Chapter 1

Structural Loads, Determinacy and Stability

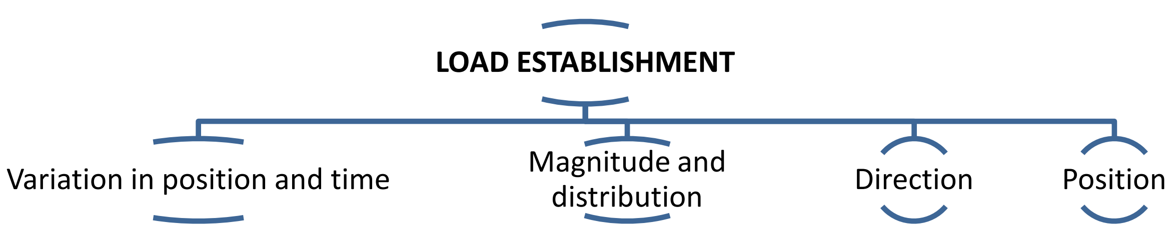
Structural Loads

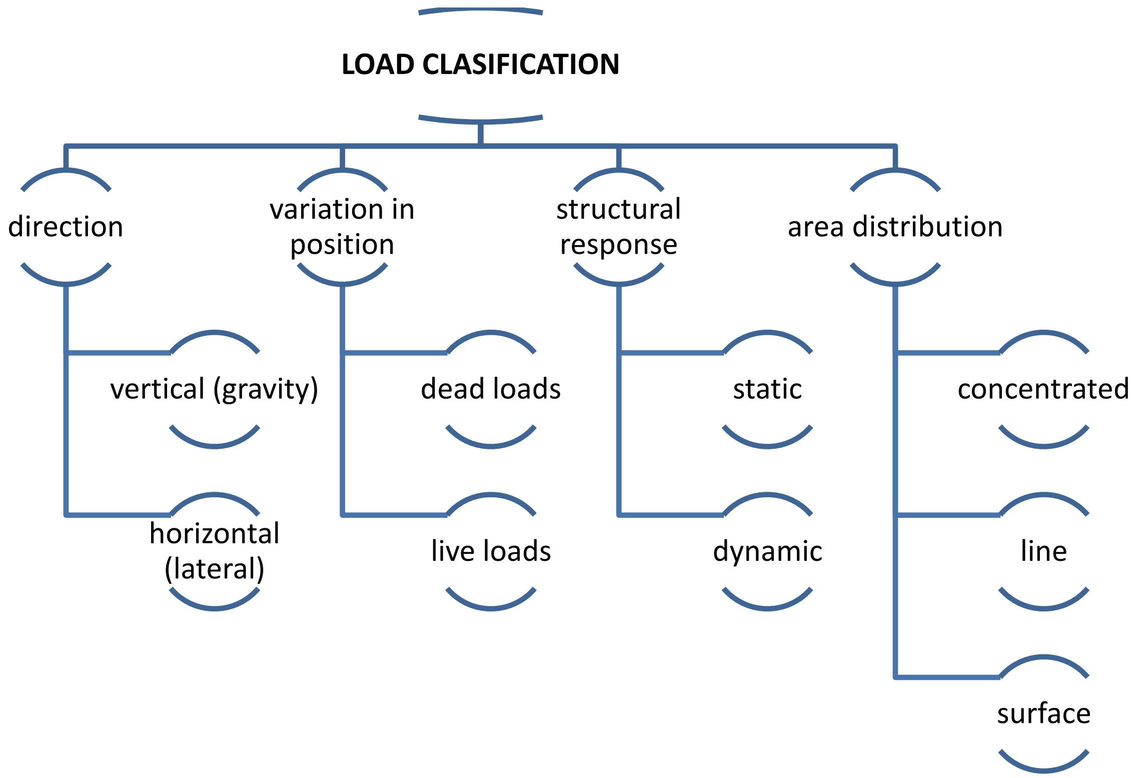
Introductory discussion points:

- How do you see the increasing interest and utility of computers in analysis and design of structures?
    - Matrix methods
    - Lengthy calculations
    - Accuracy
    - Deeper understanding
    - Reliance (cross checking of results)

- Consider the above points in comparison to the use of the classical hand calculations.

Structural engineering is the science and art of planning, designing, and constructing safe and economical structures that will serve their intended purposes. Structural analysis is an integral part of any structural engineering project, its function being the prediction of the performance of the proposed structure.



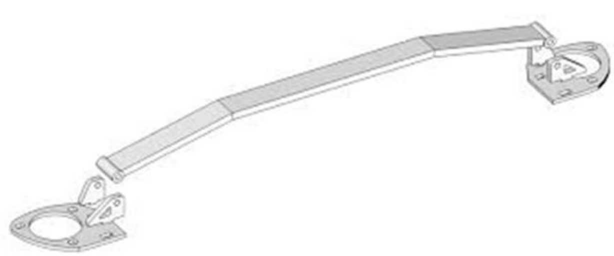


Points of discussion:

- What follows from load determination? [Think of design and analysis]
- The purpose of design majorly entails unity of functionality and economics. What other elements can you think of? [Consider aesthetics etc.]

Following are examples of structural elements:

1. Tie rods (bracing struts)
    - slender tensile members



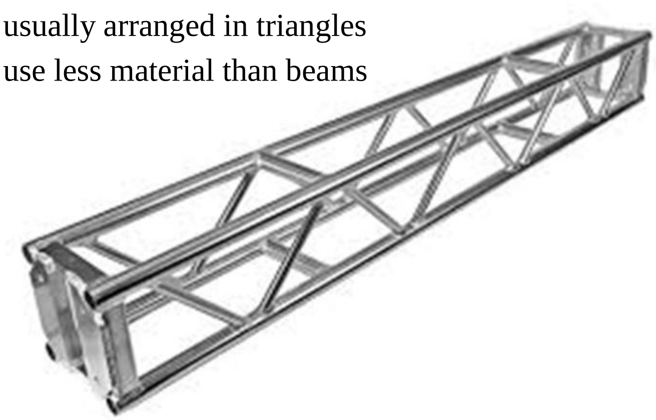
2. Beams

- carry horizontal loads
- mainly for flexural resistance
- need steel reinforcement

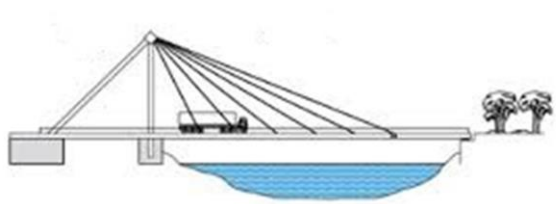
3. Columns
   - vertical members
   - resist axial compressive loads
   - beam-columns: resist both axial load and bending moment

Types of Structures

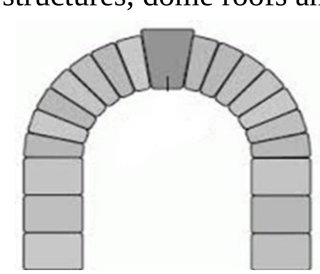
1. Trusses
   - usually arranged in triangles
   - use less material than beams



2. Cables
   - flexible
   - tension resistance only

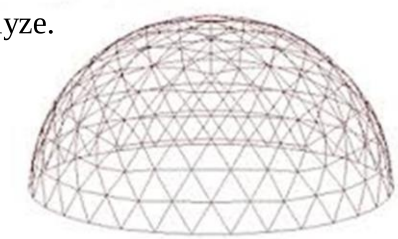


3. Arches
   - gain strength through material compression
   - used in bridge structures, dome roofs and for openings in masonry walls.



4. Frames
    - usually indeterminate ⟶ rigid joint connections
    - Economic benefit depends on efficiency of using smaller beams and larger columns
    - E.g. buildings

5. Surface structures
    - tents, air-inflated structures, plate or shell structures
    - difficult to analyze.



General building codes provide minimum loads and standards. However, detailed technical standards for structural design can be obtained from design codes.

Dead Loads

- Weight of structure + permanent fixtures
- EBCS 1
    - Densities of construction materials: E.g. normal concrete = 24 kN/m^3, cement mortar = 17 kN/m^3
    - Categories of building areas
    - Etc

Live Loads

- Movable
  - Stay for long periods of time. E.g. stored material in a warehouse
- Moving
  - E.g. vehicular loads on bridges
  - Traffic loads for bridges according to AASHTO
- Time-dependent
  - Dynamic in nature
  - E.g. load due to operation machinery

Environmental Loads

- earthquake
- Wind
- Rain

Load Combinations

- Load combinations for Ultimate Limit State (ULS)
  - E.g. Fd = 1.3Gk + 1.6Qk
    Where Fd is design load
    Gk is dead load
    Qk is live load
    1.3 and 1.6 are partial safety factors

  - Partial safety factors are provided to make up for
    - Calculation errors
    - Construction inaccuracies
    - Unforeseen increases in load
    -
- Load combinations for Serviceability Limit States (SLS)
  - E.g. Fd = Gk + Qk

Discussion points:

- Is it good to combine maximum effects of all loading conditions?
  - Consider economic repercussions of excessively conservative designs

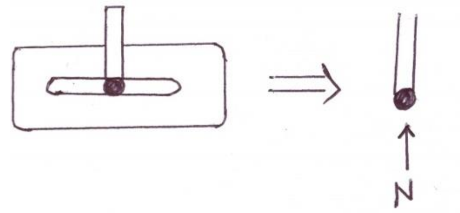
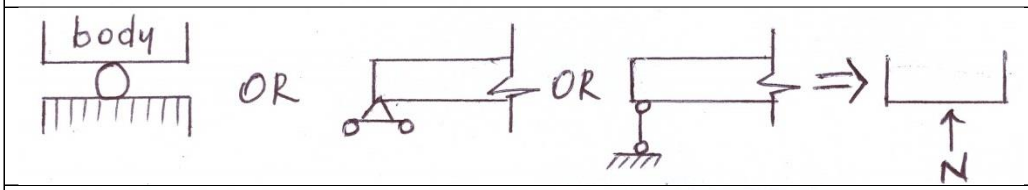
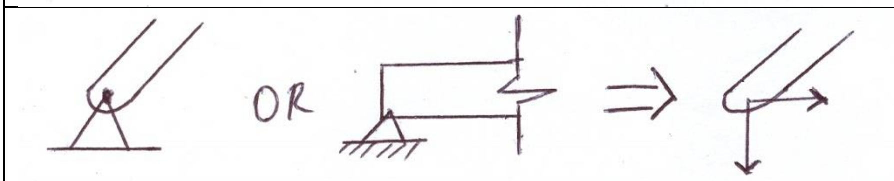
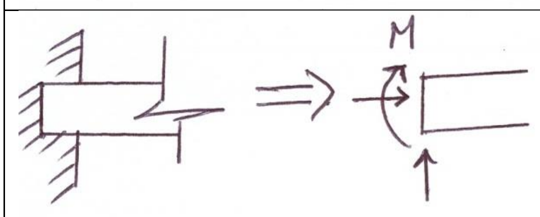
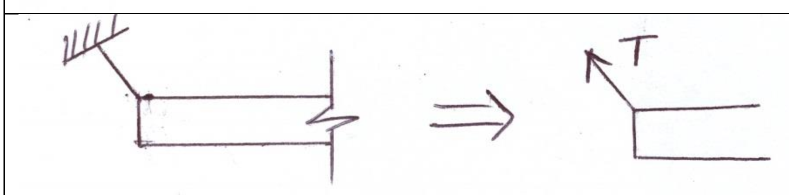
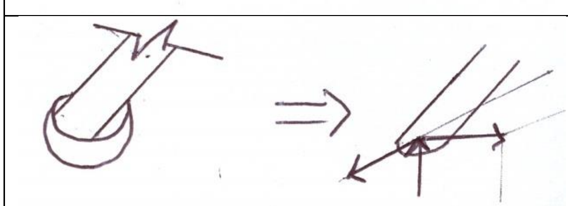
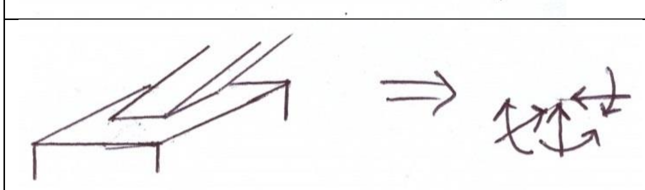
  - Discuss on general differences between structures of ancient times with those in the era of modern civil engineering?
- In the load combination Fd = 1.3Gk + 1.6Gk, the partial safety factor for live load exceeds that for live load. Brainstorm on why this is so.

Message to the instructor:

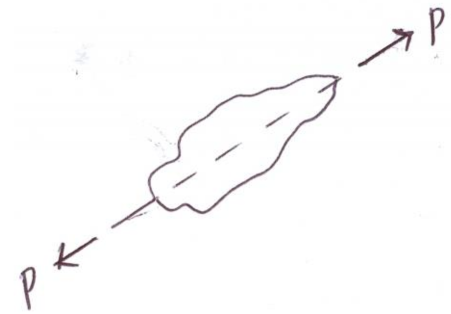
Students are encouraged to see references for a more complete understanding of loads on structures. Tip students on how to go through multiple references.

Stability and Determinacy of Structures

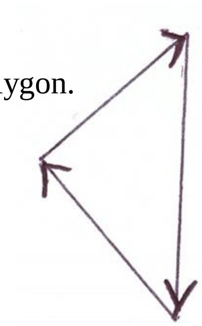
Types of Supports

	Freely sliding guide
	Roller support
	Hinge (pin) connection
	Fixed or built in support
	Cable support
	Ball and socket joint
	Rigid support in space

- Two-force members must have equal and opposite forces along the same line of action.



- Three-force members must have forces forming closed polygon.



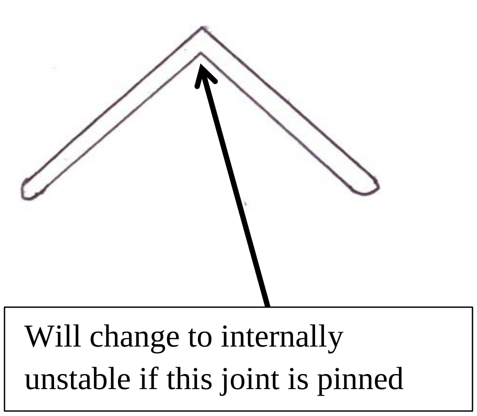
Truss: a framework composed of members joined at their ends to form a rigid structure.

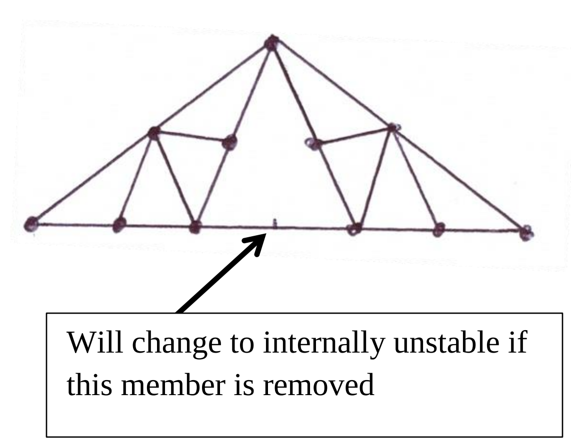
Plane truss: members lie in a plane.

Simple truss: formed from basic triangles to form a non-collapsible system.

A body is said to be **internally stable** or rigid if it maintains its shape and remains a rigid body when detached from the supports.

See the following two illustrations of internal stability:





Beam and Frame Structures

1. External stability
   a. Static determinacy of internally stable structures:
      i. If ($r < 3$), the structure is statically unstable externally.
      ii. If ($r = 3$), the structure is statically determinate externally.
      iii. If ($r > 3$), the structure is statically indeterminate externally.

   b. Static determinacy of internally unstable structures:
      i. If ($r < 3 + n$), the structure statically unstable externally.
      ii. If ($r = 3 + n$), the structure is statically determinate externally.
      iii. If ($r > 3 + n$), the structure is statically indeterminate externally.

   Where r is number of reactions; n is condition (construction) equations

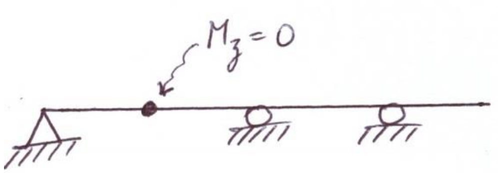
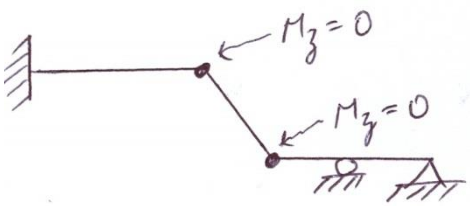
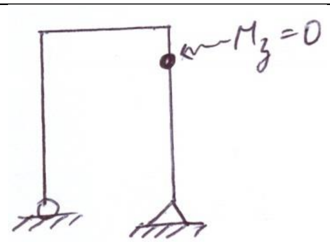
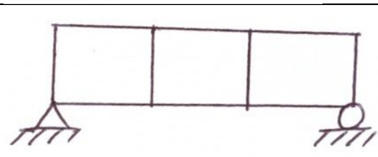
2. Overall stability
   The following situations can be observed for a planar beam-type structure:
   a. There are ($3m_a + r_a$) unknown quantities since each existing member is defined by an axial force, shear force, and bending moment. Also all reaction forces at the existing supports must be determined.

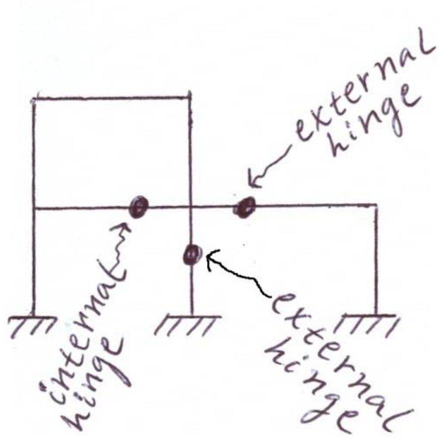
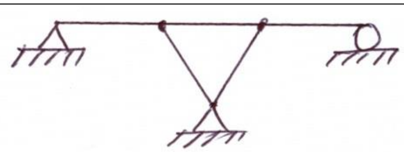
   b. There are ($3j + n$) available equations since each joint must satisfy $\sum F_x = 0$, $\sum F_y = 0$, and $\sum M_z = 0$. Additionally, there are n condition equations.

   If ($3m_a + r_a$) < ($3j + n$), then structure is statically unstable
   If ($3m_a + r_a$) = ($3j + n$), then structure is statically determinate
   If ($3m_a + r_a$) > ($3j + n$), then structure is statically indeterminate

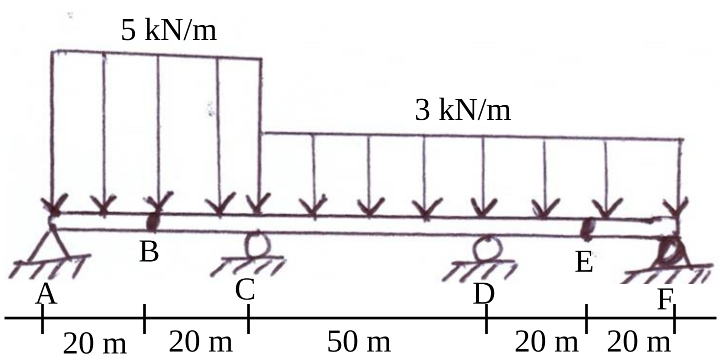
   Where r_a is number of existing reactions; j is number of joints, m_a number of existing members

Remark: Internal degree of indeterminacy can be calculated by subtracting external degree of indeterminacy from the overall degree of indeterminacy.

Example Structure		Structure Characteristics				External Classification		Overall Classification		
		j	n	m_a	r_a	r = 3 + n	Classification	(3ma + ra)	(3j + n)	Classification
1	 Free end of a cantilever beam is considered as a joint.	4	1	3	4	r = 3 + 1 = 4	$r = r_a$ Determinate; stable	9 + 4 = 13	12 + 1 = 13	Determinate Stable
2		5	2	4	6	r = 3 + 2 = 5	$r < r_a$ Indeterminate (1^{st} degree); stable	12 + 6 = 18	15 + 2 = 17	Indeterminate (1^{st} degree); stable
3		4	1	3	3	r = 3 + 1 = 4	$r > r_a$ Unstable	9 + 3 = 12	12 + 1 = 13	Unstable
4		8	0	10	3	r = 3	$r = r_a$ Determinate	30 + 3 = 33	24 + 0 = 24	Indeterminate (9^{th} degree); stable

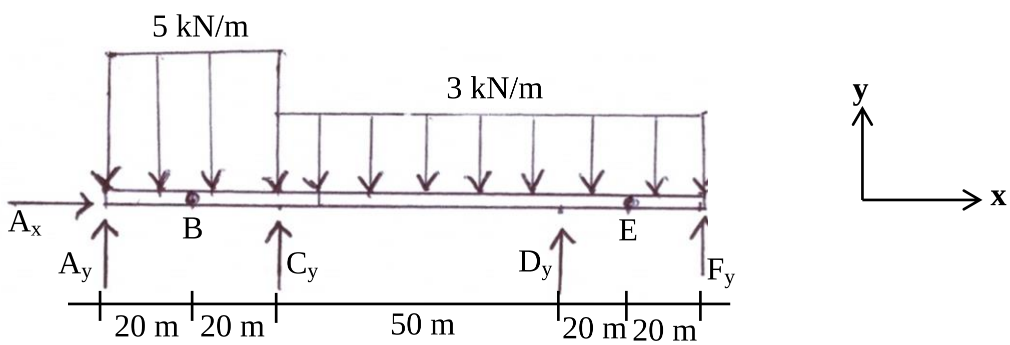
	Example Structure	**Structure Characteristics**				**External Classification**		**Overall Classification**		
		j	n	m_a	r_a	r = 3 + n	Classification	$(3m_a + r_a)$	(3j + n)	Classification
5		8	3 N.B. Only 2 are external	8	9	r = 3 + 2 = 5	$r < r_a$ Indeterminate (4^{th} degree)	24 + 9 = 33	24 + 3 = 27	Indeterminate (9^{th} degree); stable
6	 If there are p members that frame into a common pin support, (p – 1) member-end conditions must be introduced.	5	1	5	5	r = 3 + 0	$r < r_a$ Indeterminate (2^{nd} degree); stable	15 + 5 = 20	15 + 1 = 16	Indeterminate (4^{th} degree); stable

Example 1: Determine the reactions at the supports



Solution

Free Body Diagram



Take a section to the left of hinge B

$\sum M_B = 0$, CCW +ve

$(5*20*10) - (A_y*20) = 0$

$\mathbf{A_y = 50\ kN}$

Take a section to the right of hinge E

$\sum M_E = 0$, CCW +ve

$(F_y*20) - (3*20*10) = 0$

$\mathbf{Fy = 30\ kN}$

Take the entire beam

$\sum M_C = 0$, CCW +ve

$(5*40*20) - (A_y*40) - (3*90*45) + (D_y*50) + (F_y*90) = 0$

But $A_y = 50$ kN, $F_y = 30$ kN

$\mathbf{Dy = 149\ kN}$

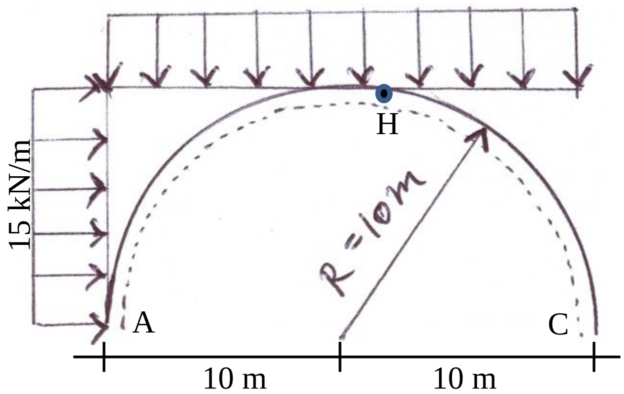
$\sum Fy = 0$, upward +ve

$-(5*40) - (3*90) + A_y + C_y + D_y + F_y = 0$

But $A_y = 50$ kN, $F_y = 30$ kN, and Dy = 149 kN

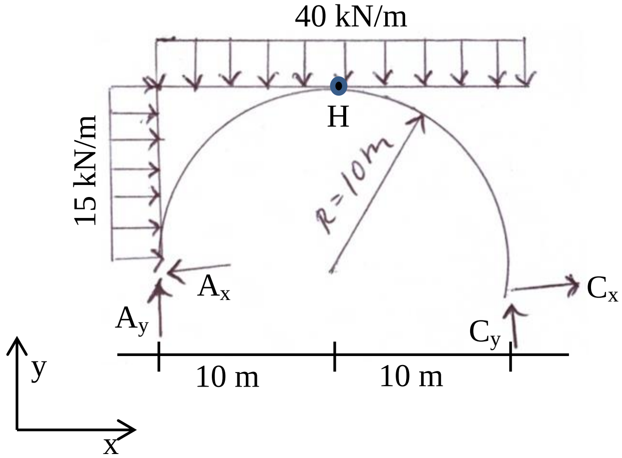
$\mathbf{C_y = 241\ kN}$

Example 2: Find the reaction at A and C



Solution

Free body diagram



Left of hinge

$\sum M_H = 0$, CCW +ve

$(40*10*5) + (15*10*5) + (A_x*10) – (A_y*10) = 0$

$2750 + 10A_x – 10A_y = 0$ ……………….. ①

Right of hinge

$\sum M_H = 0$, CCW +ve

$– (40*10*5) + (C_y*10) – (C_y*10) = 0$

$– 2000 + 10C_y + 10C_x = 0$ ……………….. ②

Entire arch

$\sum F_x = 0$, right +ve

$(15*10) + A_x + C_x = 0$ ……………….. ③

Entire arch

$\sum F_y = 0$, upward +ve

$– (40*20) + A_y + C_y = 0$ ……………….. ④

Moment at C

$\sum M_C = 0$, CCW +ve

$(40*20*10) – (15*10*5) – (A_y*20) = 0$

$– 2000 + 10C_y + 10C_x = 0$

A_y = 362.5 kN……………….. ⑤

Insert ⑤ in ④, **Cy = 437.5 kN** ……….. ⑥

Insert ⑥ in ②, **Cx = – 237.5 kN** ……….. ⑦

Insert ⑦ in ③, A**x = 87.5 kN**

Trusses

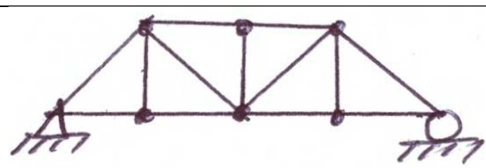
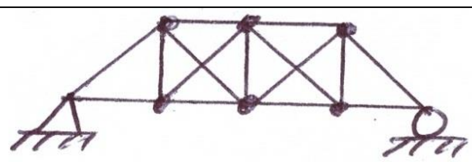
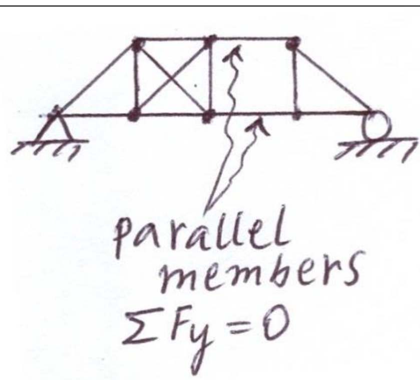
1. External stability:
   The analysis is the same as in beam and frame structures discussed above.

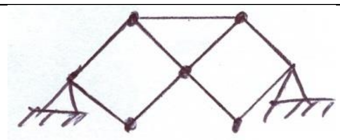
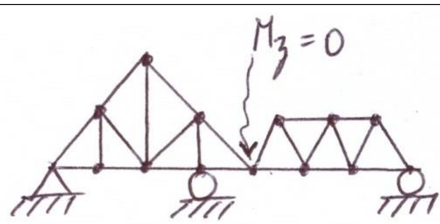
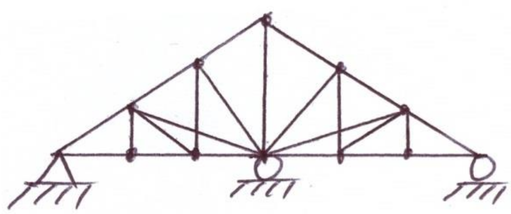
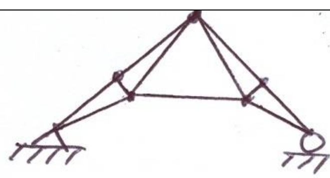
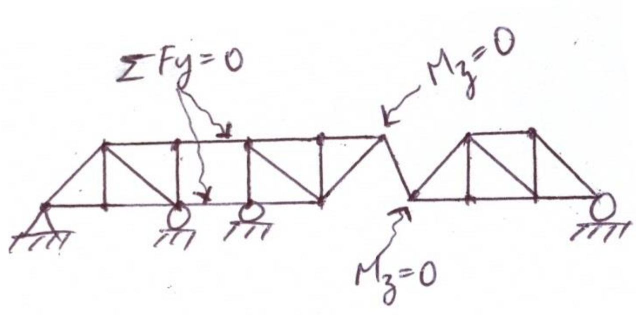
2. Internal stability:
   There are ($m + r_a$) unknown quantities where m is the number of members and r_a is the number of existing reaction forces.

   There are 2j available equations for plannar trusses, and 3j available equations for space trusses where j is the number of joints and n is the number of condition equations.

   $m + r_a = 2j$ or $m = 2j - r_a$ for statically determinate stable structures.

   If ($m_a < m$), then structure is statically unstable
   If ($m_a = m$), then structure is statically determinate
   If ($m_a < m$), then structure is statically indeterminate

Example Structure		External Classification	Internal Classification
1		$j = 8$, $m_a = 13$, $r_a = 3$, $r = 3$ $r_a = r$ Determinate; stable	$m = 2j - ra = 16 - 3 = 13$ $ma = m$ Determinate; stable
2		$j = 8$, $m_a = 15$, $r_a = 3$, $r = 3$ $r_a = r$ Determinate; stable	$m = 2j - ra = 16 - 3 = 13$ $m_a > m$ Indeterminate (2^{nd} degree); stable
3		$j = 8$, $m_a = 13$, $r_a = 3$, $r = 3 + 1 = 4$ $r_a < r$ Unstable	$m = 2j - ra = 16 - 3 = 13$ $m_a = m$ Determinate; unstable

	Example Structure	**External Classification**	**Internal Classification**
4		$j = 7$, $m_a = 9$, $r_a = 4$, $r = 3$ $r_a > r$ Indeterminate (1st degree); stable	$m = 2j - r_a = 14 - 3 = 11$ $m_a = m$ Determinate; stable
5		$j = 14$, $m_a = 24$, $r_a = 4$, $r = 3 + 1 = 4$ $r_a = r$ Determinate; stable	$m = 2j - r_a = 28 - 4 = 24$ $m_a = m$ Determinate; stable
6		$j = 12$, $m_a = 23$, $r_a = 3$, $r = 3$ $r_a > r$ Indeterminate (1st degree); stable	$m = 2j - r_a = 14 - 3 = 11$ $m_a = m$ Determinate; stable
7		$j = 7$, $m_a = 11$, $r_a = 3$, $r = 3$ $r_a = r$ Determinate; stable	$m = 2j - r_a = 14 - 3 = 11$ $m_a = m$ Determinate; stable
8		$j = 16$, $m_a = 26$, $r_a = 6$, $r = 3 + 3 = 6$ $r_a = r$ Determinate; stable	$m = 2j - r_a = 32 - 6 = 26$ $m_a = m$ Determinate; stable

Analysis of Trusses

Points of discussion:

1. The method of joints and the method of sections for truss analysis have been dealt with in engineering mechanics I (statics). Make quick revision if necessary.
2. What are the major differences between the method of joints and the method of sections?
3. Discuss qualitatively and quantitatively how to go about solving example 8 above using both methods.