Chapter 1: Inductive & Deductive Reasoning

Section 1.1: Making Conjectures

- Patterns are used widely in mathematics to reach logical conclusions.
- This is called **inductive reasoning**
- Example: Predict the next number in each list

    1, 5, 25, 125, ___________

    -5, -2, 4, 13, _____________

    3, 12, 27, 48, _____________

Inductive Reasoning

- Drawing a general conclusion (**ie a conjecture**) by observing patterns and identifying specific properties in specific examples

Conjecture

- Testable hypothesis based on available evidence not yet proven
- Conjectures can be tested and those that appear valid allow us to make conclusions

Examples

- A math class consists of 20 boys and 10 girls. Can a conjecture be made about the composition of the school?

  Conjecture:

Ex: Determine a possible relationship between the figure number and number of triangles present in the figure.

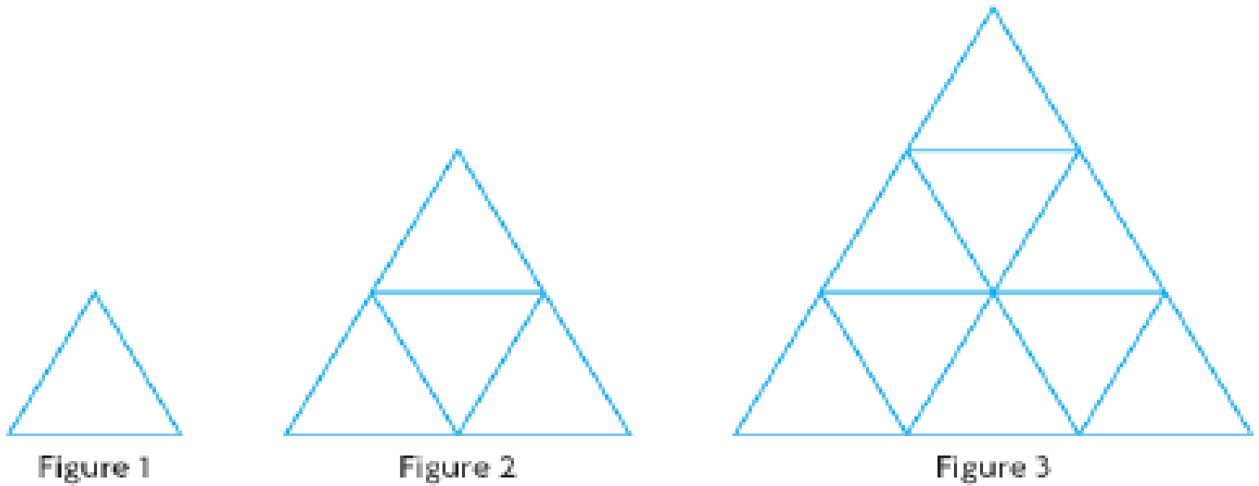


Figure #	1	2	3	4	5
# of small triangles					

- Xander predicts there will be 10 triangles in the 10th figure.
- Can you come up with a conjecture?
- How many triangles do you think will be in the 12th figure?

- What conjecture can be made about the **product of two odd integers?**

3 x 5 = 15

-5 x 7 = -35

-9 x -3 = 27

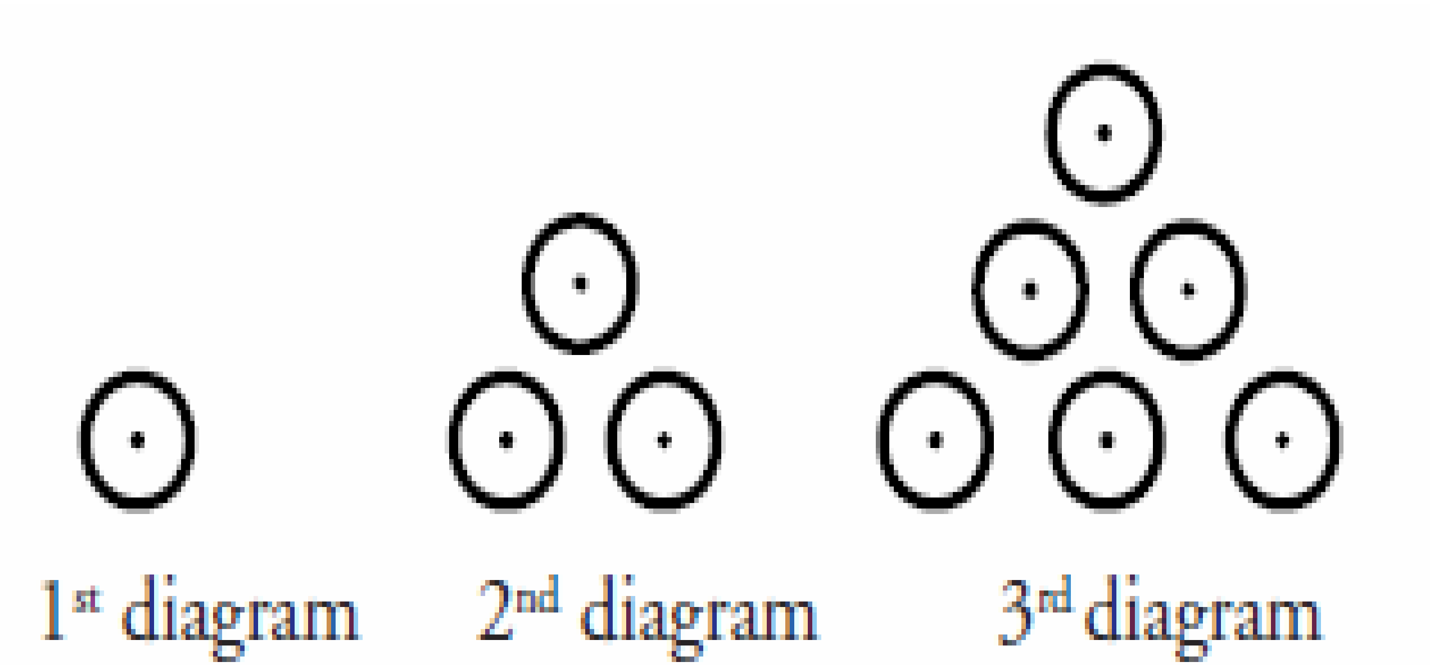
7 x -9 = -63

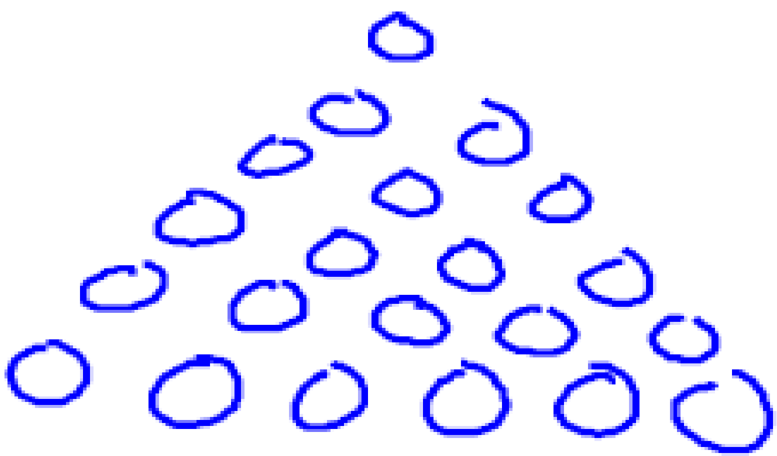
Conjecture: The product of two odd integers is an odd integer

- Mr. Vasquez wore a black tie the last four days it was raining. It's raining today. Make a conjecture about the color of his tie.

  Conjecture:

- What would be the 6th diagram in the following sequence?





Make a conjecture about the product of two consecutive integers

... , -3, -2, -1, 0, 1, 2, 3, ...

5 x 6 = 30

7 x 8 = 42

-3 x -4 = 12

Conjecture: The product of two consecutive integers is always even

2. Tomas gathered the following evidence and noticed a pattern.

$17(11) = \mathbf{187}$ $\quad 23(11) = \mathbf{253}$

$41(11) = \mathbf{451}$ $\quad 62(11) = \mathbf{682}$

Tomas made this conjecture: When you multiply a two-digit number by 11, the first and last digits of the product are the digits of the original number. Is Tomas's conjecture reasonable? Develop evidence to test his conjecture and determine whether it is reasonable.

NO

11 x 19 = 209

11 x 69 = 759

Paula claims that when you square an odd integer, the result is an odd integer? Is this true?

$3^2 = 9$

$17^2 = 289$

$21^2 = 441$

Important Note

- Multiple examples are needed to make a conjecture. The more examples one has = the more valid a conjecture is
- However, strength **does not equal** proof.
- Proof only occurs when **ALL** cases have been considered.

- Predict the missing values

9 x 9 + 7 = 88

98 x 9 + 6 = 888

987 x 9 +5 =

9876 x 9 +4 =

98765 x 9 +3 =

- Predict the missing values

$9^2 = 81$

$99^2 = 9801$

$999^2 = 998001$

$9999^2 =$

$99999^2 =$

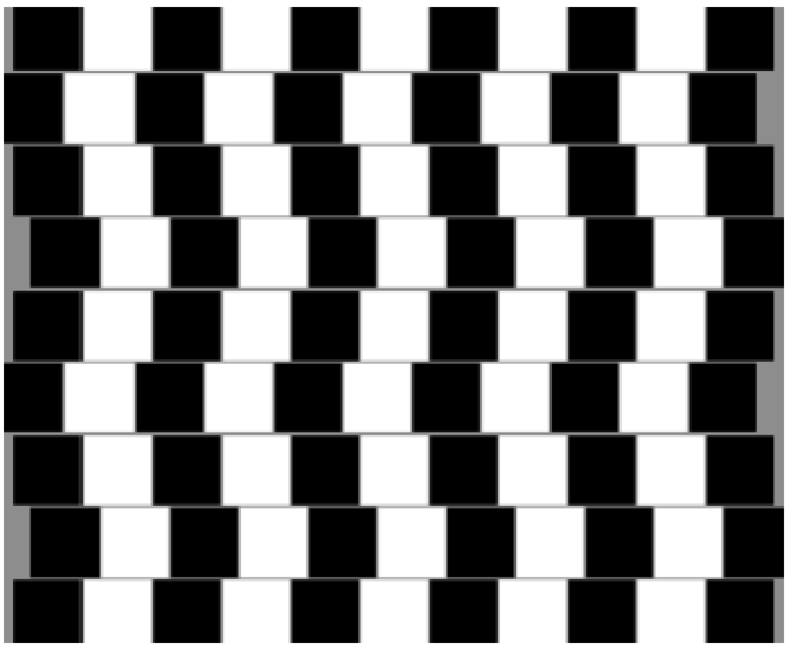
Example: Study the pattern and predict the next two terms.

a)	2, 3, 5, 8, 13, ____, ____	(21, 34 add previous)
b)	20, 25, 31, 38, 46, ____, ____	(55, 65 add 5, 6, 7...)
c)	10, 7, 12, 9, 14, ____, ____	(11, 16 add 2 every 2nd #)
d)	3, 6, 11, 18, 27, 38, ____, ____	(51, 64 add 3, 5, 7, ...)
e)	2, 6, 15, 31, 56, ____, ____	(92, 141 add squares)
f)	2, 6, 12, 20, 30, ____, ____	(42, 56 add 4, 6, 8, ...)
g)	15, 19, 25, 33, 43, ____, ____	(55, 69, add 4, 6, 8, ...)
h)	1, 2, 5, 14, 41, ____, ____	(122, 365, add 1, 3, 9, 27)
i)	3, 5, 11, 29, 83, ____, ____	(245,731, add 2, 6, 18, 54)
j)	59, 52, 55, 48, 51, 44, 47, ____, ____	(40, 43,subtract 4 every 2nd one)

Section 1.2: Exploring the Validity of Conjectures

- Some conjectures seem valid at first, but shown invalid after more evidence is gathered
- To show a conjecture is invalid, all one needs to do is show **one example** where it's not valid
- We should be careful reaching conclusions with inductive reasoning. **A conjecture can be reviewed, based on new evidence.**

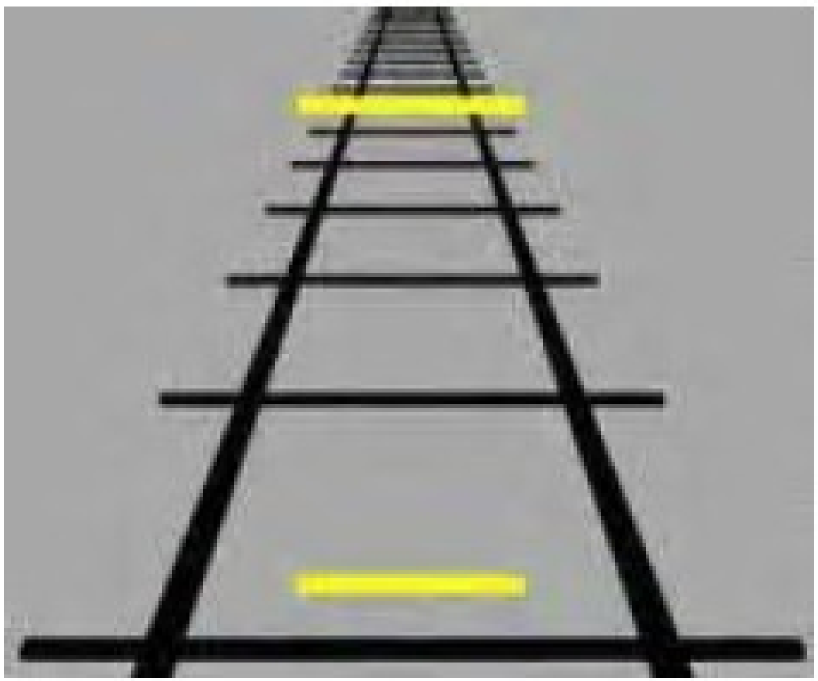
- Optical illusions are useful examples to disprove initial conjectures
- Example: Make a **conjecture** about the lines in this picture



Conjecture: The lines in the picture are not straight.

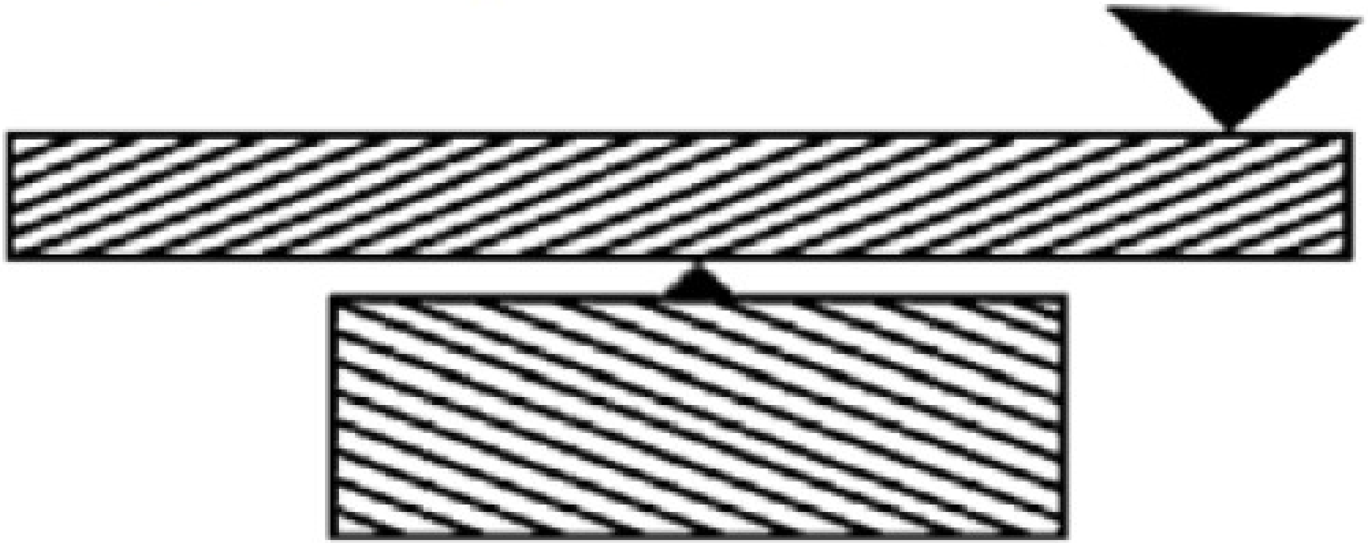
Prove Validity: Use a ruler to discover that the lines are actually straight

- Example: Make a conjecture about the lines in this picture



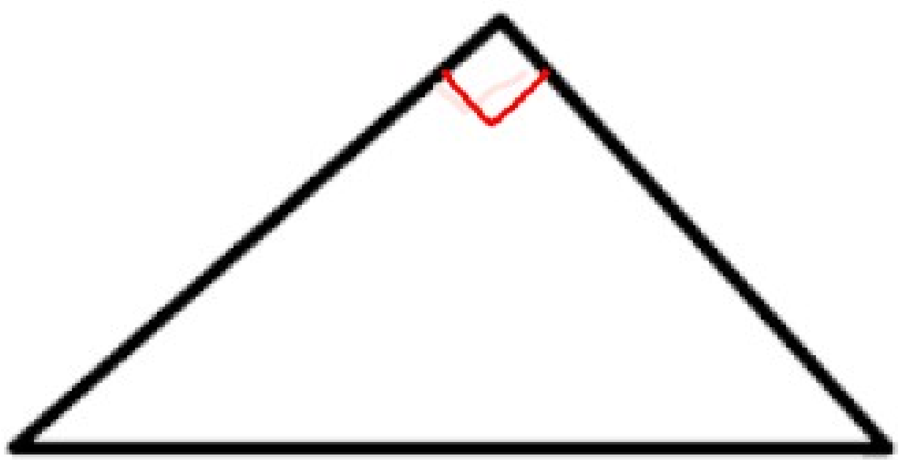
The upper line appears much larger because it spans a greater apparent distance between the rails, which our mind assumes is parallel.

(i) Susie conjectures that this balance is not level. Do you agree or disagree? Justify.



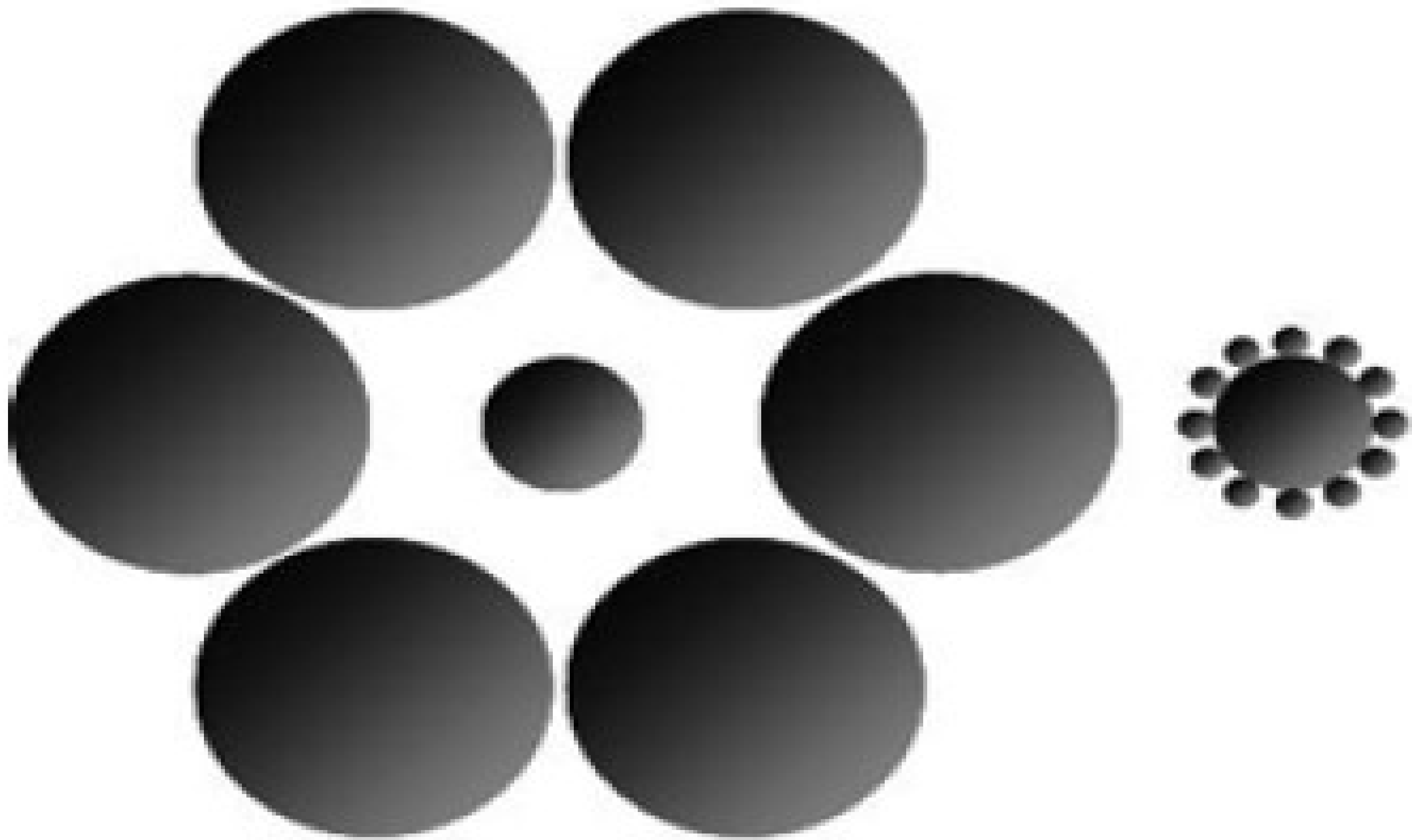
(NL1.1, NL1.3)

(ii) Grace conjectures that the triangle below is a right triangle. Do you agree or disagree? Explain.

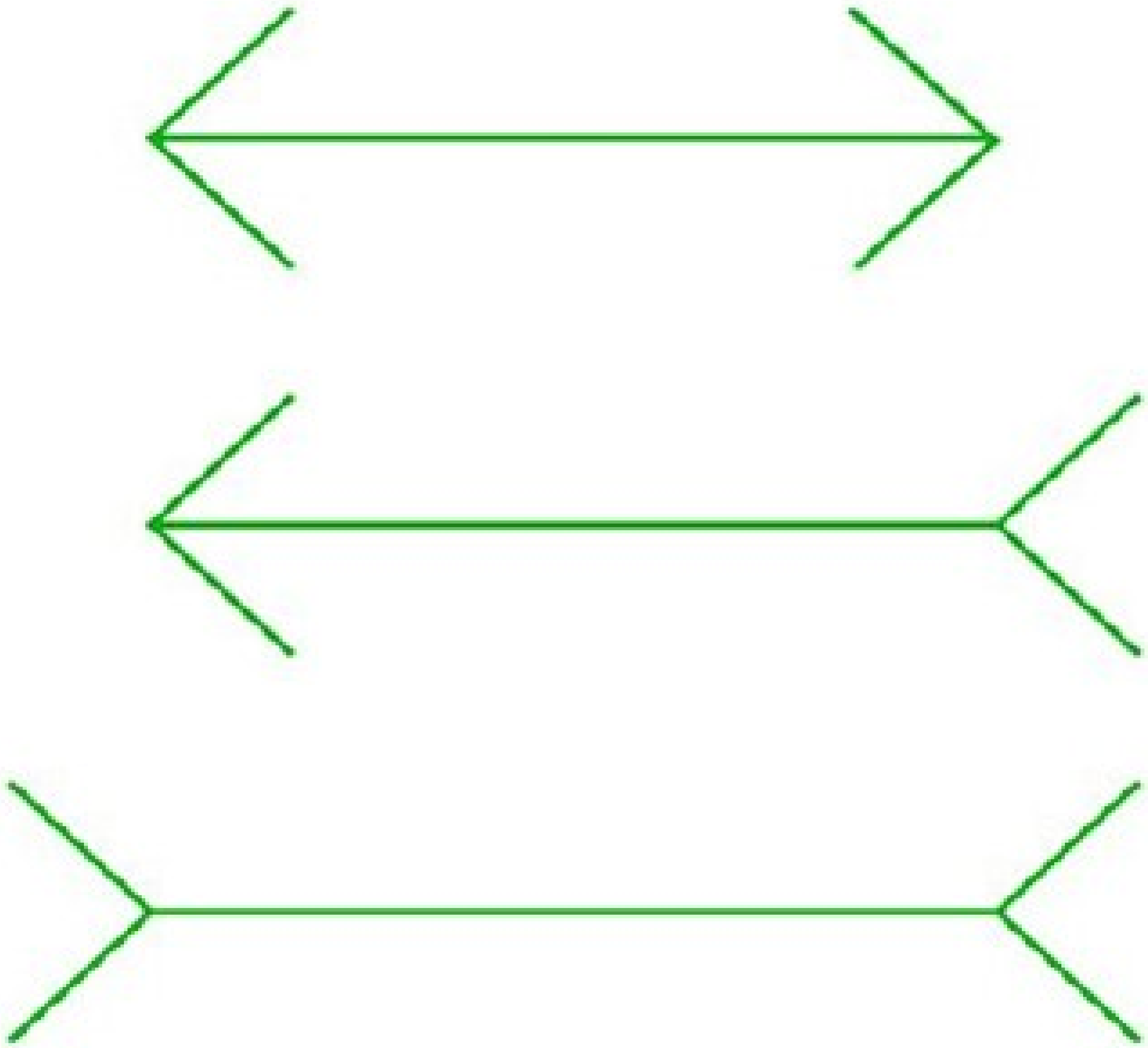


(NL1.1, NL1.3)

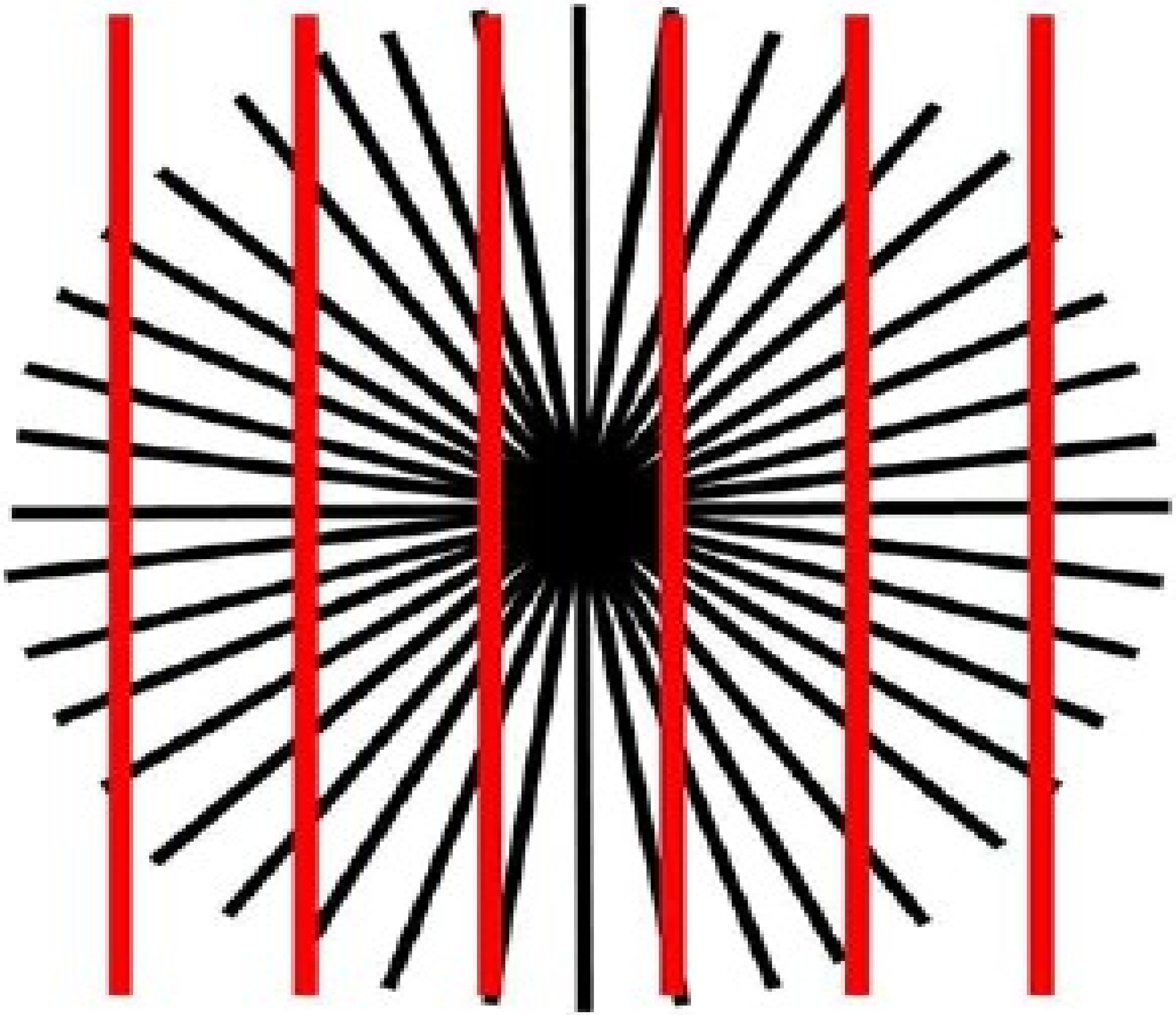
- Example: Make a conjecture about the size of the circles in the middle.



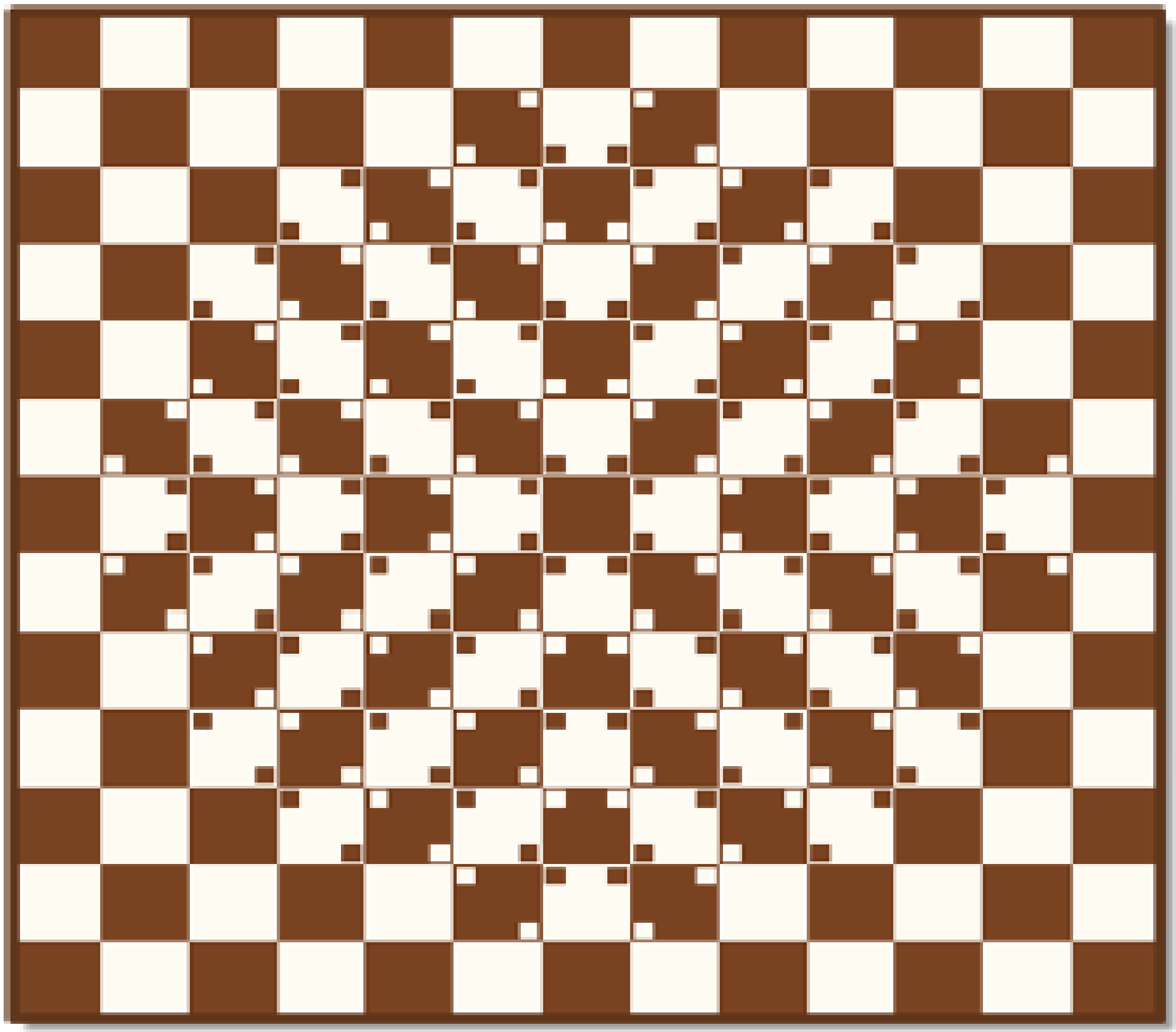
- Example: Which is the longest line?



- Example: Make a conjecture about the lines in the picture



- Example: Make a conjecture about the lines in the picture



Example: Aoccdrnig to rscheearch at Cmabrigde Uinervtisy, it deosn't mttaer in waht oredr the ltteers in a wrod are, the olny iprmoatnt tihng is taht the frist and lsat ltteer be at the rghit pclae. The rset can be a toatl mses and you can sitll raed it wouthit a porbelm. Tihs is bcuseae the huamn mnid deos not raed ervey lteter by istlef, but the wrod as a wlohe.

- Example: If possible, find a counterexample to each of the following assumptions.

a) Every prime number is odd

b) Multiplying **always** leads to a larger number

c) If a number is divisible by 2, then it is divisible by 4

d) If $x + 4 > 0$ then x is a positive number

Summary

- Some conjectures seem initially valid, but are shown to be not valid after more evidence is gathered
- All we can say about a conjecture reached inductively is that there is evidence to either support or deny it
- A conjecture may be revised, based on new evidence

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#1 and 2

Section 1.3: Using Reasoning to find a Counterexample to a Conjecture

- An important thing to remember about conjectures is that they may or may not be right.
- As with any logical statement, take the time to look for counterexamples, and also verify that the conjecture works on the examples it was based on.
- In order to disprove a conjecture we only need to find **<u>ONE</u>** example of it being false.

- A **counterexample** is an example that invalidates or proves a conjecture to be false in this particular instance
- If you can find **one example** of a conjecture being false, then you have proven this conjecture to be false

- Come up with counterexamples to the following statements:
  - Any piece of furniture having four legs is a table
  - Everything in the house with hands is a clock
  - If the grass is wet then it is raining
  - Everything that is cold is snow
  - All basketball players are taller than 6 feet
  - If it is a cell phone, then it has a touch screen
  - No triangles have 2 sides of the same length

- Conjecture: All but one of the vowels (a, e, i, o, u, y) are used to spell numbers (zero, one, two, ...)
- Is this conjecture true?
- The conjecture appears to be true until “thousand”
- Revised Conjecture:
  - The letter “a” is the only letter not used to spell words for numbers from 0 to 999

- Conjecture: The sum of two prime numbers is an even number
  - Find two examples that support this conjecture
  - Find an example makes this conjecture false

- Conjecture: The difference between consecutive perfect squares is always a prime number
  - Find two examples that support this conjecture
  - Find an example that makes this conjecture false

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- Matt found an interesting number pattern:

  $1 \times 8 + 1 = 9$

  $12 \times 8 + 2 = 98$

  $123 \times 8 + 3 = 987$

  $1234 \times 8 + 4 = 9876$

- Matt thinks this pattern will continue forever
- Find an counterexample to this pattern

1	1 x 8 + 1	9
2	12 x 8 + 2	98
3	123 x 8 + 3	987
4	1234 x 8 + 4	9876
5	12345 x 8 + 5	98765
6	123456 x 8 + 6	987654
7	1234567 x 8 + 7	9876543
8	12345678 x 8 + 8	98765432
9	123456789 x 8 + 9	987654321
10	12345678910 x 8 + 10	98765431290

- Is Matt's conjecture correct?
- Nope because the tenth entry for the pattern broke the pattern
- This counterexample disproved his original conjecture
- Revised Conjecture: This pattern holds when the value at the end/being added is from 0-9

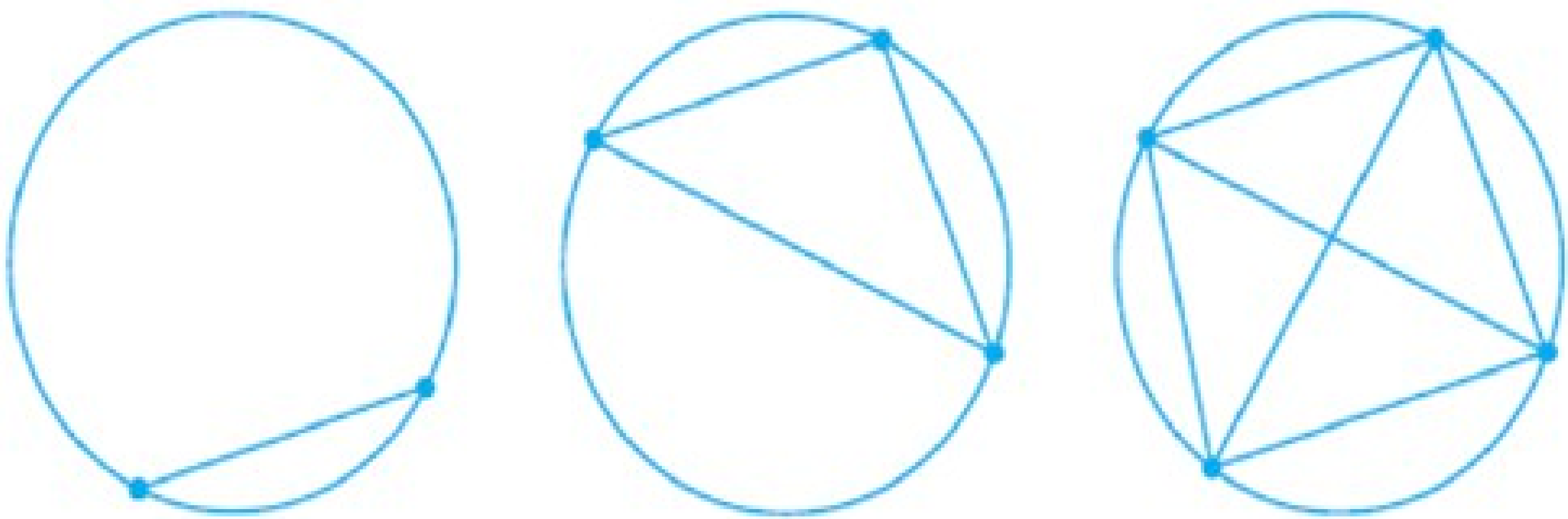
Important Note

- If a counterexample cannot be found that does not mean the conjecture is disproven
- This simply means it cannot be proven to be false
- Goldbach Conjecture states that every even number greater than 2 can be wrote as the sum of two prime numbers
  - $4 = 2 + 2$, $6 = 3 + 3$, $100 = 17 + 83$
- This conjecture has not been proven true and a counterexample has yet to be found

Example: (page 18)

Kerry created a series of cirlces.

Each circle has points marked on its circumference and joined by chords.



As the number of points on the circumference increased, Kerry noticed a pattern for the number of regions created by the chords.

Number of points	2	3	4	5	6
Number of regions	2	4	8	16	31

Conjecture: As the number of connected points on the circumference of the circle increases by 1, the number of regions created within the circle increases by a factor of 2

Testing a conjecture

To the right is another diagram with five points on its circumference. The pairs of points are joined with chords. The diagram has <u>16</u> regions. This supported Kerry's conjecture because the pattern for the resulting regions was $2^1, 2^2, 2^3, 2^4$.

Draw another circle with six points on its circumference. Join the pairs of points with chords and count the number of regions.

Does this support Kerry's conjecture? Explain.

No it does not. There are only 31 regions and there should be 32 according to Kerry's conjecture. Since this does not increase by a factor of 2, it disproves Kerry's conjecture.

This is a counterexample, since it does not increase by a factor of 2.

Section 1.3 Summary

- Finding a counterexample to a conjecture disproves the conjecture
- You might be able to revise the original conjecture with the counterexample
- **Inductive reasoning** is used to make a **conjecture** which is supported by **evidence** and can be proven false by a **counterexample**

For you to do

- Pg. 22-23 # 1(except c & e), 2, 3, 4, 5, 16

Sections 1.1-1.3 Summary

- Inductive reasoning is a kind of logical reasoning which involves drawing a general conclusion, called a conjecture, based on a specific set of observations.
- In this process, specific examples are examined for a pattern, and then the pattern is generalized by assuming it will continue in unseen examples.
- Conjectures and predictions can then be made.

- One of the disadvantages of inductive reasoning is that a conjecture found by inductive reasoning may or may not always be true.
  - This is common when we overgeneralize, that is, when we use a small number of observations and try to apply them to a much wider situation.
  - If all we can do is test individual examples, it's difficult, if not impossible, to say that there may not be a counterexample that we haven't found yet.
- One counterexample is sufficient to disprove a conjecture.
- Finding a counterexample doesn't call for despair—we may be able to use the new information to revise our conjecture.

Section 1.4: Proving Conjectures Deductive Reasoning

Definitions

- Proof – A mathematical argument that says a statement is valid in all cases, or that no counterexample exists
- Generalization – A principle, statement or idea that has a general application
- To prove a conjecture is true for all cases, we use deductive reasoning

- Deductive Reasoning
  - Drawing a specific conclusion through logical reasoning by starting with general assumptions that are known to be valid

Review of Mathematical Properties

Arithmetic properties:

- Addition
  - When you add two numbers together you find how many you have in all.
- Subtraction
  - Subtraction is removing some objects from a group.
  - The meaning of 5-3=2 is "Three objects are taken away from a group of five objects and two objects remain".
- Adding (or Subtracting) 0
  - Any number added (subtracted) by 0 equals that number
- (Identity Property for Addition/Subtraction)

- Multiplication
  - The result of multiplication is the total number (product) that would be obtained by combining several groups of similar size.
  - The same result can be obtained by repeated addition.

- Multiplying by 0
  - Any number multiplied by 0 equals 0.
  - (Zero Property for Multiplication)

- Multiplying by 1
  - Any number multiplied by 1 equals that number.
  - (Identity Property for Multiplication)

- Division
  - Dividing is separating a number into several equal groups.
  - When we divide 6 by 3 we are separating 6 into 3 equal groups of 2.
  - There are two common ways to write the sign for division.
  - The number 6 divided by 3 could be written as 6/3 or 6 ÷ 3.

- Dividing by 0
  - Numbers cannot be divided by 0 because it is impossible to make 0 groups of a number. (Undefined)
- Dividing by 1
  - Any number divided by 1 equals that number. If you divide by 1 you have one group and so everything is in that group.
  - (Identity Property for Division)

Other Properties

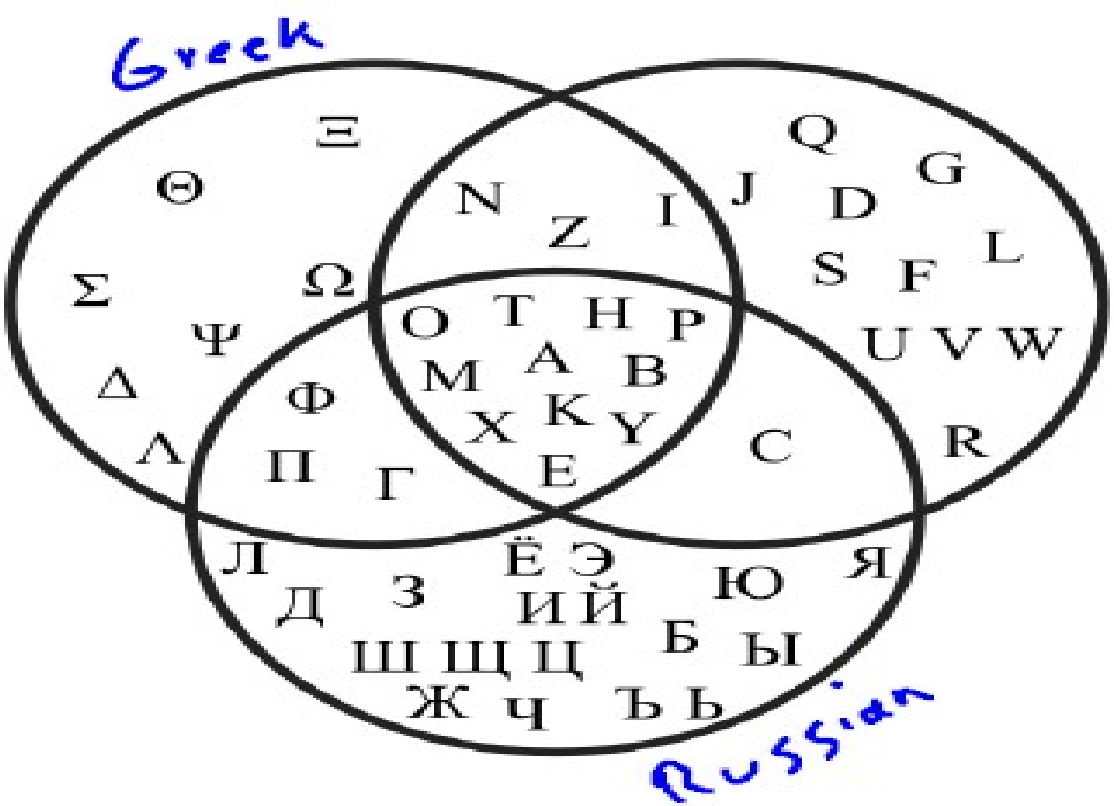
- Commutative property (for addition):
  - When two numbers are added, the sum is the same regardless of the order of the numbers.
  - For example 4 + 2 = 2 + 4
- Associative Property (for addition):
  - When three or more numbers are added, the sum is the same regardless of the grouping of the numbers.
  - For example (2 + 3) + 4 = 2 + (3 + 4)
- Distributive property:
  - The sum of two numbers times a third number is equal to the sum of each number times the third number.
  - For example 4 * (6 + 3) = 4*6 + 4*3

3 ways to prove conjectures

- Venn Diagrams (visual representations)
- Number Theory Proofs (choosing a variable to algebraically represent a situation)
- Two-Column Proofs – using statements and reasoning in an organized list (*next unit)

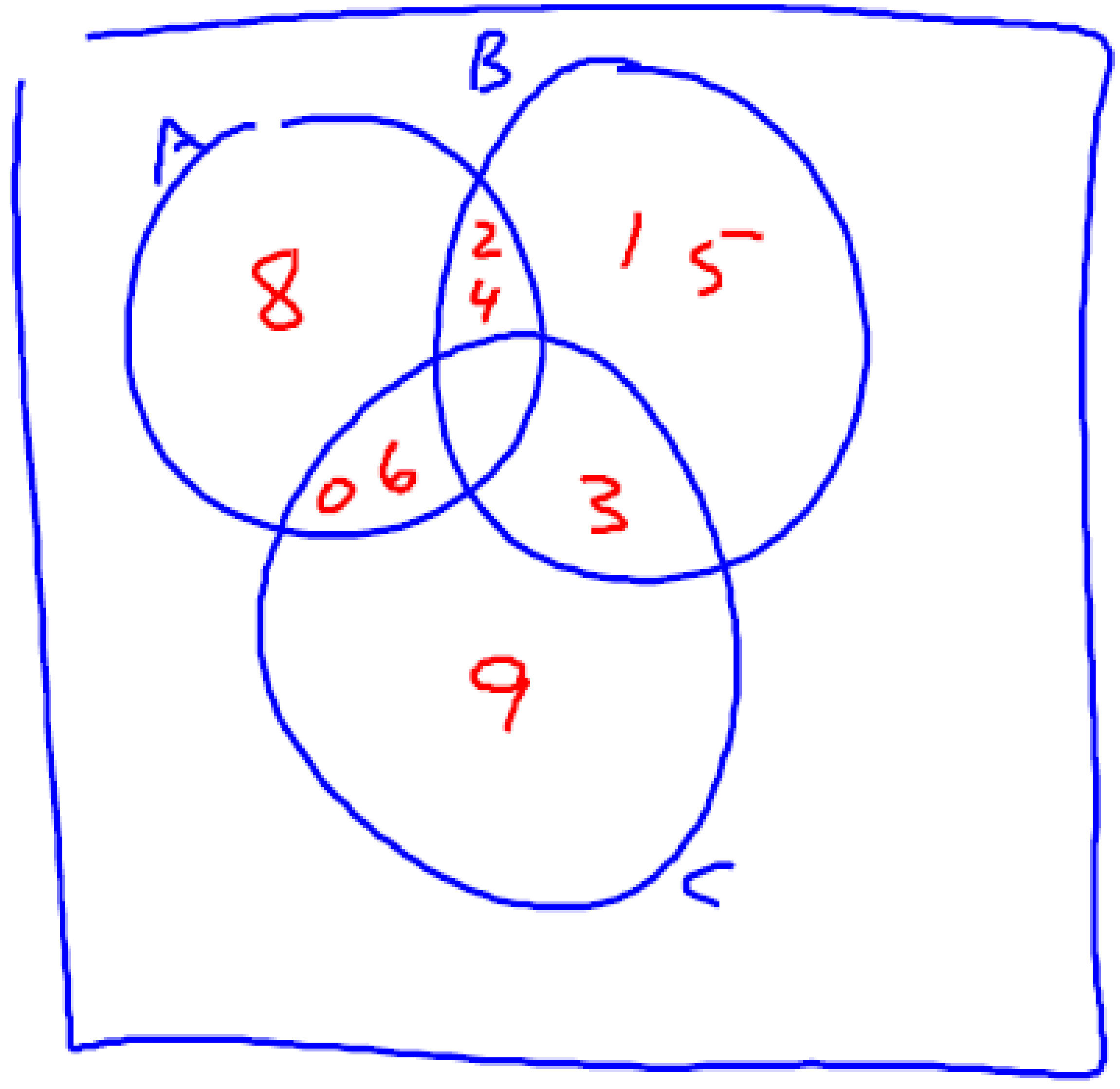
Venn Diagrams

- An illustration that uses overlapping or non-overlapping circles to show the relationship between groups of things
- Example: Venn Diagram of 3 Alphabets (Greek, English and Russian)



- Which letters are common to all three languages?
  - O, T, H, P, M, A, B, X, K, Y, E
- Which letter is not in the Greek alphabet, but in the Russian and English alphabet?
  - C

- Put the following sets into a Venn Diagram
  - A: {0, 2, 4, 6, 8}
  - B: {1, 2, 3, 4, 5}
  - C: {0, 3, 6, 9}

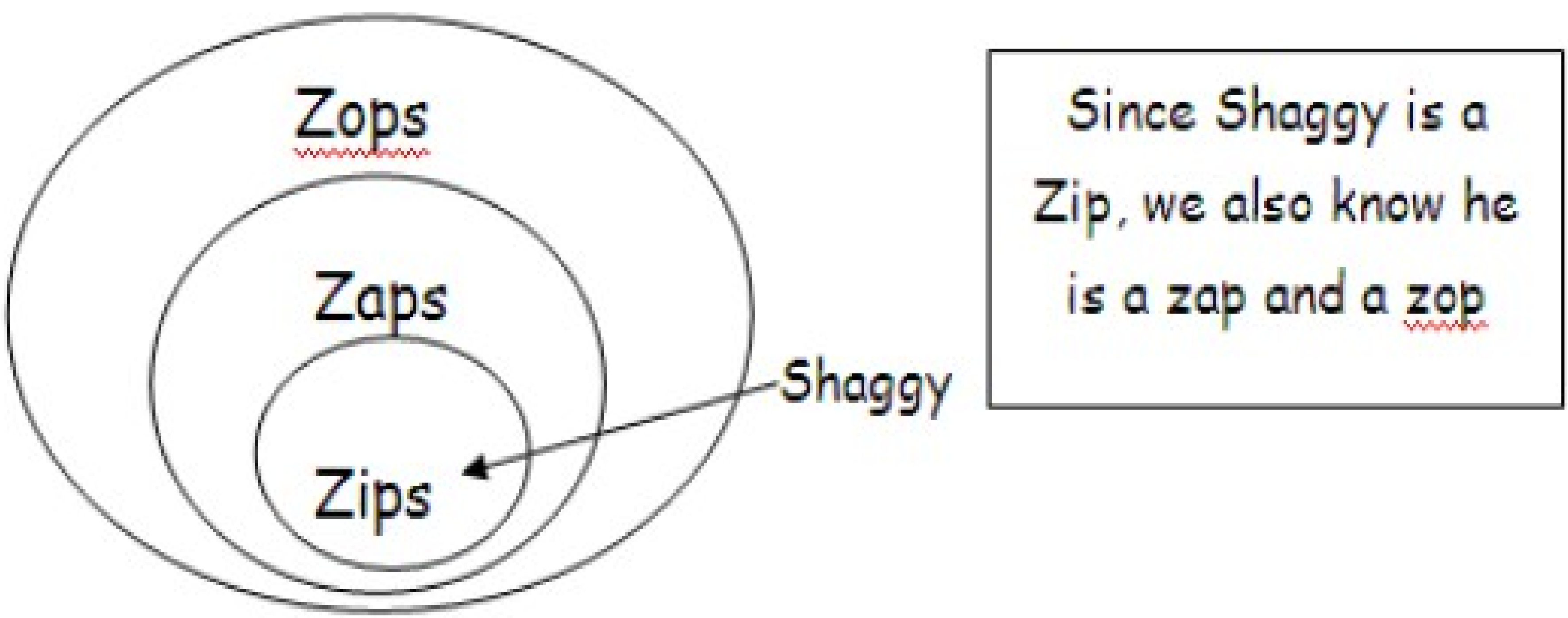


Put the following sets in a Venn Diagram

- All dogs are mammals
- All mammals are vertebrates
- Lula is a dog
  - What can be deduced about Lula?

Example

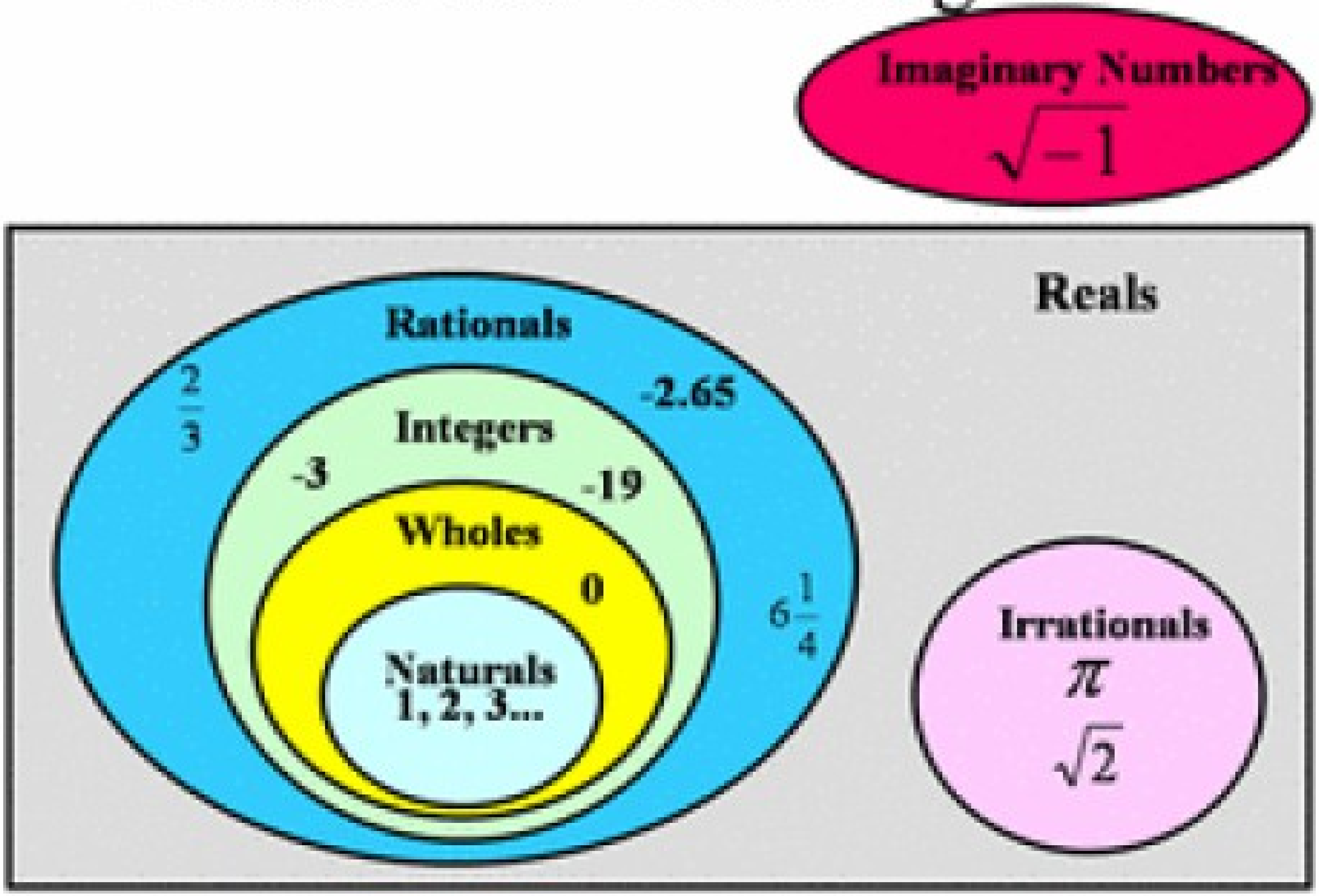
- All zips are zaps
- All zaps are zops
- Shaggy is a zip
  - What can be deduced about Shaggy?

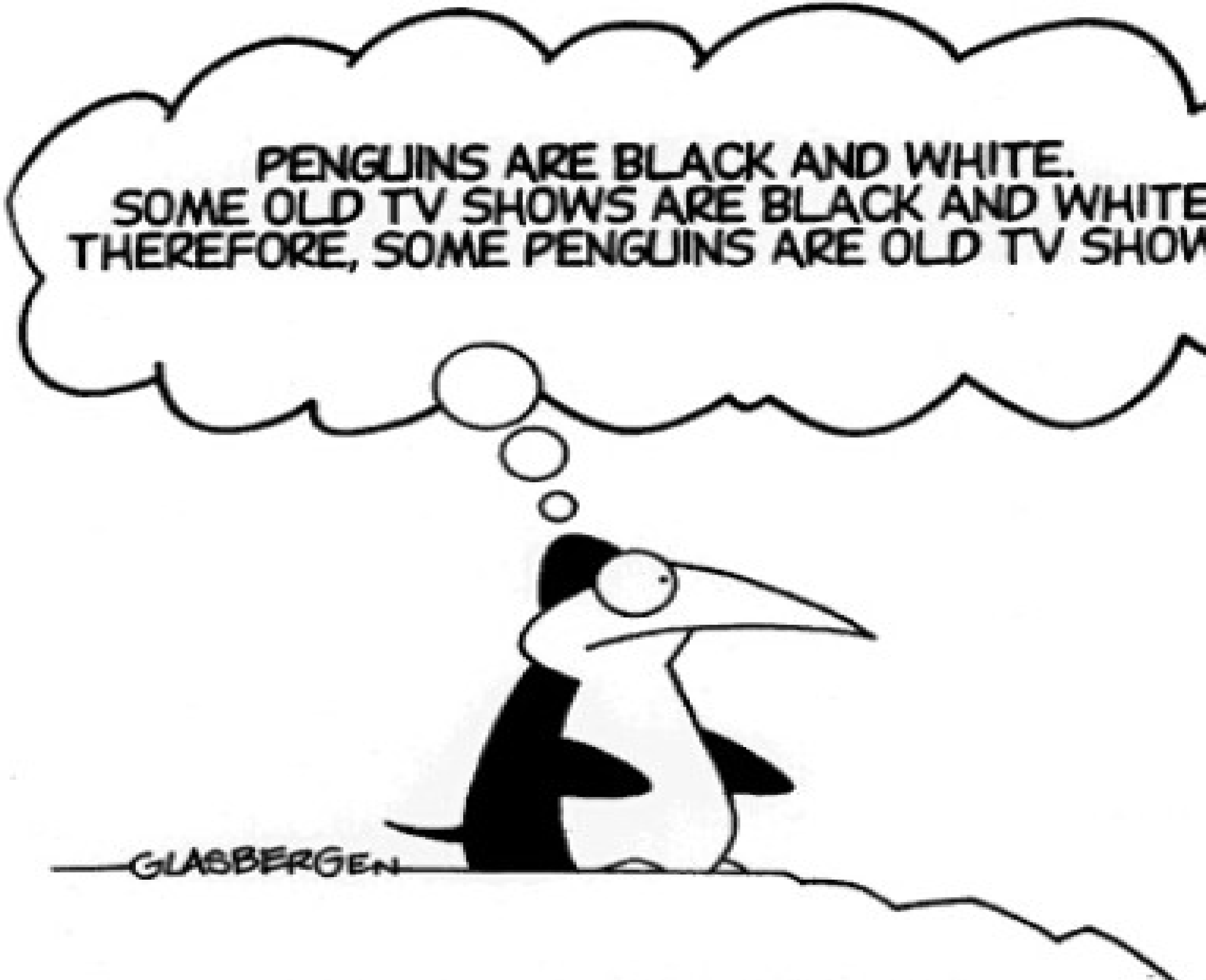


Simple logic examples

- Casey voted in the last election. Only people 18 years old voted? What can we say about Casey?
- All students love to read. Mona is a student. Therefore Mona loves to read.
- All multiples of 10 are divisible by 2. 6 is divisible by 2. Therefore 6 is a multiple of 10.
- Mammals have fur/hair. Lions are mammals. Therefore mammals have fur.

Number Sets Venn Diagram





Logic: another thing that penguins aren't very good at.

Brief description of number systems

- Natural Numbers (N)
  - They start with 1 and go onward (1, 2, 3, 4, 5,.....)
  - Zero is not included
  - Only whole positive numbers are natural
    - No decimal or negative numbers
- Whole Numbers
  - Includes the Natural Numbers and 0
- Integers (I or Z)
  - Includes the Whole Numbers and their negative counterparts
    - (..., -3, -2, -1, 0, 1, 2, 3, 4,......)

- Rational Numbers (Q)
  - All numbers that can be expressed as a ratio of two integers
  - Often expressed as a decimal
  - Either a decimal with all 0's (ex: 5 or 5/1)
  - Terminating decimal (ex: 2.4 or 24/10)
  - Repeating decimals (ex: 2.333... or 7/3)
- Keep in mind that how a decimal behaves is just simply what happens when one divides an integer by another integer (definition of a rational number)

- Irrational Numbers $\bar{Q}$
  - These are numbers that cannot be expressed as an integer divided by an integer
  - Excluding imaginary/complex numbers
  - Examples included π, e, and $\sqrt{2}$

- Real Numbers (R)
  - Includes the rational and irrational sets of numbers
  - Set of all numbers found on a number line

Special Types of Numbers

- Prime Numbers
  - Is a natural number bigger than 1 that can only be divided evenly by itself and 1
- List of all prime numbers less than 100
  - 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47
  - 53, 59, 61, 67, 71, 73, 79, 83, 89, 97
- A natural number greater than 1 that is not a prime number is called a composite number

Number Theory

- This strategy involves choosing a variable(s) to algebraically represent a situation
- Even Integers: 2n, n is any integer (all cases)
- Odd Integers: 2n + 1
- Consecutive Integers: n, n + 1, n + 2, etc
- Consecutive Odd Integers: 2n + 1, 2n + 3, 2n + 5, etc
- Consecutive Even Integers: 2n, 2n +2, 2n + 4, etc

- Write an expression for the following:
- An Even Number
- An Odd Number
- The sum of two consecutive even integers
- The sum of the squares of two consecutive integers
- The difference of squares of two consecutive integers
- A two digit number
- The product of two consecutive integers
- The product of an even and odd integer

Examples of Deductive Reasoning

- Prove deductively that the sum of two even integers is an even integer
  - Let one integer be 2n
  - Let the other integer be 2m
  - 2n + 2m = 2(n +m) --> 2 times any number is even
  - Therefore the sum of two even integers is always an even integer
  - m, n are integers

- Prove deductively that the sum of two odd integers is always an even integer
  - One odd integer: $2m + 1$
  - One odd integer: $2n + 1$
  - Need to show that $(2m + 1) + (2n + 1)$ = even integer
  - $(2m + 1) + (2n + 1)$

    $= 2m + 2n + 2$

    $= 2\ (m + n + 1)$

    $= 2$ (integer) --> even integer
- Therefore the sum of two odd integers is always an even integer

- Prove deductively that the sum of an odd and even number is odd
  - Let $2m + 1$ = odd number
  - Let $2n$ = even number
  - m & n are integers
  - $2m + 1 + 2n$

    $= 2m + 2n + 1$

    $= 2(m + n) +1$

    = odd number
- Therefore the sum of an odd and even number is always odd

- Prove deductively that the product of two odd integers is always an odd integer
  - Let $2m + 1$ = odd number
  - Let $2n + 1$ = odd number
  - m & n are integers
  - $(2m + 1)(2n + 1)$

    $= 4mn + 2m + 2n + 1$

    $= 2(2mn + m + n) + 1$

    $= 2(\text{integer}) + 1$

    = odd integer
- Therefore the product of two odd integers is always an odd integer

- Prove deductively the difference of the squares of two consecutive numbers is odd
  - Let x = a number
  - Let x + 1 = next number
  - x = an integer
  - $(x+1)^2 – x^2$

    $= (x+1)(x+1) – x^2$

    $= x^2 + x + x + 1 – x^2$

    $= 2x + 1$

    = odd number

- Prove that the sum of five consecutive integers is 5 times bigger than the median
  - Let x = first number
  - $x + (x+1) + (x+2) + (x+3) + (x+4)$

    $= 5x + 10$

    $= 5(x+2)$

    = 5 times the median
- This proves the above conjecture

- Conjecture: The sum of four consecutive integers is equal to the sum of the first and last integer, then multiplied by two

- Conjecture: The square of the sum of two positive integers is greater than the sum of the squares of the same two integers

- Prove the above conjectures deductively

Number tricks

- Think of a number. Multiply that number by 2, then add 6 and divide the result by 2. Next subtract the original number. What is the resulting number?
- Let x = a number
  - Number: x
  - Multiply by 2: 2x
  - Add 6: 2x + 6
  - Divide by 2: x + 3
  - Subtract the original number: 3
  - Result: 3
  - This shows the number trick always results in 3 for whatever number x is

Number tricks

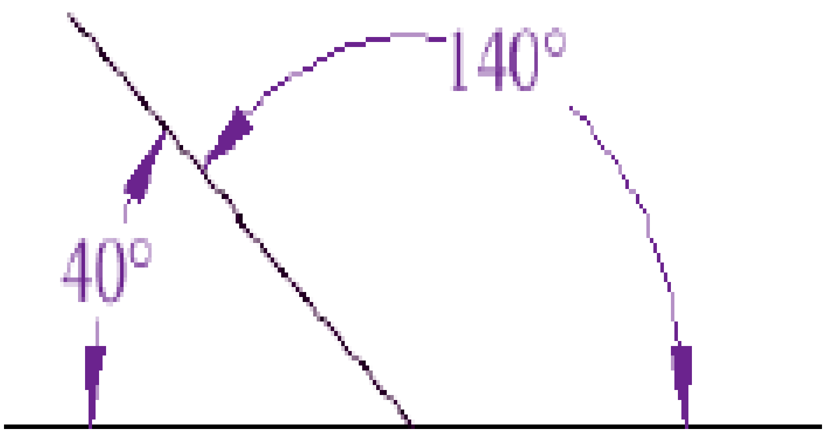
- Think of a number. Add 50 to that number, then multiply the result by 2. Next subtract the original number. What is the resulting number?
- Let x = a number
  - Number: x
  - Add 50: x + 50
  - Multiply by 2: 2x + 100
  - Subtract the original number: x + 100
  - Result: x + 100
  - This shows the number trick always results in a number that is 100 more than whatever number x is.

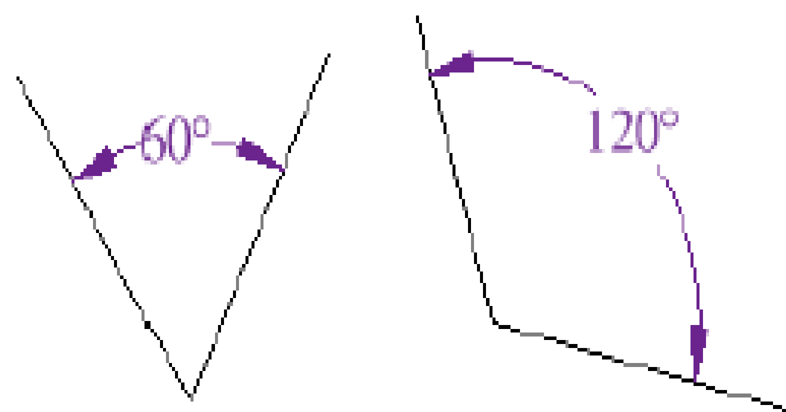
For you to do!

- Pg. 31-32 #'s 1, 2, 4, 5, 7, 11, 13

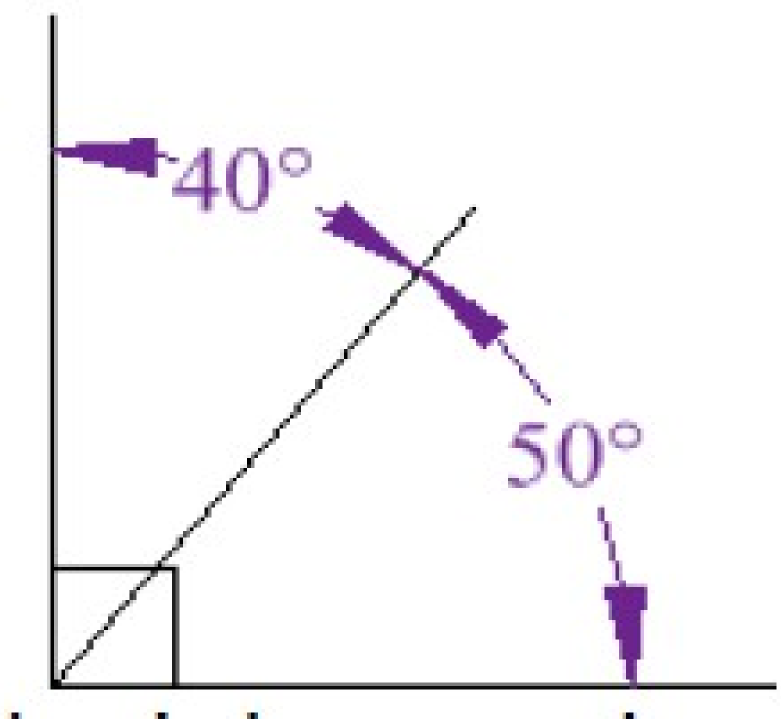
Good to know Geometric Properties

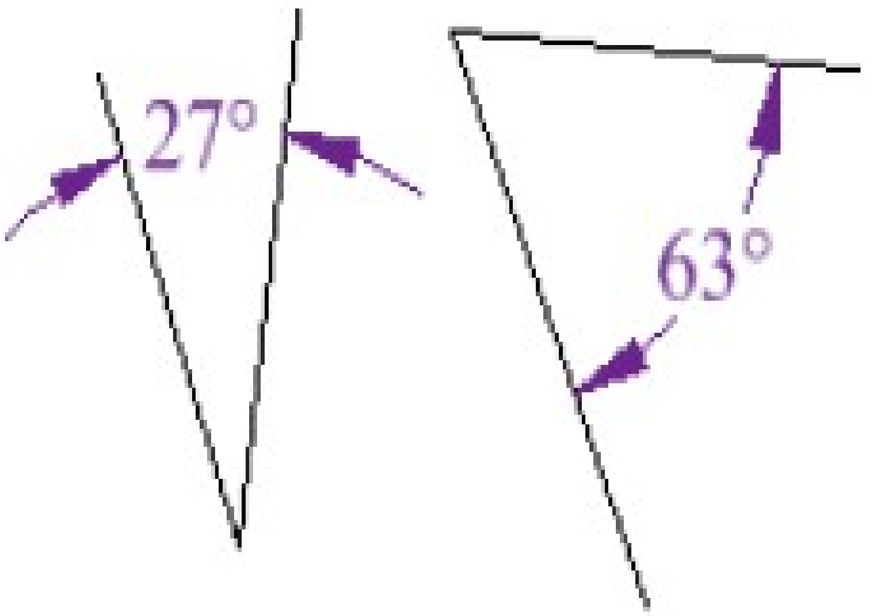
- Supplementary Angles
  - Angles that add up to 180 degrees (Straight Angle)
  - They don't have to be together to be supplementary, just so long as they add up to 180
  -





- Complementary Angles
  - Angles that add up to 90 degrees (right angle)
  - They don't have to be together to be complementary





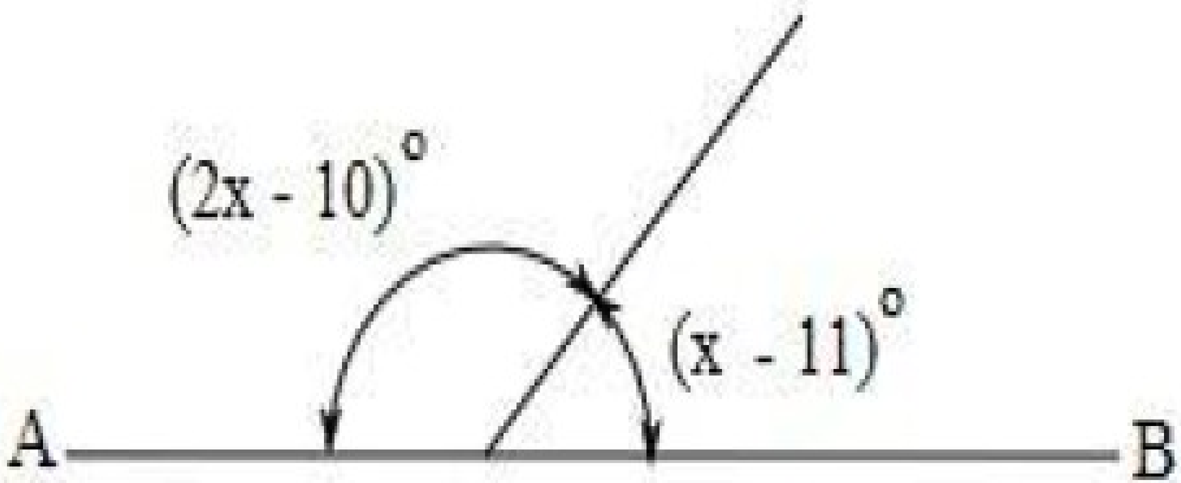
Practice

- Two angles are supplementary and one of them is 31 degrees. What's the other angle?
  - A: 31 degrees
  - B: 59 degrees
  - C: 121 degrees
  - D: 149 degrees

Practice

- Two angles are complementary and one of them is 77 degrees. What's the other angle?
  - A: 13 degrees
  - B: 23 degrees
  - C: 77 degrees
  - D: 103 degrees

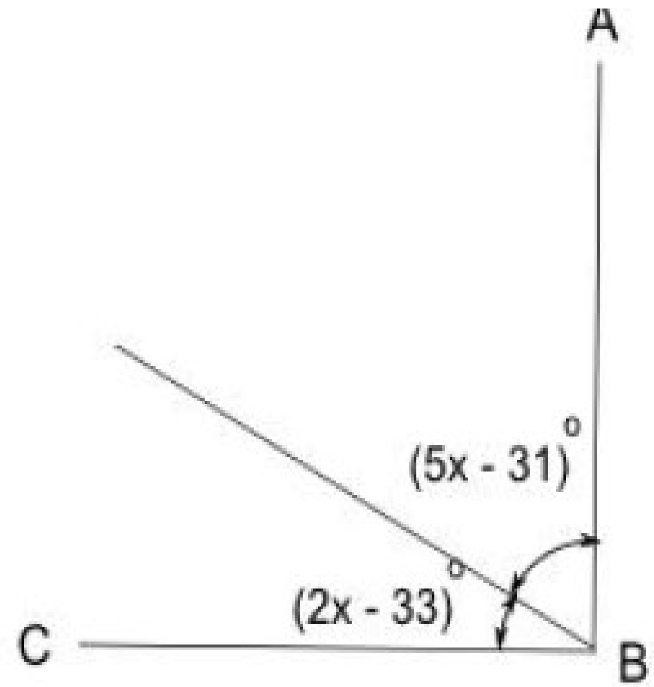
3.



AB is a straight line. What is the value of x?

A) 53° B) 56° C) 67° D) 124°

4.



ABC is a right angle. What is the value of x?

A x = 22	B x = 30
C x = 32	D x = 34.9

- Two angles are supplementary and one is 3 times bigger than the other. What's the size of the smaller angle? (45 degrees)

- Two angles are complementary and one is 4 times bigger than the other. What's the size of the bigger angle? (72 degrees)

Transitive Property of Equality

- The Transitive Property states that:
  - If a = b & b = c then a = c
  - In other words, “things that are equal to the same thing are equal to each other”
- Examples
  - 5 = 3 + 2; 3 + 2 = 1 + 4 ----> 5 = 1 + 4
  - a = 3; 3 = b ------> a = b

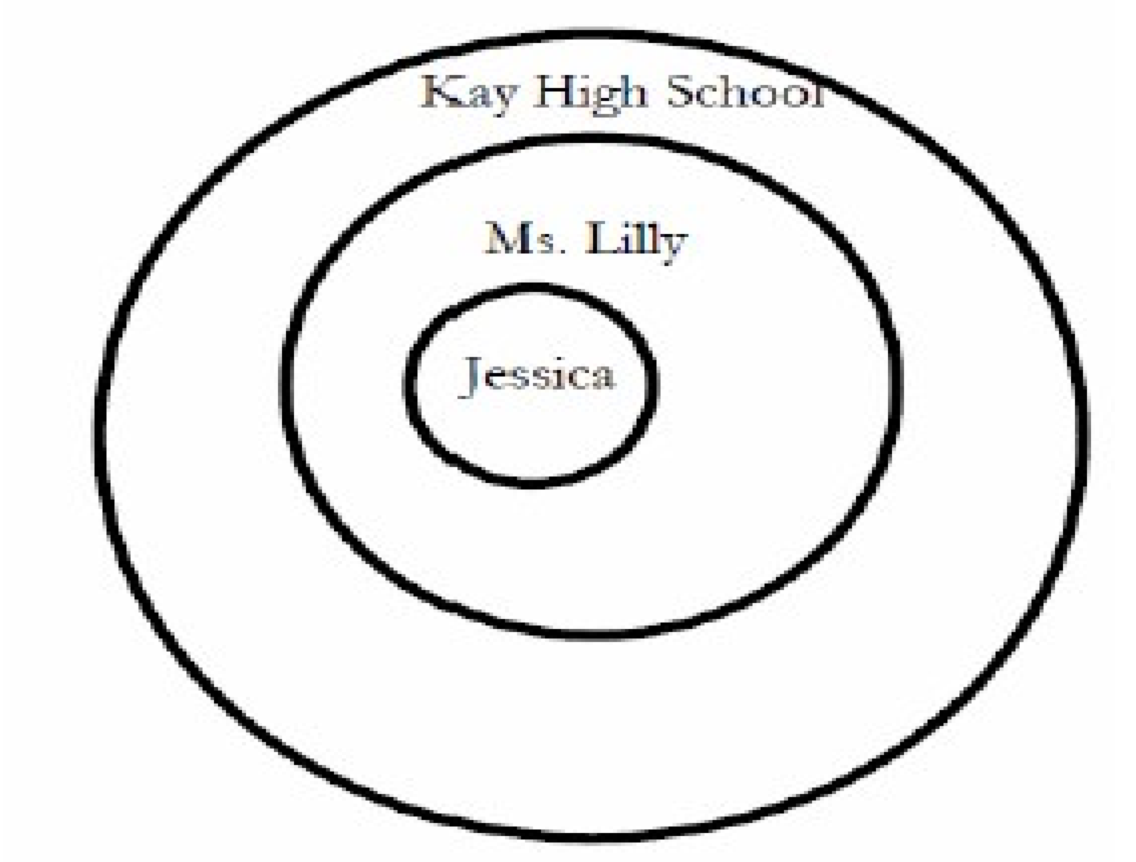
- See page 29 for a geometric proof example of the Transitive Property

Section 1.5: Proofs that are not Valid

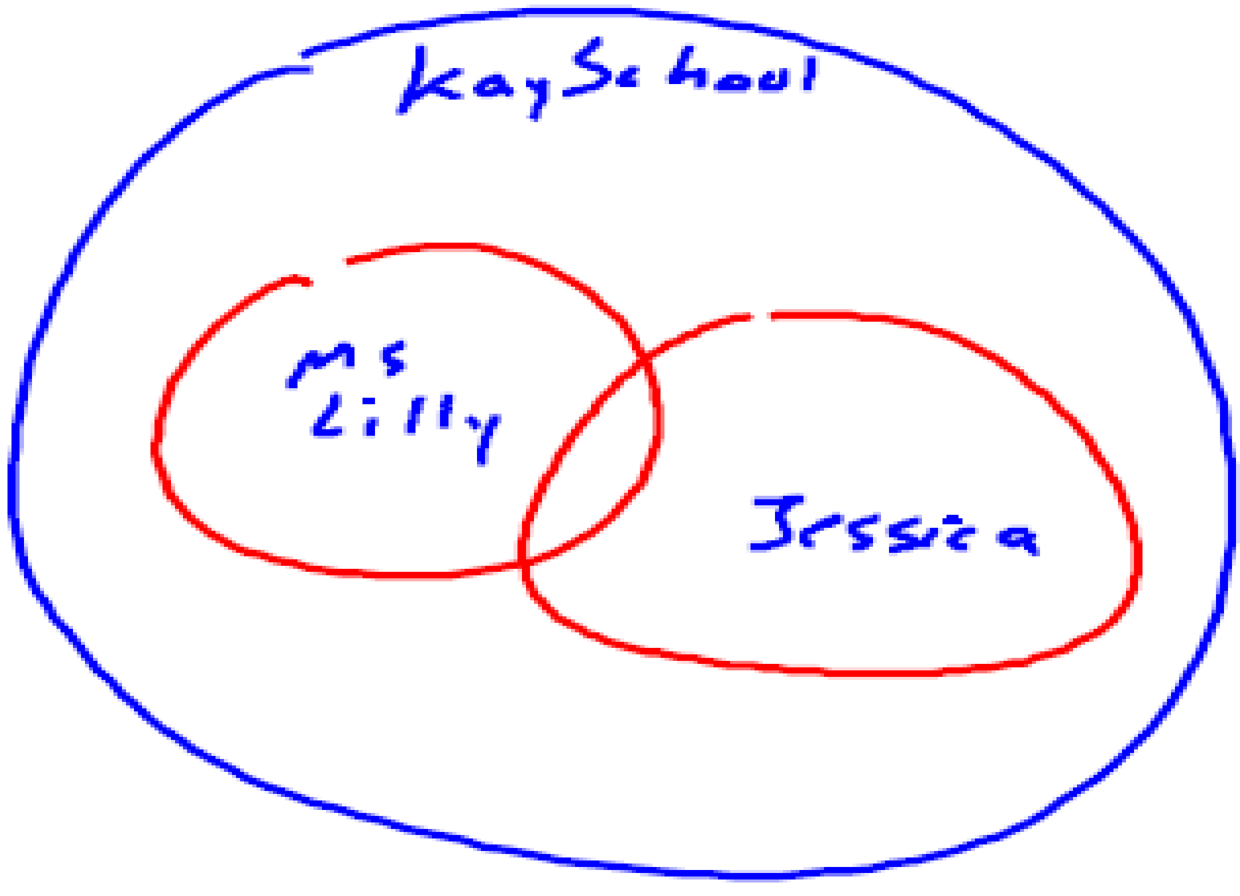
- Invalid Proof
  - Contains an error in reasoning or an incorrect assumption
- Typical Errors in Proofs include:
  - Beginning with a false statement
  - Algebraic errors
  - Division by zero
  - Circular reasoning
    - Uses the conclusion in the proof itself

Examples

- Ms. Lilly is a teacher at a school. Jane is a student at that school. A possible conclusion can be that Jane is a student of Ms. Lilly.



- There is an error in this conclusion
  - Wrong assumption in that there is nothing given to suggest Jane is a student of Ms. Lilly



- Xander claims he can prove that 3 = 4. Xavier scoffs at that notion.
  - a + b = c
  - (4a – 3a) + (4b – 3b) = 4c – 3c
  - 4a + 4b – 4c = 3a + 3b – 3c
  - 4(a + b – c) = 3(a + b -c)
  - 4 = 3
- This looks good but in step 4 both sides are divided by (a + b – c). In step 1 it was established that a + b = c or a + b – c = 0. So step 4 is division by 0 which is disallowed.

Circular Reasoning

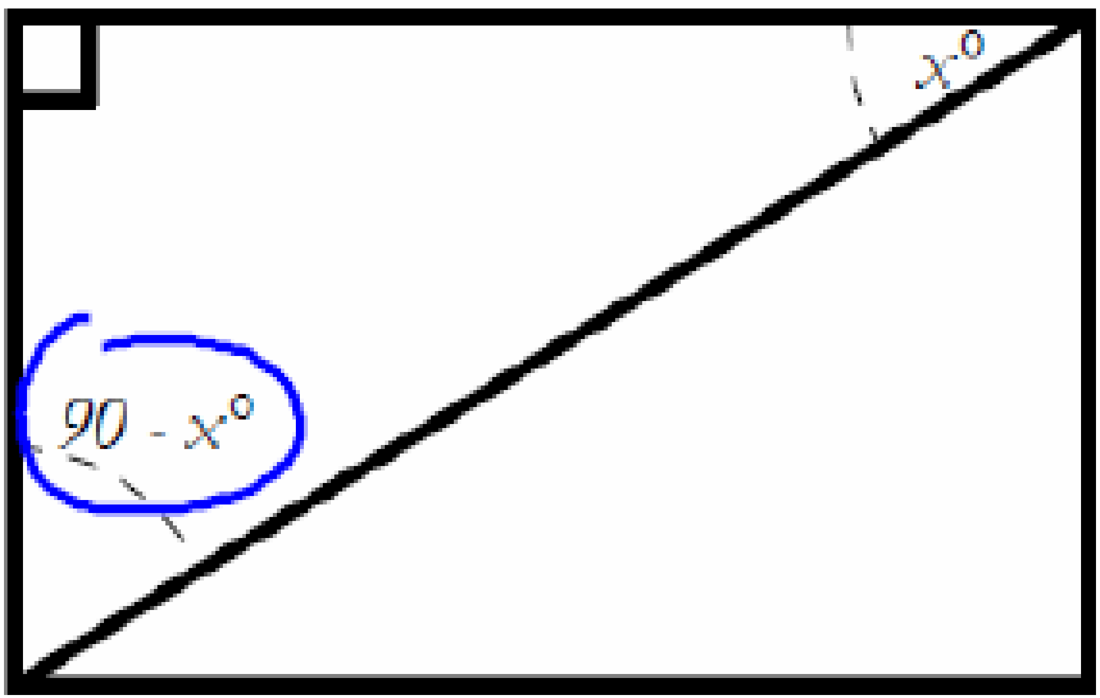
- Xavier claims that he can prove that -5 = 5. Xander scoffs at that notion.
- Proof
  - Assume that –5 = 5
  - Square both sides ---> $(-5)^2 = 5^2$
  - End up with 25 = 25 -----> Proof is good
- The problem is that the proof started with a false assumption that -5 = 5, which is clearly wrong.

Note

- If an assumption is false, then any argument based on the false assumption is also wrong
- Starting with an error and then ending by saying the error has been proven is arguing in a circle

Circular Reasoning

- Darren claims he can prove that the sum of the interior angles in a triangle is 180 degrees.



- Proof
  - He drew a rectangle with a diagonal
  - He knew all the angles were 90 degrees
  - He labeled one of the angles as x
  - Other angle is 180 – 90 – x = 90 – x
  - Then 90 + x + (90 – x) = 180
- His error is based on circular reasoning
  - He assumed true what he was trying to prove

Example

- Choose a number. Add 3. Double it. Add 4. Divide by 2. Subtract the original number.
  - N Choose any number
  - N + 3 Add 3
  - 2n + 6 Double it
  - 2n + 10 Add 4
  - N + 10 Divide by 2
  - 10 Subtract the original number
- Where is the error in the above proof

- In step 4, 2n + 10 was to be divided by 2
- However the result has the 10 not being divided by 2
- This is known as an algebraic error

For you to do!

- Pg. 42-44 #'s 1, 2, 3, 5, 7

Section 1.6: Reasoning to Solve Problems

- Matt, Jon, Mary, Jessica are sitting around a table.
- People with names starting with the same letter are sitting across from each other
- Mary is sitting to the right of Jon
- What's the sitting order?
- 

- Ted, Ken, Allyson and Jen (married couples) each have a favorite sport: swimming, running, biking and golf. Who likes which based on:
  - Ted dislikes golf
  - Each woman's favorite sport is featured in a triathlon
  - Ken and his wife hate running
  - Allyson bought her husband a bike for his sport

	swimming	running	biking	golf
TED	x	x	✓	x
KEN		x		✓
Allyson		✓		x
Janie	✓	x		x

- If one has a 5L and a 3L bottle and plenty of water, how can one get 4L of water into the 5L bottle accurately?

- Fill the 3-liter bottle and pour it into the empty 5-liter bottle.
- Fill the 3-liter bottle again, and pour enough to fill 5-liter bottle. This leaves exactly 1 liter in the 3-liter bottle.
- Empty the 5-liter bottle; pour the remaining 1 liter from the 3-liter bottle into the 5-liter bottle.
- Fill the 3-liter bottle and pour it into the 5-liter bottle. The 5-liter bottle now has exactly 4 liters

- Fill the 5-liter bottle and pour water from it into the 3-liter bottle until it is full.
- This leaves 2 liters in the 5-liter bottle.
- Empty the 3-liter bottle and pour the 2 liters of water from the 5-liter bottle into the 3-liter bottle.
- Fill the 5-liter bottle again.
- Fill the 3-liter bottle from the 5-liter bottle. Since the 3-liter bottle had 2 liters of water, only one liter is transferred leaving exactly 4 liters of water in the 5-liter jug.

- The members of a team have a meeting and shake hands. There are 12 players and 2 coaches. How many handshakes happened?
- Solution
  - 14 people in total, 1st person shakes hands with 13 other people
  - 2nd person shakes hands with first person (already counted) and 12 other people
  - Pattern repeats
- 13+12+11+10+9+8+7+6+5+4+3+2+1 = 91 handshakes

- Inductive reasoning was used to solve this problem b/c we determined the new number of handshakes based on the pattern found in the first two cases

- 8 people signed up for fun night. The people are Dave, Angie, Josh, Tanya, Joy, Stu, Linus and Sue. They were paired up male-female. At start, Dave and his partner were to the left of Stu (female). Across from Dave was Sue, who was to the right of Josh. Dave's brother's partner, Tanya, was across from Stu. Joy was not on Stu's right. Name the four pairs of partners.
- Answer found on page 47-48
- Deductive reasoning was used b/c given info was used to deduce the arrangement.
  If ... then = deductive.

- The 3 little pigs build houses of straw, sticks and bricks. Figure out who built which house and where it was built.
- Penny Pig did not build a brick house
- The straw house was not medium sized
- Peter's house was made of sticks and was neither medium or small
- Patricia pig's house is in Pleasantville
- The house is Hillsdale is large
- One house is in Riverview

	Town	Size	Material
Patricia Pig			
Penny Pig			
Peter Pig			

- At the end of a banquet 10 people shake hands with each other. How many handshakes will there be in total?
  - A: 100
  - B: 20
  - C: 45
  - D: 50
  - E: 90

- John, Jenita, Maria & Julie picked a ball with a certain color. The colors are yellow, violet, green and red. Who picked what color?
- John and Maria both don't like violet
- The person who likes red is a girl
- Jenita likes green, but it's not her favorite color
- Jenita's favorite color is either yellow or green, but she can have only one favorite color
- John likes red, but not his favorite color

- Jenita: Yellow
- John: Green
- Maria: Red
- Julie: Violet

For you to do!

Page 48-51 #'s 1, 6, 7, 8, 16