Chapter 1: Fundamentals of Atomic and Nuclear Physics

Slide set of 44 slides based on the chapter authored by
K. H. Ng and D. R. Dance
of the IAEA publication (ISBN 978-92-0-131010-1):

Diagnostic Radiology Physics:
A Handbook for Teachers and Students

Objective:

To familiarize students with basic principles of atomic and nuclear Physics used in diagnostic radiology

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Chapter 1. TABLE OF CONTENTS

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1.1 INTRODUCTION

Knowledge of the

- structure of the atom
- elementary nuclear physics
- the nature of electromagnetic radiation
- production of X-rays

is fundamental to the understanding of the physics of medical imaging and radiation protection. This, the first chapter of the Handbook, summarises those aspects of these areas which, being part of the foundation of modern physics, underpin the remainder of the book

Radiation may be classified as:

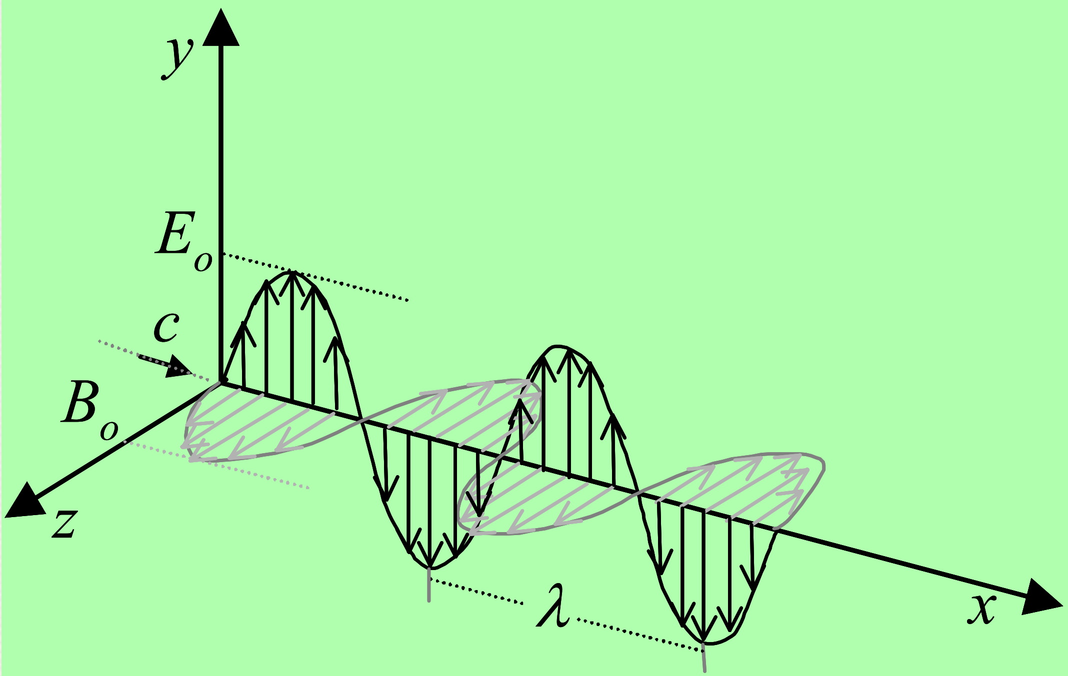
Electromagnetic radiation

- radiofrequency
- infrared
- visible light
- ultraviolet
- X rays
- gamma rays

Particulate radiation

- electrons
- positrons
- protons
- neutrons

1.2. CLASSIFICATION OF RADIATION
1.2.1. Electromagnetic radiation



Electromagnetic waves consist of oscillating electric and magnetic fields, which are at right angles to each other and also to the direction of wave propagation

They are characterized by their:

- amplitudes E_o and B_o
- wavelength (λ)
- frequency (ν) and
- speed $c = \lambda\ \nu$

In vacuum, $c = 3\times10^8$m/s

For X rays:

- **wavelength is usually expressed in nanometre (nm) (1 nm = 10^{-9}m) and**
- **frequency is expressed in hertz (Hz) (1 Hz = 1 cycle/sec = 1 sec^{-1})**

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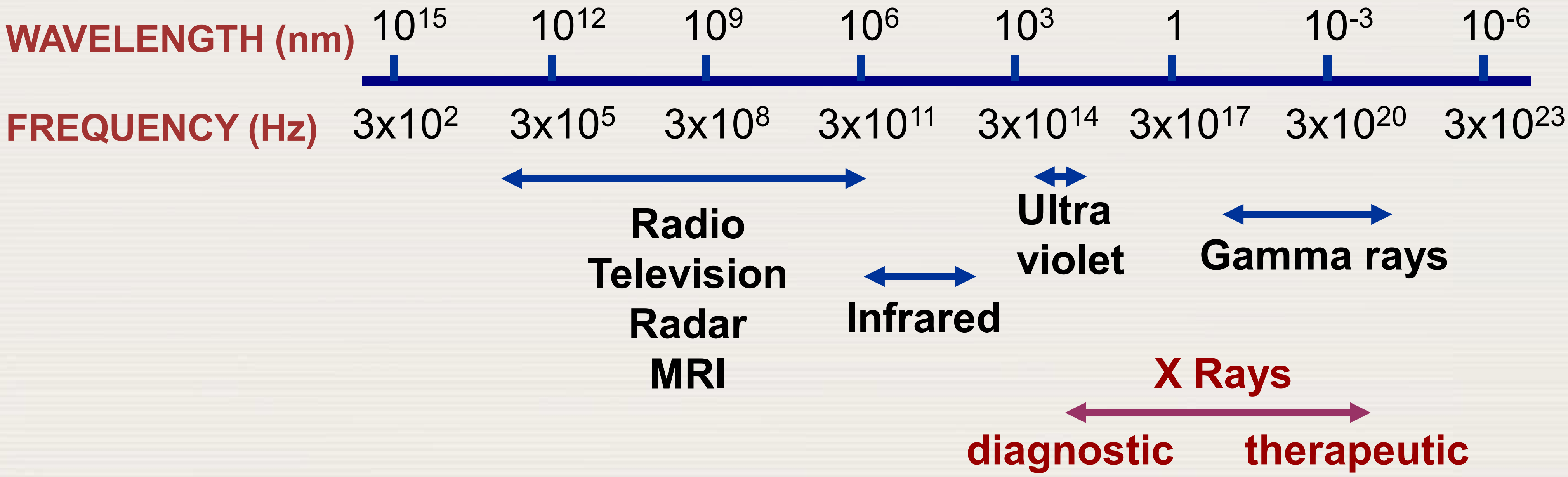
1.2. CLASSIFICATION OF RADIATION

1.2.1. Electromagnetic radiation



Electromagnetic spectrum as a function of:

- **wavelength (nm)**
- **frequency (Hz)**

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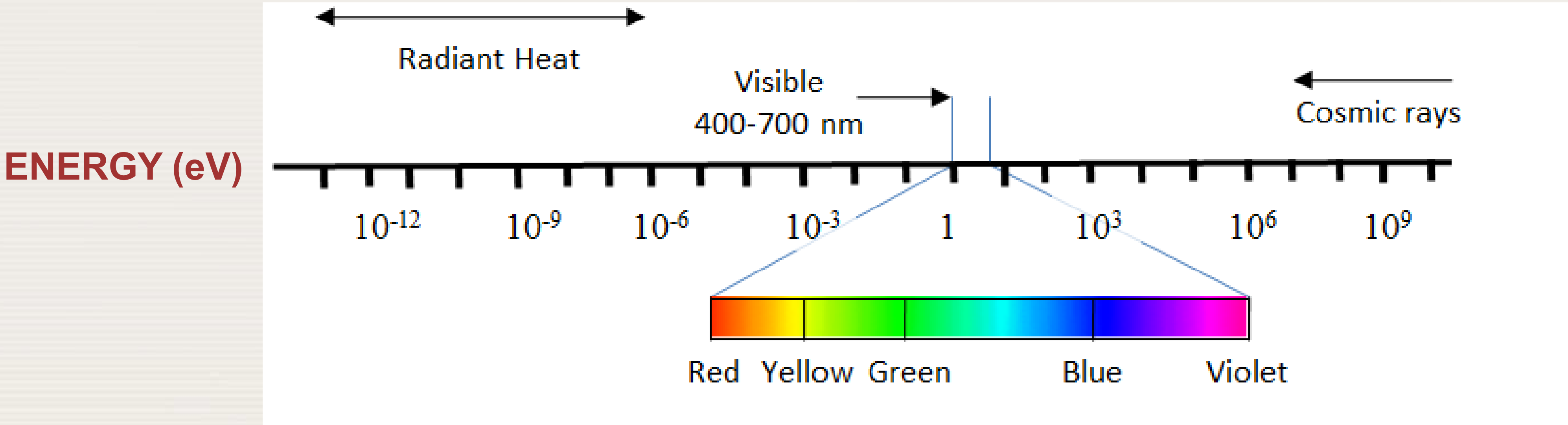
1.2. CLASSIFICATION OF RADIATION

1.2.1. Electromagnetic radiation



Electromagnetic spectrum as a function of:

- **photon energy (eV)**

1.2 CLASSIFICATION OF RADIATION

1.2.1. *Electromagnetic radiation*

When interactions with matter are considered, electromagnetic radiation is generally treated as series of individual particles, known as photons. The energy E of each photon is given by:

$$E = h\nu = hc/\lambda$$

h (Planck's constant) = 6.63×10^{-34} J·s = 4.14×10^{-15} eV·s
1 eV = 1.6×10^{-19} J, is the energy given to an electron by accelerating it through 1 volt of electric potential difference

ν (Hz = s^{-1}) is the frequency of electromagnetic wave
λ (m) is the wavelength of electromagnetic wave

In diagnostic radiology the photon energy is usually expressed in units of keV. 1 keV = 1000 eV

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In diagnostic radiology, the only particulate radiation that needs to be considered is the electron

rest mass of electron = 9.109×10^{-31} kg

rest energy of electron = 511 keV = 0.511 MeV

1.2.3. Ionizing and non-ionizing radiation

Non-ionizing radiation - cannot ionize matter: (electromagnetic radiation with energy below the far-ultraviolet region, e.g. visible light, infrared and radiofrequency)

Ionizing radiation - can ionize matter: (fast charged particles, X rays, gamma rays and neutrons)

Ionizing radiation - can ionize matter either:

Directly:

fast charged particles that deposit their energy in matter directly, through many small Coulomb (electrostatic) interactions with orbital electrons along the particle track

Indirectly:

X- or gamma- ray photons or neutrons that first transfer their energy to fast charged particles released in one or a few interactions in the matter through which they pass. The resulting fast charged particles then deposit their energy directly in the matter

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1.2.3. Ionizing and non-ionizing radiation

Ionization potential is the minimum energy required to ionize an atom. For elements its magnitude ranges from a few eV for alkali metals to 24.5 eV for helium. For water it is 12.6 eV

Element	Ionization potential (eV)
H	13.6
C	11.3
O	13.6
Mo	7.1
W	7.9

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An atom is composed of a central nucleus surrounded by a cloud of negatively charged electrons

Most of the mass of the atom is concentrated in the atomic nucleus which consists of:

- ***Z* protons and**
- **(*A* – *Z*) = N neutrons**

***Z*: Atomic number**
***A*: Atomic mass number**

Unified atomic mass unit μ: a unit used for specifying the masses of atoms

1 μ = 1/12 of the mass of the ^{12}C atom or 931.5 MeV/c^2

1.3. ATOMIC AND NUCLEAR STRUCTURE

1.3.1. Basic definitions

Particle	Charge (C)	Rest energy (MeV)
Electron (e)	-1.602×10^{-19}	0.511
Proton (p)	$+1.602\times10^{-19}$	938.28
Neutron (n)	0	939.57

In a non-ionised atom:
number of electrons = number of protons

Radius of an atom ≈ 0.1 nm
Radius of the nucleus ≈ 10^{-5} nm

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1.3. ATOMIC AND NUCLEAR STRUCTURE

1.3.1. Basic definitions

Protons and neutrons are referred to as nucleons

They are bound in the nucleus with the strong force

The strong force between two nucleons is a very short-range force, active only at distances of the order of a few femtometer (fm). 1 fm = 10^{-15} m

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1.3. ATOMIC AND NUCLEAR STRUCTURE

1.3.1. Basic definitions

Atomic mass number = Z+N

Chemical symbol for the element

$$^{A}_{Z}\mathrm{X} \quad \text{or} \quad \mathrm{X\text{-}A}$$

Atomic number

$^{60}_{27}\mathrm{Co}$ (Co-60) nucleus of Cobalt-60 with 27 protons and 33 neutrons

$^{137}_{55}\mathrm{Cs}$ (Cs-137) nucleus of Cesium-137 with 55 protons and 82 neutrons

$^{226}_{88}\mathrm{Ra}$ (Ra-226) nucleus of Radium-226 with 88 protons and 138 neutrons

Empirical relation between A and Z

$$Z = \frac{\mathrm{A}}{1.98 + 0.0155\mathrm{A}^{2/3}}$$

Isotopes of a given element have in the nucleus :

- **same number of protons, but**
- **different numbers of neutrons**

ordinary hydrogen	${}^{1}_{1}\mathrm{H}$
deuterium	${}^{2}_{1}\mathrm{H}$
tritium	${}^{3}_{1}\mathrm{H}$

Isotopes of chemical element hydrogen (Z = 1)

${}^{12}_{6}\mathrm{C}$

${}^{13}_{6}\mathrm{C}$

${}^{14}_{6}\mathrm{C}$

Isotopes of chemical element carbon (Z = 6)

1.3. ATOMIC AND NUCLEAR STRUCTURE

1.3.1. Basic definitions

Atomic weight A_r is a dimensionless physical quantity

$$A_r = \frac{\text{average mass of the atoms of an element}}{\text{unified atomic mass unit}}$$

The average is a weighted mean over all the isotopes of the particular element taking account of their relative abundance

Atomic mass *M* is expressed in unified atomic mass unit

The atomic mass *M* for a particular isotope is smaller than the sum of the individual masses of constituent particles because of the intrinsic energy associated with binding the particles (nucleons) within the nucleus

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1.3.1. Basic definitions

Atomic g-atom (gram-atom) is the number of grams of an atomic substance that contains a number of atoms exactly equal to one Avogadro's constant ($N_A = 6.022 \times 10^{23}$ atoms/g-atom)

Atomic weight definition means that A_r grams of each element contain exactly N_A atoms. For a single isotope M grams contain N_A atoms

Example:

- 1 gram-atom of Cobalt- 60 is 59.93 g of Co-60
- 1 gram-atom of Radium-226 is 226.03 g of Ra-226

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1.3. ATOMIC AND NUCLEAR STRUCTURE

1.3.1. Basic definitions

Molecular g-mole (gram-mole) is defined as the number of grams of a molecular compound that contains exactly one Avogadro's constant of molecules ($N_A = 6.022 \times 10^{23}$ molecule/g-mole)

The mass of a molecule is the sum of the masses of the atoms that make up the molecule

Example:

- 1 gram-mole of water is ≈18 g of water
- 1 gram-mole of carbon dioxide is ≈ 44 g of carbon dioxide

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1.3.1. Basic definitions

Number of atoms per unit mass of an element:

$$N_{am} = \frac{N_A}{A_r}$$

Number of electrons per unit mass of an element:

$$ZN_{am} = \frac{Z}{A_r} N_A$$

Number of electrons per unit volume of an element:

$$ZN_{aV} = \rho ZN_{am} = \rho Z \frac{N_A}{A_r}$$

N_A: Avogadro constant, Z : atomic number
A_r : atomic weight, ρ : density

Note that $(Z/A_r) \approx 0.5$ for all elements, except for hydrogen, for which $(Z/A_r) = 1$. Actually, (Z/A_r) slowly decreases from 0.5 for low Z elements to 0.4 for high Z elements

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Modern quantum mechanical model of the atom is built on the work of many physicists

The idea of a dense central nucleus surrounded by orbiting electrons was first proposed by Ernest Rutherford in 1911

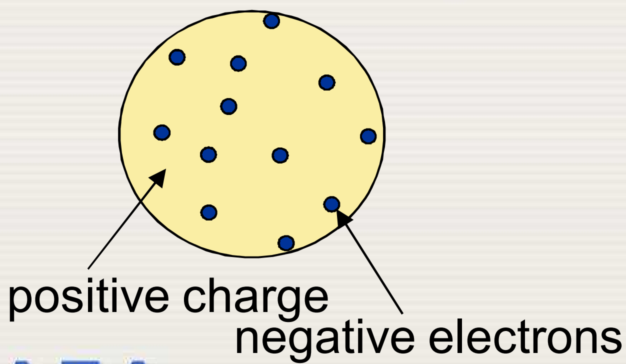
Rutherford's atomic model is based on results of the Geiger- Marsden experiment of 1909 with α particles emitted from Radium C, scattered on thin gold foils with a thickness of 0.00004 cm

Geiger and Marsden found that:

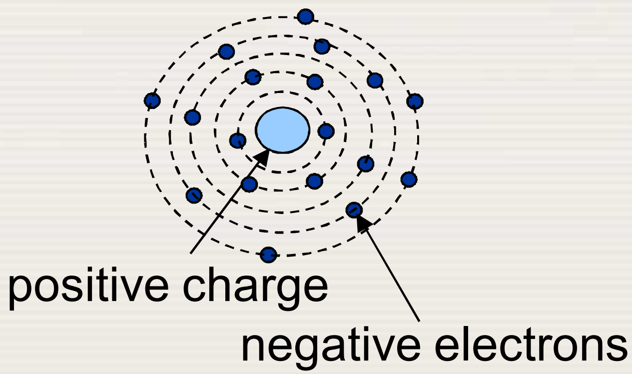
- more than 99% of the α particles incident on the gold foil were scattered at scattering angles less than 3°
- roughly 1 in 10^4 alpha particles was scattered with a scattering angle exceeding 90°

This finding (1 in 10^4) was in drastic disagreement with the theoretical prediction of one in 10^{3500} resulting from Thomson's atomic model

Thomson atomic model



Rutherford atomic model

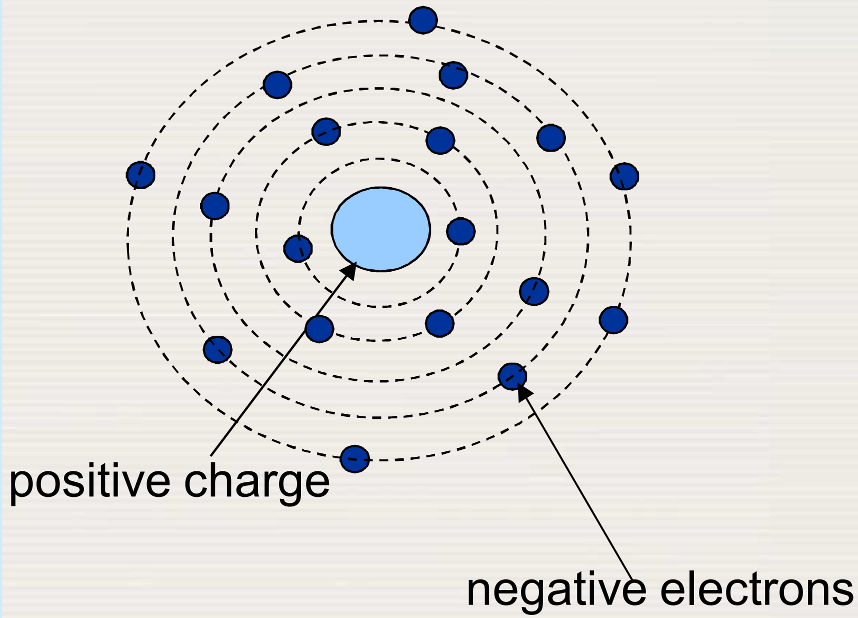


1.3.2. Atomic structure

Rutherford proposed that:

- **mass and positive charge of the atom are concentrated in the nucleus of the size of the order of 10^{-15} m**
- **negatively charged electrons revolve about the nucleus with a radius of the order of 10^{-10} m**

Rutherford atomic model



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1.3. ATOMIC AND NUCLEAR STRUCTURE

1.3.2. Atomic structure

The Rutherford atomic model, however, had a number of unsatisfactory features

For example, it could not explain the observed emission spectra of the elements

Visible lines of emission spectrum for Hydrogen

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In 1913, Niels Bohr elaborated the model of hydrogen atom, based on four postulates:

- the electron revolves in circular allowed orbit about the proton under the influence of the Coulomb force of attraction being balanced by the centripetal force arising from the orbital motion
- while in orbit, the electron does not lose any energy in spite of being constantly accelerated
- the angular momentum of the electron in an allowed orbit is quantized and only takes values of $n\hbar$, where n is an integer and $\hbar = h/2\pi$, where h is Planck's constant
- an atom emits radiation when an electron makes a transition from an initial orbit with quantum number n_i to a final orbit with quantum number n_f for $n_i > n_f$.

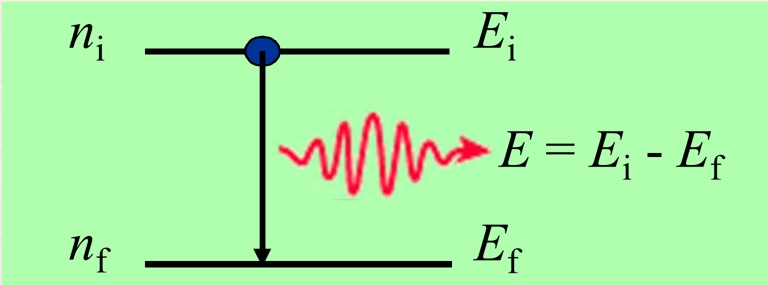
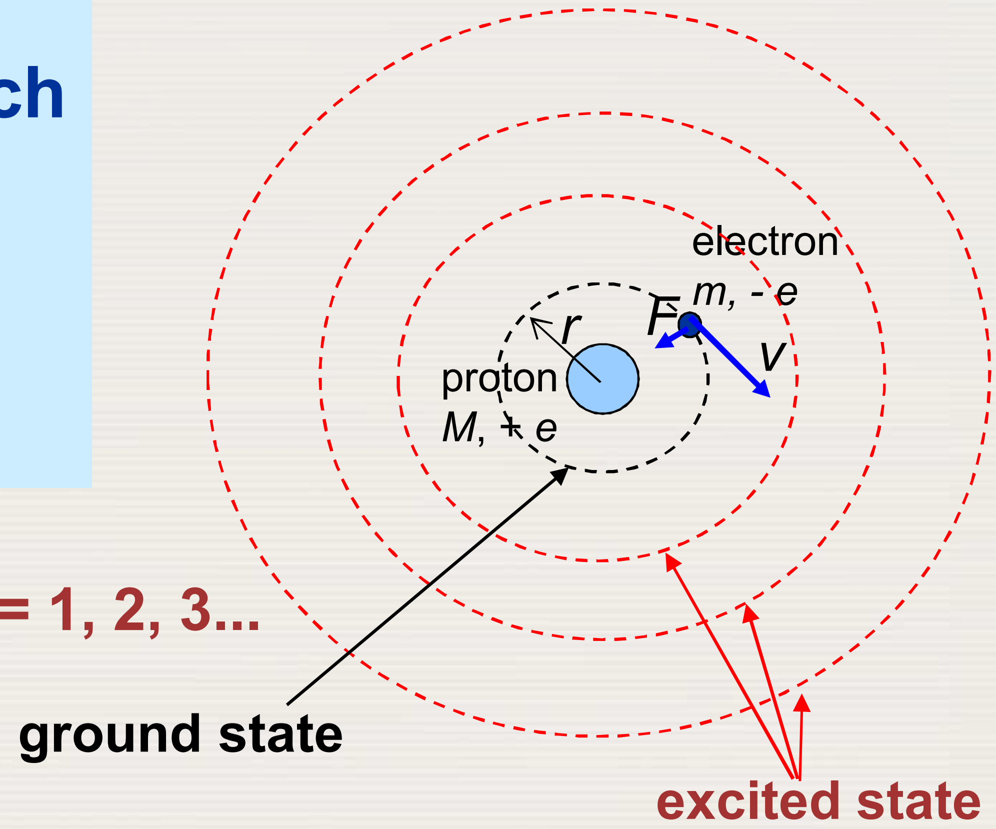


Diagram representing Bohr's model of the hydrogen atom, in which the orbiting electron is allowed to be only in specific orbits of discrete radii



Quantization of energy, with n = 1, 2, 3...

$$E_n(\text{eV}) = -\frac{13.6}{n^2}$$

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Whilst the work of Bohr was a major breakthrough, successfully explaining aspects of the behaviour of the hydrogen atom, the singly ionized helium atom, and the doubly ionized lithium atom, etc., the story did not stop there

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Through the work of Heisenberg, Schrödinger, Dirac, Pauli and others the theory of quantum mechanics was developed. In this theory, the electrons occupy individual energy states defined by four quantum numbers as follows:

- **the principal quantum number, *n*, which can take integer values and specifies the main energy shell**
- **the azimuthal quantum number, l, which can take integer values between 0 and $n - 1$**
- **the magnetic quantum number, *m*, which can take integer values between $-l$ and $+l$**
- **the spin quantum number, *s*, which takes values -1/2 or +1/2 and specifies a component of the spin angular momentum of the electron**

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According to the Pauli Exclusion Principle, no two electrons can occupy the same state and it follows that the number of electron states that can share the same principal quantum number n is equal to $2n^2$

The energy levels associated with n = 1, 2, 3 etc. are known as the K, L, M etc. bands

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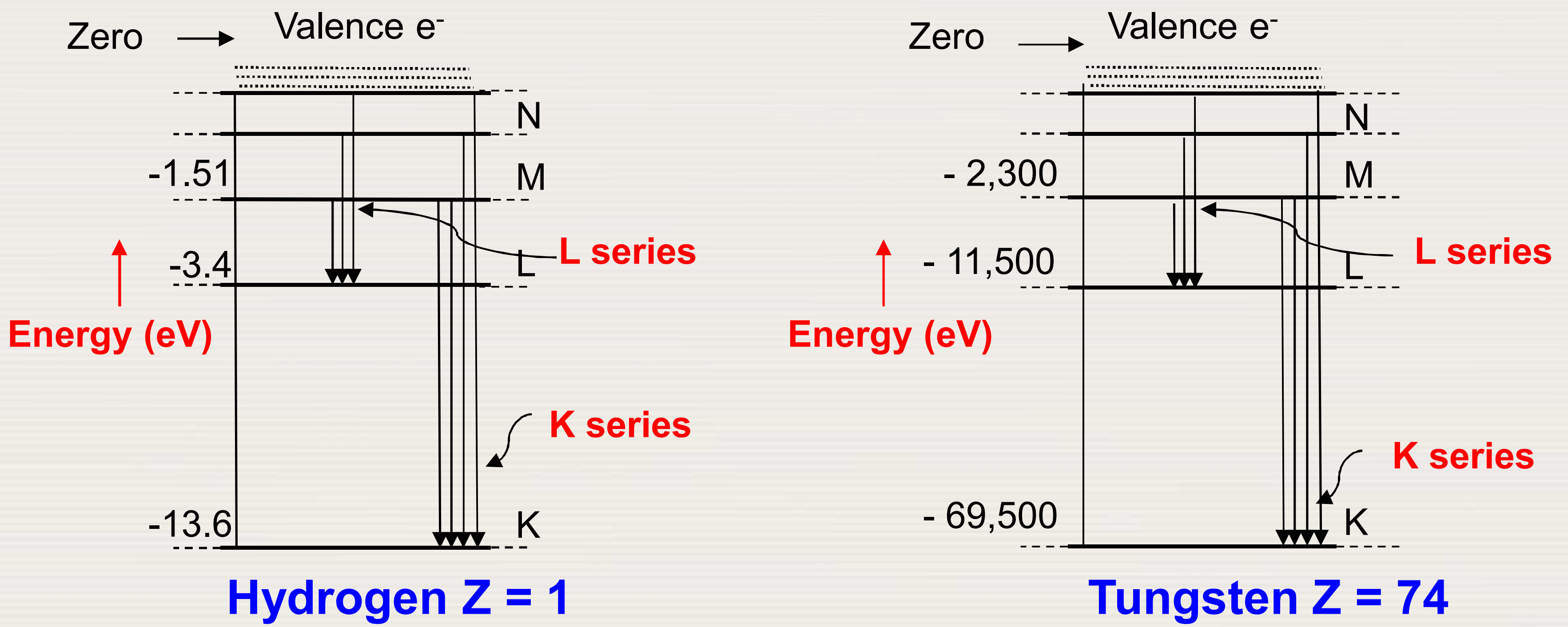
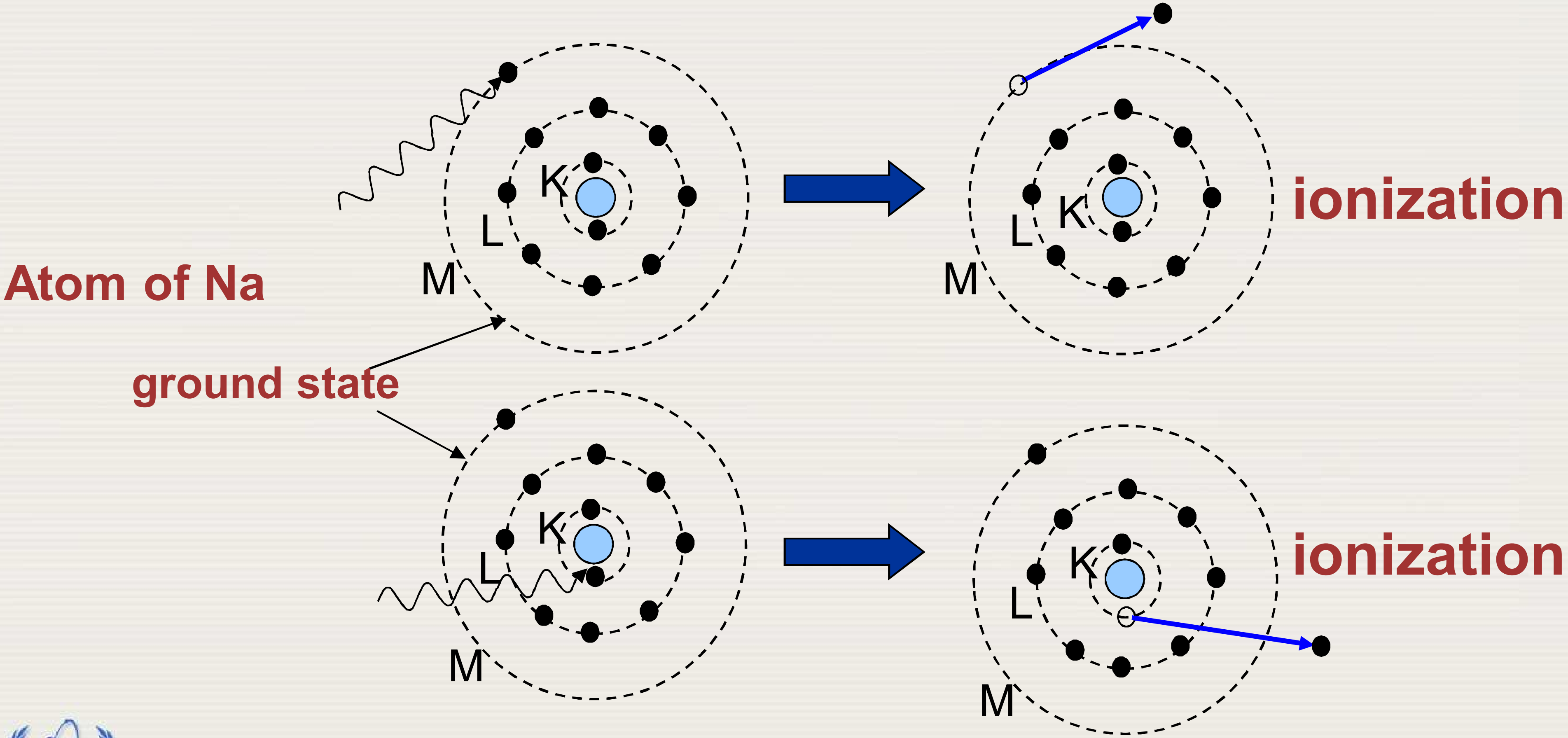


Energy levels for hydrogen and tungsten. Possible transitions between the various energy levels are shown with arrows

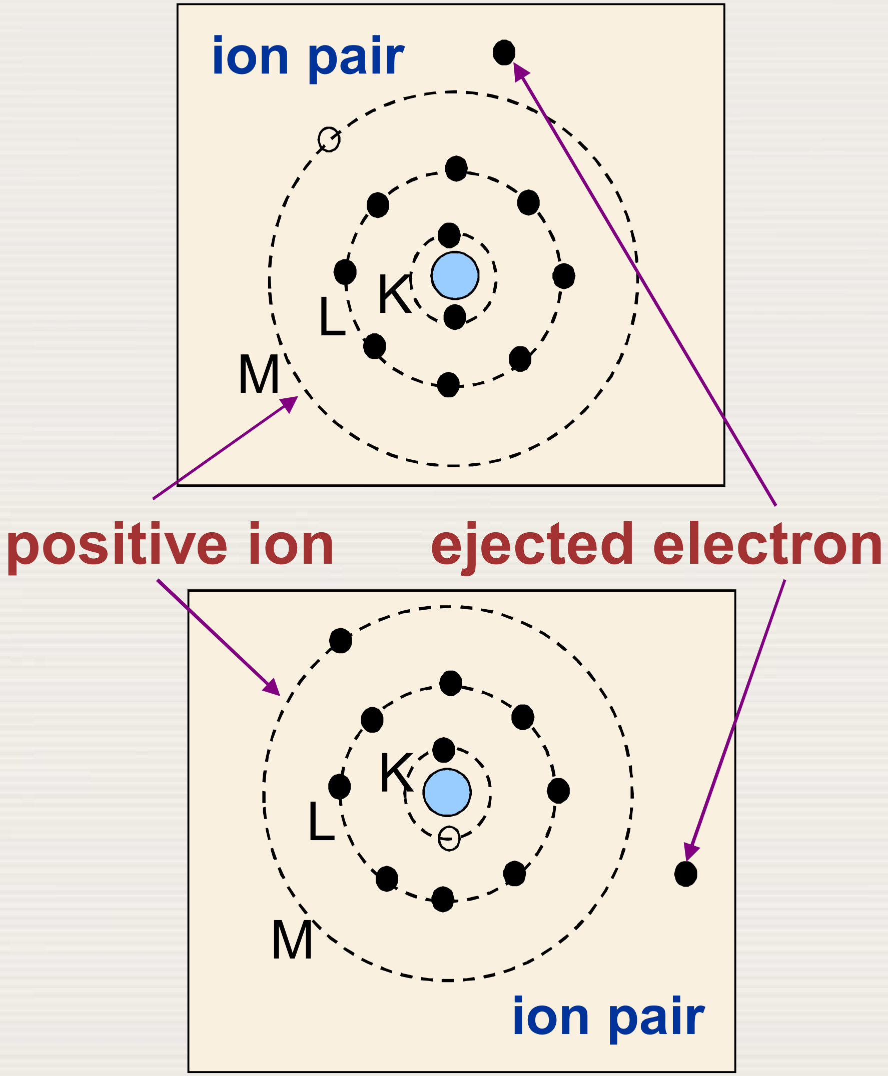
1.4. X RAYS

1.4.1. The production of characteristic X rays and Auger electrons

When charged particles pass through matter they interact with the atomic electrons and lose energy through the processes of ionization and excitation



1.4.1. The production of characteristic X rays and Auger electrons



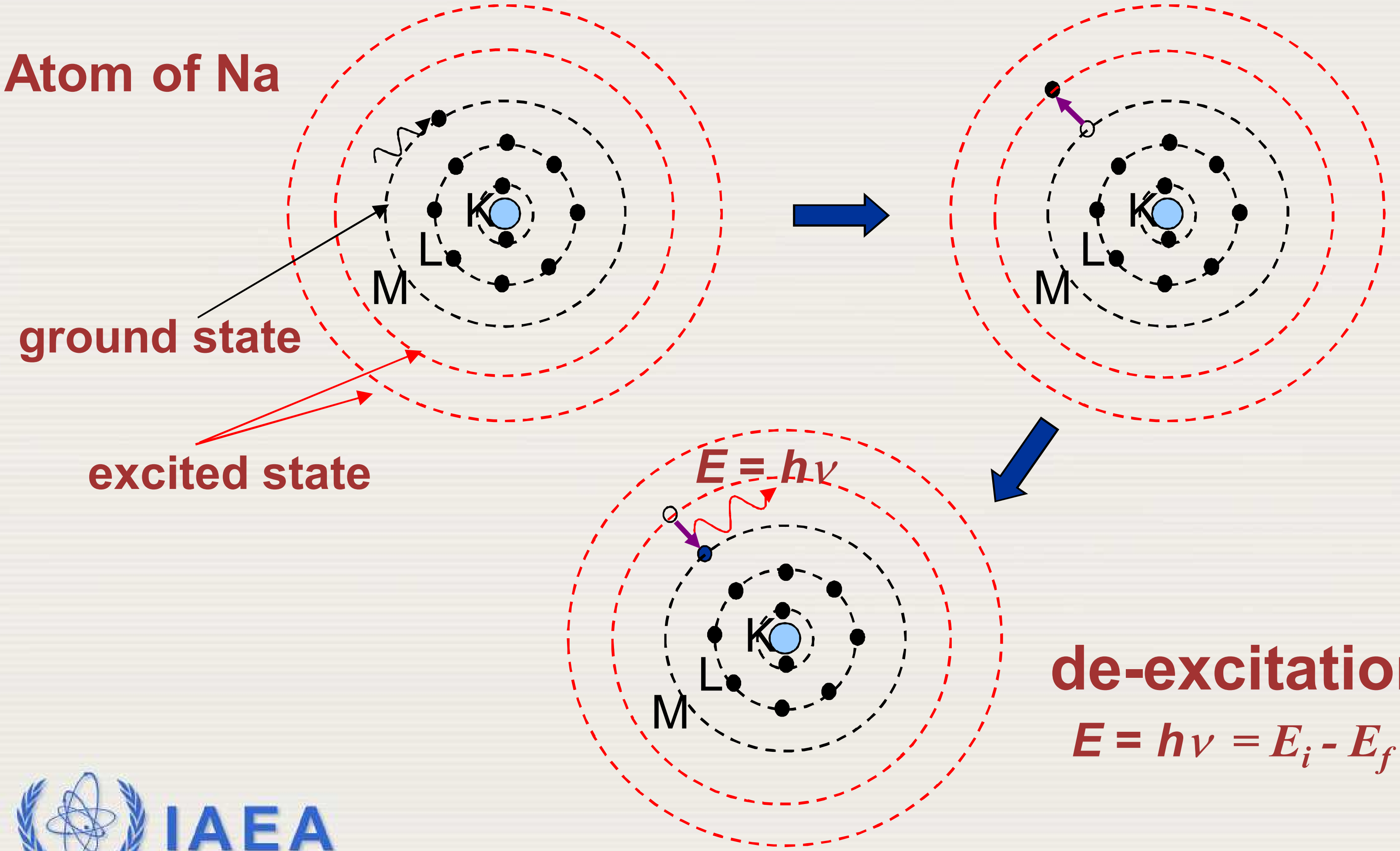
If the transferred energy exceeds the binding energy of the electron, ionization occurs, resulting in the electron ejected from the atom. An ion pair consisting of the ejected electron and the ionized, positively charged atom is then formed

The average energy required to produce an ion pair in air or soft tissue for electrons is equal to 33.97 eV

1.4. X RAYS

1.4.1. The production of characteristic X rays and Auger electrons

When charged particles pass through matter they interact with the atomic electrons and lose energy through the processes of ionization and excitation



excitation

de-excitation

$E = h\nu = E_i - E_f$

Whenever a vacancy is created in an inner electronic shell, it is filled by an electron from a more distant (outer) shell

This results in a vacancy in this second outer shell which is then filled by an electron (if available) from an even more distant outer shell and the whole process repeats producing a cascade of transitions

The energy released in each transition is carried away by:

- **the emission of electromagnetic radiation**

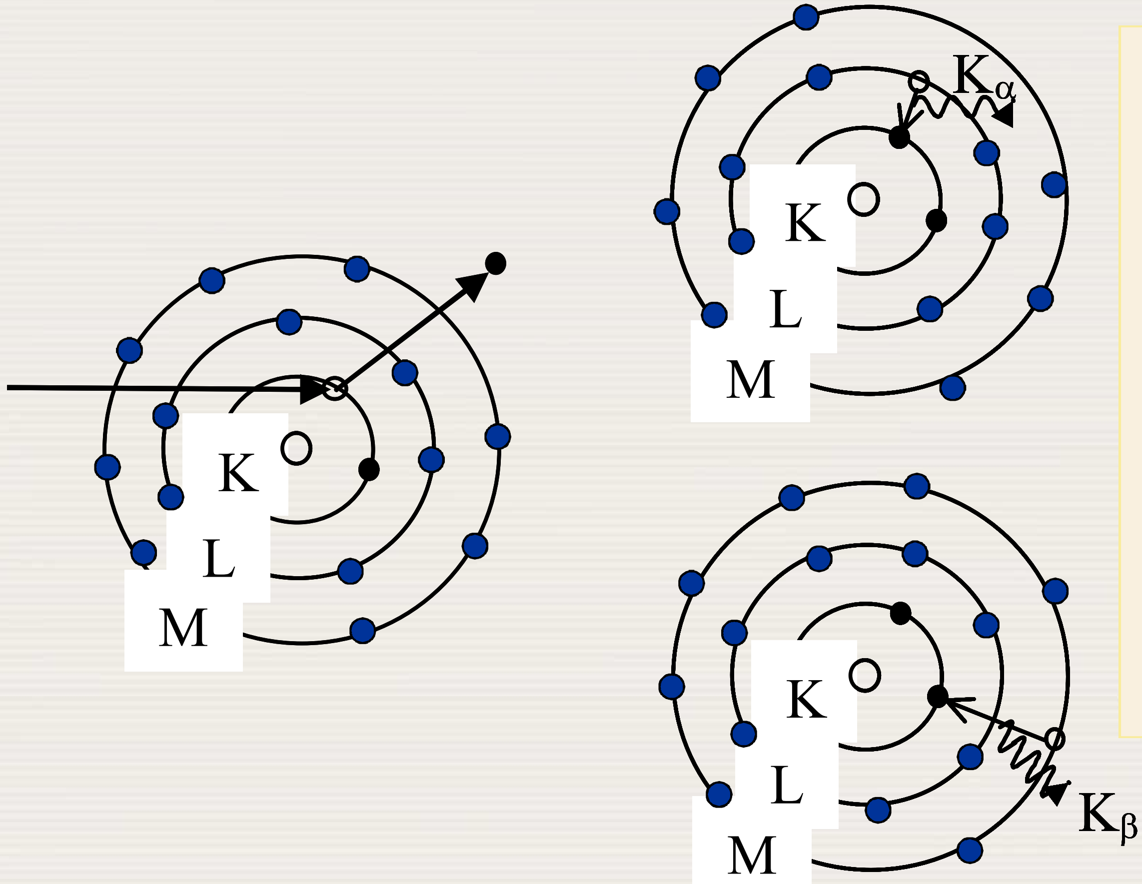
depending on the atomic number of the material, and the electronic shells involved, this radiation may be in the visible, ultraviolet, and X ray portions of the spectrum

in case of X rays, they are known as characteristic or fluorescent X rays

- **an electron ejected from another outer shell, known as Auger electron**

1.4. X RAYS

1.4.1. The production of characteristic X rays



- K_α X ray is emitted for a transition between L and K shells

- K_β X ray is emitted for a transition between M or N and K shells

1.4.1. The production of characteristic X rays

For tungsten (W):

the energies of the K_α and K_β characteristic X rays are given by:

$$E\,(K_{\alpha 1}) = E_{LIII} - E_K = -10.2 - (-69.5) = 59.3 \text{ keV}$$

$$E\,(K_{\alpha 2}) = E_{LI} - E_K = -11.5 - (-69.5) = 58.0 \text{ keV}$$

$$E\,(K_{\beta 1}) = E_{MIII} - E_K = -2.3 - (-69.5) = 67.2 \text{ keV}$$

$$E\,(K_{\beta 2}) = E_{NIII} - E_K = -0.4 - (-69.5) = 69.1 \text{ keV}$$

For molybdenum (Mo):

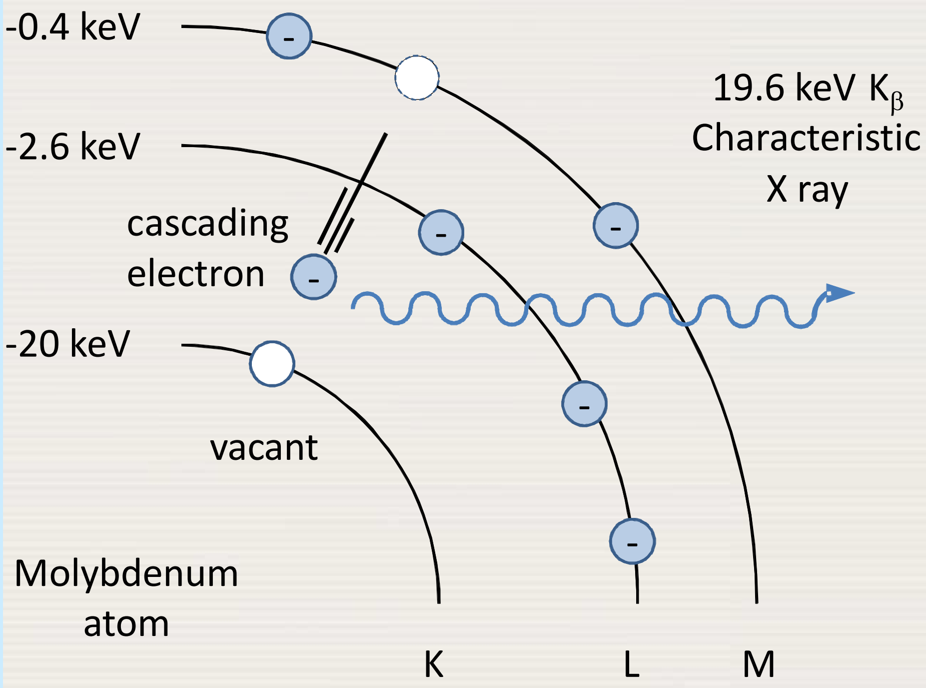
Energies of K, L and M shell are:

E_K = - 20.0 keV

E_L = - 2.6 keV

E_M = - 0.4 keV

the energies of the K_α and K_β characteristic X rays are given by:



$$E(K_\alpha) = E_L - E_K = -2.6 - (-20.0) = 17.4 \text{ keV}$$

$$E(K_\beta) = E_M - E_K = -0.4 - (-20.0) = 19.6 \text{ keV}$$

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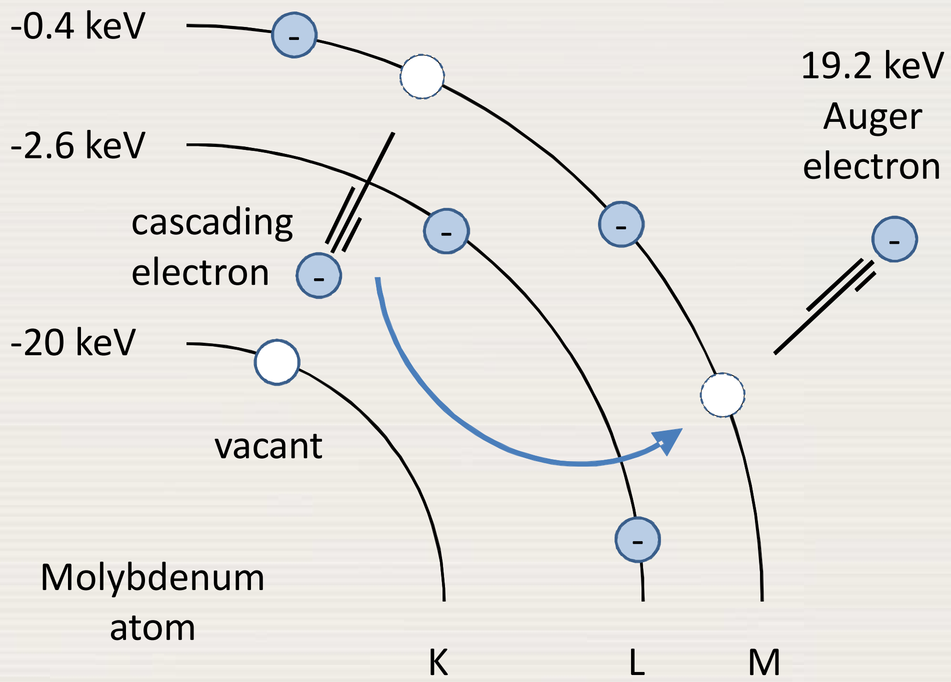
1.4.1. The production of Auger electrons

If the initial transition is from an M to a K shell, and the Auger electron is also emitted from the M shell, there will be two resultant vacancies in the M shell

The kinetic energy of the Auger electron is thus determined by the difference between the binding energy of the shell with the initial vacancy and the sum of the binding energies associated with the two vacancies which are created. In case of molybdenum atom, the energy of the Auger electron is given by:



$$E\,(\text{Auger}) = E_M + E_M - E_K = -0.4 - 0.4 - (-20.0) = 19.2\ \text{keV}$$

When considering energy deposition in matter it is important to know whether a fluorescent X ray or an Auger electron is emitted

The probability of emission of a fluorescent X ray is known as the fluorescent yield, denoted ω and the probability of emitting an Auger electron is 1- ω

Auger electron emission is more important for materials of low atomic number and for transitions amongst outer shells

The K-fluorescence yield is close to zero for low atomic number materials, but increases with atomic number and is 0.007, 0.17, 0.60 and 0.93 for oxygen, calcium, selenium and gadolinium respectively

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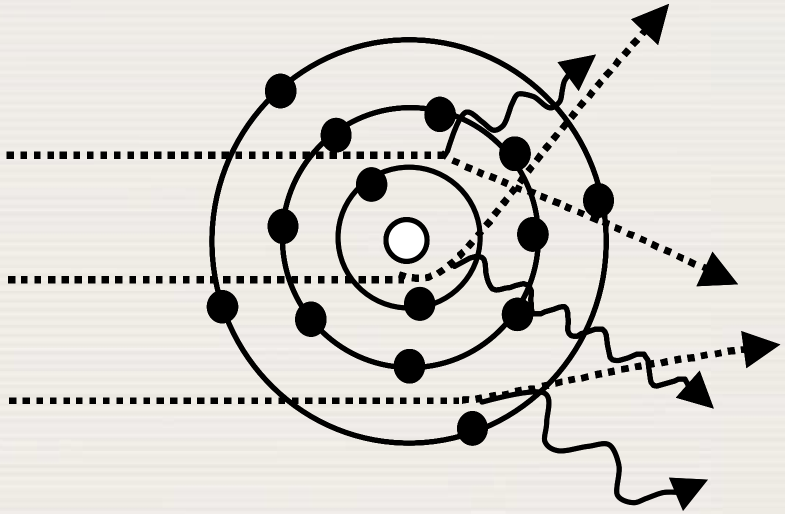
1.4. X RAYS

1.4.2. Radiation from an accelerated charge, Bremsstrahlung

In inelastic interactions of the fast electrons with atomic nuclei as they pass through matter, the electron path is deflected and energy is transferred to a photon, which is emitted

The emitted photon is known as Bremsstrahlung, which means "brake radiation", in German

The energy of the emitted photon can take any value from zero up to the energy of the initial electron, producing a continuous spectrum



Bremsstrahlung photons are the major component of the X ray spectrum emitted by X ray tubes

1.4.2. Radiation from an accelerated charge, Bremsstrahlung

The probability of Bremsstrahlung emission is proportional to Z^2. But even for tungsten ($Z = 74$) the efficiency of Bremsstrahlung production is less than 1% for 100 keV electrons

The angle of emission of the Bremsstrahlung photons depends upon the electron energy:

- for electron energies much greater than the rest energy of the electron, the angular distribution is peaked in the forward direction
- when the electron energy is low, the radiation is mainly emitted between 60 and 90 degrees to the forward direction

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