Chapter 1

Concept of Stress

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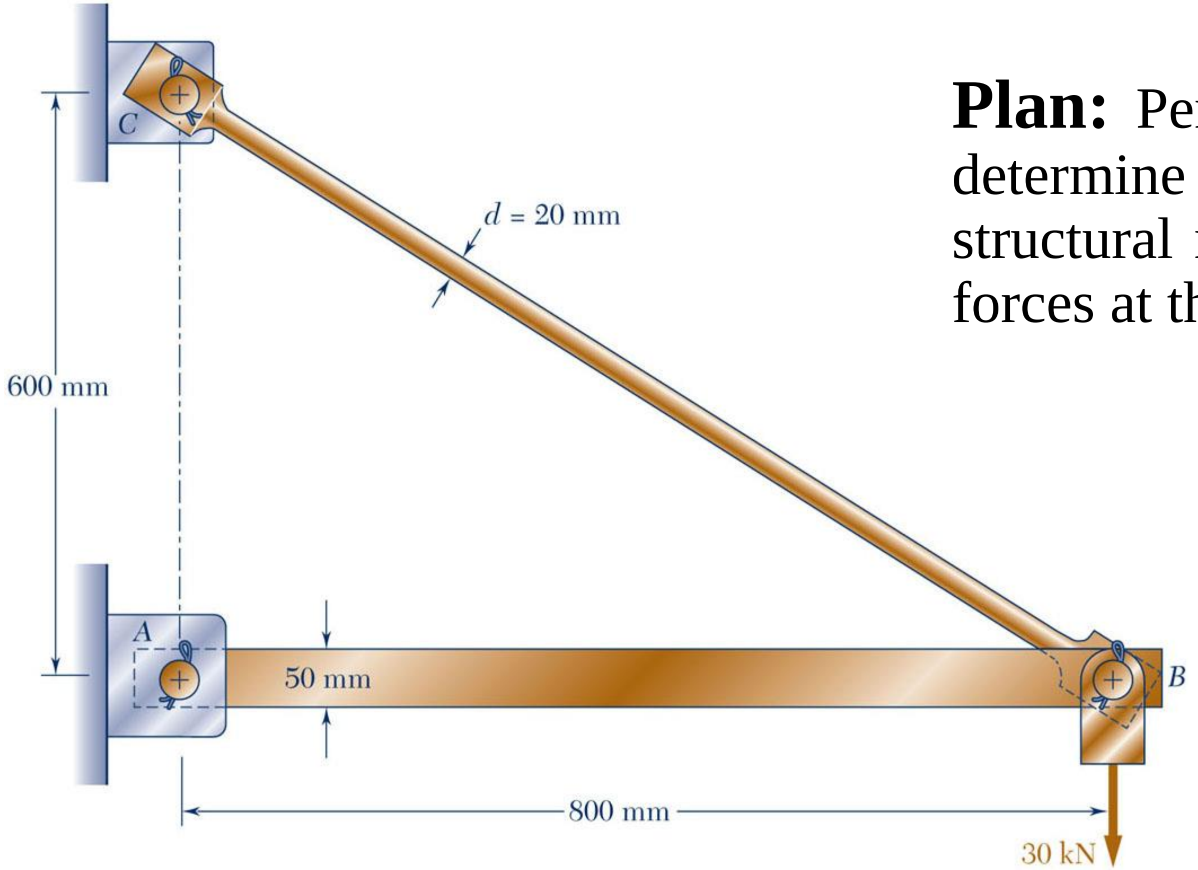
Objectives

The main objective of the study of mechanics of materials is to provide the future engineer with the means of analyzing and designing various machines and load bearing structures.

Both the analysis and design of a given structure involve the determination of **stresses** and **deformations**. This chapter is devoted to the concept of stress.

Statics Review

Example: Consider the structure shown below and assume that it is designed to support a 30 kN load. The structure consists of a boom and rod joined by pins (zero moment connections) at the junctions and supports.



Plan: Perform a static analysis to determine the internal force in each structural member and the reaction forces at the supports

Statics Review

FBD: free body diagram
Structure is detached from supports
loads and reaction forces are indicated

Conditions for static equilibrium

$$\sum M_C = 0 = A_x(0.6\text{m}) - (30\text{kN})(0.8\text{m})$$

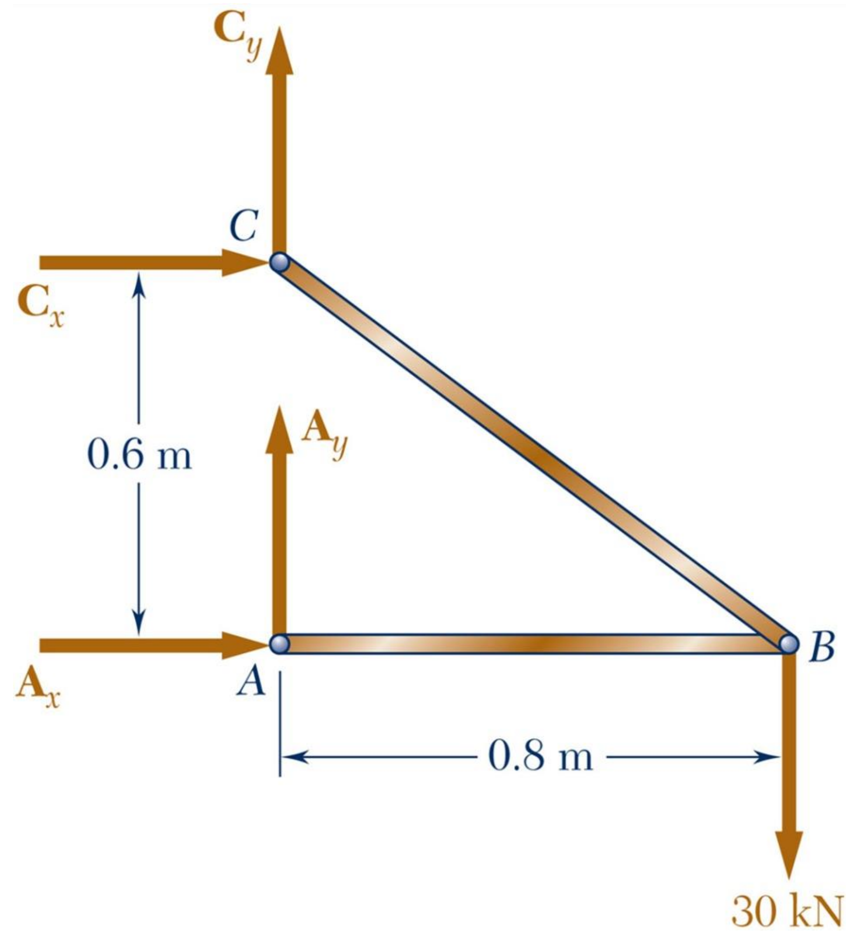
$$A_x = 40\text{kN}$$

$$\sum F_x = 0 = A_x + C_x$$

$$C_x = -A_x = -40\text{kN}$$

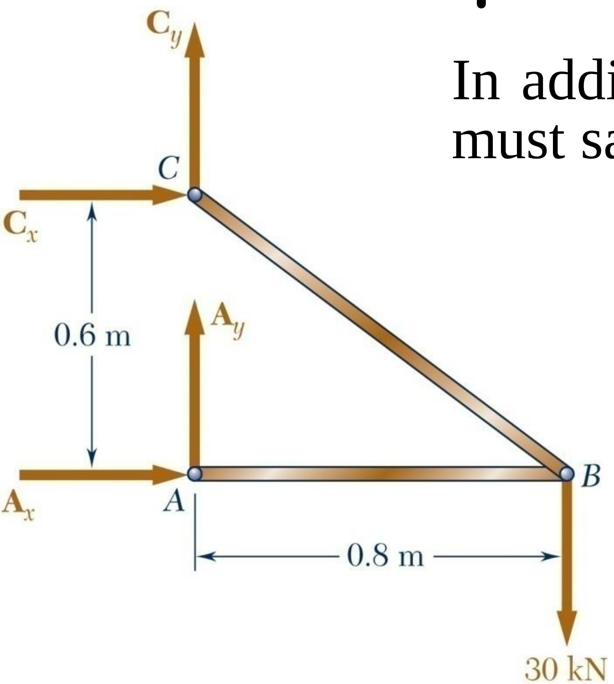
$$\sum F_y = 0 = A_y + C_y - 30\text{kN} = 0$$

$$A_y + C_y = 30\text{kN}$$



A_y and C_y cannot be determined from these equations need to find new equations

Component Free-Body Diagram



In addition to the complete structure, each component must satisfy the conditions for static equilibrium

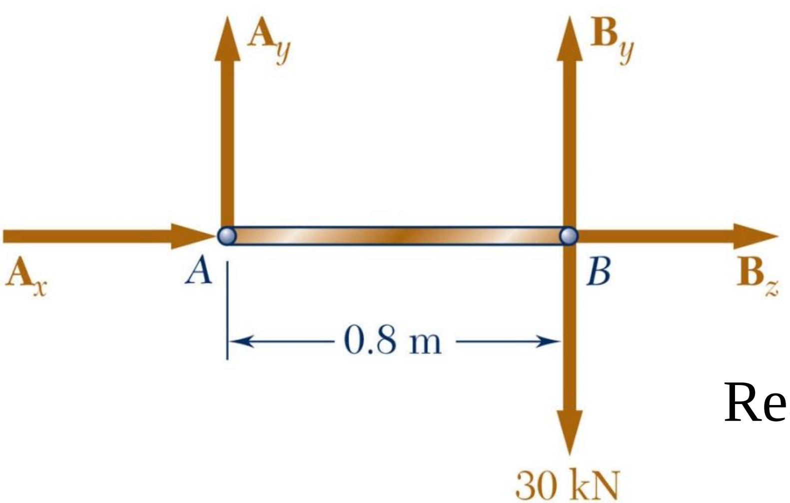
Consider the **FBD** for the boom:

$$\sum M_B = 0 = -A_y(0.8\,\text{m})$$

$$A_y = 0$$

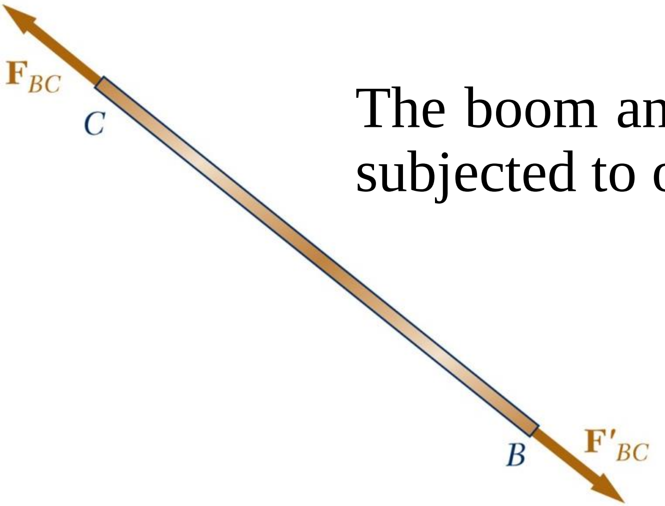
Now the structure equilibrium equation yields

$$A_x = 40\,\text{kN} \rightarrow$$
$$C_x = 40\,\text{kN} \leftarrow$$
$$C_y = 30\,\text{kN} \uparrow$$



Reaction forces are directed along boom and rod

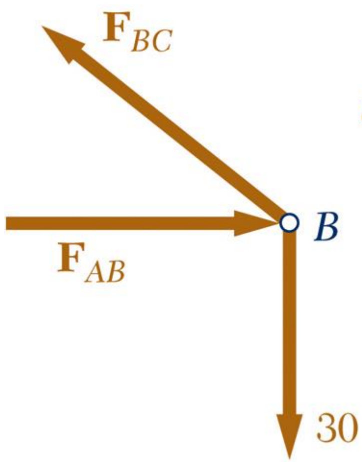
Method of Joints

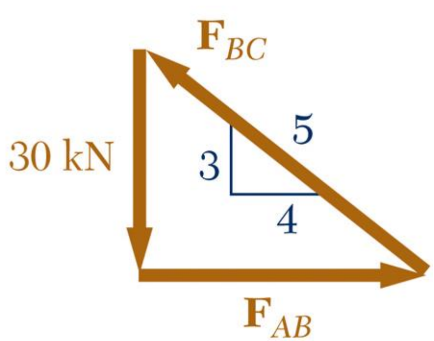


The boom and rod are 2-force members, i.e., the members are subjected to only two forces which are applied at member ends

For equilibrium, the forces must be parallel to an axis passing through the force application points, equal in magnitude, and in opposite directions

Joints must satisfy the conditions for static equilibrium which may be expressed in the form of a force triangle:





$$\sum \vec{F}_B = 0$$

$$\frac{F_{AB}}{4} = \frac{F_{BC}}{5} = \frac{30\text{ kN}}{3}$$

$$F_{AB} = 40\text{ kN} \qquad F_{BC} = 50\text{ kN}$$

Definition: Stress

Stress : in a member, stress is defined as force per unit area, or intensity of the forces distributed over a given section.

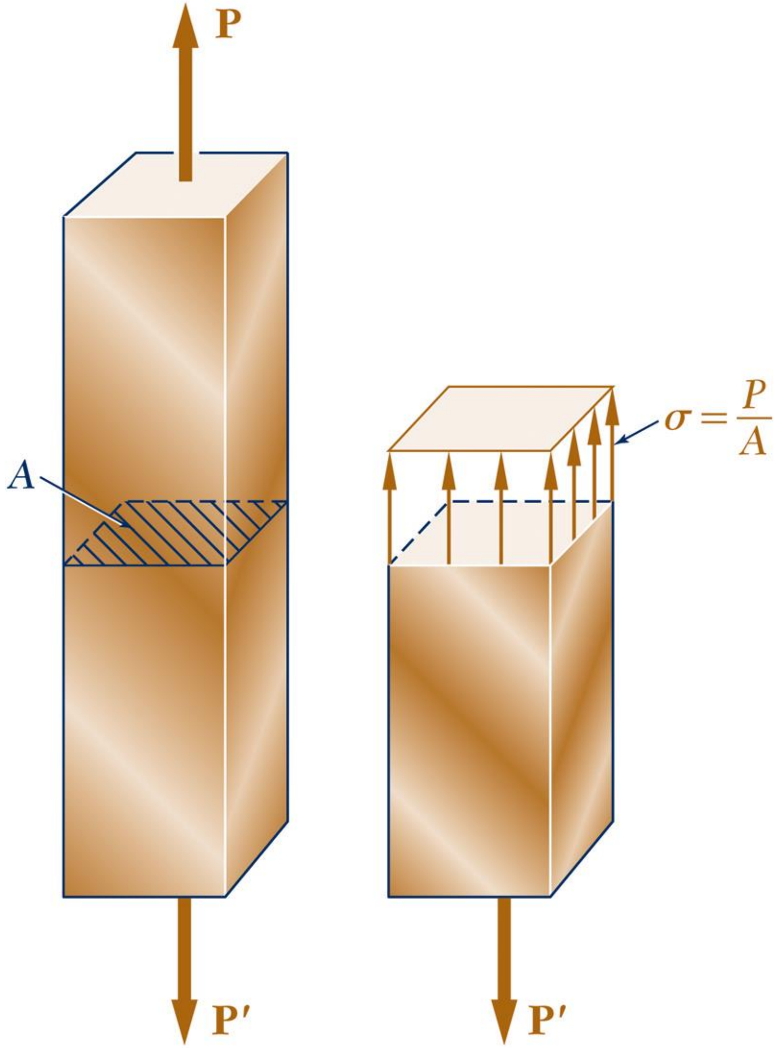
$$\sigma = \frac{P}{A}$$

Units:

P in N

A in m^2

σ in N/m^2 = Pa (Pascal)

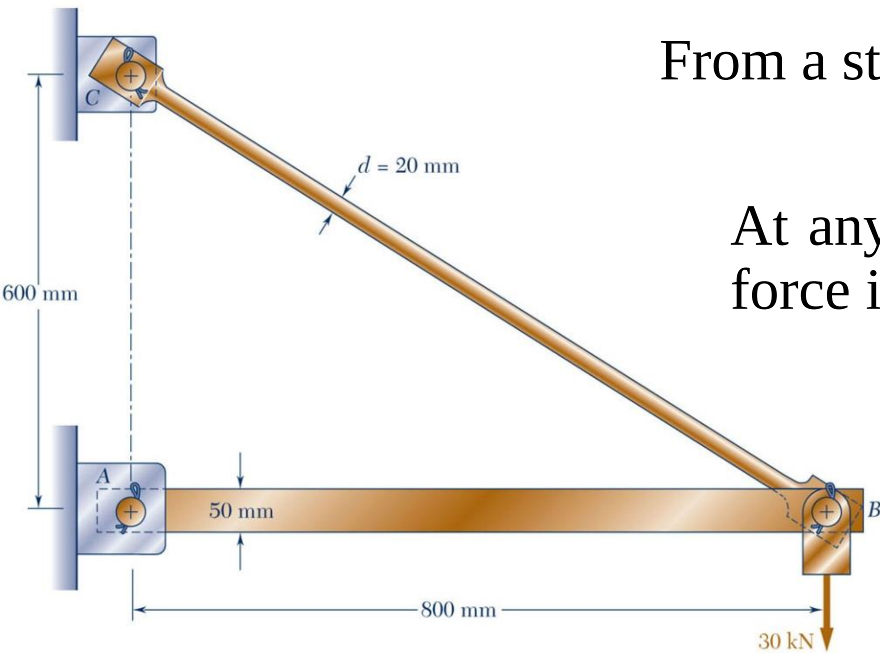


Positive stress: member in tension

Negative stress: member in compression

Stress Analysis

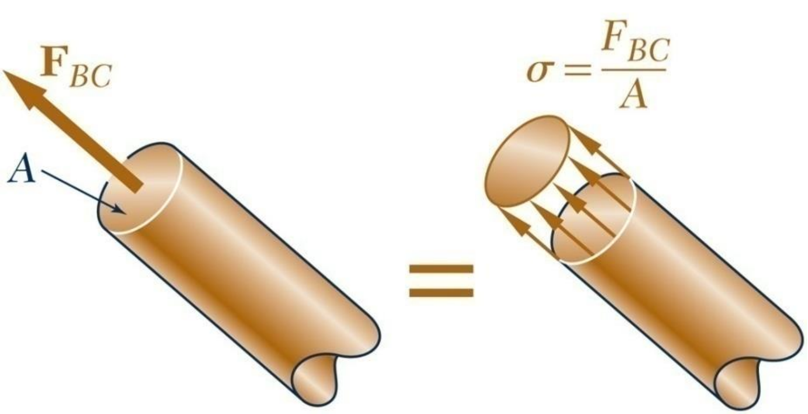
Can the structure safely support the 30 kN load ?



From a statics analysis: $F_{AB} = 40$ kN (compression)
$F_{BC} = 50$ kN (tension)

At any section through member BC, the internal force is 50 kN with a force intensity or **stress** of

$$\sigma_{BC} = \frac{P}{A} = \frac{50\times10^{3}\,\mathrm{N}}{314\times10^{-6}\,\mathrm{m}^2} = 159\,\mathrm{MPa}$$



From the material properties of steel, the allowable stress is

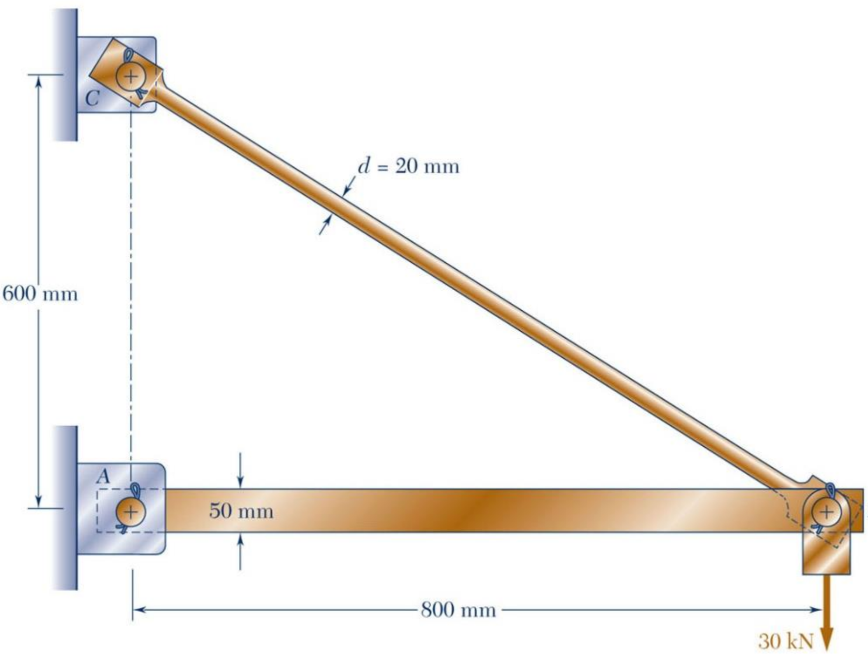
$$\sigma_{all} = 165\,\mathrm{MPa}$$

Conclusion: the strength of member *BC* is adequate

Design

Design of new structures requires selection of appropriate materials and component dimensions to meet performance requirements

For reasons based on cost, weight, availability, etc., the choice is made to construct the rod from aluminum (**σ_{all} = 100 MPa**). What is an appropriate choice for the rod diameter?



$$\sigma_{all} = \frac{P}{A}$$

$$A = \frac{P}{\sigma_{all}} = \frac{50 \times 10^3\,\text{N}}{100 \times 10^6\,\text{Pa}} = 500 \times 10^{-6}\,\text{m}^2$$

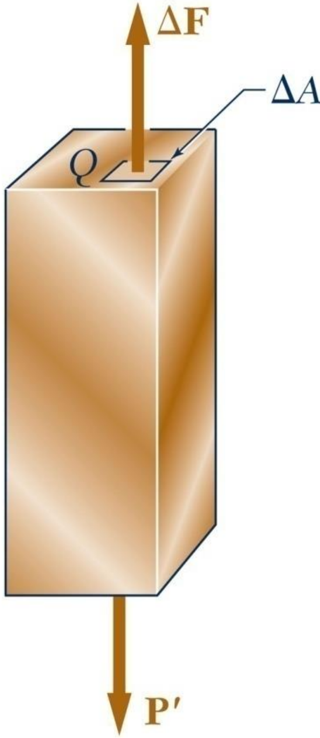
$$A = \pi \frac{d^2}{4}$$

$$d = \sqrt{\frac{4A}{\pi}} = \sqrt{\frac{4\left(500 \times 10^{-6}\,\text{m}^2\right)}{\pi}}$$

$$d = 2.52 \times 10^{-2}\,\text{m} = 25.2\,\text{mm}$$

An aluminum rod **26 mm or more** in diameter is **adequate**

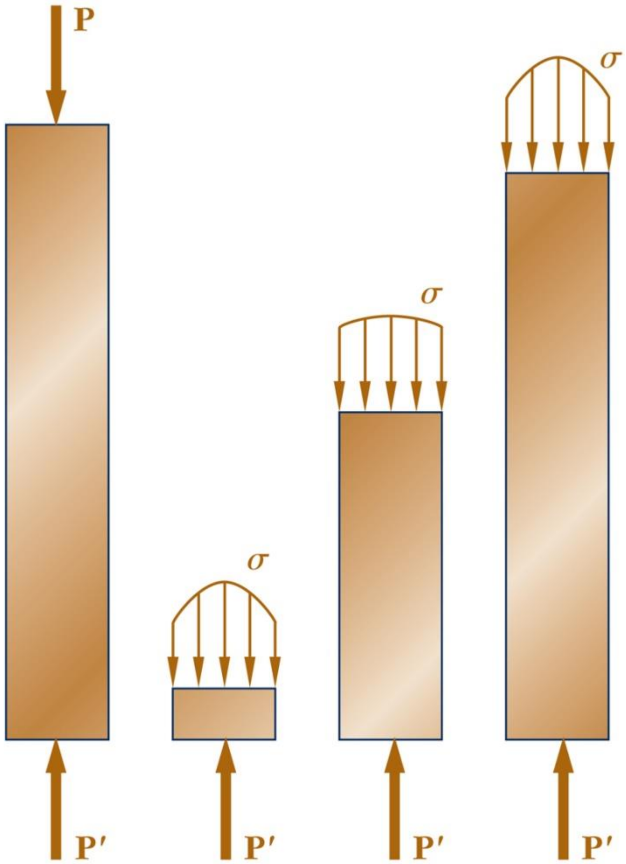
Axial Loading: Normal Stress



The resultant of the internal forces for an axially loaded member is **normal** to a section cut perpendicular to the member axis.

The force intensity on that section is defined as the normal stress.

$$\sigma = \lim_{\Delta A \to 0} \frac{\Delta F}{\Delta A} \qquad \sigma_{ave} = \frac{P}{A}$$

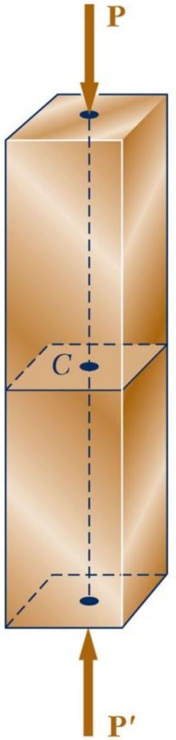


The normal stress at a particular point may not be equal to the average stress but the resultant of the stress distribution must satisfy

$$P = \sigma_{ave} A = \int dF = \int_A \sigma \, dA$$

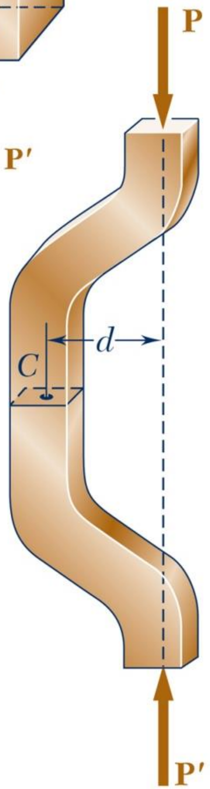
The detailed distribution of stress is statically indeterminate, i.e., cannot be found from statics alone.

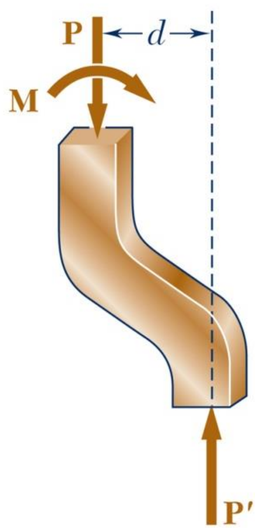
Centric & Eccentric Loading



A uniform distribution of stress in a section infers that the line of action for the resultant of the internal forces passes through the centroid of the section.

A uniform distribution of stress is only possible if the concentrated loads on the end sections of two-force members are applied at the section centroids. This is referred to as **centric loading**.

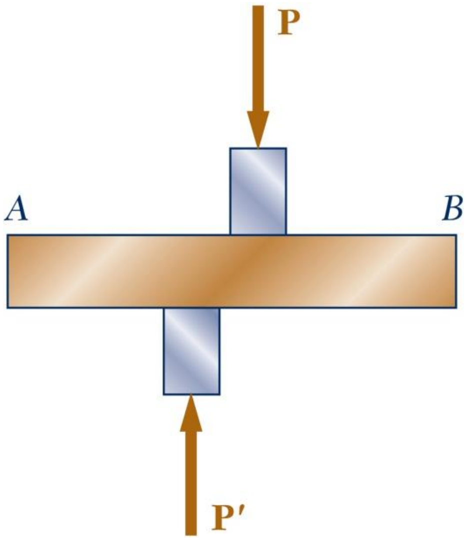




If a two-force member is **eccentrically loaded**, then the resultant of the stress distribution in a section must yield an axial force and a moment.

The stress distributions in eccentrically loaded members cannot be uniform or symmetric.

Shearing Stress

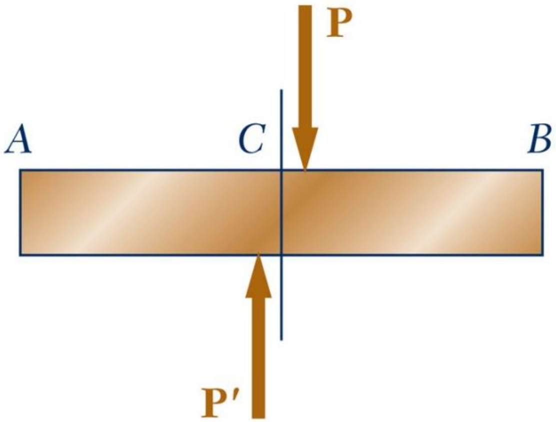


Forces **P** and **P’** are applied transversely to the member AB.

Corresponding internal forces act in the plane of section C and are called **shearing** forces. The resultant of the internal shear force distribution is defined as the **shear** of the section and is equal to the load **P**.

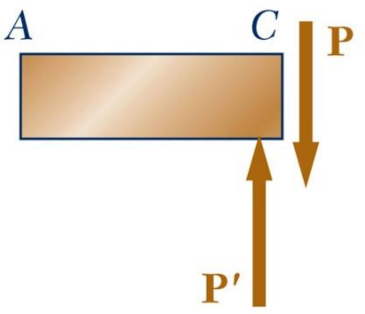
The corresponding average shear stress is,

$$\tau_{\text{ave}} = \frac{P}{A}$$



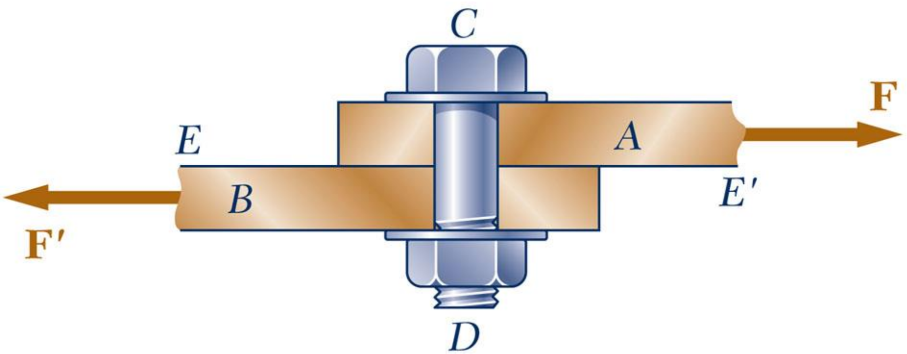
Shear stress distribution varies from zero at the member surfaces to maximum values that may be much larger than the average value.

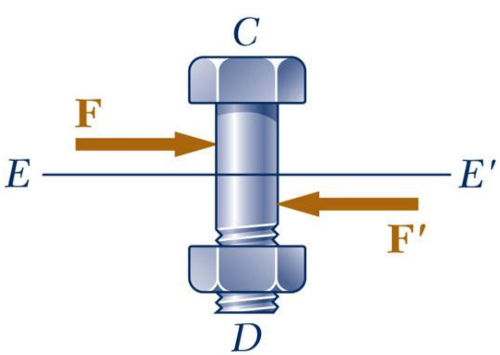
The shear stress distribution cannot be assumed to be uniform.

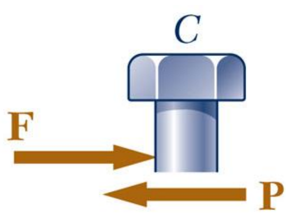


Shearing Stress Examples

Single Shear

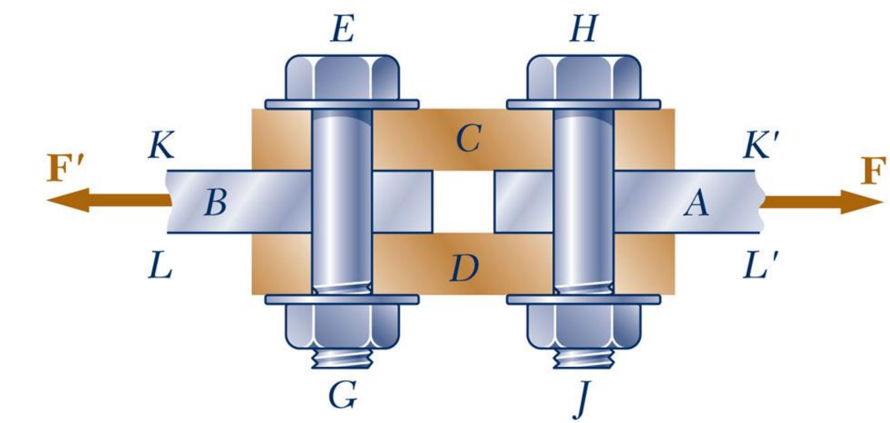


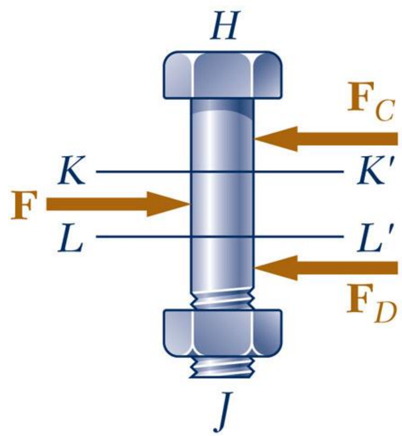


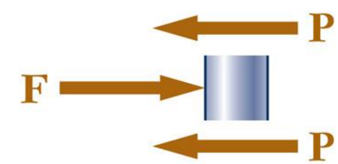


$$\tau_{ave} = \frac{P}{A} = \frac{F}{A}$$

Double Shear

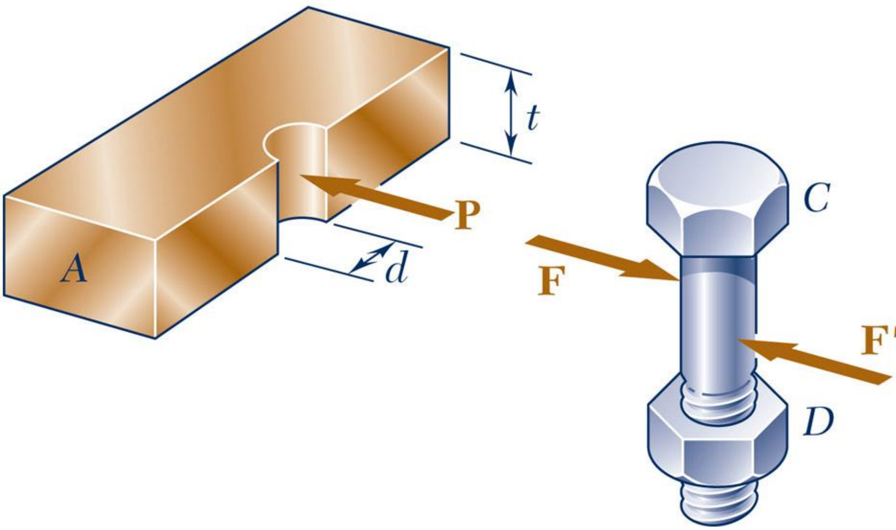






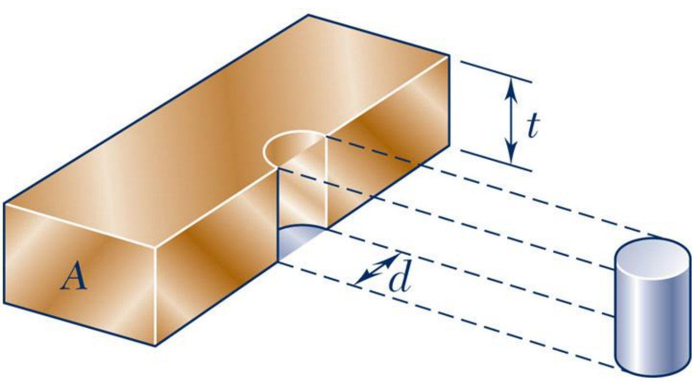
$$\tau_{ave} = \frac{P}{A} = \frac{F}{2A}$$

Bearing Stress in Connections



Bolts, rivets, and pins create stresses on the points of contact or **bearing surfaces** of the members they connect.

The resultant of the force distribution on the surface is equal and opposite to the force exerted on the pin.



Corresponding average force intensity is called the bearing stress,

$$\sigma_b = \frac{P}{A} = \frac{P}{td}$$

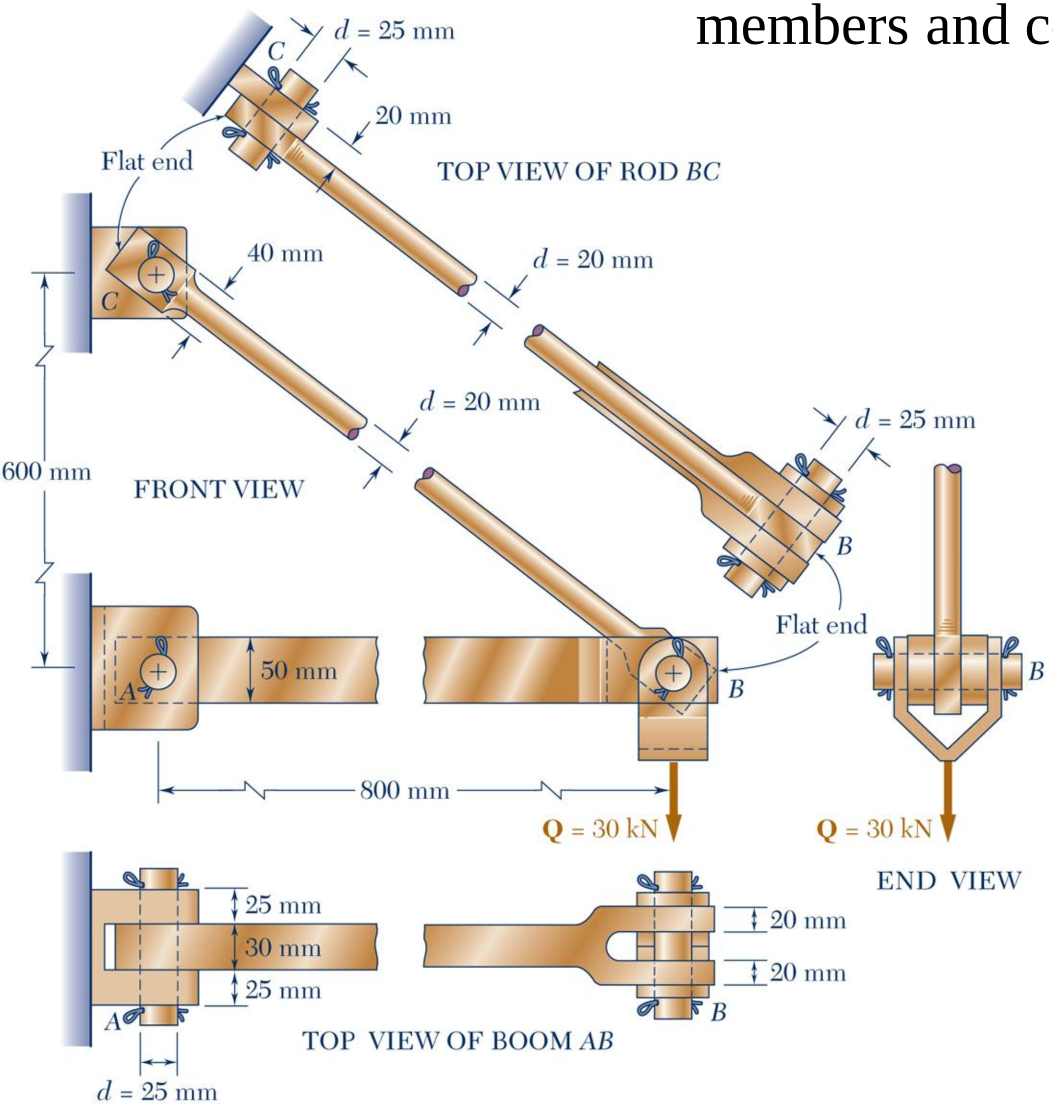
Procedure for Analysis

The statement of the problem should be clear and precise. It should contain the given data and indicate what information is required. A simplified drawing showing all essential quantities involved should be included.

1. Write the equilibrium equations of the system, this will require the drawing of one or several **FBDs**.
2. Determine the reactions at supports and internal forces and couples.
3. Compute the required stresses and deformations.
4. After the answer has been obtained, it should be carefully checked. It can be done by substituting the numerical values obtained into an equation which has not yet been used and verifying that the equation is satisfied.

Stress Analysis & Design Example

Example: determine the stresses in the members and connections of the structure shown.



From a statics analysis:

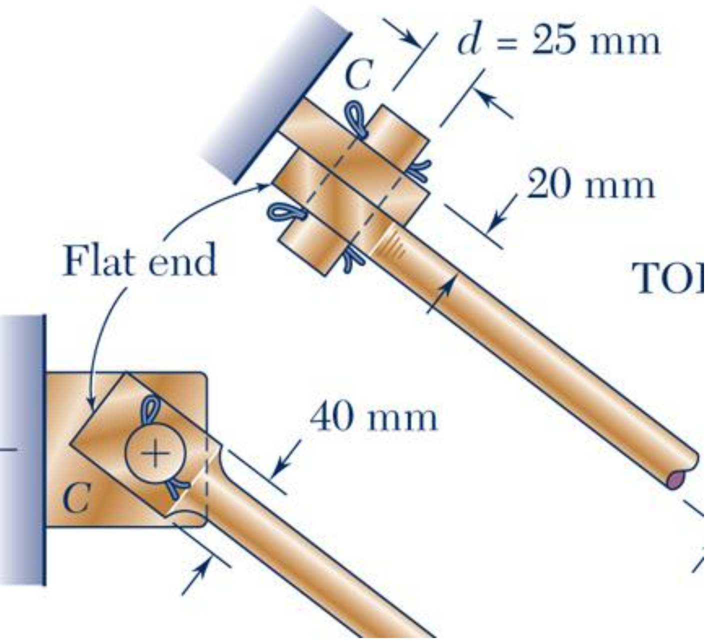
F_{AB} = 40 kN (compression)

F_{BC} = 50 kN (tension)

Must consider maximum normal stresses in AB and BC, and the shearing stress and bearing stress at each pinned connection

Rod & Boom Normal Stresses

The rod is in tension with an axial force of P = 50 kN and circular cross-sectional area $A_{BC} = 314\times10^{-6}\text{m}^2$



At the rod center, the average normal stress in the circular cross-section is $\sigma_{BC} = +159$ MPa.

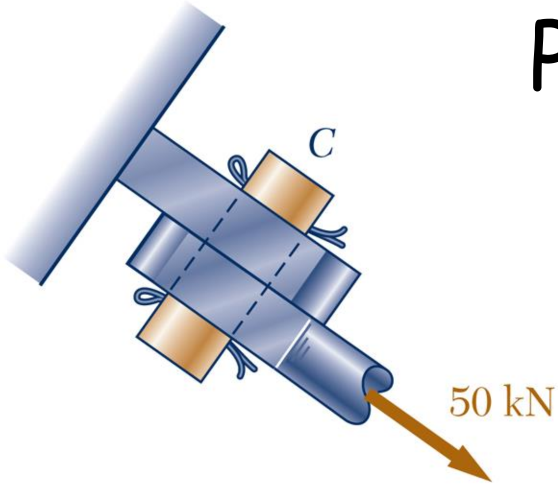
At the flattened rod ends, the smallest cross-sectional area occurs at the pin centerline,

$$A = (20\text{ mm})(40\text{ mm} - 25\text{ mm}) = 300\times10^{-6}\text{m}^2$$

$$\sigma_{BC,end} = \frac{P}{A} = \frac{50\times10^{3} N}{300\times10^{-6}\text{m}^2} = 167\text{ MPa}$$

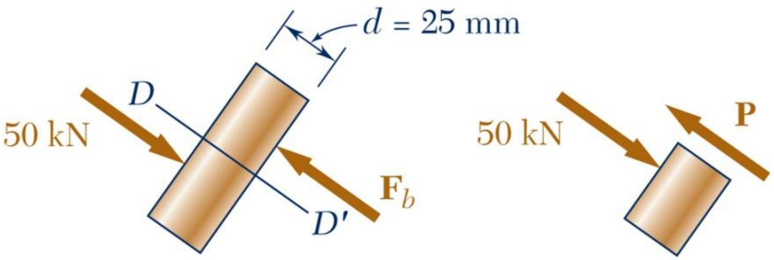
The boom is in compression with an axial force of 40 kN and average normal stress of –26.7 MPa. The **minimum area sections** at the boom ends are **unstressed** since the boom is in compression.

Pin Shearing Stresses



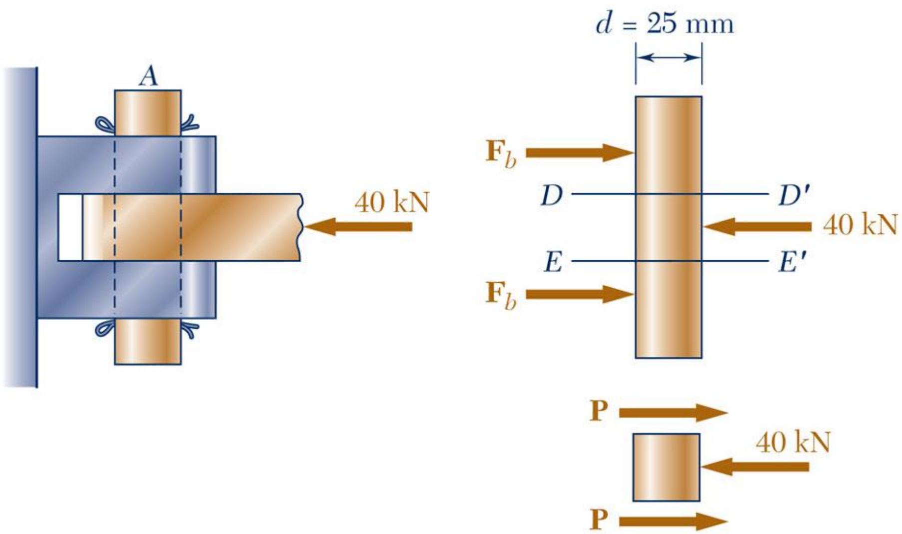
The cross-sectional area for pins at A, B, and C,

$$A = \pi r^2 = \pi\left(\frac{25\,\text{mm}}{2}\right)^2 = 491\times10^{-6}\,\text{m}^2$$



The force on the pin at C is equal to the force exerted by the rod BC,

$$\tau_{C,ave} = \frac{P}{A} = \frac{50\times10^{3}\,\text{N}}{491\times10^{-6}\,\text{m}^2} = 102\,\text{MPa}$$

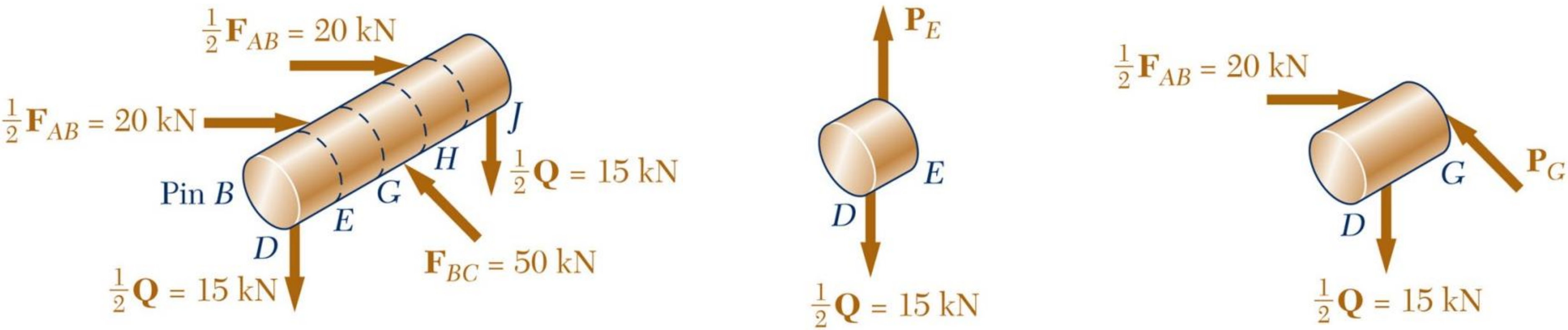


The pin at A is in double shear with a total force equal to the force exerted by the boom AB,

$$\tau_{A,ave} = \frac{P}{A} = \frac{20\,\text{kN}}{491\times10^{-6}\,\text{m}^2} = 40.7\,\text{MPa}$$

Pin Shearing Stresses

Divide the pin at *B* into sections to determine the section with the largest shear force



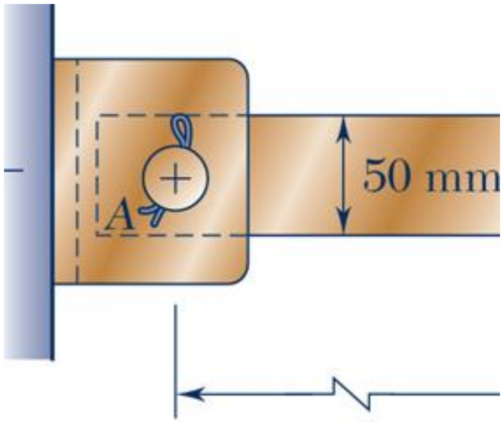
$$P_E = 15\text{ kN}$$

$$P_G = 25\text{ kN (larges}$$

Evaluate the corresponding average shearing stress,

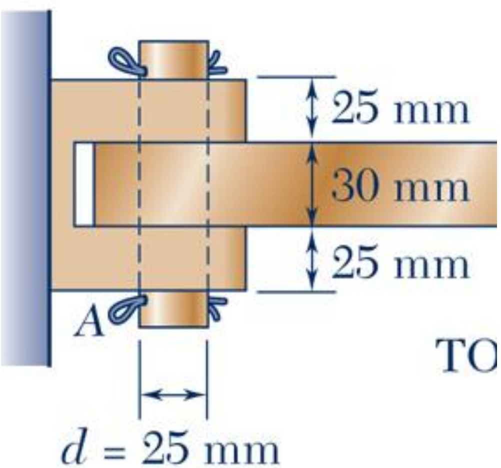
$$\tau_{B,ave} = \frac{P_G}{A} = \frac{25\text{ kN}}{491\times10^{-6}\text{ m}^2} = 50.9\text{ MPa}$$

Pin Bearing Stresses



To determine the bearing stress at *A* in the boom *AB*, we have t = 30 mm and d = 25 mm,

$$\sigma_b = \frac{P}{td} = \frac{40\,\text{kN}}{(30\,\text{mm})(25\,\text{mm})} = 53.3\,\text{MPa}$$

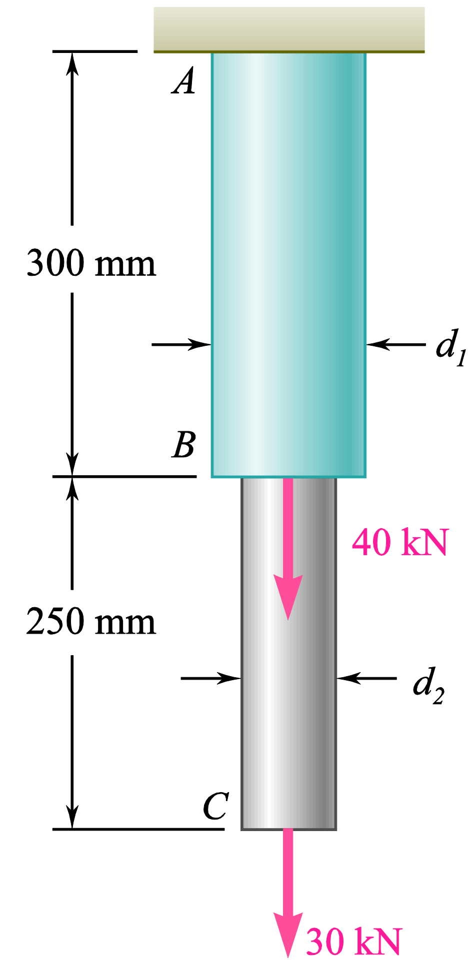


To determine the bearing stress at *A* in the bracket, we have t = 2(25 mm) = 50 mm and d = 25 mm,

$$\sigma_b = \frac{P}{td} = \frac{40\,\text{kN}}{(50\,\text{mm})(25\,\text{mm})} = 32.0\,\text{MPa}$$

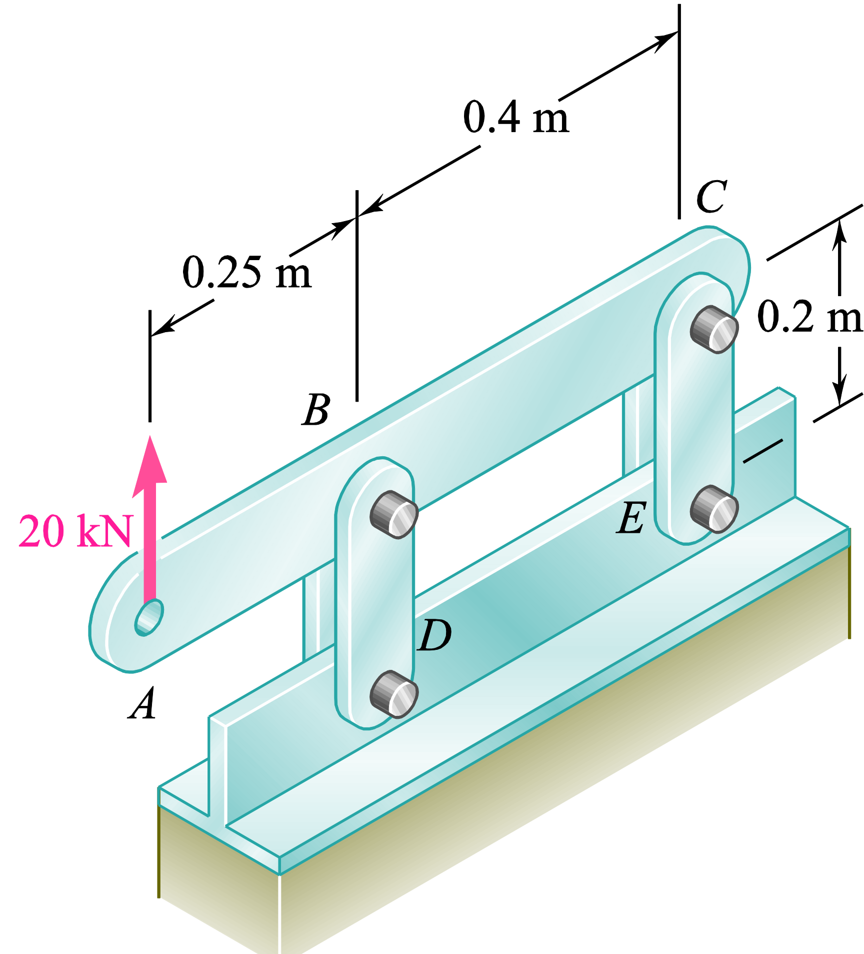
Example 1

1-4: Two solid cylindrical rods AB and BC are welded together at B and loaded as shown. Knowing that d_1=50 mm and d_2=30 mm, find the average normal stress at the midsection of (a) rod AB, (b) rod BC



Example 2

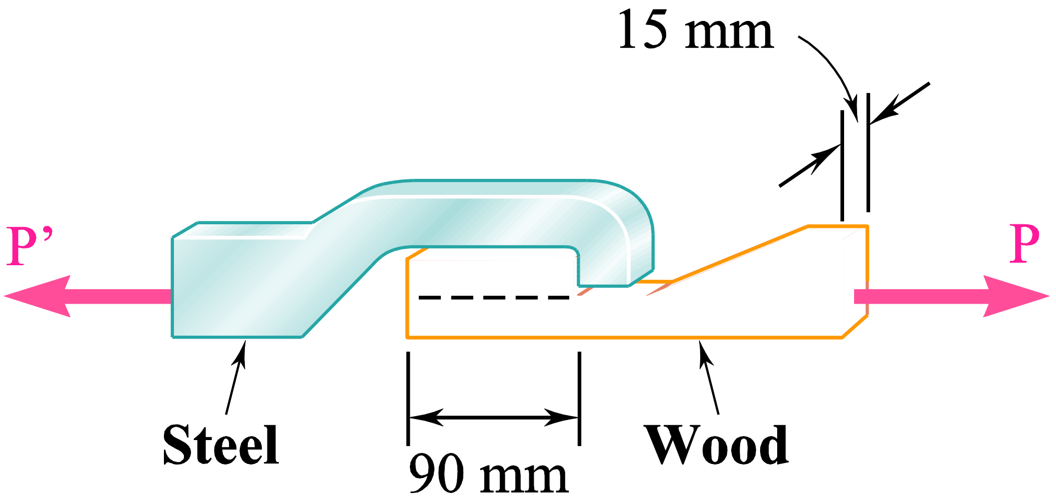
1-7: Each of the four vertical links has an 8×36-mm uniform rectangular cross section and each of the four pins has a 16-mm diameter. Determine the maximum value of the average normal stress in the links connecting (a) points B and D, (b) points C and E.



Example 2

Example 3

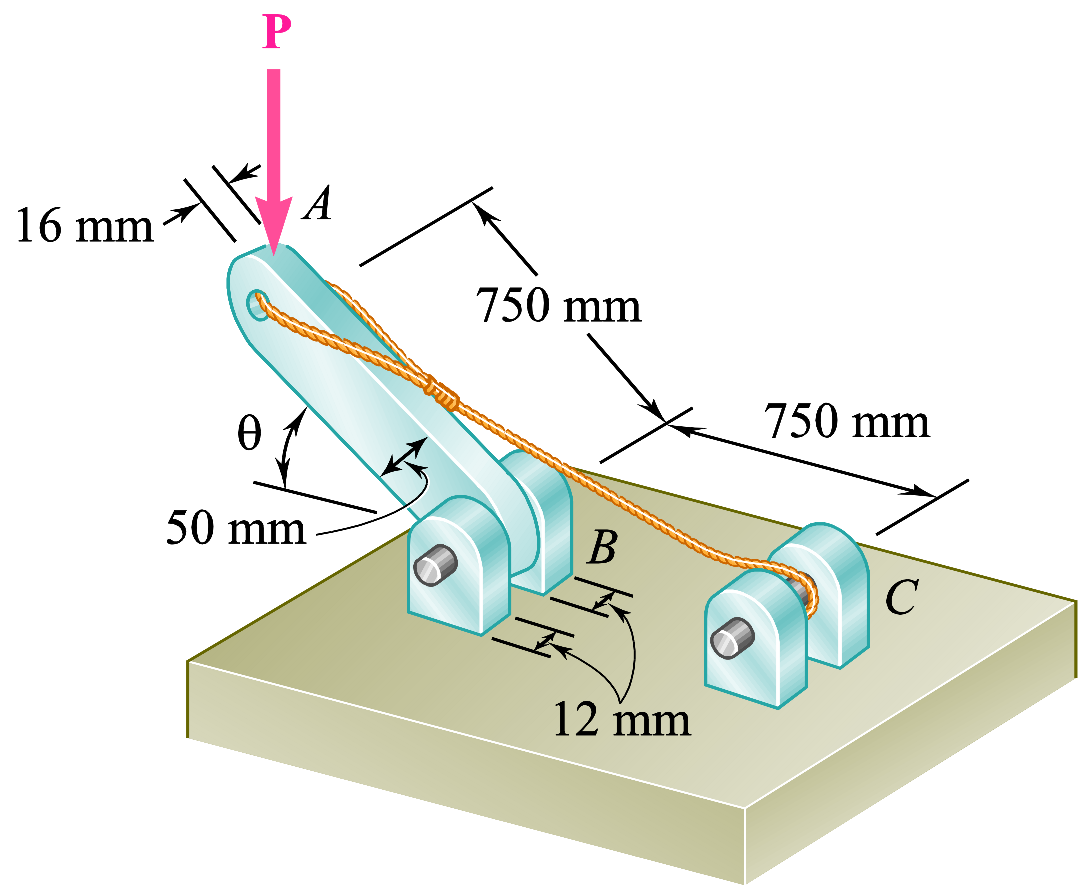
1-17: When the force P reaches 8 kN, the wooden specimen shown failed in shear along the surface indicated by the dashed line. Determine the average shearing stress along that surface at the time of failure.



Example 3

Example 4

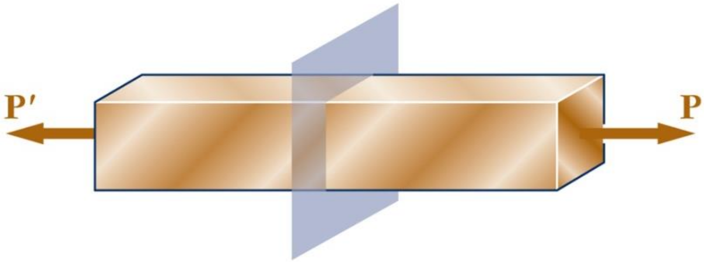
1-25: Knowing that θ = 40° and P = 9 kN, determine **(a)** the smallest allowable diameter of the pin at B if the average shearing stress in the pin is not to exceed 120 MPa, **(b)** the corresponding average bearing stress in member AB at B, **(c)** the corresponding average bearing stress in each of the support brackets at B.



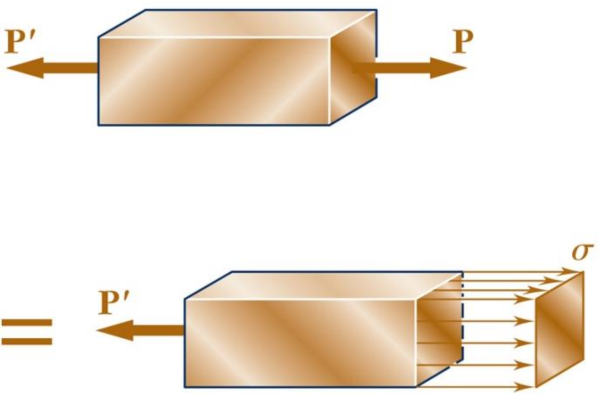
Example 4

Example 4

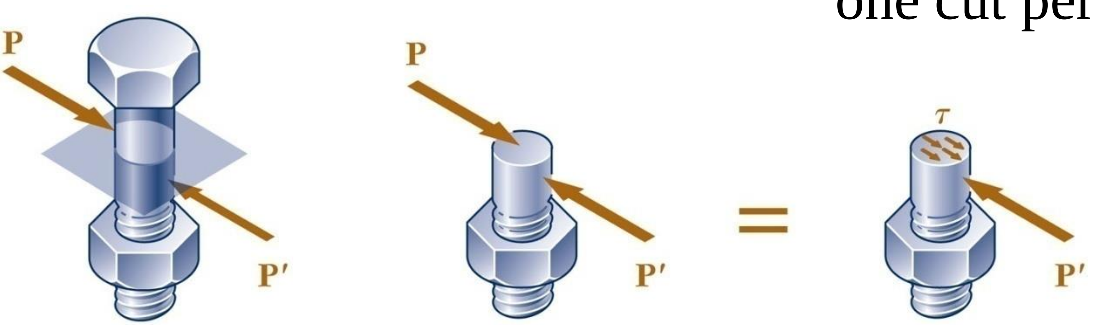
Stress in Two Force Members



(a)



(b)

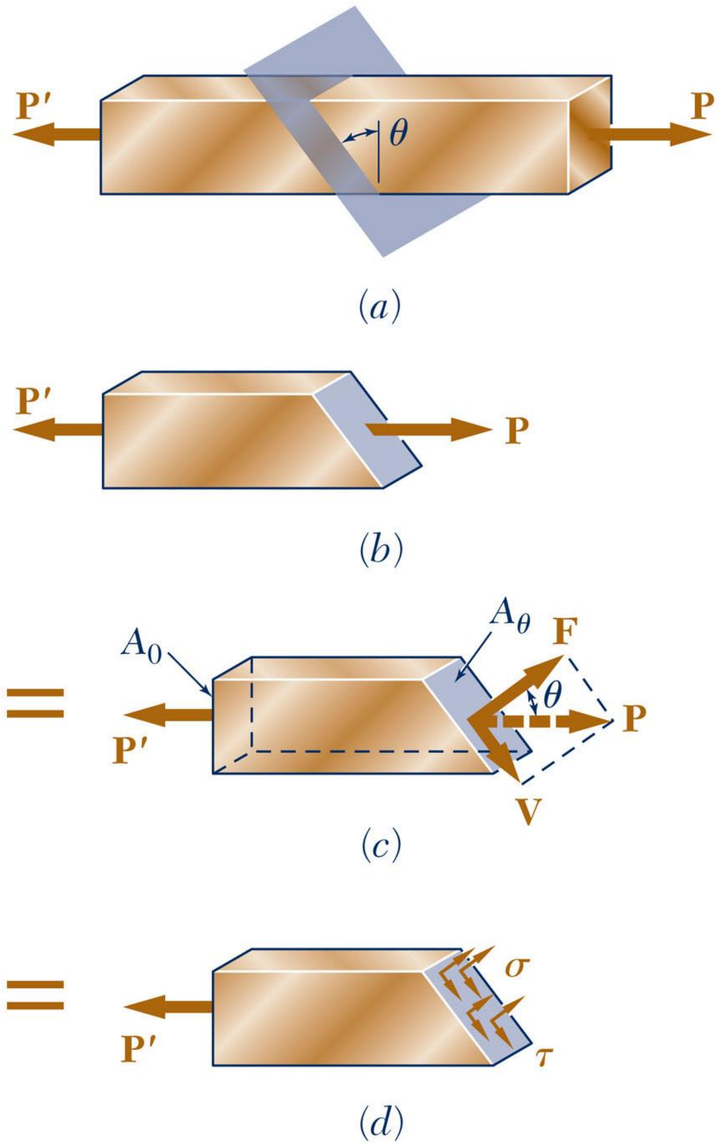


Axial forces on a two force member result in only normal stresses on a plane cut perpendicular to the member axis.

Transverse forces on bolts and pins result in only shear stresses on the plane perpendicular to bolt or pin axis.

We will show that either axial or transverse forces may produce both normal and shear stresses with respect to a plane other than one cut perpendicular to the member axis.

Stress on an Oblique Plane



Pass a section through the member forming an angle θ with the normal plane.

From equilibrium conditions, the distributed forces (stresses) on the plane must be equivalent to the force P.

Resolve P into components normal and tangential to the oblique section,

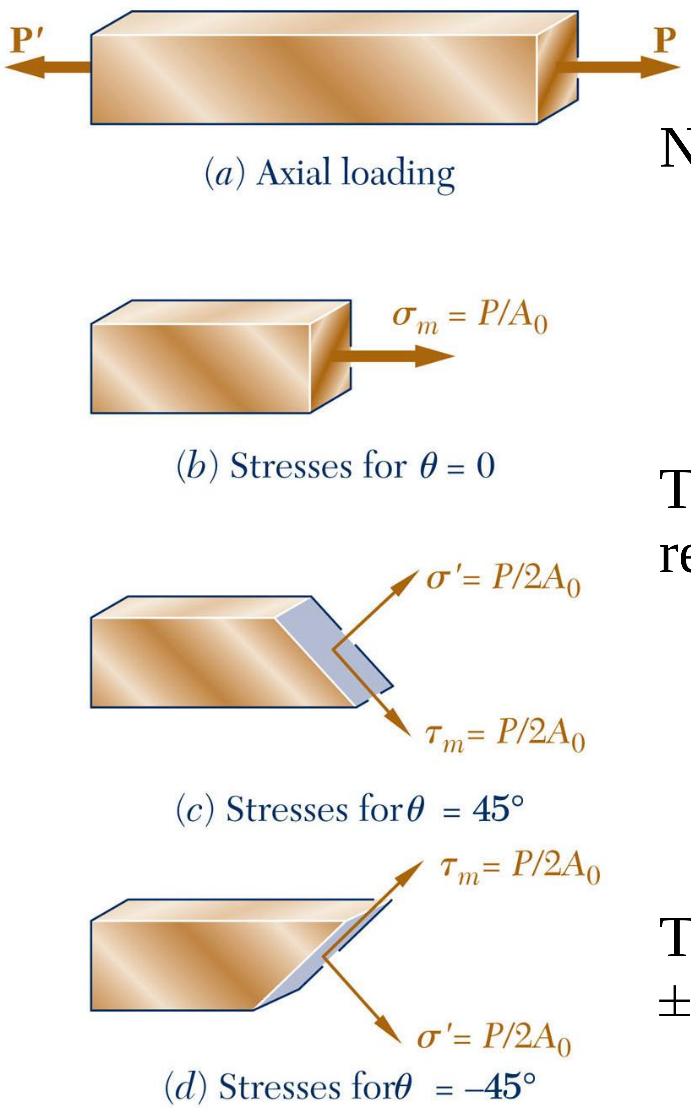
$$F = P\cos\theta \qquad V = P\sin\theta$$

The average normal and shear stresses on the oblique plane are

$$\sigma = \frac{F}{A_\theta} = \frac{P\cos\theta}{A_0/\cos\theta} = \frac{P}{A_0}\cos^2\theta$$

$$\tau = \frac{V}{A_\theta} = \frac{P\sin\theta}{A_0/\cos\theta} = \frac{P}{A_0}\sin\theta\cos\theta$$

Maximum Stresses



Normal and shearing stresses on an oblique plane

$$\sigma = \frac{P}{A_0}\cos^2\theta \quad \tau = \frac{P}{A_0}\sin\theta\cos\theta$$

The maximum normal stress occurs when the reference plane is perpendicular to the member axis,

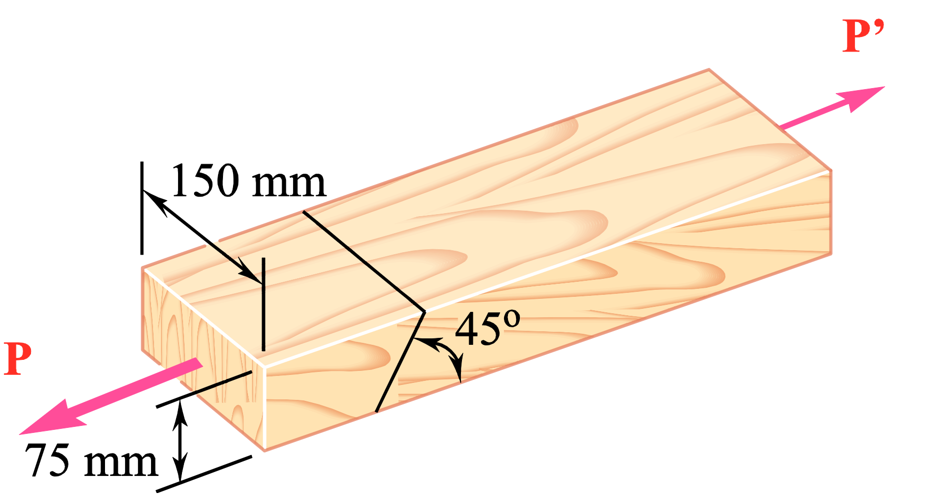
$$\sigma_m = \frac{P}{A_0}, \quad \tau' = 0$$

The maximum shear stress occurs for a plane at ±45° with respect to the axis,

$$\tau_m = \frac{P}{A_0}\sin 45\cos 45 = \frac{P}{2A_0} = \sigma'$$

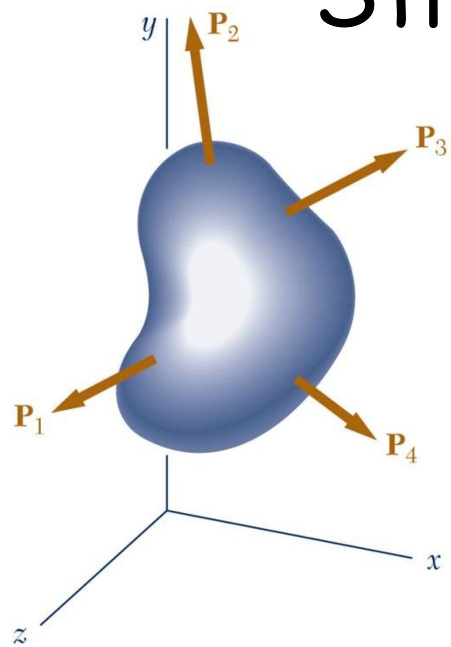
Example 5

1-30: Two wooden members of uniform rectangular cross section are joined by the simple glued scarf splice shown. Knowing that the maximum allowable shearing stress in the glued splice is 620 kPa, determine **(a)** the largest load **P** that can be safely applied, **(b)** the corresponding tensile stress in the splice.



Example 5

Stress Under General Loadings

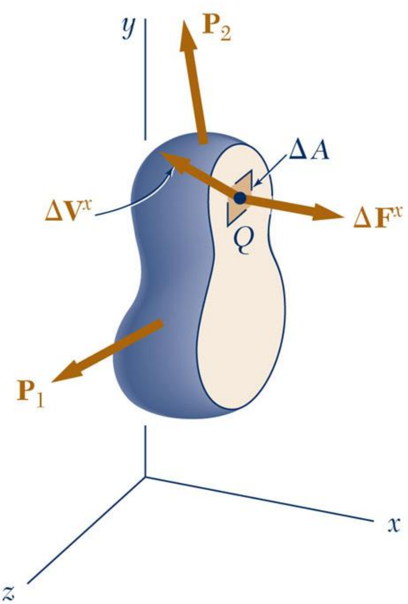


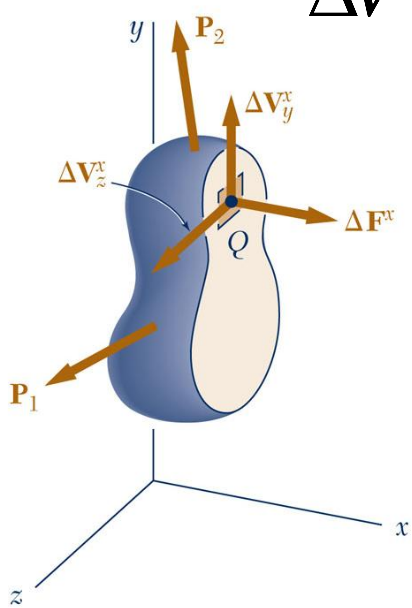
A member subjected to a general combination of loads is cut into two segments by a plane passing through *Q*. The distribution of internal force components may be defined as,

ΔF^x normal force acting on a small area ΔA

ΔV^x shearing force acting on a small area ΔA

$$\Delta V^x = \Delta V^x_y + \Delta V^x_z$$

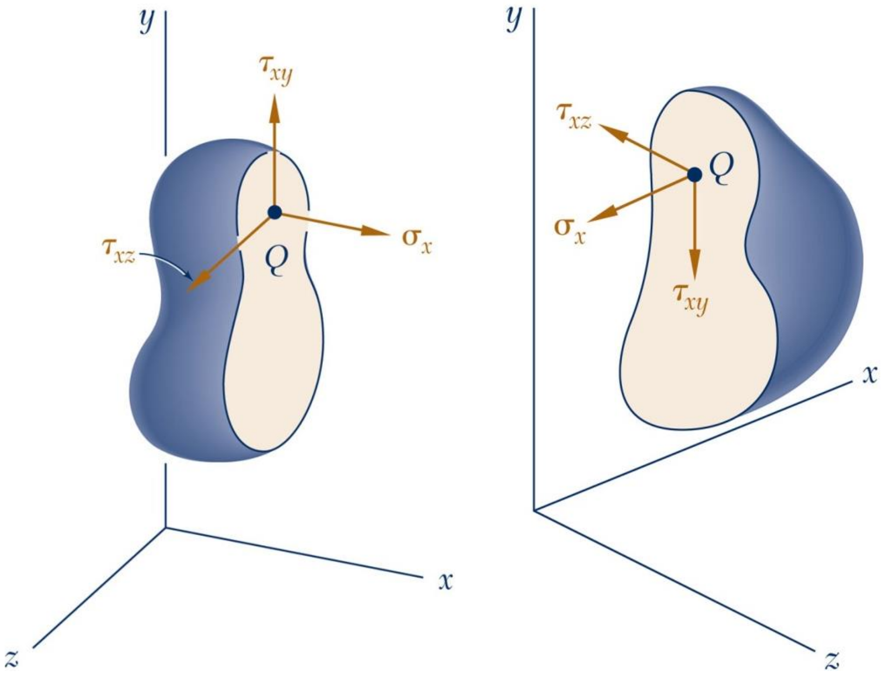




ΔV^x_y component parallel to the y-axis

ΔV^x_z component parallel to the z-axis

Stress Under General Loadings



The distribution of internal stress components may be defined as,

$$\sigma_x = \lim_{\Delta A \to 0} \frac{\Delta F^x}{\Delta A}$$

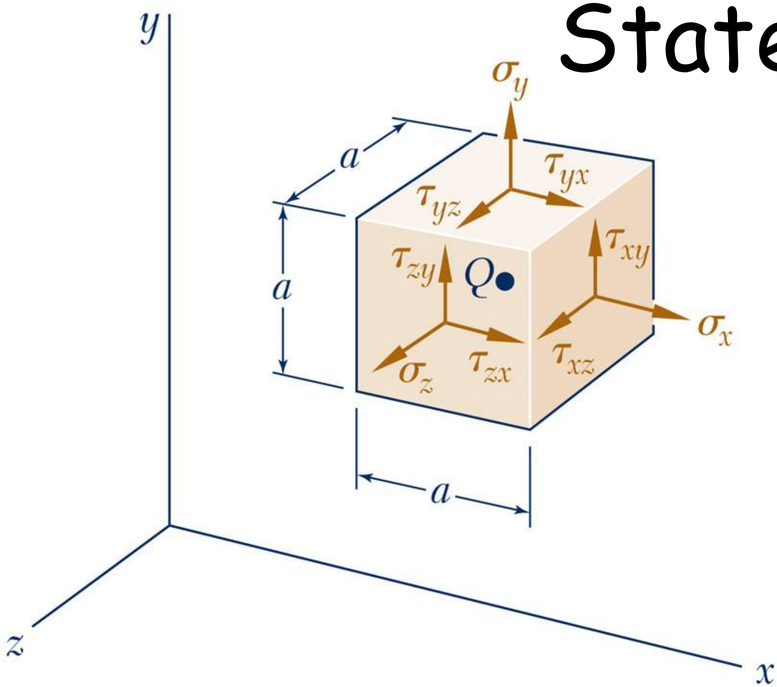
$$\tau_{xy} = \lim_{\Delta A \to 0} \frac{\Delta V_y^x}{\Delta A} \qquad \tau_{xz} = \lim_{\Delta A \to 0} \frac{\Delta V_z^x}{\Delta A}$$

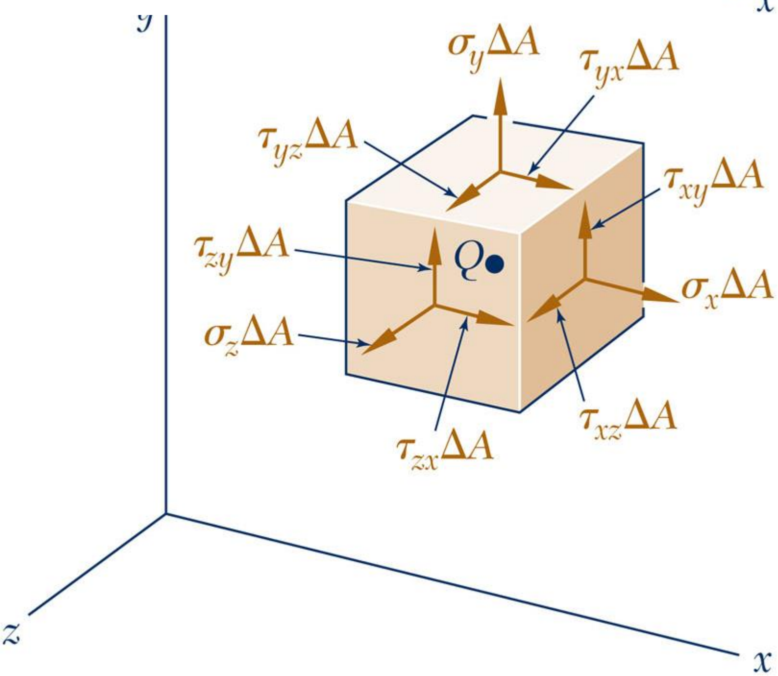
τ_{xy} The y component of the shear stress exerted on the face perpendicular to the x-axis

τ_{xz} The z component of the shear stress exerted on the face perpendicular to the x-axis

For equilibrium, an equal and opposite internal force and stress distribution must be exerted on the other segment of the member.

State of Stress





Stress components are defined for the planes cut parallel to the *x*, *y* and *z* axes.

For equilibrium, equal and opposite stresses are exerted on the hidden planes.

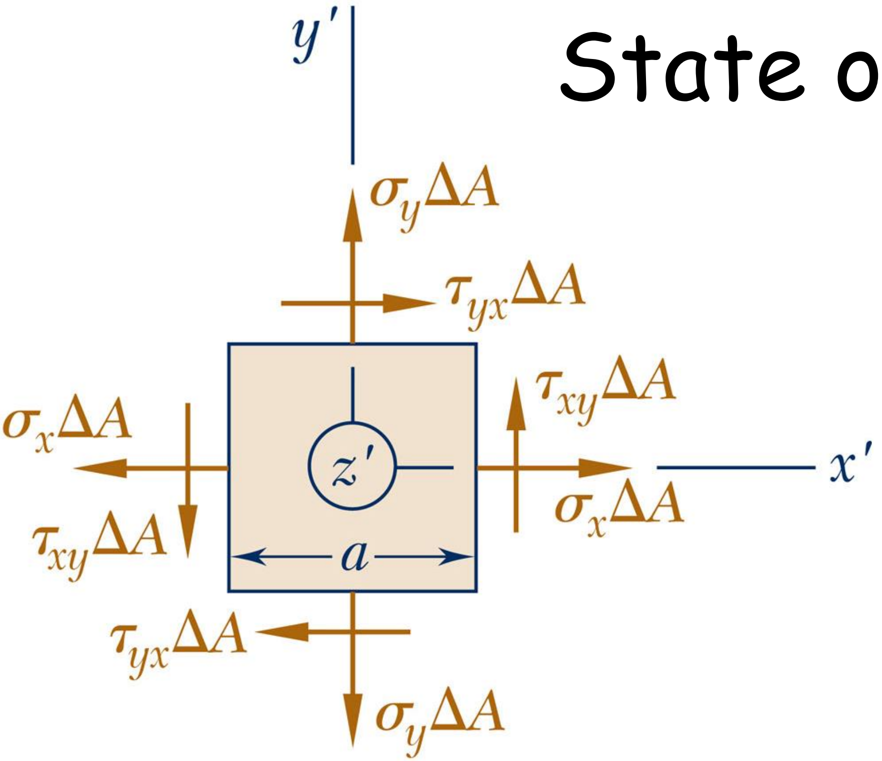
Stress components are defined for the planes cut parallel to the *x*, *y* and *z* axes. For equilibrium, equal and opposite stresses are exerted on the hidden planes.

The combination of forces generated by the stresses must satisfy the conditions for equilibrium:

$$\sum F_x = \sum F_y = \sum F_z = 0$$

$$\sum M_x = \sum M_y = \sum M_z = 0$$

State of Stress



Consider the moments about the *z* axis:

$$\sum M_z = 0 = (\tau_{xy} \Delta A)a - (\tau_{yx} \Delta A)a$$

$$\tau_{xy} = \tau_{yx}$$

Similarly $\tau_{yz} = \tau_{zy}$

$\tau_{xz} = \tau_{zx}$

It follows that only **6 components of stress** are required to define the complete state of stress

$$\sigma_x, \sigma_y, \sigma_z, \tau_{xy}, \tau_{yz} \text{ and } \tau_{zx}$$

Factor of Safety

Structural members or machines must be designed such that the working stresses are less than the ultimate strength of the material.

Factor of safety considerations:

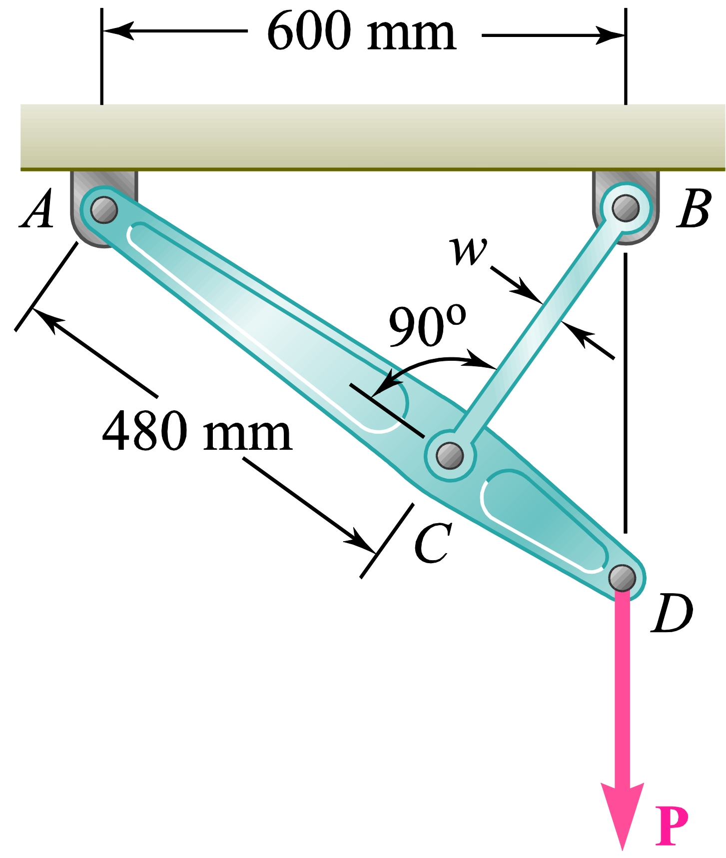
- Uncertainty in material properties
- Uncertainty of loadings
- Uncertainty of analyses
- Number of loading cycles
- Types of failure
- Maintenance requirements
- Deterioration effects

$$FS = \text{Factor of safety}$$

$$FS = \frac{\sigma_u}{\sigma_{all}} = \frac{\text{ultimate stress}}{\text{allowable stress}}$$

Example 6

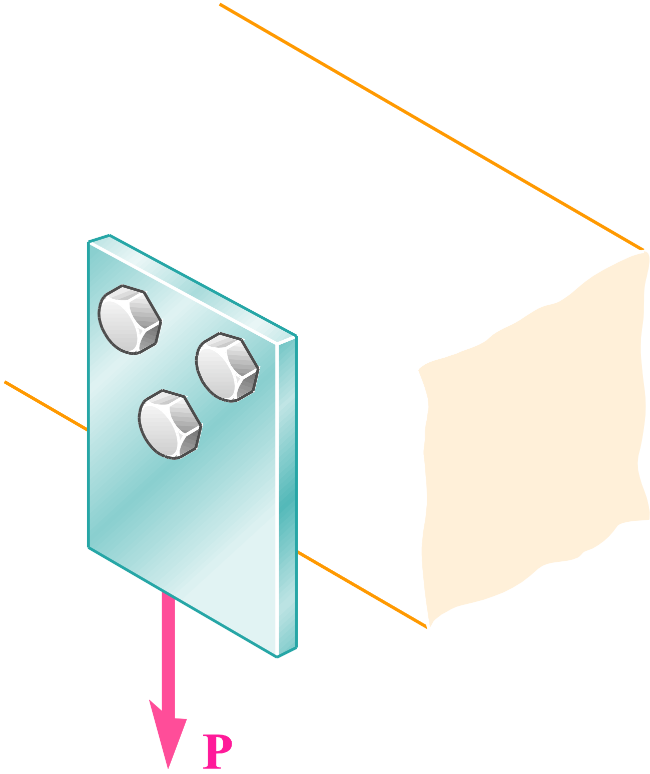
1-38: Link *BC* is 6 mm thick, has a width w = 25 mm, and is made of a steel with a 480 MPa ultimate strength in tension. What was the safety factor used if the structure shown was designed to support a 16 kN load **P**?



Example 6

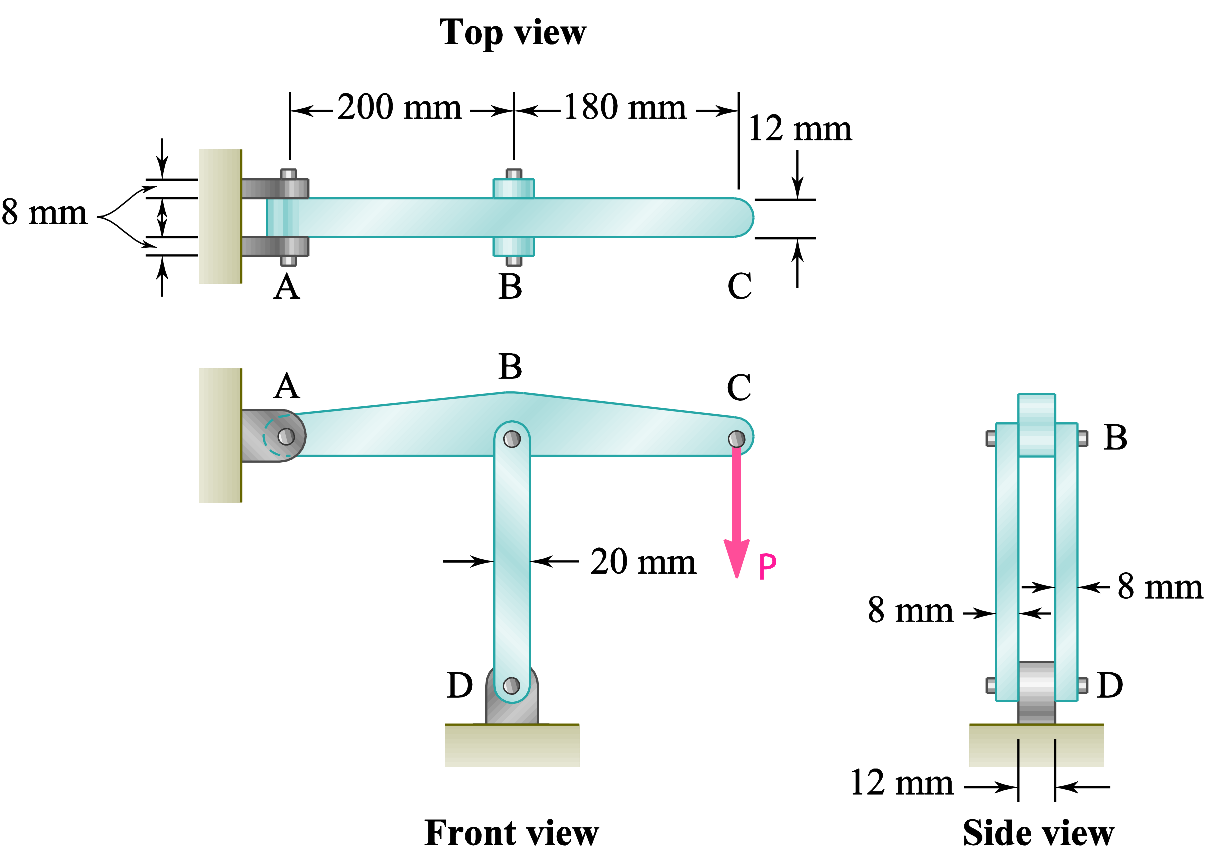
Example 7

1-46: Three steel bolts are to be used to attach the steel plate shown to a wooden beam. Knowing that the plate will support a 110 kN load, that the ultimate shearing stress for the steel used is 360 MPa, and that a factor of safety of 3.35 is desired, determine the required diameter of the bolts.



Example 8

1-55: In the structure shown, an 8 mm diameter pin is used at *A*, and 12-mm-diameter pins are used at *B* and *D*. Knowing that the ultimate shearing stress is 100 MPa at all connections and that the ultimate normal stress is 250 MPa in each of the two links joining *B* and *D*, determine the allowable load **P** if an overall factor of safety of 3.0 is desired.



Example 8

Example 8