

**GLENCOE  
MATHEMATICS**

# Algebra 1

## Chapter 1 Resource Masters



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## Consumable Workbooks

Many of the worksheets contained in the Chapter Resource Masters booklets are available as consumable workbooks in both English and Spanish.

<i>Study Guide and Intervention Workbook</i>	0-07-827753-1
<i>Study Guide and Intervention Workbook (Spanish)</i>	0-07-827754-X
<i>Skills Practice Workbook</i>	0-07-827747-7
<i>Skills Practice Workbook (Spanish)</i>	0-07-827749-3
<i>Practice Workbook</i>	0-07-827748-5
<i>Practice Workbook (Spanish)</i>	0-07-827750-7

**ANSWERS FOR WORKBOOKS** The answers for Chapter 1 of these workbooks can be found in the back of this Chapter Resource Masters booklet.

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*Algebra 1*  
*Chapter 1 Resource Masters*

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# Teacher's Guide to Using the Chapter 1 Resource Masters

The **Fast File** Chapter Resource system allows you to conveniently file the resources you use most often. The *Chapter 1 Resource Masters* includes the core materials needed for Chapter 1. These materials include worksheets, extensions, and assessment options. The answers for these pages appear at the back of this booklet.

All of the materials found in this booklet are included for viewing and printing in the *Algebra 1 TeacherWorks* CD-ROM.

**Vocabulary Builder** Pages vii–viii include a student study tool that presents up to twenty of the key vocabulary terms from the chapter. Students are to record definitions and/or examples for each term. You may suggest that students highlight or star the terms with which they are not familiar.

**WHEN TO USE** Give these pages to students before beginning Lesson 1-1. Encourage them to add these pages to their Algebra Study Notebook. Remind them to add definitions and examples as they complete each lesson.

## Study Guide and Intervention

Each lesson in *Algebra 1* addresses two objectives. There is one Study Guide and Intervention master for each objective.

**WHEN TO USE** Use these masters as reteaching activities for students who need additional reinforcement. These pages can also be used in conjunction with the Student Edition as an instructional tool for students who have been absent.

**Skills Practice** There is one master for each lesson. These provide computational practice at a basic level.

**WHEN TO USE** These masters can be used with students who have weaker mathematics backgrounds or need additional reinforcement.

**Practice** There is one master for each lesson. These problems more closely follow the structure of the Practice and Apply section of the Student Edition exercises. These exercises are of average difficulty.

**WHEN TO USE** These provide additional practice options or may be used as homework for second day teaching of the lesson.

## Reading to Learn Mathematics

One master is included for each lesson. The first section of each master asks questions about the opening paragraph of the lesson in the Student Edition. Additional questions ask students to interpret the context of and relationships among terms in the lesson. Finally, students are asked to summarize what they have learned using various representation techniques.

**WHEN TO USE** This master can be used as a study tool when presenting the lesson or as an informal reading assessment after presenting the lesson. It is also a helpful tool for ELL (English Language Learner) students.

**Enrichment** There is one extension master for each lesson. These activities may extend the concepts in the lesson, offer an historical or multicultural look at the concepts, or widen students' perspectives on the mathematics they are learning. These are not written exclusively for honors students, but are accessible for use with all levels of students.

**WHEN TO USE** These may be used as extra credit, short-term projects, or as activities for days when class periods are shortened.

## Assessment Options

The assessment masters in the *Chapter 1 Resources Masters* offer a wide range of assessment tools for intermediate and final assessment. The following lists describe each assessment master and its intended use.

## Chapter Assessment

### CHAPTER TESTS

- *Form 1* contains multiple-choice questions and is intended for use with basic level students.
- *Forms 2A and 2B* contain multiple-choice questions aimed at the average level student. These tests are similar in format to offer comparable testing situations.
- *Forms 2C and 2D* are composed of free-response questions aimed at the average level student. These tests are similar in format to offer comparable testing situations. Grids with axes are provided for questions assessing graphing skills.
- *Form 3* is an advanced level test with free-response questions. Grids without axes are provided for questions assessing graphing skills.

All of the above tests include a free-response Bonus question.

- The **Open-Ended Assessment** includes performance assessment tasks that are suitable for all students. A scoring rubric is included for evaluation guidelines. Sample answers are provided for assessment.
- A **Vocabulary Test**, suitable for all students, includes a list of the vocabulary words in the chapter and ten questions assessing students' knowledge of those terms. This can also be used in conjunction with one of the chapter tests or as a review worksheet.

## Intermediate Assessment

- Four free-response **quizzes** are included to offer assessment at appropriate intervals in the chapter.
- A **Mid-Chapter Test** provides an option to assess the first half of the chapter. It is composed of both multiple-choice and free-response questions.

## Continuing Assessment

- The **Cumulative Review** provides students an opportunity to reinforce and retain skills as they proceed through their study of Algebra 1. It can also be used as a test. This master includes free-response questions.
- The **Standardized Test Practice** offers continuing review of algebra concepts in various formats, which may appear on the standardized tests that they may encounter. This practice includes multiple-choice, grid-in, and quantitative-comparison questions. Bubble-in and grid-in answer sections are provided on the master.

## Answers

- Page A1 is an answer sheet for the Standardized Test Practice questions that appear in the Student Edition on pages 64–65. This improves students' familiarity with the answer formats they may encounter in test taking.
- The answers for the lesson-by-lesson masters are provided as reduced pages with answers appearing in red.
- Full-size answer keys are provided for the assessment masters in this booklet.

## 1

**Reading to Learn Mathematics*****Vocabulary Builder***

**This is an alphabetical list of the key vocabulary terms you will learn in Chapter 1. As you study the chapter, complete each term's definition or description. Remember to add the page number where you found the term. Add these pages to your Algebra Study Notebook to review vocabulary at the end of the chapter.**

Vocabulary Term	Found on Page	Definition/Description/Example
coefficient KOH-uh-FIH-shuhnt		
conclusion		
conditional statement		
coordinate system		
counterexample		
deductive reasoning dih-DUHK-tihv		
dependent variable		
domain		
equation		
function		

(continued on the next page)

## 1

**Reading to Learn Mathematics****Vocabulary Builder** *(continued)*

Vocabulary Term	Found on Page	Definition/Description/Example
hypothesis <u>                    </u> hy-PAH-thuh-suhs		
independent variable		
inequality		
like terms		
order of operations		
power		
range		
replacement set		
solving an open sentence		
variables		

## 1-1

**Study Guide and Intervention*****Variables and Expressions***

**Write Mathematical Expressions** In the **algebraic expression**,  $lw$ , the letters  $l$  and  $w$  are called **variables**. In algebra, a variable is used to represent unspecified numbers or values. Any letter can be used as a variable. The letters  $l$  and  $w$  are used above because they are the first letters of the words *length* and *width*. In the expression  $lw$ ,  $l$  and  $w$  are called factors, and the result is called the **product**.

**Example 1**

**Write an algebraic expression for each verbal expression.**

**a. four more than a number  $n$**

The words *more than* imply addition.

four more than a number  $n$

$$4 + n$$

The algebraic expression is  $4 + n$ .

**b. the difference of a number squared and 8**

The expression *difference of* implies subtraction.

the difference of a number squared and 8

$$n^2 - 8$$

The algebraic expression is  $n^2 - 8$ .

**Example 2**

**Evaluate each expression.**

**a.  $3^4$**

$$\begin{aligned} 3^4 &= 3 \cdot 3 \cdot 3 \cdot 3 && \text{Use 3 as a factor 4 times.} \\ &= 81 && \text{Multiply.} \end{aligned}$$

**b. five cubed**

*Cubed* means raised to the third power.

$$\begin{aligned} 5^3 &= 5 \cdot 5 \cdot 5 && \text{Use 5 as a factor 3 times.} \\ &= 125 && \text{Multiply.} \end{aligned}$$

**Exercises**

**Write an algebraic expression for each verbal expression.**

1. a number decreased by 8

2. a number divided by 8

3. a number squared

4. four times a number

5. a number divided by 6

6. a number multiplied by 37

7. the sum of 9 and a number

8. 3 less than 5 times a number

9. twice the sum of 15 and a number

10. one-half the square of  $b$

11. 7 more than the product of 6 and a number

12. 30 increased by 3 times the square of a number

**Evaluate each expression.**

13.  $5^2$

14.  $3^3$

15.  $10^4$

16.  $12^2$

17.  $8^3$

18.  $2^8$



**1-1 Study Guide and Intervention** *(continued)****Variables and Expressions***

**Write Verbal Expressions** Translating algebraic expressions into verbal expressions is important in algebra.

**Example**

Write a verbal expression for each algebraic expression.

a.  $6n^2$

the product of 6 and  $n$  squared

b.  $n^3 - 12m$

the difference of  $n$  cubed and twelve times  $m$

**Exercises**

Write a verbal expression for each algebraic expression.

1.  $w - 1$

2.  $\frac{1}{3}a^3$

3.  $81 + 2x$

4.  $12c$

5.  $8^4$

6.  $6^2$

7.  $2n^2 + 4$

8.  $a^3 \cdot b^3$

9.  $2x^3 - 3$

10.  $\frac{6k^3}{5}$

11.  $\frac{1}{4}b^2$

12.  $7n^5$

13.  $3x + 4$

14.  $\frac{2}{3}k^5$

15.  $3b^2 + 2a^3$

16.  $4(n^2 + 1)$

17.  $3^2 + 2^3$

18.  $6n^2 + 3$

## 1-1

**Skills Practice*****Variables and Expressions***

**Write an algebraic expression for each verbal expression.**

1. the sum of a number and 10

2. 15 less than  $k$

3. the product of 18 and  $q$

4. 6 more than twice  $m$

5. 8 increased by three times a number

6. the difference of 17 and 5 times a number

7. the product of 2 and the second power of  $y$

8. 9 less than  $g$  to the fourth power

**Evaluate each expression.**

9.  $8^2$

10.  $3^4$

11.  $5^3$

12.  $3^3$

13.  $10^2$

14.  $2^4$

15.  $7^2$

16.  $4^4$

17.  $7^3$

18.  $11^3$

**Write a verbal expression for each algebraic expression.**

19.  $9a$

20.  $5^2$

21.  $c + 2d$

22.  $4 - 5h$

23.  $2b^2$

24.  $7x^3 - 1$

25.  $p^4 + 6q$

26.  $3n^2 - x$

## 1-1

## Practice

*Variables and Expressions*

Write an algebraic expression for each verbal expression.

1. the difference of 10 and  $u$
2. the sum of 18 and a number
3. the product of 33 and  $j$
4. 74 increased by 3 times  $y$
5. 15 decreased by twice a number
6. 91 more than the square of a number
7. three fourths the square of  $b$
8. two fifths the cube of a number

Evaluate each expression.

9.  $11^2$
10.  $8^3$
11.  $5^4$
12.  $4^5$
13.  $9^3$
14.  $6^4$
15.  $10^5$
16.  $12^3$
17.  $100^4$

Write a verbal expression for each algebraic expression.

18.  $23f$
19.  $7^3$
20.  $5m^2 + 2$
21.  $4d^3 - 10$
22.  $x^3 \cdot y^4$
23.  $b^2 - 3c^3$
24.  $\frac{k^5}{6}$
25.  $\frac{4n^2}{7}$

**26. BOOKS** A used bookstore sells paperback fiction books in excellent condition for \$2.50 and in fair condition for \$0.50. Write an expression for the cost of buying  $e$  excellent-condition paperbacks and  $f$  fair-condition paperbacks.

**27. GEOMETRY** The surface area of the side of a right cylinder can be found by multiplying twice the number  $\pi$  by the radius times the height. If a circular cylinder has radius  $r$  and height  $h$ , write an expression that represents the surface area of its side.

## 1-1

**Reading to Learn Mathematics*****Variables and Expressions***

**Pre-Activity** What expression can be used to find the perimeter of a baseball diamond?

Read the introduction to Lesson 1-1 at the top of page 6 in your textbook. Then complete the description of the expression  $4s$ .

In the expression  $4s$ , 4 represents the \_\_\_\_\_ of sides and  $s$  represents the \_\_\_\_\_ of each side.

**Reading the Lesson**

1. Why is the symbol  $\times$  avoided in algebra?
2. What are the factors in the algebraic expression  $3xy$ ?
3. In the expression  $x^n$ , what is the base? What is the exponent?
4. Write the Roman numeral of the algebraic expression that best matches each phrase.
 

a. three more than a number $n$ _____	I. $5(x - 4)$
b. five times the difference of $x$ and 4 _____	II. $x^4$
c. one half the number $r$ _____	III. $\frac{1}{2}r$
d. the product of $x$ and $y$ divided by 2 _____	IV. $n + 3$
e. $x$ to the fourth power _____	V. $\frac{xy}{2}$

**Helping You Remember**

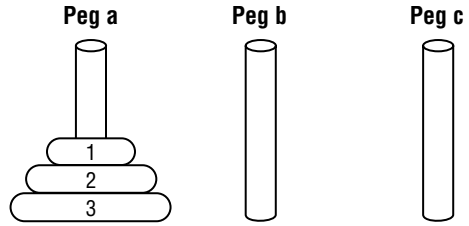
5. Multiplying 5 times 3 is not the same as raising 5 to the third power. How does the way you write “5 times 3” and “5 to the third power” in symbols help you remember that they give different results?

# 1-1 Enrichment

## The Tower of Hanoi

The diagram at the right shows the Tower of Hanoi puzzle. Notice that there are three pegs, with a stack of disks on peg a. The object is to move all of the disks to another peg. You may move only one disk at a time and a larger disk may never be put on top of a smaller disk.

As you solve the puzzle, record each move in the table shown. The first two moves are recorded.



### Solve.

- Complete the table to solve the Tower of Hanoi puzzle for three disks.
- Another way to record each move is to use letters. For example, the first two moves in the table can be recorded as 1c, 2b. This shows that disk 1 is moved to peg c, and then disk 2 is moved to peg b. Record your solution using letters.
- On a separate sheet of paper, solve the puzzle for four disks. Record your solution.
- Solve the puzzle for five disks. Record your solution.
- Suppose you start with an odd number of disks and you want to end with the stack on peg c. What should be your first move?
- Suppose you start with an even number of disks and you want to end with the stack on peg b. What should be your first move?

Peg a	Peg b	Peg c
1 2 3		
2 3		1
3	2	1

# 1-2 Study Guide and Intervention

## Order of Operations

**Evaluate Rational Expressions** Numerical expressions often contain more than one operation. To evaluate them, use the rules for order of operations shown below.

<b>Order of Operations</b>	<p><b>Step 1</b> Evaluate expressions inside grouping symbols.</p> <p><b>Step 2</b> Evaluate all powers.</p> <p><b>Step 3</b> Do all multiplication and/or division from left to right.</p> <p><b>Step 4</b> Do all addition and/or subtraction from left to right.</p>
----------------------------	---

### Example 1 Evaluate each expression.

**a.  $7 + 2 \cdot 4 - 4$**

$$7 + 2 \cdot 4 - 4 = 7 + 8 - 4 \quad \text{Multiply 2 and 4.}$$

$$= 15 - 4 \quad \text{Add 7 and 8.}$$

$$= 11 \quad \text{Subtract 4 from 15.}$$

**b.  $3(2) + 4(2 + 6)$**

$$3(2) + 4(2 + 6) = 3(2) + 4(8) \quad \text{Add 2 and 6.}$$

$$= 6 + 32 \quad \text{Multiply left to right.}$$

$$= 38 \quad \text{Add 6 and 32.}$$

### Example 2 Evaluate each expression.

**a.  $3[2 + (12 \div 3)^2]$**

$$3[2 + (12 \div 3)^2] = 3(2 + 4^2) \quad \text{Divide 12 by 3.}$$

$$= 3(2 + 16) \quad \text{Find 4 squared.}$$

$$= 3(18) \quad \text{Add 2 and 16.}$$

$$= 54 \quad \text{Multiply 3 and 18.}$$

**b.  $\frac{3 + 2^3}{4^2 \cdot 3}$**

$$\frac{3 + 2^3}{4^2 \cdot 3} = \frac{3 + 8}{4^2 \cdot 3} \quad \text{Evaluate power in numerator.}$$

$$= \frac{11}{4^2 \cdot 3} \quad \text{Add 3 and 8 in the numerator.}$$

$$= \frac{11}{16 \cdot 3} \quad \text{Evaluate power in denominator.}$$

$$= \frac{11}{48} \quad \text{Multiply.}$$

### Exercises

Evaluate each expression.

- |   |  |   |
|---|--|---|
| 1. $(8 - 4) \cdot 2$                              | 2. $(12 + 4) \cdot 6$                                | 3. $10 + 2 \cdot 3$                             |
| 4. $10 + 8 \cdot 1$                               | 5. $15 - 12 \div 4$                                  | 6. $\frac{15 + 60}{30 - 5}$                     |
| 7. $12(20 - 17) - 3 \cdot 6$                      | 8. $24 \div 3 \cdot 2 - 3^2$                         | 9. $8^2 \div (2 \cdot 8) + 2$                   |
| 10. $3^2 \div 3 + 2^2 \cdot 7 - 20 \div 5$        | 11. $\frac{4 + 3^2}{12 + 1}$                         | 12. $\frac{8(2) - 4}{8 \div 4}$                 |
| 13. $250 \div [5(3 \cdot 7 + 4)]$                 | 14. $\frac{2 \cdot 4^2 - 8 \div 2}{(5 + 2) \cdot 2}$ | 15. $\frac{4 \cdot 3^2 - 3 \cdot 2}{3 \cdot 5}$ |
| 16. $\frac{4(5^2) - 4 \cdot 3}{4(4 \cdot 5 + 2)}$ | 17. $\frac{5^2 - 3}{20(3) + 2(3)}$                   | 18. $\frac{8^2 - 2^2}{(2 \cdot 8) + 4}$         |

**1-2 Study Guide and Intervention** *(continued)***Order of Operations**

**Evaluate Algebraic Expressions** Algebraic expressions may contain more than one operation. Algebraic expressions can be evaluated if the values of the variables are known. First, replace the variables by their values. Then use the order of operations to calculate the value of the resulting numerical expression.

**Example**

**Evaluate  $x^3 + 5(y - 3)$  if  $x = 2$  and  $y = 12$ .**

$$\begin{aligned} x^3 + 5(y - 3) &= 2^3 + 5(12 - 3) && \text{Replace } x \text{ with } 2 \text{ and } y \text{ with } 12. \\ &= 8 + 5(12 - 3) && \text{Evaluate } 2^3. \\ &= 8 + 5(9) && \text{Subtract } 3 \text{ from } 12. \\ &= 8 + 45 && \text{Multiply } 5 \text{ and } 9. \\ &= 53 && \text{Add } 8 \text{ and } 45. \end{aligned}$$

The solution is 53.

**Exercises**

**Evaluate each expression if  $x = 2$ ,  $y = 3$ ,  $z = 4$ ,  $a = \frac{4}{5}$ , and  $b = \frac{3}{5}$ .**

1.  $x + 7$

2.  $3x - 5$

3.  $x + y^2$

4.  $x^3 + y + z^2$

5.  $6a + 8b$

6.  $23 - (a + b)$

7.  $\frac{y^2}{x^2}$

8.  $2xyz + 5$

9.  $x(2y + 3z)$

10.  $(10x)^2 + 100a$

11.  $\frac{3xy - 4}{7x}$

12.  $a^2 + 2b$

13.  $\frac{z^2 - y^2}{x^2}$

14.  $6xz + 5xy$

15.  $\frac{(z - y)^2}{x}$

16.  $\frac{25ab + y}{xz}$

17.  $\frac{5a^2b}{y}$

18.  $(z \div x)^2 + ax$

19.  $\left(\frac{x}{z}\right)^2 + \left(\frac{y}{z}\right)^2$

20.  $\frac{x + z}{y + 2z}$

21.  $\left(\frac{z \div x}{y}\right) + \left(\frac{y \div x}{z}\right)$

# 1-2 Skills Practice

## Order of Operations

Evaluate each expression.

1.  $(5 + 4) \cdot 7$

2.  $(9 - 2) \cdot 3$

3.  $4 + 6 \cdot 3$

4.  $28 - 5 \cdot 4$

5.  $12 + 2 \cdot 2$

6.  $(3 + 5) \cdot 5 + 1$

7.  $9 + 4(3 + 1)$

8.  $2 + 3 \cdot 5 + 4$

9.  $30 - 5 \cdot 4 + 2$

10.  $10 + 2 \cdot 6 + 4$

11.  $14 \div 7 \cdot 5 - 3^2$

12.  $6 \div 3 \cdot 7 + 2^3$

13.  $4[30 - (10 - 2) \cdot 3]$

14.  $5 + [30 - (6 - 1)^2]$

15.  $2[12 + (5 - 2)^2]$

16.  $[8 \cdot 2 - (3 + 9)] + [8 - 2 \cdot 3]$

Evaluate each expression if  $x = 6$ ,  $y = 8$ , and  $z = 3$ .

17.  $xy + z$

18.  $yz - x$

19.  $2x + 3y - z$

20.  $2(x + z) - y$

21.  $5z + (y - x)$

22.  $5x - (y + 2z)$

23.  $x^2 + y^2 - 10z$

24.  $z^3 + (y^2 - 4x)$

25.  $\frac{y + xz}{2}$

26.  $\frac{3y + x^2}{z}$



# 1-2 Practice

## Order of Operations

Evaluate each expression.

1.  $(15 - 5) \cdot 2$

2.  $9 \cdot (3 + 4)$

3.  $5 + 7 \cdot 4$

4.  $12 + 5 - 6 \cdot 2$

5.  $7 \cdot 9 - 4(6 + 7)$

6.  $8 \div (2 + 2) \cdot 7$

7.  $4(3 + 5) - 5 \cdot 4$

8.  $22 \div 11 \cdot 9 - 3^2$

9.  $6^2 + 3 \cdot 7 - 9$

10.  $3[10 - (27 \div 9)]$

11.  $2[5^2 + (36 \div 6)]$

12.  $162 \div [6(7 - 4)^2]$

13.  $\frac{5^2 \cdot 4 - 5 \cdot 4^2}{5(4)}$

14.  $\frac{(2 \cdot 5)^2 + 4}{3^2 - 5}$

15.  $\frac{7 + 3^2}{4^2 \cdot 2}$

Evaluate each expression if  $a = 12$ ,  $b = 9$ , and  $c = 4$ .

16.  $a^2 + b - c^2$

17.  $b^2 + 2a - c^2$

18.  $2c(a + b)$

19.  $4a + 2b - c^2$

20.  $(a^2 \div 4b) + c$

21.  $c^2 \cdot (2b - a)$

22.  $\frac{bc^2 + a}{c}$

23.  $\frac{2c^3 - ab}{4}$

24.  $\frac{2(a - b)^2}{5c}$

25.  $\frac{b^2 - 2c^2}{a + c - b}$

**CAR RENTAL** For Exercises 26 and 27, use the following information.

Ann Carlyle is planning a business trip for which she needs to rent a car. The car rental company charges \$36 per day plus \$0.50 per mile over 100 miles. Suppose Ms. Carlyle rents the car for 5 days and drives 180 miles.

26. Write an expression for how much it will cost Ms. Carlyle to rent the car.

27. Evaluate the expression to determine how much Ms. Carlyle must pay the car rental company.

**GEOMETRY** For Exercises 28 and 29, use the following information.

The length of a rectangle is  $3n + 2$  and its width is  $n - 1$ . The perimeter of the rectangle is twice the sum of its length and its width.

28. Write an expression that represents the perimeter of the rectangle.

29. Find the perimeter of the rectangle when  $n = 4$  inches.

## 1-2

**Reading to Learn Mathematics*****Order of Operations*****Pre-Activity** How is the monthly cost of internet service determined?

Read the introduction to Lesson 1-2 at the top of page 11 in your textbook.

In the expression  $4.95 + 0.99(117 - 100)$ , \_\_\_\_\_ represents the regular monthly cost of internet service, \_\_\_\_\_ represents the cost of each additional hour after 100 hours, and \_\_\_\_\_ represents the number of hours over 100 used by Nicole in a given month.

**Reading the Lesson**

- The first step in evaluating an expression is to evaluate inside grouping symbols. List four types of grouping symbols found in algebraic expressions.
- What does *evaluate powers* mean? Use an example to explain.
- Read the order of operations on page 11 in your textbook. For each of the following expressions, write *addition*, *subtraction*, *multiplication*, *division*, or *evaluate powers* to tell what operation to use first when evaluating the expression.

a.  $400 - 5[12 + 9]$

b.  $26 - 8 + 14$

c.  $17 + 3 \cdot 6$

d.  $69 + 57 \div 3 + 16 \cdot 4$

e.  $\frac{19 + 3 \cdot 4}{6 \div 2}$

f.  $\frac{51 \div 729}{9^2}$

**Helping You Remember**

- The sentence *Please Excuse My Dear Aunt Sally* (PEMDAS) is often used to remember the order of operations. The letter P represents parentheses and other grouping symbols. Write what each of the other letters in PEMDAS means when using the order of operations.

# 1-2 Enrichment

## *The Four Digits Problem*

One well-known mathematic problem is to write expressions for consecutive numbers beginning with 1. On this page, you will use the digits 1, 2, 3, and 4. Each digit is used only once. You may use addition, subtraction, multiplication (not division), exponents, and parentheses in any way you wish. Also, you can use two digits to make one number, such as 12 or 34.

**Express each number as a combination of the digits 1, 2, 3, and 4.**

$$1 = (3 \times 1) - (4 - 2) \quad 18 = \underline{\hspace{2cm}} \quad 35 = 2^{(4+1)} + 3$$

$$2 = \underline{\hspace{2cm}} \quad 19 = 3(2 + 4) + 1 \quad 36 = \underline{\hspace{2cm}}$$

$$3 = \underline{\hspace{2cm}} \quad 20 = \underline{\hspace{2cm}} \quad 37 = \underline{\hspace{2cm}}$$

$$4 = \underline{\hspace{2cm}} \quad 21 = \underline{\hspace{2cm}} \quad 38 = \underline{\hspace{2cm}}$$

$$5 = \underline{\hspace{2cm}} \quad 22 = \underline{\hspace{2cm}} \quad 39 = \underline{\hspace{2cm}}$$

$$6 = \underline{\hspace{2cm}} \quad 23 = 31 - (4 \times 2) \quad 40 = \underline{\hspace{2cm}}$$

$$7 = \underline{\hspace{2cm}} \quad 24 = \underline{\hspace{2cm}} \quad 41 = \underline{\hspace{2cm}}$$

$$8 = \underline{\hspace{2cm}} \quad 25 = \underline{\hspace{2cm}} \quad 42 = \underline{\hspace{2cm}}$$

$$9 = \underline{\hspace{2cm}} \quad 26 = \underline{\hspace{2cm}} \quad 43 = 42 + 1^3$$

$$10 = \underline{\hspace{2cm}} \quad 27 = \underline{\hspace{2cm}} \quad 44 = \underline{\hspace{2cm}}$$

$$11 = \underline{\hspace{2cm}} \quad 28 = \underline{\hspace{2cm}} \quad 45 = \underline{\hspace{2cm}}$$

$$12 = \underline{\hspace{2cm}} \quad 29 = \underline{\hspace{2cm}} \quad 46 = \underline{\hspace{2cm}}$$

$$13 = \underline{\hspace{2cm}} \quad 30 = \underline{\hspace{2cm}} \quad 47 = \underline{\hspace{2cm}}$$

$$14 = \underline{\hspace{2cm}} \quad 31 = \underline{\hspace{2cm}} \quad 48 = \underline{\hspace{2cm}}$$

$$15 = \underline{\hspace{2cm}} \quad 32 = \underline{\hspace{2cm}} \quad 49 = \underline{\hspace{2cm}}$$

$$16 = \underline{\hspace{2cm}} \quad 33 = \underline{\hspace{2cm}} \quad 50 = \underline{\hspace{2cm}}$$

$$17 = \underline{\hspace{2cm}} \quad 34 = \underline{\hspace{2cm}}$$

Does a calculator help in solving these types of puzzles? Give reasons for your opinion.

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# 1-3 Study Guide and Intervention

## Open Sentences

**Solve Equations** A mathematical sentence with one or more variables is called an **open sentence**. Open sentences are **solved** by finding replacements for the variables that result in true sentences. The set of numbers from which replacements for a variable may be chosen is called the **replacement set**. The set of all replacements for the variable that result in true statements is called the **solution set** for the variable. A sentence that contains an equal sign, =, is called an **equation**.

**Example 1** Find the solution set of  $3a + 12 = 39$  if the replacement set is {6, 7, 8, 9, 10}.

Replace  $a$  in  $3a + 12 = 39$  with each value in the replacement set.

$$3(6) + 12 \stackrel{?}{=} 39 \rightarrow 30 \neq 39 \quad \text{false}$$

$$3(7) + 12 \stackrel{?}{=} 39 \rightarrow 33 \neq 39 \quad \text{false}$$

$$3(8) + 12 \stackrel{?}{=} 39 \rightarrow 36 \neq 39 \quad \text{false}$$

$$3(9) + 12 \stackrel{?}{=} 39 \rightarrow 39 = 39 \quad \text{true}$$

$$3(10) + 12 \stackrel{?}{=} 39 \rightarrow 42 \neq 39 \quad \text{false}$$

Since  $a = 9$  makes the equation  $3a + 12 = 39$  true, the solution is 9.

The solution set is {9}.

**Example 2** Solve  $\frac{2(3 + 1)}{3(7 - 4)} = b$ .

$$\frac{2(3 + 1)}{3(7 - 4)} = b \quad \text{Original equation}$$

$$\frac{2(4)}{3(3)} = b \quad \text{Add in the numerator; subtract in the denominator.}$$

$$\frac{8}{9} = b \quad \text{Simplify.}$$

The solution is  $\frac{8}{9}$ .

### Exercises

Find the solution of each equation if the replacement sets are  $X = \left\{\frac{1}{4}, \frac{1}{2}, 1, 2, 3\right\}$  and  $Y = \{2, 4, 6, 8\}$ .

1.  $x + \frac{1}{2} = \frac{5}{2}$

2.  $x + 8 = 11$

3.  $y - 2 = 6$

4.  $x^2 - 1 = 8$

5.  $y^2 - 2 = 34$

6.  $x^2 + 5 = 5\frac{1}{16}$

7.  $2(x + 3) = 7$

8.  $\frac{1}{4}(y + 1)^2 = \frac{9}{4}$

9.  $y^2 + y = 20$

Solve each equation.

10.  $a = 2^3 - 1$

11.  $n = 6^2 - 4^2$

12.  $w = 6^2 \cdot 3^2$

13.  $\frac{1}{4} + \frac{5}{8} = k$

14.  $\frac{18 - 3}{2 + 3} = p$

15.  $s = \frac{15 - 6}{27 - 24}$

16.  $18.4 - 3.2 = m$

17.  $k = 9.8 + 5.7$

18.  $c = 3\frac{1}{2} + 2\frac{1}{4}$

**1-3 Study Guide and Intervention** *(continued)***Open Sentences**

**Solve Inequalities** An open sentence that contains the symbol  $<$ ,  $\leq$ ,  $>$ , or  $\geq$  is called an **inequality**. Inequalities can be solved the same way that equations are solved.

**Example**

**Find the solution set for  $3a - 8 > 10$  if the replacement set is  $\{4, 5, 6, 7, 8\}$ .**

Replace  $a$  in  $3a - 8 > 10$  with each value in the replacement set.

$$3(4) - 8 \stackrel{?}{>} 10 \rightarrow 4 > 10 \quad \text{false}$$

$$3(5) - 8 \stackrel{?}{>} 10 \rightarrow 7 > 10 \quad \text{false}$$

$$3(6) - 8 \stackrel{?}{>} 10 \rightarrow 10 > 10 \quad \text{false}$$

$$3(7) - 8 \stackrel{?}{>} 10 \rightarrow 13 > 10 \quad \text{true}$$

$$3(8) - 8 \stackrel{?}{>} 10 \rightarrow 16 > 10 \quad \text{true}$$

Since replacing  $a$  with 7 or 8 makes the inequality  $3a - 8 > 10$  true, the solution set is  $\{7, 8\}$ .

**Exercises**

**Find the solution set for each inequality if the replacement set is  $X = \{0, 1, 2, 3, 4, 5, 6, 7\}$ .**

1.  $x + 2 > 4$

2.  $x \div 3 < 6$

3.  $3x > 18$

4.  $\frac{x}{3} > 1$

5.  $\frac{x}{5} \geq 2$

6.  $\frac{3x}{8} \leq 2$

7.  $3x - 4 > 5$

8.  $3(8 - x) + 1 \leq 6$

9.  $4(x + 3) \geq 20$

**Find the solution set for each inequality if the replacement sets are**

$X = \left\{\frac{1}{4}, \frac{1}{2}, 1, 2, 3, 5, 8\right\}$  and  $Y = \{2, 4, 6, 8, 10\}$

10.  $x + 3 > 5$

11.  $y \div 3 < 6$

12.  $8y + 3 \geq 51$

13.  $\frac{x}{2} < 4$

14.  $\frac{y}{4} \geq 2$

15.  $\frac{2y}{5} \leq 2$

16.  $4x + 1 \geq 4$

17.  $3x + 3 \geq 12$

18.  $2(y + 1) \geq 18$

19.  $3x - \frac{1}{4} < 2$

20.  $3y + 2 \leq 8$

21.  $\frac{1}{2}(6 - 2x) + 2 \leq 3$

**1-3 Skills Practice****Open Sentences**

Find the solution of each equation if the replacement sets are  $A = \{4, 5, 6, 7, 8\}$  and  $B = \{9, 10, 11, 12, 13\}$ .

1.  $5a - 9 = 26$

2.  $4a - 8 = 16$

3.  $7a + 21 = 56$

4.  $3b + 15 = 48$

5.  $4b - 12 = 28$

6.  $\frac{36}{b} - 3 = 0$

Find the solution of each equation using the given replacement set.

7.  $\frac{1}{2} + x = \frac{5}{4}; \left\{ \frac{1}{2}, \frac{3}{4}, 1, \frac{5}{4} \right\}$

8.  $x + \frac{2}{3} = \frac{13}{9}; \left\{ \frac{4}{9}, \frac{5}{9}, \frac{2}{3}, \frac{7}{9} \right\}$

9.  $\frac{1}{4}(x + 2) = \frac{5}{6}; \left\{ \frac{2}{3}, \frac{3}{4}, \frac{5}{4}, \frac{4}{3} \right\}$

10.  $0.8(x + 5) = 5.2; \{1.2, 1.3, 1.4, 1.5\}$

Solve each equation.

11.  $10.4 - 6.8 = x$

12.  $y = 20.1 - 11.9$

13.  $\frac{46 - 15}{3 + 28} = a$

14.  $c = \frac{6 + 18}{31 - 25}$

15.  $\frac{2(4) + 4}{3(3 - 1)} = b$

16.  $\frac{6(7 - 2)}{3(8) + 6} = n$

Find the solution set for each inequality using the given replacement set.

17.  $a + 7 < 13; \{3, 4, 5, 6, 7\}$

18.  $9 + y < 17; \{7, 8, 9, 10, 11\}$

19.  $x - 2 \leq 2; \{2, 3, 4, 5, 6, 7\}$

20.  $2x > 12; \{0, 2, 4, 6, 8, 10\}$

21.  $4b + 1 > 12; \{0, 3, 6, 9, 12, 15\}$

22.  $2c - 5 \leq 11; \{8, 9, 10, 11, 12, 13\}$

23.  $\frac{y}{2} \geq 5; \{4, 6, 8, 10, 12\}$

24.  $\frac{x}{3} > 2; \{3, 4, 5, 6, 7, 8\}$

**1-3 Practice****Open Sentences**

Find the solution of each equation if the replacement sets are  $A = \left\{0, \frac{1}{2}, 1, \frac{3}{2}, 2\right\}$  and  $B = \{3, 3.5, 4, 4.5, 5\}$ .

1.  $a + \frac{1}{2} = 1$

2.  $4b - 8 = 6$

3.  $6a + 18 = 27$

4.  $7b - 8 = 16.5$

5.  $120 - 28a = 78$

6.  $\frac{28}{b} + 9 = 16$

Find the solution of each equation using the given replacement set.

7.  $\frac{7}{8} + x = \frac{17}{12}; \left\{\frac{1}{2}, \frac{13}{24}, \frac{7}{12}, \frac{5}{8}, \frac{2}{3}\right\}$

8.  $\frac{3}{4}(x + 2) = \frac{27}{8}; \left\{\frac{1}{2}, 1, 1\frac{1}{2}, 2, 2\frac{1}{2}\right\}$

9.  $1.4(x + 3) = 5.32; \{0.4, 0.6, 0.8, 1.0, 1.2\}$

10.  $12(x + 4) = 76.8; \{2, 2.4, 2.8, 3.2, 3.6\}$

Solve each equation.

11.  $x = 18.3 - 4.8$

12.  $w = 20.2 - 8.95$

13.  $\frac{37 - 9}{18 - 11} = d$

14.  $\frac{97 - 25}{41 - 23} = k$

15.  $y = \frac{4(22 - 4)}{3(6) + 6}$

16.  $\frac{5(2^2) + 4(3)}{4(2^3 - 4)} = p$

Find the solution set for each inequality using the given replacement set.

17.  $a + 7 < 10; \{2, 3, 4, 5, 6, 7\}$

18.  $3y \geq 42; \{10, 12, 14, 16, 18\}$

19.  $4x - 2 < 5; \{0.5, 1, 1.5, 2, 2.5\}$

20.  $4b - 4 > 3; \{1.2, 1.4, 1.6, 1.8, 2.0\}$

21.  $\frac{3y}{5} \leq 2; \{0, 2, 4, 6, 8, 10\}$

22.  $4a \geq 3; \left\{\frac{1}{8}, \frac{1}{4}, \frac{3}{8}, \frac{1}{2}, \frac{5}{8}, \frac{3}{4}\right\}$

**23. TEACHING** A teacher has 15 weeks in which to teach six chapters. Write and then solve an equation that represents the number of lessons the teacher must teach per week if there is an average of 8.5 lessons per chapter.

**LONG DISTANCE** For Exercises 24 and 25, use the following information.

Gabriel talks an average of 20 minutes per long-distance call. During one month, he makes eight in-state long-distance calls averaging \$2.00 each. A 20-minute state-to-state call costs Gabriel \$1.50. His long-distance budget for the month is \$20.

**24.** Write an inequality that represents the number of 20 minute state-to-state calls Gabriel can make this month.

**25.** What is the maximum number of 20-minute state-to-state calls that Gabriel can make this month?

## 1-3

**Reading to Learn Mathematics****Open Sentences****Pre-Activity** How can you use open sentences to stay within a budget?

Read the introduction to Lesson 1-3 at the top of page 16 in your textbook.

How is the open sentence different from the expression  $15.50 + 5n$ ?**Reading the Lesson**

- How can you tell whether a mathematical sentence is or is not an open sentence?
- How would you read each inequality symbol in words?

Inequality Symbol	Words
$<$	
$>$	
$\leq$	
$\geq$	

- Consider the equation  $3n + 6 = 15$  and the inequality  $3n + 6 \leq 15$ . Suppose the replacement set is  $\{0, 1, 2, 3, 4, 5\}$ .
  - Describe how you would find the solutions of the equation.
  - Describe how you would find the solutions of the inequality.
  - Explain how the solution set for the equation is different from the solution set for the inequality.

**Helping You Remember**

- Look up the word *solution* in a dictionary. What is one meaning that relates to the way we use the word in algebra?



# 1-3 Enrichment

## Solution Sets

Consider the following open sentence.

*It* is the name of a month between March and July.

You know that a replacement for the variable *It* must be found in order to determine if the sentence is true or false. If *It* is replaced by either April, May, or June, the sentence is true. The set {April, May, June} is called the solution set of the open sentence given above. This set includes all replacements for the variable that make the sentence true.

**Write the solution set for each open sentence.**

1. It is the name of a state beginning with the letter A.
2. It is a primary color.
3. Its capital is Harrisburg.
4. It is a New England state.
5.  $x + 4 = 10$
6. It is the name of a month that contains the letter *r*.
7. During the 1990s, she was the wife of a U.S. President.
8. It is an even number between 1 and 13.
9.  $31 = 72 - k$
10. It is the square of 2, 3, or 4.

**Write an open sentence for each solution set.**

11. {A, E, I, O, U}
12. {1, 3, 5, 7, 9}
13. {June, July, August}
14. {Atlantic, Pacific, Indian, Arctic}

# 1-4 Study Guide and Intervention

## Identity and Equality Properties

**Identity and Equality Properties** The identity and equality properties in the chart below can help you solve algebraic equations and evaluate mathematical expressions.

<b>Additive Identity</b>	For any number $a$ , $a + 0 = a$ .
<b>Multiplicative Identity</b>	For any number $a$ , $a \cdot 1 = a$ .
<b>Multiplicative Property of 0</b>	For any number $a$ , $a \cdot 0 = 0$ .
<b>Multiplicative Inverse Property</b>	For every number $\frac{a}{b}$ , $a, b \neq 0$ , there is exactly one number $\frac{b}{a}$ such that $\frac{a}{b} \cdot \frac{b}{a} = 1$ .
<b>Reflexive Property</b>	For any number $a$ , $a = a$ .
<b>Symmetric Property</b>	For any numbers $a$ and $b$ , if $a = b$ , then $b = a$ .
<b>Transitive Property</b>	For any numbers $a$ , $b$ , and $c$ , if $a = b$ and $b = c$ , then $a = c$ .
<b>Substitution Property</b>	If $a = b$ , then $a$ may be replaced by $b$ in any expression.

**Example 1** Name the property used in each equation. Then find the value of  $n$ .

a.  $8n = 8$

Multiplicative Identity Property

$n = 1$ , since  $8 \cdot 1 = 8$

b.  $n \cdot 3 = 1$

Multiplicative Inverse Property

$n = \frac{1}{3}$ , since  $\frac{1}{3} \cdot 3 = 1$

**Example 2** Name the property used to justify each statement.

a.  $5 + 4 = 5 + 4$

Reflexive Property

b. If  $n = 12$ , then  $4n = 4 \cdot 12$ .

Substitution Property

### Exercises

Name the property used in each equation. Then find the value of  $n$ .

1.  $6n = 6$

2.  $n \cdot 1 = 8$

3.  $6 \cdot n = 6 \cdot 9$

4.  $9 = n + 9$

5.  $n + 0 = \frac{3}{8}$

6.  $\frac{3}{4} \cdot n = 1$

Name the property used in each equation.

7. If  $4 + 5 = 9$ , then  $9 = 4 + 5$ .

8.  $0 + 21 = 21$

9.  $0(15) = 0$

10.  $(1)94 = 94$

11. If  $3 + 3 = 6$  and  $6 = 3 \cdot 2$ , then  $3 + 3 = 3 \cdot 2$ .

12.  $4 + 3 = 4 + 3$

13.  $(14 - 6) + 3 = 8 + 3$

**1-4 Study Guide and Intervention** *(continued)****Identity and Equality Properties***

**Use Identity and Equality Properties** The properties of identity and equality can be used to justify each step when evaluating an expression.

***Example***

**Evaluate  $24 \cdot 1 - 8 + 5(9 \div 3 - 3)$ . Name the property used in each step.**

$$\begin{array}{ll}
 24 \cdot 1 - 8 + 5(9 \div 3 - 3) = 24 \cdot 1 - 8 + 5(3 - 3) & \text{Substitution; } 9 \div 3 = 3 \\
 = 24 \cdot 1 - 8 + 5(0) & \text{Substitution; } 3 - 3 = 0 \\
 = 24 - 8 + 5(0) & \text{Multiplicative Identity; } 24 \cdot 1 = 24 \\
 = 24 - 8 + 0 & \text{Multiplicative Property of Zero; } 5(0) = 0 \\
 = 16 + 0 & \text{Substitution; } 24 - 8 = 16 \\
 = 16 & \text{Additive Identity; } 16 + 0 = 16
 \end{array}$$

***Exercises***

**Evaluate each expression. Name the property used in each step.**

1.  $2\left[\frac{1}{4} + \left(\frac{1}{2}\right)^2\right]$

2.  $15 \cdot 1 - 9 + 2(15 \div 3 - 5)$

3.  $2(3 \cdot 5 \cdot 1 - 14) - 4 \cdot \frac{1}{4}$

4.  $18 \cdot 1 - 3 \cdot 2 + 2(6 \div 3 - 2)$

5.  $10 \div 5 - 2^2 \div 2 + 13$

6.  $3(5 - 5 \cdot 1^2) + 21 \div 7$

**1-4 Skills Practice*****Identity and Equality Properties***

Name the property used in each equation. Then find the value of  $n$ .

1.  $n + 0 = 19$

2.  $1 \cdot n = 8$

3.  $28 \cdot n = 0$

4.  $0 + n = 22$

5.  $\frac{1}{4} \cdot n = 1$

6.  $n \cdot 9 = 9$

7.  $5 = n + 5$

8.  $2 \cdot n = 2 \cdot 3$

9.  $2(9 - 3) = 2(n)$

10.  $(7 \cdot 3) + 4 = n + 4$

11.  $5 + 4 = n + 4$

12.  $n = 14 \cdot 0$

13.  $3n = 1$

14.  $11 - (18 \div 2) = 11 - n$

Evaluate each expression. Name the property used in each step.

15.  $7(16 \div 4^2)$

16.  $2[5 - (15 \div 3)]$

17.  $4 - 3[7 - (2 \cdot 3)]$

18.  $4[8 - (4 \cdot 2)] + 1$

19.  $6 + 9[10 - 2(2 + 3)]$

20.  $2(6 \div 3 - 1) \cdot \frac{1}{2}$

**1-4 Practice*****Identity and Equality Properties***

Name the property used in each equation. Then find the value of  $n$ .

1.  $n + 9 = 9$

2.  $(8 + 7)(4) = n(4)$

3.  $5n = 1$

4.  $n \cdot 0.5 = 0.1 \cdot 0.5$

5.  $49n = 0$

6.  $12 = 12 \cdot n$

Evaluate each expression. Name the property used in each step.

7.  $2 + 6(9 - 3^2) - 2$

8.  $5(14 - 39 \div 3) + 4 \cdot \frac{1}{4}$

**SALES** For Exercises 9 and 10, use the following information.

Althea paid \$5.00 each for two bracelets and later sold each for \$15.00. She paid \$8.00 each for three bracelets and sold each of them for \$9.00.

9. Write an expression that represents the profit Althea made.

10. Evaluate the expression. Name the property used in each step.

**GARDENING** For Exercises 11 and 12, use the following information.

Mr. Katz harvested 15 tomatoes from each of four plants. Two other plants produced four tomatoes each, but Mr. Katz only harvested one fourth of the tomatoes from each of these.

11. Write an expression for the total number of tomatoes harvested.

12. Evaluate the expression. Name the property used in each step.

## 1-4

**Reading to Learn Mathematics*****Identity and Equality Properties*****Pre-Activity** How are identity and equality properties used to compare data?

Read the introduction to Lesson 1-4 at the top of page 21 in your textbook.

Write an open sentence to represent the change in rank  $r$  of the University of Miami from December 11 to the final rank. Explain why the solution is the same as the solution in the introduction.

**Reading the Lesson**

1. Write the Roman numeral of the sentence that best matches each term.

- |  |   |
|--|---|
| a. additive identity _____               | I. $\frac{5}{7} \cdot \frac{7}{5} = 1$                      |
| b. multiplicative identity _____         | II. $18 = 18$   |
| c. Multiplicative Property of Zero _____ | III. $3 \cdot 1 = 3$  |
| d. Multiplicative Inverse Property _____ | IV. If $12 = 8 + 4$ , then $8 + 4 = 12$ .                   |
| e. Reflexive Property _____              | V. $6 + 0 = 6$  |
| f. Symmetric Property _____              | VI. If $2 + 4 = 5 + 1$ and $5 + 1 = 6$ , then $2 + 4 = 6$ . |
| g. Transitive Property _____             | VII. If $n = 2$ , then $5n = 5 \cdot 2$ .                   |
| h. Substitution Property _____           | VIII. $4 \cdot 0 = 0$                                       |

**Helping You Remember**

2. The prefix *trans-* means “across” or “through.” Explain how this can help you remember the meaning of the Transitive Property of Equality.

# 1-4 Enrichment

## Closure

A *binary operation* matches two numbers in a set to just one number. Addition is a binary operation on the set of whole numbers. It matches two numbers such as 4 and 5 to a single number, their sum.

If the result of a binary operation is always a member of the original set, the set is said to be *closed* under the operation. For example, the set of whole numbers is closed under addition because  $4 + 5$  is a whole number. The set of whole numbers is not closed under subtraction because  $4 - 5$  is not a whole number.

**Tell whether each operation is binary. Write *yes* or *no*.**

1. the operation  $\downarrow$ , where  $a \downarrow b$  means to choose the lesser number from  $a$  and  $b$
2. the operation  $\odot$ , where  $a \odot b$  means to cube the sum of  $a$  and  $b$
3. the operation  $sq$ , where  $sq(a)$  means to square the number  $a$
4. the operation  $exp$ , where  $exp(a, b)$  means to find the value of  $a^b$
5. the operation  $\uparrow$ , where  $a \uparrow b$  means to match  $a$  and  $b$  to any number greater than either number
6. the operation  $\Rightarrow$ , where  $a \Rightarrow b$  means to round the product of  $a$  and  $b$  up to the nearest 10

**Tell whether each set is closed under addition. Write *yes* or *no*. If your answer is *no*, give an example.**

- |                   |                      |
|-------------------|----------------------|
| 7. even numbers   | 8. odd numbers       |
| 9. multiples of 3 | 10. multiples of 5   |
| 11. prime numbers | 12. nonprime numbers |

**Tell whether the set of whole numbers is closed under each operation. Write *yes* or *no*. If your answer is *no*, give an example.**

- |                                  |                                   |
|----------------------------------|-----------------------------------|
| 13. multiplication: $a \times b$ | 14. division: $a \div b$          |
| 15. exponentiation: $a^b$        | 16. squaring the sum: $(a + b)^2$ |

## 1-5

**Study Guide and Intervention*****The Distributive Property***

**Evaluate Expressions** The Distributive Property can be used to help evaluate expressions.

<b>Distributive Property</b>	For any numbers $a$ , $b$ , and $c$ , $a(b + c) = ab + ac$ and $(b + c)a = ba + ca$ and $a(b - c) = ab - ac$ and $(b - c)a = ba - ca$ .
------------------------------	---

**Example 1**

**Rewrite  $6(8 + 10)$  using the Distributive Property. Then evaluate.**

$$\begin{aligned} 6(8 + 10) &= 6 \cdot 8 + 6 \cdot 10 && \text{Distributive Property} \\ &= 48 + 60 && \text{Multiply.} \\ &= 108 && \text{Add.} \end{aligned}$$

**Example 2**

**Rewrite  $-2(3x^2 + 5x + 1)$  using the Distributive Property. Then simplify.**

$$\begin{aligned} -2(3x^2 + 5x + 1) &= -2(3x^2) + (-2)(5x) + (-2)(1) && \text{Distributive Property} \\ &= -6x^2 + (-10x) + (-2) && \text{Multiply.} \\ &= -6x^2 - 10x - 2 && \text{Simplify.} \end{aligned}$$

**Exercises**

**Rewrite each expression using the Distributive Property. Then simplify.**

1.  $2(10 - 5)$

2.  $6(12 - t)$

3.  $3(x - 1)$

4.  $6(12 + 5)$

5.  $(x - 4)3$

6.  $-2(x + 3)$

7.  $5(4x - 9)$

8.  $3(8 - 2x)$

9.  $12\left(6 - \frac{1}{2}x\right)$

10.  $12\left(2 + \frac{1}{2}x\right)$

11.  $\frac{1}{4}(12 - 4t)$

12.  $3(2x - y)$

13.  $2(3x + 2y - z)$

14.  $(x - 2)y$

15.  $2(3a - 2b + c)$

16.  $\frac{1}{4}(16x - 12y + 4z)$

17.  $(2 - 3x + x^2)3$

18.  $-2(2x^2 + 3x + 1)$



**1-5 Study Guide and Intervention** *(continued)****The Distributive Property***

**Simplify Expressions** A **term** is a number, a variable, or a product or quotient of numbers and variables. **Like terms** are terms that contain the same variables, with corresponding variables having the same powers. The Distributive Property and properties of equalities can be used to simplify expressions. An expression is in **simplest form** if it is replaced by an **equivalent** expression with no like terms or parentheses.

***Example*****Simplify  $4(a^2 + 3ab) - ab$ .**

$$\begin{aligned}
 4(a^2 + 3ab) - ab &= 4(a^2 + 3ab) - 1ab && \text{Multiplicative Identity} \\
 &= 4a^2 + 12ab - 1ab && \text{Distributive Property} \\
 &= 4a^2 + (12 - 1)ab && \text{Distributive Property} \\
 &= 4a^2 + 11ab && \text{Substitution}
 \end{aligned}$$

***Exercises*****Simplify each expression. If not possible, write *simplified*.**

1.  $12a - a$

2.  $3x + 6x$

3.  $3x - 1$

4.  $12g - 10g + 1$

5.  $-2x - 12$

6.  $4x^2 + 3x + 7$

7.  $20a + 12a - 8$

8.  $3x^2 + 2x^2$

9.  $-6x + 3x^2 + 10x^2$

10.  $2p + \frac{1}{2}q$

11.  $10xy - 4(xy + xy)$

12.  $21c + 18c + 31b - 3b$

13.  $3x - 2x - 2y + 2y$

14.  $xy - 2xy$

15.  $12a - 12b + 12c$

16.  $4x + \frac{1}{4}(16x - 20y)$

17.  $2 - 1 - 6x + x^2$

18.  $4x^2 + 3x^2 + 2x$

**1-5 Skills Practice*****The Distributive Property*****Rewrite each expression using the Distributive Property. Then simplify.**

1.  $4(3 + 5)$

2.  $2(6 + 10)$

3.  $5(7 - 4)$

4.  $(6 - 2)8$

5.  $(a + 7)2$

6.  $7(h - 10)$

7.  $3(m + n)$

8.  $(x - y)6$

9.  $2(x - y + 1)$

10.  $3(a + b - 1)$

**Use the Distributive Property to find each product.**

11.  $5 \cdot 89$

12.  $9 \cdot 99$

13.  $15 \cdot 104$

14.  $15\left(2\frac{1}{3}\right)$

15.  $12\left(1\frac{1}{4}\right)$

16.  $8\left(3\frac{1}{8}\right)$

**Simplify each expression. If not possible, write *simplified*.**

17.  $2x + 8x$

18.  $17g + g$

19.  $16m - 10m$

20.  $12p - 8p$

21.  $2x^2 + 6x^2$

22.  $7a^2 - 2a^2$

23.  $3y^2 - 2y$

24.  $2(n + 2n)$

25.  $4(2b - b)$

26.  $3q^2 + q - q^2$

**1-5 Practice*****The Distributive Property*****Rewrite each expression using the Distributive Property. Then simplify.**

1.  $9(7 + 8)$

2.  $7(6 - 4)$

3.  $6(b + 4)$

4.  $(9 - p)3$

5.  $(5y - 3)7$

6.  $15\left(f + \frac{1}{3}\right)$

7.  $16(3b - 0.25)$

8.  $m(n + 4)$

9.  $(c - 4)d$

**Use the Distributive Property to find each product.**

10.  $9 \cdot 499$

11.  $7 \cdot 110$

12.  $21 \cdot 1004$

13.  $12 \cdot 2.5$

14.  $27\left(2\frac{1}{3}\right)$

15.  $16\left(4\frac{1}{4}\right)$

**Simplify each expression. If not possible, write *simplified*.**

16.  $w + 14w - 6w$

17.  $3(5 + 6h)$

18.  $14(2r - 3)$

19.  $12b^2 + 9b^2$

20.  $25t^3 - 17t^3$

21.  $c^2 + 4d^2 - d^2$

22.  $3a^2 + 6a + 2b^2$

23.  $4(6p + 2q - 2p)$

24.  $x + \frac{2}{3}x + \frac{x}{3}$

**DINING OUT For Exercises 25 and 26, use the following information.**

The Ross family recently dined at an Italian restaurant. Each of the four family members ordered a pasta dish that cost \$11.50, a drink that cost \$1.50, and dessert that cost \$2.75.

25. Write an expression that could be used to calculate the cost of the Ross' dinner before adding tax and a tip.

26. What was the cost of dining out for the Ross family?

**ORIENTATION For Exercises 27 and 28, use the following information.**

Madison College conducted a three-day orientation for incoming freshmen. Each day, an average of 110 students attended the morning session and an average of 160 students attended the afternoon session.

27. Write an expression that could be used to determine the total number of incoming freshmen who attended the orientation.

28. What was the attendance for all three days of orientation?

## 1-5

**Reading to Learn Mathematics*****The Distributive Property*****Pre-Activity** How can the Distributive Property be used to calculate quickly?

Read the introduction to Lesson 1-5 at the top of page 26 in your textbook.

How would you find the amount spent by each of the first eight customers at Instant Replay Video Games on Saturday?

**Reading the Lesson**

1. Explain how the Distributive Property could be used to rewrite  $3(1 + 5)$ .
2. Explain how the Distributive Property can be used to rewrite  $5(6 - 4)$ .
3. Write three examples of each type of term.

Term	Example
number	
variable	
product of a number and a variable	
quotient of a number and variable	

4. Tell how you can use the Distributive Property to write  $12m + 8m$  in simplest form. Use the word *coefficient* in your explanation.

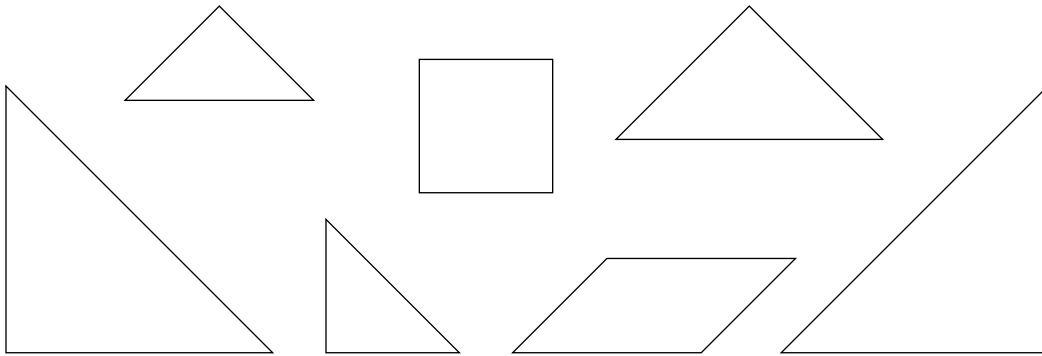
**Helping You Remember**

5. How can the everyday meaning of the word *identity* help you to understand and remember what the additive identity is and what the multiplicative identity is?

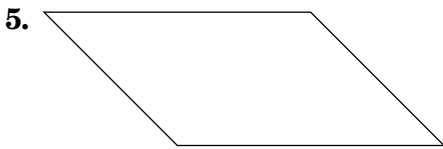
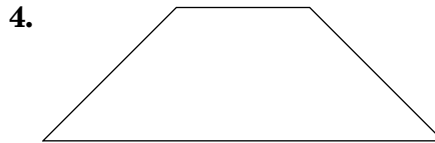
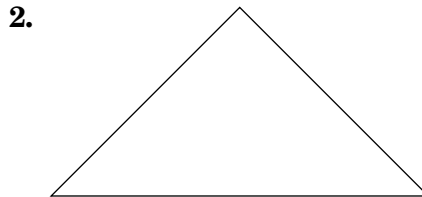
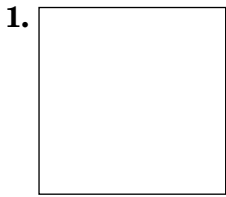
# 1-5 Enrichment

## Tangram Puzzles

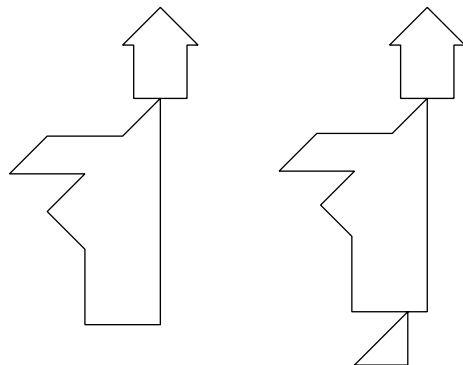
The seven geometric figures shown below are called **tans**. They are used in a very old Chinese puzzle called **tangrams**.



Glue the seven tans on heavy paper and cut them out. Use all seven pieces to make each shape shown. Record your solutions below.



6. Each of the two figures shown at the right is made from all seven tans. They seem to be exactly alike, but one has a triangle at the bottom and the other does not. Where does the second figure get this triangle?



## 1-6

## Study Guide and Intervention

## Commutative and Associative Properties

**Commutative and Associative Properties** The Commutative and Associative Properties can be used to simplify expressions. The Commutative Properties state that the order in which you add or multiply numbers does not change their sum or product. The Associative Properties state that the way you group three or more numbers when adding or multiplying does not change their sum or product.

<b>Commutative Properties</b>	For any numbers $a$ and $b$ , $a + b = b + a$ and $a \cdot b = b \cdot a$ .
<b>Associative Properties</b>	For any numbers $a$ , $b$ , and $c$ , $(a + b) + c = a + (b + c)$ and $(ab)c = a(bc)$ .

**Example 1****Evaluate  $6 \cdot 2 \cdot 3 \cdot 5$ .**

$$\begin{aligned}
 6 \cdot 2 \cdot 3 \cdot 5 &= 6 \cdot 3 \cdot 2 \cdot 5 && \text{Commutative Property} \\
 &= (6 \cdot 3)(2 \cdot 5) && \text{Associative Property} \\
 &= 18 \cdot 10 && \text{Multiply.} \\
 &= 180 && \text{Multiply.}
 \end{aligned}$$

The product is 180.

**Example 2****Evaluate** **$8.2 + 2.5 + 2.5 + 1.8$ .**

$$\begin{aligned}
 8.2 + 2.5 + 2.5 + 1.8 \\
 &= 8.2 + 1.8 + 2.5 + 2.5 && \text{Commutative Prop.} \\
 &= (8.2 + 1.8) + (2.5 + 2.5) && \text{Associative Prop.} \\
 &= 10 + 5 && \text{Add.} \\
 &= 15 && \text{Add.}
 \end{aligned}$$

The sum is 15.

**Exercises****Evaluate each expression.**

1.  $12 + 10 + 8 + 5$

2.  $16 + 8 + 22 + 12$

3.  $10 \cdot 7 \cdot 2.5$

4.  $4 \cdot 8 \cdot 5 \cdot 3$

5.  $12 + 20 + 10 + 5$

6.  $26 + 8 + 4 + 22$

7.  $3\frac{1}{2} + 4 + 2\frac{1}{2} + 3$

8.  $\frac{3}{4} \cdot 12 \cdot 4 \cdot 2$

9.  $3.5 + 2.4 + 3.6 + 4.2$

10.  $4\frac{1}{2} + 5 + \frac{1}{2} + 3$

11.  $0.5 \cdot 2.8 \cdot 4$

12.  $2.5 + 2.4 + 2.5 + 3.6$

13.  $\frac{4}{5} \cdot 18 \cdot 25 \cdot \frac{2}{9}$

14.  $32 \cdot \frac{1}{5} \cdot \frac{1}{2} \cdot 10$

15.  $\frac{1}{4} \cdot 7 \cdot 16 \cdot \frac{1}{7}$

16.  $3.5 + 8 + 2.5 + 2$

17.  $18 \cdot 8 \cdot \frac{1}{2} \cdot \frac{1}{9}$

18.  $\frac{3}{4} \cdot 10 \cdot 16 \cdot \frac{1}{2}$

**1-6 Study Guide and Intervention** *(continued)***Commutative and Associative Properties**

**Simplify Expressions** The Commutative and Associative Properties can be used along with other properties when evaluating and simplifying expressions.

**Example****Simplify  $8(y + 2x) + 7y$ .**

$$\begin{aligned} 8(y + 2x) + 7y &= 8y + 16x + 7y && \text{Distributive Property} \\ &= 8y + 7y + 16x && \text{Commutative (+)} \\ &= (8 + 7)y + 16x && \text{Distributive Property} \\ &= 15y + 16x && \text{Substitution} \end{aligned}$$

The simplified expression is  $15y + 16x$ .

**Exercises**

**Simplify each expression.**

1.  $4x + 3y + x$

2.  $3a + 4b + a$

3.  $8rs + 2rs^2 + 7rs$

4.  $3a^2 + 4b + 10a^2$

5.  $6(x + y) + 2(2x + y)$

6.  $6n + 2(4n + 5)$

7.  $6(a + b) + a + 3b$

8.  $5(2x + 3y) + 6(y + x)$

9.  $5(0.3x + 0.1y) + 0.2x$

10.  $\frac{2}{3} + \frac{1}{2}(x + 10) + \frac{4}{3}$

11.  $z^2 + 9x^2 + \frac{4}{3}z^2 + \frac{1}{3}x^2$

12.  $6(2x + 4y) + 2(x + 9)$

**Write an algebraic expression for each verbal expression. Then simplify.**

13. twice the sum of  $y$  and  $z$  is increased by  $y$

14. four times the product of  $x$  and  $y$  decreased by  $2xy$

15. the product of five and the square of  $a$ , increased by the sum of eight,  $a^2$ , and 4

16. three times the sum of  $x$  and  $y$  increased by twice the sum of  $x$  and  $y$

## 1-6

**Skills Practice*****Commutative and Associative Properties*****Evaluate each expression.**

1.  $16 + 8 + 14 + 12$

2.  $36 + 23 + 14 + 7$

3.  $32 + 14 + 18 + 11$

4.  $5 \cdot 3 \cdot 4 \cdot 3$

5.  $2 \cdot 4 \cdot 5 \cdot 3$

6.  $5 \cdot 7 \cdot 10 \cdot 4$

7.  $1.7 + 0.8 + 1.3$

8.  $1.6 + 0.9 + 2.4$

9.  $4\frac{1}{2} + 6 + 5\frac{1}{2}$

**Simplify each expression.**

10.  $2x + 5y + 9x$

11.  $a + 9b + 6a$

12.  $2p + 3q + 5p + 2q$

13.  $r + 3s + 5r + s$

14.  $5m^2 + 3m + m^2$

15.  $6k^2 + 6k + k^2 + 9k$

16.  $2a + 3(4 + a)$

17.  $5(7 + 2g) + 3g$

**Write an algebraic expression for each verbal expression. Then simplify, indicating the properties used.**18. three times the sum of  $a$  and  $b$  increased by  $a$ 19. twice the sum of  $p$  and  $q$  increased by twice the sum of  $2p$  and  $3q$



## 1-6

## Practice

**Commutative and Associative Properties**

Evaluate each expression.

1.  $13 + 23 + 12 + 7$

2.  $6 \cdot 5 \cdot 10 \cdot 3$

3.  $7.6 + 3.2 + 9.4 + 1.3$

4.  $3.6 \cdot 0.7 \cdot 5$

5.  $7\frac{1}{9} + 2 + 1\frac{2}{9}$

6.  $3\frac{3}{4} \cdot 3\frac{1}{3} \cdot 16$

Simplify each expression.

7.  $9s^2 + 3t + s^2 + t$

8.  $(p + 2n) + 7p$

9.  $6y + 2(4y + 6)$

10.  $2(3x + y) + 5(x + 2y)$

11.  $3(2c + d) + 4(c + 4d)$

12.  $6s + 2(t + 3s) + 5(s + 4t)$

13.  $5(0.6b + 0.4c) + b$

14.  $\frac{1}{2}q + 2\left(\frac{1}{4}q + \frac{1}{2}r\right)$

15. Write an algebraic expression for *four times the sum of  $2a$  and  $b$  increased by twice the sum of  $6a$  and  $2b$* . Then simplify, indicating the properties used.

**SCHOOL SUPPLIES** For Exercises 16 and 17, use the following information.

Kristen purchased two binders that cost \$1.25 each, two binders that cost \$4.75 each, two packages of paper that cost \$1.50 per package, four blue pens that cost \$1.15 each, and four pencils that cost \$.35 each.

16. Write an expression to represent the total cost of supplies before tax.

17. What was the total cost of supplies before tax?

**GEOMETRY** For Exercises 18 and 19, use the following information.

The lengths of the sides of a pentagon in inches are 1.25, 0.9, 2.5, 1.1, and 0.25.

18. Using the commutative and associative properties to group the terms in a way that makes evaluation convenient, write an expression to represent the perimeter of the pentagon.

19. What is the perimeter of the pentagon?

## 1-6

**Reading to Learn Mathematics*****Commutative and Associative Properties*****Pre-Activity** How can properties help you determine distances?

Read the introduction to Lesson 1-6 at the top of page 32 in your textbook.

How are the expressions  $0.4 + 1.5$  and  $1.5 + 0.4$  alike? different?

**Reading the Lesson**

1. Write the Roman numeral of the term that best matches each equation.

a.  $3 + 6 = 6 + 3$  \_\_\_\_\_

I. Associative Property of Addition

b.  $2 + (3 + 4) = (2 + 3) + 4$  \_\_\_\_\_

II. Associative Property of Multiplication

c.  $2 \cdot (3 \cdot 4) = (2 \cdot 3) \cdot 4$  \_\_\_\_\_

III. Commutative Property of Addition

d.  $2 \cdot (3 \cdot 4) = 2 \cdot (4 \cdot 3)$  \_\_\_\_\_

IV. Commutative Property of Multiplication

2. What property can you use to change the order of the terms in an expression?

3. What property can you use to change the way three factors are grouped?

4. What property can you use to combine two like terms to get a single term?

5. To use the Associative Property of Addition to rewrite the sum of a group of terms, what is the least number of terms you need?

**Helping You Remember**

6. Look up the word *commute* in a dictionary. Find an everyday meaning that is close to the mathematical meaning and explain how it can help you remember the mathematical meaning.

# 1-6 Enrichment

## Properties of Operations

Let's make up a new operation and denote it by  $\otimes$ , so that  $a \otimes b$  means  $b^a$ .

$$2 \otimes 3 = 3^2 = 9$$

$$(1 \otimes 2) \otimes 3 = 2^1 \otimes 3 = 3^2 = 9$$

1. What number is represented by  $2 \otimes 3$ ?
2. What number is represented by  $3 \otimes 2$ ?
3. Does the operation  $\otimes$  appear to be commutative?
4. What number is represented by  $(2 \otimes 1) \otimes 3$ ?
5. What number is represented by  $2 \otimes (1 \otimes 3)$ ?
6. Does the operation  $\otimes$  appear to be associative?

Let's make up another operation and denote it by  $\oplus$ , so that  $a \oplus b = (a + 1)(b + 1)$ .

$$3 \oplus 2 = (3 + 1)(2 + 1) = 4 \cdot 3 = 12$$

$$(1 \oplus 2) \oplus 3 = (2 \cdot 3) \oplus 3 = 6 \oplus 3 = 7 \cdot 4 = 28$$

7. What number is represented by  $2 \oplus 3$ ?
8. What number is represented by  $3 \oplus 2$ ?
9. Does the operation  $\oplus$  appear to be commutative?
10. What number is represented by  $(2 \oplus 3) \oplus 4$ ?
11. What number is represented by  $2 \oplus (3 \oplus 4)$ ?
12. Does the operation  $\oplus$  appear to be associative?
13. What number is represented by  $1 \otimes (3 \oplus 2)$ ?
14. What number is represented by  $(1 \otimes 3) \oplus (1 \otimes 2)$ ?
15. Does the operation  $\otimes$  appear to be distributive over the operation  $\oplus$ ?
16. Let's explore these operations a little further. What number is represented by  $3 \otimes (4 \oplus 2)$ ?
17. What number is represented by  $(3 \otimes 4) \oplus (3 \otimes 2)$ ?
18. Is the operation  $\otimes$  actually distributive over the operation  $\oplus$ ?

## 1-7

**Study Guide and Intervention****Logical Reasoning**

**Conditional Statements** A **conditional statement** is a statement of the form *If A, then B*. Statements in this form are called **if-then statements**. The part of the statement immediately following the word *if* is called the **hypothesis**. The part of the statement immediately following the word *then* is called the **conclusion**.

**Example 1** Identify the hypothesis and conclusion of each statement.

- a. **If it is Wednesday, then Jerri has aerobics class.**

Hypothesis: it is Wednesday

Conclusion: Jerri has aerobics class

- b. **If  $2x - 4 < 10$ , then  $x < 7$ .**

Hypothesis:  $2x - 4 < 10$

Conclusion:  $x < 7$

**Example 2** Identify the hypothesis and conclusion of each statement. Then write the statement in if-then form.

- a. **You and Marylynn can watch a movie on Thursday.**

Hypothesis: it is Thursday

Conclusion: you and Marylynn can watch a movie  
If it is Thursday, then you and Marylynn can watch a movie.

- b. **For a number  $a$  such that  $3a + 2 = 11$ ,  $a = 3$ .**

Hypothesis:  $3a + 2 = 11$

Conclusion:  $a = 3$

If  $3a + 2 = 11$ , then  $a = 3$ .

**Exercises**

**Identify the hypothesis and conclusion of each statement.**

- If it is April, then it might rain.
- If you are a sprinter, then you can run fast.
- If  $12 - 4x = 4$ , then  $x = 2$ .
- If it is Monday, then you are in school.
- If the area of a square is 49, then the square has side length 7.

**Identify the hypothesis and conclusion of each statement. Then write the statement in if-then form.**

- A quadrilateral with equal sides is a rhombus.
- A number that is divisible by 8 is also divisible by 4.
- Karlyn goes to the movies when she does not have homework.

**1-7 Study Guide and Intervention** *(continued)***Logical Reasoning**

**Deductive Reasoning and Counterexamples** **Deductive reasoning** is the process of using facts, rules, definitions, or properties to reach a valid conclusion. To show that a conditional statement is false, use a **counterexample**, one example for which the conditional statement is false. You need to find only one counterexample for the statement to be false.

**Example 1** Determine a valid conclusion from the statement *If two numbers are even, then their sum is even for the given conditions. If a valid conclusion does not follow, write no valid conclusion and explain why.*

**a. The two numbers are 4 and 8.**

4 and 8 are even, and  $4 + 8 = 12$ . Conclusion: The sum of 4 and 8 is even.

**b. The sum of two numbers is 20.**

Consider 13 and 7.  $13 + 7 = 20$

However,  $12 + 8$ ,  $19 + 1$ , and  $18 + 2$  all equal 20. There is no way to determine the two numbers. Therefore there is no valid conclusion.

**Example 2** Provide a counterexample to this conditional statement. *If you use a calculator for a math problem, then you will get the answer correct.*

Counterexample: If the problem is  $475 \div 5$  and you press  $475 - 5$ , you will not get the correct answer.

**Exercises**

Determine a valid conclusion that follows from the statement *If the last digit of a number is 0 or 5, then the number is divisible by 5 for the given conditions. If a valid conclusion does not follow, write no valid conclusion and explain why.*

1. The number is 120.

2. The number is a multiple of 4.

3. The number is 101.

**Find a counterexample for each statement.**

4. If Susan is in school, then she is in math class.

5. If a number is a square, then it is divisible by 2.

6. If a quadrilateral has 4 right angles, then the quadrilateral is a square.

7. If you were born in New York, then you live in New York.

8. If three times a number is greater than 15, then the number must be greater than six.

9. If  $3x - 2 \leq 10$ , then  $x < 4$ .

## 1-7

**Skills Practice*****Logical Reasoning***

**Identify the hypothesis and conclusion of each statement.**

1. If it is Sunday, then mail is not delivered.
2. If you are hiking in the mountains, then you are outdoors.
3. If  $6n + 4 > 58$ , then  $n > 9$ .

**Identify the hypothesis and conclusion of each statement. Then write the statement in if-then form.**

4. Martina works at the bakery every Saturday.
5. Ivan only runs early in the morning.
6. A polygon that has five sides is a pentagon.

**Determine whether a valid conclusion follows from the statement *If Hector scores an 85 or above on his science exam, then he will earn an A in the class for the given condition. If a valid conclusion does not follow, write no valid conclusion and explain why.***

7. Hector scored an 86 on his science exam.
8. Hector did not earn an A in science.
9. Hector scored 84 on the science exam.
10. Hector studied 10 hours for the science exam.

**Find a counterexample for each statement.**

11. If the car will not start, then it is out of gas.
12. If the basketball team has scored 100 points, then they must be winning the game.
13. If the Commutative Property holds for addition, then it holds for subtraction.
14. If  $2n + 3 < 17$ , then  $n \leq 7$ .

## 1-7

**Practice*****Logical Reasoning***

Identify the hypothesis and conclusion of each statement.

1. If it is raining, then the meteorologist's prediction was accurate.
2. If  $x = 4$ , then  $2x + 3 = 11$ .

Identify the hypothesis and conclusion of each statement. Then write the statement in if-then form.

3. When Joseph has a fever, he stays home from school.
4. Two congruent triangles are similar.

Determine whether a valid conclusion follows from the statement *If two numbers are even, then their product is even* for the given condition. If a valid conclusion does not follow, write *no valid conclusion* and explain why.

5. The product of two numbers is 12.
6. Two numbers are 8 and 6.

Find a counterexample for each statement.

7. If the refrigerator stopped running, then there was a power outage.
8. If  $6h - 7 < 5$ , then  $h \leq 2$ .

**GEOMETRY** For Exercises 9 and 10, use the following information.

If the perimeter of a rectangle is 14 inches, then its area is 10 square inches.

9. State a condition in which the hypothesis and conclusion are valid.
10. Provide a counterexample to show the statement is false.
11. **ADVERTISING** A recent television commercial for a car dealership stated that "no reasonable offer will be refused." Identify the hypothesis and conclusion of the statement. Then write the statement in if-then form.

## 1-7

**Reading to Learn Mathematics*****Logical Reasoning*****Pre-Activity** How is logical reasoning helpful in cooking?

Read the introduction to Lesson 1-7 at the top of page 37 in your textbook.  
What are the two possible reasons given for the popcorn burning?

**Reading the Lesson**

- Write *hypothesis* or *conclusion* to tell which part of the if-then statement is underlined.
  - If it is Tuesday, then it is raining.
  - If our team wins this game, then they will go to the playoffs.
  - I can tell you your birthday if you tell me your height.
  - If  $3x + 7 = 13$ , then  $x = 2$ .
  - If  $x$  is an even number, then  $x \div 2$  is an odd number.
- What does the term *valid conclusion* mean?
- Give a counterexample for the statement *If a person is famous, then that person has been on television*. Tell how you know it really is a counterexample.

**Helping You Remember**

- Write an example of a conditional statement you would use to teach someone how to identify an hypothesis and a conclusion.



# 1-7 Enrichment

## Counterexamples

Some statements in mathematics can be proven false by **counterexamples**. Consider the following statement.

For any numbers  $a$  and  $b$ ,  $a - b = b - a$ .

You can prove that this statement is false in general if you can find one example for which the statement is false.

Let  $a = 7$  and  $b = 3$ . Substitute these values in the equation above.

$$7 - 3 \stackrel{?}{=} 3 - 7$$

$$4 \neq -4$$

In general, for any numbers  $a$  and  $b$ , the statement  $a - b = b - a$  is false. You can make the equivalent verbal statement: subtraction is *not* a commutative operation.

**In each of the following exercises  $a$ ,  $b$ , and  $c$  are any numbers. Prove that the statement is false by counterexample.**

1.  $a - (b - c) \stackrel{?}{=} (a - b) - c$

2.  $a \div (b \div c) \stackrel{?}{=} (a \div b) \div c$

3.  $a \div b \stackrel{?}{=} b \div a$

4.  $a \div (b + c) \stackrel{?}{=} (a \div b) + (a \div c)$

5.  $a + (bc) \stackrel{?}{=} (a + b)(a + c)$

6.  $a^2 + a^2 \stackrel{?}{=} a^4$

7. Write the verbal equivalents for Exercises 1, 2, and 3.

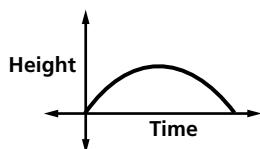
8. For the distributive property  $a(b + c) = ab + ac$  it is said that multiplication distributes over addition. Exercises 4 and 5 prove that some operations do not distribute. Write a statement for each exercise that indicates this.

# 1-8 Study Guide and Intervention

## Graphs and Functions

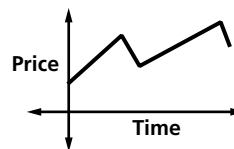
**Interpret Graphs** A **function** is a relationship between input and output values. In a function, there is exactly one output for each input. The input values are associated with the **independent variable**, and the output values are associated with the **dependent variable**. Functions can be graphed without using a scale to show the general shape of the graph that represents the function.

**Example 1** The graph below represents the height of a football after it is kicked downfield. Identify the independent and the dependent variable. Then describe what is happening in the graph.



The independent variable is time, and the dependent variable is height. The football starts on the ground when it is kicked. It gains altitude until it reaches a maximum height, then it loses altitude until it falls to the ground.

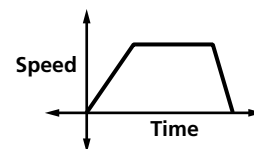
**Example 2** The graph below represents the price of stock over time. Identify the independent and dependent variable. Then describe what is happening in the graph.



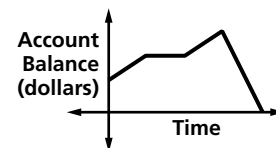
The independent variable is time and the dependent variable is price. The price increases steadily, then it falls, then increases, then falls again.

### Exercises

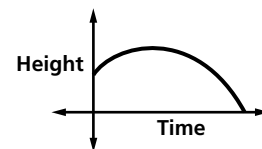
1. The graph represents the speed of a car as it travels to the grocery store. Identify the independent and dependent variable. Then describe what is happening in the graph.



2. The graph represents the balance of a savings account over time. Identify the independent and the dependent variable. Then describe what is happening in the graph.



3. The graph represents the height of a baseball after it is hit. Identify the independent and the dependent variable. Then describe what is happening in the graph.



# 1-8 Study Guide and Intervention *(continued)*

## Graphs and Functions

**Draw Graphs** You can represent the graph of a function using a **coordinate system**. Input and output values are represented on the graph using **ordered pairs** of the form  $(x, y)$ . The  $x$ -value, called the  **$x$ -coordinate**, corresponds to the  $x$ -axis, and the  $y$ -value, or  **$y$ -coordinate** corresponds to the  $y$ -axis. Graphs can be used to represent many real-world situations.

**Example**

A music store advertises that if you buy 3 CDs at the regular price of \$16, then you will receive one CD of the same or lesser value free.

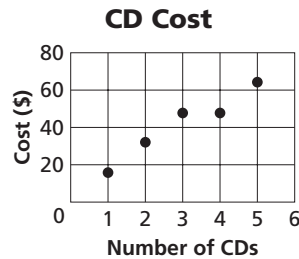
- a. Make a table showing the cost of buying 1 to 5 CDs.

Number of CDs	1	2	3	4	5
Total Cost (\$)	16	32	48	48	64

- b. Write the data as a set of ordered pairs.

$(1, 16), (2, 32), (3, 48), (4, 48), (5, 64)$

- c. Draw a graph that shows the relationship between the number of CDs and the total cost.

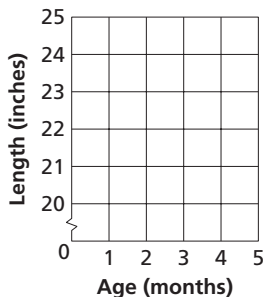


**Exercises**

1. The table below represents the length of a baby versus its age in months.

Age (months)	0	1	2	3	4
Length (inches)	20	21	23	23	24

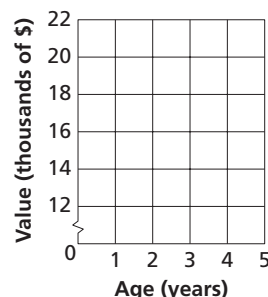
- Identify the independent and dependent variables.
- Write a set of ordered pairs representing the data in the table.
- Draw a graph showing the relationship between age and length.



2. The table below represents the value of a car versus its age.

Age (years)	0	1	2	3	4
Value (\$)	20,000	18,000	16,000	14,000	13,000

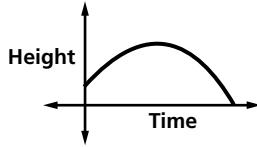
- Identify the independent and dependent variables.
- Write a set of ordered pairs representing the data in the table.
- Draw a graph showing the relationship between age and value.



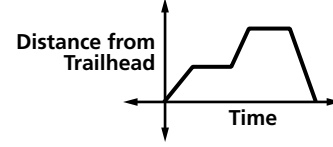
# 1-8 Skills Practice

## Graphs and Functions

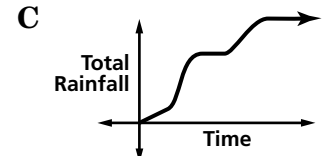
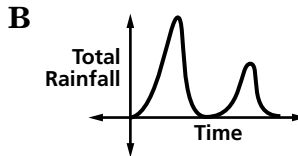
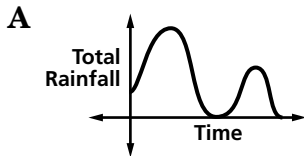
1. The graph below represents the path of a football thrown in the air. Describe what is happening in the graph.



2. The graph below represents a puppy exploring a trail. Describe what is happening in the graph.



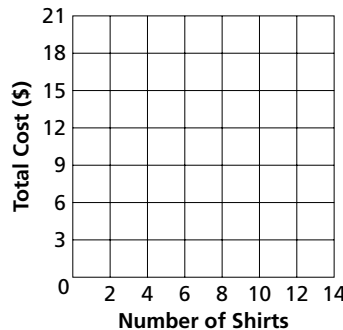
3. **WEATHER** During a storm, it rained lightly for a while, then poured heavily, and then stopped for a while. Then it rained moderately for a while before finally ending. Which graph represents this situation?



**LAUNDRY** For Exercises 4–7, use the table that shows the charges for washing and pressing shirts at a cleaners.

Number of Shirts	2	4	6	8	10	12
Total Cost (\$)	3	6	9	12	15	18

- Identify the independent and dependent variables.
- Write the ordered pairs the table represents.
- Draw a graph of the data.

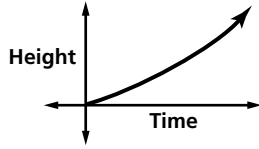


7. Use the data to predict the cost for washing and pressing 16 shirts.

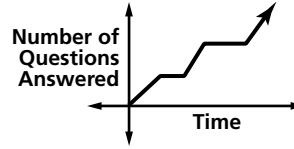
# 1-8 Practice

## Graphs and Functions

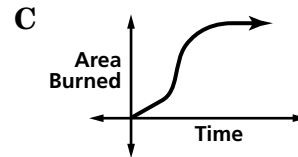
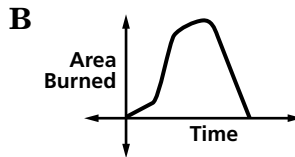
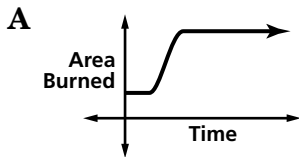
1. The graph below represents the height of a tsunami (tidal wave) as it approaches shore. Describe what is happening in the graph.



2. The graph below represents a student taking an exam. Describe what is happening in the graph.



3. **FOREST FIRES** A forest fire grows slowly at first, then rapidly as the wind increases. After firefighters answer the call, the fire grows slowly for a while, but then the firefighters contain the fire before extinguishing it. Which graph represents this situation?

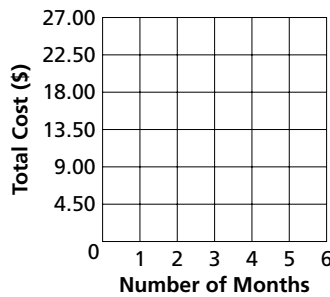


**INTERNET NEWS SERVICE** For Exercises 4–6, use the table that shows the monthly charges for subscribing to an independent news server.

Number of Months	1	2	3	4	5
Total Cost (\$)	4.50	9.00	13.50	18.00	22.50

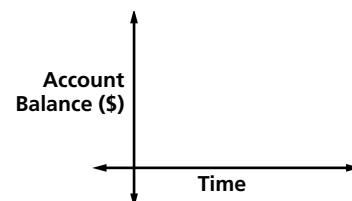
4. Write the ordered pairs the table represents.

5. Draw a graph of the data.



6. Use the data to predict the cost of subscribing for 9 months.

7. **SAVINGS** Jennifer deposited a sum of money in her account and then deposited equal amounts monthly for 5 months, nothing for 3 months, and then resumed equal monthly deposits. Sketch a reasonable graph of the account history.



# 1-8 Reading to Learn Mathematics

## Graphs and Functions

### Pre-Activity How can real-world situations be modeled using graphs and functions?

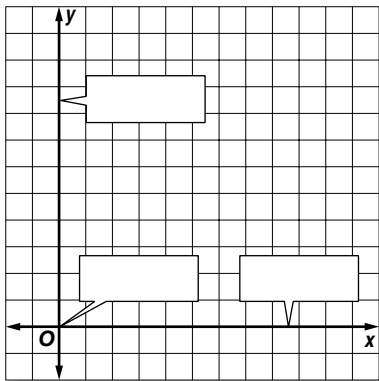
Read the introduction to Lesson 1-8 at the top of page 43 in your textbook.

The numbers 25%, 50% and 75% represent the

\_\_\_\_\_ and the numbers 0 through 10 represent the \_\_\_\_\_.

### Reading the Lesson

- Write another name for each term.
  - coordinate system
  - horizontal axis
  - vertical axis
- Identify each part of the coordinate system.



- In your own words, tell what is meant by the terms *dependent variable* and *independent variable*. Use the example below.

dependent variable		independent variable
the distance it takes to stop a motor vehicle	is a function of	the speed at which the vehicle is traveling
$d$		$s$

### Helping You Remember

- In the alphabet,  $x$  comes before  $y$ . Use this fact to describe a method for remembering how to write ordered pairs.

# 1-8 Enrichment

## The Digits of $\pi$

The number  $\pi$  (pi) is the ratio of the circumference of a circle to its diameter. It is a nonrepeating and nonterminating decimal. The digits of  $\pi$  never form a pattern. Listed at the right are the first 200 digits that follow the decimal point of  $\pi$ .

3.14159 26535 89793 23846  
 69399 37510 58209 74944  
 86280 34825 34211 70679  
 09384 46095 50582 23172  
 84102 70193 85211 05559  
 26433 83279 50288 41971  
 59230 78164 06286 20899  
 82148 08651 32823 06647  
 53594 08128 34111 74502  
 64462 29489 54930 38196

**Solve each problem.**

- Suppose each of the digits in  $\pi$  appeared with equal frequency. How many times would each digit appear in the first 200 places following the decimal point?
- Complete this frequency table for the first 200 digits of  $\pi$  that follow the decimal point.

Digit	Frequency (Tally Marks)	Frequency (Number)	Cumulative Frequency
0			
1			
2			
3			
4			
5			
6			
7			
8			
9			

- Explain how the cumulative frequency column can be used to check a project like this one.
- Which digit(s) appears most often?
- Which digit(s) appears least often?

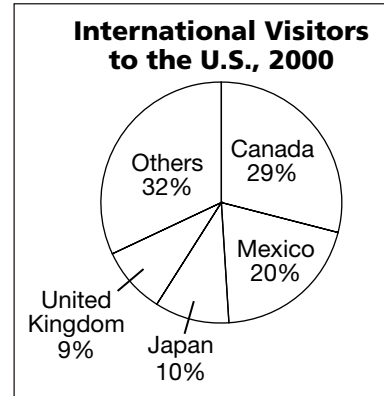
1-9

# Study Guide and Intervention

## Statistics: Analyzing Data by Using Tables and Graphs

**Analyze Data** Graphs or tables can be used to display data. A **bar graph** compares different categories of data, while a **circle graph** compares parts of a set of data as a percent of the whole set. A **line graph** is useful to show how a data set changes over time.

**Example** The circle graph at the right shows the number of international visitors to the United States in 2000, by country.

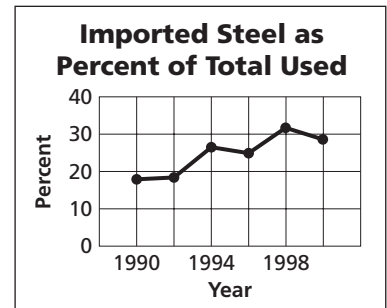


Source: Tlnet

- If there were a total of 50,891,000 visitors, how many were from Mexico?  
 $50,891,000 \times 20\% = 10,178,200$
- If the percentage of visitors from each country remains the same each year, how many visitors from Canada would you expect in the year 2003 if the total is 59,000,000 visitors?  
 $59,000,000 \times 29\% = 17,110,000$

### Exercises

- The graph shows the use of imported steel by U. S. companies over a 10-year period.
  - Describe the general trend in the graph.
  - What would be a reasonable prediction for the percentage of imported steel used in 2002?



Source: Chicago Tribune

- The table shows the percentage of change in worker productivity at the beginning of each year for a 5-year period.
  - Which year shows the greatest percentage increase in productivity?
  - What does the negative percent in the first quarter of 2001 indicate?

Year (1st Qtr.)	% of Change
1997	+1
1998	+4.6
1999	+2
2000	+2.1
2001	-1.2

Source: Chicago Tribune



# 1-9 Study Guide and Intervention *(continued)*

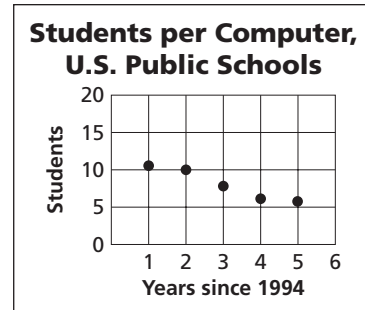
## Statistics: Analyzing Data by Using Tables and Graphs

**Misleading Graphs** Graphs are very useful for displaying data. However, some graphs can be confusing, easily misunderstood, and lead to false assumptions. These graphs may be mislabeled or contain incorrect data. Or they may be constructed to make one set of data appear greater than another set.

**Example**

The graph at the right shows the number of students per computer in the U.S. public schools for the school years from 1995 to 1999. Explain how the graph misrepresents the data.

The values are difficult to read because the vertical scale is too condensed. It would be more appropriate to let each unit on the vertical scale represent 1 student rather than 5 students and have the scale go from 0 to 12.

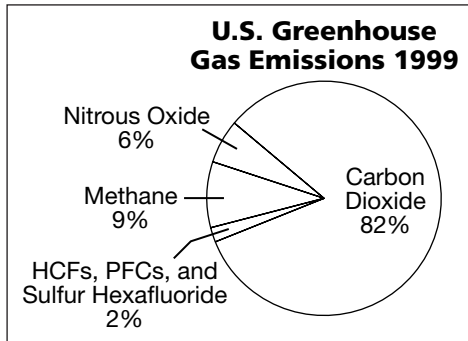


Source: *The World Almanac*

**Exercises**

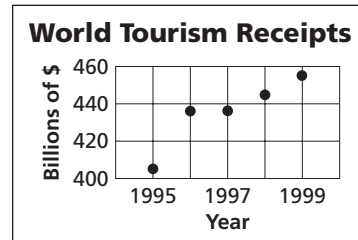
Explain how each graph misrepresents the data.

- The graph below shows the U.S. greenhouse gases emissions for 1999.



Source: Department of Energy

- The graph below shows the amount of money spent on tourism for 1998-99.

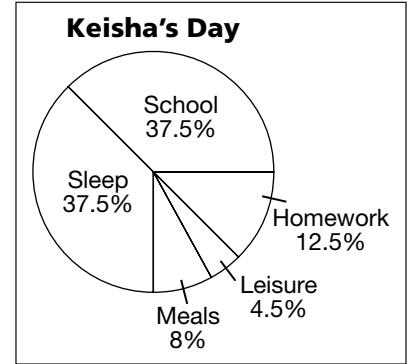


Source: *The World Almanac*

# 1-9 Skills Practice

## Statistics: Analyzing Data by Using Tables and Graphs

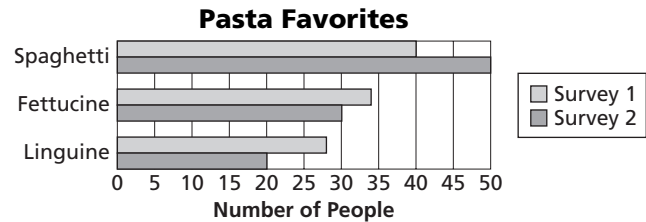
**DAILY LIFE** For Exercises 1–3, use the circle graph that shows the percent of time Keisha spends on activities in a 24-hour day.



1. What percent of her day does Keisha spend in the combined activities of school and doing homework?
2. How many hours per day does Keisha spend at school?
3. How many hours does Keisha spend on leisure and meals?

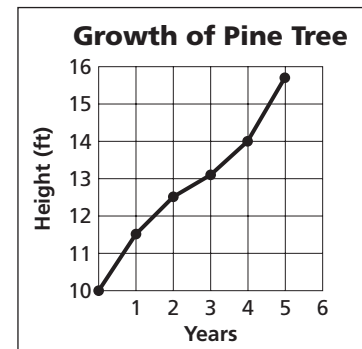
**PASTA FAVORITES** For Exercises 4–8, use the table and bar graph that show the results of two surveys asking people their favorite type of pasta.

	Spaghetti	Fettuccine	Linguine
Survey 1	40	34	28
Survey 2	50	30	20



4. According to the graph, what is the ranking for favorite pasta in both surveys?
5. In Survey 1, the number of votes for spaghetti is twice the number of votes for which pasta in Survey 2?
6. How many more people preferred spaghetti in Survey 2 than preferred spaghetti in Survey 1?
7. How many more people preferred fettuccine to linguine in Survey 1?
8. If you want to know the exact number of people who preferred spaghetti over linguine in Survey 1, which is a better source, the table or the graph? Explain.

**PLANT GROWTH** For Exercises 9 and 10, use the line graph that shows the growth of a Ponderosa pine over 5 years.



9. Explain how the graph misrepresents the data.
10. How can the graph be redrawn so that it is not misleading?

# 1-9 Practice

## Statistics: Analyzing Data by Using Tables and Graphs

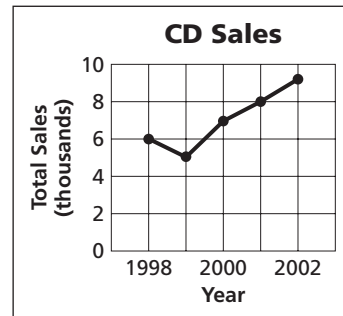
**MINERAL IDENTIFICATION** For Exercises 1–4, use the following information.

The table shows *Moh's hardness scale*, used as a guide to help identify minerals. If mineral A scratches mineral B, then A's hardness number is greater than B's. If B cannot scratch A, then B's hardness number is less than or equal to A's.

Mineral	Hardness
Talc	1
Gypsum	2
Calcite	3
Fluorite	4
Apatite	5
Orthoclase	6
Quartz	7
Topaz	8
Corundum	9
Diamond	10

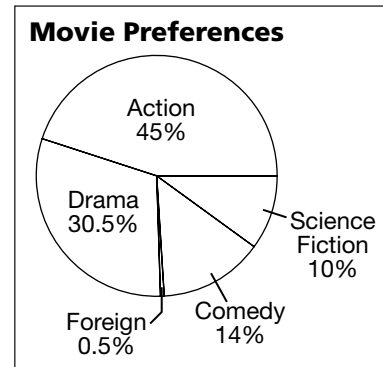
- Which mineral(s) will fluorite scratch?
- A fingernail has a hardness of 2.5. Which mineral(s) will it scratch?
- Suppose quartz will not scratch an unknown mineral. What is the hardness of the unknown mineral?
- If an unknown mineral scratches all the minerals in the scale up to 7, and corundum scratches the unknown, what is the hardness of the unknown?

**SALES** For Exercises 5 and 6, use the line graph that shows CD sales at Berry's Music for the years 1998–2002.



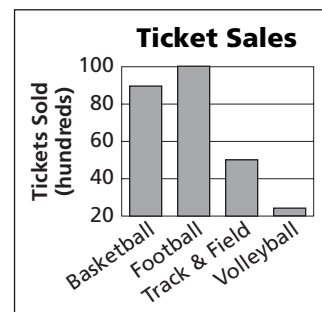
- Which one-year period shows the greatest growth in sales?
- Describe the sales trend.

**MOVIE PREFERENCES** For Exercises 7–9, use the circle graph that shows the percent of people who prefer certain types of movies.



- If 400 people were surveyed, how many chose action movies as their favorite?
- Of 1000 people at a movie theater on a weekend, how many would you expect to prefer drama?
- What percent of people chose a category other than action or drama?

**TICKET SALES** For Exercises 10 and 11, use the bar graph that compares annual sports ticket sales at Mars High.



- Describe why the graph is misleading.
- What could be done to make the graph more accurate?

## 1-9

**Reading to Learn Mathematics****Statistics: Analyzing Data by Using Tables and Graphs****Pre-Activity Why are graphs and tables used to display data?**

Read the introduction to Lesson 1-9 at the top of page 50 in your textbook.

Compare your reaction to the statement, *A stack containing George W. Bush's votes from Florida would be 970.1 feet tall, while a stack of Al Gore's votes would be 970 feet tall* with your reaction to the graph shown in the introduction. Write a brief description of which presentation works best for you.

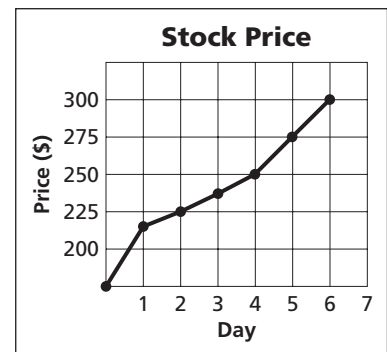
**Reading the Lesson**

1. Choose from the following types of graphs as you complete each statement.

bar graph      circle graph      line graph

- A \_\_\_\_\_ compares parts of a set of data as a percent of the whole set.
- \_\_\_\_\_ are useful when showing how a set of data changes over time.
- \_\_\_\_\_ are helpful when making predictions.
- \_\_\_\_\_ can be used to display multiple sets of data in different categories at the same time.
- The percents in a \_\_\_\_\_ should always have a sum of 100%.
- A \_\_\_\_\_ compares different categories of numerical information, or data.

2. Explain how the graph is misleading.

**Helping You Remember**

- Describe something in your daily routine that you can connect with bar graphs and circle graphs to help you remember their special purpose.

# 1-9 Enrichment

## Percentiles

The table at the right shows test scores and their frequencies. The frequency is the number of people who had a particular score. The cumulative frequency is the total frequency up to that point, starting at the lowest score and adding up.

Score	Frequency	Cumulative Frequency
95	1	50
90	2	49
85	5	47
80	6	42
75	7	36
70	8	29
65	7	21
60	6	14
55	4	8
50	3	4
45	1	1

### Example 1 What score is at the 16th percentile?

A score at the 16th percentile means the score just above the lowest 16% of the scores.

16% of the 50 scores is 8 scores.

The 8th score is 55.

The score just above this is 56.

So, the score at the 16th percentile is 56.

Notice that no one had a score of 56 points.

Use the table above to find the score at each percentile.

- 42nd percentile
- 70th percentile
- 33rd percentile
- 90th percentile
- 58th percentile
- 80th percentile

### Example 2 At what percentile is a score of 75?

There are 29 scores below 75.

Seven scores are at 75. The fourth of these seven is the midpoint of this group.

Adding 4 scores to the 29 gives 33 scores.

33 out of 50 is 66%.

Thus, a score of 75 is at the 66th percentile.

Use the table above to find the percentile of each score.

- a score of 50
- a score of 77
- a score of 85
- a score of 58
- a score of 62
- a score of 81

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1-1

Study Guide and Intervention

Variables and Expressions

**Write Mathematical Expressions** In the algebraic expression,  $\ell w$ , the letters  $\ell$  and  $w$  are called **variables**. In algebra, a variable is used to represent unspecified numbers or values. Any letter can be used as a variable. The letters  $\ell$  and  $w$  are used above because they are the first letters of the words *length* and *width*. In the expression  $\ell w$ ,  $\ell$  and  $w$  are called factors, and the result is called the **product**.

**Example 1**

**Write an algebraic expression for each verbal expression.**

- a. **four more than a number  $n$**   
The words *more than* imply addition.  
four more than a number  $n$   
 $4 + n$   
The algebraic expression is  $4 + n$ .
- b. **the difference of a number squared and 8**  
The expression *difference of* implies subtraction.  
the difference of a number squared and 8  
 $n^2 - 8$   
The algebraic expression is  $n^2 - 8$ .

**Example 2**

**Evaluate each expression.**

- a.  **$3^4$**   
 $3^4 = 3 \cdot 3 \cdot 3 \cdot 3$  Use 3 as a factor 4 times.  
 $= 81$  Multiply.
- b. **five cubed**  
*Cubed* means raised to the third power.  
 $5^3 = 5 \cdot 5 \cdot 5$  Use 5 as a factor 3 times.  
 $= 125$  Multiply.

**Exercises**

**Write an algebraic expression for each verbal expression.**

- 1. a number decreased by 8  **$b - 8$**
- 2. a number divided by 8  **$\frac{n}{8}$**
- 3. a number squared  **$n^2$**
- 4. four times a number  **$4n$**
- 5. a number divided by 6  **$\frac{n}{6}$**
- 6. a number multiplied by 37  **$37n$**
- 7. the sum of 9 and a number  **$9 + n$**
- 8. 3 less than 5 times a number  **$5n - 3$**
- 9. twice the sum of 15 and a number  **$2(15 + n)$**
- 10. one-half the square of  $b$   **$\frac{1}{2}b^2$**
- 11. 7 more than the product of 6 and a number  **$6n + 7$**
- 12. 30 increased by 3 times the square of a number  **$30 + 3n^2$**

**Evaluate each expression.**

- 13.  $5^2$  **25**
- 14.  $3^3$  **27**
- 16.  $12^2$  **144**
- 17.  $8^3$  **512**
- 15.  $10^4$  **10,000**
- 18.  $2^8$  **256**

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Glencoe Algebra 1

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1-1

Study Guide and Intervention

Variables and Expressions

**Write Verbal Expressions** Translating algebraic expressions into verbal expressions is important in algebra.

**Example**

**Write a verbal expression for each algebraic expression.**

- a.  **$6n^2$**   
the product of 6 and  $n$  squared
- b.  **$n^3 - 12n$**   
the difference of  $n$  cubed and twelve times  $n$

**Exercises**

**Write a verbal expression for each algebraic expression. 1–18. Sample answers are given.**

- 1.  $w - 1$  **one less than  $w$**
- 2.  $\frac{1}{3}a^3$  **one third the cube of  $a$**
- 3.  $81 + 2x$  **eighty-one increased by twice  $x$**
- 4.  $12c$  **12 times  $c$**
- 5.  $8^2$  **eight to the fourth power**
- 6.  $6^2$  **the square of 6**
- 7.  $2n^2 + 4$  **the sum of 4 and twice the square of  $n$**
- 8.  $a^3 \cdot b^3$   **$a$  cubed times  $b$  cubed**
- 9.  $2x^3 - 3$  **the difference of twice a number cubed and 3**
- 10.  $\frac{6k^3}{5}$  **6 times the cube of  $k$  divided by 5**
- 11.  $\frac{1}{4}b^2$  **one-fourth the square of  $b$**
- 12.  $7n^5$  **seven times the fifth power of  $n$**
- 13.  $3x + 4$  **the sum of three times a number and 4**
- 14.  $\frac{2}{3}k^5$  **two-thirds the fifth power of  $k$**
- 15.  $3b^2 + 2d^3$  **3 times  $b$  squared plus 2 times  $d$  cubed**
- 16.  $4(n^2 + 1)$  **4 times the sum of the square of  $n$  and 1**
- 17.  $3^2 + 2^3$  **3 squared plus 2 cubed**
- 18.  $6n^2 + 3$  **the sum of 6 times  $n$  squared and 3**

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Glencoe Algebra 1

NAME _____	DATE _____	PERIOD _____
<b>1-1 Skills Practice</b> <b>Variables and Expressions</b>	<b>1-1 Practice (Average)</b> <b>Variables and Expressions</b>	
Write an algebraic expression for each verbal expression.		
1. the sum of a number and 10 <b><math>x + 10</math></b>	1. the difference of 10 and $u$ <b><math>10 - u</math></b>	2. the sum of 18 and a number <b><math>18 + x</math></b>
3. the product of 18 and $q$ <b><math>18q</math></b>	3. the product of 33 and $j$ <b><math>33j</math></b>	4. 74 increased by 3 times $y$ <b><math>74 + 3y</math></b>
5. 8 increased by three times a number <b><math>8 + 3x</math></b>	5. 15 decreased by twice a number <b><math>15 - 2x</math></b>	6. 91 more than the square of a number <b><math>x^2 + 91</math></b>
7. the product of 2 and the second power of $y$ <b><math>2y^2</math></b>	7. three fourths the square of $b$ <b><math>\frac{3}{4}b^2</math></b>	8. two fifths the cube of a number <b><math>\frac{2}{5}x^3</math></b>
Evaluate each expression.	Evaluate each expression.	
9. $8^2$ <b>64</b>	9. $11^2$ <b>121</b>	10. $8^3$ <b>512</b>
11. $5^3$ <b>125</b>	12. $4^5$ <b>1024</b>	13. $9^3$ <b>729</b>
13. $10^2$ <b>100</b>	15. $10^5$ <b>100,000</b>	16. $12^3$ <b>1728</b>
15. $7^2$ <b>49</b>	17. $100^4$ <b>100,000,000</b>	
17. $7^3$ <b>343</b>		
Write a verbal expression for each algebraic expression. 19–26. Sample answers are given.	Write a verbal expression for each algebraic expression. 18–25. Sample answers are given.	
19. $9a$ the product of 9 and $a$	18. $23f$ the product of 23 and $f$	19. $7^3$ seven cubed
21. $c + 2d$ the sum of $c$ and twice $d$	20. $5m^2 + 2$ 2 more than 5 times $m$ squared	21. $4d^3 - 10$ 4 times $d$ cubed minus 10
23. $2b^2$ 2 times $b$ squared	22. $x^3 \cdot y^4$ $x$ cubed times $y$ to the fourth power	23. $b^2 - 3c^3$ $b$ squared minus 3 times $c$ cubed
25. $p^4 + 6q$ $p$ to the fourth power plus 6 times $q$	24. $\frac{k^5}{6}$ one sixth of the fifth power of $k$	25. $\frac{4t^2}{7}$ one seventh of 4 times $t$ squared
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3	4	4

Lesson 1-1

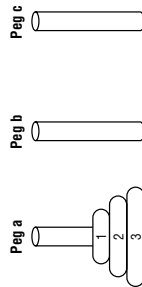


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## 1-1 Enrichment

### The Tower of Hanoi

The diagram at the right shows the Tower of Hanoi puzzle. Notice that there are three pegs, with a stack of disks on peg a. The object is to move all of the disks to another peg. You may move only one disk at a time and a larger disk may never be put on top of a smaller disk. As you solve the puzzle, record each move in the table shown. The first two moves are recorded.



	Peg a	Peg b	Peg c
1.	1		
2.	2		
3.	3		
4.	2		
5.	3	1	1
6.	3	2	1
7.	3	1	2
8.	3	2	3
9.	1	2	3
10.	1	2	3
11.	1	2	3

#### Solve.

- Complete the table to solve the Tower of Hanoi puzzle for three disks.
- Another way to record each move is to use letters. For example, the first two moves in the table can be recorded as 1c, 2b. This shows that disk 1 is moved to peg c, and then disk 2 is moved to peg b. Record your solution using letters.  
**1c, 2b, 1b, 3c, 1a, 2c, 1c**
- On a separate sheet of paper, solve the puzzle for four disks. Record your solution.  
**1c, 2b, 1b, 3c, 1a, 2c, 1c, 4b, 1b, 2a, 1a, 3b, 1c, 2b, 1b**
- Solve the puzzle for five disks. Record your solution.  
**1c, 2b, 1b, 3c, 1a, 2c, 1c, 4b, 1b, 2a, 1a, 3b, 1c, 2b, 1b, 5c, 1a, 2c, 1c, 3a, 1b, 2a, 1a, 4c, 1c, 2b, 1b, 3c, 1a, 2c, 1c**
- Suppose you start with an odd number of disks and you want to end with the stack on peg c. What should be your first move?  
**1c**
- Suppose you start with an even number of disks and you want to end with the stack on peg b. What should be your first move?  
**1c**

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## 1-1 Reading to Learn Mathematics

### Variables and Expressions

**Pre-Activity** What expression can be used to find the perimeter of a baseball diamond?

Read the introduction to Lesson 1-1 at the top of page 6 in your textbook. Then complete the description of the expression  $4s$ .  
In the expression  $4s$ , 4 represents the **number** of sides and  $s$  represents the **length** of each side.

#### Reading the Lesson

- Why is the symbol  $\times$  avoided in algebra?  
**It is easily confused with the variable  $x$ .**
- What are the factors in the algebraic expression  $3xy$ ?  
**3,  $x$ ,  $y$**
- In the expression  $x^n$ , what is the base? What is the exponent?  
 **$x$ ;  $n$**
- Write the Roman numeral of the algebraic expression that best matches each phrase.
  - three more than a number  $n$  **IV**
  - five times the difference of  $x$  and 4 **I**
  - one half the number  $r$  **III**
  - the product of  $x$  and  $y$  divided by 2 **V**
  - $x$  to the fourth power **II**

#### Helping You Remember

- Multiplying 5 times 3 is not the same as raising 5 to the third power. How does the way you write "5 times 3" and "5 to the third power" in symbols help you remember that they give different results?  
**Sample answer: "5 times 3" is written with the numbers 5 and 3 on the same level, as in  $5 \cdot 3$  or  $5(3)$ . "5 to the third power" is written as  $5^3$ , with the exponent 3 on a higher level than the number 5.**

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Glencoe Algebra 1



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## 1-2 Study Guide and Intervention (continued)

### Order of Operations

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**Evaluate Rational Expressions** Numerical expressions often contain more than one operation. To evaluate them, use the rules for order of operations shown below.

**Order of Operations**

**Step 1** Evaluate expressions inside grouping symbols.

**Step 2** Evaluate all powers.

**Step 3** Do all multiplication and/or division from left to right.

**Step 4** Do all addition and/or subtraction from left to right.

**Example 1** Evaluate each expression.

**a.**  $7 + 2 \cdot 4 - 4$   
 $7 + 2 \cdot 4 - 4 = 7 + 8 - 4$  Multiply 2 and 4.  
 $= 15 - 4$  Add 7 and 8.  
 $= 11$  Subtract 4 from 15.

**b.**  $3(2) + 4(2 + 6)$   
 $3(2) + 4(2 + 6) = 3(2) + 4(8)$  Add 2 and 6.  
 $= 6 + 32$  Multiply left to right.  
 $= 38$  Add 6 and 32.

**Example 2** Evaluate each expression.

**a.**  $3[2 + (12 \div 3)^2]$   
 $3[2 + (12 \div 3)^2] = 3(2 + 4^2)$  Divide 12 by 3.  
 $= 3(2 + 16)$  Find 4 squared.  
 $= 3(18)$  Add 2 and 16.  
 $= 54$  Multiply 3 and 18.

**b.**  $\frac{3 + 2^3}{4^2 \cdot 3}$   
 $\frac{3 + 2^3}{4^2 \cdot 3} = \frac{3 + 8}{4^2 \cdot 3}$  Evaluate power in numerator.  
 $= \frac{11}{4^2 \cdot 3}$  Add 3 and 8 in the numerator.  
 $= \frac{11}{16 \cdot 3}$  Evaluate power in denominator.  
 $= \frac{11}{48}$  Multiply.

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## 1-2 Study Guide and Intervention

### Order of Operations

NAME \_\_\_\_\_ DATE \_\_\_\_\_ PERIOD \_\_\_\_\_

**Evaluate Algebraic Expressions** Algebraic expressions may contain more than one operation. Algebraic expressions can be evaluated if the values of the variables are known. First, replace the variables by their values. Then use the order of operations to calculate the value of the resulting numerical expression.

**Example** Evaluate  $x^3 + 5(y - 3) + 5(y - 3)$  if  $x = 2$  and  $y = 12$ .

$x^3 + 5(y - 3) = 2^3 + 5(12 - 3)$  Replace  $x$  with 2 and  $y$  with 12.  
 $= 8 + 5(12 - 3)$  Evaluate  $2^3$ .  
 $= 8 + 5(9)$  Subtract 3 from 12.  
 $= 8 + 45$  Multiply 5 and 9.  
 $= 53$  Add 8 and 45.

The solution is 53.

**Exercises**

Evaluate each expression if  $x = 2$ ,  $y = 3$ ,  $z = 4$ ,  $a = \frac{4}{5}$ , and  $b = \frac{3}{5}$ .

1.  $x + 7$  **9**

4.  $x^3 + y + z^2$  **27**

7.  $\frac{y^2}{x^2} \cdot 4$  **9**

10.  $(10x)^2 + 100a$  **480**

13.  $\frac{z^2 - y^2}{x^2} \cdot 4$  **7**

16.  $\frac{25ab + y}{xz}$  **18**

19.  $\left(\frac{x}{z}\right)^2 + \left(\frac{y}{z}\right)^2$   **$\frac{13}{16}$**

2.  $3x - 5$  **1**

5.  $6a + 8b$   **$9\frac{3}{5}$**

8.  $2xyz + 5$  **53**

11.  $\frac{3xy - 4}{7x}$  **1**

14.  $6xz + 5xy$  **78**

17.  $\frac{5a^2b}{y}$   **$\frac{16}{25}$**

20.  $\frac{x + z}{y + 2z}$   **$\frac{6}{11}$**

3.  $x + y^2$  **11**

6.  $23 - (a + b)$   **$21\frac{3}{5}$**

9.  $x(2y + 3z)$  **36**

12.  $a^2 + 2b$   **$1\frac{21}{25}$**

15.  $\frac{(z - y)^2}{x}$   **$\frac{1}{2}$**

18.  $(z \div x)^2 + ax$   **$5\frac{3}{5}$**

21.  $\left(\frac{z \div x}{y}\right) + \left(\frac{y \div x}{z}\right)$   **$1\frac{1}{24}$**

<div style="display: flex; justify-content: space-between;"> <span>NAME _____</span> <span>DATE _____</span> <span>PERIOD _____</span> </div> <div style="display: flex; justify-content: center; align-items: center; margin-top: 10px;"> <div style="border: 1px solid black; border-radius: 50%; width: 40px; height: 40px; display: flex; align-items: center; justify-content: center; margin-right: 10px;"> <span style="font-size: 24px; font-weight: bold;">1-2</span> </div> <div style="text-align: center;"> <h2 style="margin: 0;">Practice (Average)</h2> <h3 style="margin: 0;">Order of Operations</h3> </div> </div> <p>Evaluate each expression.</p>	<div style="display: flex; justify-content: space-between;"> <span>NAME _____</span> <span>DATE _____</span> <span>PERIOD _____</span> </div> <div style="display: flex; justify-content: center; align-items: center; margin-top: 10px;"> <div style="border: 1px solid black; border-radius: 50%; width: 40px; height: 40px; display: flex; align-items: center; justify-content: center; margin-right: 10px;"> <span style="font-size: 24px; font-weight: bold;">1-2</span> </div> <div style="text-align: center;"> <h2 style="margin: 0;">Skills Practice</h2> <h3 style="margin: 0;">Order of Operations</h3> </div> </div> <p>Evaluate each expression.</p>
<p>1. <math>(15 - 5) \cdot 2</math> <b>20</b></p> <p>2. <math>9 \cdot (3 + 4)</math> <b>63</b></p> <p>3. <math>5 + 7 \cdot 4</math> <b>33</b></p> <p>4. <math>12 + 5 - 6 \cdot 2</math> <b>5</b></p> <p>5. <math>7 \cdot 9 - 4(6 + 7)</math> <b>11</b></p> <p>6. <math>8 \div (2 + 2) \cdot 7</math> <b>14</b></p> <p>7. <math>4(3 + 5) - 5 \cdot 4</math> <b>12</b></p> <p>8. <math>22 \div 11 \cdot 9 - 3^2</math> <b>9</b></p> <p>9. <math>6^2 + 3 \cdot 7 - 9</math> <b>48</b></p> <p>10. <math>3[10 - (27 \div 9)]</math> <b>21</b></p> <p>11. <math>2[5^2 + (36 \div 6)]</math> <b>62</b></p> <p>12. <math>162 \div [6(7 - 4)^2]</math> <b>3</b></p> <p>13. <math>\frac{5^2 \cdot 4 - 5 \cdot 4^2}{5(4)}</math> <b>1</b></p> <p>14. <math>\frac{(2 \cdot 5)^2 + 4}{3^2 - 5}</math> <b>26</b></p> <p>15. <math>\frac{7 + 3^2}{4^2 \cdot 2}</math> <b>2</b></p> <p>Evaluate each expression if <math>a = 12</math>, <math>b = 9</math>, and <math>c = 4</math>.</p> <p>16. <math>a^2 + b - c^2</math> <b>137</b></p> <p>17. <math>b^2 + 2a - c^2</math> <b>89</b></p> <p>18. <math>2c(a + b)</math> <b>168</b></p> <p>19. <math>4a + 2b - c^2</math> <b>50</b></p> <p>20. <math>(a^2 \div 4b) + c</math> <b>8</b></p> <p>21. <math>c^2 \cdot (2b - a)</math> <b>96</b></p> <p>22. <math>\frac{bc^2 + a}{c}</math> <b>39</b></p> <p>23. <math>\frac{2c^3 - ab}{4}</math> <b>5</b></p> <p>24. <math>\frac{2(a - b)^2}{5c}</math> <b><math>\frac{9}{10}</math></b></p> <p>25. <math>\frac{b^2 - 2c^2}{a + c - b}</math> <b>7</b></p>	<p>1. <math>(5 + 4) \cdot 7</math> <b>63</b></p> <p>2. <math>(9 - 2) \cdot 3</math> <b>21</b></p> <p>3. <math>4 + 6 \cdot 3</math> <b>22</b></p> <p>4. <math>28 - 5 \cdot 4</math> <b>8</b></p> <p>5. <math>12 + 2 \cdot 2</math> <b>16</b></p> <p>6. <math>(3 + 5) \cdot 5 + 1</math> <b>41</b></p> <p>7. <math>9 + 4(3 + 1)</math> <b>25</b></p> <p>8. <math>2 + 3 \cdot 5 + 4</math> <b>21</b></p> <p>9. <math>30 - 5 \cdot 4 + 2</math> <b>12</b></p> <p>10. <math>10 + 2 \cdot 6 + 4</math> <b>26</b></p> <p>11. <math>14 \div 7 \cdot 5 - 3^2</math> <b>1</b></p> <p>12. <math>6 + 3 \cdot 7 + 2^3</math> <b>22</b></p> <p>13. <math>4[30 - (10 - 2) \cdot 3]</math> <b>24</b></p> <p>14. <math>5 + [30 - (6 - 1)^2]</math> <b>10</b></p> <p>15. <math>2[12 + (5 - 2)^2]</math> <b>42</b></p> <p>16. <math>[8 \cdot 2 - (3 + 9)] + [8 - 2 \cdot 3]</math> <b>6</b></p> <p>Evaluate each expression if <math>x = 6</math>, <math>y = 8</math>, and <math>z = 3</math>.</p> <p>17. <math>xy + z</math> <b>51</b></p> <p>18. <math>yz - x</math> <b>18</b></p> <p>19. <math>2x + 3y - z</math> <b>33</b></p> <p>20. <math>2(x + z) - y</math> <b>10</b></p> <p>21. <math>5z + (y - x)</math> <b>17</b></p> <p>22. <math>5x - (y + 2z)</math> <b>16</b></p> <p>23. <math>x^2 + y^2 - 10z</math> <b>70</b></p> <p>24. <math>z^3 + (y^2 - 4x)</math> <b>67</b></p> <p>25. <math>\frac{y + xz}{2}</math> <b>13</b></p> <p>26. <math>\frac{3y + x^2}{z}</math> <b>20</b></p>
<p><b>CAR RENTAL For Exercises 26 and 27, use the following information.</b></p> <p>Ann Carlyle is planning a business trip for which she needs to rent a car. The car rental company charges \$36 per day plus \$0.50 per mile over 100 miles. Suppose Ms. Carlyle rents the car for 5 days and drives 180 miles.</p> <p>26. Write an expression for how much it will cost Ms. Carlyle to rent the car. <b><math>5(36) + 0.5(180 - 100)</math></b></p> <p>27. Evaluate the expression to determine how much Ms. Carlyle must pay the car rental company. <b>\$220.00</b></p> <p><b>GEOMETRY For Exercises 28 and 29, use the following information.</b></p> <p>The length of a rectangle is <math>3n + 2</math> and its width is <math>n - 1</math>. The perimeter of the rectangle is twice the sum of its length and its width.</p> <p>28. Write an expression that represents the perimeter of the rectangle. <b><math>2[(3n + 2) + (n - 1)]</math></b></p> <p>29. Find the perimeter of the rectangle when <math>n = 4</math> inches. <b>34 in.</b></p>	<p>© Glencoe/McGraw-Hill</p> <p style="text-align: right;">9</p> <p style="text-align: right;">Glencoe Algebra 1</p>
<p>© Glencoe/McGraw-Hill</p> <p style="font-size: 24px; font-weight: bold;">A6</p>	<p>© Glencoe/McGraw-Hill</p> <p style="font-size: 24px; font-weight: bold;">10</p> <p style="text-align: right;">Glencoe Algebra 1</p>

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## 1-2 Reading to Learn Mathematics

### Order of Operations

#### Pre-Activity How is the monthly cost of internet service determined?

Read the introduction to Lesson 1-2 at the top of page 11 in your textbook.  
 In the expression  $4.95 + 0.99(117 - 100)$ , **4.95** represents the regular monthly cost of internet service, **0.99** represents the cost of each additional hour after 100 hours, and **(117 - 100)** represents the number of hours over 100 used by Nicole in a given month.

#### Reading the Lesson

1. The first step in evaluating an expression is to evaluate inside grouping symbols. List four types of grouping symbols found in algebraic expressions.  
**parentheses, brackets, braces, and fraction bars**

2. What does *evaluate powers* mean? Use an example to explain.

**Sample answer: To evaluate a power means to find the value of the power. To evaluate  $4^3$ , find the value of  $4 \times 4 \times 4$ .**

3. Read the order of operations on page 11 in your textbook. For each of the following expressions, write *addition*, *subtraction*, *multiplication*, *division*, or *evaluate powers* to tell what operation to use first when evaluating the expression.

- a.  $400 - 5(12 + 9)$  **addition**
- b.  $26 - 8 + 14$  **subtraction**
- c.  $17 + 3 \cdot 6$  **multiplication**
- d.  $69 + 57 \div 3 + 16 \cdot 4$  **division**
- e.  $\frac{19 + 3 \cdot 4}{6 \div 2}$  **multiplication**
- f.  $\frac{51 \div 729}{9^2}$  **evaluate powers**

#### Helping You Remember

4. The sentence *Please Excuse My Dear Aunt Sally* (PEMDAS) is often used to remember the order of operations. The letter P represents parentheses and other grouping symbols. Write what each of the other letters in PEMDAS means when using the order of operations.

**E—exponents (powers), M—multiply, D—divide, A—add, S—subtract**

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## 1-2 Enrichment

### The Four Digits Problem

One well-known mathematic problem is to write expressions for consecutive numbers beginning with 1. On this page, you will use the digits 1, 2, 3, and 4. Each digit is used only once. You may use addition, subtraction, multiplication (not division), exponents, and parentheses in any way you wish. Also, you can use two digits to make one number, such as 12 or 34.

**Express each number as a combination of the digits 1, 2, 3, and 4. Answers will vary. Sample answers are given.**

- 1 =  $(3 \times 1) - (4 - 2)$      18 =  $(2 \times 3) \times (4 - 1)$      35 =  $2^{(4+1)} + 3$
- 2 =  $(4 - 3) + (2 - 1)$      19 =  $3(2 + 4) + 1$      36 =  $34 + (2 \times 1)$
- 3 =  $(4 - 3) + (2 \times 1)$      20 =  $21 - (4 - 3)$      37 =  $31 + 2 + 4$
- 4 =  $(4 - 2) + (3 - 1)$      21 =  $(4 + 3) \times (2 + 1)$      38 =  $42 - (3 + 1)$
- 5 =  $(4 - 2) + (3 \times 1)$      22 =  $21 + (4 - 3)$      39 =  $42 - (3 \times 1)$
- 6 =  $4 + 3 + 1 - 2$      23 =  $31 - (4 \times 2)$      40 =  $41 - (3 - 2)$
- 7 =  $3(4 - 1) - 2$      24 =  $(2 + 4) \times (3 + 1)$      41 =  $43 - (2 \times 1)$
- 8 =  $4 + 3 + 2 - 1$      25 =  $(2 + 3) \times (4 + 1)$      42 =  $43 - (2 - 1)$
- 9 =  $4 + 2 + (3 \times 1)$      26 =  $24 + (3 - 1)$      43 =  $42 + 1^3$
- 10 =  $4 + 3 + 2 + 1$      27 =  $3^2 \times (4 - 1)$      44 =  $43 + (2 - 1)$
- 11 =  $(4 \times 3) - (2 - 1)$      28 =  $21 + 3 + 4$      45 =  $43 + (2 \times 1)$
- 12 =  $(4 \times 3) \times (2 - 1)$      29 =  $2^{(4+1)} - 3$      46 =  $43 + (2 + 1)$
- 13 =  $(4 \times 3) + (2 - 1)$      30 =  $(2 \times 3) \times (4 + 1)$      47 =  $31 + 4^2$
- 14 =  $(4 \times 3) + (2 \times 1)$      31 =  $34 - (2 + 1)$      48 =  $4^2 \times (3 \times 1)$
- 15 =  $2(3 + 4) + 1$      32 =  $4^2 \times (3 - 1)$      49 =  $41 + 2^3$
- 16 =  $(4 \times 2) \times (3 - 1)$      33 =  $21 + (3 \times 4)$      50 =  $41 + 3^2$
- 17 =  $3(2 + 4) - 1$      34 =  $2 \times (14 + 3)$

Does a calculator help in solving these types of puzzles? Give reasons for your opinion.

**Answers will vary. Using a calculator is a good way to check your solutions.**

# 1-3 Study Guide and Intervention (continued)

## Open Sentences

**Solve Inequalities** An open sentence that contains the symbol  $<$ ,  $\leq$ ,  $>$ , or  $\geq$  is called an **inequality**. Inequalities can be solved the same way that equations are solved.

**Example** Find the solution set for  $3a - 8 > 10$  if the replacement set is  $\{4, 5, 6, 7, 8\}$ .

Replace  $a$  in  $3a - 8 > 10$  with each value in the replacement set.

- $3(4) - 8 \not> 10 \rightarrow 4 > 10$  false
- $3(5) - 8 \not> 10 \rightarrow 7 > 10$  false
- $3(6) - 8 \not> 10 \rightarrow 10 > 10$  false
- $3(7) - 8 \not> 10 \rightarrow 13 > 10$  true
- $3(8) - 8 \not> 10 \rightarrow 16 > 10$  true

Since replacing  $a$  with 7 or 8 makes the inequality  $3a - 8 > 10$  true, the solution set is  $\{7, 8\}$ .

### Exercises

Find the solution set for each inequality if the replacement set is  $X = \{0, 1, 2, 3, 4, 5, 6, 7\}$ .

- 1.  $x + 2 > 4$   **$\{3, 4, 5, 6, 7\}$**
- 2.  $x + 3 < 6$   **$\{0, 1, 2, 3, 4, 5, 6, 7\}$**
- 3.  $3x > 18$   **$\{7\}$**
- 4.  $\frac{x}{3} > 1$   **$\{4, 5, 6, 7\}$**
- 5.  $\frac{3x}{5} \geq 2$  **no numbers**
- 6.  $\frac{3x}{8} \leq 2$   **$\{0, 1, 2, 3, 4, 5\}$**
- 7.  $3x - 4 > 5$   **$\{4, 5, 6, 7\}$**
- 8.  $3(8 - x) + 1 \leq 6$   **$\{2, 3, 4, 5, 6, 7\}$**

Find the solution set for each inequality if the replacement sets are  $X = \left\{\frac{1}{4}, \frac{1}{2}, 1, 2, 3, 5, 8\right\}$  and  $Y = \{2, 4, 6, 8, 10\}$

- 10.  $x + 3 > 5$   **$\{3, 5, 8\}$**
- 11.  $y \div 3 < 6$   **$\{2, 4, 6, 8, 10\}$**
- 12.  $8y + 3 \geq 51$   **$\{6, 8, 10\}$**
- 13.  $\frac{x}{2} < 4$   **$\{1, \frac{1}{2}, 1, 2, 3, 5\}$**
- 14.  $\frac{y}{4} \geq 2$   **$\{8, 10\}$**
- 15.  $\frac{2y}{5} \leq 2$   **$\{2, 4\}$**
- 16.  $4x + 1 \geq 4$   **$\{1, 2, 3, 5, 8\}$**
- 17.  $3x + 3 \geq 12$   **$\{8, 10\}$**
- 18.  $2(y + 1) \geq 18$   **$\{8, 10\}$**
- 19.  $3x - \frac{1}{4} < 2$   **$\{2\}$**
- 20.  $3y + 2 \leq 8$   **$\{2\}$**
- 21.  $\frac{1}{2}(6 - 2x) + 2 \leq 3$   **$\{2, 3, 5, 8\}$**

# 1-3 Study Guide and Intervention

## Open Sentences

**Solve Equations** A mathematical sentence with one or more variables is called an **open sentence**. Open sentences are **solved** by finding replacements for the variables that result in true sentences. The set of numbers from which replacements for a variable may be chosen is called the **replacement set**. The set of all replacements for the variable that result in true statements is called the **solution set** for the variable. A sentence that contains an equal sign,  $=$ , is called an **equation**.

**Example 1** Find the solution set of  $3a + 12 = 39$  if the replacement set is  $\{6, 7, 8, 9, 10\}$ .

- Replace  $a$  in  $3a + 12 = 39$  with each value in the replacement set.
- $3(6) + 12 \neq 39 \rightarrow 30 \neq 39$  false
- $3(7) + 12 \neq 39 \rightarrow 33 \neq 39$  false
- $3(8) + 12 \neq 39 \rightarrow 36 \neq 39$  false
- $3(9) + 12 = 39 \rightarrow 39 = 39$  true
- $3(10) + 12 \neq 39 \rightarrow 42 \neq 39$  false

Since  $a = 9$  makes the equation  $3a + 12 = 39$  true, the solution is 9. The solution set is  $\{9\}$ .

### Exercises

Find the solution of each equation if the replacement sets are  $X = \left\{\frac{1}{4}, \frac{1}{2}, 1, 2, 3\right\}$  and  $Y = \{2, 4, 6, 8\}$ .

- 1.  $x + \frac{1}{2} = \frac{5}{2}$   **$\{2\}$**
- 2.  $x + 8 = 11$   **$\{3\}$**
- 3.  $y - 2 = 6$   **$\{8\}$**
- 4.  $x^2 - 1 = 8$   **$\{3\}$**
- 5.  $y^2 - 2 = 34$   **$\{6\}$**
- 6.  $x^2 + 5 = 5\frac{1}{16}$   **$\left\{\frac{1}{4}\right\}$**
- 7.  $2(x + 3) = 7$   **$\left\{\frac{1}{2}\right\}$**
- 8.  $\frac{1}{4}(y + 1)^2 = \frac{9}{4}$   **$\{2\}$**
- 9.  $y^2 + y = 20$   **$\{4\}$**
- 10.  $a = 2^3 - 1$  **7**
- 11.  $n = 6^2 - 4^2$  **20**
- 12.  $w = 6^2 \cdot 3^2$  **324**
- 13.  $\frac{1}{4} + \frac{5}{8} = k$   **$\frac{7}{8}$**
- 14.  $\frac{18 - 3}{2 + 3} = p$  **3**
- 15.  $s = \frac{15 - 6}{27 - 24}$  **3**
- 16.  $18.4 - 3.2 = m$  **15.2**
- 17.  $k = 9.8 + 5.7$  **15.5**
- 18.  $c = 3\frac{1}{2} + 2\frac{1}{4}$  **5 $\frac{3}{4}$**

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### 1-3 Skills Practice

#### Open Sentences

Find the solution of each equation if the replacement sets are  $A = \{4, 5, 6, 7, 8\}$  and  $B = \{9, 10, 11, 12, 13\}$ .

1.  $5a - 9 = 26$  **7**                      2.  $4a - 8 = 16$  **6**

3.  $7a + 21 = 56$  **5**                      4.  $3b + 15 = 48$  **11**

5.  $4b - 12 = 28$  **10**                    6.  $\frac{36}{b} - 3 = 0$  **12**

**Find the solution of each equation using the given replacement set.**

7.  $\frac{1}{2} + x = \frac{5}{4}; \left\{ \frac{1}{2}, \frac{3}{4}, 1, \frac{5}{4} \right\}$   **$\frac{3}{4}$**

9.  $\frac{1}{4}(x + 2) = \frac{5}{6}; \left\{ \frac{2}{3}, \frac{3}{4}, \frac{5}{4} \right\}$   **$\frac{4}{3}$**

**Solve each equation.**

11.  $10.4 - 6.8 = x$  **3.6**

13.  $\frac{46 - 15}{3 + 28} = a$  **1**

15.  $\frac{2(4) + 4}{3(3 - 1)} = b$  **2**

### 1-3 Practice (Average)

#### Open Sentences

Find the solution of each equation if the replacement sets are  $A = \left\{ 0, \frac{1}{2}, 1, \frac{3}{2}, 2 \right\}$  and  $B = \{3, 3.5, 4, 4.5, 5\}$ .

1.  $a + \frac{1}{2} = 1$   **$\frac{1}{2}$**                       2.  $4b - 8 = 6$  **3.5**                      3.  $6a + 18 = 27$   **$\frac{3}{2}$**

4.  $7b - 8 = 16.5$  **3.5**                      5.  $120 - 28a = 78$   **$\frac{3}{2}$**                       6.  $\frac{28}{b} + 9 = 16$  **4**

**Find the solution of each equation using the given replacement set.**

7.  $\frac{7}{8} + x = \frac{17}{12}; \left\{ \frac{1}{2}, \frac{13}{24}, \frac{7}{12}, \frac{5}{8}, \frac{2}{3} \right\}$   **$\frac{13}{24}$**                       8.  $\frac{3}{4}(x + 2) = \frac{27}{8}; \left\{ \frac{1}{2}, 1, \frac{1}{2}, 2, \frac{1}{2} \right\}$   **$2\frac{1}{2}$**

9.  $1.4(x + 3) = 5.32; \{0.4, 0.6, 0.8, 1.0, 1.2\}$  **0.8**                      10.  $12(x + 4) = 76.8; \{2, 2.4, 2.8, 3.2, 3.6\}$  **2.4**

**Solve each equation.**

11.  $x = 18.3 - 4.8$  **13.5**                      12.  $w = 20.2 - 8.95$  **11.25**                      13.  $\frac{37 - 9}{18 - 11} = d$  **4**

14.  $\frac{97 - 25}{41 - 23} = k$  **4**                      15.  $y = \frac{4(22 - 4)}{3(6) + 6}$  **3**                      16.  $\frac{5(2^2) + 4(3)}{4(2^3 - 4)} = p$  **2**

**Find the solution set for each inequality using the given replacement set.**

17.  $a + 7 < 10; \{2, 3, 4, 5, 6, 7\}$   **$\{2\}$**                       18.  $3y \geq 42; \{10, 12, 14, 16, 18\}$   **$\{14, 16, 18\}$**

19.  $4x - 2 < 5; \{0.5, 1, 1.5, 2, 2.5\}$   **$\{0.5, 1, 1.5\}$**                       20.  $4b - 4 > 3; \{1.2, 1.4, 1.6, 1.8, 2.0\}$   **$\{1.8, 2.0\}$**

21.  $\frac{3y}{5} \leq 2; \{0, 2, 4, 6, 8, 10\}$   **$\{0, 2\}$**                       22.  $4a \geq 3; \left\{ \frac{1}{8}, \frac{1}{4}, \frac{3}{8}, \frac{1}{2}, \frac{5}{8}, \frac{3}{4} \right\}$   **$\left\{ \frac{3}{4} \right\}$**

**23. TEACHING** A teacher has 15 weeks in which to teach six chapters. Write and then solve an equation that represents the number of lessons the teacher must teach per week if there is an average of 8.5 lessons per chapter.  **$n = \frac{5(8.5)}{15}; 3.4$**

**LONG DISTANCE** For Exercises 24 and 25, use the following information.  
Gabriel talks an average of 20 minutes per long-distance call. During one month, he makes eight in-state long-distance calls averaging \$2.00 each. A 20-minute state-to-state call costs Gabriel \$1.50. His long-distance budget for the month is \$20.

24. Write an inequality that represents the number of 20 minute state-to-state calls Gabriel can make this month.  **$8(2) + 1.5s \leq 20$**

25. What is the maximum number of 20-minute state-to-state calls that Gabriel can make this month? **2**

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### 1-3 Skills Practice

#### Open Sentences

Find the solution of each equation if the replacement sets are  $A = \{4, 5, 6, 7, 8\}$  and  $B = \{9, 10, 11, 12, 13\}$ .

1.  $5a - 9 = 26$  **7**                      2.  $4a - 8 = 16$  **6**

3.  $7a + 21 = 56$  **5**                      4.  $3b + 15 = 48$  **11**

5.  $4b - 12 = 28$  **10**                    6.  $\frac{36}{b} - 3 = 0$  **12**

**Find the solution of each equation using the given replacement set.**

7.  $\frac{1}{2} + x = \frac{5}{4}; \left\{ \frac{1}{2}, \frac{3}{4}, 1, \frac{5}{4} \right\}$   **$\frac{3}{4}$**

9.  $\frac{1}{4}(x + 2) = \frac{5}{6}; \left\{ \frac{2}{3}, \frac{3}{4}, \frac{5}{4} \right\}$   **$\frac{4}{3}$**

**Solve each equation.**

11.  $10.4 - 6.8 = x$  **3.6**

13.  $\frac{46 - 15}{3 + 28} = a$  **1**

15.  $\frac{2(4) + 4}{3(3 - 1)} = b$  **2**

### Lesson 1-3

12.  $y = 20.1 - 11.9$  **8.2**

14.  $c = \frac{6 + 18}{31 - 25}$  **4**

16.  $\frac{6(7 - 2)}{3(8) + 6} = n$  **1**

**Find the solution set for each inequality using the given replacement set.**

17.  $a + 7 < 13; \{3, 4, 5, 6, 7\}$   **$\{3, 4, 5\}$**                       18.  $9 + y < 17; \{7, 8, 9, 10, 11\}$   **$\{7\}$**

19.  $x - 2 \leq 2; \{2, 3, 4, 5, 6, 7\}$   **$\{2, 3, 4\}$**                       20.  $2x > 12; \{0, 2, 4, 6, 8, 10\}$   **$\{8, 10\}$**

21.  $4b + 1 > 12; \{0, 3, 6, 9, 12, 15\}$   **$\{3, 6, 9, 12, 15\}$**

23.  $\frac{y}{2} \geq 5; \{4, 6, 8, 10, 12\}$   **$\{10, 12\}$**

24.  $\frac{x}{3} > 2; \{3, 4, 5, 6, 7, 8\}$   **$\{7, 8\}$**

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## Reading to Learn Mathematics

### Open Sentences

#### Pre-Activity

How can you use open sentences to stay within a budget?

Read the introduction to Lesson 1-3 at the top of page 16 in your textbook.

How is the open sentence different from the expression  $15.50 + 5n$ ? **The open sentence has two expressions joined by the  $\geq$  symbol.**

#### Reading the Lesson

1. How can you tell whether a mathematical sentence is or is not an open sentence?  
**An open sentence must contain one or more variables.**

2. How would you read each inequality symbol in words?

Inequality Symbol	Words
$<$	<b>is less than</b>
$>$	<b>is greater than</b>
$\leq$	<b>is less than or equal to</b>
$\geq$	<b>is greater than or equal to</b>

3. Consider the equation  $3n + 6 = 15$  and the inequality  $3n + 6 \leq 15$ . Suppose the replacement set is  $\{0, 1, 2, 3, 4, 5\}$ .

- Describe how you would find the solutions of the equation.  
**Replace  $n$  with each member of the replacement set. The members of the replacement set that make the inequality true are the solutions.**
- Describe how you would find the solutions of the inequality.  
**Replace  $n$  with each member of the replacement set. The members of the replacement set that make the equation true are the solutions.**
- Explain how the solution set for the equation is different from the solution set for the inequality.  
**The solution set for the equation contains only one number, 3. The solution set for the inequality contains the four numbers 0, 1, 2, and 3.**

#### Helping You Remember

4. Look up the word *solution* in a dictionary. What is one meaning that relates to the way we use the word in algebra?

**Sample answer: answer to a problem**

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## Enrichment

### Solution Sets

Consider the following open sentence.

*It* is the name of a month between March and July.

You know that a replacement for the variable *It* must be found in order to determine if the sentence is true or false. If *It* is replaced by either April, May, or June, the sentence is true. The set  $\{\text{April, May, June}\}$  is called the solution set of the open sentence given above. This set includes all replacements for the variable that make the sentence true.

**Write the solution set for each open sentence.**

- It is the name of a state beginning with the letter A.  
 **$\{\text{Alabama, Alaska, Arizona, Arkansas}\}$**
- It is a primary color.  
 **$\{\text{red, yellow, blue}\}$**
- Its capital is Harrisburg.  
 **$\{\text{Pennsylvania}\}$**
- It is a New England state.  
 **$\{\text{Maine, New Hampshire, Vermont, Massachusetts, Rhode Island, Connecticut}\}$**
- $x + 4 = 10$   
 **$\{6\}$**
- It is the name of a month that contains the letter *r*.  
 **$\{\text{Jan, Feb, Mar, Apr, Sept, Oct, Nov, Dec}\}$**
- During the 1990s, she was the wife of a U.S. President.  
 **$\{\text{Barbara Bush, Hillary Clinton}\}$**
- It is an even number between 1 and 13.  
 **$\{2, 4, 6, 8, 10, 12\}$**
- $31 = 72 - k$   
 **$\{41\}$**
- It is the square of 2, 3, or 4.  
 **$\{4, 9, 16\}$**

**Write an open sentence for each solution set.**

- $\{A, E, I, O, U\}$   
**It is a vowel.**
- $\{1, 3, 5, 7, 9\}$   
**It is an odd number between 0 and 10.**
- $\{\text{June, July, August}\}$   
**It is a summer month.**
- $\{\text{Atlantic, Pacific, Indian, Arctic}\}$   
**It is an ocean.**

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## 1-4 Study Guide and Intervention

### Identity and Equality Properties

**Identity and Equality Properties** The identity and equality properties in the chart below can help you solve algebraic equations and evaluate mathematical expressions.

<b>Additive Identity</b>	For any number $a$ , $a + 0 = a$ .
<b>Multiplicative Identity</b>	For any number $a$ , $a \cdot 1 = a$ .
<b>Multiplicative Property of 0</b>	For any number $a$ , $a \cdot 0 = 0$ .
<b>Multiplicative Inverse Property</b>	For every number $\frac{a}{b}$ , $a, b \neq 0$ , there is exactly one number $\frac{b}{a}$ such that $\frac{a}{b} \cdot \frac{b}{a} = 1$ .
<b>Reflexive Property</b>	For any number $a$ , $a = a$ .
<b>Symmetric Property</b>	For any numbers $a$ and $b$ , if $a = b$ , then $b = a$ .
<b>Transitive Property</b>	For any numbers $a, b$ , and $c$ , if $a = b$ and $b = c$ , then $a = c$ .
<b>Substitution Property</b>	If $a = b$ , then $a$ may be replaced by $b$ in any expression.

**Example 1** Name the property used in each equation. Then find the value of  $n$ .

a.  $8n = 8$   
Multiplicative Inverse Property  
 $n = 1$ , since  $8 \cdot 1 = 8$

b.  $n \cdot 3 = 1$   
Multiplicative Inverse Property  
 $n = \frac{1}{3}$ , since  $\frac{1}{3} \cdot 3 = 1$

**Exercises**

Name the property used in each equation. Then find the value of  $n$ .

1.  $6n = 6$       **Mult. Identity; 1**  
 2.  $n \cdot 1 = 8$       **Mult. Identity; 8**  
 3.  $6 \cdot n = 6 \cdot 9$       **Substitution Property; 9**  
 4.  $9 = n + 9$       **Add. Identity; 0**  
 5.  $n + 0 = \frac{3}{8}$       **Add. Identity;  $\frac{3}{8}$**   
 6.  $\frac{3}{4} \cdot n = 1$       **Mult. Inverse;  $\frac{4}{3}$**

Name the property used in each equation.

7. If  $4 + 5 = 9$ , then  $9 = 4 + 5$ .      **Symmetric Property**  
 8.  $0 + 21 = 21$       **Add. Identity**  
 9.  $0(15) = 0$       **Mult. Prop. of Zero**  
 10.  $(1)94 = 94$       **Mult. Identity**  
 11. If  $3 + 3 = 6$  and  $6 = 3 \cdot 2$ , then  $3 + 3 = 3 \cdot 2$ .      **Transitive Property**  
 12.  $4 + 3 = 4 + 3$       **Reflexive Property**  
 13.  $(14 - 6) + 3 = 8 + 3$       **Substitution Property**

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## 1-4 Study Guide and Intervention

### Identity and Equality Properties

**Use Identity and Equality Properties** The properties of identity and equality can be used to justify each step when evaluating an expression.

**Example** Evaluate  $24 \cdot 1 - 8 + 5(9 + 3 - 3)$ . Name the property used in each step.

$$\begin{aligned}
 24 \cdot 1 - 8 + 5(9 + 3 - 3) &= 24 \cdot 1 - 8 + 5(3 - 3) && \text{Substitution; } 9 + 3 = 3 \\
 &= 24 \cdot 1 - 8 + 5(0) && \text{Substitution; } 3 - 3 = 0 \\
 &= 24 - 8 + 5(0) && \text{Multiplicative Identity; } 24 \cdot 1 = 24 \\
 &= 24 - 8 + 0 && \text{Multiplicative Property of Zero; } 5(0) = 0 \\
 &= 16 + 0 && \text{Substitution; } 24 - 8 = 16 \\
 &= 16 && \text{Additive Identity; } 16 + 0 = 16
 \end{aligned}$$

**Exercises**

Evaluate each expression. Name the property used in each step.

1.  $2\left[\frac{1}{4} + \left(\frac{1}{2}\right)\right]$   
 $= 2\left(\frac{1}{4} + \frac{1}{4}\right)$       **Substitution**  
 $= 2\left(\frac{1}{2}\right)$       **Substitution**  
 $= 1$       **Mult. Inverse**
2.  $15 \cdot 1 - 9 + 2(15 \div 3 - 5)$   
 $= 15 \cdot 1 - 9 + 2(5 - 5)$       **Substitution**  
 $= 15 \cdot 1 - 9 + 2(0)$       **Substitution**  
 $= 15 \cdot 1 - 9 + 0$       **Mult. Prop. Zero**  
 $= 15 - 9 + 0$       **Mult. Identity**  
 $= 6 - 0$       **Substitution**  
 $= 6$       **Substitution**
3.  $2(3 \cdot 5 \cdot 1 - 14) - 4 \cdot \frac{1}{4}$   
 $= 2(15 \cdot 1 - 14) - 4 \cdot \frac{1}{4}$       **Subst.**  
 $= 2(15 - 14) - 4 \cdot \frac{1}{4}$       **Mult. Identity**  
 $= 2(1) - 4 \cdot \frac{1}{4}$       **Substitution**  
 $= 2 - 4 \cdot \frac{1}{4}$       **Mult. Identity**  
 $= 2 - 1$       **Mult. Inverse**  
 $= 1$       **Substitution**
4.  $18 \cdot 1 - 3 \cdot 2 + 2(6 \div 3 - 2)$   
 $= 18 \cdot 1 - 3 \cdot 2 + 2(2 - 2)$       **Subst.**  
 $= 18 \cdot 1 - 3 \cdot 2 + 2(0)$       **Substitution**  
 $= 18 - 3 \cdot 2 + 2(0)$       **Mult. Identity**  
 $= 18 - 6 + 2(0)$       **Substitution**  
 $= 18 - 6 + 0$       **Mult. Prop. Zero**  
 $= 12 + 0$       **Substitution**  
 $= 12$       **Add. Identity**
5.  $10 \div 5 - 9^2 \div 2 + 13$   
 $= 10 \div 5 - 4 + 2 + 13$       **Subst.**  
 $= 2 - 4 + 2 + 13$       **Substitution**  
 $= 0 + 2 + 13$       **Substitution**  
 $= 0 + 13$       **Additive Identity**  
 $= 13$       **Additive Identity**
6.  $3(5 - 5 \cdot 1^2) + 21 \div 7$   
 $= 3(5 - 5 \cdot 1) + 21 \div 7$       **Subst.**  
 $= 3(5 - 5) + 21 \div 7$       **Mult. Identity**  
 $= 3(0) + 21 \div 7$       **Substitution**  
 $= 0 + 21 \div 7$       **Mult. Prop. Zero**  
 $= 0 + 3$       **Substitution**  
 $= 3$       **Additive Identity**

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## 1-4 Skills Practice

### Identity and Equality Properties

Name the property used in each equation. Then find the value of  $n$ .

- $n + 0 = 19$   
**Additive Identity; 19**
- $28 \cdot n = 0$   
**Multiplicative Prop. of Zero; 0**
- $\frac{1}{4} \cdot n = 1$   
**Multiplicative Inverse; 4**
- $5 = n + 5$   
**Additive Identity; 0**
- $2(9 - 3) = 2(n)$   
**Substitution Prop.; 6**
- $5 + 4 = n + 4$   
**Reflexive Prop.; 5**
- $3n = 1$   
**Multiplicative Inverse;  $\frac{1}{3}$**
- $1 \cdot n = 8$   
**Multiplicative Identity; 8**
- $4 \cdot 0 + n = 22$   
**Additive Identity; 22**
- $n \cdot 9 = 9$   
**Multiplicative Identity; 1**
- $2 \cdot n = 2 \cdot 3$   
**Reflexive Prop.; 3**
- $(7 \cdot 3) + 4 = n + 4$   
**Substitution Prop.; 21**
- $n = 14 \cdot 0$   
**Multiplicative Prop. of Zero; 0**
- $11 - (18 \div 2) = 11 - n$   
**Substitution Prop.; 9**

Evaluate each expression. Name the property used in each step.

- $7(16 \div 4^2)$   
 $= 7(16 \div 16)$  Substitution  
 $= 7(1)$  Substitution  
 $= 7$  Multiplicative Identity
- $4 - 3(7 - 6)$  Substitution  
 $= 4 - 3(1)$  Substitution  
 $= 4 - 3$  Multiplicative Identity  
 $= 1$  Substitution
- $6 + 9[10 - 2(2 + 3)]$   
 $= 6 + 9[10 - 2(5)]$  Substitution  
 $= 6 + 9(10 - 10)$  Substitution  
 $= 6 + 9(0)$  Substitution  
 $= 6 + 0$  Mult. Prop. of Zero  
 $= 6$  Additive Identity
- $4[8 - (4 \cdot 2)] + 1$   
 $= 4(8 - 8) + 1$  Substitution  
 $= 4(0) + 1$  Substitution  
 $= 0 + 1$  Mult. Prop. of Zero  
 $= 1$  Additive Identity
- $2(6 \div 3 - 1) \cdot \frac{1}{2}$   
 $= 2(2 - 1) \cdot \frac{1}{2}$  Substitution  
 $= 2(1) \cdot \frac{1}{2}$  Substitution  
 $= 2 \cdot \frac{1}{2}$  Multiplicative Identity  
 $= 1$  Multiplicative Inverse

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## 1-4 Practice (Average)

### Identity and Equality Properties

Name the property used in each equation. Then find the value of  $n$ .

- $n + 9 = 9$   
**Additive Identity; 0**
- $(8 + 7)(4) = n(4)$   
**Substitution Prop.; 15**
- $5n = 1$   
**Multiplicative Inverse;  $\frac{1}{5}$**
- $n \cdot 0.5 = 0.1 \cdot 0.5$   
**Reflexive Prop.; 0.1**
- $49n = 0$   
**Multiplicative Prop. of Zero; 0**
- $12 = 12 \cdot n$   
**Multiplicative Identity; 1**

Evaluate each expression. Name the property used in each step.

- $7 \cdot 2 + 6(9 - 3^2) - 2$   
 $= 2 + 6(9 - 9) - 2$  Substitution  
 $= 2 + 6(0) - 2$  Substitution  
 $= 2 + 0 - 2$  Mult. Prop. of Zero  
 $= 2 - 2$  Additive Identity  
 $= 0$  Substitution
- $5(14 - 39 \div 3) + 4 \cdot \frac{1}{4}$   
 $= 5(14 - 13) + 4 \cdot \frac{1}{4}$  Substitution  
 $= 5(1) + 4 \cdot \frac{1}{4}$  Substitution  
 $= 5 + 4 \cdot \frac{1}{4}$  Multiplicative Identity  
 $= 5 + 1$  Multiplicative Inverse  
 $= 6$  Substitution

**SALES** For Exercises 9 and 10, use the following information.

Althea paid \$5.00 each for two bracelets and later sold each for \$15.00. She paid \$8.00 each for three bracelets and sold each of them for \$9.00.

- Write an expression that represents the profit Althea made.  $2(15 - 5) + 3(9 - 8)$
- Evaluate the expression. Name the property used in each step.  
 $2(15 - 5) + 3(9 - 8) = 2(10) + 3(1)$  Substitution  
 $= 20 + 3(1)$  Substitution  
 $= 20 + 3$  Multiplicative Identity  
 $= 23$  Substitution

**GARDENING** For Exercises 11 and 12, use the following information.

Mr. Katz harvested 15 tomatoes from each of four plants. Two other plants produced four tomatoes each, but Mr. Katz only harvested one fourth of the tomatoes from each of these.

- Write an expression for the total number of tomatoes harvested.  $4(15) + 2(4 \cdot \frac{1}{4})$
- Evaluate the expression. Name the property used in each step.  
 $4(15) + 2(4 \cdot \frac{1}{4}) = 60 + 2(4 \cdot \frac{1}{4})$  Substitution  
 $= 60 + 2(1)$  Multiplicative Inverse  
 $= 60 + 2$  Multiplicative Identity  
 $= 62$  Substitution

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# 1-5 Study Guide and Intervention (continued)

## The Distributive Property

**Simplify Expressions** A term is a number, a variable, or a product or quotient of numbers and variables. **Like terms** are terms that contain the same variables, with corresponding variables having the same powers. The Distributive Property and properties of equalities can be used to simplify expressions. An expression is in **simplest form** if it is replaced by an **equivalent** expression with no like terms or parentheses.

**Example** Simplify  $4(a^2 + 3ab) - ab$ .

$$\begin{aligned} 4(a^2 + 3ab) - ab &= 4a^2 + 3ab - 1ab && \text{Multiplicative Identity} \\ &= 4a^2 + 12ab - 1ab && \text{Distributive Property} \\ &= 4a^2 + (12 - 1)ab && \text{Distributive Property} \\ &= 4a^2 + 11ab && \text{Substitution} \end{aligned}$$

### Exercises

Simplify each expression. If not possible, write *simplified*.

- $12a - a$       **11a**
- $3x + 6x$       **9x**
- $3x - 1$       **simplified**
- $12g - 10g + 1$       **2g + 1**
- $-2x - 12$       **simplified**
- $4x^2 + 3x + 7$       **simplified**
- $20a + 12a - 8$       **32a - 8**
- $3x^2 + 2x^2$       **5x^2**
- $-6x + 3x^2 + 10x^2$       **-6x + 13x^2**
- $2p + \frac{1}{2}q$       **simplified**
- $10xy - 4(xy + xy)$       **2xy**
- $21c + 18c + 31b - 3b$       **39c + 28b**
- $3x - 2x - 2y + 2y$       **x**
- $xy - 2xy$       **-xy**
- $12a - 12b + 12c$       **simplified**
- $4x + \frac{1}{4}(16x - 20y)$       **8x - 5y**
- $2 - 1 - 6x + x^2$       **1 - 6x + x^2**
- $4x^2 + 3x^2 + 2x$       **7x^2 + 2x**

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# 1-5 Study Guide and Intervention

## The Distributive Property

**Evaluate Expressions** The Distributive Property can be used to help evaluate expressions.

**Distributive Property** For any numbers  $a$ ,  $b$ , and  $c$ ,  $a(b + c) = ab + ac$  and  $(b + c)a = ba + ca$  and  $a(b - c) = ab - ac$  and  $(b - c)a = ba - ca$ .

**Example 1** Rewrite  $6(8 + 10)$  using the Distributive Property. Then evaluate.

$$\begin{aligned} 6(8 + 10) &= 6 \cdot 8 + 6 \cdot 10 && \text{Distributive Property} \\ &= 48 + 60 && \text{Multiply.} \\ &= 108 && \text{Add.} \end{aligned}$$

**Example 2** Rewrite  $-2(3x^2 + 5x + 1)$  using the Distributive Property. Then simplify.

$$\begin{aligned} -2(3x^2 + 5x + 1) &= -2(3x^2) + (-2)(5x) + (-2)(1) && \text{Distributive Property} \\ &= -6x^2 + (-10x) + (-2) && \text{Multiply.} \\ &= -6x^2 - 10x - 2 && \text{Simplify.} \end{aligned}$$

### Exercises

Rewrite each expression using the Distributive Property. Then simplify.

- $2(10 - 5)$       **10**
- $6(12 - t)$       **72 - 6t**
- $3(x - 1)$       **3x - 3**
- $6(12 + 5)$       **102**
- $(x - 4)3$       **3x - 12**
- $-2(x + 3)$       **-2x - 6**
- $5(4x - 9)$       **20x - 45**
- $3(8 - 2x)$       **24 - 6x**
- $12(6 - \frac{1}{2}x)$       **72 - 6x**
- $2(2 + \frac{1}{2}x)$       **24 + 6x**
- $\frac{1}{4}(12 - 4t)$       **3 - t**
- $3(2x - y)$       **6x - 3y**
- $2(3x + 2y - z)$       **6x + 4y - 2z**
- $(x - 2)y$       **xy - 2y**
- $2(3a - 2b + c)$       **6a - 4b + 2c**
- $\frac{1}{4}(16x - 12y + 4z)$       **4x - 3y + z**
- $(2 - 3x + x^2)3$       **6 - 9x + 3x^2**
- $4x - 3y + z$       **4x^2 - 6x - 2**

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Lesson 1-5

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**1-5 Skills Practice**

**The Distributive Property**

Rewrite each expression using the Distributive Property. Then simplify.

1.  $4(3 + 5)$  **4 · 3 + 4 · 5; 32**
2.  $2(6 + 10)$  **2 · 6 + 2 · 10; 32**
3.  $5(7 - 4)$  **5 · 7 - 5 · 4; 15**
4.  $(6 - 2)8$  **6 · 8 - 2 · 8; 32**
5.  $(a + 7)2$  **a · 2 + 7 · 2; 2a + 14**
6.  $7(h - 10)$  **7 · h - 7 · 10; 7h - 70**
7.  $3(m + n)$  **3 · m + 3 · n; 3m + 3n**
8.  $(x - y)6$  **x · 6 - y · 6; 6x - 6y**
9.  $2(x - y + 1)$  **2(x) - 2(y) + 2(1); 2x - 2y + 2**
10.  $3(a + b - 1)$  **3(a) + 3(b) - 3(1); 3a + 3b - 3**

Use the Distributive Property to find each product.

11.  $5 \cdot 89$  **445**
12.  $9 \cdot 99$  **891**
13.  $15 \cdot 104$  **1560**
14.  $15\left(\frac{2}{3}\right)$  **35**
15.  $12\left(1\frac{1}{4}\right)$  **15**
16.  $8\left(\frac{1}{8}\right)$  **25**

Simplify each expression. If not possible, write *simplified*.

17.  $2x + 8x$  **10x**
18.  $17g + g$  **18g**
19.  $16m - 10m$  **6m**
20.  $12p - 8p$  **4p**
21.  $2x^2 + 6x^2$  **8x^2**
22.  $7a^2 - 2a^2$  **5a^2**
23.  $3y^2 - 2y$  **simplified**
24.  $2(n + 2n)$  **6n**
25.  $4(2b - b)$  **4b**
26.  $3q^2 + q - q^2$  **2q^2 + q**

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**1-5 Practice (Average)**

**The Distributive Property**

Rewrite each expression using the Distributive Property. Then simplify.

1.  $9(7 + 8)$  **9 · 7 + 9 · 8; 135**
2.  $7(6 - 4)$  **7 · 6 - 7 · 4; 14**
3.  $6(b + 4)$  **6 · b + 6 · 4; 6b + 24**
4.  $(9 - p)3$  **9 · 3 - p · 3; 27 - 3p**
5.  $(5y - 3)7$  **5y · 7 - 3 · 7; 35y - 21**
6.  $15\left(f + \frac{1}{3}\right)$  **15 · f + 15 ·  $\frac{1}{3}$ ; 15f + 5**
7.  $16(3b - 0.25)$  **16 · 3b - 16 · 0.25; 48b - 4**
8.  $m(n + 4)$  **m · n + m · 4; mn + 4m**
9.  $(c - 4)d$  **c · d - 4 · d; cd - 4d**

Use the Distributive Property to find each product.

10.  $9 \cdot 499$  **4491**
11.  $7 \cdot 110$  **770**
12.  $21 \cdot 1004$  **21,084**
13.  $12 \cdot 2.5$  **30**
14.  $27\left(\frac{1}{3}\right)$  **63**
15.  $16\left(\frac{1}{4}\right)$  **68**

Simplify each expression. If not possible, write *simplified*.

16.  $w + 14w - 6w$  **9w**
17.  $3(5 + 6t)$  **15 + 18h**
18.  $14(2r - 3)$  **28r - 42**
19.  $12b^2 + 9b^2$  **21b^2**
20.  $25t^3 - 17t^3$  **8t^3**
21.  $c^2 + 4d^2 - d^2$  **c^2 + 3d^2**
22.  $3a^2 + 6a + 2b^2$  **simplified**
23.  $4(6p + 2q - 2p)$  **16p + 8q**
24.  $x + \frac{2}{3}x + \frac{x}{3}$  **2x**

**DINING OUT For Exercises 25 and 26, use the following information.**

The Ross family recently dined at an Italian restaurant. Each of the four family members ordered a pasta dish that cost \$11.50, a drink that cost \$1.50, and dessert that cost \$2.75.

25. Write an expression that could be used to calculate the cost of the Ross' dinner before adding tax and a tip. **4(11.5 + 1.5 + 2.75)**
26. What was the cost of dining out for the Ross family? **\$63.00**

**ORIENTATION For Exercises 27 and 28, use the following information.**

Madison College conducted a three-day orientation for incoming freshmen. Each day, an average of 110 students attended the morning session and an average of 160 students attended the afternoon session.

27. Write an expression that could be used to determine the total number of incoming freshmen who attended the orientation. **3(110 + 160)**
28. What was the attendance for all three days of orientation? **810**

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## 1-5

## Reading to Learn Mathematics

### The Distributive Property

**Pre-Activity** How can the Distributive Property be used to calculate quickly?

Read the introduction to Lesson 1-5 at the top of page 26 in your textbook.  
How would you find the amount spent by each of the first eight customers at Instant Replay Video Games on Saturday?

**Add \$14.95 and \$34.95.**

### Reading the Lesson

1. Explain how the Distributive Property could be used to rewrite  $3(1 + 5)$ .  
**Find the sum of 3 times 1 and 3 times 5.**

2. Explain how the Distributive Property can be used to rewrite  $5(6 - 4)$ .

**Write the difference of 5 times 6 and 5 times 4, that is  $5 \cdot 6 - 5 \cdot 4$ .**

3. Write three examples of each type of term. **Sample answers are given.**

Term	Example
number	<b>3, 17, 0.25</b>
variable	<b><math>w</math>, <math>t^2</math>, <math>x</math></b>
product of a number and a variable	<b><math>4y</math>, <math>0.78z</math>, <math>\frac{1}{8}r</math></b>
quotient of a number and variable	<b><math>\frac{x}{3}</math>, <math>\frac{2s}{7}</math>, <math>\frac{6}{5t}</math></b>

4. Tell how you can use the Distributive Property to write  $12m + 8m$  in simplest form. Use the word *coefficient* in your explanation.

**Sample answer:** Add the coefficients of the two terms and multiply by  $m$ .

### Helping You Remember

5. How can the everyday meaning of the word *identity* help you to understand and remember what the additive identity is and what the multiplicative identity is?

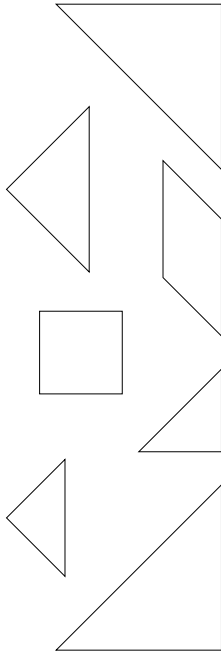
**Sample answer:** When you add 0 (the additive identity) to a number, the result is the very same number you started with. The same is true if you multiply the number by 1 (the multiplicative identity).

## 1-5

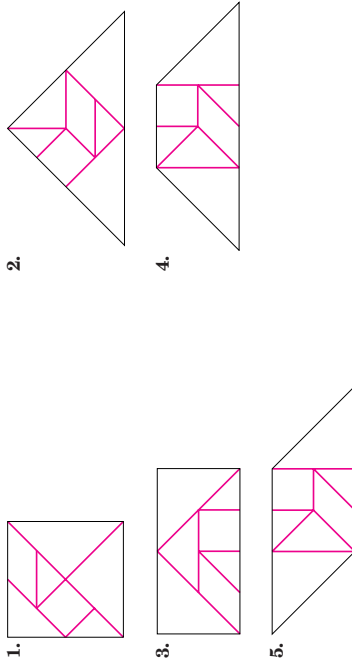
## Enrichment

### Tangram Puzzles

The seven geometric figures shown below are called **tans**. They are used in a very old Chinese puzzle called **tangrams**.

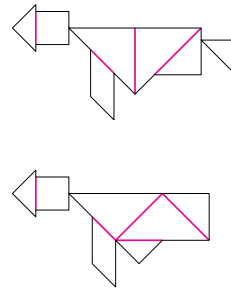


Glue the seven tans on heavy paper and cut them out. Use all seven pieces to make each shape shown. Record your solutions below.



6. Each of the two figures shown at the right is made from all seven tans. They seem to be exactly alike, but one has a triangle at the bottom and the other does not. Where does the second figure get this triangle?

**In the left figure, the body is made from 4 pieces rather than 3. The extra piece becomes the triangle at the bottom in the right figure.**



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## 1-6 Study Guide and Intervention

### Commutative and Associative Properties

**Commutative and Associative Properties** The Commutative and Associative Properties can be used to simplify expressions. The Commutative Properties state that the order in which you add or multiply numbers does not change their sum or product. The Associative Properties state that the way you group three or more numbers when adding or multiplying does not change their sum or product.

<b>Commutative Properties</b>	For any numbers $a$ and $b$ , $a + b = b + a$ and $a \cdot b = b \cdot a$ .
<b>Associative Properties</b>	For any numbers $a$ , $b$ , and $c$ , $(a + b) + c = a + (b + c)$ and $(ab)c = a(bc)$ .

#### Example 1

$$\begin{aligned}
 6 \cdot 2 \cdot 3 \cdot 5 &= 6 \cdot 3 \cdot 2 \cdot 5 && \text{Commutative Property} \\
 &= (6 \cdot 3)(2 \cdot 5) && \text{Associative Property} \\
 &= 18 \cdot 10 && \text{Multiply.} \\
 &= 180 && \text{Multiply.}
 \end{aligned}$$

The product is 180.

#### Example 2

$$\begin{aligned}
 8.2 + 2.5 + 2.5 + 1.8 &&& \text{Evaluate} \\
 &= 8.2 + 1.8 + 2.5 + 2.5 && \text{Commutative Prop.} \\
 &= (8.2 + 1.8) + (2.5 + 2.5) && \text{Associative Prop.} \\
 &= 10 + 5 && \text{Add.} \\
 &= 15 && \text{Add.}
 \end{aligned}$$

The sum is 15.

#### Exercises

Evaluate each expression.

- $12 + 10 + 8 + 5$     **35**
- $16 + 8 + 22 + 12$     **58**
- $10 \cdot 7 \cdot 2.5$     **175**
- $4 \cdot 8 \cdot 5 \cdot 3$     **480**
- $12 + 20 + 10 + 5$     **47**
- $26 + 8 + 4 + 22$     **60**
- $3\frac{1}{2} + 4 + 2\frac{1}{2} + 3$     **13**
- $\frac{3}{4} \cdot 12 \cdot 4 \cdot 2$     **72**
- $3.5 + 2.4 + 3.6 + 4.2$     **13.7**
- $4\frac{1}{2} + 5 + \frac{1}{2} + 3$     **13**
- $0.5 \cdot 2.8 \cdot 4$     **5.6**
- $2.5 + 2.4 + 2.5 + 3.6$     **11**
- $4 \cdot 18 \cdot 25 \cdot \frac{2}{9}$     **80**
- $32 \cdot \frac{1}{5} \cdot \frac{1}{2} \cdot 10$     **32**
- $\frac{1}{4} \cdot 7 \cdot 16 \cdot \frac{1}{4}$     **4**
- $3.5 + 8 + 2.5 + 2$     **16**
- $18 \cdot 8 \cdot \frac{1}{9}$     **8**
- $\frac{3}{4} \cdot 10 \cdot 16 \cdot \frac{1}{2}$     **60**

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## 1-6 Study Guide and Intervention

### Commutative and Associative Properties

**Simplify Expressions** The Commutative and Associative Properties can be used along with other properties when evaluating and simplifying expressions.

#### Example

$$\begin{aligned}
 8(y + 2x) + 7y &= 8y + 16x + 7y && \text{Distributive Property} \\
 &= 8y + 7y + 16x && \text{Commutative (+)} \\
 &= (8 + 7)y + 16x && \text{Distributive Property} \\
 &= 15y + 16x && \text{Substitution}
 \end{aligned}$$

Simplify  $8(y + 2x) + 7y$ .

The simplified expression is  $15y + 16x$ .

#### Exercises

Simplify each expression.

- $4x + 3y + x$      **$5x + 3y$**
- $3a + 4b + a$      **$4a + 4b$**
- $8rs + 2rs^2 + 7rs$      **$15rs + 2rs^2$**
- $3a^2 + 4b + 10a^2$      **$13a^2 + 4b$**
- $6(x + y) + 2(2x + y)$      **$10x + 8y$**
- $6n + 2(4n + 5)$      **$14n + 10$**
- $6(a + b) + a + 3b$      **$7a + 9b$**
- $5(2x + 3y) + 6(y + x)$      **$16x + 21y$**
- $5(0.3x + 0.1y) + 0.2x$      **$1.7x + 0.5y$**
- $\frac{2}{3} + \frac{1}{2}(x + 10) + \frac{4}{3}$      **$11 \cdot z^2 + 9x^2 + \frac{4}{3}z^2 + \frac{1}{3}x^2$**
- $6(2x + 4y) + 2(x + 9)$      **$14x + 24y + 18$**
- $7 + \frac{1}{2}x$      **$\frac{7}{3}z^2 + \frac{28}{3}x^2$**

Write an algebraic expression for each verbal expression. Then simplify.

- twice the sum of  $y$  and  $z$  is increased by  $y$      **$3y + 2z$**
- four times the product of  $x$  and  $y$  decreased by  $2xy$      **$2xy$**
- the product of five and the square of  $a$ , increased by the sum of eight,  $a^2$ , and 4     **$6a^2 + 12$**
- three times the sum of  $x$  and  $y$  increased by twice the sum of  $x$  and  $y$      **$5x + 5y$**

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<div style="text-align: center;"> <h2 style="margin: 0;">1-6 Skills Practice</h2> <h3 style="margin: 0;">Commutative and Associative Properties</h3> </div> <p>Evaluate each expression.</p> <p>10. <math>1.6 + 8 + 14 + 12</math> <b>50</b>      2. <math>36 + 23 + 14 + 7</math> <b>80</b>      3. <math>32 + 14 + 18 + 11</math> <b>75</b></p> <p>4. <math>5 \cdot 3 \cdot 4 \cdot 3</math> <b>180</b>      5. <math>2 \cdot 4 \cdot 5 \cdot 3</math> <b>120</b>      6. <math>5 \cdot 7 \cdot 10 \cdot 4</math> <b>1400</b></p> <p>7. <math>1.7 + 0.8 + 1.3</math> <b>3.8</b>      8. <math>1.6 + 0.9 + 2.4</math> <b>4.9</b>      9. <math>4\frac{1}{2} + 6 + 5\frac{1}{2}</math> <b>16</b></p> <p><b>Simplify each expression.</b></p> <p>10. <math>2x + 5y + 9x</math> <b><math>11x + 5y</math></b>      11. <math>a + 9b + 6a</math> <b><math>7a + 9b</math></b></p> <p>12. <math>2p + 3q + 5p + 2q</math> <b><math>7p + 5q</math></b>      13. <math>r + 3s + 5r + s</math> <b><math>6r + 4s</math></b></p> <p>14. <math>5m^2 + 3m + m^2</math> <b><math>6m^2 + 3m</math></b>      15. <math>6k^2 + 6k + k^2 + 9k</math> <b><math>7k^2 + 15k</math></b></p> <p>16. <math>2a + 3(4 + a)</math> <b><math>5a + 12</math></b>      17. <math>5(7 + 2g) + 3g</math> <b><math>35 + 13g</math></b></p> <p><b>Write an algebraic expression for each verbal expression. Then simplify, indicating the properties used.</b></p> <p>18. three times the sum of <math>a</math> and <math>b</math> increased by <math>a</math></p> $3(a + b) + a$ $= 3(a) + 3(b) + a \quad \text{Distributive Property}$ $= 3a + 3b + a \quad \text{Multiply.}$ $= 3a + a + 3b \quad \text{Commutative (+)}$ $= (3a + a) + 3b \quad \text{Associative (+)}$ $= (3 + 1)a + 3b \quad \text{Distributive Property}$ $= 4a + 3b \quad \text{Substitution}$ <p>19. twice the sum of <math>p</math> and <math>q</math> increased by twice the sum of <math>2p</math> and <math>3q</math></p> $2(p + q) + 2(2p + 3q)$ $= 2(p) + 2(q) + 2(2p) + 2(3q) \quad \text{Distributive Property}$ $= 2p + 2q + 4p + 6q \quad \text{Multiply.}$ $= 2p + 4p + 2q + 6q \quad \text{Commutative (+)}$ $= (2p + 4p) + (2q + 6q) \quad \text{Associative (+)}$ $= (2 + 4)p + (2 + 6)q \quad \text{Distributive Property}$ $= 6p + 8q \quad \text{Substitution}$	<div style="text-align: center;"> <h2 style="margin: 0;">1-6 Practice (Average)</h2> <h3 style="margin: 0;">Commutative and Associative Properties</h3> </div> <p>Evaluate each expression.</p> <p>1. <math>13 + 23 + 12 + 7</math> <b>55</b>      2. <math>6 \cdot 5 \cdot 10 \cdot 3</math> <b>900</b></p> <p>3. <math>7.6 + 3.2 + 9.4 + 1.3</math> <b>21.5</b>      4. <math>3.6 \cdot 0.7 \cdot 5</math> <b>12.6</b></p> <p>5. <math>7\frac{1}{9} + 2 + 1\frac{2}{9}</math> <b><math>10\frac{1}{3}</math></b>      6. <math>3\frac{3}{4} \cdot 3\frac{1}{3} \cdot 16</math> <b>200</b></p> <p><b>Simplify each expression.</b></p> <p>7. <math>9s^2 + 3t + s^2 + t</math> <b><math>10s^2 + 4t</math></b>      8. <math>(p + 2n) + 7p</math> <b><math>8p + 2n</math></b></p> <p>9. <math>6y + 2(4y + 6)</math> <b><math>14y + 12</math></b>      10. <math>2(3x + y) + 5(x + 2y)</math> <b><math>11x + 12y</math></b></p> <p>11. <math>3(2c + d) + 4(c + 4d)</math> <b><math>10c + 19d</math></b>      12. <math>6s + 2(t + 3s) + 5(s + 4t)</math> <b><math>17s + 22t</math></b></p> <p>13. <math>5(0.6b + 0.4c) + b</math> <b><math>4b + 2c</math></b>      14. <math>\frac{1}{2}q + 2\left(\frac{1}{4}q + \frac{1}{2}r\right)</math> <b><math>q + r</math></b></p> <p>15. Write an algebraic expression for <i>four times the sum of <math>2a</math> and <math>b</math> increased by twice the sum of <math>6a</math> and <math>2b</math></i>. Then simplify, indicating the properties used.</p> $4(2a + b) + 2(6a + 2b)$ $= 4(2a) + 4(b) + 2(6a) + 2(2b) \quad \text{Distributive Property}$ $= 8a + 4b + 12a + 4b \quad \text{Multiply.}$ $= 8a + 12a + 4b + 4b \quad \text{Commutative (+)}$ $= (8 + 12)a + (4b + 4b) \quad \text{Associative (+)}$ $= (8 + 12)a + (4 + 4)b \quad \text{Distributive Property}$ $= 20a + 8b \quad \text{Substitution}$ <p><b>SCHOOL SUPPLIES</b> For Exercises 16 and 17, use the following information.</p> <p>Kristen purchased two binders that cost \$1.25 each, two binders that cost \$4.75 each, two packages of paper that cost \$1.50 per package, four blue pens that cost \$1.15 each, and four pencils that cost \$.35 each.</p> <p>16. Write an expression to represent the total cost of supplies before tax.</p> $2(1.25 + 4.75 + 1.50) + 4(1.15 + 0.35)$ <p>17. What was the total cost of supplies before tax? <b>\$21.00</b></p> <p><b>GEOMETRY</b> For Exercises 18 and 19, use the following information.</p> <p>The lengths of the sides of a pentagon in inches are 1.25, 0.9, 2.5, 1.1, and 0.25.</p> <p>18. Using the commutative and associative properties to group the terms in a way that makes evaluation convenient, write an expression to represent the perimeter of the pentagon. <b>Sample answer: <math>(1.25 + 0.25) + (0.9 + 1.1) + 2.5</math></b></p> <p>19. What is the perimeter of the pentagon? <b>6 in.</b></p>
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**1-6** **Reading to Learn Mathematics**  
**Commutative and Associative Properties**

**Pre-Activity** How can properties help you determine distances?

Read the introduction to Lesson 1-6 at the top of page 32 in your textbook.  
How are the expressions  $0.4 + 1.5$  and  $1.5 + 0.4$  alike? different?

**The numbers and the operation are the same; the order of the numbers is different.**

**Reading the Lesson**

1. Write the Roman numeral of the term that best matches each equation.

- a.  $3 + 6 = 6 + 3$  **III** **I.** Associative Property of Addition
- b.  $2 + (3 + 4) = (2 + 3) + 4$  **I** **II.** Associative Property of Multiplication
- c.  $2 \cdot (3 \cdot 4) = (2 \cdot 3) \cdot 4$  **II** **III.** Commutative Property of Addition
- d.  $2 \cdot (3 \cdot 4) = 2 \cdot (4 \cdot 3)$  **IV** **IV.** Commutative Property of Multiplication

2. What property can you use to change the order of the terms in an expression?

**Commutative Property of Addition**

3. What property can you use to change the way three factors are grouped?

**Associative Property of Multiplication**

4. What property can you use to combine two like terms to get a single term?

**Distributive Property**

5. To use the Associative Property of Addition to rewrite the sum of a group of terms, what is the least number of terms you need? **three**

**Helping You Remember**

6. Look up the word *commute* in a dictionary. Find an everyday meaning that is close to the mathematical meaning and explain how it can help you remember the mathematical meaning.

**Sample answer: To travel back and forth, as between a suburb and a city; in the Commutative Property of Addition,  $a + b = b + a$ , the quantities  $a$  and  $b$  are switched back and forth.**

**1-6** **Enrichment**

**Properties of Operations**

Let's make up a new operation and denote it by  $\oplus$ , so that  $a \oplus b$  means  $b^a$ .

$2 \oplus 3 = 3^2 = 9$   
 $(1 \oplus 2) \oplus 3 = 2^1 \oplus 3 = 3^2 = 9$

- 1. What number is represented by  $2 \oplus 3$ ?  **$3^2 = 9$**
- 2. What number is represented by  $3 \oplus 2$ ?  **$2^3 = 8$**
- 3. Does the operation  $\oplus$  appear to be commutative? **no**
- 4. What number is represented by  $(2 \oplus 1) \oplus 3$ ? **3**
- 5. What number is represented by  $2 \oplus (1 \oplus 3)$ ? **9**
- 6. Does the operation  $\oplus$  appear to be associative? **no**

Let's make up another operation and denote it by  $\oplus$ , so that  $a \oplus b = (a + 1)(b + 1)$ .

$3 \oplus 2 = (3 + 1)(2 + 1) = 4 \cdot 3 = 12$   
 $(1 \oplus 2) \oplus 3 = (2 \cdot 3) \oplus 3 = 6 \oplus 3 = 7 \cdot 4 = 28$

- 7. What number is represented by  $2 \oplus 3$ ? **12**
- 8. What number is represented by  $3 \oplus 2$ ? **12**
- 9. Does the operation  $\oplus$  appear to be commutative? **yes**
- 10. What number is represented by  $(2 \oplus 3) \oplus 4$ ? **65**
- 11. What number is represented by  $2 \oplus (3 \oplus 4)$ ? **63**
- 12. Does the operation  $\oplus$  appear to be associative? **no**
- 13. What number is represented by  $1 \oplus (3 \oplus 2)$ ? **12**
- 14. What number is represented by  $(1 \oplus 3) \oplus (1 \oplus 2)$ ? **12**
- 15. Does the operation  $\oplus$  appear to be distributive over the operation  $\oplus$ ? **yes**

16. Let's explore these operations a little further. What number is represented by  $3 \oplus (4 \oplus 2)$ ? **3375**

17. What number is represented by  $(3 \oplus 4) \oplus (3 \oplus 2)$ ? **585**

18. Is the operation  $\oplus$  actually distributive over the operation  $\oplus$ ? **no**

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1-7

Study Guide and Intervention

Logical Reasoning

**Conditional Statements** A conditional statement is a statement of the form *if A, then B*. Statements in this form are called **if-then statements**. The part of the statement immediately following the word *if* is called the **hypothesis**. The part of the statement immediately following the word *then* is called the **conclusion**.

**Example 1**

Identify the hypothesis and conclusion of each statement.

- a. If it is Wednesday, then Jerri has aerobics class.  
Hypothesis: it is Wednesday  
Conclusion: Jerri has aerobics class

- b. If  $2x - 4 < 10$ , then  $x < 7$ .  
Hypothesis:  $2x - 4 < 10$   
Conclusion:  $x < 7$

**Example 2**

Identify the hypothesis and conclusion of each statement. Then write the statement in if-then form.

- a. You and Marylynn can watch a movie on Thursday.  
Hypothesis: it is Thursday  
Conclusion: you and Marylynn can watch a movie  
If it is Thursday, then you and Marylynn can watch a movie.

- b. For a number  $a$  such that  $3a + 2 = 11$ ,  $a = 3$ .  
Hypothesis:  $3a + 2 = 11$   
Conclusion:  $a = 3$   
If  $3a + 2 = 11$ , then  $a = 3$ .

**Exercises**

Identify the hypothesis and conclusion of each statement.

- If it is April, then it might rain. **H: it is April; C: it might rain**
- If you are a sprinter, then you can run fast. **H: you are a sprinter; C: you can run fast**
- If  $12 - 4x = 4$ , then  $x = 2$ . **H:  $12 - 4x = 4$ ; C:  $x = 2$**
- If it is Monday, then you are in school. **H: it is Monday; C: you are in school**
- If the area of a square is 49, then the square has side length 7. **H: the area of a square is 49; C: the square has side length 7**

Identify the hypothesis and conclusion of each statement. Then write the statement in if-then form.

- A quadrilateral with equal sides is a rhombus. **H: a quadrilateral has equal sides; C: the figure is a rhombus; if a quadrilateral has equal sides, then the quadrilateral is a rhombus.**
- A number that is divisible by 8 is also divisible by 4. **H: a number is divisible by 8; C: the number is divisible by 4; if a number is divisible by 8, then it is divisible by 4.**
- Karlyn goes to the movies when she does not have homework. **H: Karlyn does not have homework. C: Karlyn goes to the movies; if Karlyn does not have homework, then Karlyn goes to the movies.**

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Study Guide and Intervention

Logical Reasoning

**Deductive Reasoning and Counterexamples** Deductive reasoning is the process of using facts, rules, definitions, or properties to reach a valid conclusion. To show that a conditional statement is false, use a **counterexample**, one example for which the conditional statement is false. You need to find only one counterexample for the statement to be false.

**Example 1**

Determine a valid conclusion from the statement *If two numbers are even, then their sum is even* for the given conditions. If a valid conclusion does not follow, write *no valid conclusion* and explain why.

- a. The two numbers are 4 and 8.  
4 and 8 are even, and  $4 + 8 = 12$ . Conclusion: The sum of 4 and 8 is even.
- b. The sum of two numbers is 20.  
Consider 13 and 7.  $13 + 7 = 20$   
However,  $12 + 8$ ,  $19 + 1$ , and  $18 + 2$  all equal 20. There is no way to determine the two numbers. Therefore there is no valid conclusion.

**Example 2**

Provide a counterexample to this conditional statement. *If you use a calculator for a math problem, then you will get the answer correct.*  
Counterexample: If the problem is  $475 \div 5$  and you press  $475 \div 5$ , you will not get the correct answer.

**Exercises**

Determine a valid conclusion that follows from the statement *If the last digit of a number is 0 or 5, then the number is divisible by 5* for the given conditions. If a valid conclusion does not follow, write *no valid conclusion* and explain why.

- The number is 120. **Conclusion: 120 is divisible by 5.**
- The number is a multiple of 4. **No valid conclusion; a multiple of 4 need not end in 0 and never ends in 5.**
- The number is 101. **No valid conclusion because the number does not end in 0 or 5**

Find a counterexample for each statement.

- If Susan is in school, then she is in math class. **Susan is in school and she is in history class.**
- If a number is a square, then it is divisible by 2. **25 is a square that is not divisible by 2.**
- If a quadrilateral has 4 right angles, then the quadrilateral is a square. **A rectangle with  $\ell = 5$  and  $w = 6$**
- If you were born in New York, then you live in New York. **You could be born in New York and then live in California.**
- If three times a number is greater than 15, then the number must be greater than six.  **$5.5; 3(5.5)$  is greater than 15, but  $5.5$  is less than 6.**
- If  $3x - 2 \leq 10$ , then  $x < 4$ .  **$3(4) - 2 \leq 10$ , but 4 is not less than 4.**

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1-7

**Skills Practice****Logical Reasoning**

Identify the hypothesis and conclusion of each statement.

- If it is Sunday, then mail is not delivered.  
**H: it is Sunday, C: mail is not delivered**
- If you are hiking in the mountains, then you are outdoors.  
**H: you are hiking in the mountains, C: you are outdoors**
- If  $6n + 4 > 58$ , then  $n > 9$ . **H:  $6n + 4 > 58$ , C:  $n > 9$**

Identify the hypothesis and conclusion of each statement. Then write the statement in if-then form.

- Martina works at the bakery every Saturday.  
**H: it is Saturday, C: Martina works at the bakery; if it is Saturday, then Martina works at the bakery.**
- Ivan only runs early in the morning.  
**H: Ivan is running, C: it is early in the morning; if Ivan is running, it is early in the morning.**
- A polygon that has five sides is a pentagon.  
**H: a polygon has five sides, C: it is a pentagon; if a polygon has five sides, then it is a pentagon.**

Determine whether a valid conclusion follows from the statement *If Hector scores an 85 or above on his science exam, then he will earn an A in the class for the given condition. If a valid conclusion does not follow, write no valid conclusion and explain why.*

- Hector scored an 86 on his science exam. **Hector earned an A in science.**
  - Hector did not earn an A in science. **Hector scored less than 85 on the exam.**
  - Hector scored 84 on the science exam. **Hector did not earn an A in science.**
  - Hector studied 10 hours for the science exam. **No valid conclusion; the conditional statement does not mention the number of hours Hector studied.**
- Find a counterexample for each statement. 11–14. Sample answers are given.**
- If the car will not start, then it is out of gas. **The battery could be dead.**
  - If the basketball team has scored 100 points, then they must be winning the game. **The other team could have scored 101 points.**
  - If the Commutative Property holds for addition, then it holds for subtraction.  **$4 - 1 \neq 1 - 4$**
  - If  $2n + 3 < 17$ , then  $n \leq 7$ . **When  $n = 7$ ,  $2n + 3$  is equal to 17, not less than 17.**

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1-7

**Practice (Average)****Logical Reasoning**

Identify the hypothesis and conclusion of each statement.

- If it is raining, then the meteorologist's prediction was accurate.  
**H: it is raining, C: the meteorologist's prediction was accurate**
- If  $x = 4$ , then  $2x + 3 = 11$ . **H:  $x = 4$ , C:  $2x + 3 = 11$**

Identify the hypothesis and conclusion of each statement. Then write the statement in if-then form.

- When Joseph has a fever, he stays home from school.  
**H: Joseph has a fever, C: he stays home from school; if Joseph has a fever, then he stays home from school.**
- Two congruent triangles are similar.  
**H: two triangles are congruent, C: they are similar; if two triangles are congruent, then they are similar.**

Determine whether a valid conclusion follows from the statement *If two numbers are even, then their product is even for the given condition. If a valid conclusion does not follow, write no valid conclusion and explain why.*

- The product of two numbers is 12. **No valid conclusion; The product is even, but one of the numbers could be odd, such as 4 : 3.**
  - Two numbers are 8 and 6. **The product of the numbers is even.**
- Find a counterexample for each statement. 7–8. Sample answers are given.**
- If the refrigerator stopped running, then there was a power outage.  
**Perhaps someone accidentally unplugged it while cleaning.**
  - If  $6h - 7 < 5$ , then  $h \leq 2$ .  
**When  $h = 2$ , then  $6h - 7 = 5$ , and so is not less than 5.**

**GEOMETRY** For Exercises 9 and 10, use the following information. **9–10. Sample answers are given.**  
If the perimeter of a rectangle is 14 inches, then its area is 10 square inches.

- State a condition in which the hypothesis and conclusion are valid.  
**A rectangle has a length of 5 in. and a width of 2 in.**
- Provide a counterexample to show the statement is false. **A rectangle with a length of 6 in. and a width of 1 in. has a perimeter of 14 in. and an area of 6 in<sup>2</sup>.**

**11. ADVERTISING** A recent television commercial for a car dealership stated that “no reasonable offer will be refused.” Identify the hypothesis and conclusion of the statement. Then write the statement in if-then form.**H: there is a reasonable offer, C: it will not be refused; if there is a reasonable offer, then it will not be refused.**

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## 1-7 Enrichment

### Counterexamples

Some statements in mathematics can be proven false by **counterexamples**. Consider the following statement.

For any numbers  $a$  and  $b$ ,  $a - b = b - a$ .

You can prove that this statement is false in general if you can find one example for which the statement is false.

Let  $a = 7$  and  $b = 3$ . Substitute these values in the equation above.

$$7 - 3 \stackrel{?}{=} 3 - 7$$

$$4 \neq -4$$

In general, for any numbers  $a$  and  $b$ , the statement  $a - b = b - a$  is false. You can make the equivalent verbal statement: subtraction is *not* a commutative operation.

In each of the following exercises  $a$ ,  $b$ , and  $c$  are any numbers. Prove that the statement is false by counterexample. **Sample answers are given.**

1.  $a - (b - c) \stackrel{?}{=} (a - b) - c$ 

$$6 - (4 - 2) \stackrel{?}{=} (6 - 4) - 2$$

$$6 - 2 \stackrel{?}{=} 2 - 2$$

$$4 \neq 0$$
2.  $a \div (b \div c) \stackrel{?}{=} (a \div b) \div c$ 

$$6 \div (4 \div 2) \stackrel{?}{=} (6 \div 4) \div 2$$

$$\frac{6}{2} \stackrel{?}{=} \frac{1.5}{2}$$

$$3 \neq 0.75$$
3.  $a \div b \stackrel{?}{=} b \div a$ 

$$6 \div 4 \stackrel{?}{=} 4 \div 6$$

$$\frac{3}{2} \neq \frac{2}{3}$$
4.  $a \div (b + c) \stackrel{?}{=} (a \div b) + (a \div c)$ 

$$6 \div (4 + 2) \stackrel{?}{=} (6 \div 4) + (6 \div 2)$$

$$6 \div 6 \stackrel{?}{=} 1.5 + 3$$

$$1 \neq 4.5$$
5.  $a + (bc) \stackrel{?}{=} (a + b)(a + c)$ 

$$6 + (4 \cdot 2) \stackrel{?}{=} (6 + 4)(6 + 2)$$

$$6 + 8 \stackrel{?}{=} (10)(8)$$

$$14 \neq 80$$
6.  $a^2 + a^2 \stackrel{?}{=} a^4$ 

$$6^2 + 6^2 \stackrel{?}{=} 6^4$$

$$36 + 36 \stackrel{?}{=} 1296$$

$$72 \neq 1296$$

7. Write the verbal equivalents for Exercises 1, 2, and 3.

1. **Subtraction is not an associative operation.**
2. **Division is not an associative operation.**
3. **Division is not a commutative operation.**

8. For the distributive property  $a(b + c) = ab + ac$  it is said that multiplication distributes over addition. Exercises 4 and 5 prove that some operations do not distribute. Write a statement for each exercise that indicates this.

4. **Division does not distribute over addition.**
5. **Addition does not distribute over multiplication.**

## Lesson 1-7

## 1-7 Reading to Learn Mathematics

### Logical Reasoning

**Pre-Activity** How is logical reasoning helpful in cooking?

Read the introduction to Lesson 1-7 at the top of page 37 in your textbook.

What are the two possible reasons given for the popcorn burning?

**The heat was too high, or the kernels heated unevenly.**

### Reading the Lesson

1. Write *hypothesis* or *conclusion* to tell which part of the if-then statement is underlined.
  - a. If it is Tuesday, then it is raining. **conclusion**
  - b. If our team wins this game, then they will go to the playoffs. **conclusion**
  - c. I can tell you your birthday if you tell me your height. **hypothesis**
  - d. If  $3x + 7 = 13$ , then  $x = 2$ . **hypothesis**
  - e. If  $x$  is an even number, then  $x \div 2$  is an odd number. **conclusion**

2. What does the term *valid conclusion* mean?

**Sample answer: A valid conclusion is a statement that has to be true if you used true statements and correct reasoning to obtain the conclusion.**

3. Give a counterexample for the statement *If a person is famous, then that person has been on television*. Tell how you know it really is a counterexample.

**Sample answer: President Abraham Lincoln was and still is famous, but he was never on television. There was no television when Lincoln was alive.**

### Helping You Remember

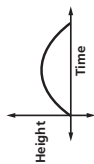
4. Write an example of a conditional statement you would use to teach someone how to identify an hypothesis and a conclusion. **See students' work.**

## 1-8 Study Guide and Intervention

### Graphs and Functions

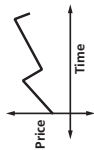
**Interpret Graphs** A function is a relationship between input and output values. In a function, there is exactly one output for each input. The input values are associated with the **independent variable**, and the output values are associated with the **dependent variable**. Functions can be graphed without using a scale to show the general shape of the graph that represents the function.

**Example 1** The graph below represents the height of a football after it is kicked downfield. Identify the independent and the dependent variable. Then describe what is happening in the graph.



The independent variable is time, and the dependent variable is height. The football starts on the ground when it is kicked. It gains altitude until it reaches a maximum height, then it loses altitude until it falls to the ground.

**Example 2** The graph below represents the price of stock over time. Identify the independent and dependent variable. Then describe what is happening in the graph.



The independent variable is time and the dependent variable is price. The price increases steadily, then it falls, then increases, then falls again.

## 1-8 Study Guide and Intervention

### Graphs and Functions

## 1-8 Study Guide and Intervention

### (continued)

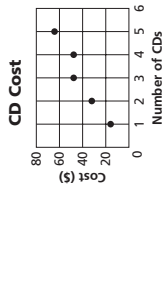
**Draw Graphs** You can represent the graph of a function using a **coordinate system**. Input and output values are represented on the graph using **ordered pairs** of the form  $(x, y)$ . The  $x$ -value, called the  **$x$ -coordinate**, corresponds to the  $x$ -axis, and the  $y$ -value, or  **$y$ -coordinate**, corresponds to the  $y$ -axis. Graphs can be used to represent many real-world situations.

**Example** A music store advertises that if you buy 3 CDs at the regular price of \$16, then you will receive one CD of the same or lesser value free.

a. Make a table showing the cost of buying 1 to 5 CDs.  
 b. Write the data as a set of ordered pairs.

Number of CDs	1	2	3	4	5
Total Cost (\$)	16	32	48	48	64

c. Draw a graph that shows the relationship between the number of CDs and the total cost.

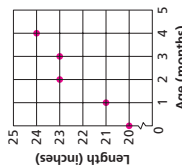


### Exercises

1. The table below represents the length of a baby versus its age in months.

Age (months)	0	1	2	3	4
Length (inches)	20	21	23	23	24

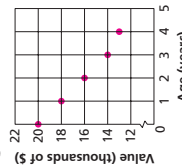
- a. Identify the independent and dependent variables.  
**ind: age; dep: length**
- b. Write a set of ordered pairs representing the data in the table.  
**(0, 20), (1, 21), (2, 23), (3, 23), (4, 24)**
- c. Draw a graph showing the relationship between age and length.



2. The table below represents the value of a car versus its age.

Age (years)	0	1	2	3	4
Value (\$)	20,000	18,000	16,000	14,000	13,000

- a. Identify the independent and dependent variables.  
**ind: age; dep: value**
- b. Write a set of ordered pairs representing the data in the table.  
**(0, 20,000), (1, 18,000), (2, 16,000), (3, 14,000), (4, 13,000)**
- c. Draw a graph showing the relationship between age and value.



## 1-8 Study Guide and Intervention

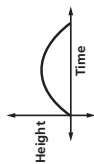
### Graphs and Functions

## 1-8 Study Guide and Intervention

### Graphs and Functions

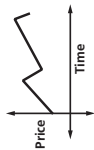
**Interpret Graphs** A function is a relationship between input and output values. In a function, there is exactly one output for each input. The input values are associated with the **independent variable**, and the output values are associated with the **dependent variable**. Functions can be graphed without using a scale to show the general shape of the graph that represents the function.

**Example 1** The graph below represents the height of a baseball after it is hit. Identify the independent and the dependent variable. Then describe what is happening in the graph.



The independent variable is time, and the dependent variable is height. The ball starts on the ground when it is hit. It gains altitude until it reaches a maximum height, then it loses altitude until it falls to the ground.

**Example 2** The graph below represents the price of stock over time. Identify the independent and dependent variable. Then describe what is happening in the graph.



The independent variable is time and the dependent variable is price. The price increases steadily, then it falls, then increases, then falls again.

### Lesson 1-8

### Exercises

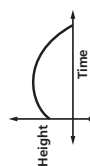
1. The graph represents the speed of a car as it travels to the grocery store. Identify the independent and dependent variable. Then describe what is happening in the graph.  
**ind: time; dep: speed. The car starts from a standstill, accelerates, then travels at a constant speed for a while. Then it slows down and stops.**



2. The graph represents the balance of a savings account over time. Identify the independent and the dependent variable. Then describe what is happening in the graph.  
**ind: time; dep: balance. The account balance has an initial value then it increases as deposits are made. It then stays the same for a while, again increases, and lastly goes to 0 as withdrawals are made.**



3. The graph represents the height of a baseball after it is hit. Identify the independent and the dependent variable. Then describe what is happening in the graph.  
**ind: time; dep: height. The ball is hit a certain height above the ground. The height of the ball increases until it reaches its maximum value, then the height decreases until the ball hits the ground.**



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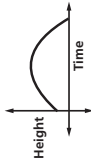
PERIOD \_\_\_\_\_

1-8

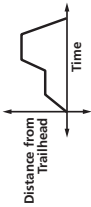
Skills Practice

Graphs and Functions

1. The graph below represents the path of a football thrown in the air. Describe what is happening in the graph.

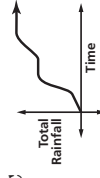
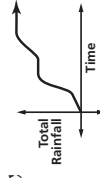
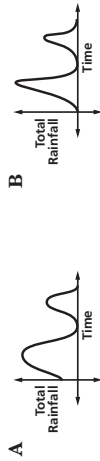


The football is thrown upward from above the ground, reaches its maximum height, and then falls downward until it hits the ground.



The puppy goes a distance on the trail, stays there for a while, goes ahead some more, stays there for a while, then goes back to the beginning of the trail.

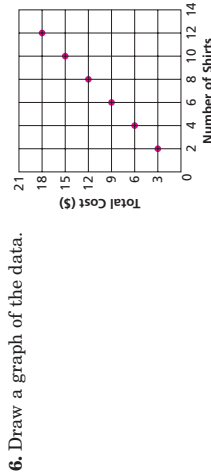
2. The graph below represents a puppy exploring a trail. Describe what is happening in the graph.



3. **WEATHER** During a storm, it rained lightly for a while, then poured heavily, and then stopped for a while. Then it rained moderately for a while before finally ending. Which graph represents this situation? **C**

Number of Shirts	2	4	6	8	10	12
Total Cost (\$)	3	6	9	12	15	18

4. Identify the independent and dependent variables.  
**independent : number of shirts; dependent: total cost**
5. Write the ordered pairs the table represents.  
**(2, 3), (4, 6), (6, 9), (8, 12), (10, 15), (12, 18)**



6. Draw a graph of the data.

7. Use the data to predict the cost for washing and pressing 16 shirts. **\$24**

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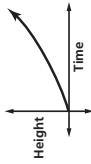
PERIOD \_\_\_\_\_

1-8

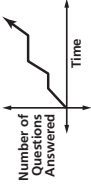
Practice (Average)

Graphs and Functions

1. The graph below represents the height of a tsunami (tidal wave) as it approaches shore. Describe what is happening in the graph.

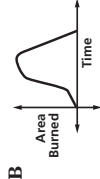
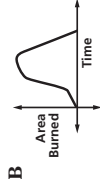


As the tsunami approaches shore, the height of the wave increases more and more quickly.



The student steadily answers questions, then pauses, resumes answering, pauses again, then resumes answering.

2. The graph below represents a student taking an exam. Describe what is happening in the graph.



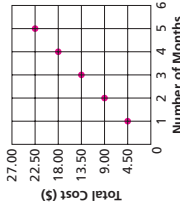
3. **FOREST FIRES** A forest fire grows slowly at first, then rapidly as the wind increases. After firefighters answer the call, the fire grows slowly for a while, but then the firefighters contain the fire before extinguishing it. Which graph represents this situation? **B**

4. **INTERNET NEWS SERVICE** For Exercises 4–6, use the table that shows the monthly charges for subscribing to an independent news server.

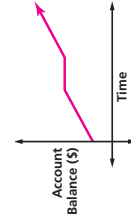
Number of Months	1	2	3	4	5
Total Cost (\$)	4.50	9.00	13.50	18.00	22.50

5. Write the ordered pairs the table represents. **(1, 4.5), (2, 9), (3, 13.5), (4, 18), (5, 22.5)**

6. Draw a graph of the data.



7. **SAVINGS** Jennifer deposited a sum of money in her account and then deposited equal amounts monthly for 5 months, nothing for 3 months, and then resumed equal monthly deposits. Sketch a reasonable graph of the account history.



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## 1-8 Reading to Learn Mathematics Graphs and Functions

### Pre-Activity

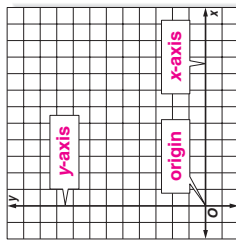
How can real-world situations be modeled using graphs and functions?

Read the introduction to Lesson 1-8 at the top of page 43 in your textbook.

The numbers 25%, 50% and 75% represent the \_\_\_\_\_ percent of blood flow to the brain \_\_\_\_\_ and the numbers 0 through 10 represent the \_\_\_\_\_ number of days after the concussion \_\_\_\_\_.

### Reading the Lesson

- Write another name for each term.
  - coordinate system **coordinate plane**
  - horizontal axis **x-axis**
  - vertical axis **y-axis**
- Identify each part of the coordinate system.



- In your own words, tell what is meant by the terms *dependent variable* and *independent variable*. Use the example below.

<b>dependent variable</b> the distance it takes to stop a motor vehicle	<b>independent variable</b> the speed at which the vehicle is traveling
$d$	$s$

**Sample answer:** The value of the dependent variable is a result of the value of the independent variable. Since  $d$  is a result of  $s$ ,  $d$  is the dependent variable and  $s$  is the independent variable.

### Helping You Remember

- In the alphabet,  $x$  comes before  $y$ . Use this fact to describe a method for remembering how to write ordered pairs. **Sample answer:** Since  $x$  comes before  $y$ , when writing ordered pairs, write the  $x$  value before the  $y$  value.

## 1-8 Enrichment

### The Digits of $\pi$

The number  $\pi$  ( $\pi$ ) is the ratio of the circumference of a circle to its diameter. It is a nonrepeating and nonterminating decimal. The digits of  $\pi$  never form a pattern. Listed at the right are the first 200 digits that follow the decimal point of  $\pi$ .

3.14159 26535 89793 23846  
69399 37510 58209 74944  
86280 34825 34211 70679  
09384 46095 50582 23172  
84102 70193 85211 05559  
26433 83279 50288 41971  
59230 78164 06286 20899  
82148 08651 32823 06647  
53594 08128 34111 74502  
64462 29489 54930 38196

### Solve each problem.

- Suppose each of the digits in  $\pi$  appeared with equal frequency. How many times would each digit appear in the first 200 places following the decimal point? **20**
- Complete this frequency table for the first 200 digits of  $\pi$  that follow the decimal point.

Digit	Frequency (Tally Marks)	Frequency (Number)	Cumulative Frequency
0		19	19
1		20	39
2		24	63
3		20	83
4		22	105
5		20	125
6		16	141
7		12	153
8		24	177
9		23	200

- Explain how the cumulative frequency column can be used to check a project like this one. **The last number should be 200, the number of items being counted.**
- Which digit(s) appears most often? **2 and 8**
- Which digit(s) appears least often? **7**



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1-9

Study Guide and Intervention

Statistics: Analyzing Data by Using Tables and Graphs

**Analyze Data** Graphs or tables can be used to display data. A bar graph compares different categories of data, while a circle graph compares parts of a set of data as a percent of the whole set. A line graph is useful to show how a data set changes over time.

**Example**

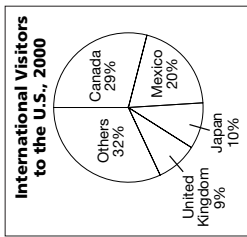
The circle graph at the right shows the number of international visitors to the United States in 2000, by country.

a. If there were a total of 50,891,000 visitors, how many were from Mexico?

$$50,891,000 \times 20\% = 10,178,200$$

b. If the percentage of visitors from each country remains the same each year, how many visitors from Canada would you expect in the year 2003 if the total is 59,000,000 visitors?

$$59,000,000 \times 29\% = 17,110,000$$



Source: Time

**Exercises**

1. The graph shows the use of imported steel by U.S. companies over a 10-year period.

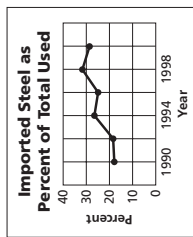
a. Describe the general trend in the graph. **The general trend is an increase in the use of imported steel over the 10-year period, with slight decreases in 1996 and 2000.**

b. What would be a reasonable prediction for the percentage of imported steel used in 2002? **about 30%**

2. The table shows the percentage of change in worker productivity at the beginning of each year for a 5-year period.

a. Which year shows the greatest percentage increase in productivity? **1998**

b. What does the negative percent in the first quarter of 2001 indicate? **Worker productivity decreased in this period, as compared to the productivity one year earlier.**



Source: Chicago Tribune

Year (1st Qtr.)	% of Change
1997	+1
1998	+4.6
1999	+2
2000	+2.1
2001	-1.2

Source: Chicago Tribune

NAME \_\_\_\_\_

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1-9

Study Guide and Intervention

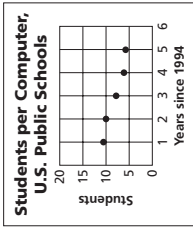
Statistics: Analyzing Data by Using Tables and Graphs

**Misleading Graphs** Graphs are very useful for displaying data. However, some graphs can be confusing, easily misunderstood, and lead to false assumptions. These graphs may be mislabeled or contain incorrect data. Or they may be constructed to make one set of data appear greater than another set.

**Example**

The graph at the right shows the number of students per computer in the U.S. public schools for the school years from 1995 to 1999. Explain how the graph misrepresents the data.

The values are difficult to read because the vertical scale is too condensed. It would be more appropriate to let each unit on the vertical scale represent 1 student rather than 5 students and have the scale go from 0 to 12.

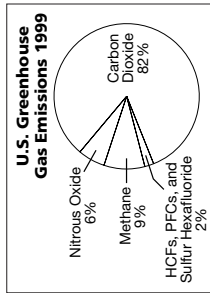


Source: The World Almanac

**Exercises**

Explain how each graph misrepresents the data.

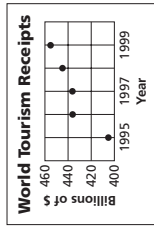
1. The graph below shows the U.S. greenhouse gases emissions for 1999.



Source: Department of Energy

The graph is misleading because the sum of the percentages is not 100%. Another section needs to be added to account for the missing 1%, or 3.6%.

2. The graph below shows the amount of money spent on tourism for 1998-99.



Source: The World Almanac

The graph is misleading because the vertical axis starts at 400 billion. This gives the impression that \$400 billion is a minimum amount spent on tourism.

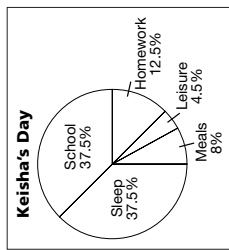
NAME \_\_\_\_\_ DATE \_\_\_\_\_ PERIOD \_\_\_\_\_

**1-9 Skills Practice**

**Statistics: Analyzing Data by Using Tables and Graphs**

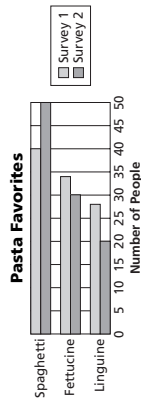
**DAILY LIFE** For Exercises 1–3, use the circle graph that shows the percent of time Keisha spends on activities in a 24-hour day.

1. What percent of her day does Keisha spend in the combined activities of school and doing homework? **50%**
2. How many hours per day does Keisha spend at school? **9 h**
3. How many hours does Keisha spend on leisure and meals? **3 h**



**PASTA FAVORITES** For Exercises 4–8, use the table and bar graph that show the results of two surveys asking people their favorite type of pasta.

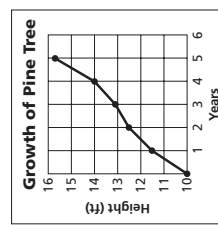
	Spaghetti	Fettuccine	Linguine
Survey 1	40	34	28
Survey 2	50	30	20



4. According to the graph, what is the ranking for favorite pasta in both surveys?  
**The ranking is the same for both: spaghetti, fettuccine, linguine.**
5. In Survey 1, the number of votes for spaghetti is twice the number of votes for which pasta in Survey 2? **linguine**
6. How many more people preferred spaghetti in Survey 2 than preferred spaghetti in Survey 1? **10 people**
7. How many more people preferred fettuccine to linguine in Survey 1? **6 people**
8. If you want to know the exact number of people who preferred spaghetti over linguine in Survey 1, which is a better source, the table or the graph? Explain.  
**The table, because it gives exact numbers.**

**PLANT GROWTH** For Exercises 9 and 10, use the line graph that shows the growth of a Ponderosa pine over 5 years.

9. Explain how the graph misrepresents the data.  
**The vertical axis begins at 10, making it appear that the tree grew much faster compared to its initial height than it actually did.**
10. How can the graph be redrawn so that it is not misleading?  
**To reflect accurate proportions, the vertical axis should begin at 0.**



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**1-9 Practice (Average)**

**Statistics: Analyzing Data by Using Tables and Graphs**

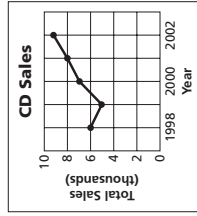
**MINERAL IDENTIFICATION** For Exercises 1–4, use the following information.

The table shows *Moh's hardness scale*, used as a guide to help identify minerals. If mineral A scratches mineral B, then A's hardness number is greater than B's. If B cannot scratch A, then B's hardness number is less than or equal to A's.

1. Which mineral(s) will fluorite scratch? **talc, gypsum, calcite**
2. A fingernail has a hardness of 2.5. Which mineral(s) will it scratch? **talc, gypsum**
3. Suppose quartz will not scratch an unknown mineral. What is the hardness of the unknown mineral? **at least 7**
4. If an unknown mineral scratches all the minerals in the scale up to 7, and corundum scratches the unknown, what is the hardness of the unknown? **between 7 and 9**

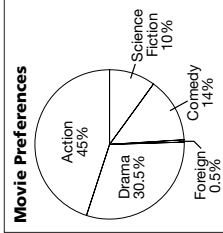
Mineral	Hardness
Talc	1
Gypsum	2
Calcite	3
Fluorite	4
Apatite	5
Orthoclase	6
Quartz	7
Topaz	8
Corundum	9
Diamond	10

**SALES** For Exercises 5 and 6, use the line graph that shows CD sales at Berry's Music for the years 1998–2002.



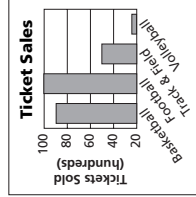
5. Which one-year period shows the greatest growth in sales?  
**from 1999 to 2000**
6. Describe the sales trend. **Sales started off at about 6000 in 1998, then dipped in 1999, showed a sharp increase in 2000, then a steady increase to 2002.**

**MOVIE PREFERENCES** For Exercises 7–9, use the circle graph that shows the percent of people who prefer certain types of movies.



7. If 400 people were surveyed, how many chose action movies as their favorite? **180**
8. Of 1000 people at a movie theater on a weekend, how many would you expect to prefer drama? **305**
9. What percent of people chose a category other than action or drama? **24.5%**

**TICKET SALES** For Exercises 10 and 11, use the bar graph that compares annual sports ticket sales at Mars High.



10. Describe why the graph is misleading. **Beginning the vertical axis at 20 instead of 0 makes the relative sales for volleyball and track and field seem low.**
11. What could be done to make the graph more accurate?  
**Start the vertical axis at 0.**

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## 1-9 Enrichment

### Percentiles

The table at the right shows test scores and their frequencies. The frequency is the number of people who had a particular score. The cumulative frequency is the total frequency up to that point, starting at the lowest score and adding up.

Score	Frequency	Cumulative Frequency
95	1	50
90	2	49
85	5	47
80	6	42
75	7	36
70	8	29
65	7	21
60	6	14
55	4	8
50	3	4
45	1	1

#### Example 1 What score is at the 16th percentile?

A score at the 16th percentile means the score just above the lowest 16% of the scores.

16% of the 50 scores is 8 scores.

The 8th score is 55.

The score just above this is 56.

So, the score at the 16th percentile is 56.

Notice that no one had a score of 56 points.

Use the table above to find the score at each percentile.

1. 42nd percentile **66**
2. 70th percentile **76**
3. 33rd percentile **66**
4. 90th percentile **86**
5. 58th percentile **71**
6. 80th percentile **81**

#### Example 2 At what percentile is a score of 75?

There are 29 scores below 75.

Seven scores are at 75. The fourth of these seven is the midpoint of this group.

Adding 4 scores to the 29 gives 33 scores.

33 out of 50 is 66%.

Thus, a score of 75 is at the 66th percentile.

Use the table above to find the percentile of each score.

7. a score of 50 **6th**
8. a score of 77 **72nd**
9. a score of 85 **90th**
10. a score of 58 **16th**
11. a score of 62 **28th**
12. a score of 81 **84th**

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## 1-9 Reading to Learn Mathematics

### Statistics: Analyzing Data by Using Tables and Graphs

#### Pre-Activity Why are graphs and tables used to display data?

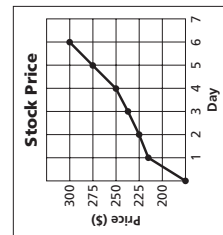
Read the introduction to Lesson 1-9 at the top of page 50 in your textbook.

Compare your reaction to the statement, *A stack containing George W.*

*Bush's votes from Florida would be 970.1 feet tall, while a stack of Al Gore's votes would be 970 feet tall* with your reaction to the graph shown in the introduction. Write a brief description of which presentation works best for you. **See students' work.**

#### Reading the Lesson

1. Choose from the following types of graphs as you complete each statement.  
 bar graph    circle graph    line graph
- a. A **circle graph** compares parts of a set of data as a percent of the whole set.
- b. **Line graphs** are useful when showing how a set of data changes over time.
- c. **Line graphs** are helpful when making predictions.
- d. **Bar graphs** can be used to display multiple sets of data in different categories at the same time.
- e. The percents in a **circle graph** should always have a sum of 100%.
- f. A **bar graph** compares different categories of numerical information, or data.



2. Explain how the graph is misleading. **Sample answer:** The first interval is from 0-200 and all other intervals are in units of 25, so the price rise appears steeper than it is.

#### Helping You Remember

3. Describe something in your daily routine that you can connect with bar graphs and circle graphs to help you remember their special purpose. **Sample answer: circle graphs—parts of a pizza; bar graphs—number of slices left in a loaf of bread**

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