

Chapter

1

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Learning Objectives

- 1 What is an inductive statement and how can the truth of an inductive statement be analyzed?
- 2 What are examples of inductive statements that you deal with every day?
- 3 What is research design and how does it relate to statistical reasoning?
- 4 What is the “Pygmalion effect”? What three characteristics of the experimental/statistical method do experimental explorations of the Pygmalion effect illustrate?
- 5 What is the difference between a population and a sample? Between a parameter and a statistic?
- 6 How do we determine the probability that an event will occur?



An inductive statement is one whose truth or falsehood can be assessed by collecting and analyzing data. Statistics is the best tool available for analyzing data, and mastering statistical analysis is worth the effort involved. We’ll consider studies of the Pygmalion effect, classic experiments that rely on statistical analyses. These studies highlight the distinction between populations and samples. We also discuss the basic notions of probability.

Pygmalion (pig·MAIL·ee·un)
 inductive
 deductive
 placebo (pluh·SEE·bo)
 syllogism (SILL·oh·jizm)
 blind
 population
 sample
 probability of an event
 random
 lectlet (LECT·let)
 SILI (SILL·lee) note

A principal aim of all education is to become skilled in differentiating true statements from false ones. That may seem obvious, but learning about truth is not simple. Part of the difficulty is that *truth* has different meanings in different contexts. Consider these four statements, all of which use the word *true*:

1. The conclusion of this syllogism is true:
 Major premise: All men are mortal.
 Minor premise: Socrates is a man.
 Conclusion: Therefore Socrates is mortal.
2. William Tell’s arrow sped true to its mark and split the apple in two.
3. “For aught that I could ever read,
 Could ever hear by tale or history,
 The course of true love never did run smooth.”¹
4. It is true that one can get across town faster on Elm Street than on Pine Street.

All four statements use the word *true*, but with four different meanings. Philosophers have tried unsuccessfully to determine whether there is a fundamental meaning of *truth* that underlies all four of those examples. Statistics focuses on the kind of truth illustrated in statement 4.

1.1 Inductive Statements

inductive statements
 Statements whose truth can be assessed by collecting and analyzing data

Statement 4 is an example of an *inductive statement*—a statement whose truth is assessed by observing a series of examples, by collecting and analyzing data.

¹ W. Shakespeare, *A Midsummer Night’s Dream*, act 1, sc. 1, line 132.



Inductive statements are evaluated using statistical reasoning.

Inductive statements are frequent:

- I use fluoridated toothpaste because it is said to be more effective than nonfluoridated.
- I choose to fly because I believe air travel is safer than driving.
- The Surgeon General says that smoking cigarettes causes cancer.
- In basketball, the home team has an advantage, but I wonder how big that advantage is.
- I believe that most beer drinkers can't distinguish between Miller and Bud; that is, most "preference" is really the result of advertising hype.
- I think the ozone layer is deteriorating.

Some of these situations are trivial, some have life-or-death implications, some of the opinions are true and some are false, but all are based on (implicit or explicit) evaluations of inductive statements. In fact, most of what we know about the world is the result of inductive processes. Like it or not, aware of it or not, skillful at it or not, we all engage in inductive reasoning almost constantly.



Personal Trainer

Lectlets

REMINDER: "Lectlets" are short audio lectures synchronized to displays that appear on your computer screen. In *Personal Trainer* click **Lectlets**. Then click **1A** for an introduction to the study of statistics. Click **1B** for a discussion of Sections 1.1 through 1.6.



lectlet (LECTlet)

1.2 Statistical Reasoning

Statistics is the best set of tools available for deciding whether inductive statements should be considered "true." Statement 4 and all the inductive statements in the preceding section are best supported or discarded by using statistical procedures. Every time we exercise an inductive process (hundreds of times a day), we use statistical reasoning more or less skillfully. Statistical reasoning is not something foreign to us—we do it all the time, so this book is intended simply to increase your skill at doing what you already do.

Consider the Elm Street/Pine Street trips. I state that the Elm Street route is faster than Pine Street. You doubt my conclusion, so you propose a "race": We will leave at the same time, you will take Pine Street and I will take Elm, and we both agree to observe the speed limits.

Suppose I win this race. Should we conclude that the Elm Street route is in fact faster than Pine Street? "Perhaps," you say, "but not for certain." Maybe I was just lucky with the traffic lights; maybe you were unlucky because that truck blocked traffic while it backed up into the supermarket; maybe it depends on the time of day. You conclude, "We need more races to decide for sure."

When you think like that, you are reasoning like a statistician. You are recognizing that travel time is a "variable" that can take on different values, some longer and some shorter. You are recognizing that many unpredictable things (traffic-lights, trucks) influence that variable, increasing or decreasing the magnitude of the variable (the travel time itself). In the language of statistics, you are recognizing that the travel-time variable is "distributed" (see Chapter 3).



The science of statistics is simply a refinement of techniques that we use every day.

You know already that we need more races; in this book you will learn something about *how many more* races we need to conclude rationally that the Elm Street route is faster than Pine Street.

1.3 Rational Decision Making

So rational inductive decision making requires statistical reasoning. Behavioral science has developed *two* specialties that are part of the rational inductive process: research design and statistical reasoning. Research design is the science of collecting data, making observations about the real world, considering how many observations to make and under what conditions to make them. Statistical reasoning begins with the collected data and prescribes the rules by which rational statements about those data can be made. Research design and statistical reasoning are intertwined, dependent skills; one cannot be a good research designer without being a good statistician, and vice versa. They are, however, usually taught as two separate skills, and this book follows that practice. We will focus on statistical reasoning and not discuss research design here.

Although inductive rationality (and therefore skill in statistical reasoning) is indeed valuable in many situations, it *cannot* claim to be the ultimate form of human truth seeking. Statements 1, 2, and 3 of the four statements near the beginning of the chapter do *not* involve inductive (statistical) reasoning. For example, statistics is of no use in determining whether your own or someone else's love is true, and yet the truth in such a situation may be of vital importance. That determination must be made on some grounds other than statistical. Thus, it seems to me, it is wise to be skillfully rational when the situation calls for it and to be artfully irrational when some other situation calls for that.

Although objective rationality is not necessarily the primary access to ultimate truth, it is our primary access to the truth about the real events in our world, and it is therefore one of the most valued skills we know. That skill requires using statistical reasoning competently, and this book is dedicated toward that end.



Statistics is not about ultimate truth.

1.4 A Classic Example: Pygmalion in the Classroom



Pygmalion (pig-MAIL-ee-un)

You see, really and truly, apart from the things anyone can pick up (the dressing and the proper way of speaking, and so on), the difference between a lady and a flower girl is not how she behaves, but how she's treated. I shall always be a flower girl to Professor Higgins, because he always treats me as a flower girl, and always will; but I know I can be a lady to you, because you always treat me as a lady, and always will.

—Eliza Doolittle in George Bernard Shaw's *Pygmalion*

Pygmalion effect People act in accordance with others' expectations

George Bernard Shaw was an acute observer of human nature, and Eliza Doolittle's important observation has come to be known as the "*Pygmalion effect*": People act in accordance with others' expectations.

However, the Pygmalion effect, though compelling, may or may not in fact exist. To decide whether it exists requires the skillful rationality of the experimental method and its competent statistical analysis. Let's consider an example.

Psychologist Robert Rosenthal and his colleagues explored whether the Pygmalion effect exists in a variety of settings. For example, they told one group of students in an experimental psychology laboratory course the rats they were training were “maze bright”—bred to be extremely quick at learning to run a maze. They told another group of students in the same class that their rats were “maze-dull” (Rosenthal & Fode, 1963). Actually, the rats had been *randomly* assigned to the students and were neither particularly maze-bright nor maze-dull. The rats whose handlers thought they were maze-bright did in fact learn their mazes faster than those whose handlers thought they were maze-dull. The students were not intentionally trying to speed up or slow down their rats' learning; nonetheless, through some apparently unconscious mechanism, they did influence that learning. Thus, the Pygmalion effect applies in the rat lab: If rat handlers expect quick learning, they get quick learning.

Rosenthal is perhaps best known for a study of the Pygmalion effect at “Oak School.” Did teachers' expectations of children affect the performance of those children?² In the spring of 1964, they administered the “Harvard Test of Inflected Acquisition” (“HTIA”) to all the children of Oak School who might return the following fall. They explained to each teacher that the HTIA was a new test that could predict future “academic spurts” in students, that students who scored in the top 20% on the HTIA were likely to “spurt” or “bloom” in the next year. They explained that they were administering this test at Oak School as a final check on the validity of the HTIA, a project sponsored by the National Science Foundation.

That fall, when the students returned to school, Rosenthal and Jacobson gave teachers a list of “the top 20% scorers on the HTIA. . . . Teachers were told only that they might find it of interest to know which of their children were about to bloom. They were also cautioned not to discuss the test findings with their pupils or the children's parents” (p. 70).

This study involved deception: The “top 20% scorers” were *not* really the highest scorers on the HTIA but were actually selected by using a table of random numbers; that is, students were chosen with no information whatever about their performance on the HTIA, their actual ability, or their previous performance in the classroom. (We'll discuss the use of tables of random numbers in Chapter 7.)

Thus teachers were manipulated into thinking that a particular 20% of their students would likely bloom and the other 80% would not. Actually, there was no difference between the bloomers and the others; a totally arbitrary random procedure had assigned the label “bloomer” to some children and the label “other” to the remaining children.

You will recall that the HTIA was administered to all the students as part of the cover story for assigning the label “bloomer.” Actually, the “HTIA” was a standardized test of intelligence called the Tests of General Ability (TOGA) that yielded an IQ score



This is an influential study in psychology. We'll return to it frequently throughout the book.



The bloomers and others were selected at random.

² We have simplified our explanation by presenting only a portion of Rosenthal and Jacobson's study.

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 “Intellectual growth” =
 posttest IQ minus pretest IQ.

i
 Does being labeled a bloomer
 lead to intellectual growth?

i
Remember: The bloomers
 were selected at random.

for each child. The teachers were unaware that the TOGA (aka “HTIA”) was actually an IQ test, and they were not given the children’s actual TOGA IQ scores.

A year later, Rosenthal and Jacobson administered the TOGA to the same children again. They defined “intellectual growth” as the difference between a child’s current (“posttest”) IQ and his or her “pretest” IQ from the original testing (positive scores indicate an increase in IQ). Pretest IQ, posttest IQ, and intellectual growth scores for the bloomers are listed in Table 1.1. The first student identified as a bloomer was Kathy. Her IQ on the original TOGA was 105, her IQ on the TOGA a year later was 125, so her intellectual growth (or IQ gain) was $125 - 105 = 20$ IQ points. A similar table exists for the other children, but it is too long to show here.

Rosenthal’s question was whether there was more intellectual growth in the bloomers than in the other children. Finding the answer requires inductive reasoning: the collection and analysis of data such as those in Table 1.2. That table shows the intellectual growth (IQ gain) scores for both the bloomers and the other children (note that the first column of Table 1.2 is the same as the last column of Table 1.1). The highest intellectual growth score (Mario’s 69) is in the bloomer group, which might lead us to think that being labeled a bloomer *does* improve intellectual growth. However, the next three highest intellectual growth scores (31, 30, and 26) are in the others group, which might lead us to think that *not* being labeled a bloomer improves intellectual growth. Furthermore, we are actually interested in *all* the children, not just the highest or lowest scorers, and there is a lot of overlap between the intellectual growth scores for bloomers and other children. For example, intellectual growth scores of 20, 19, 14, 13, 12, 11, 1, -4 , and -6 occur in both the bloomers and the other children, which again would lead us to conclude that being labeled a bloomer does *not* have a particular advantage for improving intellectual growth.

Inspection of the data, then, does not lead to an obvious answer to the question of whether positive expectations lead to greater intellectual growth. You may feel that the bloomer scores are clearly higher as a group, but someone else might think that the two sets of scores are about the same. Statistics is a set of tools designed to give us

TABLE 1.1 Intellectual growth (IQ gain) of Oak School second-grade bloomers*

Student	Pretest IQ	Posttest IQ	Intellectual Growth (IQ Gain)
Kathy	105	125	$125 - 105 = 20$
Tony	109	123	14
Mario	133	202	69
Louise	101	114	13
Juan	123	117	-6
Able	109	134	25
Patricia	89	90	1
Douglas	111	107	-4
Baker	108	132	24
Charlie	89	101	12
Delta	72	91	19
Echo	75	86	11

* These values can be inferred from statistics provided in Rosenthal and Jacobson (1968, pp. 75, 85–93, 187, 190, and 193). Rosenthal and Jacobson provided names for only seven of these students: I added Able, Baker, Charlie, Delta, and Echo for the missing names.

TABLE 1.2 Intellectual growth (IQ gain) of Oak School second-grade bloomers and other children*

Bloomers	Others			
20	3	-3	-10	-4
14	-2	4	15	-3
69	-15	20	15	6
13	1	8	8	11
-6	31	-6	6	13
25	10	-6	30	18
1	5	6	-5	4
-4	9	10	17	0
24	1	11	7	26
12	12	13	4	-1
19	14	14	19	14
11	-11	7	3	

* These values are from Rosenthal and Jacobson (1968). The Bloomers data are exact (see Table 1.1). The Others data are manufactured to match the classroom means and standard deviations that Rosenthal and Jacobson (p. 193) provided (they did not provide original data).

i
Do bloomers have greater intellectual growth than do the others?

rational ways of deciding between such alternatives rather than relying on your own or someone else’s feelings.

Do these data actually imply that the Pygmalion effect exists in the classroom? We shall see in Chapter 11 that the rational/statistical answer is yes.

In fact, many studies in many different situations have used statistical methods to demonstrate the existence of the Pygmalion effect. As a result, the Pygmalion effect (also called “experimenter bias”) is now an accepted fact, no longer in need of further empirical demonstration. Thus, for example, when researchers attempt to demonstrate the effectiveness of a new drug compared with a *placebo*, we require that experimenter bias be eliminated by keeping the researchers *blind* to which participants receive the actual drug and which participants receive a placebo. Many school districts, as another example, have eliminated “tracking” (where students are divided into groups according to scores on IQ tests) because such groupings may affect teacher bias; some school districts have eliminated the use of IQ tests entirely. Thus, the experimental/statistical examination of the Pygmalion effect has led to important social consequences. Our lives today are in fact substantially altered by yesterday’s application of statistical tests.

This classic example illustrates three characteristics of the experimental/statistical method. First, the inspiration for an experiment is a pre-experimental observation: George Bernard Shaw (and, of course, others) observed what he took to be a characteristic of human nature. Only later did Rosenthal and Jacobson seek to verify experimentally whether that observation was correct.

Second, Rosenthal and Jacobson’s main interest was not in the intellectual growth patterns of the particular 59 students (12 bloomers plus 47 others) who participated in this study, but rather in the intellectual growth patterns of all students. Our statistical tools, then, must distinguish between samples (e.g., Rosenthal’s 59 second-graders) and populations (e.g., all second-graders in the United States). We will begin to discuss the distinction between samples and populations in Section 1.5, and most of the text (Chapters 7–18) will discuss the general question of what can be inferred about populations when all we know is about samples.

≡
placebo (pluh·SEE·bo)

placebo In a drug study, a substance that looks like the drug being tested but actually has no effect

blind Not knowing which participants are assigned to which experimental condition

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One of the most important questions in statistics is What can we say about a population when all we know about is a sample?

The third characteristic of the statistical method is that it must allow us to derive meaningful results from data that are not perfectly consistent—that fluctuate from one person to another or from one occasion to another. The Pygmalion bloomers are a random sample from all the second-graders. We wonder whether our conclusions would be the same if a different random sample were used. Understanding this aspect of statistics requires us to understand some of the concepts of probability, which are discussed in Section 1.6, and the characteristics of random samples, which are described in Chapter 7. Furthermore, our statistical toolbox must include methods to measure the differences between individuals. We will discuss those tools starting in Chapter 3.

Nowadays we take the Pygmalion effect for granted because many experiments have collected data (not only in the rat lab and the classroom) and analyzed them as discussed in this textbook. Statistics is important!

1.5 Samples from Populations

We just saw that one of the main tasks of statistics is to use small samples to infer characteristics about larger populations: Rosenthal and Jacobson used the characteristics of their sample of 59 second-graders from Oak School to infer something about the characteristics of the entire population of millions of second-graders. Let's be clear about the distinction between *populations* and *samples*.

A *population* includes *all* the members of the group under consideration. Populations can be large (such as the residents of the United States, 295 million) or small (such as the residents who live on my street, 33). Populations can be of people (such as piano players), of objects (such as the stars in our galaxy), or of events (such as thunderstorms). What makes a particular group a population is *not* a characteristic of the group itself, but rather a characteristic of *our interest*. If we are interested in a group for its own sake, *not* as being representative of or selected from some other larger group, then we call that group a population.

A *sample* is a subset of a population, a group that is interesting to us *not* on its own merits but because it somehow represents the larger population. For example, if we are interested in the voting patterns of the U.S. electorate and contact by telephone 1200 voters and inquire how they plan to vote, then the 1200 voters we contacted are not interesting to us *for themselves*, but only because they may be representative of the whole electorate. The 1200 voters are a sample from the population of all voters.

The Yankees have 40 players. Is that a population or a sample? It depends on our interest. If the Yankees are our target of interest, then the Yankees are a population. But if professional baseball players are our interest, then the Yankees are a sample that might reveal something about professional baseball players in general.

To be clear about the distinction between populations and samples, we use the term *statistic* to refer to any measurement on a sample and the term *parameter* to refer to any measurement on (or assumption about) a population.

Most often, it is too difficult, too costly, or impossible to measure all the elements of a population, so we select a sample of the population and measure just those elements. Almost always (but not necessarily), samples are much smaller than the parent populations.

population All the members of the group under consideration

sample Some subset of the group under consideration



Our interest determines whether a group is a population or a sample.

statistic Any measurement on a sample

parameter Any measured (or assumed) characteristic of a population



Populations: We are primarily interested in them but (usually) can't measure them.

Samples: We can measure them but are not primarily interested in them.

Much of the science of statistics can be thought of as the procedures for using relatively small samples to infer the characteristics of large populations. Rosenthal and Jacobson, for example, used their small (59 person) sample to draw conclusions about the effect of teacher expectancies on the second-grade population in general. They might have preferred to have measured the intellectual growth of all the millions of second-graders, but that would have been impractical.

1.6 Probability

random Unpredictable given our current knowledge

We have observed that in Rosenthal's data, the amount of intellectual growth differs from student to student. Some extraneous influences that might have affected intellectual growth during the year are help from parents with homework (or lack of it), good or bad parent models, and health or illness during testing. There are actually many such influences—far too many for us to be able to take directly into account. Because we can't measure them directly, we lump all such influences together into what we call "random" influences. *Random* means unpredictable at our current level of understanding. Perhaps if we knew how much homework help students got from their parents, and if we knew how stable their parents' interpersonal patterns were, and if we knew how healthy the students were, and so on, we could reduce the size of this random effect. But there are almost always far too many things to measure, so our data almost always contain random effects.

The science that deals with the nature of randomness is called *probability theory*: thus, probability is a measure of our ignorance or uncertainty about the outcomes of events in the world. Nearly all statistical tests rely on probability concepts. Probability theory is a fascinating study in its own right, and its mastery can easily be the exclusive topic of a textbook. For our present purposes, we need only to understand its most basic elements.

Random procedures have several possible *outcomes* or results. One random procedure is drawing a single card from a shuffled standard deck.³ One possible outcome of this procedure would be "ace of spades," another would be "6 of diamonds," and so on. An *event* is a set of possible outcomes; thus, an event might be defined as "drawing a heart." The event of drawing a heart can be satisfied by any of the 13 outcomes ace of hearts, king of hearts, . . . , 2 of hearts.

If we assume that each outcome is equally likely, then the *probability of an event* E can be obtained from this formula:

$$P(E) = \frac{\text{number of outcomes favorable to } E}{\text{total number of possible outcomes}} \quad (1.1)$$

For example, what is the probability of shuffling a standard deck and then drawing the Jack of diamonds? The event E is drawing the Jack of diamonds. There is only one

probability of an event
The number of outcomes favorable to that event divided by total number of possible outcomes

Probability of an event

³ A standard deck has 52 cards divided into four suits (spades, hearts, diamonds, and clubs). Spades and clubs are black. Hearts and diamonds are red. Each suit has 13 cards (2, 3, 4, 5, 6, 7, 8, 9, 10, jack, queen, king, and ace).

outcome “favorable” to that event—the drawn card is in fact the Jack of diamonds. Therefore the numerator of $P(E)$ is 1. There are 52 total possible outcomes, so the probability $P(E) = 1/52 = .02$.

Another example: What is the probability of shuffling the deck and drawing any heart? The event E is drawing a heart. There are 13 outcomes “favorable” to that event—the drawn card is the 2 of hearts, the 3 of hearts, the 4 of hearts, . . . , the King of hearts, or the Ace of hearts. Therefore the numerator of $P(E)$ is 13. There are still 52 total possible outcomes, so the probability $P(E) = 13/52 = 1/4 = .25$. Similarly:

$$P(\text{drawing the 15 of hearts}) = \frac{0}{52} = 0$$

$$P(\text{drawing a heart or a spade or a diamond or a club})$$

$$= \frac{13 + 13 + 13 + 13}{52} = \frac{52}{52} = 1$$

These examples illustrate three important characteristics of probability:

- $0 \leq P(E) \leq 1$; probabilities lie between 0 and 1.
- The probability of an event that *cannot* occur is 0.
- The probability of an event that *must* occur is 1.

Unusual events have probabilities close to 0; for example, P (drawing the queen of spades) = $1/52 = .02$. Very likely events have probabilities close to 1; for example, P (drawing any card *except* the queen of spades) = $51/52 = .98$. Probabilities are close to .5 for events that occur about half the time; for example, P (drawing a red card) = $(13 + 13)/52 = .5$.

When we determine probabilities, we must count *all* the possible outcomes of an experiment. That may seem obvious, but in practice such counting can be tricky. Consider the rolling of dice, where $P(1)$ is the probability of rolling a 1. A die is called *fair* if $P(1) = P(2) = \dots = P(6) = 1/6$. Now suppose you roll *two* fair dice. What is the probability that the two dice will sum to 4? Answering that question requires that we count the number of ways of rolling a sum of 4 on two dice. It may seem (incorrectly) that there are two such ways: rolling 1 and 3, and rolling 2 and 2. However, the correct answer is three: rolling 1 and 3, rolling 2 and 2, and rolling 3 and 1. Because there are 36 total possible ways of rolling two dice (1–1, 2–1, 3–1, . . . , 6–1; 1–2, 2–2, . . . , 6–2; 1–3, etc.), we see that P (rolling a 4 with two dice) = $3/36 = 1/12$.

If you know the probability of an event, you can determine how often that event is expected to occur by multiplying the probability by the number of occasions. For example, if the probability of drawing a heart is .25 and you perform 200 draws (replacing the card and shuffling each time), you can expect to draw a heart approximately $.25 \times 200 = 50$ times. Exactly how many hearts you actually draw will depend on chance.

Sometimes we put restrictions or *conditions* on the range of possible outcomes of a procedure. If we do so, we call it *conditional probability*. For example, we may ask: What is the probability of drawing a heart *given that* or *on the condition that* the card is known to be red? This condition requires that the card is either a heart or a diamond; therefore the number of possible outcomes (the denominator of the probability

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Hint: Imagine one die is green and the other red. Green 1 and red 3 is different from green 3 and red 1, but both sum to 4.

i
The expected number of outcomes favorable to an event is the probability of the event times the total number of outcomes.

conditional probability The probability of an event given that another event has occurred

formula) is $13 + 13 = 26$. Thus, the conditional probability of drawing a heart on the condition that (or *given that*) the card is red is $P(\text{heart} \mid \text{red card}) = 13/26 = .5$. Note that we symbolize “given that” by a vertical line.


The probabilities that we have been considering so far are “discrete” in the sense that we could explicitly count all the possible outcomes. We can extend the notion of probability to situations where such counting is impossible or impractical. For example, suppose we are about to measure a person’s intelligence quotient (IQ). We might ask: What is $P(\text{her IQ is higher than } 130)$? To answer that question using discrete probability, we would have to know the number of individuals whose IQs are higher than 130 and then divide that number by the total number of individuals. That would be impractical in real life; we will develop methods of approximating this probability in subsequent chapters. When we make a statement such as $P(\text{IQ is higher than } 130) = .02$, we mean that if we were to measure all people’s IQs (which we won’t because it is too impractical), we would find that 2% of all those individuals would have IQs over 130.

For reasons that will become clearer in Chapters 8 and 9, probabilities of .05 and .95 are the most widely mentioned values in statistics, so let’s explore these values explicitly. Suppose we have an urn that contains 1000 identical balls except that 950 are red and 50 are white. We thoroughly mix the balls and then draw out one of them. What is the probability that the ball is white? Equation (1.1) indicates that $P(\text{white}) = 50/1000 = .05$ and $P(\text{red}) = 950/1000 = .95$.

Drawing a white ball out of such an urn is an “unusual” event. It is not impossible, but it won’t happen very often—about 5 times out of every 100 attempts on the average. Drawing a red ball is “usual”; it will happen about 95 times out of every 100 over the long run.

Much of statistics depends on a definition of *unusual* such as this. Statisticians generally take *unusual* to mean “occurring with probability less than .05”; however, in some situations, we may prefer a stricter definition of *unusual*—perhaps a probability of less than .01.

Click **Resources** and then **1A** in the *Personal Trainer* for a true story about how failure to understand basic probability concepts can lead to important, possibly disastrous mistakes.


 .05 is a frequently used probability. .05 indicates, on average, 50 successes out of every 1000 chances.

 Convention:
.05 is “unusual”
.95 is “usual”

 **Personal Trainer**
Resources

1.7 A Note to the Student

 distribution
distribution of means
test statistic

 Worth memorizing—three main concepts:

1. Distribution of a variable
2. Distribution of means
3. Test statistic

I would like to impress upon you at the very beginning that statistics is fundamentally quite simple. In fact, there are basically only three major concepts to be mastered in this text. I’ll state them here, even though the terms might not mean much to you yet: (1) what a “distribution of a variable” is and how to describe it, (2) what a “distribution of means” is and how it is related to the distribution of the variable, and (3) what a “test statistic” is and how it is related to the distribution of means. There are, to be sure, many important details to be learned, but once you grasp the three major concepts, the rest of statistics follows rather straightforwardly. It’s worth memorizing those three concepts now (even if it is mere rote memory for the moment) to begin building the cognitive structure that we will elaborate throughout the remainder of the textbook.

Studying statistics—attempting to acquire statistical skills—is a valuable exercise. It is worthwhile to develop your ability to think rationally about empirical events. But skill acquisition requires work and practice, work and practice. I urge you to look forward positively to the prospect of engaging in this exercise. If I were your basketball coach, I would try to make running laps interesting (by using races, relays, music), but I would require that you run the laps regardless of whether you found them fun. I would try to impress upon you that good basketball players do not avoid calisthenics but instead look upon them as a discipline, a self-challenge.

The same is true about the effort required to learn statistics. As your statistics “coach,” I have gone to great lengths to make the work of learning statistics as interesting, challenging, informative, and rewarding as possible, but you may still find that some “calisthenics” are involved. I urge you not to avoid that work but to use it as a way to strengthen your mental discipline.

This mental discipline is itself worth striving for, as the world’s greatest thinkers have maintained since the beginning of recorded history. For example, Buddha held 2500 years ago:

The mind is wavering and restless, difficult to guard and restrain: let the wise man straighten his mind as a maker of arrows makes his arrows straight.

Let a wise man remove impurities from himself even as a silversmith removes impurities from the silver: one after one, little by little, again and again.⁴

I urge you to recognize that the burn of annoyance when a computation does not work out is a sign of mental undiscipline, not the result of statistical ignorance, and to recognize that such undiscipline can be overcome with consistent practice.

 **Personal Trainer**
Algebra

Click **Algebra** in the *Personal Trainer* for a quick review of concepts in basic algebra. Use the three paths **Inequality, Squares, etc.**, and **Signed Values**.

 **Personal Trainer**
Errata

Nothing is more frustrating than trying to understand a statistics concept only to find that the textbook contains an error. We have worked hard to make this textbook error-free, but errors may still occur. I maintain a website that contains an up-to-the-minute listing of all errors that are reported. Please take a few minutes to correct those errors in your textbook—it may save you time later. In the *Personal Trainer*, click **Errata**. By the way, I and other students would greatly appreciate your reporting any new errors. There is an error report form on the Errata website.

 **Personal Trainer**
ReviewMaster

Click **ReviewMaster** and then **Chapter 1** in the *Personal Trainer* for an electronic interactive review of the concepts in Chapter 1.

 **Personal Trainer**
Labs

Click **Labs** and then **Chapter 1** in the *Personal Trainer* for interactive practice of the skills in Chapter 1 and a quiz to test your understanding.

⁴ *The Dhannapada: The Path of Perfection*, vv. 33 and 239. translated from the Pali by Juan Mascara (New York: Penguin Books, 1973).

CHAPTER 1 EXERCISES

Section A: Basic Exercises

(Answers in Appendix D, page 575)

1. Define inductive statement.

EXERCISE 1 WORKED OUT

(Throughout the textbook, the first exercise of each chapter is worked out for the student.) Here, Exercise 1 calls for a simple definition. Section 1.1 states that an inductive statement is a statement whose truth can be assessed by collecting and analyzing data.

2. Which of the following are inductive statements?
 - (a) More Democrats favor socialized medicine than do Republicans.
 - (b) That poem truly expresses how I feel.
 - (c) Taking vitamin C reduces the frequency of colds.
 - (d) I prefer new music to rap.
3. True or false: Statistical reasoning provides access to the ultimate truth.
4. Name the three major concepts to be mastered in this text.
5. What is the “Pygmalion effect”? What three characteristics of the experimental/statistical method do experimental explorations of the Pygmalion effect illustrate?
6. Define *sample* and *population*.
7. The University of Nevada, Las Vegas (UNLV) has 30,000 students, and I would like to know how much time UNLV students spend doing homework. My statistics class has 40 students in it, and I assume that they are representative of UNLV students in general. I ask students in my statistics class how many hours they spent doing homework during the last week. Is my statistics class a sample or a population? Why?
8. My statistics class has 40 students in it, and I would like to know how much time my students spend doing homework. I ask them how many hours they spent doing homework during the last week. Is my statistics class a sample or a population? Why?
9. A standard deck of cards has 52 cards in four suits: spades, hearts, diamonds, and clubs. Spades and clubs are black; hearts and diamonds are red. Each suit has 13 cards: 2 through 10, Jack, Queen, King, Ace. The “face cards” are the Jack, Queen, and King. Robby Billity shuffles a standard deck and draws one card. Then Robby replaces the card, shuffles again, and draws another card. He repeats this procedure for a total of 1000 draws. About how many times would we expect Robby to draw these cards?
 - (a) The ace of spades
 - (b) The 2 of clubs
 - (c) A face card in spades
 - (d) A face card in any suit
10. (a) In the dice game called craps, the shooter rolls two dice and wins immediately if he rolls either 7 or 11 as the sum of two dice. What is $P(\text{an immediate win})$?
 - (b) In craps, the shooter loses immediately if he “craps out”—that is, rolls 2, 3, or 12 as the sum of two dice. What is $P(\text{crapping out})$?
11. Suppose we wish to select a birthday from the year 2005 “at random.” We take 365 Ping-Pong balls (2005 is not a leap year), put them in a barrel, and mix them thoroughly. Find these probabilities:
 - (a) $P(\text{selecting July 7})$
 - (b) $P(\text{selecting any July day})$
 - (c) $P(\text{selecting any February day})$
 - (d) $P(\text{selecting any Friday})$ [*Hint: The first day of 2005 is a Saturday.*]
 - (e) $P(\text{selecting any Saturday})$
12. If we draw a card from a standard deck, find these probabilities:
 - (a) $P(\text{drawing any king})$
 - (b) $P(\text{drawing any king} \mid \text{the draw is a face card})$
 - (c) $P(\text{drawing any king} \mid \text{the draw is a spade})$

Section B: Supplementary Exercises

13. South of Phoenix is the Ak-Chin Indian community. They own a casino called the Phoenix Ak-Chin Casino (I’m not making this up). Inside are roulette

wheels, circular wheels with 38 indentations equally spaced around the circumference. The indentations are numbered 1 through 36, with the remaining two indentations numbered 0 and 00. Half of the numbers 1 through 36 are colored red, whereas the other half are colored black. The 0 and 00 indentations are green. The wheel is rotated in one direction, and a ball is rolled in the opposite direction until it comes to rest in one of the indentations, which is the winner. Assuming the Ak-Chin wheels and balls are fair, find these values:

- (a) $P(\text{spinning black})$
 - (b) $P(\text{spinning red})$
 - (c) $P(\text{spinning green})$
 - (d) $P(\text{spinning "1"})$
 - (e) $P(\text{spinning "23"})$
 - (f) $P(\text{"3"} \mid \text{the outcome is odd})$
 - (g) $P(\text{"3"} \mid \text{the outcome is even})$
 - (h) $P(\text{"0"} \mid \text{the outcome is green})$
14. In roulette, what is the probability of spinning "3" on both of the next two spins?
 15. Suppose you approach a roulette table where the winner was just "3." What is the probability that the next winner will also be "3"?
 16. The players win when the spin is either red or black; the house wins when the spin is green. Out of every 1000 spins, about how often does the house win?