Chapter 1
Mathematical Modeling

Mathematical Overview

This chapter opens the door for students to better understand the use of mathematical modeling when presented with a situation or problem to solve. They begin by examining a presentation of different forms of mathematical models and proceed to using ratios and proportions to create a model for estimating animal populations. Then students explore how proportions can be used to model a variety of other real-world situations. In the last lesson of this chapter, students examine the use of words, graphs, and tables as models to relate one real-world quantity to another. Understanding which model best describes a situation, looking closely at that model, discovering patterns in the model, and describing the patterns mathematically are steps students will use throughout this book to solve problems.

Lesson Summaries

Lesson 1.1 ***Activity: Animal Populations***

In this Activity, students use beans to represent a wild horse population. Each bean represents one wild horse in a population to be estimated. During the Activity, students simulate a technique known as *capture-recapture* to estimate how many horses there are without actually counting each horse. Students write ratios, set up proportions using a variable to represent the total number in the horse population, and then solve for the variable.

Lesson 1.2 ***Proportions as Models***

In this lesson, students extend their knowledge of proportions by representing and solving a variety of real-world situations. The first real-world situation they examine is the cost of driving a given distance when the cost per mile is constant. Then students examine using a scale from a scale drawing to find the actual width of a room. A third situation draws on students' recall of geometry. Students use corresponding sides of similar polygons to calculate the scale factor, set up a proportion, and then solve for an unknown length for one of the polygons.

Lesson 1.3 ***R.A.P.***

In this lesson, students **R**eview **A**nd **P**ractice solving problems that require the use of skills and concepts taught in previous math levels. The skills reviewed in this lesson are skills that are needed as a basis for solving problems throughout this course.

Lesson 1.4 ***Investigation: Patterns and Explanations***

In this Investigation, students are given a situation and asked to choose a graph or a table that best models the relationship between the variables in the situation. They discuss the features of several qualitative graphs and identify the two variables in each situation. They also examine patterns in graphs and tables to better understand how to use mathematics to describe the relationship between the variables in a given situation.

Lesson Guide

Lesson/Objectives	Materials	
Chapter 1 Opener: What Is a Mathematical Model? • recognize that many different representations can be used to model real-world situations.		
1.1 Activity: Animal Populations • use ratios and proportions to create mathematical models. • use mathematical models to estimate the sizes of populations. • solve proportions.	*Per group:* • white beans or other small objects that can be marked with a marker (about 150) • small paper bag or other container for holding the beans • permanent marker	*Optional:* • TRM table shell for Question 5.
1.2 Proportions as Models • use proportions to model real-world situations. • solve problems that involve scale drawings. • solve problems that involve similar polygons.		*Optional:* • TRM shell for the Vocabulary Organizer • an item that shows a scale (map, blueprint, model car, etc.)
1.3 R.A.P. (Review and Practice) • solve problems that require previously learned concepts and skills.		
1.4 Investigation: Patterns and Explanations • use multiple representations to model real-world situations.		

Pacing Guide

	Day 1	Day 2	Day 3	Day 4	Day 5	Day 6
Basic	p. 2, 1.1	1.2	1.3	1.4	project	review
Standard	p. 2, 1.1	1.2	1.3	1.4	project	review
Block	p. 2, 1.1, 1.2	1.3, 1.4	project, review			

Supplement Support

See the Book Companion Website at www.highschool.bfwpub.com/ModelingwithMathematics and the Teacher's Resource Materials (TRM) for additional resources.

CHAPTER 1

Mathematical Modeling

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CHAPTER 1 OPENER

5e Engage

Lesson Objective

- recognize that many different representations can be used to model real-world situations.

Vocabulary

- mathematical modeling
- mathematical models

Description

This chapter sets the stage for the entire course. The chapter is purposely short, yet it shows most of the different types of lessons: Activities, Investigations, R.A.P. lessons, Modeling Projects, and Chapter Reviews. The mathematical skills reviewed in this chapter are necessary for student success in future chapters.

This reading introduces students to the process of mathematical modeling and the different forms that mathematical models can take.

TEACHING TIP

After students have read the Chapter Opener and examined the examples of the different types of representations discussed in the reading, lead a whole-class discussion asking students to give other examples of each type of model.

What Is a Mathematical Model?

The process of starting with a situation or problem and gaining understanding about the situation through the use of mathematics is known as **mathematical modeling**. The mathematical descriptions obtained in the process are called **mathematical models**. These models are often built to explain why things happen in a certain way. They are also created to make predictions about the future.

Mathematical models can take many different forms. Among them are:

verbal descriptions

"The U.S. Postal Service accepts a package for mailing only if the sum of its length and girth is not more than 108 inches."

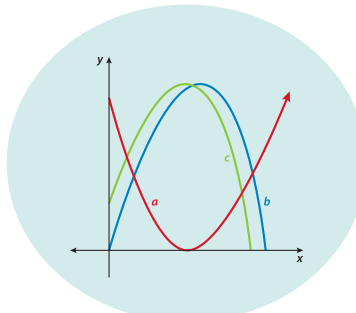
tables of information

Time (minutes)	Temperature (°F)
0	40
5	48
10	54
15	59
20	62

equations

$h = 2 + 1.5n$

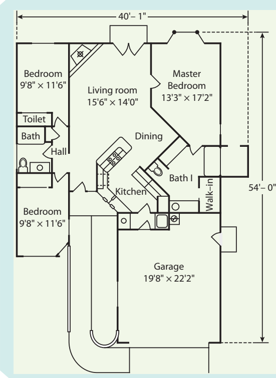
graphs



formulas

$A = \pi r^2$

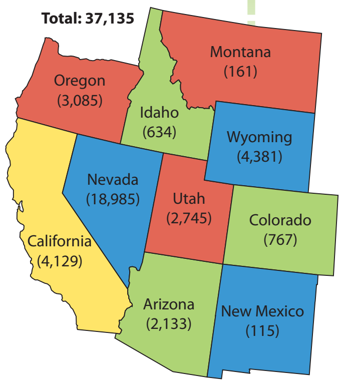
drawings and diagrams



physical models

A good mathematical model is one that helps you better understand the situation under investigation.

Lesson 1.1 ACTIVITY: Animal Populations



Recall that a **ratio** is a comparison of two numbers by division. A ratio can be written in the form of a fraction. In this lesson, you will use ratios to create a model for estimating animal populations.

In 1900, there were about 2 million *mustangs* (wild horses) in the western United States. By 1950, there were fewer than 100,000, and at the start of the 21st century, only about 37,000 remain.

It would be almost impossible to actually count the number of horses in a given region. Instead, a technique called *capture-recapture* (or *mark-recapture*) is used.

In this process, a number of animals are captured and marked in some way. Horses are often branded on an easily-seen part of the body. Then the marked animals are released. After they have had time to mix in with the rest of the animals in a region, a second group is captured. Finding the number of marked horses in this group makes it possible to make an estimate of the entire population.

In this Investigation, you will use beans or other small objects in a container to represent a wild horse population.

1. Each bean in your container represents one horse in a population to be estimated. Scoop out some of the beans and count them. How many beans did you scoop out of your container?
2. Let the beans that you scooped out represent the horses that will be marked. To simulate marking horses, mark each bean that was removed from the container with a permanent marker. Then put the marked beans back in the container with the "uncaptured" beans. Mix the beans in the container well. This is equivalent to letting marked horses mix in with the population of unmarked horses. How many marked beans did you put back in your container?
3. After the beans have been thoroughly mixed, scoop out a second group of beans and count the number in this group. How many did you scoop out?
4. How many of these beans are marked beans that have been "recaptured?"

Lesson 1.1 Activity Answers

1. Sample answer: 37
2. Sample answer: 37, the same number that were taken out
3. Sample answer: 42
4. Sample answer: 5

CONNECTION

Methods similar to the *capture-recapture* method used in this lesson are used to estimate the population of homeless people in large cities. A known quantity of people acting as decoys is planted in the street population. Then the number of decoys later spotted during a search for homeless people is recorded.

5e Engage

Lesson Objectives

- use ratios and proportions to create mathematical models.
- use mathematical models to estimate the sizes of populations.
- solve proportions.

Vocabulary

- proportion
- ratio
- variable

Materials List

Per group:

- white beans or other small objects that can be marked with a marker (about 150)
- small paper bag or other container for holding the beans
- permanent marker

Description

Preparation:

This lesson is designed as a whole class/small group activity (2–4 students). Prior to class, place an unknown number of white beans in each bag (one bag of about 120–150 beans per group).

During the Activity:

Give each group one bag of beans and point out that the number of beans is unknown. Explain that the object is to determine the number of beans in the bag without counting them.

As students remove some of their beans, have them count and mark them with a permanent marker. Once done, they should place all of the marked beans back into the bag.

Closing the Activity:

Remind students that this procedure is used when you are physically unable to count an unknown population. Reinforce the modeling aspect of this Activity by asking students to explain why they think this procedure provides them with a reasonable estimate of the number of beans in their bag.

5. Sample answer:

First Captured Group	
Number captured and marked	37
Total population size	Unknown (p)
Second Captured Group	
Number that were marked	5
Number captured	42

6. Sample answer: $\frac{5}{42}$

7. Sample answer: $\frac{5}{42} = \frac{37}{p}$

8. Sample answer: $p = 310.8$ or about 311 beans

9. Sample answer: First estimate: 42 beans, 5 marked; estimated population of 311. Second estimate: 40 beans, 6 marked; estimated population of 247. The estimates vary by 64 beans.

5. Complete the table to summarize your findings so far.

First Captured Group	
Number captured and marked	
Total population size	Unknown (p)
Second Captured Group	
Number that were marked	
Number captured	

6. What is the ratio of the number of marked (recaptured) beans to the total number of beans in the second captured group?

7. If the marked beans were well mixed with the unmarked beans, any captured group should contain about the same $\frac{\text{marked beans}}{\text{total beans captured}}$ ratio as the entire population.

A statement that two ratios are equal is called a **proportion**.

Complete the proportion below by comparing the ratio of marked beans to total beans captured for the second captured group and the ratio of marked beans to total beans for the whole population.

$$\frac{\text{marked beans (in second captured group)}}{\text{total beans captured (in second captured group)}} = \frac{\text{marked beans (in whole population)}}{\text{total beans (in whole population)}}$$

$$\frac{?}{?} = \frac{?}{p}$$

Recall

To solve proportions, use cross products and Properties of Equality. For example,

Original equation	$\frac{x}{8} = \frac{3}{4}$
Find the cross products.	$4(x) = 3(8)$
Simplify.	$4x = 24$
Divide each side by 4.	$\frac{4x}{4} = \frac{24}{4}$
Simplify.	$x = 6$

When values of quantities are unknown, **variables** can be used to represent their values. Notice that the variable p is used to represent the total number of beans in the whole population because that number is unknown.

8. To estimate the total number of beans in the container, solve your proportion.

9. Repeat Questions 3–8 to find a second estimate of the bean population. Is the result similar to your first estimate?

Practice for Lesson 1.1

Solve each proportion. If necessary, round any decimal answers to the nearest tenth.

1. $\frac{15}{y} = \frac{5}{6}$

2. $\frac{c}{12} = \frac{2}{7}$

3. $\frac{10}{2.8} = \frac{a}{4.2}$

4. $\frac{3}{4} = \frac{7}{n}$

5. $\frac{x}{2} = \frac{15}{6}$

6. $\frac{7.1}{3} = \frac{t}{2}$

7. Suppose a similar *capture-recapture* procedure is used to find the number of horses in a large grassland. Twenty horses are captured and marked. Then they are released into the grassland. After a week, 80 horses are captured. Five of those horses are found to be marked.

a. Write a proportion that models this situation.

b. Use your proportion to estimate the population of horses in this region.

Practice for Lesson 1.1 Answers

1. 18

2. $\frac{24}{7}$ or $3\frac{3}{7}$

3. 15

4. $9\frac{1}{3}$

5. 5

6. 4.7

7a. $\frac{5}{80} = \frac{20}{p}$

7b. 320 horses

COMMON ERROR

Exercise 7 If students incorrectly write the order of the quantities in their proportion, suggest that they state the units aloud. For example,

$$\frac{\text{marked in captured group}}{\text{total number in captured group}} = \frac{\text{marked in total population}}{\text{total number in total population}}$$

LESSON 1.2

5e Explain

Lesson Objectives

- use proportions to model real-world situations.
- solve problems that involve scale drawings.
- solve problems that involve similar polygons.

Vocabulary

- congruent
- polygon
- scale
- scale factor
- sides
- similar figures
- similar polygons
- vertex

Description

In this lesson students explore writing proportions to solve problems. Special attention is given to writing and solving proportions for scale models, drawings, and maps. Solving for an unknown side in similar polygons is also investigated.

TEACHING TIP

Guide students as they work through each of the three examples. Use the following additional examples as extra in-class practice.

ADDITIONAL EXAMPLE 1

Walking at a fast pace burns 5.6 Calories per minute. How many minutes of walking at a fast pace are needed to burn the 500 Calories consumed by eating a dish of ice cream? about 89 minutes

Lesson 1.2 Proportions as Models

As you saw in the previous lesson, proportions can be used as mathematical models to help estimate animal populations. In this lesson, you will explore how proportions can be used to model a variety of other real-world situations.

WRITING AND SOLVING PROPORTIONS

When you write a proportion to represent a given situation, be sure that the quantities in each ratio are written in the same order. For example, you know that there are 12 inches in 1 foot and there are 36 inches in 3 feet. You can write a proportion to model how these quantities are related.

$$\begin{matrix}\text{inches} \rightarrow \\ \text{feet} \rightarrow\end{matrix} \quad \frac{12 \text{ inches}}{1 \text{ foot}} = \frac{36 \text{ inches}}{3 \text{ feet}} \quad \begin{matrix}\leftarrow \text{inches} \\ \leftarrow \text{feet}\end{matrix}$$

Notice that because the ratio on the left is expressed as "inches to feet," the ratio on the right must also be expressed as "inches to feet."

EXAMPLE 1

According to the American Automobile Association (AAA), the overall cost of owning and operating a passenger vehicle averages $7,834 based on 15,000 miles of driving. If the cost per mile is constant, about what would it cost to drive 12,000 miles?

Solution:

Let c represent the cost of driving 12,000 miles.

Write a proportion for the problem.

$$\begin{matrix}\text{average cost} \rightarrow \\ \text{number of miles} \rightarrow\end{matrix} \quad \frac{\$7{,}834}{15{,}000} = \frac{c}{12{,}000} \quad \begin{matrix}\leftarrow \text{average cost} \\ \leftarrow \text{number of miles}\end{matrix}$$

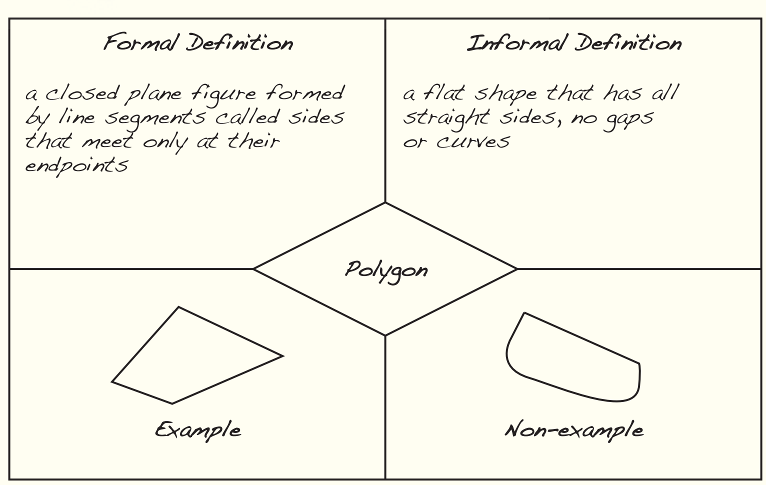
Solve for c.

Original equation	$\frac{7{,}834}{15{,}000} = \frac{c}{12{,}000}$
Find the cross products.	$15{,}000c = (7{,}834)(12{,}000)$
Simplify.	$15{,}000c = 94{,}008{,}000$
Divide each side by 15,000.	$\frac{15{,}000c}{15{,}000} = \frac{94{,}008{,}000}{15{,}000}$
Simplify.	$c = 6{,}267.20$

So, the average cost of driving 12,000 miles is about $6,267.

TEACHING TIP

Vocabulary organizers, such as the one below, are particularly helpful for this chapter.

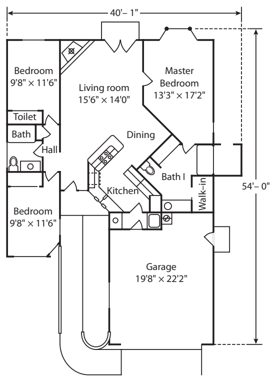


SCALE DRAWINGS

Scale drawings are used in many types of design work to accurately model the shapes of objects. A **scale** is a ratio that compares the size of a model to the actual size of an object. Scales are often found on drawings, maps, and models.

EXAMPLE 2

A typical scale for a house plan is $\frac{1}{4}$ inch to 1 foot. If the width of a room on such a plan measures $3\frac{1}{2}$ inches, what is the actual width of the room?



Solution:

Let w represent the actual width of the room.

Write a proportion to model the situation.

$$\begin{array}{lcc} \text{drawing (in.)} \rightarrow & \dfrac{\frac{1}{4}}{1} = \dfrac{3\frac{1}{2}}{w} & \leftarrow \text{drawing (in.)} \\ \text{actual room (ft)} \rightarrow & & \leftarrow \text{actual room (ft)} \end{array}$$

Solve for w.

Original equation	$\dfrac{\frac{1}{4}}{1} = \dfrac{3\frac{1}{2}}{w}$
Find the cross products.	$\frac{1}{4}w = 3\frac{1}{2}(1)$
Multiply each side by 4.	$(4)\frac{1}{4}w = (4)\left(3\frac{1}{2}\right)$
Simplify.	$w = 14$

So, the width of the room is 14 feet.

Recall

A **polygon** is a closed plane figure formed by line segments called **sides** that meet only at their endpoints. Each point where the sides meet is called a **vertex**.

SIMILAR POLYGONS

Two figures that have the same shape, but not necessarily the same size, are said to be **similar.**

Two polygons are **similar polygons** if their corresponding angles are equal in measure and the lengths of their corresponding sides are proportional.

ADDITIONAL EXAMPLE 2

A plan for an office building uses a scale of $\frac{1}{16}$ inch to 1 foot. How long would a 35-foot wall appear on the plan? $2\frac{3}{16}$ in.

TEACHING TIP

When a scale is written as a ratio, it usually takes this form:

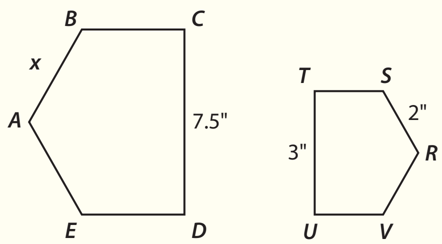
$$\text{scale} = \frac{\text{dimensions of model}}{\text{dimensions of actual object}}$$

CONNECTION

Bring to class an item that shows a scale (map, blueprint, model car, etc.) or suggest that students share items they might have.

ADDITIONAL EXAMPLE 3

Given: $ABCDE \sim RSTUV$. Find the value of x. 5 in.



Recall

Congruent figures have the same size and shape.

It is also the case that if two polygons are similar, then you know that the corresponding angles are congruent and the corresponding sides are proportional.

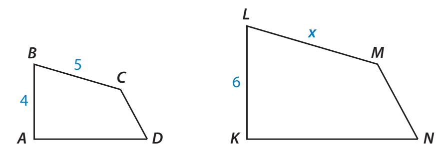
If two polygons are similar, the ratio of the lengths of two corresponding sides is called the **scale factor**.

Recall

When similar polygons are named, the corresponding vertices are listed in the same order.

EXAMPLE 3

Given: $ABCD \sim KLMN$



a. What is the scale factor of $ABCD$ to $KLMN$?

b. Find the value of x.

Solution:

a. $\overline{AB}$ and $\overline{KL}$ are corresponding sides of the two quadrilaterals. So, the scale factor is $\frac{AB}{KL} = \frac{4}{6} = \frac{2}{3}$.

b. Since the polygons are similar, you know the following:

$\angle A \cong \angle K$, $\angle B \cong \angle L$, $\angle C \cong \angle M$, and $\angle D \cong \angle N$.

Also, $\frac{AB}{KL} = \frac{BC}{LM} = \frac{CD}{MN} = \frac{DA}{NK}$.

To find the value of x, write a proportion and solve.

Corresponding sides of similar polygons are proportional.	$\frac{AB}{KL} = \frac{BC}{LM}$
$AB = 4, BC = 5, KL = 6, LM = x$	$\frac{4}{6} = \frac{5}{x}$
Find the cross products.	$4x = (5)(6)$
Simplify.	$4x = 30$
Divide each side by 4.	$\frac{4x}{4} = \frac{30}{4}$
Simplify.	$x = 7.5$

Practice for Lesson 1.2

For Exercises 1–3, choose the correct answer.

1. Which proportion *cannot* be used to solve the following problem?

How many milligrams (mg) of medication should you give to a 120-pound person if you should give 50 mg for every 10 pounds?

A. $\frac{50\text{ mg}}{10\text{ lb}} = \frac{x}{120\text{ lb}}$ **B.** $\frac{10\text{ lb}}{50\text{ mg}} = \frac{120\text{ lb}}{x}$

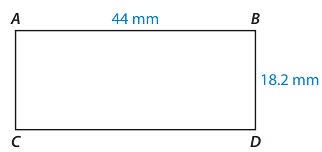
C. $\frac{50\text{ mg}}{x} = \frac{120\text{ lb}}{10\text{ lb}}$ **D.** $\frac{10\text{ lb}}{120\text{ lb}} = \frac{50\text{ mg}}{x}$

2. Triangles *ABC* and *XYZ* are similar. Which statement is *not* true?

A. $\frac{AB}{XY} = \frac{BC}{YZ}$ **B.** $\frac{XZ}{AC} = \frac{YZ}{BC}$

C. $\frac{CB}{ZY} = \frac{AC}{XZ}$ **D.** $\frac{XY}{AB} = \frac{ZY}{CA}$

3. *ABCD* is a rectangle.



Which set of dimensions produces a rectangle that is similar to rectangle *ABCD*?

A. 36.4 mm, 11 mm **B.** 44 mm, 9.1 mm
C. 176 mm, 72.8 mm **D.** 91 mm, 66 mm

4. If your new car goes 320 miles on 10 gallons of gas, how far will it go on 6 gallons of gas?

5. The Tannery Mall in Massachusetts is partially powered by an array of 375 solar panels. They produce 60 kilowatts of electrical power. How many panels would be needed to produce 84 kilowatts of power?

Practice for Lesson 1.2 Answers

1. C
2. D
3. C
4. 192 miles
5. 525 panels

TEACHING TIP

Exercises 8 and 9 The scale of a drawing can be written in more than one way. For example, if 1 inch on a drawing represents an actual length of 12 feet, the scale can be written as

1 in. : 12 ft, $\frac{1 \text{ in.}}{12 \text{ ft}}$, or 1 in. = 12 ft.

6. 87.5 acres
7. 31.5 inches
8. 18 miles
9. 1.25 inches
10a. $\frac{3}{1.5}$ or $\frac{2}{1}$
10b. 2 cm
11. 1.2 m

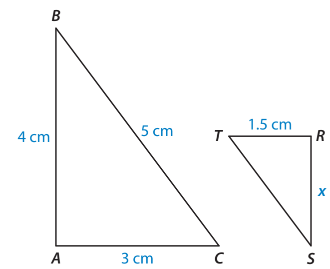
6. An airplane sprays 16 gallons of liquid fertilizer on 5 acres of crops. If the plane's tank can hold 280 gallons, how many acres of crops can be sprayed?

7. Most conventional TV screens have a width : height ratio of 4 : 3. If a screen has a width of 42 inches, what is its height?

8. The scale on a map is 1 inch : 6 miles. Find the actual length of a road if it is 3 inches long on the map.

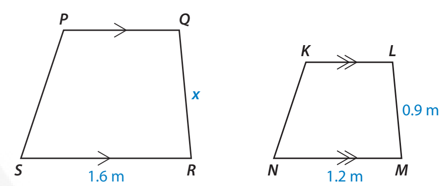
9. A drawing's scale is 0.5 inch : 20 feet. If a banquet room's length is 50 feet, what is the length of the room in the drawing?

10. Given: $\triangle ABC \sim \triangle RST$



a. What is the scale factor of triangle *ABC* to triangle *RST*?
b. Find the value of *x*.

11. Trapezoid *PQRS* is similar to trapezoid *KLMN*. Find the value of *x*.



12. The ratio of the corresponding sides of two similar rectangles is 4 : 9. The length of the smaller rectangle is 16 cm and its width is 12 cm. What is the perimeter of the larger rectangle?

13. Suppose that a, b, c, and d represent four numbers that form the proportion $\frac{a}{b} = \frac{c}{d}$. If a is doubled while b remains the same, how would c or d have to change for the proportion to stay true?

12. 126 cm

13. Sample answer: c could be doubled; d could be divided by 2.

LESSON 1.3

5e Evaluate

Lesson Objective

- solve problems that require previously learned concepts and skills.

Exercise Reference

Exercises 1–3: Lesson 1.1
Exercise 4: Appendix J
Exercises 5–8: Appendix A
Exercises 9–12: Appendix H
Exercises 13–14: Appendix G
Exercises 15–20: Appendix I
Exercises 21–22: Appendix J
Exercises 23–25: Appendix L
Exercises 26–29: Lesson 1.1
Exercise 30: Appendix C
Exercise 31: Lesson 1.1
Exercises 32–33: Lesson 1.2

Lesson 1.3 R.A.P.

Fill in the blank.

1. A comparison of two numbers by division is called a(n) ________.
2. A statement that two ratios are equal is called a(n) ___________.

Choose the correct answer.

3. Which ratio is equivalent to $\frac{3}{4}$?

   A. $\frac{4}{3}$ **B.** $\frac{6}{8}$ **C.** $\frac{6}{4}$ **D.** $\frac{3}{8}$

4. Which expression is equivalent to $3(5x - 6)$?

   A. $15x - 6$ **B.** $18x - 15$ **C.** $15x + 30$ **D.** $15x - 18$

Add or subtract. Write your answer in simplest form.

5. $3\frac{1}{2} + 2\frac{7}{8}$
6. $\frac{3}{5} + \frac{7}{8}$
7. $6\frac{1}{8} - 2\frac{1}{4}$
8. $\frac{1}{2} - \frac{4}{9}$

Evaluate the expression.

9. $10 + 8 \div 4$
10. $(2 + 18) \div 5 + 12$
11. $(8 - 3)^2 - 100 \div 4$
12. $71 - 24 + (-3)$
13. $|-8| + 2$
14. $-|-2|$

Add, subtract, multiply, or divide.

15. $17 + (-23)$
16. $-5 - (-17)$
17. $14(-8)$
18. $-18 \div (-6)$
19. $5 + (-3) - 18$
20. $4(-6) + 2(8)$

Identify the property illustrated in each equation.

21. $5(3 + x) = 15 + 5x$
22. $11 + 0 = 11$

Solve.

23. $2x + 7 = 23$
24. $\frac{2n}{3} = 48$
25. $18 = 5t - 32$

Lesson 1.3 R.A.P. Answers

1. ratio
2. proportion
3. B
4. D
5. $6\frac{3}{8}$
6. $1\frac{19}{40}$
7. $3\frac{7}{8}$
8. $\frac{1}{18}$
9. 12
10. 16
11. 0
12. 44
13. 10
14. -2
15. -6
16. 12
17. -112
18. 3
19. -16
20. -8
21. Distributive Property
22. Identity Property of Addition
23. 8
24. 72
25. 10

Solve the proportion.

26. $\frac{6}{n} = \frac{18}{33}$

27. $\frac{20}{8} = \frac{x}{16}$

28. $\frac{a}{7.2} = \frac{1.8}{5.4}$

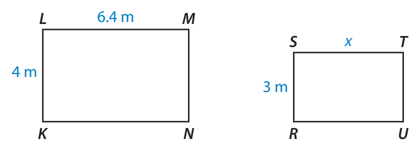
29. $\frac{6}{4} = \frac{8}{y}$

30. There are 8 males and 14 females in the school choir.
- **a.** Write the ratio of the number of males to the number of females.
- **b.** Write the ratio of the number of females to the total number of students in the choir.

31. Animal biologists wanted to estimate the deer population in a large wildlife area. Initially, 15 deer were captured and tagged. Then the tagged deer were returned to the area. After a week, 60 deer were observed by the biologist. Four of those deer were found to be tagged. About how many deer were in the region?

32. If 50 milliliters of water are used for 100 grams of plaster to make a dental model, how much water should be used for 150 grams of plaster?

33. Rectangle *KLMN* is similar to rectangle *RSTU*. Find the value of x.



26. 11
27. 40
28. 2.4
29. $5\frac{1}{3}$
30a. $\frac{8}{14} = \frac{4}{7}$
30b. $\frac{14}{22} = \frac{7}{11}$
31. 225 deer
32. 75 ml
33. 4.8 m

LESSON 1.4

5e Explore

Lesson Objective

- use multiple representations to model real-world situations.

Vocabulary

none

Materials List

none

Description

This lesson is designed as a whole class/small group investigation (2–4 students). Have students read the information about finding patterns. Then, as a class, discuss the graph in the Example. Review the parts of graphs as needed so that students will recognize that the scale on the vertical axis is missing. Also discuss the two variables shown on the graph. Point out the pattern and other features of the graph that provide the information needed to answer the question.

In groups, have students read through the three questions that they need to answer for each of the four situations on page 15.

Once all groups have completed the Investigation, discuss their results for **Questions 1–4** by talking about why they chose a particular graph and discounted others. Pay particular attention to the identification of the two variables in each situation. Ask how they know which variable is on the horizontal axis and which is on the vertical axis.

Wrapping Up the Investigation:

Point out that they have seen two different types of models in this lesson, graphs and tables. Ask students why they think that graphs are often used as models. Reinforce that graphs are used to provide visual models of situations and that patterns often become apparent when information is shown in a graph or table.

Lesson 1.4 **INVESTIGATION: Patterns and Explanations**

When a mathematical model and a real-world situation are well matched, the information obtained from the model is meaningful in the real-world situation. In this lesson, you will explore several situations and the graphs and tables that model them.

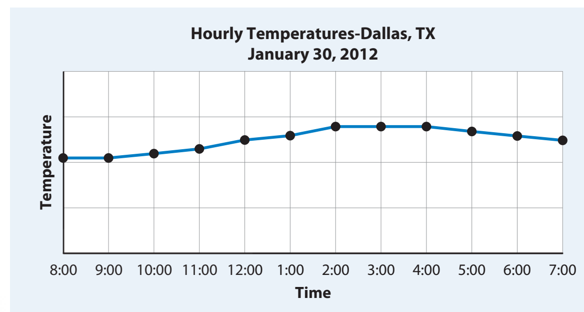
FINDING PATTERNS

When developing a model, modelers often look for patterns in the real world. Frequently these patterns involve numbers. Describing these patterns mathematically helps produce useful information.

Among the simplest patterns are those that relate one real-world quantity to another. Sometimes these patterns are more obvious if they are shown on a graph.

EXAMPLE

Does the line graph below show hourly daytime temperatures (8:00 a.m. – 7:00 p.m.) or hourly nighttime temperatures (8:00 p.m. – 7:00 a.m.)? Explain.



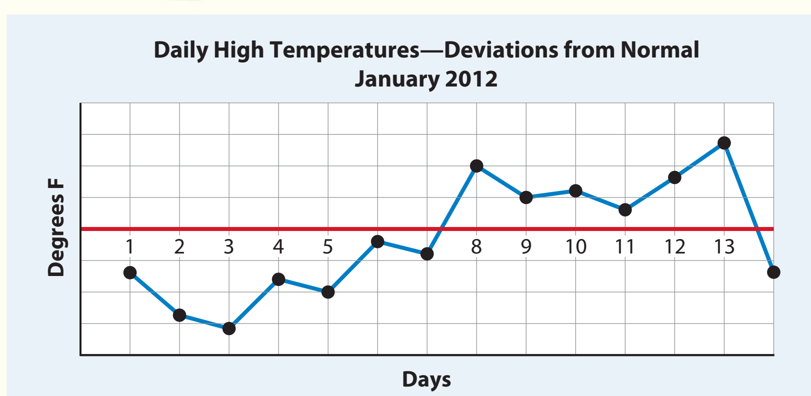
Solution:

Even though the graph does not give you the exact temperatures, the pattern of the graph is apparent at a glance. The graph shows that the temperatures rise, and then fall. Since daytime temperatures usually increase around midday, and are followed by a drop in temperature in the evening, it is likely that this graph shows daytime temperatures (8:00 a.m. – 7:00 p.m.).

14 Chapter 1 MATHEMATICAL MODELING

ADDITIONAL EXAMPLE

Describe the situation that might be modeled by the line graph.

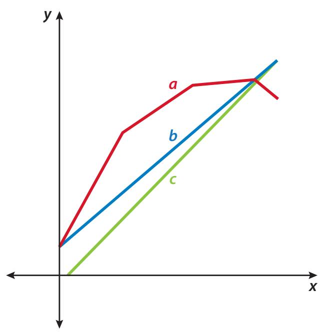


A sample answer for the Additional Example can be found on page 15.

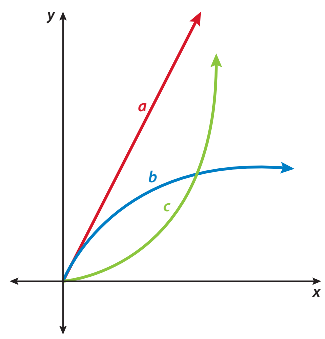
For Questions 1–4, a context and a figure showing three graphs are given. After discussing the context with a partner or group, answer the following questions:

- **i.** Which graph, *a*, *b*, or *c*, best models the given situation?
- **ii.** What features made you choose that particular graph? What features made you discount the other graphs?
- **iii.** What are the two quantities or variables in the given situation?

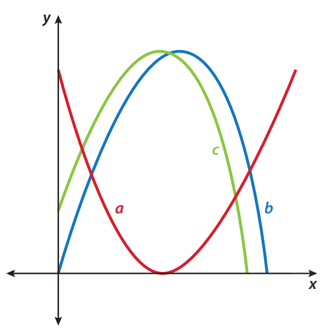
1. the height of a person over his or her lifetime



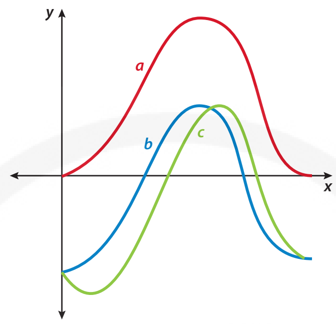
2. the circumference of a circle as its radius changes



3. the height of a ball as it is thrown in the air



4. the daily average low temperature in degrees Fahrenheit over the course of one year in Fairbanks, Alaska



Lesson 1.4 Investigation Answers

1 i. Graph *a*
ii. Sample answer: A person's growth slows with age and might even decrease as shown in graph *a*.
iii. height, time, or age

2 i. Graph *a*
ii. Sample answer: The circumference of a circle increases at a constant rate as the radius increases.
iii. radius, circumference

TEACHING TIP

Since time is not specifically mentioned in **Question 3**, students may have difficulty determining that time is the second variable. If that is the case, toss an object into the air and have the student describe in words what happened.

3 i. Graph *c*
ii. Sample answer: The ball is thrown upward, so graph *a* is not correct, and it is thrown from a location above the ground, so graph *b* is not correct.
iii. height, time

4 i. Graph *c*
ii. Sample answer: Temperatures in January in Fairbanks will probably be below zero, so graph *a* is not correct. It is unlikely that all January and February temperatures will be warmer than the temperature on January 1, so graph *b* is not correct.
iii. time, temperature

ADDITIONAL EXAMPLE

Sample answer: The temperatures during the first week in January were below normal. During the second week of the month, temperatures were above normal until the last day of the week. The high temperature on Day 6 was the closest to normal. Overall, temperatures for about half the days were below normal and half were above normal.

5. Sample answer: Table 1 best describes the growth of a plant. As time increases, the height of the plant should increase, not decrease as shown in Table 2.

Practice for Lesson 1.4 Answers

1. Sample answers: Important features include: when the graph is increasing and when it is decreasing, whether the graph goes through the origin, and whether it ever has negative values.

2. Sample answers: In Question 1, people eventually die; in Question 3, the ball hits the ground and no longer moves; and in Question 4, a specific time frame was given in the situation.

3. In Question 4, temperature can be negative.

4a. Graph *c*

TEACHING TIP

Exercise 4 If students have difficulty determining which graph best represents the growing mold, have them examine each graph individually and discuss what that graph would indicate. For example, Graph *b* would indicate that the mold grew very slowly for a while. Then the amount of mold would decrease slowly and then more rapidly.

5. Which of the tables below better models the height of a kudzu plant over time? Explain.

Table 1

Time (days)	Height (cm)
0	0
4	3
8	6
12	9
16	12

Table 2

Time (days)	Height (cm)
0	12
4	9
8	6
12	3
16	0

Practice for Lesson 1.4

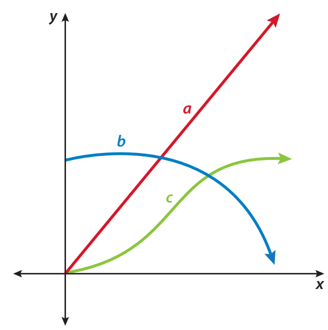
1. Examine the graphs in Questions 1–4 of the Investigation. List some of the important features of the graphs that helped you choose the one that best modeled the given situation.

2. In Question 2, arrows were drawn on the ends of the graphs to show that the graphs continue indefinitely. Explain why arrows were not always used in Questions 1, 3, and 4.

3. In Questions 1–4, you identified the variables. For which of those situations does it make sense for either of the variables to have a negative value? Explain.

4. Consider the relationship between the amount of observable mold on a piece of bread and the time from when it was baked until several months later.

a. Which graph, *a*, *b*, or *c*, best models the given situation?



b. What features made you choose that graph, and what features made you discount the other graphs?

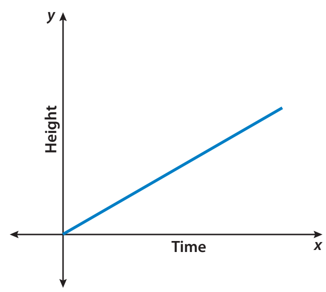
c. What are the two quantities or variables in the given situation?

5. Water is pumped from a plastic cylinder at a constant rate. Which representation shown below, *words*, *graph*, or *table*, best models this situation? Explain.

Words

The height of the water decreases for a few minutes, stays at the same height for a while, then increases again.

Graph



Table

Time (min)	Height (in.)
0	18
1	14
2	10
3	6
4	2
5	0

4b. Sample answer: The mold grows rapidly at first then slows down as it covers the bread. So, only graph *c* can be correct.

4c. time, amount of mold

5. The table best describes the situation because the height of the water will decrease until there is no water left in the container.

MODELING PROJECT

5e Elaborate

Materials List

- magazines and newspapers

Description

This project is designed to reinforce the idea that there are multiple ways to represent real-world situations. In this case, students are looking for and making connections between words and tables or graphs.

Students are asked to look through magazines and newspapers for models in the form of tables or graphs. Once they find a graph or table of interest, they are to write about how their particular representation reflects what the article is trying to say.

Once the projects are complete, you may want to have students share what they found with the entire class. This project works best when students work individually.

Sample Answer

Answers will vary depending on the articles chosen.

CHAPTER 1

Modeling Project

A Picture Is Worth a Thousand Words

Look through magazines and newspapers for articles that contain tables and/or graphs. Once you have found a table or graph of interest, look at the article to see how the table or graph represents the ideas in the article.

Then write a short explanation about how a particular graph or table represents or is connected to the words in the article. Bring both a copy of your article and your written explanation to class.

Chapter 1 Review

You Should Be Able to:

Lesson 1.1
- use ratios and proportions to create mathematical models.
- use mathematical models to estimate the sizes of populations.
- solve proportions.

Lesson 1.2
- use proportions to model real-world situations.
- solve problems that involve scale drawings.
- solve problems that involve similar polygons.

Lesson 1.3
- solve problems that require previously learned concepts and skills.

Lesson 1.4
- use multiple representations to model real-world situations.

Key Vocabulary

mathematical modeling (p. 2)
mathematical models (p. 2)
ratio (p. 3)
proportion (p. 4)
variable (p. 4)
scale (p. 7)
similar figures (p. 7)
similar polygons (p. 7)
polygon (p. 7)
sides (p. 7)
vertex (p. 7)
congruent (p. 8)
scale factor (p. 8)

Chapter 1 Test Review

Fill in the blank.

1. A(n) _________________ is the comparison of two numbers by division.

2. If two figures have the same shape and size, then they are ____________.

3. A statement that two ratios are equal is called a(n) _________________.

Solve each proportion.

4. $\frac{8}{x} = \frac{2}{3}$ **5.** $\frac{n}{0.3} = \frac{9}{0.6}$ **6.** $\frac{5}{3} = \frac{11}{a}$

CHAPTER REVIEW

5e Evaluate

Chapter 1 Test Review Answers

1. ratio
2. congruent
3. proportion
4. 12
5. 4.5
6. $6\frac{3}{5}$

CHAPTER REVIEW

7a. $\frac{7}{56} = \frac{42}{p}$

7b. 336 turtles

8. $16.20

9. 360 ft

10. 65 mm

11. about 6.2 cm

12a. $\frac{5}{3}$

12b. $11\frac{2}{3}$

13. Sample answer: Most likely the times shown on the graph are 5:00 a.m. to 3:00 p.m. as traffic is slowed during the rush hours in the early morning.

7. The *capture-recapture* method is used to find the number of turtles in a small stream. Forty-two turtles are captured. Their shells are marked with green paint. Then the turtles are released back into the stream. Later in the summer, 56 turtles are captured. Of those captured, 7 had green paint on their shells.

a. Write a proportion that models this situation.

b. About how many turtles are in the stream?

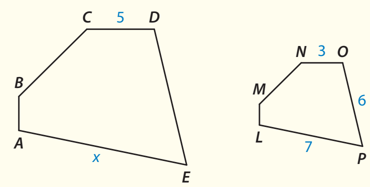
8. If 18 greeting cards cost $24.30, what is the cost of 12 cards?

9. The floor plan for an office building has a scale of $\frac{1}{8}$ in. = 1 ft. If the length of the main hallway measures 45 inches on the drawing, how long is the actual hallway?

10. The ratio of the corresponding sides of two similar triangles is 2 : 5. The sides of the smaller triangle are 6 mm, 8 mm, and 12 mm. What is the perimeter of the larger triangle?

11. A computer image-processing program can be used to change the size of a digital photograph. If a 4.0 cm × 5.5 cm photo is enlarged so that its length is 8.5 cm, what is its new width?

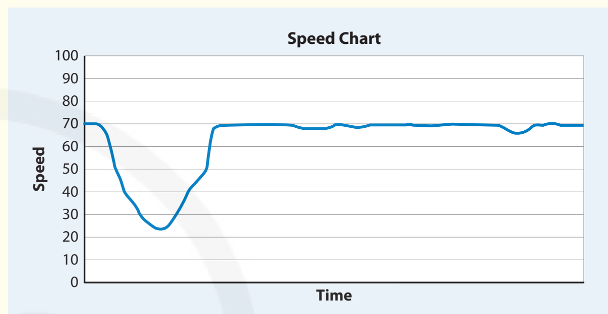
12. *ABCDE* ~ *LMNOP*



a. What is the scale factor of *ABCDE* to *LMNOP*?

b. Find the value of x.

13. The graph below shows the average speed of vehicles on a freeway of a large city at specific times of day.



Does the graph show times from 5:00 a.m. to 3:00 p.m. or from 10 a.m. to 8 p.m.? Explain.