADIATION PHYSICS

- The radius  $r$  of the nucleus is estimated from:

$$r = r_0 \sqrt[3]{A} \quad (1.24)$$

where  $r_0$  is a constant ( $\sim 1.4$  fm) assumed equal to  $\frac{1}{2}$  of  $r_e$ , the classical electron radius.

- Protons and neutrons are commonly referred to as nucleons and are bound in the nucleus with the strong force. In contrast to electrostatic and gravitational forces, which are inversely proportional to the square of the distance between two particles, the strong force between two nucleons is a very short range force, active only at distances of the order of a few femtometres. At these short distances the strong force is the predominant force, exceeding other forces by several orders of magnitude.
- The binding energy  $E_B$  per nucleon in a nucleus varies slowly with the number of nucleons  $A$ , is of the order of  $\sim 8$  MeV/nucleon and exhibits a broad maximum of 8.7 MeV/nucleon at  $A \approx 60$ . For a given nucleus it may be calculated from the energy equivalent of the mass deficit  $\Delta m$  as follows:

$$\frac{E_B}{\text{nucleon}} = \Delta mc^2/A = [Zm_p c^2 + (A - Z)m_n c^2 - Mc^2]/A \quad (1.25)$$

where

$M$  is the nuclear mass in atomic mass units  $u$  (note that  $uc^2 = 931.5$  MeV);

$m_p c^2$  is the proton rest energy;

$m_n c^2$  is the neutron rest energy.

### 1.2.6. Nuclear reactions

Much of the present knowledge of the structure of nuclei comes from experiments in which a particular nuclide  $A$  is bombarded with a projectile  $a$ . The projectile undergoes one of three possible interactions: (i) elastic scattering (no energy transfer occurs; however, the projectile changes trajectory); (ii) inelastic scattering (the projectile enters the nucleus and is re-emitted with less energy and in a different direction); or (iii) nuclear reaction (the projectile  $a$  enters the nucleus  $A$ , which is transformed into nucleus  $B$  and a different particle  $b$  is emitted).

## CHAPTER 1

- Nuclear reactions are designated as follows:



- A number of physical quantities are rigorously conserved in all nuclear reactions. The most important of these quantities are charge, mass number, linear momentum and mass–energy.
- The threshold energy for a nuclear reaction is defined as the smallest value of a projectile's kinetic energy at which a nuclear reaction can take place. The threshold kinetic energy  $E_K^{\text{thr}}(a)$  of projectile  $a$  is derived from relativistic conservation of energy and momentum as:

$$E_K^{\text{thr}}(a) = \frac{(m_B c^2 + m_b c^2)^2 - (m_A c^2 + m_a c^2)^2}{2m_A c^2} \quad (1.27)$$

where  $m_A$ ,  $m_a$ ,  $m_B$  and  $m_b$  are the rest masses of the target  $A$ , projectile  $a$  and products  $B$  and  $b$ , respectively.

### 1.2.7. Radioactivity

Radioactivity is characterized by a transformation of an unstable nucleus into a more stable entity that may be unstable and will decay further through a chain of decays until a stable nuclear configuration is reached. The exponential laws that govern the decay and growth of radioactive substances were first formulated by Rutherford and Soddy in 1902 and then refined by Bateman in 1910.

- The activity  $\mathcal{A}(t)$  of a radioactive substance at time  $t$  is defined as the product of the decay constant  $\lambda$  and the number of radioactive nuclei  $N(t)$ :

$$\mathcal{A}(t) = \lambda N(t) \quad (1.28)$$

- The simplest radioactive decay is characterized by a radioactive parent nucleus  $P$  decaying with a decay constant  $\lambda_p$  into a stable daughter nucleus  $D$ :



– The number of radioactive parent nuclei  $N_p(t)$  as a function of time  $t$  is governed by the following relationship:



## BASIC RADIATION PHYSICS

$$N_p(t) = N_p(0)e^{-\lambda_p t} \quad (1.30)$$

where  $N_p(0)$  is the initial number of parent nuclei at time  $t = 0$ .

– Similarly, the activity of parent nuclei  $\mathcal{A}_p(t)$  at time  $t$  is given as:

$$\mathcal{A}_p(t) = \mathcal{A}_p(0)e^{-\lambda_p t} \quad (1.31)$$

where  $\mathcal{A}_p(0)$  is the initial activity of parent nuclei at time  $t = 0$ .

- The half-life  $t_{1/2}$  of a radioactive substance is the time during which the number of radioactive nuclei decays to half of the initial value  $N_p(0)$  present at time  $t = 0$ :

$$N_p(t = t_{1/2}) = (1/2)N_p(0) = N_p(0)e^{-\lambda_p t_{1/2}} \quad (1.32)$$

- The decay constant  $\lambda_p$  and half-life  $(t_{1/2})_p$  for the parent are thus related as follows:

$$\lambda_p = \frac{\ln 2}{t_{1/2}} \quad (1.33)$$

- The specific activity  $a$  is defined as the parent's activity per unit mass:

$$a = \frac{\mathcal{A}_p}{m} = \frac{\lambda_p N}{m} = \lambda_p \frac{N_A}{A_p} = \frac{N_A \ln 2}{A_p (t_{1/2})_p} \quad (1.34)$$

where  $N_A$  is Avogadro's number and  $A_p$  is the parent's atomic mass number.

- The average (mean) life  $\tau_p$  of a radioactive substance represents the average life expectancy of all parent radioactive atoms in the substance at time  $t = 0$ :

$$\mathcal{A}_p(0)\tau_p = \int_0^{\infty} \mathcal{A}_p(0)e^{-\lambda_p t} dt = \frac{\mathcal{A}_p(0)}{\lambda_p} \quad (1.35)$$

- The decay constant  $\lambda_p$  and average life  $\tau_p$  are thus related as follows:

$$\lambda_p = 1/\tau_p \quad (1.36)$$

resulting in the following relationship between  $(t_{1/2})_p$  and  $\tau_p$ :

## CHAPTER 1

$$(t_{1/2})_P = \tau_P \ln 2 \quad (1.37)$$

- A more complicated radioactive decay occurs when a radioactive parent nucleus P decays with a decay constant  $\lambda_P$  into a daughter nucleus D which in turn is radioactive and decays with a decay constant  $\lambda_D$  into a stable granddaughter G:



– The activity of the daughter  $\mathcal{A}_D(t)$  may then be expressed as:

$$\mathcal{A}_D(t) = \frac{\lambda_D}{\lambda_D - \lambda_P} \mathcal{A}_P(0)(e^{-\lambda_P t} - e^{-\lambda_D t}) \quad (1.39)$$

where  $\mathcal{A}_P(0)$  is the initial activity of the parent nuclei present at time  $t = 0$  (i.e.  $\mathcal{A}_P(0) = \lambda_P N_P(0)$ , where  $N_P(0)$  is the number of parent nuclei at  $t = 0$ ).

– The maximum activity of daughter nuclei occurs at time  $t_{\max}$  given by:

$$t_{\max} = \frac{\ln(\lambda_D/\lambda_P)}{\lambda_D - \lambda_P} \quad (1.40)$$

under the condition that  $N_D = 0$  at time  $t = 0$ .

- Special considerations in parent  $\rightarrow$  daughter  $\rightarrow$  granddaughter relationships:

– For  $\lambda_D < \lambda_P$  or  $(t_{1/2})_D > (t_{1/2})_P$  we obtain the following general relationship:

$$\frac{\mathcal{A}_D}{\mathcal{A}_P} = \frac{\lambda_D}{\lambda_D - \lambda_P} [1 - e^{-(\lambda_D - \lambda_P)t}] \quad (1.41)$$

– For  $\lambda_D > \lambda_P$  or  $(t_{1/2})_D < (t_{1/2})_P$  we obtain transient equilibrium with:

$$\frac{\mathcal{A}_D}{\mathcal{A}_P} = \frac{\lambda_D}{\lambda_D - \lambda_P} \quad \text{for } t \gg t_{\max} \quad (1.42)$$

– For  $\lambda_D \gg \lambda_P$  or  $(t_{1/2})_D \ll (t_{1/2})_P$  we obtain secular equilibrium and

$$\mathcal{A}_D/\mathcal{A}_P \approx 1 \quad (1.43)$$

**1.2.8. Activation of nuclides**

Activation of nuclides occurs when a stable parent isotope P is bombarded with neutrons in a nuclear reactor and transforms into a radioactive daughter D that decays into a granddaughter G:



The probability for activation is determined by the cross-section  $\sigma$  for the nuclear reaction, usually expressed in barns per atom, where 1 barn =  $10^{-24}$  cm<sup>2</sup>.

- Activity of the daughter  $\mathcal{A}_D(t)$  is expressed as:

$$\mathcal{A}_D(t) = \frac{\sigma\phi\lambda_D}{\lambda_D - \sigma\phi} N_P(0)(e^{-\sigma\phi t} - e^{-\lambda_D t}) \quad (1.45)$$

where  $N_P(0)$  is the initial number of parent nuclei.

- This result is similar to the P → D → G relationship above (Eq. (1.39)) in which an unstable parent P decays into an unstable daughter D that in turn decays into granddaughter G. However, the decay constant  $\lambda_p$  in the P → D → G decay relationship is replaced by  $\sigma\phi$ , where  $\sigma$  is the cross-section for activation of the parent nuclei (cm<sup>2</sup>/atom) and  $\phi$  is the fluence rate of neutrons in the reactor (cm<sup>-2</sup>.s<sup>-1</sup>).
- The time  $t_{\max}$  at which the maximum activity  $\mathcal{A}_D$  occurs in the activation process is then, similarly to Eq. (1.40), given by:

$$t_{\max} = \frac{\ln \frac{\lambda_D}{\sigma\phi}}{\lambda_D - \sigma\phi} \quad (1.46)$$

- In situations where  $\sigma\phi \ll \lambda_D$ , the daughter activity relationship of Eq. (1.45) transforms into a simple exponential growth relationship:

$$\mathcal{A}_D(t) = \sigma\phi N_P(0)(1 - e^{-\lambda_D t}) \quad (1.47)$$

- An important example of nuclear activation is the production of the <sup>60</sup>Co isotope by bombarding <sup>59</sup>Co with thermal neutrons in a nuclear reactor:



## CHAPTER 1

or in shorthand notation  ${}^{59}_{27}\text{Co}(n, \gamma){}^{60}_{27}\text{Co}$ , with an activation cross-section  $\sigma$  of  $37 \times 10^{-24} \text{ cm}^2/\text{atom}$  (37 barn/atom with 1 barn =  $10^{-24} \text{ cm}^2$ ) and typical reactor neutron fluence rates  $\phi$  of the order of  $10^{13} \text{ cm}^{-2}\cdot\text{s}^{-1}$ .

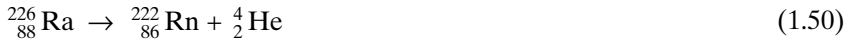
### 1.2.9. Modes of radioactive decay

A radioactive parent X with atomic number  $Z$  and atomic mass number  $A$  decays into a daughter Y through the following possible modes of decay:  $\alpha$ ,  $\beta^-$ ,  $\beta^+$ , electron capture  $\gamma$  and internal conversion.

*$\alpha$  decay:*



where  ${}^4_2\text{He}(\alpha)$  is a  ${}^4\text{He}$  nucleus referred to as an  $\alpha$  particle. An example of  $\alpha$  decay is the decay of  ${}^{226}_{88}\text{Ra}$  into  ${}^{222}_{86}\text{Rn}$  with a half-life of 1600 years:



*$\beta^-$  decay:*



A neutron transforms into a proton, and an electron  $\beta^-$  and antineutrino  $\bar{\nu}_e$ , sharing the available energy, are ejected from the nucleus. An example of  $\beta^-$  decay is the decay of  ${}^{60}\text{Co}$  nuclei into excited  ${}^{60}\text{Ni}$  nuclei with a half-life of 5.26 years:



*$\beta^+$  decay:*



A proton transforms into a neutron, and a positron  $\beta^+$  and neutrino  $\nu_e$ , sharing the available energy, are ejected from the nucleus. An example of  $\beta^+$  decay is the decay of  ${}^{13}\text{N}$  into  ${}^{13}\text{C}$ :



## BASIC RADIATION PHYSICS

*Electron capture:*



The nucleus captures one of its own K shell orbital electrons, a proton transforms into a neutron and a neutrino  $\nu_e$  is ejected. An example of electron capture is the decay of  ${}^{125}\text{I}$  into  ${}^{125}\text{Te}$  in an excited state, which decays to the  ${}^{125}\text{Te}$  ground state through  $\gamma$  decay and internal conversion:



The resulting K shell vacancy is filled with a higher level orbital electron and the transition energy is emitted from the atom in the form of characteristic photons or Auger electrons.

*$\gamma$  decay:*



An excited nucleus  ${}^A_Z\text{X}^*$ , generally produced through  $\beta^-$  or  $\beta^+$  decay, attains its ground state  ${}^A_Z\text{X}$  through emission of one or several  $\gamma$  photons. An example of  $\gamma$  decay is the transition of the excited  ${}^{60}_{28}\text{Ni}^*$ , resulting from the  $\beta^-$  decay of  ${}^{60}\text{Co}$ , into stable  ${}^{60}_{28}\text{Ni}$  through an emission of two  $\gamma$  rays with energies of 1.17 and 1.33 MeV.

*Internal conversion:*



Rather than being emitted as a  $\gamma$  photon, the nuclear excitation energy may be transferred to a K shell orbital electron that is ejected with a kinetic energy equal to the excitation energy less the orbital electron binding energy. The resulting K shell vacancy is filled with a higher level orbital electron and the transition energy is emitted in the form of characteristic photons or Auger electrons. An example of internal conversion is the decay of excited  ${}^{125}\text{Te}$ , which results from an electron capture decay of  ${}^{125}\text{I}$ , into stable  ${}^{125}\text{Te}$  through emission of 35 keV  $\gamma$  rays (7%) and internal conversion electrons (93%).

## 1.3. ELECTRON INTERACTIONS

As an energetic electron traverses matter, it interacts with matter through Coulomb interactions with atomic orbital electrons and atomic nuclei. Through these collisions the electrons may lose their kinetic energy (collision and radiative losses) or change their direction of travel (scattering). Energy losses are described by stopping power; scattering is described by scattering power.

The collisions between the incident electron and an orbital electron or nucleus of an atom may be elastic or inelastic. In an elastic collision the electron is deflected from its original path but no energy loss occurs, while in an inelastic collision the electron is deflected from its original path and some of its energy is transferred to an orbital electron or emitted in the form of bremsstrahlung. Energetic electrons experience thousands of collisions as they traverse an absorber, hence their behaviour is described by a statistical theory of multiple scattering embracing the individual elastic and inelastic collisions with orbital electrons and nuclei.

The type of interaction that the electron undergoes with a particular atom of radius  $a$  depends on the impact parameter  $b$  of the interaction, defined as the perpendicular distance between the electron direction before the interaction and the atomic nucleus (see Fig. 1.4).

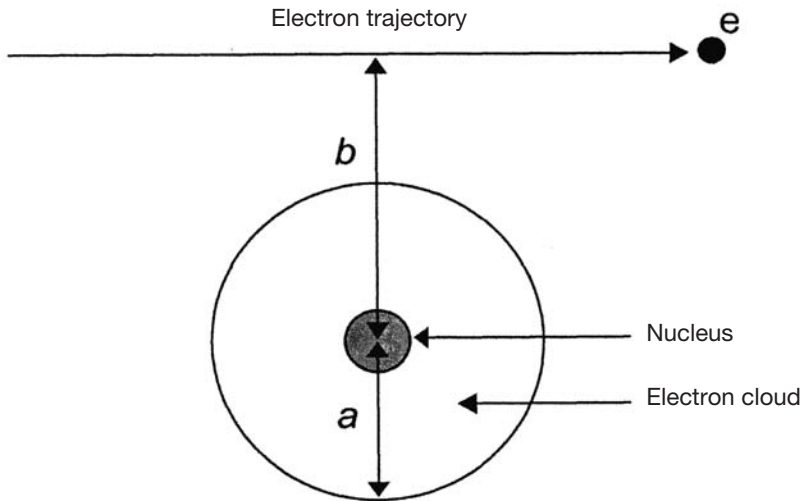


FIG. 1.4. Interaction of an electron with an atom, where  $a$  is the atomic radius and  $b$  is the impact parameter.

## BASIC RADIATION PHYSICS

- For  $b \gg a$  the electron will undergo a soft collision with the whole atom and only a small amount of energy will be transferred from the incident electron to orbital electrons.
- For  $b \approx a$  the electron will undergo a hard collision with an orbital electron and an appreciable fraction of the electron's kinetic energy will be transferred to the orbital electron.
- For  $b \ll a$  the incident electron undergoes a radiative interaction (collision) with the atomic nucleus. The electron will emit a photon (bremsstrahlung) with energy between zero and the incident electron kinetic energy. The energy of the emitted bremsstrahlung photon depends on the magnitude of the impact parameter  $b$ ; the smaller the impact parameter, the higher the energy of the bremsstrahlung photon.

### 1.3.1. Electron–orbital electron interactions

- Coulomb interactions between the incident electron and orbital electrons of an absorber result in ionizations and excitations of absorber atoms:
  - Ionization: ejection of an orbital electron from the absorber atom;
  - Excitation: transfer of an orbital electron of the absorber atom from an allowed orbit to a higher allowed orbit (shell).
- Atomic excitations and ionizations result in collisional energy losses and are characterized by collision (ionization) stopping powers.

### 1.3.2. Electron–nucleus interactions

- Coulomb interactions between the incident electron and nuclei of the absorber atom result in electron scattering and energy loss of the electron through production of X ray photons (bremsstrahlung). These types of energy loss are characterized by radiative stopping powers.
- Bremsstrahlung production is governed by the Larmor relationship, which states that the power  $P$  emitted in the form of photons from an accelerated charged particle is proportional to the square of the particle acceleration  $a$  and the square of the particle charge  $q$ , or:

$$P = \frac{q^2 a^2}{6\pi\epsilon_0 c^3} \quad (1.59)$$

- The angular distribution of the emitted photons (bremsstrahlung) is proportional to  $\sin^2 \theta / (1 - \beta \cos \theta)^5$ , where  $\theta$  is the angle between the acceleration of the charged particle and a unit vector connecting the charge with the point of observation and  $\beta$  is the standard relativistic  $v/c$ .

- At small velocities  $v$  of the charged particle ( $\beta \rightarrow 0$ ) the angular distribution goes as  $\sin^2 \theta$  and exhibits a maximum at  $\theta = 90^\circ$ . However, as the velocity of the charged particle increases from 0 towards  $c$ , the angular distribution of the emitted photons becomes increasingly more forward peaked.
- The angle at which the photon emission intensity is maximum can be calculated from the following relationship:

$$\theta_{\max} = \arccos \left[ \frac{1}{3\beta} (\sqrt{1+15\beta^2} - 1) \right] \quad (1.60)$$

that for  $\beta \rightarrow 0$  gives  $\theta_{\max} = \pi/2$  and for  $\beta \rightarrow 1$  gives  $\theta_{\max} = 0$ , indicating that in the diagnostic radiology energy range (orthovoltage beams) most X ray photons are emitted at  $90^\circ$  to the electron path, while in the megavoltage range (linac beams) most photons are emitted in the direction of the electron beam striking the target.

- The energy loss by radiation and the radiative yield  $g$  increase directly with the absorber atomic number  $Z$  and the kinetic energy of electrons. The radiation yield for X ray targets in the diagnostic radiology energy range ( $\sim 100$  keV) is of the order of 1%, while in the megavoltage energy range it amounts to 10–20%.

### 1.3.3. Stopping power

The inelastic energy losses by an electron moving through a medium with density  $\rho$  are described by the total mass–energy stopping power  $(S/\rho)_{\text{tot}}$ , which represents the kinetic energy  $E_K$  loss by the electron per unit path length  $x$ , or:

$$(S/\rho)_{\text{tot}} = \frac{1}{\rho} \frac{dE_K}{dx} \quad (\text{MeV} \cdot \text{cm}^2/\text{g}) \quad (1.61)$$

$(S/\rho)_{\text{tot}}$  consists of two components: the mass collision stopping power  $(S/\rho)_{\text{col}}$ , resulting from electron–orbital electron interactions (atomic excitations and ionizations), and the mass radiative stopping power  $(S/\rho)_{\text{rad}}$ , resulting from electron–nucleus interactions (bremsstrahlung production):

$$(S/\rho)_{\text{tot}} = (S/\rho)_{\text{col}} + (S/\rho)_{\text{rad}} \quad (1.62)$$

- $(S/\rho)_{\text{col}}$  has an important role in radiation dosimetry, since the dose  $D$  in the medium may be expressed as:



$$D = \phi(S/\rho)_{\text{col}} \quad (1.63)$$

where  $\phi$  is the fluence of electrons.

- $(S/\rho)_{\text{tot}}$  is used in the calculation of electron range  $R$  as follows:

$$R = \int_0^{E_{\text{ki}}} \left( \frac{S}{\rho} (E_{\text{K}}) \right)_{\text{tot}}^{-1} dE_{\text{K}} \quad (1.64)$$

where  $E_{\text{ki}}$  is the initial kinetic energy of the electron.

- Both  $(S/\rho)_{\text{rad}}$  and  $(S/\rho)_{\text{tot}}$  are used in the determination of radiation yield (also referred to as bremsstrahlung efficiency)  $Y$  as:

$$Y = \frac{1}{E_{\text{Ki}}} \int_0^{E_{\text{ki}}} \frac{(S/\rho)_{\text{rad}}}{(S/\rho)_{\text{tot}}} dE_{\text{K}} \quad (1.65)$$

- The stopping power focuses on the energy loss by an electron moving through a medium. When attention is focused on the absorbing medium, one is interested in the linear rate of energy absorption by the absorbing medium as the electron traverses the medium. The rate of energy absorption, called the linear energy transfer (LET), is defined as the average energy locally imparted to the absorbing medium by an electron of specified energy in traversing a given distance in the medium.
- In radiation dosimetry the concept of restricted stopping power ( $S_{\Delta}/\rho$ ) is introduced, which accounts for that fraction of the collisional stopping power  $(S/\rho)_{\text{col}}$  that includes all the soft collisions plus those hard collisions that result in delta rays with energies less than a cut-off value  $\Delta$ . In radiation dosimetry this cut-off energy is usually taken as 10 keV, an energy that allows an electron just to traverse an ionization chamber gap of 1 mm in air. Delta rays are defined as electrons that acquire sufficiently high kinetic energies through hard collisions so as to enable them to carry this energy a significant distance away from the track of the primary particle and produce their own ionizations of absorber atoms.

#### 1.3.4. Mass scattering power

When a beam of electrons passes through an absorbing medium, the electrons undergo multiple scattering through Coulomb interactions between the incident electrons and nuclei of the absorber. The angular and spatial spread of a pencil electron beam can be approximated by a Gaussian distribution. The multiple scattering of electrons traversing a path length  $l$  through an absorbing medium is commonly described by the mean square angle of

## CHAPTER 1

scattering  $\overline{\theta^2}$  that is proportional to the mass thickness  $\rho l$  of the absorber. Analogously to the definition of stopping power, the International Commission on Radiation Units and Measurements (ICRU) defines the mass scattering power  $T/\rho$  as:

$$\frac{T}{\rho} = \frac{1}{\rho} \frac{d\overline{\theta^2}}{dl} \quad \text{or} \quad \frac{T}{\rho} = \frac{\overline{\theta^2}}{\rho l} \quad (1.66)$$

The scattering power varies approximately as the square of the absorber atomic number and inversely as the square of the electron kinetic energy.

### 1.4. PHOTON INTERACTIONS

#### 1.4.1. Types of indirectly ionizing photon radiation

Depending on their origin, the indirectly ionizing photon radiations fall into one of the following four categories:

- Bremsstrahlung (continuous X rays), emitted through electron–nucleus interactions.
- Characteristic X rays (discrete), emitted in transitions of orbital electrons from one allowed orbit to a vacancy in another allowed orbit.
- $\gamma$  rays (discrete), emitted through nuclear transitions in  $\gamma$  decay.
- Annihilation radiation (discrete, typically 0.511 MeV), emitted through positron–electron annihilation.

#### 1.4.2. Photon beam attenuation

The intensity  $I(x)$  of a narrow monoenergetic photon beam, attenuated by an attenuator of thickness  $x$ , is given as:

$$I(x) = I(0)e^{-\mu(h\nu, Z)x} \quad (1.67)$$

where

$I(0)$  is the original intensity of the unattenuated beam;  
 $\mu(h\nu, Z)$  is the linear attenuation coefficient, which depends on photon energy  $h\nu$  and attenuator atomic number  $Z$ .

## BASIC RADIATION PHYSICS

- The half-value layer (HVL or  $x_{1/2}$ ) is defined as that thickness of the attenuator that attenuates the photon beam intensity to 50% of its original value:

$$x_{1/2} = \text{HVL} = (\ln 2)/\mu \quad (1.68)$$

- Similarly, the tenth-value layer (TVL or  $x_{1/10}$ ) is defined as that thickness of the attenuator that attenuates the photon beam intensity to 10% of its original value:

$$x_{1/10} = \text{TVL} = (\ln 10)/\mu \quad (1.69)$$

- HVL and TVL are thus related as follows:

$$x_{1/10} = x_{1/2} \frac{\ln 10}{\ln 2} = 3.3x_{1/2} \quad (1.70)$$

- The mass attenuation coefficient  $\mu_m$ , atomic attenuation coefficient  ${}_a\mu$  and electronic attenuation coefficient  ${}_e\mu$  are proportional to the linear attenuation coefficient  $\mu$  through the following relationships:

$$\mu = \rho\mu_m = \frac{\rho N_A}{A} {}_a\mu = \frac{\rho N_A Z}{A} {}_e\mu \quad (1.71)$$

where  $\rho$ ,  $Z$  and  $A$  are the density, atomic number and atomic mass number, respectively, of the attenuator.

- Typical units for the linear, mass, atomic and electronic attenuation coefficients are:  $\text{cm}^{-1}$ ,  $\text{cm}^2/\text{g}$ ,  $\text{cm}^2/\text{atom}$  and  $\text{cm}^2/\text{electron}$ , respectively, implying that thickness  $x$  in the exponent ( $-\mu x$ ) must be given in  $\text{cm}$ ,  $\text{g}/\text{cm}^2$ ,  $\text{atoms}/\text{cm}^2$  and  $\text{electrons}/\text{cm}^2$ , respectively.
- For use in radiation dosimetry two additional attenuation coefficients are defined: the energy transfer coefficient  $\mu_{\text{tr}}$  and the energy absorption coefficient  $\mu_{\text{ab}}$  (often designated as  $\mu_{\text{en}}$ ). The two coefficients are related to  $\mu$  as follows:

$$\mu_{\text{tr}} = \mu \frac{\bar{E}_{\text{tr}}}{h\nu} \quad (1.72)$$

and

$$\mu_{\text{ab}} = \mu \frac{\bar{E}_{\text{ab}}}{h\nu} \quad (1.73)$$

where

$\bar{E}_{\text{tr}}$  is the average energy transferred to charged particles (electrons and positrons) in the attenuator;

$\bar{E}_{\text{ab}}$  is the average energy deposited by charged particles in the attenuator.

- The energy transfer coefficient  $\mu_{\text{tr}}$  and the energy absorption coefficient  $\mu_{\text{ab}}$  are related through the radiative fraction  $g$  as follows:

$$\mu_{\text{ab}} = \mu_{\text{tr}}(1 - g) \quad (1.74)$$

### 1.4.3. Types of photon interaction

Photons may undergo various possible interactions with the atoms of an attenuator; the probability or cross-section for each interaction depends on the energy  $h\nu$  of the photon and on the atomic number  $Z$  of the attenuator.

- The photon interactions may be with a tightly bound electron (i.e. with an atom as a whole (photoelectric effect, coherent scattering)), with the field of the nucleus (pair production) or with an essentially free orbital electron (Compton effect, triplet production).
- In the context of photon interactions, a tightly bound electron is an orbital electron with a binding energy of the order of, or slightly larger than, the photon energy, while a free electron is an electron with a binding energy that is much smaller than the photon energy.
- During the interaction the photon may completely disappear (photoelectric effect, pair production, triplet production) or it may be scattered coherently (coherent scattering) or incoherently (Compton effect).

### 1.4.4. Photoelectric effect

In the photoelectric effect (sometimes referred to as the photoeffect) the photon interacts with a tightly bound orbital electron of an attenuator and disappears, while the orbital electron is ejected from the atom as a photoelectron with a kinetic energy  $E_K$  given as:

$$E_K = h\nu - E_B \quad (1.75)$$

where  $h\nu$  is the incident photon energy and  $E_B$  is the binding energy of the electron.

- The atomic attenuation coefficient for the photoelectric effect  ${}_a\tau$  is proportional to  $Z^4/(h\nu)^3$ , while the mass attenuation coefficient for the photoelectric effect  $\tau_m$  is proportional to  $(Z/h\nu)^3$ , where  $Z$  is the atomic number of the attenuator and  $h\nu$  is the photon energy.
- In addition to a steady decrease in  $\tau_m$  with an increasing  $h\nu$ , the plot of  $\tau_m$  versus  $h\nu$  also shows sharp discontinuities in  $\tau_m$  when  $h\nu$  equals the binding energy for a particular electronic shell of the attenuator. These discontinuities, called absorption edges, reflect the fact that for  $h\nu$  less than the binding energy photons cannot undergo the photoelectric effect with electrons in that particular shell, while for  $h\nu$  greater than or equal to the binding energy they can.
- The average energy transferred from the photon with energy  $h\nu > E_B(K)$  to electrons  $(\bar{E}_K)_{tr}^{PE}$  in the photoelectric effect is given as follows:

$$(\bar{E}_K)_{tr}^{PE} = h\nu - P_K \omega_K E_B(K) \quad (1.76)$$

where  $E_B(K)$  is the binding energy of the K shell orbital electron (photoelectron),  $P_K$  is the fraction of all photoelectric effect interactions that occur in the K shell and  $\omega_K$  is the fluorescent yield for the K shell. The range of  $P_K$  is from 1.0 at low atomic numbers  $Z$  to 0.8 at high atomic numbers (see Fig. 1.9).

#### 1.4.5. Coherent (Rayleigh) scattering

In coherent (Rayleigh) scattering the photon interacts with a bound orbital electron (i.e. with the combined action of the whole atom). The event is elastic in the sense that the photon loses essentially none of its energy and is scattered through only a small angle. Since no energy transfer occurs from the photon to charged particles, Rayleigh scattering plays no role in the energy transfer coefficient; however, it contributes to the attenuation coefficient.

- The atomic cross-section for Rayleigh scattering  ${}_a\sigma_R$  is proportional to  $(Z/h\nu)^2$  and the mass attenuation coefficient  $\sigma_R/\rho$  is proportional to  $Z/(h\nu)^2$ .
- In tissue and tissue equivalent materials the relative importance of Rayleigh scattering in comparison with other photon interactions is small, as it contributes only a few per cent or less to the total attenuation coefficient.

**1.4.6. Compton effect (incoherent scattering)**

The Compton effect (incoherent scattering) represents a photon interaction with an essentially ‘free and stationary’ orbital electron. The incident photon energy  $h\nu$  is much larger than the binding energy of the orbital electron. The photon loses part of its energy to the recoil (Compton) electron and is scattered as photon  $h\nu'$  through a scattering angle  $\theta$ , as shown schematically in Fig. 1.5. Angle  $\phi$  represents the angle between the incident photon direction and the direction of the recoil electron.

- The change in photon wavelength  $\Delta\lambda$  is given by the well known Compton relationship:

$$\Delta\lambda = \lambda_C(1 - \cos \theta) \quad (1.77)$$

where  $\lambda_C$  is the Compton wavelength of the electron, expressed as:

$$\lambda_C = \frac{h}{m_e c} = 0.024 \text{ \AA} \quad (1.78)$$

- The relationship for  $\Delta\lambda$  is calculated from equations representing conservation of energy and momentum in the Compton process:

$$h\nu + m_e c^2 = h\nu' + m_e c^2 + E_K \quad (1.79)$$

$$\frac{h\nu}{c} = \frac{h\nu'}{c} \cos \theta + \frac{m_e v}{\sqrt{1 - \left(\frac{v}{c}\right)^2}} \cos \phi \quad (1.80)$$

and

$$0 = \frac{h\nu'}{c} \sin \theta - \frac{m_e v}{\sqrt{1 - \left(\frac{v}{c}\right)^2}} \sin \phi \quad (1.81)$$

where  $\varepsilon$  is the normalized incident photon energy:

$$\varepsilon = \frac{h\nu}{m_e c^2}$$

## BASIC RADIATION PHYSICS

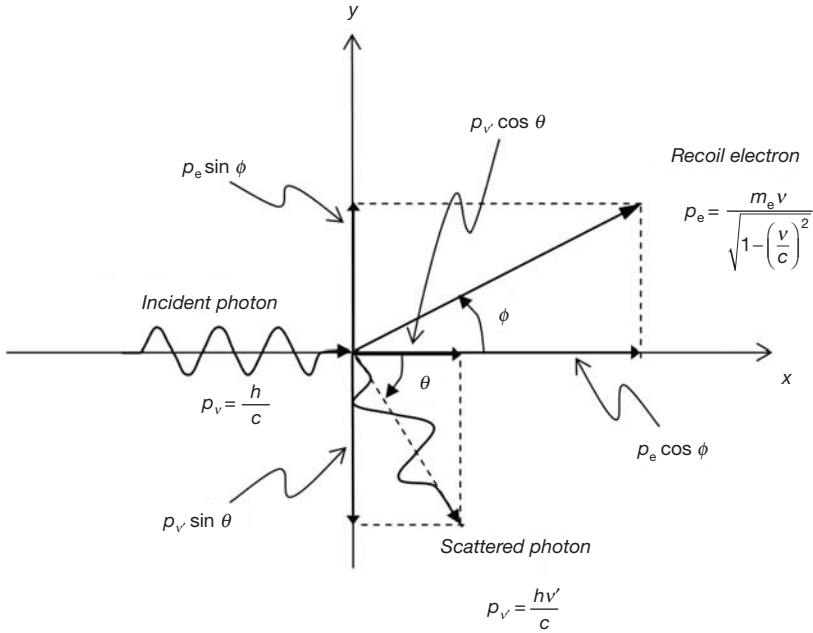


FIG. 1.5. Schematic diagram of Compton scattering. An incident photon with energy  $h\nu$  interacts with a loosely bound (essentially free) atomic electron. The electron is ejected from the atom as a recoil (Compton) electron with kinetic energy  $E_K$  and a scattered photon with energy  $h\nu' = h\nu - E_K$  is produced (see Eq. (1.79)).

and  $E_K$  is the kinetic energy of the recoil electron. Equation (1.79) represents conservation of energy; Eqs (1.80) and (1.81) represent conservation of momentum along the x axis and y axis, respectively, of Fig. 1.5.

- The scattering angle  $\theta$  and the recoil electron angle  $\phi$  are related through the following relationship:

$$\cot \phi = (1 + \epsilon) \tan(\theta/2) \quad (1.82)$$

From Eq. (1.82) it is evident that the range of angle  $\phi$  is between 0 for  $\theta = \pi$  (photon backscattering) and  $\pi/2$  for  $\theta = 0$  (photon forward scattering) for any arbitrary photon energy. For a given  $\theta$ , the higher the incident photon energy, the smaller is the recoil electron angle  $\phi$ .

- The Compton interaction represents a photon interaction with an essentially free and stationary electron ( $h\nu \gg E_B$ ). Consequently, the atomic Compton attenuation coefficient  ${}_a\sigma_C$  depends linearly on the atomic

## CHAPTER 1

number  $Z$  of the attenuator, while  ${}_e\sigma_C$  and  $\sigma_C/\rho$ , the electronic and mass Compton attenuation coefficients, respectively, are independent of  $Z$ .

- The electronic Compton attenuation coefficient  ${}_e\sigma_C$  steadily decreases with  $h\nu$  from a value of  $0.665 \times 10^{-24}$  cm<sup>2</sup>/electron at low photon energies to  $0.21 \times 10^{-24}$  cm<sup>2</sup>/electron at  $h\nu = 1$  MeV;  $0.051 \times 10^{-24}$  cm<sup>2</sup>/electron at  $h\nu = 10$  MeV; and  $0.008 \times 10^{-24}$  cm<sup>2</sup>/electron at  $h\nu = 100$  MeV.
- The scattered photon energy  $h\nu'$  and the kinetic energy of the Compton electron  $E_K$  are given as follows:

$$h\nu' = h\nu \frac{1}{1 + \varepsilon(1 - \cos \theta)} \quad \text{and} \quad E_K = h\nu \frac{\varepsilon(1 - \cos \theta)}{1 + \varepsilon(1 - \cos \theta)} \quad (1.83)$$

- The energy of photons scattered at  $90^\circ$  and  $180^\circ$  is thus given as:

$$h\nu'(\theta = 90^\circ) = \frac{h\nu}{1 + \varepsilon} \quad \text{and} \quad h\nu'(\theta = 180^\circ) = \frac{h\nu}{1 + 2\varepsilon} \quad (1.84)$$

which for large incident photon energies ( $\varepsilon = h\nu/(m_e c^2) \rightarrow \infty$ ) results in  $m_e c^2$  and  $0.5 m_e c^2$  for  $\theta = 90^\circ$  and  $\theta = 180^\circ$ , respectively.

- The maximum (for  $\theta = 180^\circ$  (i.e. photon backscattering)) and mean fractions of the incident photon energy transferred to the Compton recoil electron are given in Fig. 1.6. The mean fraction is used in the determination of the Compton effect contribution to the energy transfer coefficient.
- For example, from Fig. 1.6 we determine that a 1 MeV photon undergoing a Compton backscattering event would result in a recoil electron with a kinetic energy of 800 keV and a backscattered photon with an energy of 200 keV.
- On average, a 1 MeV photon undergoing Compton scattering will produce a 440 keV recoil electron and a 560 keV scattered photon; a 100 keV photon will produce a 15 keV recoil electron and a 85 keV scattered photon; a 10 MeV photon will produce a 6.9 MeV recoil electron and a 3.1 MeV scattered photon; and a 100 MeV photon will produce an 80 MeV recoil electron and a 20 MeV scattered photon.

### 1.4.7. Pair production

In pair production the photon disappears and an electron–positron pair with a combined kinetic energy equal to  $h\nu - 2m_e c^2$  is produced in the nuclear Coulomb field.



## BASIC RADIATION PHYSICS

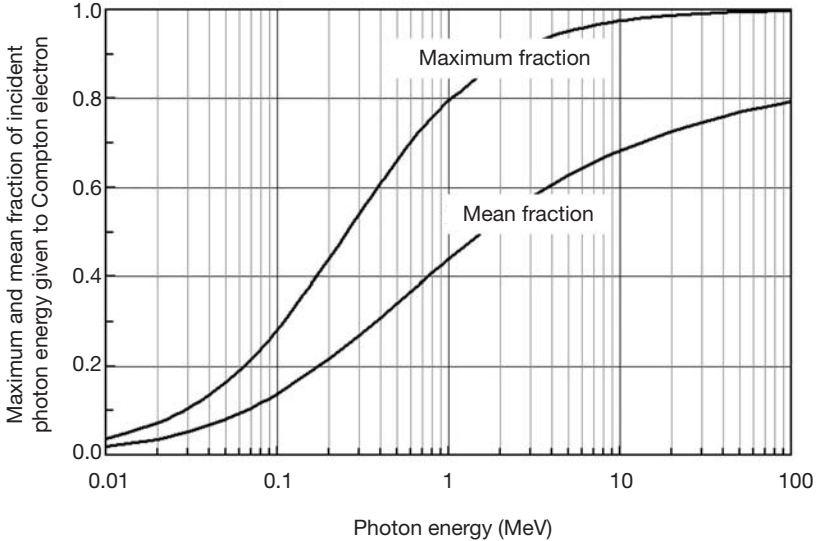


FIG. 1.6. Maximum and mean fractions of incident photon energy transferred to a Compton recoil electron in the photon energy range from 10 keV to 100 MeV. Data are obtained from the National Institute of Science and Technology (NIST) in Washington, DC ([www.nist.gov](http://www.nist.gov)).

- Since mass is produced out of photon energy in the form of an electron–positron pair, pair production has an energy threshold (minimum photon energy required for the effect to happen) of  $2m_e c^2 = 1.02$  MeV.
- When pair production occurs in the field of an orbital electron, the effect is referred to as triplet production, and three particles (an electron–positron pair and the orbital electron) share the available energy. The threshold for this effect is  $4m_e c^2$ .
- The probability for pair production is zero for photon energies below the threshold energy and increases rapidly with photon energy above the threshold.
- The atomic attenuation coefficient for pair production  ${}_a\kappa$  and the mass attenuation coefficient for pair production  $\kappa/\rho$  vary approximately as  $Z^2$  and  $Z$ , respectively, where  $Z$  is the atomic number of the attenuator.

**1.4.8. Photonuclear reactions**

Photonuclear reactions (also referred to as photodisintegration reactions) occur when a high energy photon is absorbed by the nucleus of an atom, resulting in an emission of a neutron ((x, n) reaction) or proton ((x, p) reaction) and a transformation of the nucleus into a radioactive reaction product.

- The threshold for a particular photonuclear reaction depends on the reaction and the nucleus and is of the order of 10 MeV or higher for most nuclei (with the exception of the deuteron and <sup>9</sup>Be nuclei, for which the threshold is of the order of 2 MeV).
- The probability for photonuclear reactions is much smaller than that for other photon interactions, and their contribution to the total attenuation coefficient amounts to only a few per cent at photon energies above the reaction threshold.
- While photonuclear reactions do not play an active role in photon attenuation considerations, they are of concern in high energy radiotherapy treatment rooms because of the neutron production through the (x, n) reactions and because of the radioactivity that is induced in the treatment room air and in machine components through the (x, n) reaction. Both the neutrons and the radioactivity pose a health hazard to personnel and must be dealt with in the treatment room and treatment machine design. The neutron problem is dealt with special treatment room doors incorporating borated hydrogenous materials to thermalize and absorb the neutrons, the radioactivity with adequate room ventilation (six to eight air changes per hour) and use of machine components with a low reaction cross-section and short half-life of the reaction product.

**1.4.9. Contributions to attenuation coefficients**

For a given photon energy  $h\nu$  and attenuator  $Z$ , the attenuation coefficient  $\mu$ , energy transfer coefficient  $\mu_{tr}$  and energy absorption coefficient  $\mu_{ab}$  are given as a sum of coefficients for individual photon interactions (the energy absorption coefficient is often designated as  $\mu_{en}$ ):

$$\mu = \tau + \sigma_R + \sigma_C + \kappa \tag{1.85}$$

$$\mu_{tr} = \tau_{tr} + (\sigma_C)_{tr} + \kappa_{tr} = \tau \frac{(\bar{E}_K)_{tr}^{PE}}{h\nu} + \sigma_C \frac{(\bar{E}_K)_{tr}^{CE}}{h\nu} + \kappa \frac{(\bar{E}_K)_{tr}^{PP}}{h\nu} \tag{1.86}$$

## BASIC RADIATION PHYSICS

$$\mu_{\text{ab}} = \mu_{\text{en}} = \mu_{\text{tr}}(1 - g) \quad (1.87)$$

where  $g$  is the radiative fraction, and the average energies transferred to charged particles (electrons and positrons) for the photoelectric effect, the Compton effect and pair production are designated as  $(\bar{E}_{\text{K}})_{\text{tr}}^{\text{PE}}$ ,  $(\bar{E}_{\text{K}})_{\text{tr}}^{\text{CE}}$  and  $(\bar{E}_{\text{K}})_{\text{tr}}^{\text{PP}}$ , respectively.

- $(\bar{E}_{\text{K}})_{\text{tr}}^{\text{PE}}$  may be approximated by  $h\nu - P_{\text{K}}\omega_{\text{K}}E_{\text{B}}(\text{K})$ , where  $E_{\text{B}}(\text{K})$  is the binding energy of the K shell electron,  $P_{\text{K}}$  is the fraction of all photoelectric effect interactions that occur in the K shell and  $\omega_{\text{K}}$  is the fluorescent yield for the K shell.
- $(\bar{E}_{\text{K}})_{\text{tr}}^{\text{CE}}$  is obtained from tabulated values or from the graph shown in Fig. 1.6.
- $(\bar{E}_{\text{K}})_{\text{tr}}^{\text{PP}} = h\nu - 2m_{\text{e}}c^2$ .
- Note that in Rayleigh scattering no energy transfer occurs and therefore Rayleigh scattering contributes neither to the energy transfer coefficient nor to the energy absorption coefficient.

The individual components of the attenuation coefficients, when summed, result in the total mass attenuation, mass–energy transfer and mass–energy absorption coefficients as follows:

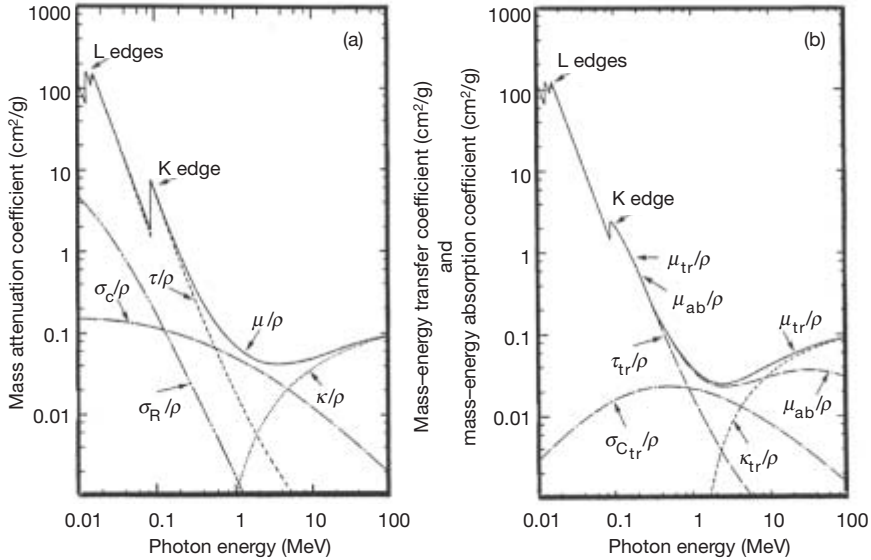
$$\frac{\mu}{\rho} = \frac{\tau}{\rho} + \frac{\sigma_{\text{R}}}{\rho} + \frac{\sigma_{\text{C}}}{\rho} + \frac{\kappa}{\rho} \quad (1.88)$$

$$\begin{aligned} \frac{\mu_{\text{tr}}}{\rho} &= \frac{\tau_{\text{tr}}}{\rho} + \frac{(\sigma_{\text{C}})_{\text{tr}}}{\rho} + \frac{\kappa_{\text{tr}}}{\rho} \\ &= \frac{1}{\rho} \left( \tau \frac{h\nu - P_{\text{K}}\omega_{\text{K}}E_{\text{B}}(\text{K})}{h\nu} + \sigma_{\text{C}} \frac{(\bar{E}_{\text{K}})_{\text{tr}}^{\text{CE}}}{h\nu} + \kappa \frac{h\nu - 2m_{\text{e}}c^2}{h\nu} \right) \end{aligned} \quad (1.89)$$

$$\frac{\mu_{\text{ab}}}{\rho} = \frac{\mu_{\text{tr}}}{\rho}(1 - g) \quad (1.90)$$

Figure 1.7 shows the mass attenuation coefficient  $\mu/\rho$  in (a) and the mass–energy transfer coefficient  $(\mu_{\text{tr}}/\rho)$  and mass–energy absorption coefficient  $(\mu_{\text{ab}}/\rho)$  in (b) for lead in the photon energy range from 10 keV to 100 MeV.

## CHAPTER 1



*FIG. 1.7. Mass attenuation coefficient  $\mu/\rho$  (a); mass–energy transfer coefficient  $\mu_{tr}/\rho$  and mass–energy absorption coefficient  $\mu_{ab}/\rho$  (b) for lead in the photon energy range between 10 keV and 100 MeV. The dotted–dashed curves represent contributions of individual effects, while the solid curves represent the sum of the contributions of the individual effects as given by Eq. (1.88) for  $\mu/\rho$ , Eq. (1.89) for  $\mu_{tr}/\rho$  and Eq. (1.90) for  $\mu_{ab}/\rho$ . For photon energies below 2 MeV,  $\mu_{tr}/\rho \approx \mu_{ab}/\rho$ , because the radiative fraction  $g$  in this energy region is negligible. Above 2 MeV,  $g$  increases with photon energy, causing the divergence between the mass–energy transfer and mass–energy absorption coefficients.*

### 1.4.10. Relative predominance of individual effects

The probability for a photon to undergo any one of the various interaction phenomena with an attenuator depends on the energy  $h\nu$  of the photon and on the atomic number  $Z$  of the attenuating material. In general, the photoelectric effect predominates at low photon energies, the Compton effect at intermediate energies and pair production at high photon energies.

Figure 1.8 shows the regions of relative predominance of the three most important individual effects with  $h\nu$  and  $Z$  as parameters. The two curves display the points in the  $(h\nu, Z)$  diagram for which  ${}_a\sigma_C = {}_a\tau$  or  ${}_a\sigma_C = {}_a\kappa$  and thus delineate the regions of photoelectric effect predominance at low photon energies, Compton effect predominance at intermediate energies and pair

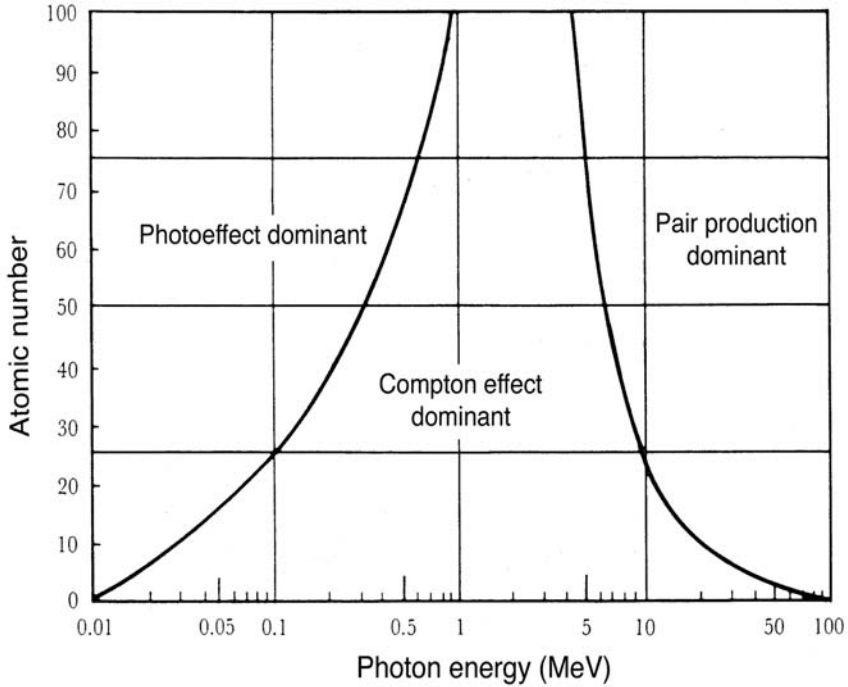


FIG. 1.8. Regions of relative predominance of the three main forms of photon interaction with matter. The left curve represents the region where the atomic coefficients for the photoelectric effect and Compton effect are equal ( ${}_a\tau = {}_a\sigma_C$ ), the right curve is for the region where the atomic Compton coefficient equals the atomic pair production coefficient ( ${}_a\sigma_C = {}_a\kappa$ ).

production predominance at high photon energies. For example, a 100 keV photon will interact with lead ( $Z = 82$ ) predominantly through the photoelectric effect and with soft tissue ( $Z_{\text{eff}} = 7.5$ ) predominantly through the Compton effect. A 10 MeV photon, on the other hand, will interact with lead predominantly through pair production and with tissue predominantly through the Compton effect.

#### 1.4.11. Effects following photon interactions

In the photoelectric effect, the Compton effect and triplet production, vacancies are produced in atomic shells through the ejection of orbital electrons. For the orthovoltage and megavoltage photons used in the diagnosis

## CHAPTER 1

and treatment of disease with radiation, the shell vacancies occur mainly in inner atomic shells and are followed by characteristic X rays or Auger electrons, the probability for the former given by the fluorescent yield  $\omega$  (see Fig. 1.9), while the probability for the Auger effect is  $1 - \omega$

Pair production and triplet production are followed by the annihilation of the positron with a 'free' and stationary electron, producing two annihilation quanta, most commonly with energies of 0.511 MeV each and emitted at  $180^\circ$  from each other to satisfy the conservation of charge, momentum and energy. An annihilation of a positron before it has expended all of its kinetic energy is referred to as annihilation in flight and produces photons with energies exceeding 0.511 MeV.

### 1.4.12. Summary of photon interactions

Table 1.4 summarizes the main characteristics of the photoeffect, Rayleigh scattering, the Compton effect and pair production.

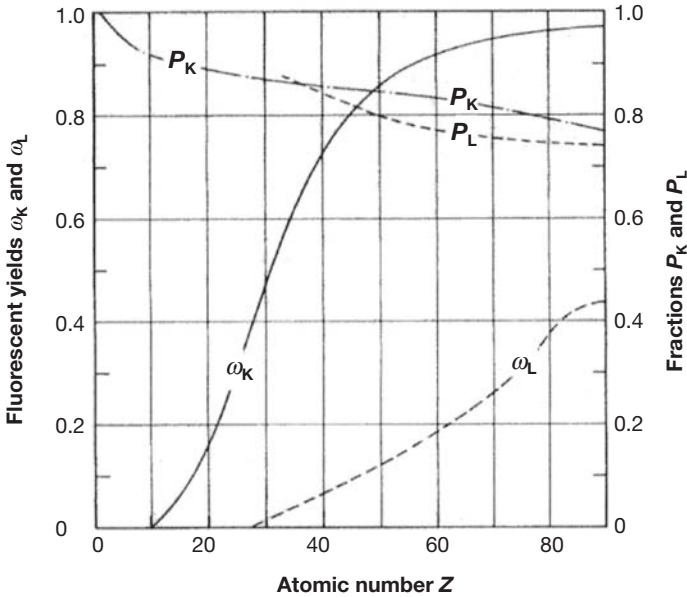


FIG. 1.9. Fluorescent yields  $\omega_K$  for  $h\nu > (E_B)_K$  and  $\omega_L$  for  $(E_B)_L < h\nu < (E_B)_K$  as well as fractions  $P_K$  for  $h\nu > (E_B)_K$  and  $P_L$  for  $(E_B)_L < h\nu < (E_B)_K$  against the atomic number Z. Data were obtained from F.H. Attix, *Introduction to Radiological Physics and Radiation Dosimetry*, Wiley, New York (1986).

## BASIC RADIATION PHYSICS

TABLE 1.4. MAIN CHARACTERISTICS OF THE PHOTOELECTRIC EFFECT, RAYLEIGH SCATTERING, THE COMPTON EFFECT AND PAIR PRODUCTION

	Photoelectric effect	Rayleigh scattering	Compton effect	Pair production
Photon interaction	With whole atom (bound electron)	With bound electrons	With free electrons	With nuclear Coulomb field
Mode of photon interaction	Photon disappears	Photon scattered	Photon scattered	Photon disappears
Energy dependence	$\frac{1}{(h\nu)^3}$	$\frac{1}{(h\nu)^2}$	Decreases with energy	Increases with energy
Threshold	No	No	No	$2m_e c^2$
Linear attenuation coefficient	$\tau$	$\sigma_R$	$\sigma_C$	$\kappa$
Particles released	Photoelectron	None	Compton (recoil) electron	Electron–positron pair
Atomic coefficient dependence on $Z$	${}_a\tau \propto Z^4$	${}_a\sigma_R \propto Z^2$	${}_a\sigma_C \propto Z$	${}_a\kappa \propto Z^2$
Mass coefficient dependence on $Z$	$\frac{\tau}{\rho} \propto Z^3$	$\frac{\sigma_R}{\rho} \propto Z$	Independent	$\frac{\kappa}{\rho} \propto Z$
Average energy transferred	$h\nu - P_K \omega_K E_B(K)$	0	$(\bar{E}_K)_{tr}^{CE}$ (see Fig. 1.6)	$h\nu - 2m_e c^2$
Subsequent effect	Characteristic X ray, Auger effect	None	Characteristic X ray, Auger effect	Annihilation radiation
Significant energy region for water	<20 keV	<20 keV	20 keV–10 MeV	>10 MeV

**1.4.13. Example of photon attenuation**

For 2 MeV photons in lead ( $Z = 82$ ;  $A = 207.2$  g/g-atom;  $\rho = 11.36$  g/cm<sup>3</sup>) the photoelectric effect, coherent scattering, the Compton effect and pair production linear attenuation coefficients are:  $\tau = 0.055$  cm<sup>-1</sup>,  $\sigma_R = 0.008$  cm<sup>-1</sup>,  $\sigma_C = 0.395$  cm<sup>-1</sup> and  $\kappa = 0.056$  cm<sup>-1</sup>. The average energy transferred to charged particles  $(\bar{E}_K)_{tr} = 1.13$  MeV and the average energy absorbed in lead is  $(\bar{E}_K)_{ab} = 1.04$  MeV.

Calculate the linear attenuation coefficient  $\mu$ ; mass attenuation coefficient  $\mu_m$ ; atomic attenuation coefficient  ${}_a\mu$ ; mass-energy transfer coefficient  $\mu_{tr}/\rho$ ; mass-energy absorption coefficient  $\mu_{ab}/\rho$ ; and radiative fraction  $g$ :

$$\mu = \tau + \sigma_R + \sigma_C + \kappa = (0.055 + 0.008 + 0.395 + 0.056) \text{ cm}^{-1} = 0.514 \text{ cm}^{-1} \quad (1.91)$$

$$\mu_m = \frac{\mu}{\rho} = \frac{0.514 \text{ cm}^{-1}}{11.36 \text{ g/cm}^3} = 0.0453 \text{ cm}^2/\text{g} \quad (1.92)$$

$$\begin{aligned} {}_a\mu &= \left( \frac{\rho N_A}{A} \right)^{-1} \mu = \frac{207.2 \text{ g/g-atom} \times 0.514 \text{ cm}^{-1}}{11.36 \text{ g/cm}^3 \times 6.022 \times 10^{23} \text{ atom/g-atom}} \\ &= 1.56 \times 10^{-23} \text{ cm}^2/\text{atom} \end{aligned} \quad (1.93)$$

$$\frac{\mu_{tr}}{\rho} = \frac{(\bar{E}_K)_{tr}}{h\nu} \frac{\mu}{\rho} = \frac{1.13 \text{ MeV} \times 0.0453 \text{ cm}^2/\text{g}}{2 \text{ MeV}} = 0.0256 \text{ cm}^2/\text{g} \quad (1.94)$$

$$\frac{\mu_{ab}}{\rho} = \frac{\mu_{en}}{\rho} = \frac{(\bar{E}_K)_{ab}}{h\nu} \frac{\mu}{\rho} = \frac{1.04 \text{ MeV} \times 0.0453 \text{ cm}^2/\text{g}}{2 \text{ MeV}} = 0.0236 \text{ cm}^2/\text{g} \quad (1.95)$$

$$g = \frac{(\bar{E}_K)_{tr} - (\bar{E}_K)_{ab}}{(\bar{E}_K)_{tr}} = 1 - \frac{(\bar{E}_K)_{ab}}{(\bar{E}_K)_{tr}} = 1 - \frac{1.04 \text{ MeV}}{1.13 \text{ MeV}} = 0.08 \quad (1.96)$$

or



## BASIC RADIATION PHYSICS

$$g = 1 - \frac{\mu_{\text{ab}}/\rho}{\mu_{\text{tr}}/\rho} = 1 - \frac{0.0236 \text{ cm}^2/\text{g}}{0.0256 \text{ cm}^2/\text{g}} = 0.08 \quad (1.97)$$

The mass–energy transfer coefficient  $\mu_{\text{tr}}/\rho$  can also be determined using Eq. (1.89) with:

$$h\nu - P_{\text{K}}\omega_{\text{K}}E_{\text{B}} = 2 \text{ MeV} - 0.8 \times 0.96 \times 0.088 \text{ MeV} = 1.93 \text{ MeV} \quad (1.98)$$

(from Fig. 1.9)

$$(\bar{E}_{\text{K}})_{\text{tr}}^{\text{CE}} = 0.53 \times 2 \text{ MeV} = 1.06 \text{ MeV} \quad (\text{from Fig. 1.6}) \quad (1.99)$$

$$h\nu - 2m_{\text{e}}c^2 = 2 \text{ MeV} - 1.02 \text{ MeV} = 0.98 \text{ MeV} \quad (1.100)$$

to obtain

$$\frac{\mu_{\text{tr}}}{\rho} = \frac{1}{11.36} \left( \frac{1.93}{2} \times 0.055 + \frac{1.06}{2} \times 0.395 + \frac{0.98}{2} \times 0.056 \right) \frac{\text{cm}^2}{\text{g}} = 0.0254 \frac{\text{cm}^2}{\text{g}} \quad (1.101)$$

in good agreement with the result obtained in Eq. (1.94).

Thus, as shown schematically in Fig. 1.10, a 2 MeV photon in lead will on average:

- Transfer 1.13 MeV to charged particles (electrons and positrons); and
- 0.87 MeV will be scattered through Rayleigh and Compton scattering.

Of the 1.13 MeV of energy transferred:

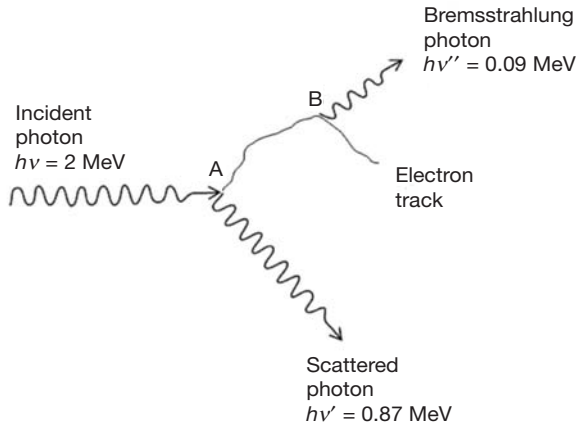
- 1.04 MeV will be absorbed in lead; and
- 0.09 MeV will be re-emitted through bremsstrahlung radiative loss.

The radiative fraction  $g$  for 2 MeV photons in lead is 0.08.

### 1.4.14. Production of vacancies in atomic shells

There are eight main means of producing vacancies in atomic shells and transforming the atom from a neutral state into an excited positive ion:

## CHAPTER 1



*FIG. 1.10. Schematic diagram of general photon interactions with an atom. In this example a 2 MeV photon  $h\nu$  interacts with a lead atom. An individual 2 MeV photon, as it encounters a lead atom at point A, may interact with the atom through the photoelectric effect, Rayleigh scattering, the Compton effect or pair production, or it may not interact at all. However, for a large number of 2 MeV photons striking lead, we may state that on average: 1.13 MeV will be transferred at point A to charged particles (mainly to fast energetic electrons, but possibly also to positrons if the interaction is pair production); 0.87 MeV will be scattered through Rayleigh and Compton scattering ( $h\nu'$ ). Of the 1.13 MeV transferred to charged particles: 1.04 MeV will be absorbed in lead over the fast charged particle tracks, and 0.09 MeV will be emitted in the form of bremsstrahlung photons ( $h\nu''$ ).*

- Coulomb interaction (1) of an energetic charged particle with an orbital electron.
- Photon interactions:
  - Photoelectric effect (2);
  - Compton effect (3);
  - Triplet production (4).
- Nuclear decay:
  - Electron capture (5);
  - Internal conversion (6).
- Positron annihilation (7).
- Auger effect (8).

## BASIC RADIATION PHYSICS

Note that pair production does not produce shell vacancies. Vacancies in inner atomic shells are not stable; they are followed by emission of characteristic photons or Auger electrons and cascade to the outer shell of the ion. The ion eventually attracts an electron from its surroundings and reverts to a neutral atom.

## BIBLIOGRAPHY

ATTIX, F.H., Introduction to Radiological Physics and Radiation Dosimetry, Wiley, New York (1986).

ATTIX, F.H., ROESCH, W.C., TOCHILIN, E., Radiation Dosimetry, Academic Press, New York (1968).

EVANS, R.D., The Atomic Nucleus, McGraw-Hill, New York (1955).

HALE, J., The Fundamentals of Radiological Science, Thomas, Springfield, IL (1974).

JOHNS, H.E., CUNNINGHAM, J.R., The Physics of Radiology, Thomas, Springfield, IL (1984).

KASE, K.R., BJARNGARD, B.E., ATTIX, F.H. (Eds), The Dosimetry of Ionizing Radiation, Academic Press, San Diego, CA (1985).

KHAN, F., The Physics of Radiation Therapy, 3rd edn, Lippincott, Williams and Wilkins, Baltimore, MD (2003).

ROHLF, J.W., Modern Physics from  $\alpha$  to  $Z^0$ , Wiley, New York (1994).

JAYARAMAN, S., LANZL, L.H., Clinical Radiotherapy Physics, CRC Press, Boca Raton, FL (1996).