CHAPTER 1
INTRODUCTION

1.1 REINFORCED CONCRETE STRUCTURES

Concrete is arguably the most important building material, playing a part in all building structures. Its virtue is its versatility, i.e. its ability to be moulded to take up the shapes required for the various structural forms. It is also very durable and fire resistant when specification and construction procedures are correct.

Concrete can be used for all standard buildings both single storey and multi-storey and for containment and retaining structures and bridges. Some of the common building structures are shown in Fig.1.1 and are as follows:

1. The single-storey portal supported on isolated footings;
2. The medium-rise framed structure which may be braced by shear walls or unbraced. The building may be supported on isolated footings, strip foundations or a raft;
3. The tall multi-storey frame and core structure where the core and rigid frames together resist wind loads. The building is usually supported on a raft which in turn may bear directly on the ground or be carried on piles or caissons. These buildings usually include a basement.

Complete designs for types 1 and 2 are given. The analysis and design for type 3 is discussed. The design of all building elements and isolated foundations is described.

1.2 STRUCTURAL ELEMENTS AND FRAMES

The complete building structure can be broken down into the following elements:

Beams horizontal members carrying lateral loads

Slabs horizontal plate elements carrying lateral loads

Columns vertical members carrying primarily axial load but generally subjected to axial load and moment

Walls vertical plate elements resisting vertical, lateral or in-plane loads

Bases and foundations pads or strips supported directly on the ground that spread the loads from columns or walls so that they can be supported by the ground without excessive settlement. Alternatively the bases may be supported on piles.

To learn about concrete design it is necessary to start by carrying out the design of separate elements. However, it is important to recognize the function of the element in the complete structure and that the complete structure or part of it needs to be analysed to obtain actions for design. The elements listed above are illustrated in Fig.1.2 which shows typical *cast-in-situ* concrete building construction.

A *cast-in-situ* framed reinforced concrete building and the rigid frames and elements into which it is idealized for analysis and design are shown in Fig.1.3. The design with regard to this building will cover

1. one-way continuous slabs
2. transverse and longitudinal rigid frames
3. foundations

Various types of floor are considered, two of which are shown in Fig.1.4. A one-way floor slab supported on primary reinforced concrete frames and secondary continuous flanged beams is shown in Fig.1.4(a). In Fig.1.4(b) only primary reinforced concrete frames are constructed and the slab spans two ways. Flat slab construction, where the slab is supported by the columns without beams, is also described. Structural design for isolated pad, strip and combined and piled foundations and retaining walls (Fig.1.5) is covered in this book.

1.3 STRUCTURAL DESIGN

The first function in design is the planning carried out by the architect to determine the arrangement and layout of the building to meet the client's requirements. The structural engineer then determines the best structural system or forms to bring the architect's concept into being. Construction in different materials and with different arrangements and systems may require investigation to determine the most economical answer. Architect and engineer should work together at this conceptual design stage.

Once the building form and structural arrangement have been finalized the design problem consists of the following:

1. idealization of the structure into load bearing frames and elements for analysis and design
2. estimation of loads
3. analysis to determine the maximum moments, thrusts and shears for design
4. design of sections and reinforcement arrangements for slabs, beams, columns and walls using the results from 3
5. production of arrangement and detail drawings and bar schedules

1.4 DESIGN STANDARDS

In the UK, design is generally to limit state theory in accordance with BS8110:1997: *Structural Use of Concrete Part 1: Code of Practice for Design and Construction*

The design of sections for strength is according to plastic theory based on behaviour at ultimate loads. Elastic analysis of sections is also covered because this is used in calculations for deflections and crack width in accordance with BS 8110:1985: *Structural Use of Concrete Part 2: Code of Practice for Special Circumstances*

The loading on structures conforms to

BS 6399–1:1996 *Loading for buildings. Code of Practice for Dead and Imposed Loads*

BS 6399–2:1997 *Loading for buildings. Code of Practice for Wind Loads*

BS 6399–3:1988 *Loading for buildings. Code of Practice for Imposed Roof Loads*

The codes set out the design loads, load combinations and partial factors of safety, material strengths, design procedures and sound construction practice. A thorough knowledge of the codes is one of the essential requirements of a designer. Thus it is important that copies of these codes are obtained and read in conjunction with the book. Generally, only those parts of clauses and tables are quoted which are relevant to the particular problem, and the reader should consult the full text.

Only the main codes involved have been mentioned above. Other codes, to which reference is necessary, will be noted as required.

1.5 CALCULATIONS, DESIGN AIDS AND COMPUTING

Calculations form the major part of the design process. They are needed to determine the loading on the elements and structure and to carry out the analysis and design of the elements. Design office calculations should be presented in accordance with

Higgins, J.B and Rogers, B.R., 1999, Designed and detailed. British Cement Association.

The need for orderly and concise presentation of calculations cannot be emphasized too strongly.

Design aids in the form of charts and tables are an important part of the designer's equipment. These aids make exact design methods easier to apply, shorten design time and lessen the possibility of making errors. Part 3 of BS 8110 consists of design charts for beams and columns, and the construction of charts is set out in this book, together with representative examples. Useful books are

Reynolds, C.E. and Steedman, J.C., 1988, Reinforced concrete designers handbook, (Spon Press).

Goodchild, C.H., 1997, Economic concrete frame elements, (Reinforced Concrete Council).

The use of computers for the analysis and design of structures is standard practice. Familiarity with the use of Spread Sheets is particularly useful. A useful reference is

Goodchild, C.H. and Webster, R.M., 2000, Spreadsheets for concrete design to BS 8110 and EC2, (Reinforced concrete council).

In analysis exact and approximate manual methods are set out but computer analysis is used where appropriate. However, it is essential that students understand the design principles involved and are able to make manual design calculations before using computer programs.

1.6 TWO CARRIAGE RETURNS DETAILING

The general arrangement drawings give the overall layout and principal dimensions of the structure. The structural requirements for the individual elements are presented in the detail drawings. The output of the design calculations are sketches giving sizes of members and the sizes, arrangement, spacing and cut-off points for reinforcing bars at various sections of the structure. Detailing translates this information into a suitable pattern of reinforcement for the structure as a whole. Detailing is presented in accordance with the

Standard Method of Detailing Structural Concrete. Institution of Structural Engineers, London, 1989.

It is essential for the student to know the conventions for making reinforced concrete drawings such as scales, methods for specifying steel bars, links, fabric, cut-off points etc. The main particulars for detailing are given for most of the worked exercises in the book. The bar schedule can be prepared on completion of the detail drawings. The form of the schedule and shape code for the bars are to conform to

BS 8666:2000: *Specification for Scheduling, Dimensioning, Bending and cutting of steel for Reinforcement for Concrete*

It is essential that the student carry out practical work in detailing and preparation of bar schedules prior to and/or during his design course in reinforced concrete. Computer detailing suites are now in general use in design offices.

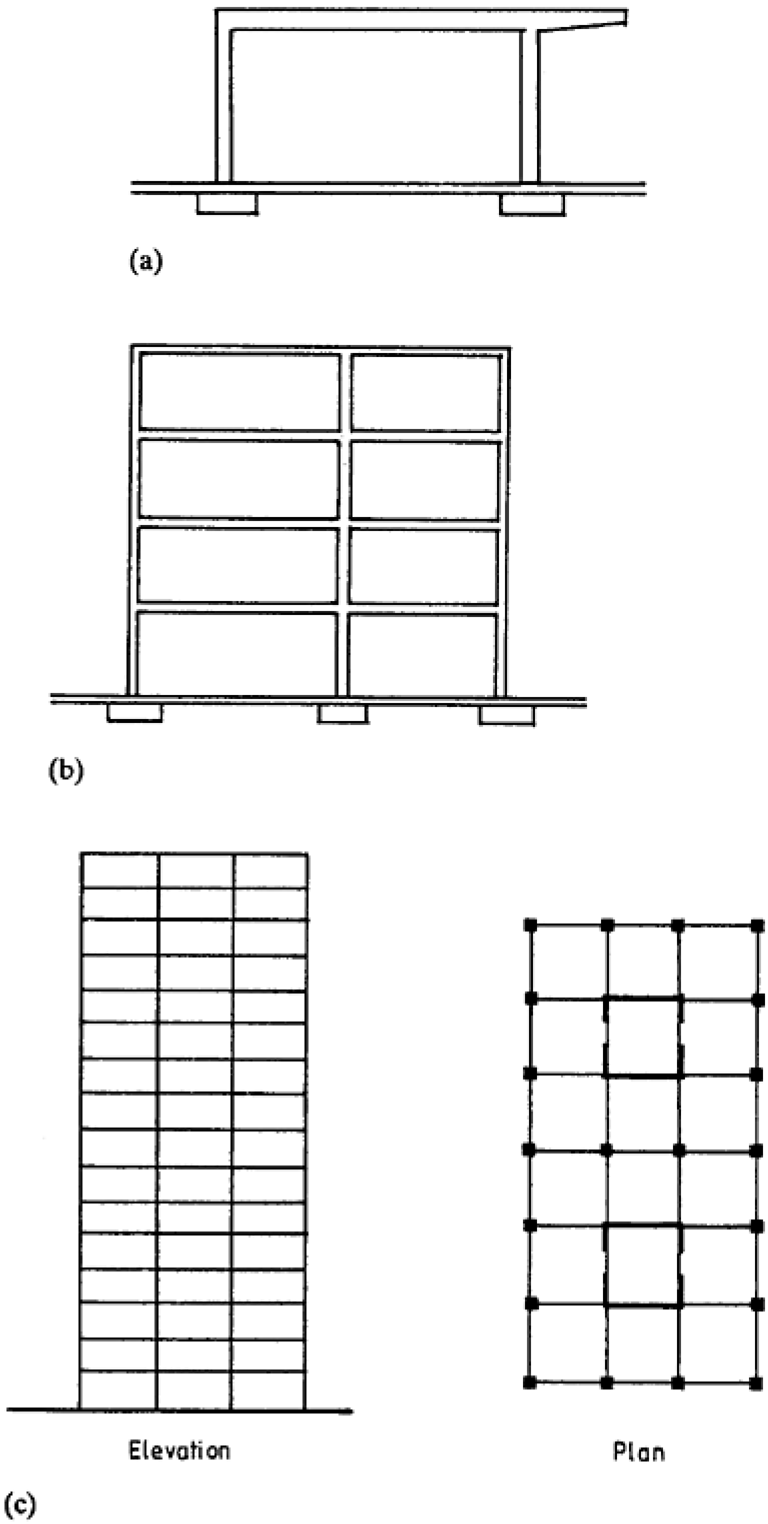


Fig 1.1 (a) Single storey portal; (b) medium-rise reinforced concrete framed building; (c) reinforced concrete frame and core structure

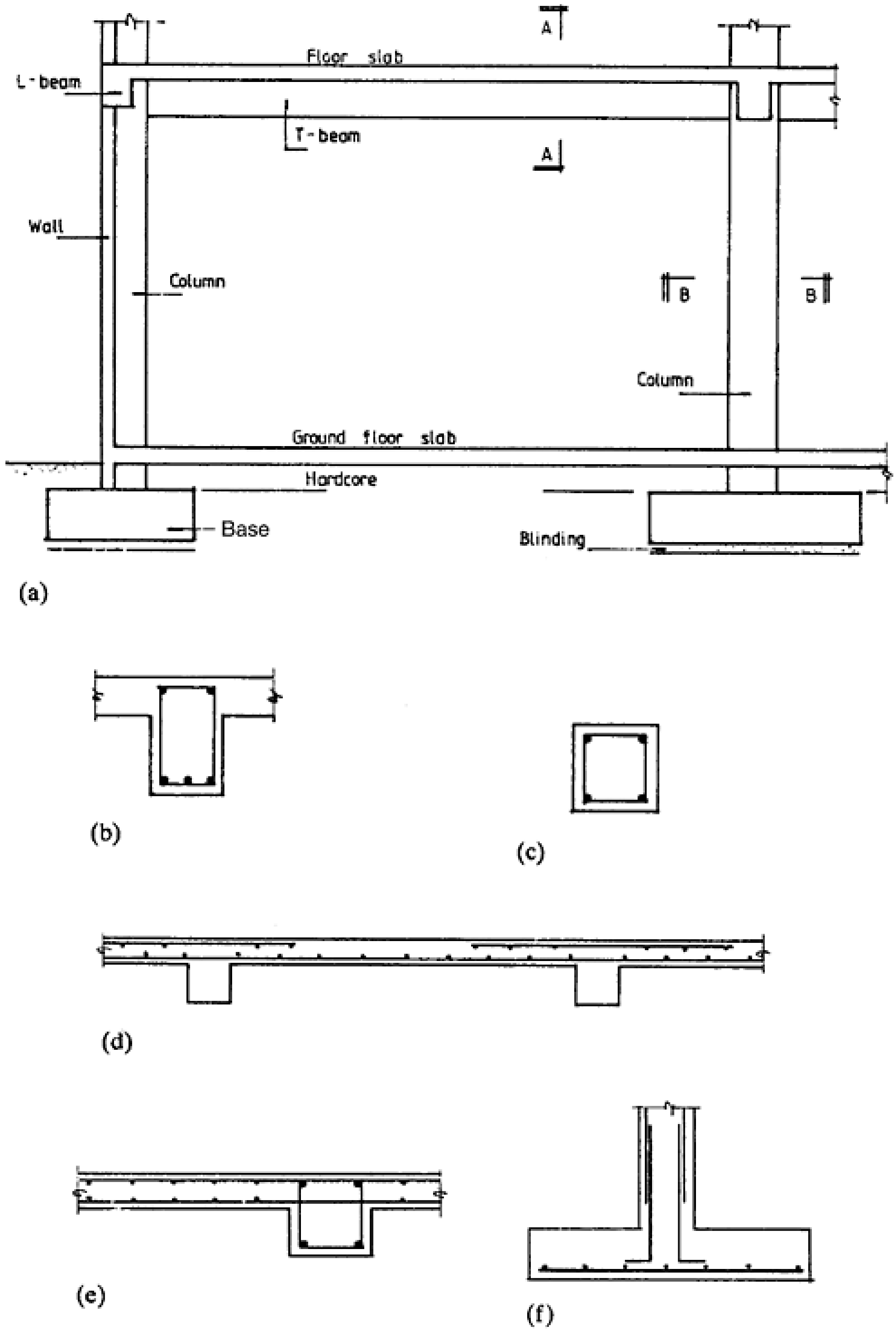


Fig 1.2 (a) Part elevation of reinforced concrete building; (b) section AA, T-beam ; (c) section BB; (d) continuous slab; (e) wall; (f) column base

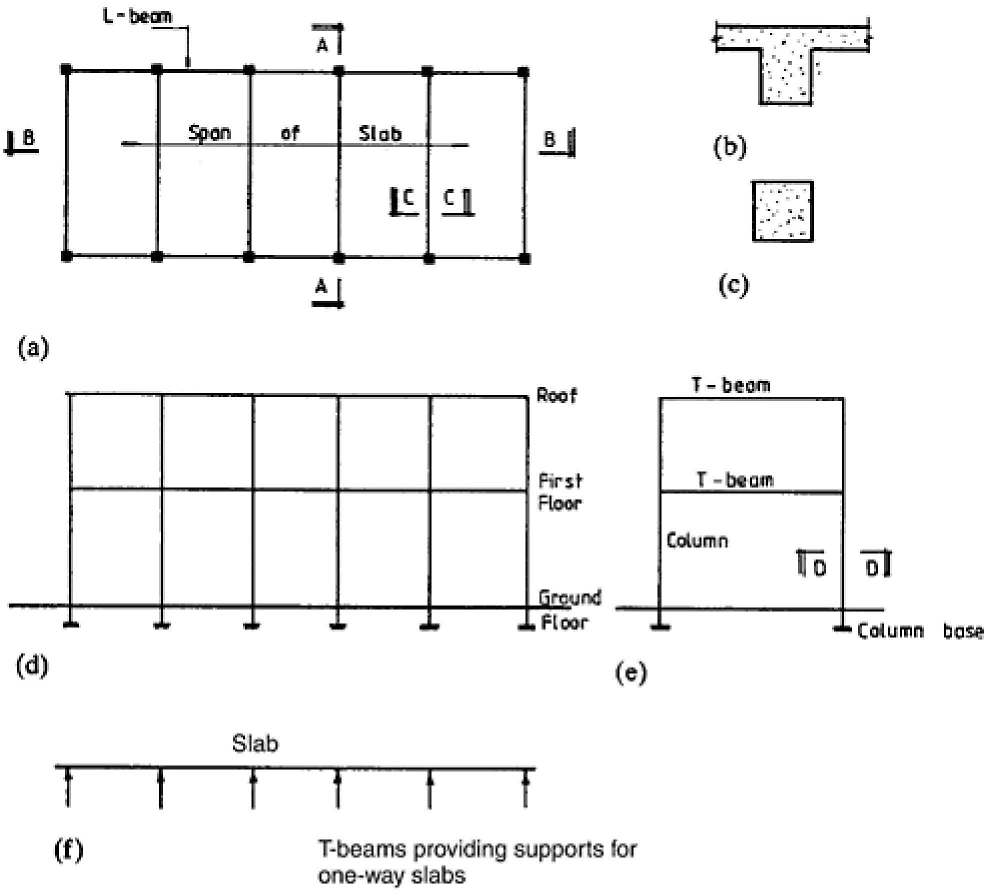


Fig 1.3 (a) Plan of roof and floor; (b) section CC, T-beam; (c) section DD, column; (d) side elevation, longitudinal frame; (e) section AA, transverse frame; (f) continuous one-way slab.

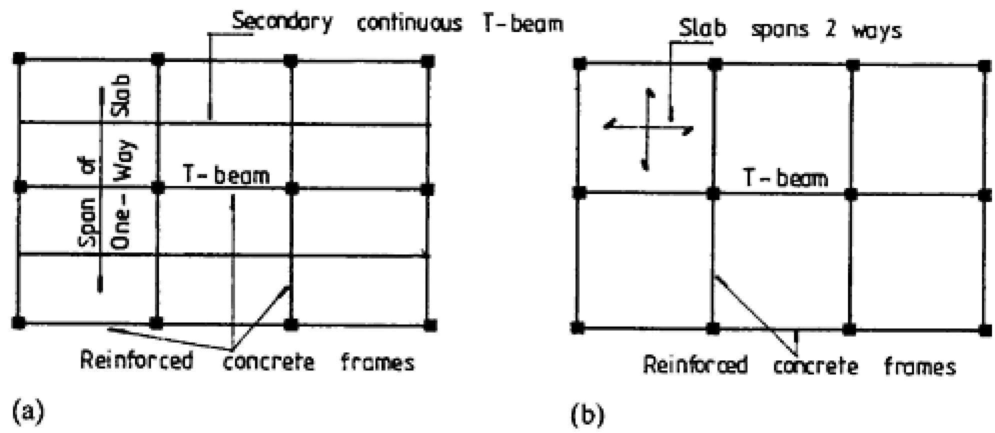


Fig.1.4 (a) One-way floor slab; (b) two-way floor slab.

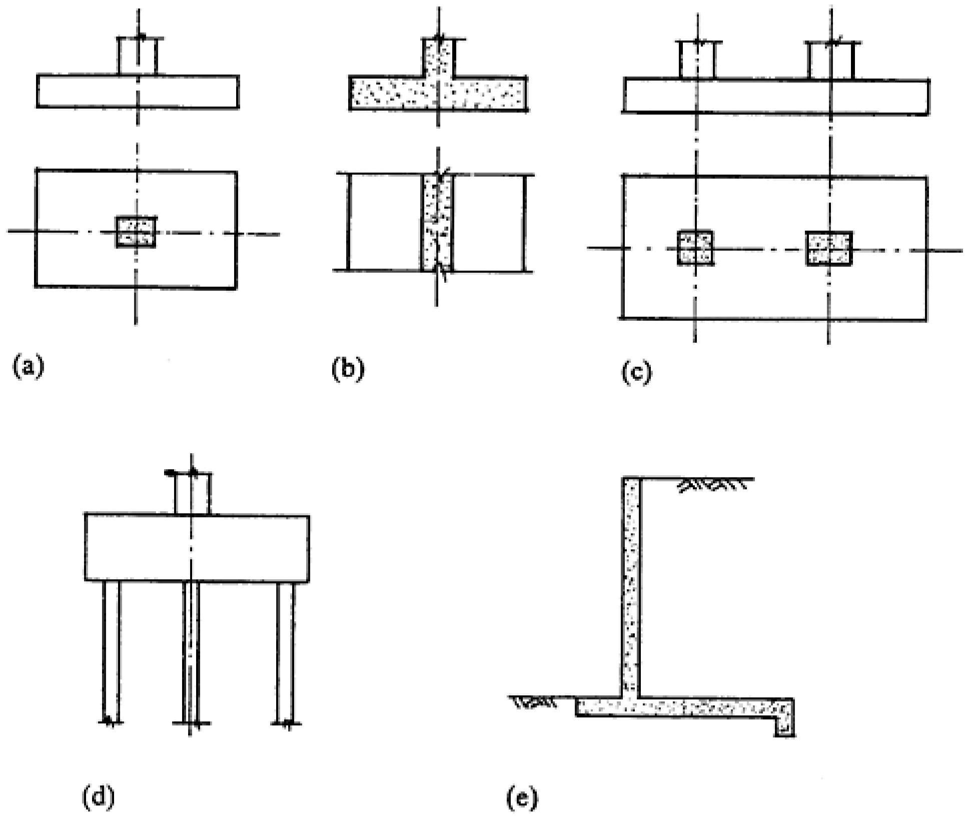


Fig.1.5 (a) Isolated base; (b) wall footing; (c) combined base; (d) piled foundation; (e) retaining wall.

CHAPTER 2
MATERIALS, STRUCTURAL FAILURES AND DURABILITY

2.1 REINFORCED CONCRETE STRUCTURES

Reinforced concrete is a composite material of steel bars embedded in a hardened concrete matrix; concrete, assisted by the steel, carries the compressive forces, while steel resists tensile forces. Concrete itself is a composite material. The dry mix consists of cement and coarse and fine aggregates. Water is added and this reacts with the cement which hardens and binds the aggregates into the concrete matrix; the concrete matrix sticks or bonds onto the reinforcing bars.

The properties of the constituents used in making concrete, mix design and the principal properties of concrete are discussed briefly. Knowledge of the properties and an understanding of the behaviour of concrete is an important factor in the design process. The types and characteristics of reinforcing steels are noted.

Deterioration of and failures in concrete structures are now of widespread concern. This is reflected in the increased prominence given in the concrete code BS 8110 to the durability of concrete structures. The types of failure that occur in concrete structures are listed and described. Finally the provisions regarding the durability of concrete structures noted in the code and the requirements for cover to prevent corrosion of the reinforcement and provide fire resistance are set out.

2.2 CONCRETE MATERIALS

2.2.1 Cement

Ordinary Portland cement (OPC) is the commonest type in use. The raw materials from which it is made are lime, silica, alumina and iron oxide. These constituents are crushed and blended in the correct proportions and burnt in a rotary kiln. The clinker is cooled, mixed with gypsum and ground to a fine powder to give cement. The main chemical compounds in cement are calcium silicates and aluminates.

When water is added to cement and the constituents are mixed to form cement paste, chemical reactions occur and the mix becomes stiffer with time and sets. The addition of gypsum mentioned above retards and controls the setting time. This ensures that the concrete does not set too quickly before it can be placed in its final position or too slowly so as to hold up construction. Two stages in the setting process are defined in

BS EN 197-1:2000: *Cement. Composition, specifications and conformity criteria for common cements*

BS EN 197-2:2000: *Cement. Conformity evaluation*

These are an initial setting time which must be a minimum of 45 min and a final set which must take place in 10 h.

Cement must be sound, i.e. it must not contain excessive quantities of certain substances such as lime, magnesia, calcium sulphate etc. that may expand on hydrating or react with other substances in the aggregate and cause the concrete to disintegrate. Tests are specified for soundness and strength of cement mortar cubes.

Many other types of cement are available some of which are:

1. Rapid hardening Portland cement: the clinker is more finely ground than for ordinary Portland cement. This is used in structures where it is necessary for the concrete to gain strength rapidly. Typical example is where the formwork needs to be removed early for reuse.
2. Low heat Portland cement: this has a low rate of heat development during hydration of the cement. This is used in situations such as thick concrete sections where it is necessary to keep the rate of heat generation due to hydration low as otherwise it could lead to serious cracking.
3. Sulphate-resisting Portland cement: this is often used for foundation concrete when the soil contains sulphates which can attack OPC concrete.

A very useful reference is

Adam M Neville, Properties of Concrete, Prentice-Hall, 4th Edition, 1996.

2.2.2 Aggregates

The bulk of concrete is aggregate in the form of sand and gravel which is bound together by cement. Aggregate is classed into the following two sizes;

1. coarse aggregate: gravel or crushed rock 5 mm or larger in size
2. fine aggregate: sand less than 5 mm in size

Natural aggregates are classified according to the rock type, e.g. basalt, granite, flint. Aggregates should be chemically inert, clean, hard and durable. Organic impurities can affect the hydration of cement and the bond between the cement and the aggregate. Some aggregates containing silica may react with alkali in the cement causing the some of the larger aggregates to expand which may lead to the concrete disintegrating. This is the alkali-silica reaction. Presence of chlorides in aggregates, e.g. salt in marine sands, will cause corrosion of the steel reinforcement. Excessive amounts of sulphate will also cause concrete to disintegrate.

To obtain a dense strong concrete with minimum use of cement, the cement paste should fill the voids in the fine aggregate while the fine aggregate and cement paste fills the voids in the coarse aggregate. Coarse and fine aggregates are graded by sieve analysis in which the percentage by weight passing a set of standard sieve sizes is determined. Grading limits for each size of coarse and fine aggregate are set out in

BS EN 12620:2002: *Aggregates for Concrete*

The grading affects the workability; a lower water-to-cement ratio can be used if the grading of the aggregate is good and therefore strength is also increased. Good grading saves cement content. It helps prevent segregation during placing and ensures a good finish.

2.2.3 Concrete Mix Design

Concrete mix design consists in selecting and proportioning the constituents to give the required strength, workability and durability. Mixes are defined in

BS 8500–1:2002: *Concrete. Methods of Specifying and guidance for the specifier*

BS 8500–2:2002: *Specifications for constituent materials and concrete*

The five types are

1. Designated concretes: This is used where concrete is intended for use such as plain and reinforced foundations, floors, paving, and other given in Table A.6 or A.7 of the code.
2. Designed concretes: This is the most flexible type of specification. The environment to which the concrete is exposed, the intended working life of the structure, the limiting values of composition are all taken account of in selecting the requirements of the concrete mix.
3. Prescribed concretes: This is used where the specifier prescribes the exact composition and constituents of the concrete. No requirements regarding concrete strength can be prescribed. This has very limited applicability.
4. Standardised prescribed concretes: This is used where concrete is site batched or obtained from a ready mixed concrete producer with no third party accreditation.
5. Proprietary concretes: Used where concrete achieves a performance using defined test methods, outside the normal requirements for concrete.

The water-to-cement ratio is the single most important factor affecting concrete strength. For full hydration cement absorbs 0.23 of its weight of water in normal conditions. This amount of water gives a very dry mix and extra water is added to give the required workability. The actual water-to-cement ratio used generally ranges from 0.45 to 0.6. The aggregate-to-cement ratio also affects workability through its influence on the water-to-cement ratio, as noted above. The mix is designed for the ‘target mean strength’ which is the characteristic strength required for design plus a specified number of times the standard deviation of the mean strength.

Several methods of mix design are used. The main factors involved are discussed briefly for mix design according to

Teychenne, R.E. Franklin and Entroy, H.C., 1988, Design of Normal Concrete Mixes. (HMSO, London).

1. Curves giving compressive strength versus water-to-cement ratio for various types of cement and ages of hardening are available. The water-to-cement ratio is selected to give the required strength.

2. Minimum cement contents and maximum free water-to-cement ratios are specified in BS8110: Part 1, Table 3.3, to meet durability requirements. The maximum cement content is also limited to avoid cracking due mainly to shrinkage.

3. In *Design of Normal Concrete Mixes,* the selection of the aggregate-to-cement ratio depends on the grading curve for the aggregate.

Trial mixes based on the above considerations are made and used to determine the final proportions for designed mixes.

2.2.4 Admixtures

Advice on admixtures is given in

BS EN 934–2:1998 *Admixtures for concrete, mortar and grout.*

The code defines admixtures as 'Materials added during the mixing process of in a quantity not more than 5% by mass of the cement content of the concrete, to modify the properties of the mix in the fresh and/or hardened state'.

Admixtures covered by British Standards are as follows:

1. set accelerators or set retarders
2. water-reducing/plasticizing admixtures which give an increase in workability with a lower water-to-cement ratio
3. air-entraining admixtures, which increase resistance to damage from freezing and thawing
4. high range water reducing agents/super plasticizers, which are more efficient than (2) above.
5. hardening accelerators which increases the early strength of concrete.

The general requirements of admixtures are given in Table 1 of the code. The effect of new admixtures should be verified by trial mixes. A useful publication on admixtures is

Hewlett, P.C (Editor). 1988, Cement Admixtures: Uses and Applications, (Longman Scientific and Technical).

2.3 CONCRETE PROPERTIES

The main properties of concrete are discussed below.

2.3.1 Compressive Strength

The compressive strength is the most important property of concrete. The characteristic strength that is the concrete grade is measured by the 28 day cube strength. Standard cubes of 150 or 100 mm for aggregate not exceeding 25 mm in size are crushed to determine the strength. The test procedure is given in

BS EN 12390:2:2000: *Testing Hardened Concrete: Making and curing specimens for strength tests*
BS EN 12390:3:2000: *Testing Hardened Concrete: Compressive strength of test specimens*

2.3.2 Tensile Strength

The tensile strength of concrete is about a tenth of the compressive strength. It is determined by loading a concrete cylinder across a diameter as shown in Fig.2.1 (a). The test procedure is given in
BS EN 12390:6:2000: *Testing Hardened Concrete: Tensile splitting strength of test specimens*

2.3.3 Modulus of Elasticity

The short-term stress-strain curve for concrete in compression is shown in Fig.2.1 (b). The slope of the initial straight portion is the initial tangent modulus. At any point P the slope of the curve is the tangent modulus and the slope of the line joining P to the origin is the secant modulus. The value of the secant modulus depends on the stress and rate of application of the load.
BS 1881–121:1983 *Testing concrete. Methods for determination of Static modulus of elasticity in compression.*
specifies both values to standardize determination of the secant or static modulus of elasticity.
The dynamic modulus is determined by subjecting a beam specimen to longitudinal vibration. The value obtained is unaffected by creep and is approximately equal to the initial tangent modulus shown in Fig.2.1 (b). The secant modulus can be calculated from the dynamic modulus.
BS 8110: Part 1 gives the following expression for the short-term modulus of elasticity in Fig.2.1, the short-term design stress-strain curve for concrete.

$$E_c = 5.5\sqrt{\frac{f_{cu}}{\gamma_m}} \text{ kN/mm}^2$$

where f_{cu}=cube strength and γ_m=material safety factor taken as 1.5. A further expression for the static modulus of elasticity is given in Part 2, section 7.2. (The idealized short-term stress-strain curve is shown in Fig.2.1.)

2.3.4 Creep

Creep in concrete is the gradual increase in strain with time in a member subjected to prolonged stress. The creep strain is much larger than the elastic strain on loading. If the specimen is unloaded there is an immediate elastic recovery and a slower recovery in the strain due to creep. Both amounts of recovery are much less than the original strains under load.

The main factors affecting creep strain are the concrete mix and strength, the type of aggregate, curing, ambient relative humidity and the magnitude and duration of sustained loading.

BS 8110: Part 2, section 7.3, specifies that the creep strain ε_{cc} is calculated from the creep coefficient Φ by the equation

$$\varepsilon_{cc} = \frac{stress}{E_t}\phi$$

where E_t is the modulus of elasticity of the concrete at the age of loading. The creep coefficient Φ depends on the effective section thickness, the age of loading and the relative ambient humidity. Values of Φ can be taken from BS 8110: Part 2, Fig.7.1. Suitable values of relative humidity to use for indoor and outdoor exposure in the UK are indicated in the figure. The creep coefficient is used in deflection calculations.

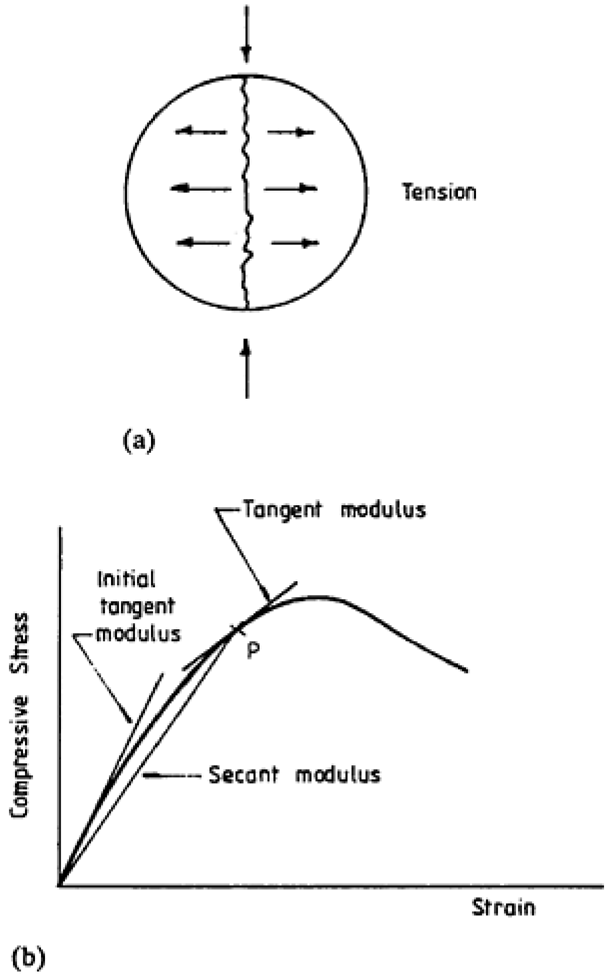


Fig.2.1 (a) Cylinder tensile test; (b) stress-strain curve for concrete.

2.3.5 Shrinkage

Shrinkage or drying shrinkage is the contraction that occurs in concrete when it dries and hardens. Drying shrinkage is irreversible but alternate wetting and drying causes expansion and contraction of concrete.

The aggregate type and content are the most important factors influencing shrinkage. The larger the size of the aggregate is, the lower is the shrinkage and the higher is the aggregate content; the lower the workability and water-to-cement ratio are, the lower is the shrinkage. Aggregates that change volume on wetting and drying, such as sandstone or basalt, give concrete with a large shrinkage strain, while non-shrinking aggregates such as granite or gravel give lower shrinkages. A decrease in the ambient relative humidity also increases shrinkage.

Drying shrinkage is discussed in BS8110: Part 2, section 7.4. The drying shrinkage strain for normal-weight concrete may be obtained from Fig.7.2 in the code for various values of effective section thickness and ambient relative humidity. Suitable values of humidity to use for indoor and outdoor exposure in the UK are indicated in the figure. Values of shrinkage strain are used in deflection calculations.

2.4 TESTS ON WET CONCRETE

2.4.1 Workability

The workability of a concrete mix gives a measure of the ease with which fresh concrete can be placed and compacted. The concrete should flow readily into the form and go around and cover the reinforcement, the mix should retain its consistency and the aggregates should not segregate. A mix with high workability is needed where sections are thin and/or reinforcement is complicated and congested.

The main factor affecting workability is the water content of the mix. Plasticizing admixtures will increase workability. The size of aggregate, its grading and shape, the ratio of coarse to fine aggregate and the aggregate-to-cement ratio also affect workability to some degree.

2.4.2 Measurement of Workability

(a) Slump test

The fresh concrete is tamped into a standard cone which is lifted off after filling and the slump is measured. The slump is 25–50 mm for low workability, 50–100 mm for medium workability and 100–175 mm for high workability. Normal reinforced concrete requires fresh concrete of medium workability. The slump test is the usual workability test specified. The following British standard covers slump test.

BS EN 12350–2: *Testing fresh concrete-Part 2: Slump Test*

(b) Compacting factor test

The degree of compaction achieved by a standard amount of work is measured. The apparatus consists of two conical hoppers placed over one another and over a cylinder. The upper hopper is filled with fresh concrete which is then dropped into the second hopper and into the cylinder which is struck off flush. The compacting factor is the ratio of the weight of concrete in the cylinder to the weight of an equal volume of fully compacted concrete. The compacting factor for concrete of medium workability is about 0.9. The following British standard covers slump test.

BS EN 12350–4: *Testing fresh concrete-Part 4: Degree of compatibility*

(c) Other tests

Other tests are specified for stiff mixes and super plasticized mixes. Reference should be made to specialist books on concrete.

2.5 TESTS ON HARDENED CONCRETE

2.5.1 Normal Tests

The main destructive tests on hardened concrete are as follows.

(a) Cube test: Refer to section 2.3.1 above.

(b) Tensile splitting test: Refer to section 2.3.2 above.

(c) Flexure test: A plain concrete specimen is tested to failure in bending. The theoretical maximum tensile stress at the bottom face at failure is calculated. This is termed the modulus of rupture. It is about 1.5 times the tensile stress determined by the splitting test. The following British standard covers testing of flexural strength.

BS EN 12390:5:2000: *Testing Hardened Concrete: Flexural strength of test specimens*

(d) Test cores: Cylindrical cores are cut from the finished structure with a rotary cutting tool. The core is soaked, capped and tested in compression to give a measure of the concrete strength in the actual structure. The ratio of core height to diameter and the location where the core is taken affect the strength. The strength is lowest at the top surface and increases with depth through the element. A ratio of core height-to-diameter of 2 gives a standard cylinder test. The following British standard covers testing of cores.

BS EN 12504–1: *Testing concrete in structures-Part 1 Cored specimens-Taking examining and testing in compression.*

2.5.2 Non-Destructive Tests

The main non-destructive tests for strength on hardened concrete are as follows.

(a) Rebound hardness test

The Schmidt hammer is used in the rebound hardness test in which a metal hammer held against the concrete is struck by another spring-driven metal mass and rebounds. The amount of rebound is recorded on a scale and this gives an indication of the concrete strength. The larger the rebound number is the higher is the concrete strength. The following British standard covers testing by Rebound hammer.

BS EN 12504-2: *Testing concrete in structures-Part 2: Non-destructive testing-Determination of rebound number.*

(b) Ultrasonic pulse velocity test

In the ultrasonic pulse velocity test the velocity of ultrasonic pulses that pass through a concrete section from a transmitter to a receiver is measured. The pulse velocity is correlated against strength. The higher the velocity is the stronger is the concrete.

(c) Other non-destructive tests

Equipment has been developed to measure

1. crack widths and depths
2. water permeability and the surface dampness of concrete
3. depth of cover and the location of reinforcing bars
4. the electrochemical potential of reinforcing bars and hence the presence of corrosion

A useful reference on testing of concrete in structures is

Bungey, J.H. and Millard, S.G., 1996, Testing Concrete in Structures (Blackie Academic and Professional), 3rd Edition.

2.5.3 Chemical Tests

A complete range of chemical tests is available to measure

1. depth of carbonation
2. the cement content of the original mix
3. the content of salts such as chlorides and sulphates that may react and cause the concrete to disintegrate or cause corrosion of the reinforcement.

The reader should consult specialist literature

2.6 REINFORCEMENT

Reinforcing bars are produced in two grades: hot rolled mild steel bars have yield strength f_y of 250 N/mm^2; hot rolled or cold worked high yield steel bars have yield strength f_y of 460 N/mm^2. Steel fabric is made from cold drawn steel wires welded to form a mesh. It has a yield strength f_y of 460 N/mm^2.

The stress-strain curves for reinforcing bars are shown in Fig.2.2. Hot rolled bars have a definite yield point. A defined proof stress is recorded for the cold worked bars. The value of Young's modulus *E* is 200 kN/mm^2. The idealized design stress-strain curve for all reinforcing bars is shown in BS8110: Part 1 (see Fig.2.2). The behaviour in tension and compression is taken to be the same.

Mild steel bars are produced as smooth round bars. High yield bars are produced as deformed bars in two types defined in the code to increase bond stress: Type 1 Square twisted cold worked bars. This type is obsolete. Type 2 Hot rolled bars with transverse ribs

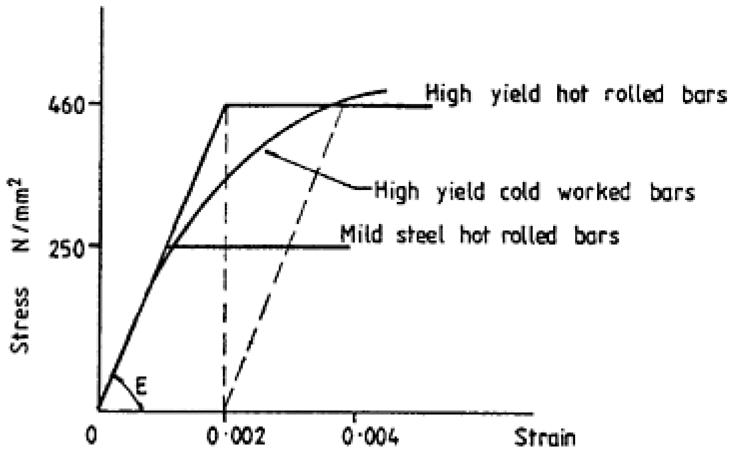


Fig.2.2 Stress-strain curves for reinforcing bars.

2.7 FAILURES IN CONCRETE STRUCTURES

2.7.1 Factors Affecting Failure

Failures in concrete structures can be due to any of the following factors:

1. incorrect selection of materials
2. errors in design calculations and detailing
3. poor construction methods and inadequate quality control and supervision
4. chemical attack
5. external physical and/or mechanical factors including alterations made to the structure

The above items are discussed in more detail below.

2.7.1.1 Incorrect Selection of Materials

The concrete mix required should be selected to meet the environmental or soil conditions where the concrete is to be placed. The minimum grade that should be used for reinforced concrete is grade 30. Higher grades should be used for some foundations and for structures near the sea or in an aggressive industrial environment. If sulphates are present in the soil or ground water, sulphate-resisting Portland cement should be used. Where freezing and thawing occurs air entrainment should be adopted. Further aspects of materials selection are discussed below.

2.7.1.2 Errors in Design Calculations and Detailing

An independent check should be made of all design calculations to ensure that the section sizes, slab thickness etc. and reinforcement sizes and spacing specified are adequate to carry the worst combination of design loads. The check should include overall stability, robustness and serviceability and foundation design.

Incorrect detailing is one of the commonest causes of failure and cracking in concrete structures. First the overall arrangement of the structure should be correct, efficient and robust. Movement joints should be provided where required to reduce or eliminate cracking. The overall detail should be such as to shed water.

Internal or element detailing must comply with the code requirements. The provisions specify the cover to reinforcement, minimum thicknesses for fire resistance, maximum and minimum steel areas, bar spacing limits and reinforcement to control cracking, lap lengths, anchorage of bars etc.

2.7.1.3 Poor Construction Methods

The main items that come under the heading of poor construction methods resulting from bad workmanship and inadequate quality control and supervision are as follows. BS 8110, clause 6.2 gives guidance on many of the aspects discussed below.

(a) Incorrect placement of steel
Incorrect placement of steel can result in insufficient cover, leading to corrosion of the reinforcement. If the bars are placed grossly out of position or in the wrong position, collapse can occur when the element is fully loaded.

(b) Inadequate cover to reinforcement
Inadequate cover to reinforcement permits ingress of moisture, gases and other substances and leads to corrosion of the reinforcement and cracking and spalling of the concrete.

(c) Incorrectly made construction joints
The main faults in construction joints are lack of preparation and poor compaction. The old concrete should be washed and a layer of rich concrete laid before pouring is continued. Poor joints allow ingress of moisture and staining of the concrete face.

(d) Grout leakage
Grout leakage occurs where formwork joints do not fit together properly. The result is a porous area of concrete that has little or no cement and fine aggregate. All formwork joints should be properly sealed.

(e) Poor compaction
If concrete is not properly compacted by ramming or vibration, the result is a portion of porous honeycomb concrete. This part must be hacked out and recast. Complete compaction is essential to give a dense, impermeable concrete.

(f) Segregation
Segregation occurs when the mix ingredients become separated. It is the result of

1. dropping the mix through too great a height in placing. Chutes or pipes should be used in such cases.
2. using a harsh mix with high coarse aggregate content
3. large aggregate sinking due to over-vibration or use of too much plasticizer Segregation results in uneven concrete texture, or porous concrete in some cases.

(g) Poor curing
A poor curing procedure can result in loss of water through evaporation. This can cause a reduction in strength if there is not sufficient water for complete hydration of the cement. Loss of water can cause shrinkage cracking. During curing the concrete should be kept damp and covered. See BS 8110, clause 6.2.3 on curing.

(h) Too high a water content
Excess water increases workability but decreases the strength and increases the porosity and permeability of the hardened concrete, which can lead to corrosion of the reinforcement. The correct water-to-cement ratio for the mix should be strictly enforced.

2.7.1.4 Chemical Attack

The main causes of chemical attack on concrete and reinforcement can be classified under the following headings.

(a) Chlorides
High concentrations of chloride ions cause corrosion of reinforcement and the products of corrosion can disrupt the concrete. Chlorides can be introduced into the concrete either during or after construction as follows.

(i) Before construction Chlorides can be admitted in admixtures containing calcium chloride, through using mixing water contaminated with salt water or improperly washed marine aggregates.

(ii) After construction Chlorides in salt or sea water, in airborne sea spray and from de-icing salts can attack permeable concrete causing corrosion of reinforcement.

(b) Sulphates

Sulphates are present in most cements and some aggregates. Sulphates may also be present in soils, groundwater and sea water, industrial wastes and acid rain. The products of sulphate attack on concrete occupy a larger space than the original material and this causes the concrete to disintegrate and permits corrosion of steel to begin. Sulphate-resisting Portland cement should be used where sulphates are present in the soil, water or atmosphere and come into contact with the concrete. Super sulphated cement, made from blast furnace slag, can also be used. This cement can resist the highest concentrations of sulphates.

(c) Carbonation

Carbonation is the process by which carbon dioxide from the atmosphere slowly transforms calcium hydroxide into calcium carbonate in concrete. The concrete itself is not harmed and increases in strength, but the reinforcement can be seriously affected by corrosion as a result of this process.

Normally the high pH value of the concrete prevents corrosion of the reinforcing bars by keeping them in a highly alkaline environment due to the release of calcium hydroxide by the cement during its hydration. Carbonated concrete has a pH value of 8.3 while the passivation of steel starts at a pH value of 9.5. The depth of Carbonation in good dense concrete is about 3 mm at an early stage and may increase to 6–10 mm after 30–40 years. Poor concrete may have a depth of carbonation of 50 mm after say 6–8 years. The rate of carbonation depends on time, cover, concrete density, cement content, water-to-cement ratio and the presence of cracks.

(d) Alkali-silica reaction

A chemical reaction can take place between alkali in cement and certain forms of silica in aggregate. The reaction produces a gel which absorbs water and expands in volume, resulting in cracking and disintegration of the concrete. The reaction only occurs when the following are present together:

1. a high moisture level in the concrete
2. cement with a high alkali content or some other source of alkali
3. aggregate containing an alkali-reactive constituent

The following precautions should be taken if uncertainty exists:

1. Reduce the saturation of the concrete;
2. Use low alkali Portland cement and limit the alkali content of the mix to a low level;
3. Use replacement cementitious materials such as blast furnace slag or pulverized fuel ash.

Most normal aggregates behave satisfactorily.

(e) Acids

Portland cement is not acid resistant and acid attack may remove part of the set cement. Acids are formed by the dissolution in water of carbon dioxide or sulphur dioxide from the atmosphere. Acids can also come from industrial wastes. Good dense concrete with adequate cover is required and sulphate-resistant cements should be used if necessary.

2.7.1.5 External Physical and/or Mechanical Factors

The main external factors causing concrete structures to fail are as follows.

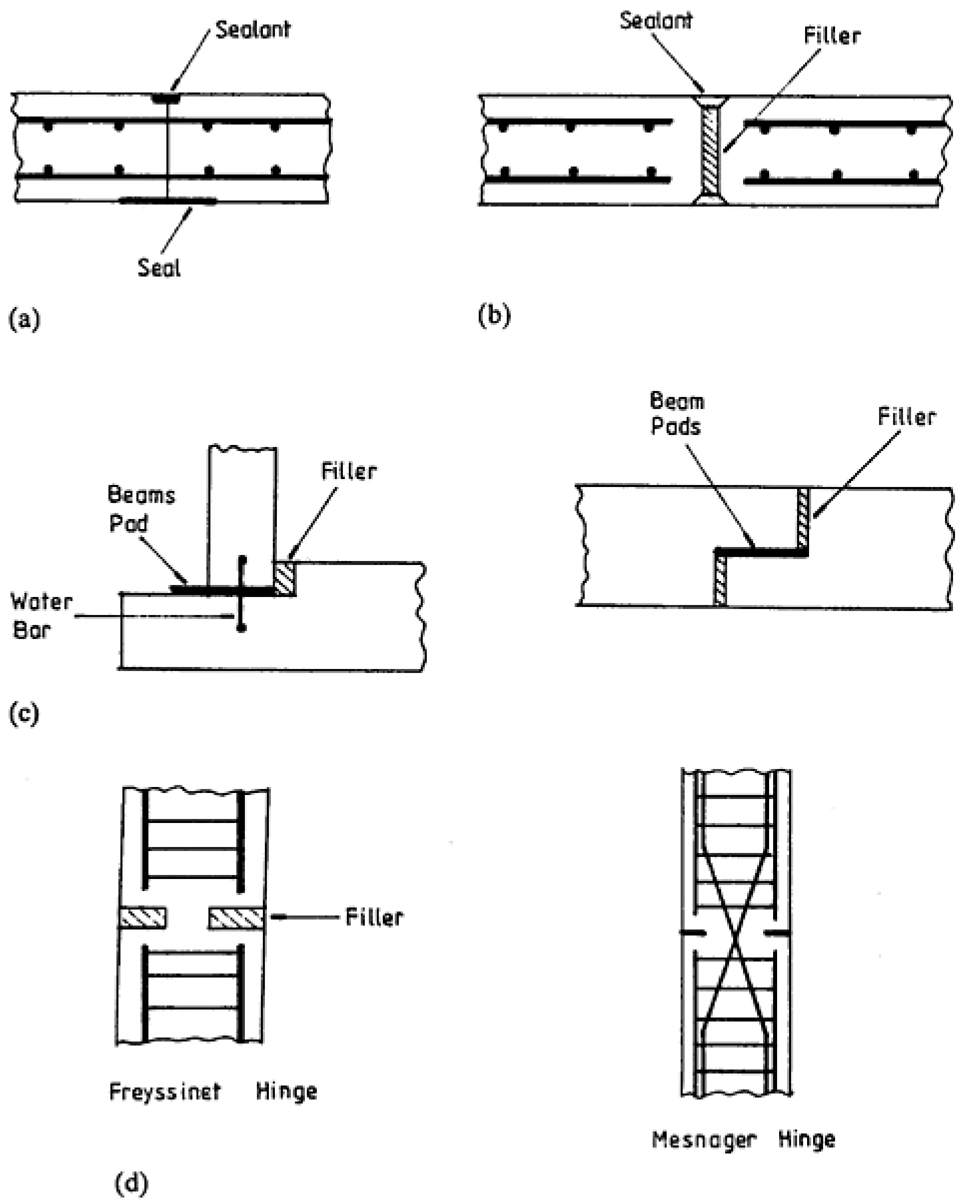


Fig.2.3 (a) Partial contraction joint; (b) expansion joint; (c) sliding joints; (d) hinge joints.

(a) Restraint against movement

Restraint against movement causes cracking. Movement in concrete is due to elastic deformation and creep under constant load, shrinkage on drying and setting, temperature changes, changes in moisture content and the settlement of foundations. The design should include sufficient movement joints to prevent serious cracking. Cracking may only detract from the appearance rather than be of structural significance but cracks permit ingress of moisture and lead to corrosion of the steel. Various proprietary substances are available to seal cracks.

Movement joints are discussed in BS 8110: Part 2, section 8. The code states that the joints should be clearly indicated for both members and structure as a whole. The joints are to permit relative movement to occur without impairing structural integrity. Types of movement joints defined in the code are as follows.

1. The **contraction joint** may be a complete or partial joint with reinforcement running through the joint. There is no initial gap and only contraction of the concrete is permitted.
2. The **expansion joint** is made with a complete discontinuity and gap between the concrete portions. Both expansion and contraction can occur. The joint must be filled with a sealer.
3. There is complete discontinuity in a **sliding joint** and the design is such as to permit movement in the plane of the joint.
4. The **hinged joint** is specially designed to permit relative rotation of members meeting at the joint. The Freyssinet hinge has no reinforcement passing through the joint.
5. The **settlement joint** permits adjacent members to settle or displace vertically as a result of foundation or other movements relative to each other. Entire parts of the building can be separated to permit relative settlement, in which case the joint must run through the full height of the structure.

Diagrams of some movement joints are shown in Fig.2.3. The location of movement joints is a matter of experience. Joints should be placed where cracks would probably develop, e.g. at abrupt changes of section, corners or locations where restraints from adjoining elements occur.

(b) Abrasion

Abrasion can be due to mechanical wear, wave action etc. Abrasion reduces cover to reinforcement. Dense concrete with hard wearing aggregate and extra cover allowing for wear are required.

(c) Wetting and drying

Wetting and drying leaches lime out of concrete and makes it more porous, which increases the risk of corrosion to the reinforcement. Wetting and drying also causes movement of the concrete which can cause cracking if restraint exists. Detail should be such as to shed water and the concrete may also be protected by impermeable membranes.

(d) Freezing and thawing

Concrete nearly always contains water which expands on freezing. The freezing-thawing cycle causes loss of strength, spalling and disintegration of the concrete. Resistance to damage is improved by using an air-entraining agent.

(e) Overloading

Extreme overloading will cause cracking and eventual collapse. Factors of safety in the original design allow for possible overloads but vigilance is always required to ensure that the structure is never grossly overloaded. A change in function of the building or room can lead to overloading, e.g. if a class room is changed to a library the imposed load can be greatly increased.

(f) Structural alterations

If major structural alterations are made to a building, the members affected and the overall integrity of the building should be rechecked. Common alterations are the removal of walls or columns to give a large clear space or provide additional doors or openings. Steel beams are inserted to carry loads from above. In such cases the bearing of the new beam on the original structure should be checked and if walls are removed the overall stability may be affected.

(g) Settlement

Differential settlement of foundations can cause cracking and failure in extreme cases. The foundation design must be adequate to carry the building loads without excessive settlement. Where a building with a large plan area is located on ground where subsidence may occur, the building should be constructed in sections on independent rafts with complete settlement joints between adjacent parts.

Many other factors can cause settlement and ground movement problems. Some problems are shrinkage of clays from ground dewatering or drying out in droughts, tree roots causing disruption, ground movement from nearby excavations, etc.

(h) Fire resistance

Concrete is a porous substance bound together by water-containing crystals. The binding material can decompose if heated to too high a temperature, with consequent loss of strength. The loss of moisture causes shrinkage and the temperature rise causes the aggregates to expand, leading to cracking and spalling of the concrete. High temperature also causes reinforcement to lose strength. At 550°C the yield stress of steel has dropped to about its normal working stress and failure occurs under service loads.

Concrete, however, is a material with very good fire resistance and protects the reinforcing steel. Fire resistance is a function of member thickness and cover. The code requirements regarding fire protection are set out below in section 2.9.2.

2.8 DURABILITY OF CONCRETE STRUCTURES

2.8.1 Code References to Durability

Frequent references are made to durability in BS8110: Part 1, section 2. The clauses referred to are as follows.

(a) Clause 2.1.3
The quality of material must be adequate for safety, serviceability and durability.

(b) Clause 2.2.1
The structure must not deteriorate unduly under the action of the environment over its design life. i.e. it must be durable.

(c) Clause 2.2.4
This states that 'integration of all aspects of design, materials and construction is required to produce a durable structure'. The main provisions in the clause are the following:

1. Environmental conditions should be defined at the design stage;
2. The design should be such as to ensure that surfaces are freely draining;
3. Cover must be adequate;
4. Concrete must be of relevant quality. Constituents that may cause durability problems should be avoided;
5. Particular types of concrete should be specified to meet special requirements;
6. Good workmanship, particularly in curing, is essential. Guidance on concrete construction such as placing and compaction, curing, etc. are set out in section 6.2 of the code.

2.9 CONCRETE COVER

2.9.1 Nominal Cover against Corrosion

The code states in section 3.3.1 that the actual cover should never be less than the nominal cover minus 5 mm. The nominal cover should protect steel against corrosion and fire. The cover to a main bar should not be less than the bar size or in the case of pairs or bundles the size of a single bar of the same cross-sectional area.

The cover depends on the exposure conditions given in Table 3.2 in the code. These are as follows.

Mild: concrete is protected against weather

Moderate:

concrete is sheltered from severe rain
concrete under non-aggressive water
concrete in non-aggressive soil

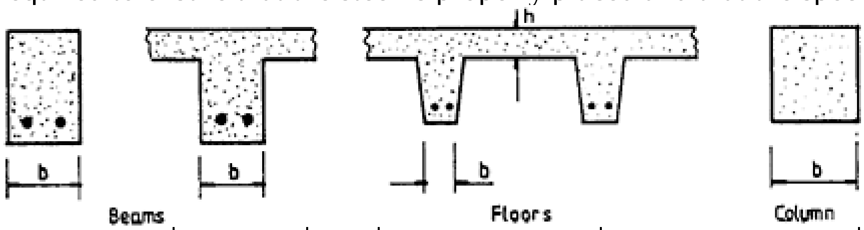
Severe: concrete exposed to severe rain, alternate wetting and drying or occasional freezing or severe condensation

Very severe: concrete occasionally exposed to sea water, de-icing salts or corrosive fumes

Most severe: concrete frequently exposed to sea water, de-icing salts or corrosive fumes

Abrasive: concrete exposed to abrasive action

Limiting values for nominal cover are given in Table 3.3 of the code and Table 2.1. Note that the water-to-cement ratio and minimum cement content are specified. Good workmanship is required to ensure that the steel is properly placed and that the specified cover is obtained.



Fire resistance (hour)	Min. Beam b	Rib b	Min. floor Thickness h	Column width Fully exposed b	Min. wall Thickness $0.4\% < p < 1.0\%$
1.0	200	125	95	200	120
1.5	200	125	110	250	140
2.0	200	125	125	300	160

Fig.2.4 Minimum dimensions for fire resistance

2.9.2 Cover as Fire Protection

Nominal cover to all reinforcement to meet a given fire resistance period for various elements in a building is given in Table 2.2 and Table 3.4 in the code. Minimum dimensions of members from Fig.3.2 in the code are shown in Fig.2.4. Reference should be made to the complete tables and figures in the code.

Table 2.1 Nominal cover to all reinforcement including links to meet durability requirements

Conditions of exposure	*Nominal cover (mm)*		
Mild	25	20	20
Moderate		35	30
Severe			40
Very severe			50
Most severe	–	–	–
Abrasive	Nominal cover+allowance for loss of cover due to brasion.		
Maximum free water-to-cement ratio	0.65	0.60	0.55
Minimum cement content (kg/m^3)	275	300	325
Lowest grade of concrete	C30	C35	C40

Table 2.2 Nominal cover to all reinforcement including links to meet specified periods of fire resistance

Fire Resistance	*Nominal Cover -mm*						
	Beams		*Floors*		*Ribs*		*Columns*
Hour	SS	C	SS	C	SS	C	
1.0	20	20	20	20	20	20	20
1.5	20	20	25	20	35	20	20
2.0	40	30	35	25	45	35	25

SS Simply supported, C Continuous

2.10 REFERENCES
REFERENCES

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CHAPTER 3
LIMIT STATE DESIGN AND STRUCTURAL ANALYSIS

3.1 STRUCTURAL DESIGN AND LIMIT STATES

3.1.1 Aims and Methods of Design

The code BS 8110, part 1 in clause 2.1.1 states that the aim of design is the achievement of an acceptable probability that the structure will perform satisfactorily during its life. It must carry the loads safely, not deform excessively and have adequate durability and resistance to the effects of misuse and fire. The clause recognizes that no structure can be made one hundred percent safe and that it is only possible to reduce the probability of failure to an acceptably low level.

Clause 2.1.2 states that the method recommended in the code is limit state design where account is taken of theory, experiment and experience. It adds that calculations alone are not sufficient to produce a safe, serviceable and durable structure. Correct selection of materials, quality control and supervision of construction are equally important.

3.1.2 Criteria for a Safe Design: Limit States

The criterion for a safe design is that the structure should not become unfit for use, i.e. that it should not reach a limit state during its design life. This is achieved, in particular, by designing the structure to ensure that it does not reach

1. The ultimate limit state (ULS): the whole structure or its elements should not collapse, overturn or buckle when subjected to the design loads

2. Serviceability limit states (SLS): the structure should not become unfit for use due to excessive deflection, cracking or vibration

The structure must also be durable, i.e. it must not deteriorate or be damaged excessively by the environment to which it is exposed or action of substances coming into contact with it. The code places particular emphasis on durability (see the discussion in Chapter 2). For reinforced concrete structures the normal practice is to design for the ultimate limit state, check for serviceability and take all necessary precautions to ensure durability.

3.1.3 Ultimate Limit State

(a) Strength

The structure must be designed to carry the most severe combination of loads to which it is subjected. Each and every section of the elements must be capable of resisting the axial loads, shears and moments derived from the analysis.

The design is made for ultimate loads and design strengths of materials with partial safety factors applied to loads and material strengths. This permits uncertainties in the estimation of loads and in the performance of materials to be assessed separately. The section strength is determined using plastic analysis based on the short-term design stress-strain curves for concrete and reinforcing steel.

(b) Stability

Clause 2.2.2.1 of the code states that the layout should be such as to give a stable and robust structure. It stresses that the engineer responsible for overall stability should ensure compatibility of design and details of parts and components.

Overall stability of a structure is provided by shear walls, lift shafts, staircases and rigid frame action or a combination of these means. The structure should be such as to transmit all loads, dead, imposed and wind, safely to the foundations.

(c) Robustness

Clause 2.2.2.2 of the code states that the planning and design should be such that damage to a small area or failure of a single element should not cause collapse of a major part of a structure. This means that the design should be resistant to progressive collapse. The clause specifies that this type of failure can be avoided by taking the following precautions.

1. The structure should be capable of resisting notional horizontal loads applied at roof level and at each floor level. The loads are 1.5% of the characteristic dead weight of the structure between mid-height of the storey below and either mid-height of the storey above or the roof surface. The wind load is not to be taken as less than the notional horizontal load.

2. All structures are to be provided with effective horizontal ties. These are
(a) peripheral ties
(b) internal ties
(c) horizontal ties to column and walls

The arrangement and design of ties is discussed in section 14.3.

3. For buildings of five or more storeys, key elements are to be identified, failure of which would cause more than a limited amount of damage. These key elements must be designed for a specially heavy ultimate load of 34 kN/m applied in any direction on the area supported by the member. Provisions regarding the application of this load are set out in BS 8110: Part 2, section 2.6.

4. For buildings of five or more storeys it must be possible to remove any vertical load bearing element other than a key element without causing more than a limited

amount of damage. This requirement is generally achieved by the inclusion of vertical ties in addition to the other provisions noted above.

3.1.4 Serviceability Limit States

The serviceability limit states are discussed in BS 8110: Part 1, section 2.2.3. The code states that account is to be taken of temperature, creep, shrinkage, sway and settlement and possibly other effects.

The main serviceability limit states and code provisions are as follows.

(a) Deflection

The deformation of the structure should not adversely affect its efficiency or appearance. Deflections may be calculated, but may tend to be complicated and in normal cases span-to-effective depth ratios can be used to check compliance with requirements.

(b) Cracking

Cracking should be kept within reasonable limits by correct detailing. Crack widths may be calculated, but may tend to be complicated and in normal cases cracking can be controlled by adhering to detailing rules with regard to bar spacing in zones where the concrete is in tension.

In analysing a section for the serviceability limit states the behaviour is assessed assuming a linear elastic relationship for steel and concrete stresses. Allowance is made for the stiffening effect of concrete in the tension zone and for creep and shrinkage.

3.2 CHARACTERISTIC AND DESIGN LOADS

The characteristic or service loads are the actual loads that the structure is designed to carry. These are normally thought of as the maximum loads which will not be exceeded during the life of the structure. In statistical terms the characteristic loads have a 95% probability of not being exceeded.

The characteristic loads used in design and defined in BS 8110: Part 1, clause 2.4.1, are as follows:

1. **The characteristic dead load G_k** is the self-weight of the structure and the weight of finishes, ceilings, services and partitions;

2. **The characteristic imposed load Q_k** is caused by people, furniture, and equipment etc. on floors and snow on roofs. Dead and imposed loads for various types of buildings are given in BS 6399: Part 1, 1996. *Loadings for buildings. Code of practice for dead and imposed loads.*

3. **The characteristic wind load W_k** depends on the location, shape and dimensions of the buildings. Wind loads are estimated using BS 6399: Part 2, 1997. *Loadings for buildings. Code of practice for wind loads.*

The code BS 8110 states that nominal earth loads $\mathbf{E}_n$ are to be obtained in accordance with normal practice. Reference should be made to BS 8004:1986: *Code of Practice for Foundations* and textbooks on Geotechnics. A useful work is

Bowles, Joseph E., 1995, Foundation analysis and design, (McGraw-Hill), 5th Edition.

The structure must also be able to resist the notional horizontal loads defined in clause 3.1.4.2 of the code. The definition for these loads was given in section 3.1.3(c) above.

design load=characteristic load×partial safety factor for load

$=F_k \gamma_f$

The partial safety factor γ_f takes account of

1. possible increases in load
2. inaccurate assessment of the effects of loads
3. unforeseen stress distributions in members
4. the importance of the limit state being considered

The code states that the values given for γ_f ensure that serviceability requirements can generally be met by simple rules. The values of γ_f to give design loads and the load combinations for the ultimate limit state are given in BS 8110: Part 1, Table 2.1. These factors are given in Table 3.1. The code states that the adverse partial safety factor is applied to a load producing more critical design conditions. The beneficial factor is applied to a load producing a less, critical design condition.

Table 3.1 Load combinations

Load combination	*Load type*					
	Dead load		*Imposed load*		*Earth and Water pressure*	*Wind*
	Adverse	*Beneficial*	*Adverse*	*Beneficial*		
1 . Dead and imposed (and earth and water pressure)	1.4	1.0	1.6	0	1.4	–
2. Dead and wind (and earth and water pressure)	1.4	1.0	–	–	1.4	1.4
3. Dead, wind and imposed (and earth and water pressure)	1.2	1.2	1.2	1.2	1.2	1.2

For example in the case of a beam with an overhang as shown in Fig.3.1, maximum upward reaction at the left hand support and maximum bending moment in the main span occur when there is minimum load on the overhang and maximum load in the main span. On the other hand, possibility of uplift and maximum bending moment in the main span causing tension at top occurs when there is minimum load on the main span and maximum load on the overhang.

In considering the effects of exceptional loads caused by misuse or accident γ_f can be taken as 1.05. The loads to be applied in this case are the dead load, one-third of the wind load and one-third of the imposed load except for storage and industrial buildings when the full imposed load is to be used.

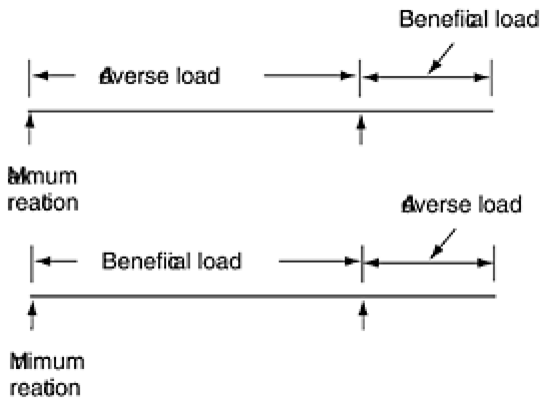


Fig.3.1 Beam with an overhang.

3.3 MATERIALS: PROPERTIES AND DESIGN STRENGTHS

The characteristic strengths or grades of materials are as follows:

Concrete: f_{cu} is the 28 day cube strength in Newtons per square millimetre. The minimum grades for reinforced concrete are given in Table 3.3 in the code. These are grades C30, C35, C40, C45 and C50 in Newtons per square millimetre.

Reinforcement: f_y is the yield or proof stress in Newtons per square millimetre. The specified characteristic strengths of reinforcement given in Table 3.1 in the code are

Hot rolled mild steel f_y=250 N/mm^2
High yield steel, hot rolled or cold worked f_y=460 N/mm^2

Clause 3.1.7.4 of the code states that a lower value may be used to reduce deflection or control cracking. The reason for this is that a lower stress in steel at SLS reduces the number and widths of cracks.

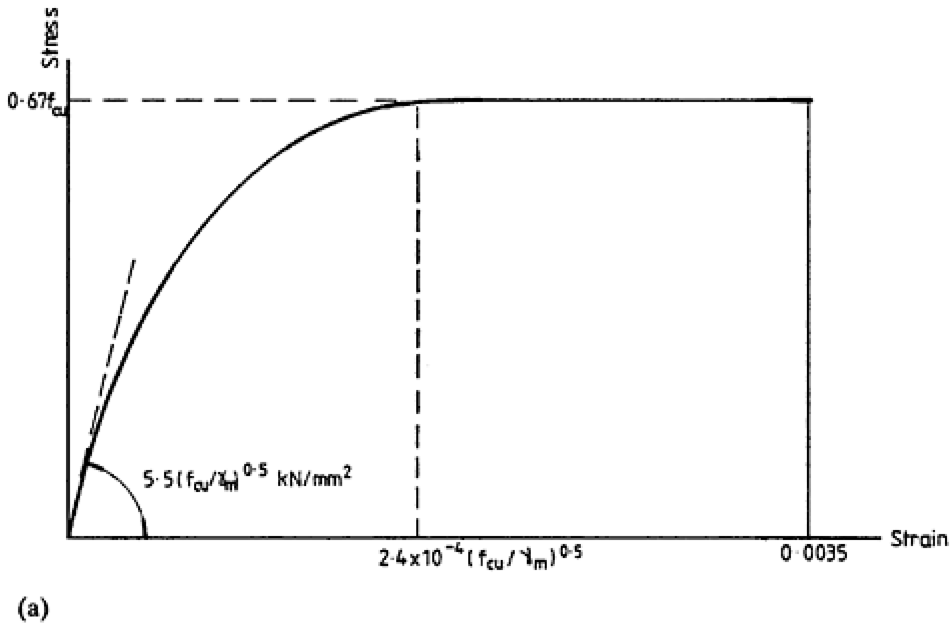
The resistance of sections to applied stresses is based on the design strength which is defined as

$$\frac{characteristic\ strength}{partial\ factor\ of\ safety\ of\ materials} = \frac{f_k}{\gamma_m}$$

The factor γ_m takes account of

1. uncertainties in the strength of materials in the structure
2. uncertainties in the accuracy of the method used to predict the behaviour of members
3. variations in member sizes and building dimensions

Values of γ_m from Table 2.2 in the code used for design for the ultimate limit state are given in Table 3.2. For exceptional loads γ_m is to be taken as 1.3 for concrete and 1.0 for steel.



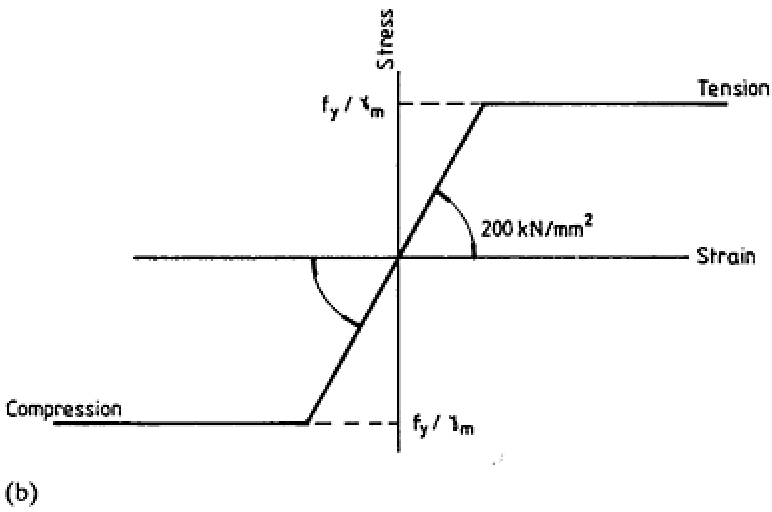


Fig.3.2 Short-term design stress-strain curve for (a) normal-weight concrete and (b) reinforcement

Table 3.2 Values of γ_m for the ultimate limit state

Reinforcement	1.05
Concrete in flexure or axial load	1.5
Shear strength without shear reinforcement	1.25
Bond strength	1.4
Others, e.g. bearing strength	≥1.5

The short-term design stress-strain curves for normal-weight concrete and reinforcement from Figs 2.1 and 2.2 in the code are shown in Figs 3.2(a) and 3.2(b) The curve for concrete in compression is an idealization of the actual compression behaviour which begins with a parabolic portion where the slope of the tangent at the origin equals the short-term value of Young's modulus. At strain ε_o which depends on the concrete grade, the stress remains constant with increasing load until a strain of 0.0035 is reached when the concrete is considered to have failed and is incapable of resisting any stress. Expressions for E_c and ε_o are given in the figure. The maximum design stress in the concrete is given as $0.67f_{cu}/\gamma_m$. The coefficient 0.67 takes account of the relation between the cube strength and the bending strength in a flexural member. It is not a partial factor of safety.

The stress-strain curve for reinforcement shown in Fig.3.2(b) is bilinear with one yield point. The behaviour and strength of reinforcement are taken to be the same in tension and compression.

3.4 STRUCTURAL ANALYSIS

3.4.1 General Provisions

The general provisions relating to analysis of the structure set out in BS 8110: Part 1, section 2.5, are discussed briefly. The methods of frame analysis outlined in section 3.2 are set out. Examples using these methods are given later in the book.

The object of analysis of the structure is to determine the axial forces, shears and moments throughout the structure. The code states that it is generally satisfactory to obtain maximum design values from moment and shear envelopes constructed from linear elastic analysis and to allow for moment redistribution if desired and for buckling effects in frames with slender columns. The code also states that plastic methods such as yield line analysis may also be used.

The member stiffness to be used in linear elastic analysis can be based on

1. the gross concrete section ignoring reinforcement: This is the method most frequently used in practice because it does not require data about the reinforcement present.

2. the gross concrete section including reinforcement on the basis of the modular ratio

3. the transformed section (the compression area of concrete and the transformed area of reinforcement in tension and compression based on the modular ratio are used)

The code states that a modular ratio of 15 may be assumed. It adds that a consistent approach should be used for all elements of the structure.

3.4.2 Methods of Frame Analysis

The complete structure may be analysed elastically using a matrix computer program adopting the basis set out above. It is normal practice to model beam elements using only the rectangular section of T-beam elements in the frame analysis (Fig. 1.3). The T-beam section is taken into account in the element design.

Approximate methods of analysis are set out in the code as an alternative to a rigorous analysis of the whole frame. These methods are discussed below.

3.4.3 Monolithic braced frame

Shear walls, lifts and staircases provide stability and resistance to horizontal loads. A braced frame is shown in Fig.3.3. The approximate methods of analysis and the critical load arrangements are as follows.

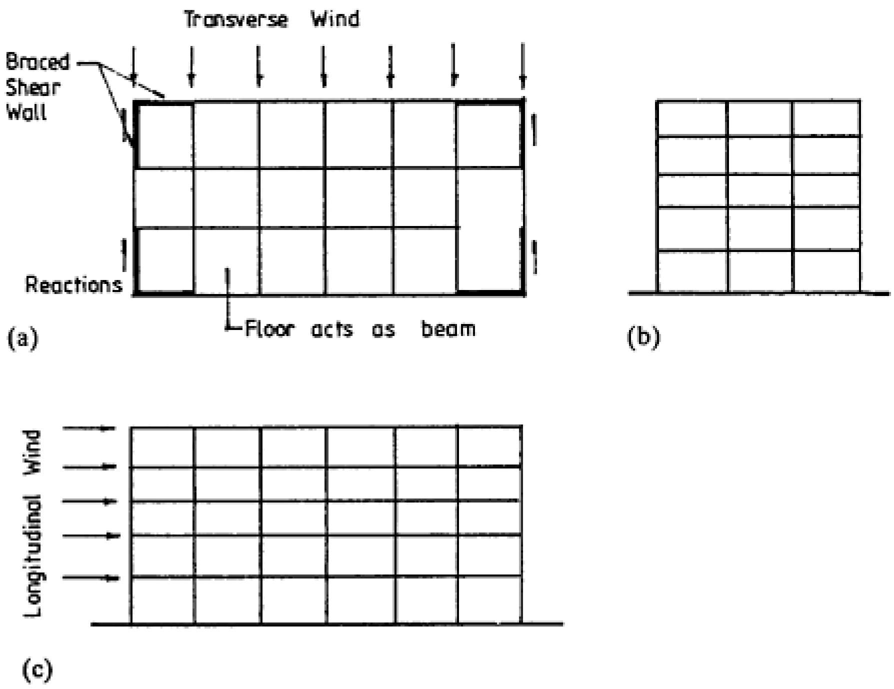


Fig.3.3 Braced multi-storey building: (a) plan; (b) rigid transverse frame; (c) side elevation.

(a) Division into subframes

The structural frame is divided into sub-frames consisting of the beams at one level and the columns above and below that level with ends taken as fixed. The moments and shears are derived from an elastic analysis (Figs 3.4(a) and 3.4(b)).

(b) Critical load arrangement

The critical arrangements of vertical load are

1. all spans loaded with the maximum design ultimate load of $1.4G_k+1.6Q_k$
2. alternate spans loaded with the maximum design ultimate load of $1.4G_k+1.6Q_k$ and all other spans loaded with the minimum design ultimate load of 1.0Gk where G_k is the total dead load on the span and Q_k is the imposed load on the span.

The load arrangements are shown in Fig.3.4(b).

(c) Simplification for individual beams and columns

The simplified sub-frame consists of the beam to be designed, the columns at the ends of the beam and the beams on either side if any. The column and beam ends remote from the beam considered are taken as fixed and the stiffness of the beams on either side should be taken as *one-half* of their actual value (Fig.3.4(c)).

The moments for design of an individual column may be found from the same sub-frame analysis provided that its central beam is the longer of the two beams framing into the column.

(d) Continuous beam simplification

The beam at the floor considered may be taken as a continuous beam over supports providing no restraint to rotation. This gives a more conservative design than the procedures set out above. Pattern loading as set out in (b) is applied to determine the critical beam moments and shear for design (Fig.3.4(d)).

(e) Asymmetrically loaded column

The asymmetrically loaded column method is to be used where the beam has been analysed on the basis of the continuous beam simplification set out in (d) above. The column moments can be calculated on the assumption that the column and beam ends remote from the junction under consideration are fixed and that the beams have one-half their actual stiffnesses. The imposed load is to be arranged to cause maximum moment in the column (Fig.3.4(e)). Examples of the application of these methods are given Chapter 14.

3.4.4 Rigid Frames Providing Lateral Stability

Where rigid frames provides lateral stability, they must be analysed for horizontal and vertical loads. Clause 3.1.4.2 of the code states that all buildings must be capable of resisting a notional horizontal load equal to 1.5% of the characteristic dead weight of the structure applied at roof level and at each floor.

The complete structure may be analysed for vertical and horizontal loads using computer analysis program. As an alternative the code gives the following

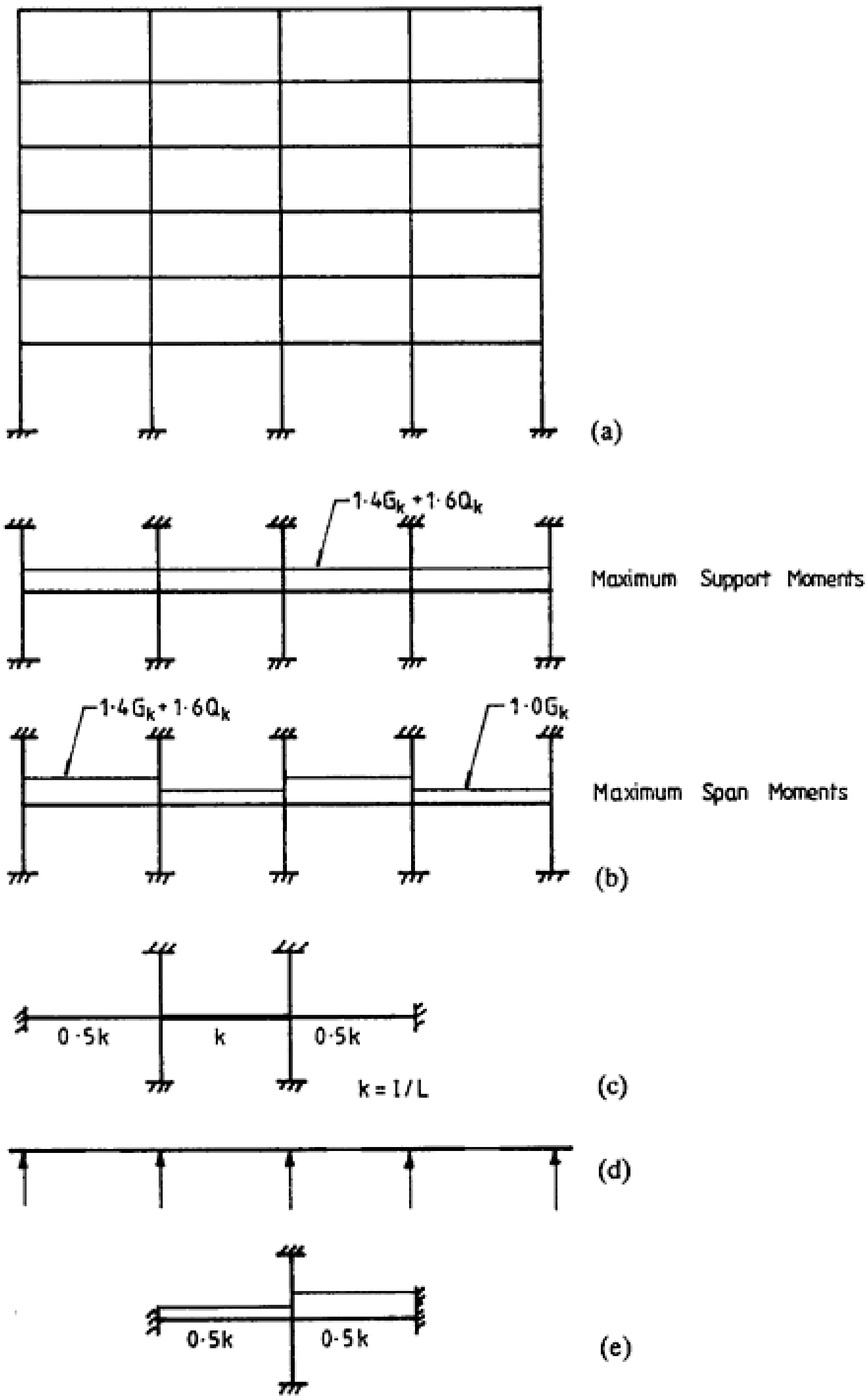


Fig 3.4 Analysis for vertical load: (a) frame elevation; (b) subframes; (c) simplified subframe; (d) continuous beam simplification; (c) column moments analysis for (d)

method for sway frames of three or more approximately equal bays (the design is to be based on the more severe of the conditions):

1. elastic analysis for vertical loads only with maximum design load $1.4G_k+1.6Q_k$ (refer to sections 3.4.3(a) and 3.4.3(b) above)
2. or the sum of the moments obtained from

(a) elastic analysis of subframes as defined in section 3.4.3(a) with all beams loaded with $1.2G_k+1.2Q_k$ (horizontal loads are ignored)

(b) elastic analysis of the complete frame assuming points of contraflexure at the centres of all beams and columns for wind load $1.2W_k$ only

The column bases may be considered as pinned if this assumption gives more realistic analyses. A sway frame subjected to horizontal load is shown in Fig.3.5. Method of analysis for horizontal load, the portal method, is discussed in Chapter 13. Examples in the use of these methods are also given.

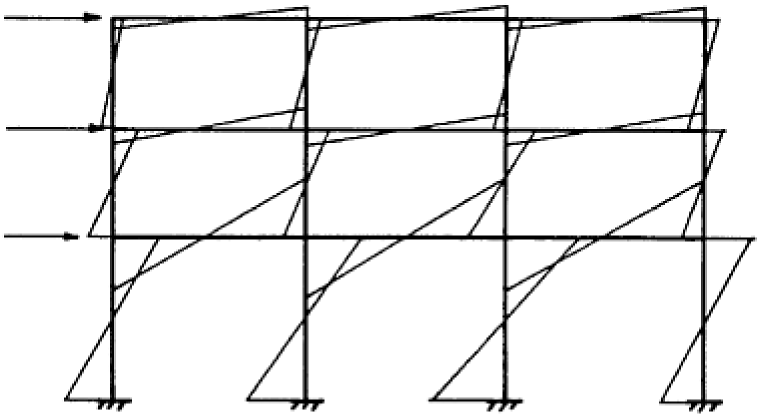


Fig.3.5 Horizontal loads.

3.4.5 Redistribution of Moments

Plastic method of analysis for steel structures based on the stress-strain curve shown in Fig.3.6(a), which gives the moment-rotation curve in Fig.3.6(b), can be used for the analysis of reinforced concrete structures provided due attention is paid to the fact reinforced concrete sections have limited ductility. In order to prevent serious cracking occurring at serviceability limit state, the code adopts a method that gives the designer control over the amount of redistribution and hence of rotation that is permitted to take place. In clause 3.2.2 the code allows a reduction of up to 30% of the peak elastic moment to be made whilst keeping internal and external forces in equilibrium. The conditions under which this can carried out are set out later in Chapter 13.

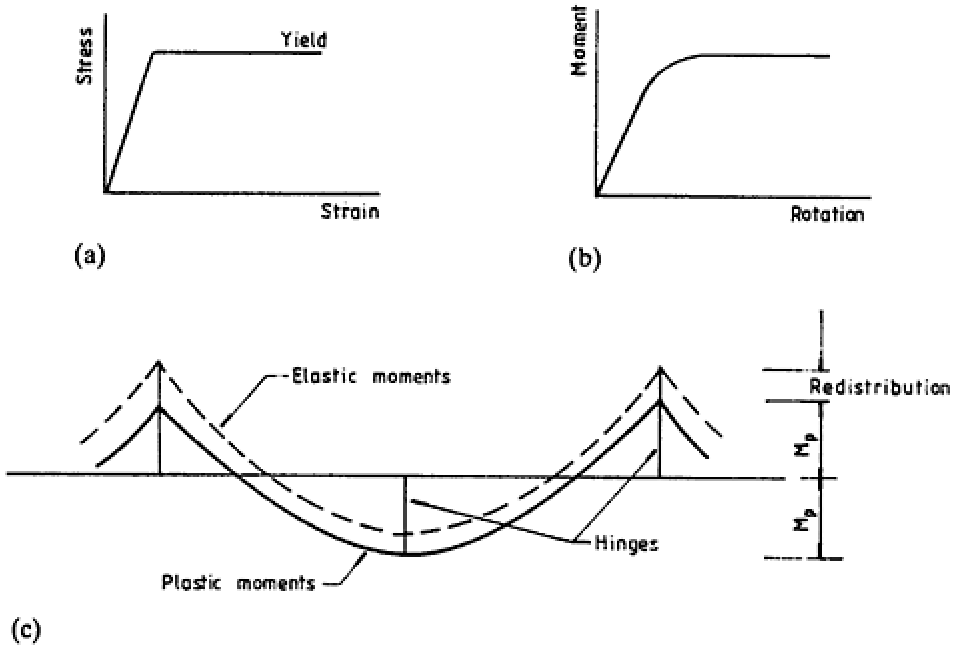


Fig.3.6 (a) Stress-strain curve; (b) moment-rotation curve; (c) elastic and plastic moment distributions.

CHAPTER 4
SECTION DESIGN FOR MOMENT

4.1 TYPES OF BEAM SECTION

The three common types of reinforced concrete beam section are

a. rectangular sections with tension steel only (this generally occurs when designing a given width of slab as a beam.)

b. rectangular sections with tension and compression steel

c. flanged sections of either T or L shape with tension steel and with or without compression steel

Beam sections are shown in Fig.4.1. It will be established later that all beams of structural importance must have steel top and bottom to carry links to resist shear.

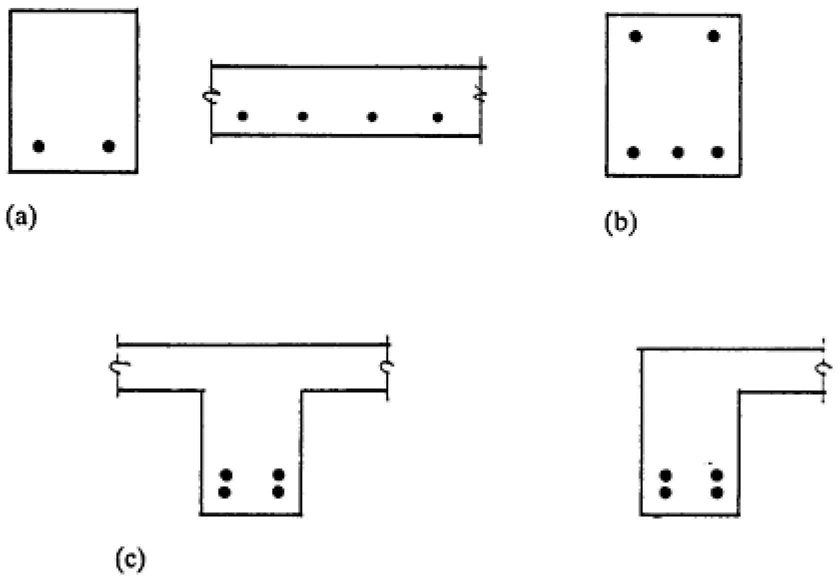


Fig.4.1 (a) Rectangular beam and slab, tension steel only; (b) rectangular beam, tension and compression steel; (c) flanged beams.

4.2 REINFORCEMENT AND BAR SPACING

Before beginning section design, reinforcement data and code requirements with regard to minimum and maximum areas of bars in beams and bar spacing are set out. This is to enable sections to be designed with practical amounts and layout of steel. Requirements for cover were discussed in section 2.9.

4.2.1 Reinforcement Data

In accordance with BS8110: Part 1, clause 3.12.4.1, bars may be placed singly or in pairs or in bundles of three or four bars in contact. For design purposes the pair or bundle is treated as a single bar of equivalent area. Bars are available with diameters of 6, 8, 10, 12, 16, 20, 25, 32 and 40 mm and in two grades with characteristic strengths f_y:

Hot rolled mild steel f_y=250 N/mm^2
High yield steel f_y=460 N/mm^2

For convenience in design calculations, areas of groups of bars are given in Table 4.1. Table 4.2 gives equivalent diameter of bundles of bars of same diameter.

Table 4.1 Areas of groups of bars

Diameter of bar in mm	Numbers of bars in group							
	1	2	3	4	5	6	7	8
6	28	57	85	113	141	170	198	226
8	50	101	151	201	251	302	352	402
10	79	157	236	314	393	471	550	628
12	113	226	339	452	566	679	792	905
16	201	402	603	804	1005	1206	1407	1609
20	314	628	943	1257	1571	1885	2109	2513
25	491	982	1473	1964	2454	2945	3436	3927
32	804	1609	2413	3217	4021	4826	5630	6434

Table 4.2 Equivalent diameters of bars in groups

Diameter in mm of bars in group	Number of bars in group			
	1	2	3	4
6	6	8.5	10.4	12
8	8	11.3	13.9	16
10	10	14.1	17.3	20
12	12	17.0	20.8	24
16	16	22.6	27.7	32
20	20	28.3	34.6	40
25	25	35.4	43.3	50
32	32	45.3	55.4	64

Detailed drawings should be prepared according to

Standard Method of Detailing Structural Concrete. Institution of Structural Engineers, London, 1989.

Bar types are specified by letters:

R mild steel bars
T high yield bars

Bars are designated on drawings as, for example, 4T25, i.e. four 25 mm diameter bars of grade 460. This system will be used to specify bars in figures.

4.2.2 Minimum and Maximum Areas of Reinforcement in Beams

The minimum areas of reinforcement in a beam section to control cracking as well as to resist tension or compression due to bending in different types of beam section are given in BS 8110: Part 1, clause 3.12.5.3 and Table 3.25. Some commonly used values are shown in Fig.4.2 and Table 4.3. Other values will be discussed in appropriate parts of the book.

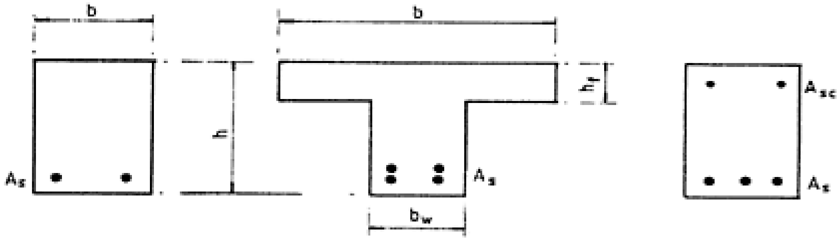


Fig.4.2 Minimum tension and compression steels

Table 4.3: Minimum steel areas

	Percentage	f_y=250 N/mm^2	fy=460 N/mm^2
Tension reinforcement			
Rectangular beam	$100A_s/A_c$	0.24	0.13
Flanged beam—Web in tension: $b_w/b<0.4$	100As/bw h	0.32	0.18
Flanged beam—Web in tension: $b_w/b\geq0.4$	100As/bw h	0.24	0.13
Compression reinforcement			
Rectangular beam	$100A_{sc}/A_c$	0.2	0.2
Flanged beam—flange in compression:	$100A_{sc}/b_wh_f$	0.2	0.2

A_c=total area of concrete, A_s=minimum area of reinforcement, A_{sc}=area of steel in compression, b, b_w, h_f=dimensions.

The maximum area of both tension and compression reinforcement in beams is specified in BS8110: Part 1, clause 3.12.6.1. Neither should exceed 4% of the gross cross-sectional area of the concrete.

4.2.3 Minimum Spacing of Bars

The minimum spacing of bars is given in BS8110: Part 1, clause 3.12.11.1. This clause states the following:

1. The horizontal distance between bars should not be less than h_{agg}+5 mm
2. Where there are two or more rows

(a) the gap between corresponding bars in each row should be vertically in line and
(b) the vertical distance between bars should not be less than $2h_{agg}/3$

where h_{agg} is the maximum size of coarse aggregate. The clause also states that if the bar size exceeds h_{agg}+5 mm the spacing should not be less than the bar size.

Note that pairs or bundles are treated as a single bar of equivalent area.

The above spacing ensures that the concrete can be properly compacted around the reinforcement. Spacing of top bars in beams should also permit the insertion of a vibrator. The information is summarized in Fig.4.3.

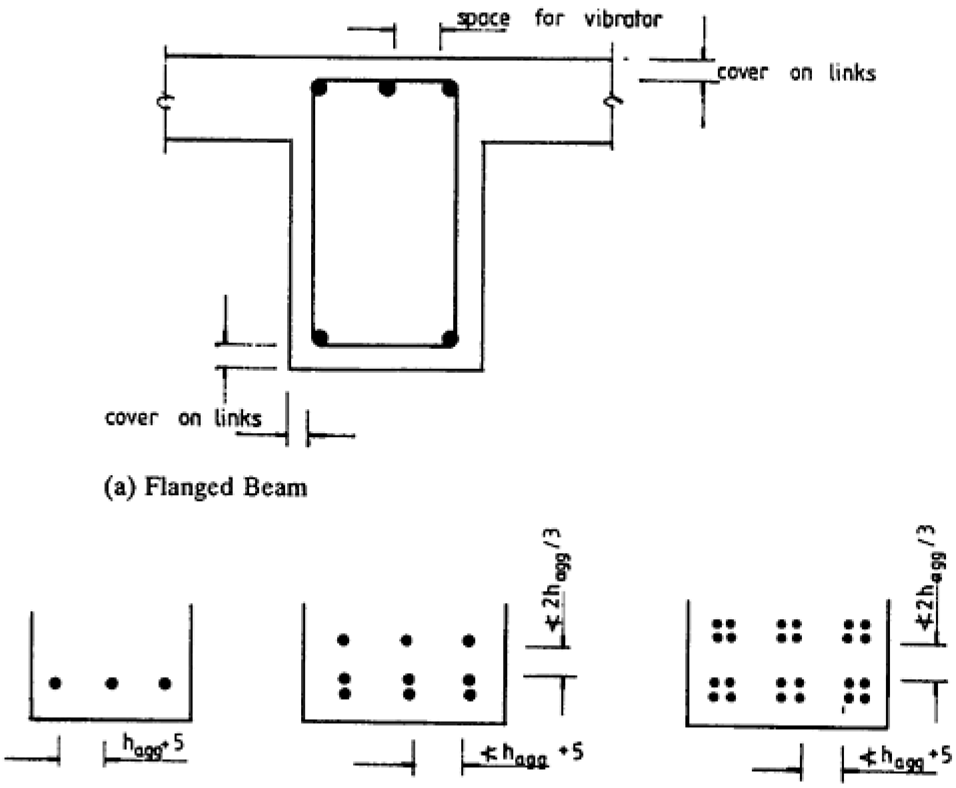


Fig.4.3 (a) Flanged beam; (b) minimum spacing.

4.3 BEHAVIOUR OF BEAMS IN BENDING

The behaviour of a cross section subjected to pure bending is studied by loading a beam at third points as shown in Fig.4.4(a). Under this system of loading, sections between the loads are subjected to pure bending. Initially the beam behaves as a monolithic elastic beam till the stresses at the bottom fibre reach the tensile strength of concrete. Because of the very low tensile strength of concrete (about 10% of its compression strength), vertical cracks appear at a fairly low load. As the load is increased, cracks lengthen and penetrate deeper towards the compression face. Simultaneously, the strain in steel also increases. The final failure depends on the amount and yield stress of steel. The three possible modes of failure are:

1. **Steel yields first:** If the tensile force capacity of steel is ‘low’, then steel yields before the strain in the concrete at the compression face reaches the maximum permissible value of 0.0035. Because steel is a ductile material, steel elongates while maintaining its strength. The beam continues to deform at constant load and the neutral axis moves up. The beam finally fails when the depth of the compression zone is too small to balance the tensile force in steel. This type of failure is the desired type because there is ample warning before failure. All beams, if overloaded, should be designed to fail in this manner. Fig.4.4(b) shows the qualitative load versus deflection curve and Fig.4.4(c) shows the stress distribution at elastic and ultimate stages.

2. **Simultaneous ‘yielding’ of steel and concrete:** If the tensile force capacity of steel is ‘moderate’, yielding of steel is simultaneously accompanied by the crushing of concrete. Unlike the failure mode where the steel yields first, there is little warning before failure. This is not a desirable mode of failure.

3. **Concrete crushes first:** If the tensile force capacity of steel is ‘high’, then steel does not yield at all before concrete crushes. Because concrete is a fairly brittle material, it fails in an explosive manner without any significant residual load bearing capacity. *This form of failure is to be avoided at all costs!*

4.4 SINGLY REINFORCED RECTANGULAR BEAMS

4.4.1 Assumptions and Stress-Strain Diagrams

The ultimate moment of resistance of a section is based on the assumptions set out in BS8110: Part 1, clause 3.4.4.1. These are as follows:

1. The strains in the concrete and reinforcement are derived assuming that plane sections remain plane;

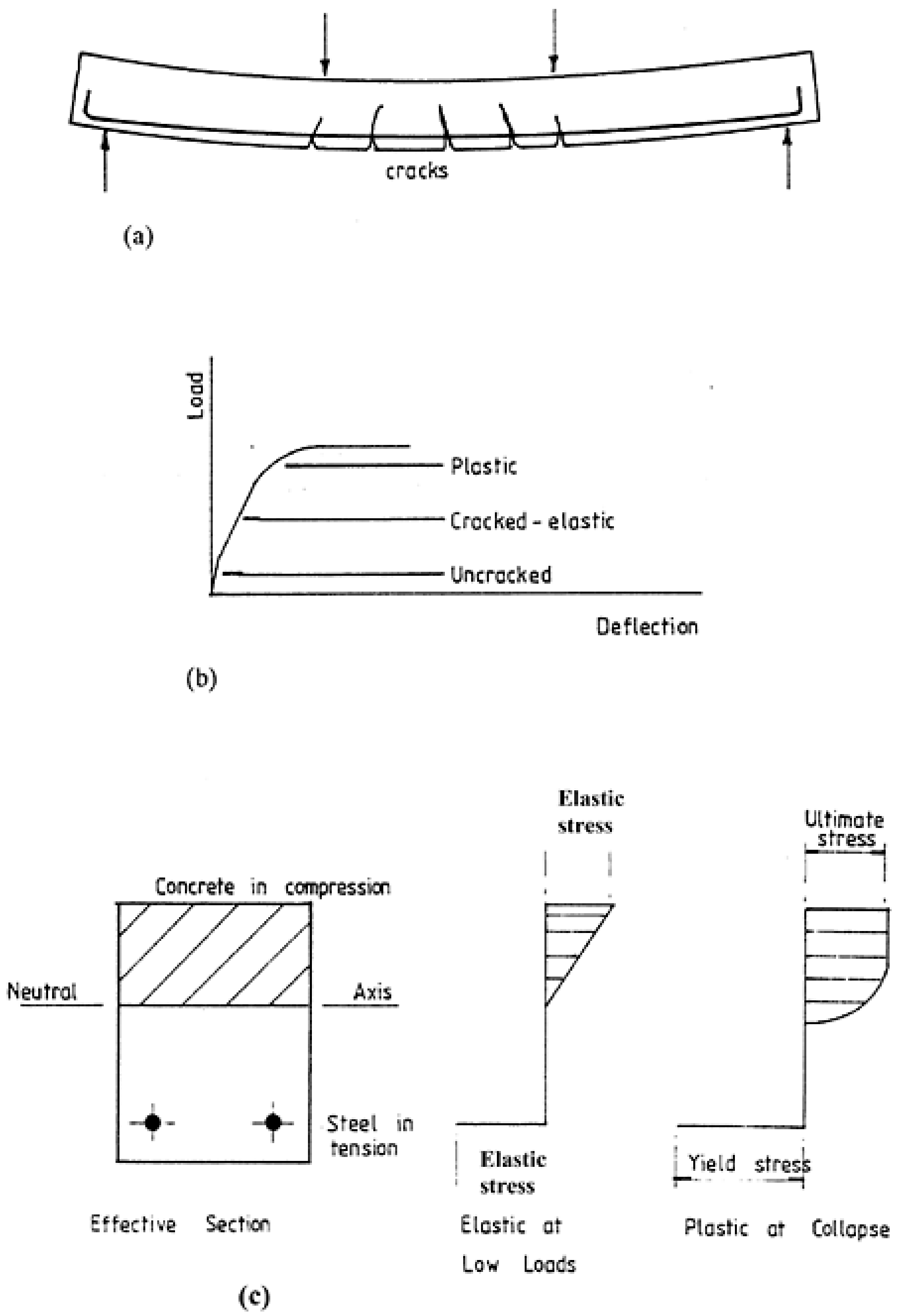


Fig.4.4 (a) Flexural cracks at collapse; (b) load-deflection curve; (c) effective section and stress distribution.

2. The stresses in the concrete in compression are derived using either:

(a) the design stress-strain curve given in Fig.4.5(a) with γ_m=1.5 or
(b) the simplified stress block shown in Fig.4.6(d) where the depth of the stress block is 0.9 of the depth to the neutral axis denoted by x.

Note that in both cases the maximum strain in the concrete at failure is 0.0035;

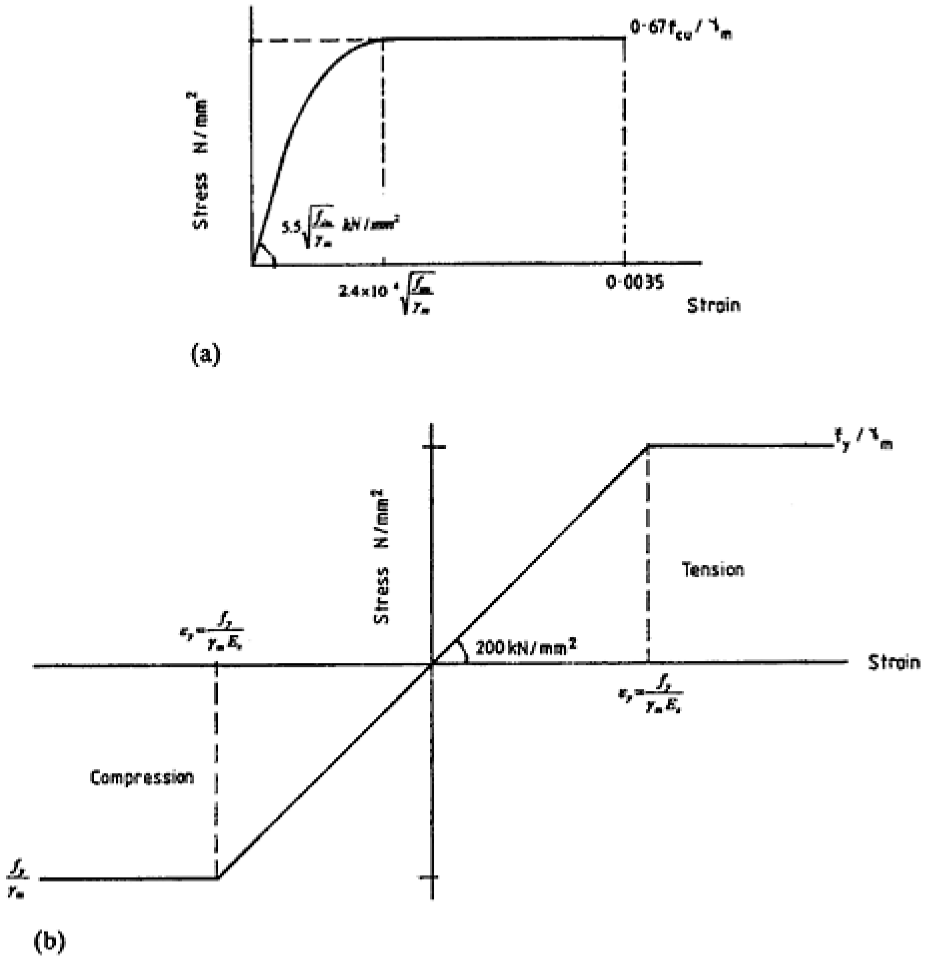


Fig.4.5 Stress-strain diagrams (a) Concrete; (b) Steel.

3. The tensile strength of the concrete is ignored;
4. The stresses in the reinforcement are derived from the stress-strain curve shown in Fig.4.5(b) where γ_m=1.05;
5. Where the section is designed to resist flexure only, the lever arm should not be assumed to be greater than 0.95 of the effective depth, d. This is because of the fact that at the top face during compaction water tends to move to the top and causes a higher water cement ratio than the rest of the beam. In addition weathering also affects the strength. Because of that a layer of concrete at the top is likely to be weak

and by limiting the value of the lever arm z, one avoids the possibility of expecting a weak layer of concrete to resist the compressive stress due to bending.

On the basis of these assumptions the strain and stress diagrams for the two alternative stress distributions for the concrete in compression are as shown in Fig.4.6, where the following symbols are used:

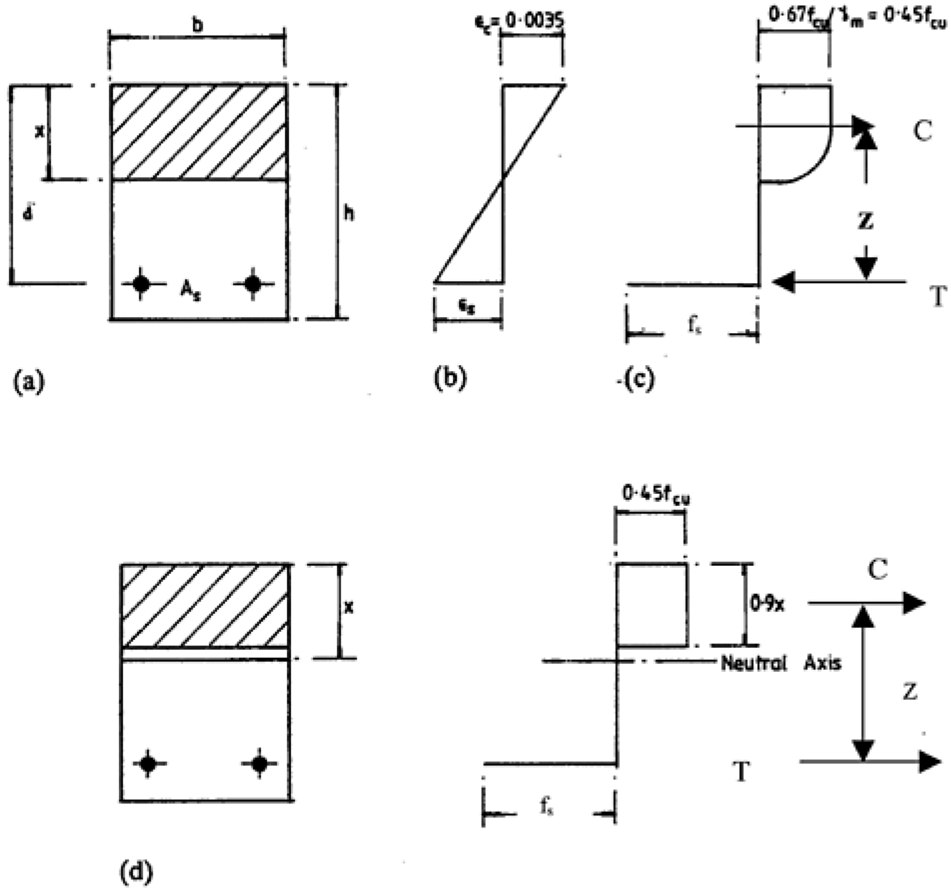


Fig.4.6 (a) Section; (b) strain; (c) rectangular parabolic stress diagram; (d) simplified stress diagram.

h overall depth of the section

d effective depth, i.e. depth from the compression face to the *centroid* of tension steel

b breadth of the section

x depth to the neutral axis

f_s stress in steel

A_s area of tension reinforcement

ε_c maximum strain in the concrete (0.0035)

ε_s strain in steel

The alternative stress distributions for the compressive stress in the concrete, the rectangular parabolic stress diagram and the simplified stress block, are shown in Figs 4.6(c) and 4.6(d) respectively.

The maximum strain in the concrete is 0.0035 and the strain ε_s in the steel depends on the depth of the neutral axis. Stress-strain curves for concrete and for steel are shown in Figs 4.5(a) and 4.5(b) respectively.

4.4.2 Moment of Resistance: Rectangular Stress Block

Fig.4.6 (d) shows the assumed stress distribution. The concrete stress is

$$0.67\ f_{cu}/\gamma_m=0.67\ f_{cu}/(\gamma_m=1.5)=0.447f_{cu}$$

which is generally rounded off to $0.45f_{cu}$.

The total compressive force C is given by

$$C=0.447\ f_{cu}\times b\times 0.9x=0.402\ b\times x\times f_{cu}$$

The lever arm z is

$$z=d-0.9x/2=d-0.45x$$

If M is the applied moment, then

$$M=C\times z=0.402\ b\times x\times f_{cu}\times(d-0.45x)$$

Setting

$$k=M/(b\ d^2\ f_{cu})$$
$$k=0.402(x/d)\ (1-0.45\ (x/d))$$

Rearranging,

$$0.1809(x/d)^2-0.402\ (x/d)+k=0$$

Solving for x/d

$$x/d=\{0.402-\sqrt{}(0.1616-0.7236\ k\}/\ 0.3618$$
$$=1.11-\sqrt{}(1.2345-5.5279k)$$
$$z/d=1-0.45(x/d)$$
$$z.d=0.5+\sqrt{}(0.25-k/0.9)$$

Total tensile force T in steel is

$$T=A_s\times f_s$$

For internal equilibrium, total tension T must be equal to total compression C. The forces T and C form a couple ata lever arm of z.

$M = T\,z = A_s\, f_s\, z$
$A_s = M/(f_s\, z)$

The stress f_s in steel depends on the strain ε_s in steel. As remarked in section 4.3, it is highly desirable that final failure is due to yielding of steel rather than due to crushing of concrete. It is useful therefore to calculate the maximum neutral axis depth in order to achieve this. Assuming that plane sections remain plane before and after bending, an assumption validated by experimental observations, if as shown in Fig.4.6(b), the maximum permitted strain in concrete at the compression face is 0.0035, then the strain ε_s in steel is calculated from the strain diagram by

$$\varepsilon_s = \frac{(d-x)}{x} 0.0035$$

Strain ε_s in steel at a stress of f_y/γ_m is given by

$$E_s\, \varepsilon_s = \frac{f_y}{\gamma_m}$$

where f_y=yield stress, γ_m=material safety factor and E_s is Young's modules for steel.
Taking $f_y=460\ N/mm^2$, $\gamma_m=1.05$, $f_y/\gamma_m=438\ N/mm^2$, $E_s=200\ kN/mm^2$, $\varepsilon_s= 0.0022$
For $\varepsilon_s=0.0022$, the depth of neutral axis x is given by

$$\varepsilon_s = 0.0022 = \frac{(d-x)}{x} 0.0035$$

$x/d=0.6140$

However in order to ensure that failure is preceded by steel yielding well before the strain in concrete reaches 0.0035 resulting in the desirable ductile form of failure, in clause 3.4.4.4, the code limits the ratio x/d to a maximum of 0.5. If x=0.5 d, then

$C=0.447\ f_{cu}\times b\times 0.9x=0.402\times b\times 0.5d\times f_{cu}$
$C=0.201\times f_{cu}\times b\times d$
$z=d-0.45x=d-0.45\times 0.5d=0.775\ d$
$M=C\times z=0.156\times b\times d^2\times f_{cu}$
$k=M/(bd^2 f_{cu})=0.156$

This is the maximum value of the applied moment that the section can resist because it utilises fully the compression capacity of the cross section. This formula can be used to calculate the *minimum effective depth* required in a singly reinforced rectangular concrete section.

$$d_{\min} = \sqrt{\frac{M}{0.156\ b\ f_{cu}}}$$

In practice the effective depth d is made larger than the required minimum consistent with the required headroom.

$$d \geq \sqrt{\frac{M}{0.156\ b\ f_{cu}}}$$

The reason for this is that with a larger depth, the neutral axis depth is smaller and hence the lever arm is larger leading for a given moment M, to a smaller amount of reinforcement. It has the additional advantage that in the event of unexpected overload, the beams will show large ductility before failure.

If $x/d \leq 0.5$, steel will always yield,

$f_s = f_y/1.05 = 0.95 f_y$
$M = T\,z = A_s\, 0.95\, f_y\, z$,
$A_s = M/(0.95\, f_y\, z)$

4.4.3 Procedure for the Design of Singly Reinforced Rectangular Beam

The steps to be followed in the design of singly reinforced rectangular beams can be summarised as follows.

- From the minimum requirements of span/depth ratio to control deflection (see Chapter 6), estimate a suitable effective depth *d*.
- Assuming the bar diameter for the main steel and links and the required cover as determined by exposure conditions, estimate an overall depth *h*.

$$h = d + \text{bar diameter} + \text{Link diameter} + \text{Cover}$$

- Assume breadth as about half the overall depth.
- Calculate the self-weight.
- Calculate the design live load and dead load moment using appropriate load factors. The load factors are normally 1.4 for dead loads and 1.6 for live loads.
- In the case of singly reinforced sections, calculate the minimum effective depth using the formula

$$d_{\min} = \sqrt{\frac{M}{0.156\, b\, f_{cu}}}$$

- Adopt an effective depth greater than the minimum depth in order to reduce the total tension reinforcement.
- Check that the new depth due to increased self-weight does not drastically affect the calculated design moment. If it does, calculate the revised ultimate moment required.
- Calculate $k = M/(b\, d^2\, f_{cu})$
- Calculate the lever arm z

$$z = d\{0.5 + \sqrt{(0.25 - k/0.9)}\} \leq 0.95d$$

Note that $z/d \leq 0.95$ if $k \geq 0.0428$

- Calculate the required steel A_s

$$A_s = M/\{0.95\, f_y\, z\}$$

- Check that the steel provided satisfies the minimum and maximum steel percentages specified in the code.

4.4.4 Examples of Design of Singly Reinforced Rectangular Sections

Example 1: A simply supported reinforced rectangular beam of 8 m span carries uniformly distributed characteristic dead load, which includes an allowance for self-weight of 7 kN/m and characteristic imposed load of 5 kN/m.

The breadth b=250 mm. Design the beam at mid-span section. Use grade 30 concrete and high yield steel reinforcement, f_y=460 N/mm^2.

Design load=(1.4×7)+(1.6×5)=17.8 kN/m

Design ultimate moment M at mid-span:

M=17.8×8^2/8=142.4 kNm

Minimum effective depth to avoid any compression steel is given by

$$d_{\min} = \sqrt{\frac{M}{0.156 \times b \times f_{cu}}} = \sqrt{\frac{142.4 \times 10^6}{0.156 \times 250 \times 30}} = 348.9 \text{ mm}$$

Using this value of d,

x=0.5d
z=d−0.45x=0.775 d.

The area of steel required is

$$A_s = \frac{M}{0.775 d \times 0.95 f_y} = \frac{142.4 \times 10^6}{0.775 \times 348.9 \times 0.95 \times 460} = 1206 \text{ mm}^2$$

However, if a value of d equal to say 400 mm, which is larger than the minimum value is used, then one can reduce the area of steel required.

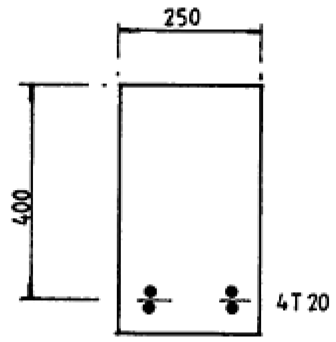


Fig.4.7: Mid-span section of the beam.

Assuming d=400 mm

$$k = \frac{M}{bd^2 f_{cu}} = \frac{142.4 \times 10^6}{250 \times 400^2 \times 30} = 0.119 < 0.156$$

$$\frac{z}{d} = 0.5 \quad + \quad \sqrt{\left(0.25 - \frac{k}{0.9}\right)} \ = 0.843 < 0.95$$

$$A_s = \frac{M}{0.843d \times 0.95f_y} = \frac{142.4\times10^6}{0.843\times400\times0.95\times460} = 967 \text{ mm}^2$$

Provide four 20 mm diameter bars in two layers as shown in Fig.4.7. From Table 4.1, A=1257 mm^2. Assuming cover of 30 mm and link diameter of 8 mm, the overall depth h of the beam is

h=400+30+8+20=458, say 460 mm.

Check that the percentage steel provided is greater than the minimum of 0.13.

100 A_s/(bh)=100×1257/(250×450)=1.12>0.13.

Note that this is only one of several possible satisfactory solutions.

Example 2: Determination of tension steel cut-off.

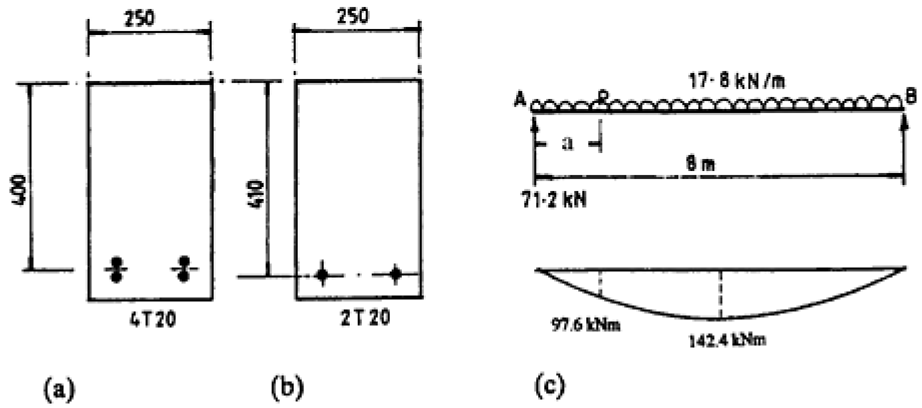


Fig 4.8 (a) Section at mid-span; (b) section at support; (c) loading and bending moment diagram.

In simply supported beams bending moment decreases towards the supports. Therefore the amount of steel required towards the support region is much less than at mid-span. For the beam in Example 4.1, determine the position along the beam where theoretically two of the four 20 mm diameter bars may be cut off.

The section at cut-off has two 20 mm diameter bars continuing: A_s=628 mm^2. The effective depth here is 410 mm (Fig.4.8(b)). The neutral axis depth can be determined by equating total compression in concrete to total tension in the beam.

$T=0.95\ f_y\ A_s=0.95\times460\times628\times10^{-3}=274.44$ kN
$C=(0.445\ fcu\ b\ 0.9x)10^{-3}$
$C=(0.445\times30\times250\times0.9x)\times10^{-3}=3x$ kN

Equating C=T,

x=91 mm
z=d−0.45x=369 mm
z/d=369/410=0.90<0.95

Moment of resistance M_R

$M_R = T\ z = 264.44 \times 369 \times 10^{-30} = 97.6$ kNm

Determine the position of p along the beam such that M=97.6 kN m (Fig.4.8c).

Left hand reaction V is

V=17.8×8/2=71.2 kN
97.6=71.2a−0.5×17.8a^2

The solutions to this equation are a=1.76 m and a=6.24 m from end A.

Table 4.4 Table to be used for calculating steel areas in slabs, walls, etc.

TOTAL REINFORCEMENT AREA (mm^2/m)

	Bar diameter mm						
Bar spacing mm	6	8	10	12	16	20	25
50	566	1010	1570	2260	4020	6280	9820
75	378	670	1050	1510	2680	4190	6550
100	283	503	785	1130	2010	3140	4910
125	226	402	628	904	1610	2510	3930
150	189	335	523	753	1340	2090	3270
175	162	288	448	646	1150	1790	2810
200	141	251	392	565	1010	1570	2460
250	113	201	314	452	804	1260	1960
300	94	167	261	376	670	1050	1640
350	81	144	224	323	574	897	1400
400	70	126	196	282	502	785	1230
450	63	112	174	251	447	697	1090
500	57	101	157	226	402	628	982

Note: A=(πd /4) {1000/(c to c spacing in mm}

Example 3: Singly reinforced one-way slab section
A slab section 1 m wide and 130 mm deep with an effective depth of 100 mm is subjected to a design ultimate moment of 10.5 kNm. Find the area of reinforcement required. The concrete is grade C30 and the reinforcement grade

$$k=\frac{M}{bd^2 f_{cu}}=\frac{10.5\times 10^6}{1000\times 100^2\times 30}=0.035<0.156$$

$$\frac{z}{d}=0.5+\sqrt{(0.25-\frac{k}{0.9})}=0.96>0.95$$

Constrain z to, z=0.95×100=95 mm

$$A_s=\frac{M}{0.95d\times 0.95 f_y}=\frac{10.5\times 10^6}{0.95\times 100\times 0.95\times 460}=253\ \text{mm}^2$$

In the case of slabs, reinforcement is not usually specified as a fixed number of bars but in terms of the diameter of the bar and its spacing. Using Table 4.4, provide 8 mm diameter bars at 175 mm centres. A_s=288 mm^2.

The percentage steel=100 A_s/(bh)=100×288/(1000×130)=0.22>0.13. The reinforcement for the slab is shown in Fig.4.9.

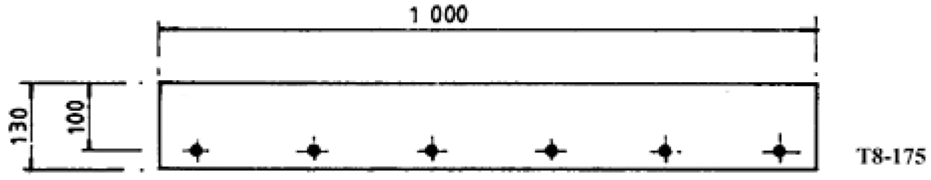


Fig.4.9 Reinforcement in slab

4.4.5 Design Chart

Using the equations developed in section 4.4.2, a chart for the design of singly reinforced rectangular beams can be constructed as follows.

- Choose a value of (x/d)≤0.5
- (z/d)={1−0.45 (x/d)}≤0.95
- C=0.45 f_{cu}bd 0.9 (x/d)
- M=C z=0.401 b d^2 f_{cu} (x/d) (z/d)
- A_s=M/(0.95 f_y z)=0.4221 b d (f_{cu}/f_y)
- $100\frac{A_s}{b\,d}\frac{f_y}{f_{cu}} = 42.21\,\frac{x}{d}$

Table 4.5 shows the calculations and the design chart is shown in Fig.4.10.

Table 4.5: Calculations for the design chart

x/d	1–0.45 x/d	z/d	M/(b d^2 f_{cu})	100(A_s/bd)(f_y/f_{cu})
0.001	1.00	0.95	0	0
0.025	0.99	0.95	0.0095	1.055
0.05	0.98	0.95	0.0191	2.111
0.10	0.96	0.95	0.0381	4.221
0.15	0.93	0.93	0.0561	6.332
0.20	0.91	0.91	0.0730	8.442
0.25	0.89	0.89	0.0890	10.553
0.30	0.87	0.87	0.1041	12.663
0.35	0.84	0.84	0.1183	14.774
0.40	0.82	0.82	0.1315	16.884
0.45	0.80	0.80	0.1439	18.995
0.50	0.78	0.78	0.1554	21.105

4.4.5.1 Examples of use of design chart

Example 1: Use the design chart to calculate the area of steel for the beam in Example 1, section 4.4.4.

$$\frac{M}{bd^2 f_{cu}} = \frac{142.4 \times 10^6}{250 \times 400^2 \times 30} = 0.119$$

From Fig.4.9,

$$100\frac{A_s}{bd}\frac{f_y}{f_{cu}} = 100\frac{A_s}{250 \times 400}\frac{460}{30} = 15.0$$

A_s=978 mm compared with 967 mm previously calculated.

Example 2: Calculate the moment of resistance for the beam in Example 2, section 4.4.4, for the section where steel is curtailed to 2T20.

$$100\frac{A_s}{bd}\frac{f_y}{f_{cu}} = 100\frac{628}{250 \times 410}\frac{460}{30} = 9.4$$

From Fig.4.10,

$$\frac{M}{bd^2 f_{cu}} = \frac{M \times 10^6}{250 \times 410^2 \times 30} = 0.08$$

M=100.9 kNm (Exact answer M=97.6 kNm)

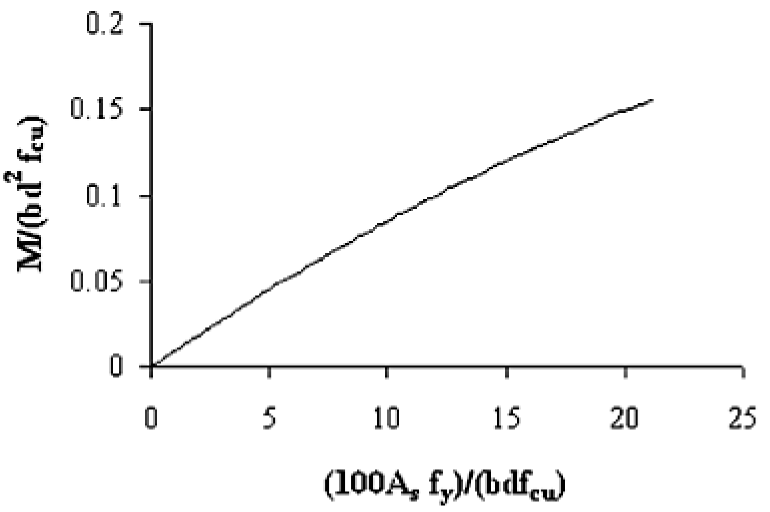


Fig.4.10 Design chart for singly reinforced rectangular concrete beams.

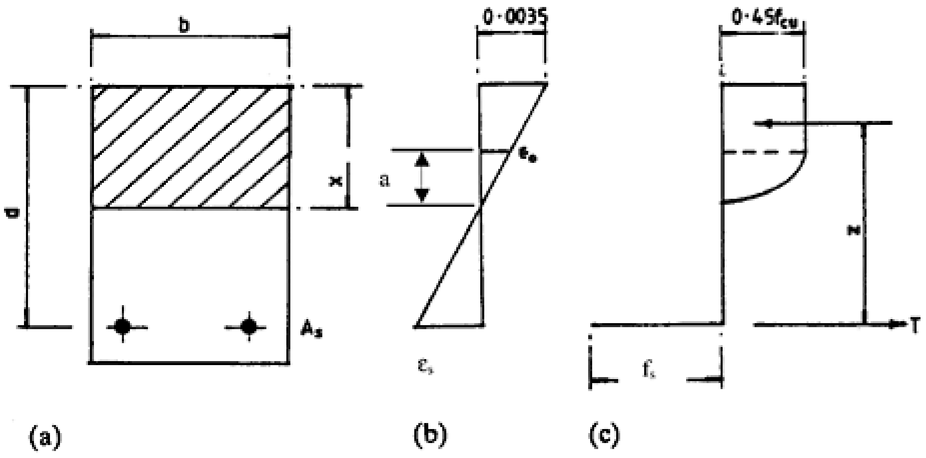


Fig.4.11 (a) section; (b) Strain diagram; (c) stress diagram.

4.4.6 Moment of Resistance Using Rectangular Parabolic Stress Block

In the previous sections, the simplified rectangular stress block was used to derive design equations. In this section, the stress-strain curve for concrete shown in Fig.4.5a will be used to derive the corresponding design equations. As shown in Fig.4.11(b), the maximum strain at the top is 0.0035. The strain ε_0 is where the parabolic part of the stress strain-strain curve ends. If the neutral axis depth is x, the distance 'a' from the neutral axis to where the strain is ε_0 is given by

$$a = x\,(\varepsilon_0/0.0035)$$

where ε_0 from Fig.4.5(a) is given by

$$\varepsilon_0 = 2.4\times10^{-4}\sqrt{\frac{f_{cu}}{\gamma_m}}$$

The compressive force C_1 in the rectangular portion of depth (x–a) of the stress block is given by

$$C_1 = 0.67\,\frac{f_{cu}}{\gamma_m}\,b\,(x-a)$$

The lever arm z_1 for C_1 from the centroid of steel area is

$$z_1 = d - 0.5(x-a)$$

Using the 'well-known' result that the area of a parabola is equal to two-thirds the area of the enclosing rectangle, the compressive force in the parabolic portion of depth 'a' of the stress block is given by

$$C_2 = \frac{2}{3}\left\{0.67\,\frac{f_{cu}}{\gamma_m}\,b\,a\right\}$$

The centroid of C_2 is at a distance of 5a/8 from the neutral axis. The lever arm z_2 for C_2 from the centroid of steel area is

$z_2 = d - x + 5a/8$

Therefore taking moments about the centroid of steel area,

$M = C_2 z_2 + C_1 z_1$

For f_{cu}=30 N/mm^2 and γ_m=1.5,

$$\varepsilon_0 = 2.4\times10^{-4}\sqrt{\frac{f_{cu}}{\gamma_m}} = 0.00107$$

$\varepsilon_0/0.0035=0.306$
$a=x(\varepsilon_0/0.0035)=0.306x$
$C_1=0.31\ f_{cu}bx$
$C_2=0.091\ f_{cu}bx$
$z_1=(d-0.347x)$
$z_2=d-0.8088x$

$$M = C_1 z_1 + C_2 z_2 = f_{cu}\ b\ d^2\ \frac{x}{d}\ \{0.4014 - 0.1814\frac{x}{d}\}$$

The corresponding equation for rectangular stress block assumption as derived in section 4.4.2 is

$$M = f_{cu}\ b\ d^2\ \frac{x}{d}\ \{0.402 - 0.1809\frac{x}{d}\}$$

The two equations differ by very little from each other. The Rectangular stress block assumption is therefore accurate for all practical calculations.

4.5 DOUBLY REINFORCED BEAMS

The normal design practice is to use singly reinforced sections. However if for any reason, for example headroom considerations, it is necessary to restrict the overall depth of a beam, then it becomes necessary to use steel in the compression zone as well because concrete alone cannot provide the necessary compression resistance.

4.5.1 Design Formulae Using the Simplified Stress Block

The formulae for the design of a doubly reinforced beam are derived using the rectangular stress block.

Let M be the design ultimate moment. As shown in section 4.4.2, a rectangular section as a *singly reinforced section* can resist a *maximum* value of the moment equal to

$M_{sr}=0.156bd^2 f_{cu}$

The corresponding neutral axis depth x=0.5 d. The compressive force C_c in concrete is

$C_c=0.45f_{cu}b0.45\ d=0.2f_{cu}bd$

The lever arm z_c

$z_c = 0.775d$

If $M > M_{sr}$, then compression steel is required. The compressive force C_s due to compression steel of area $A_s`$ is

$C_s = A_s` f_s`$

where f_s' is the stress in compression steel. As shown in Fig.4.12, the lever arm z_s for compression steel is

$z_s = (d - d`)$

The stress in the tensile steel is 0.95 f_y because the neutral axis depth is limited to 0.5d. However the stress $f_s`$ in the compressive steel depends on the corresponding strain strain ε_{sc} in concrete at steel level. ε_{sc} is given by

$$\varepsilon_{sc} = 0.0035\frac{(x - d^`)}{x}$$

If the strain ε_{sc} is equal to or greater than the yield strain in steel, then steel yields and the stress $f_s`$ in compression steel is equal to 0.95 f_y. Otherwise, the stress in compression steel is given by

$f_s` = E_s\ \varepsilon_{sc}$.

If $f_y = 460\ N/mm^2$ and $E_s = 200\ kN/mm^2$, then the yield strain in steel is equal to

$$\varepsilon_{yield} = \frac{0.95\,f_y}{E} = 0.0022$$

Therefore, steel will yield if

$$\varepsilon_{sc} = 0.0035\frac{(x - d')}{x} \geq \left(\frac{0.95 f_y}{E} = 0.0022\right)$$

$$\therefore \frac{d'}{x} \leq 0.37 \quad \text{If } x = 0.5d\text{, then } \frac{d'}{x} \leq 0.186$$

If mild steel is used, then $f_y = 250\ N/mm^2$. The above equations then become

$$\varepsilon_{yield} = \frac{0.95\,f_y}{E} = 0.0012$$

Therefore, steel will yield if

$$\varepsilon_{sc} = 0.0035\frac{(x-d^{'})}{x} \geq (\frac{0.95f_y}{E} = 0.0012)$$

$$\therefore \frac{d'}{x} \leq 0.66 \quad \text{If x = 0.5d, then } \frac{d'}{x} \leq 0.33$$

Taking moments about the tension steel,

$M=C_c z_c+C_s z_s$
$M=0.2\ f_{cu}\ bd(0.775\ d)+A_s^{`}\ f_s^{`}(d-d^{`})$
$M=0.156\ bd^2\ f_{cu}+A^{`}f_s^{`}(d-d')$
$A_s^{`}=(M-0.156\ bd^2\ f_{cu})/\{f_s^{`}(d-d^{`})\}$

From equilibrium, the tensile force T is

$T=A_s 0.95f_y=C_c+C_s$

One important point to remember is that to prevent steel bars in compression from buckling, it is necessary to restrain them using links. Clause 3.12.7 of the code says that links or ties at least one quarter of the size of the largest compression bar or 6 mm whichever is greater should be provided at a maximum spacing of 12 times the size of the smallest compression bar.

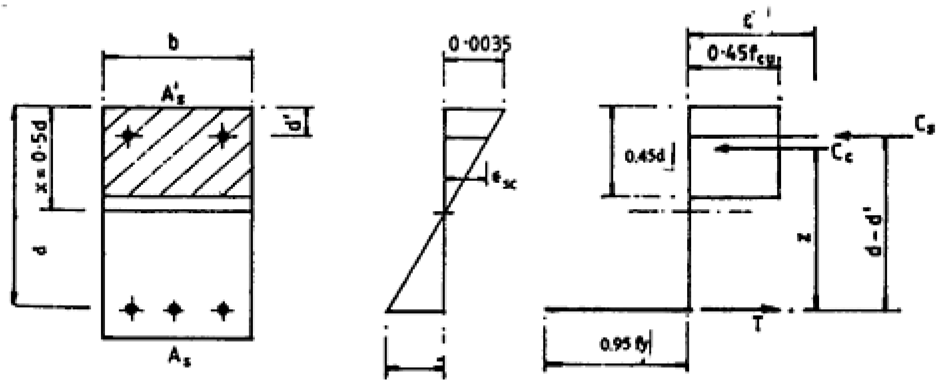


Fig.4.12 Doubly reinforced beam.

4.5.2 Examples of Rectangular Doubly Reinforced Concrete Beams

The use of the formulae developed in the previous section is illustrated by a few examples.

Example 1: A rectangular beam is simply supported over a span of 6 m and carries characteristic dead load including self-weight of 12.7 kN/m and characteristic imposed load of 6.0 kN/m. The beam is 200 mm wide by 300 mm effective depth and the inset of the compression steel is 40 mm. Design the steel for mid-span of the beam for grade C30 concrete and grade 460 reinforcement.

design load=(12.7×1.4)+(6×1.6)=27.4 kN/m

Required ultimate moment M:

$M=27.4\times6^2/8=123.3$ kN m

Maximum moment that the beam section can resist as a singly reinforced section is

$M_{sr}=0.156\times30\times200\times300^2\times10^{-6}=84.24$ kNm

$M>M_{sr}$, Compression steel is required.

$d'/x=40/150=0.27<0.37$

The compression steel yields. The stress f_s' in the compression steel is $0.95f_y$.

$A_s'=\{M-0.156\,b\,d^2\,f_{cu}\}/[0.95\,fy\,(d-d)]$
$A_s' =\{123.3-84.24\}\times 10^6/[0.95\times460\times(300-40)]=344\ mm^2$

From equilibrium:

$A_s\ 0.95\ f_y=0.2bdf_{cu}+A_s`f_s`$
$A_s\ 0.95\times460=0.2\times200\times300\times30+344\times0.95\times460$
$A_s= 1168\ mm^2$

For the tension steel (2T25+2T16) give A_s=1383 mm^2. For the compression steel 2T16 give A_s=402 mm^2. The beam section and flexural reinforcement steel are shown in Fig.4.13.

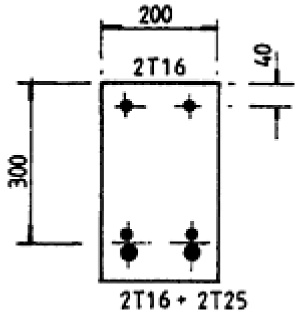


Fig.4.13 Doubly reinforced beam.

Example 2: Design the beam in Example 4.6 but with d'=60 mm.

$d^`/x=60/150=0.40>0.37$

Compression steel does not yield. Strain in compression steel

$$\varepsilon_{sc} = 0.0035\frac{(x-d^{'})}{x} = 0.0035\frac{(150-60)}{150} = 0.0021$$

Stress in compression steel is

$f_s^` = E_s\ \varepsilon_{sc} = 200\times10^3\times0.0021 = 420\ N/mm^2$

$A_s^` = \{M-0.156\ b\ d^2\ f_{cu}\}/[420\ (d-d')]$
$A_s^` = \{123.3-84.24\}\times10^6/[420\times(300-40)] = 358\ mm^2$
From equilibrium:

$A_s\ 0.95\ f_y = 0.2bdf_{cu} + A_s^`f_s^`$
$A_s\ 0.95\times460 = 0.2\times200\times300\times30+358\times420,\ A_s = 1168\ mm^2$

4.6 FLANGED BEAMS

4.6.1 General Considerations

In simple slab-beam system shown in Fig.4.14, the slab is designed to span between the beams. The beams span between external supports such as columns, walls, etc. The reactions from the slabs act as load on the beam. When a series of beams are used to support a concrete slab, because of the monolithic nature of concrete construction, the slab acts as the flange of the beams. The end beams become L-beams while the intermediate beams become T-beams. In designing the intermediate beams, it is assumed that the loads acting on half the slab on the two sides of the beam are carried by the beam. Because of the

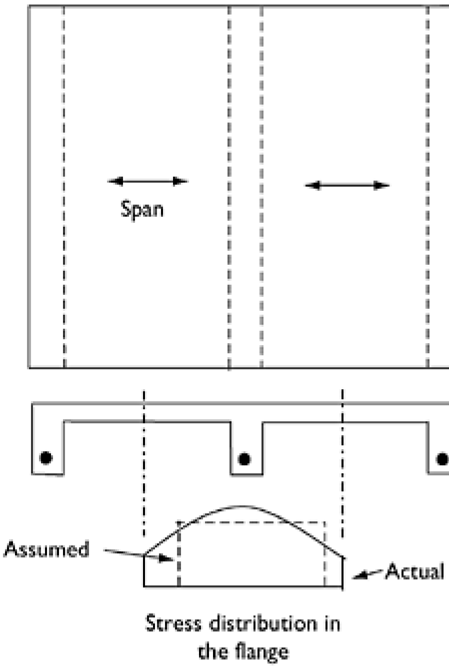


Fig.4.14 Beam-slab system.

comparatively small contact area at the junction of the flange and the rib of the beam, the distribution of the compressive stress in the flange is not uniform. It is higher at the junction and decreases away from the junction. This phenomenon is known as shear lag. For simplicity in design, it is assumed that only part of full physical flange width is considered to sustain compressive stress of uniform magnitude. This smaller width is known as effective breadth of the flange. Although the effective width actually varies even along the span as well, it is common to assume that the effective width remains constant over the entire span.

The effective breadth b of flanged beams (Fig.4.15) as given in BS 8110: Part 1, clause 3.4.1.5:

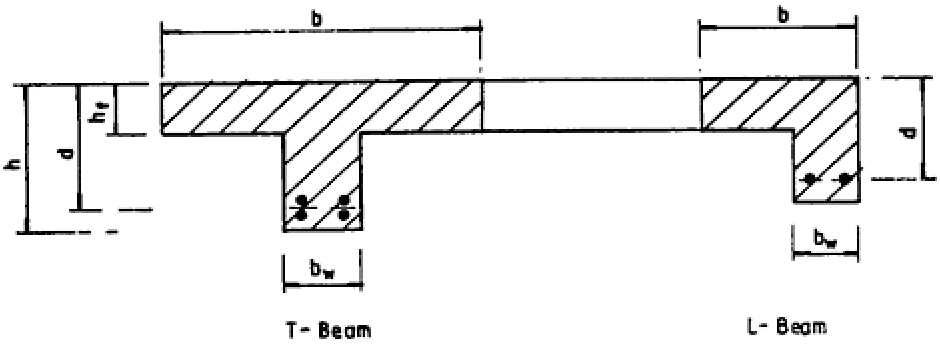


Fig.4.15: Cross section of flanged beams.

1. T-beams: $b=\{\text{web width } b_w+\ell_z/5\}$ or the actual flange width if less;
2. L-beams: $b=\{\text{web width } b_w+\ell_z/10\}$ or the actual flange width if less

where b_w is the breadth of the web of the beam and ℓ_z is the distance between points of zero moment in the beam. In simply supported beams it is the effective span where as in continuous beams ℓ_z may be taken as 0.7 times the effective span.

The design procedure for flanged beams depends on the depth of the stress block. Two possibilities need to be considered.

4.6.2 Stress Block within the Flange

If $0.9x \le h_f$, the depth of the flange (same as the total depth of the slab) then all the concrete below the flange is cracked and the beam may be treated as a rectangular beam of breadth b and effective depth d and the methods set out in sections 4.4.6 and 4.4.7 above apply. The maximum moment of resistance when $0.9x=h_f$ is equal to

$$M_{flange}=0.45\, f_{cu} b h_f (d-h_f/2)$$

Thus if the design moment $M \le M_{flange}$, then design the beam as singly reinforced rectangular section $b \times d$.

4.6.3 Stress Block Extends into the Web

As shown in Fig.4.16, the compression forces are:

In the flange of width $(b-b_w)$, the compression force C_1 is

$$C_1=0.45\, f_{cu}(b-b_w)h_f$$

In the web, the compression force C_2 is

$$C_2=0.45\, f_{cu} b_w 0.9x$$

The corresponding lever arms about the tension steel are

$$z_1=d-h_f/2$$
$$z_2=(d-0.9x/2)$$

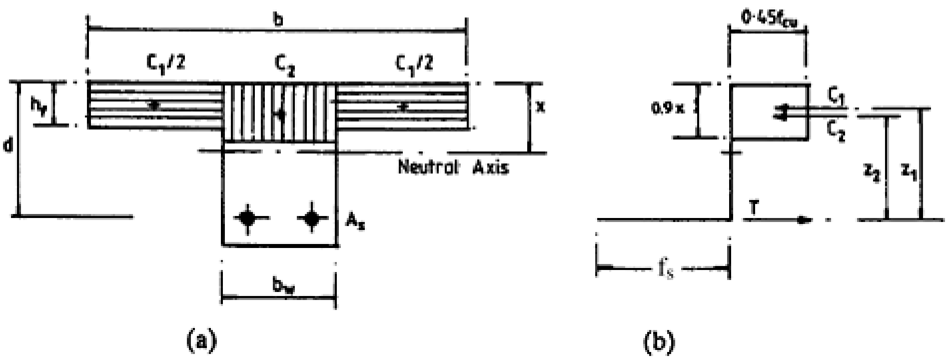


Fig.4.16: T-beam with the stress block extending into the web.

The moment of resistance M_R is given by

$M_R = C_1 z_1 + C_2 z_2$
$M_R = 0.45\ f_{cu}(b-b_w)h_f(d-h_f/2) + 0.45 f_{cu} b_w 0.9x\ (d-0.9x/2)$

$$\frac{M}{bd^2 f_{cu}} = 0.45(1-\frac{b_w}{b})\frac{h_f}{d}(1-\frac{h_f}{2d}) + 0.4\frac{b_w}{b}\frac{x}{d}(1-0.45\frac{x}{d})$$

From equilibrium,

$T = A_s f_s = C_1 + C_2$

If the amount of steel provided is sufficient to cause yielding of the steel, then $f_s = 0.95f_y$. The maximum moment of resistance without any compression steel is when x=0.5d. Substituting x=0.5d in the expression for M_R, the maximum moment of resistance is

$M_{max} = 0.45\ f_{cu}(b-bw)h_f(d-h_f/2) + 0.156\ f_{cu} b_w d^2$

$$M_{max} = bd^2 f_{cu}[\ 0.45(1-\frac{b_w}{b})\frac{h_f}{d}(1-\frac{h_f}{2d}) + 0.156\frac{b_w}{b}]$$

$$\text{or } M_{max} = \beta_f\, bd^2 f_{cu}$$

or $M_{max} = \beta_f bd^2 f_{cu}$

where

$$\beta_f = [\ 0.45(1-\frac{b_w}{b})\frac{h_f}{d}(1-\frac{h_f}{2d}) + 0.156\frac{b_w}{b}]$$

Thus if the design moment $M_{flange} < M \leq M_{max}$, then determine the value of x from

$$\frac{M}{bd^2 f_{cu}} = 0.45(1-\frac{b_w}{b})\frac{h_f}{d}(1-\frac{h_f}{2d}) + 0.4\frac{b_w}{b}\frac{x}{d}(1-0.45\frac{x}{d})$$

where $x \leq 0.5d$ and the reinforcement required is obtained from the equilibrium condition,

$A_s 0.95 f_y = C_1 + C_2$

4.6.3.1 Code formula

As an alternative, a slightly conservative formula for calculating the steel area is given in clause 3.4.4.5 of the code. The equation in the code is derived using the simplified stress block with x=0.5d (Fig.4.16).

depth of stress block=0.9x=0.45d

The concrete forces in compression are

$C_1 = 0.45 f_{cu} h_f (b-bw)$
$C_2 = 0.45 f_{cu} \times 0.45 d b_w = 0.2 f_{cu} b_w d$

The values of the lever arms for C_1 and C_2 from the steel force T are:

$z_1 = d - 0.5h_f$
$z_2 = d - 0.5x\ 0.45d = 0.775d$

The steel force in tension is

$T=0.95f_yA_s$

The moment of resistance of the section is found by taking moments about force C_1:

$M=Tz_1-C_2(z_1-z_2)$
$M=0.95f_yA_s(d-0.5h_f)-0.2f_{cu}b_wd\,(0.225d-0.5h_f)$
$M=0.95f_y\,A_s(d-0.5h_f)-0.1f_{cu}b_wd(0.45d-h_f)$

from which

$$A_s=\frac{M+0.1f_{cu}\,b_w\,d\,(0.45\,d-h_f)}{0.95\,f_y\,\,(d-0.5\,h_f)}$$

This is the expression given in the code. It gives conservative results for cases where *x* is less than 0.5d. The equation only applies when h_f is less than 0.45d, as otherwise the second term in the numerator becomes negative.

4.6.4 Steps in Reinforcement Calculation of a T-or an L-Beam

- Calculate the total design load (including self-weight) and the corresponding design moment M using appropriate load factors.
- Calculate the maximum moment M_{flange} that can be resisted, when the entire flange is in compression.

$$M_{flange}=0.45f_{cu}bh_f(d-h_f/2)$$

- Calculate the maximum moment that the section can withstand without requiring compression reinforcement.

$$M_{max}=0.45f_{cu}(b-b_w)h_f\,(d-h_f/2)+0.156f_{cu}b_wd^2$$

- If $M\leq M_{flange}$, then design as a rectangular beam of dimensions, b×d.
- If $M_{flange}<M\leq M_{max}$, then the required steel area can be determined to sufficient accuracy from the code formula

$$A_s=\frac{M+0.1f_{cu}\,b_w\,d\,(0.45\,d-h_f)}{0.95\,f_y\,\,(d-0.5\,h_f)}$$

- If $M>M_{max}$, then compression steel is required or the section has to be revised. Compression steel is rarely required in the case of flanged beams.

4.6.5 Examples of Design of Flanged Beams

Example 1: A continuous slab 100 mm thick is carried on T-beams at 2 m centres. The overall depth of the beam is 350 mm and the breadth b_w of the web is 250 mm. The beams are 6 m span and are simply supported. The characteristic dead load including self-weight and finishes is 7.4kN/m^2 and the characteristic imposed load is 5 kN/m^2. Design the beam using the simplified stress block. The materials are grade C30 concrete and grade 460 reinforcement.

Since the beams are spaced at 2 m centres, the loads a the beam are:

Dead load=7.4×2=14.8 kN/m

Live load=5×2=10 kN/m
design load=(1.4×14.8)+(1.6×10)=36.7 kN/m
ultimate moment at mid-span=$36.7\times6^2/8=165$ kN m
effective width b of flange: b=250+6000/5=1450 mm

The beam section is shown in Fig.4.17. From BS8110: Part 1, Table 3.4, the nominal cover on the links is 25 mm for grade 30 concrete. If the links are 8 mm in diameter and the main bars are 25 mm in diameter, then

d=350–25–8−12.5=304.5 mm, say 300 mm.

First of all check if the beam can be designed as a rectangular beam by calculating M_{flange}.

$M_{flange}=0.45f_{cu}bh_f(d-h_f/2)$
$M_{flange}=0.45\times30\times1450\times100\ (300-0.5\times100)\times10^{-6}=489.3$ kNm

The design moment of 165 kNm is less than M_{flange}. The beam can be designed as a rectangular beam of size 1450×300. Using the code expressions in clause 3.4.4.4

$k=M/(bd^2f_{cu})=165\times10^6/(1450\times300^2\times30)=0.042$
$z/d=\{0.5+\sqrt{(0.25-0.042/0.9)}\}=0.95$
z=0.95d=285 mm
$A_s=165\times10^6/(0.95\times460\times285)=1325\text{mm}^2$.

Provide 3T25; $A_s=1472\ \text{mm}^2$.

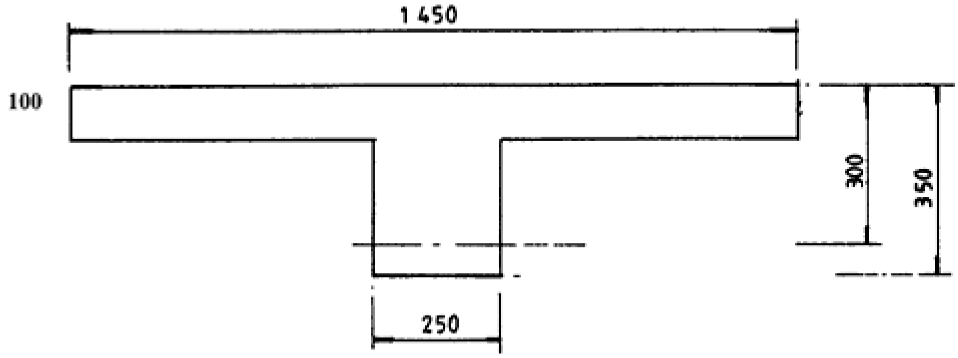


Fig.4.17 Cross section of T-beam.

Example 2: Determine the area of reinforcement required for the T-beam section shown in Fig.4.18 which is subjected to an ultimate moment of 260 kNm. The materials are grade C30 concrete and grade 460 reinforcement.
Calculate M_{flange} to check if the stress block is inside the flange or not.

$M_{flange}=0.45\times30\times600\times100(340-0.5\times100)\times10^{-6}=234.9$ kNm

The design moment of 260 kNm is greater than M_{flange}. Therefore the stress block extends into the web.
Check if compression steel is required.

$M_{max}=0.45f_{cu}(b-b_w)h_f(d-h_f/2)+0.156\ f_{cu}b_wd^2$
$M_{max}=\{0.45\times30\times(600-250)\times100\times(340-100/2)\ +0.156\times30\times250\times340^2\}\times10^{-6}$

M_{max}=(137.0+135.3)=272.3 kNm
M_{max}>(M=260 kNm)

The beam can be designed without any need for compression steel. Two approaches can be used for determining the area of tension steel required.

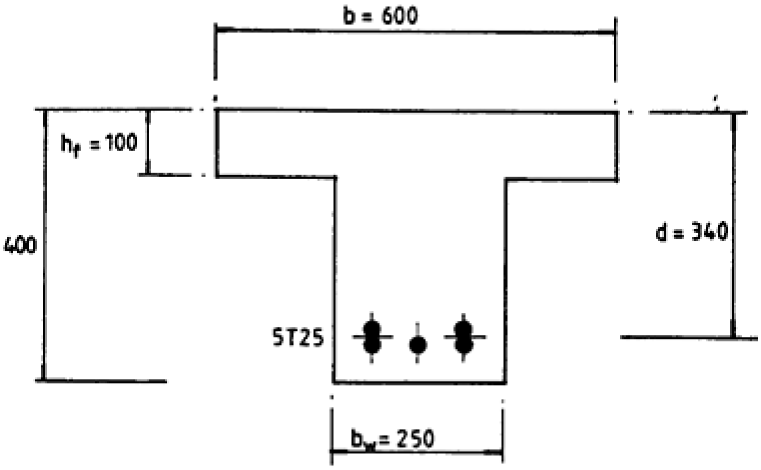


Fig.4.18 Cross section of T-beam.

(a) Exact approach
Determine the depth of the neutral axis from

$$\frac{M}{bd^2 f_{cu}} = 0.45(1-\frac{b_w}{b})\frac{h_f}{d}(1-\frac{h_f}{2d}) + 0.4\frac{b_w}{b}\frac{x}{d}(1-0.45\frac{x}{d})$$

$$\frac{260\times10^6}{600\times340^2\times30} = 0.45(1-\frac{250}{600})\frac{100}{340}(1-\frac{100}{2\times340}) + 0.4\frac{250}{600}\frac{x}{d}(1-0.45\frac{x}{d})$$

setting x/d=a

$0.1250=0.0659+0.1667\ a-0.075\ \alpha^2$

Simplifying

$\alpha^2-2.22\ \alpha+0.788=0$

Solving the quadratic in a,

a=x/d=(2.22–1.3328)/2=0.444< 0.5
×=0.444×340=151 mm
$T=0.95f_yA_s=C_1+C_2$
$T=0.45f_{cu}(b-b_w)h_f+0.45f_{cu}b_w0.9\ x$
$T=(0.45\times30\times(600-250)\times100+0.45\times30\times250\times0.9\times151)\times10^{-3}$
$T=0.95\ A_s$=(472.5+458.7)=931.2 kN
$A_s=931.2\times10^3/(0.95\times460)=2131\ mm^2$

(b) Code formula

Calculation of A_s using simplified code formula which assumes x/d=0.5

$$A_s = \frac{260\times10^6 + 0.1\times30\times250\times340\times(0.45\times340-100)}{0.95\times460\times(340-0.5\times100)} = 2159\,\text{mm}^2$$

This is only 1% more than that calculated using the exact neutral axis depth! Provide 5T25, A=2454 mm^2

4.7 CHECKING EXISTING SECTIONS

In the previous sections methods have been described for designing rectangular and flanged sections for a given moment. In practice it may be necessary to calculate the ultimate moment capacity of a given section. This situation often occurs when there is change of use in a building and the owner wants to see if the structure will be suitable for the new use. Often moment capacity can be increased either by

- Increasing the effective depth. This can be done by adding a well bonded layer of concrete at the top of the beam/slab
- Increasing the area of tension steel by bonding steel plates to the bottom of the beam.

4.7.1 Examples of Checking for Moment Capacity

Example 1: Calculate the moment of resistance of the singly reinforced beam section shown in Fig.4.19(a). The materials are grade C30 concrete and grade 460 reinforcement. The tension reinforcement is 4T20 giving A_s=1256 mm^2 Assuming that tension steel yields, total tensile force T is given by

$T=0.95f_yA_s=0.95\times460\times1256\times10^{-3}=548.7$ kN

If the neutral axis depth is x, then the compression force C is

$C=0.45f_{cu}\,(0.9x\times b)=0.45\times30\times0.9x\times250\times10^{-3}=3.0375x$ kN

For equilibrium, T=C. Solving for x

$x=181$ mm$<(0.5\,d=200$ mm$)$

Check the strain in steel

$$\varepsilon_s = \frac{0.0035}{x}(d-x) = \frac{0.0035}{181}(400-181)=0.004 > (\text{yield strain} = 0.0022)$$

Steel yields. Therefore the initial assumption is valid.

$z=d-0.45x=400-0.45\times181=310$ mm
$z/d=310/400=0.775<0.95$

Moment of resistance M

$M = T\,z = 548.7 \times 310 \times 10^{-3} = 169.9$ kNm

One can also use the Design Chart shown in Fig.4.10 to solve the problem.

$$100\frac{A_s}{bd}\frac{f_y}{f_{cu}} = 100\frac{1256}{250 \times 400}\frac{460}{30} = 19.26$$

From design chart Fig.4.10,

$$\frac{M}{bd^2 f_{cu}} = \frac{M \times 10^6}{250 \times 410^2 \times 30} = 0.145$$

M=174.0 kNm

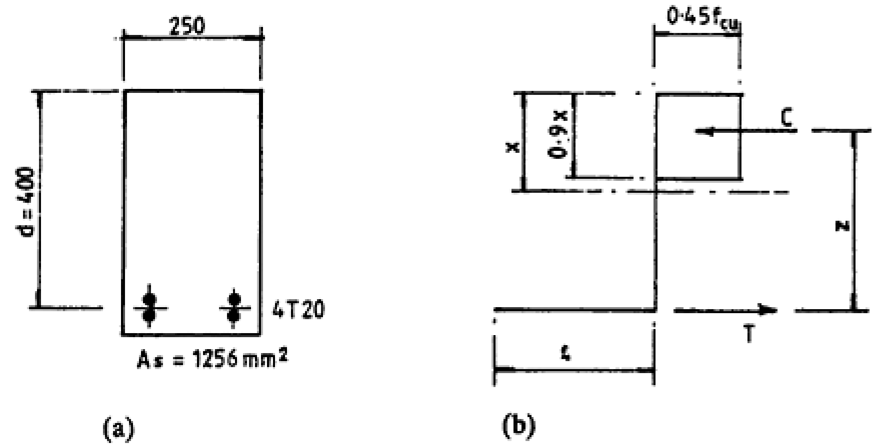


Fig 4.19 Cross section of rectangular beam.

Example 2: Determine the ultimate moment capacity of the beam in Fig.4.19, except, A_s=6T20=1885 mm^2

Proceeding as in Example 1, assume that steel yields and calculate

$T=0.95 f_y A_s=0.95\times460\times1885=8.24\times10^5$ N
$C=0.45\times f_{cu}\times0.9\times x\times b=3037.5x$ N

For equilibrium, T=C.

x=271 mm, x/d=0.68>0.5

Check strain in steel to check the validity of the initial assumption.

$$\varepsilon_s = 0.0035\frac{(d-x)}{x} = 0.0035\frac{(400-271)}{271} = 0.0017 < (\text{yield strain} = 0.0022)$$

Since the strain in steel is less than yield strain, tension steel does not yield indicating that the initial assumption is wrong. Assume that the tension steel does not yield. For an assumed value of neutral axis depth x, strain ε_s in tension steel is

$$\varepsilon_s = 0.0035\frac{(d-x)}{x}$$

Since the steel is assumed not to yield, if Young's modulus for steel is E_s=200 kN/mm^2, then stress f_s in tension steel is given by

$$f_s = E_s \times \varepsilon_s = 200\times10^3 \times 0.0035\frac{(d-x)}{x} = 700\frac{(d-x)}{x}$$

$T = A_s \times f_s = \{1885\times700\times(d-x)/x\}\times10^{-3} = 1319.5(400-x)/x$ kN

$C = 0.45\ f_{cu} b\ 0.9x = \{0.45\times30\times250\times0.9x\}\times10^{-3} = 3.0375x$ kN

For equilibrium, T=C

$1319.5\times(400-x)/x=3.0375x$

Simplifying

$x^2+434.40x-173761.3=0$

Solving the quadratic equation in x,

$x=253$ mm, $x/d=253/400=0.63>0.5$

Calculate the strain in steel.

$$\varepsilon_s = 0.0035\frac{(d-x)}{x} = 0.0035\frac{(400-253)}{253} = 0.00204$$

$f_s=E_s\times\varepsilon s=200\times10^3\times0.00204=408\ N/mm^2$
$z=d-0.45x=286$ mm
$M=T\times z=769\times286\times10^{-3}=220$ kNm

Since x/d>0.5, it is sensible to limit the permissible ultimate moment to a value less than 220 kNm. Assuming that x=0.5d=200 mm,

$C=0.45\ f_{cu}b\ 0.9x=\{0.45\times30\times250\times0.9\times200\}\times10^{-3}=607.5$ kN
Lever arm $z=0.775\ d=0.775\times400=310$ mm

Taking moments about the steel centroid, M=Cz=188.3 kNm
Example 3: Calculate the moment of resistance of the beam section shown in Fig.4.20. The materials are grade C30 concrete and grade 460 reinforcement.

$A_s=4T25=1963\ mm^2$, $A_s^`=2T20+T16–829\ mm^2$

Assume that both tension and compression steels yield and calculate the tension force T and compression force C_s in the steels.

$T=0.95f_yA_s=0.95\times460\times1963\times10^{-3}=857.8$ kN
$C_s=0.95\ f_yA_s^`=0.95\times460\times829\times10^{-3}=362.3$ kN

The compression force in concrete is

$C_c=0.45f_{cu}(0.9x\times b)=0.45\times30\times0.9x\times250\times10^{-3}=3.0375x$ kN

For equilibrium,

$C_c+C_s=T$
$3.0375x+362.3=857.8.$

Solving x=163 mm, x/d=0.47<0.5.

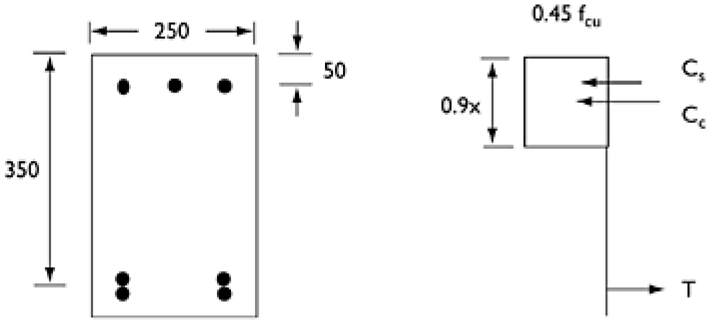


Fig.4.20 Cross section of doubly reinforced beam.

Calculate strain in tension and compression steels to verify the assumption.

$$\varepsilon_s = 0.0035\frac{(d-x)}{x} = 0.0035\frac{(350-163)}{163} = 0.004$$
$$\varepsilon_s' = 0.0035\frac{(x-d')}{x} = 0.0035\frac{(163-50)}{163} = 0.0024$$

Both strains are larger than yield strain of 0.0022. Therefore both steels yield and the initial assumption is correct.

$C_c = 0.45 f_{cu}\, b 0.9x = 0.45\times30\times250\times0.9\times163\times10^{-3} = 495.1\text{kN}$

Taking moments about the tension steel,

$M = C_c(d-0.45x) + C_s(d-d')$
$M = 495.1(350-0.45\times163)\times10^{-3} + 362.3\times(350-50)\times10^{-3} = 245.8\text{ kNm}$

4.7.2 Strain Compatibility Method

In the previous section, examples were given for calculating the moment of resistance of a given section. It required making initial assumptions about whether the steel yields or not. After calculating the neutral axis depth from equilibrium considerations, strains in tension and compression steels are calculated to validate the assumptions. The problem can become complicated if one steel yields while the other steel does not. A general approach in this case is the method of Strain Compatibility which has the advantage of avoiding the algebraic approach. The basic idea is to assume a neutral axis depth. From the assumed value of neutral axis depth, strains in steel in compression and tension are calculated. Thus

$$\varepsilon_s = \frac{0.0035}{x}(d-x),\ f_s = E\varepsilon_s \le 0.95 f_y$$
$$\varepsilon_s' = \frac{0.0035}{x}(x-d'),\ f_s' = E\varepsilon_s' \le 0.95 f_y$$

From the stresses, calculate the forces

$T = A_s f_s, C_s = A_s' f_s', C_c = 0.45 f_{cu} b 0.9x,\ C = C_s + C_c$

For equilibrium, T=C. If equilibrium is not satisfied, then adjust the value of x and repeat until equilibrium is established. Normally only two sets of calculations for neutral axis depth are required. Linear interpolation can be used to find the appropriate value of x to satisfy equilibrium. The following example illustrates the method.

4.7.2.1 Example of Strain-Compatibility Method

Example 1: Calculate the moment capacity of the section with b=250 mm, d = 350 mm, d'=50 mm,

$A_s` = 3T20 = 942.5\ mm^2$, $A_s = 6T25 = 2945.2\ mm^2$

Trial 1: Assume x=220 mm

Strain ε_s in compression steel is given by

$\varepsilon_s{}' = 0.0035(x–d`)/x = 0.0035×(220–50)/220 = 0.0027 > 0.0022$

Therefore compression steel yields and the stress f_s’ is equal to 0.95 f_y

Similarly, strain ε_s in tension steel is given by

$\varepsilon_s = 0.0035(d−x)/x = 0.0035×(350–220)/220 = 0.00207 < 0.0022$

Therefore tension steel does not yield and the stress f_s is equal to

$f_s = \varepsilon_s E_s = 0.00207 × 200 × 10^3 = 413.6\ N/mm^2$
$T = A_s × f_s = 2945.2 × 413.6 × 10^{-3} = 1218.1\ kN$
$C = 0.45\ f_{cu} × b × 0.9x + A_s` × f_s`$
$C = \{0.45×30×250×0.9×220 + 942.5×0.95×460\}×10^{-3}$
$C = (668.25 + 411.87) = 1080.1\ kN$
$T−C = 138.0\ kN$

Total tensile force T is greater than the total compressive force C. Therefore increase the value of x in order to increase the compression area of concrete and also reduce the strain in tension steel but increase the strain in compression steel.

Trial 2: Assume x=240 mm say

Strain ε_s in compression steel is given by

$\varepsilon_s` = 0.0035(x–d’)/x = 0.00277 > 0.0022$

Therefore compression steel yields and the stress f_s` is equal to 0.95 f_y

Similarly, strain ε_s in tension steel is given by

$\varepsilon_s = 0.0035(d–x)/x = 0.0016 < 0.0022$

Therefore tension steel does not yield and the stress f_s is equal to

$f_s = \varepsilon_s\ E = 0.001604×200×10^3 = 320.8\ N/mm^2$
$T = A_s × f_s = 2945.2 × 320.8 × 10^{-3} = 944.8\ kN$
$C = 0.45 f_{cu} × b × 0.9x + A_s` × f_s`$
$C = \{0.45×30×250×0.9×240 + 942.5×0.95×460\}×10^{-3}$
$C = (729.0 + 411.87) = 1140.9\ kN$
$T−C = −196.04\ kN$

As shown in Fig.4.21, linearly interpolate between x=220 and 240 to obtain the value of x giving T–C=0.

$x = 220 + (240–220)×(138.0)/(138.0+196.04) = 228\ mm$
$x/d = 228/350 = 0.65 > 0.5$

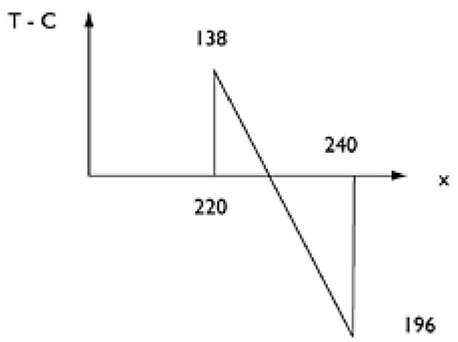


Fig.4.21 Linear interpolation

As a check calculate T and C for x=228 mm

Strain ε_s' in compression steel is given by

$\varepsilon_s^{'}$=0.0035(x−(d`)/x=0.0027>0.0022

Therefore compression steel yields and the stress f_s' is equal to 0.95 f_y

Similarly, strain ε_s in tension steel is given by

ε_s=0.0035(d−x)/x=0.00187<0.0022

Therefore tension steel does not yield and the stress f_s is equal to

$f_s=\varepsilon_s E=0.00187\times200\times10^3=374.6$ N/mm^2
$T=A_s\times f_s=2945.2\times374.6\times10^{-3}=1103.2$ kN
$C=0.45f_{cu}\times b\times 0.9x+A_s^{'}\times f_s' A_s'$
$C=\{0.45\times30\times250\times0.9\times228+942.5\times0.95\times460\}\times10^{-3}$
C=(692.6+411.87)=1104.4 kN
T−C=−1.22kN

This is close enough to be zero.

Taking moments about the tension steel, the lever arm for compression force in concrete is (d–0.45x) and for the compression force in steel it is (d–d').

$M=(692.6\times(350–0.45\times228)+411.87\times(350–50)\}\times10^{-3}=294.9$ kNm

Since x/d>0.5, it is sensible to limit the permissible ultimate moment to a value less than 294.9 kNm. Assuming that x=0.5d=175 mm,

$C_c=0.45f_{cu}b0.9x=\{0.45\times30\times250\times0.9x\ 175\}\times10^{-3}=531.6$ kN
Lever arm z_c=0.775 d=0.775×350=271 mm

Strain ε_s` in compression steel is given by $\varepsilon_s$`=0.0035(x–d')/x=0.0025>0.0022 Therefore compression steel yields and the stress f_s` is equal to 0.95 f_y

$C_s=\{942.5\times0.95\times460\}\times10^{-3}=411.9$ kN, Lever arm z_s=d−d`=300 mm

Taking moments about the steel centroid, $M=C_c z_c+C_s z_s=267.6$ kNm

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