Chapter 1 Trigonometry

Chapter 1 Prerequisite Skills

Chapter 1 Prerequisite Skills Question 1 Page 4

a)
$$x^2 = 36$$
$$x = \pm\sqrt{36}$$
$$x = \pm 6$$

b)
$$x^2 - 6 = 19$$
$$x^2 = 6 + 19$$
$$x^2 = 25$$
$$x = \pm\sqrt{25}$$
$$x = \pm 5$$

c)
$$x^2 = 64 + 36$$
$$x^2 = 100$$
$$x = \pm\sqrt{100}$$
$$x = \pm 10$$

d)
$$x^2 = 5^2 + 12^2$$
$$x^2 = 25 + 144$$
$$x^2 = 169$$
$$x = \pm\sqrt{169}$$
$$x = \pm 13$$

e)
$$7^2 + x^2 = 25^2$$
$$x^2 = 25^2 - 7^2$$
$$x^2 = 625 - 49$$
$$x^2 = 576$$
$$x = \pm\sqrt{576}$$
$$x = \pm 24$$

Chapter 1 Prerequisite Skills Question 2 Page 4

a)
$$c^2 = 8^2 + 6^2$$
$$c^2 = 64 + 36$$
$$c^2 = 100$$
$c = +\sqrt{100}$ since c is a length
$$c = 10\ cm$$

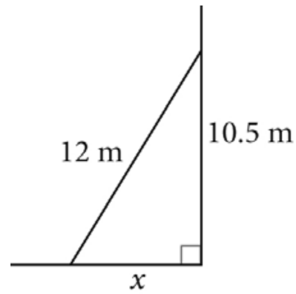
b)
$$b^2 = 13^2 - 5^2$$
$$b^2 = 169 - 25$$
$$b^2 = 144$$
$$b = +\sqrt{144}$$
$$b = 12 \text{ cm}$$

c)
$$a^2 = 25^2 - 16^2$$
$$a^2 = 625 - 256$$
$$a^2 = 369$$
$$a = +\sqrt{369}$$
$$a = 19.2 \text{ m}$$

Chapter 1 Prerequisite Skills Question 3 Page 4

Let the distance from the wall to the base of the ladder be *x* metres.

$$
\begin{aligned}
12^2 &= x^2 + 10.5^2 \\
x^2 &= 12^2 - 10.5^2 \\
x^2 &= 144 - 110.25 \\
x^2 &= 33.75 \\
x &= +\sqrt{33.75} \\
x &= 5.8
\end{aligned}
$$



The distance from the wall to the base of the ladder is 5.8 m.

Chapter 1 Prerequisite Skills Question 4 Page 4

a) $4:8 = \frac{4}{8} = \frac{4 \div 4}{8 \div 4} = \frac{1}{2} = 1:2$

b) $15:35 = \frac{15}{35} = \frac{15 \div 5}{35 \div 5} = \frac{3}{7} = 3:7$

c) $20:50 = \frac{20}{50} = \frac{20 \div 10}{50 \div 10} = \frac{2}{5} = 2:5$

Chapter 1 Prerequisite Skills Question 5 Page 4

Let the purchase price of the chip be $*x*.

$$
\begin{aligned}
\text{Then } \frac{18}{7} &= \frac{27}{x} \\
x &= \frac{27 \times 7}{18} \\
x &= \frac{3 \times 7}{2} \\
x &= 10.50
\end{aligned}
$$

The price of the chip is $10.50.

Chapter 1 Prerequisite Skills **Question 6 Page 4**

a) $\frac{x}{13} = \frac{9}{39}$

$x = \frac{9 \times 13}{39}$

$x = 3$

b) $\frac{15}{1} = \frac{45}{x}$

$x = \frac{45}{15}$

$x = 3$

c) $\frac{x}{25} = \frac{y}{5} = \frac{8}{10}$

So, $\frac{x}{25} = \frac{8}{10}$

$x = \frac{25 \times 8}{10}$

$x = 20$

and $\frac{y}{5} = \frac{8}{10}$

$y = \frac{5 \times 8}{10}$

$y = 4$

Chapter 1 Prerequisite Skills **Question 7 Page 4**

a) One unit of distance on the map represents 700 000 of the same unit of distance on the earth.

b) Since the distance on the map is 12 cm, the actual distance is:
12 × 700 000 cm
= 8 400 000 cm
= 84 000 m
= 84 km

c) The actual distance in centimetres is:
40 km = 40 000 m = 4 000 000 cm
The map distance is:
4 000 000 ÷ 700 000
= 40 ÷ 7 = 5.7 cm

Chapter 1 Prerequisite Skills **Question 8 Page 5**

a) 3.4576 = 3.46

b) 19.832 = 19.83

c) 9015.982 36 = 9015.98

Chapter 1 Prerequisite Skills Question 9 Page 5

a) $\sqrt{59} = 7.7$

b) $\sqrt{723} = 26.9$

c) $\sqrt{0.85} = 0.9$

Chapter 1 Prerequisite Skills Question 10 Page 5

a) $\angle B = 180° - (82° + 74°)$

$\angle B = 24°$

b) $\angle B = \angle C = \dfrac{(180° - 42°)}{2} = 69°$

c) $\angle A + \angle B = 90°$

$\angle A = 90° - 54°$

$\angle A = 36°$

Chapter 1 Section 1 **Revisit the Primary Trigonometric Ratios**

Chapter 1 Section 1 **Question 1 Page 13**

a) For ∠B: opposite AC or *b*; adjacent: BC or *a*; hypotenuse: AB or *c*

b) For ∠F: opposite DE or *f*; adjacent: EF or *d*; hypotenuse: DF or *e*

c) For ∠Z: opposite XY or *z*; adjacent: YZ or *x*; hypotenuse: XZ or *y*

Chapter 1 Section 1 **Question 2 Page 14**

a) $\sin 30° = 0.5000$

b) $\cos 45° = 0.7071$

c) $\tan 60° = 1.7321$

Chapter 1 Section 1 **Question 3 Page 14**

a) $\angle A = \sin^{-1}(0.2345)$
$\angle A \approx 13.6°$

b) $\angle B = \cos^{-1}(0.8765)$
$\angle B \approx 28.8°$

c) $\angle C = \tan^{-1}(1.2345)$
$\angle C \approx 51.0°$

Chapter 1 Section 1 **Question 4 Page 14**

a) $\frac{a}{25} = \cos 25°$
$a = 25(\cos 25°)$
$a \approx 23 \text{ m}$

b) $\frac{c}{25} = \sin 25°$
$c = 25(\sin 25°)$
$c \approx 11 \text{ m}$

c) $\angle A = 90° - 25°$
$\angle A = 65°$

Chapter 1 Section 1 Question 5 Page 14

a) $\sin\ A = \dfrac{10}{36}$

$\angle A = \sin^{-1}\left(\dfrac{10}{36}\right)$

$\angle A \approx 16.1°$

b) $\angle B = 90° - 16.1°$

$\angle B = 73.9°$

c) $36^2 = b^2 + 10^2$

$b^2 = 36^2 - 10^2 = 1196$

$b = +\sqrt{1196}$

$b \approx 34.58$

Side *b* measures 35 cm to the nearest centimetre.

Chapter 1 Section 1 Question 6 Page 14

a) $\dfrac{5.5}{c} = \cos 66°$

$c = \dfrac{5.5}{\cos 66°}$

$c \approx 13.5$ m

b) $\dfrac{a}{5.5} = \tan 66°$

$a = 5.5(\tan 66°)$

$a \approx 12.4$ m

Chapter 1 Section 1 Question 7 Page 14

a) $\dfrac{b}{35.5} = \tan 20°$

$b = 35.5(\tan 20°)$

$b \approx 12.9$ cm

b) $\dfrac{35.5}{c} = \sin 70°$

$c = \dfrac{35.5}{\sin 70°}$

$c \approx 37.8$ cm

Chapter 1 Section 1 Question 8 Page 14

a) $\dfrac{a}{100} = \sin 27°$

$a = 100(\sin 27°)$

$a \approx 45.4$ m

b) $\dfrac{b}{100} = \cos 27°$

$b = 100(\cos 27°)$

$b \approx 89.1$ m

Chapter 1 Section 1 Question 9 Page 15

$$\angle A = 25°$$

$$\frac{a}{15.5} = \tan 25°$$
$$a = 15.5(\tan 25°)$$
$$a \approx 7.2 \text{ cm}$$

$$\frac{15.5}{c} = \sin 65°$$
$$c = \frac{15.5}{\sin 65°}$$
$$c \approx 17.1 \text{ cm}$$

Chapter 1 Section 1 Question 10 Page 15

AD = AH + HD

$$\frac{\text{AH}}{12} = \tan 22°$$
$$\text{AH} = 12(\tan 22°)$$
$$\text{AH} \approx 4.8 \text{ m}$$

$$\frac{\text{HD}}{12} = \tan 45°$$
$$\text{HD} = 12(\tan 45°)$$
$$\text{HD} \approx 12 \text{ m}$$

AD = 4.8 m + 12 m = 16.8 m
Side AD is approximately 16.8 m.

Chapter 1 Section 1 Question 11 Page 15

$$\frac{\text{CD}}{17} = \cos 50°$$
$$\text{CD} = 17(\cos 50°)$$
$$\text{CD} \approx 10.9 \text{ m}$$

$$\frac{17}{\text{BD}} = \cos 50°$$
$$\text{BD} = \frac{17}{\cos 50°}$$
$$\text{BD} \approx 26.4 \text{ m}$$

BC = BD – CD
BC = 26.4 m – 10.9 m = 15.5 m
Side BC is approximately 15.5 m.

Chapter 1 Section 1 Question 12 Page 15

AD = AC – DC

$$\frac{75}{AC} = \tan 30°$$

$$AC = \frac{75}{\tan 30°}$$

$$AC \approx 129.9 \text{ cm}$$

$$\frac{75}{DC} = \tan 40°$$

$$DC = \frac{75}{\tan 40°}$$

$$DC \approx 89.4 \text{ cm}$$

AD = 129.9 cm – 89.4 cm = 40.5 cm
AD is approximately 40.5 cm.

Chapter 1 Section 1 Question 13 Page 15

$\angle A = 30° = \angle CBD$ (corresponding angles)

$AD = AB + BD$

$$\frac{1}{AB} = \cos 30°$$

$$AB = \frac{1}{\cos 30°}$$

$$AB \approx 1.2 \text{ mm}$$

$$\frac{7}{BD} = \sin 30°$$

$$BD = \frac{7}{\sin 30°}$$

$$BD = 14 \text{ mm}$$

AD = 1.2 mm + 14 mm $\approx$ 15 mm
AD is approximately 15 mm.

Chapter 1 Section 1 Question 14 Page 15

The area of trapezoid ACDE is given by $\frac{\text{AE} \times (\text{AC} + \text{ED})}{2}$.

AE = BD = 12 cm
AB = ED = 22 cm
AC = AB + BC

To find BC,

$$\frac{\text{BD}}{\text{BC}} = \tan 70°$$

$$\text{BC} = \frac{12}{\tan 70°}$$

$$\text{BC} \approx 4.4 \text{ cm}$$

To find the area, *A*:

$$A = \frac{12(22 + 4.4 + 22)}{2}$$

$$A = 6(48.4)$$

$$A = 290.4$$

The area of trapezoid ACDE is approximately 290 cm^2.

Chapter 1 Section 2 **Solve Problems Using Trigonometric Ratios**

Chapter 1 Section 2 **Question 1 Page 21**

Let the angle of the ramp be $x°$.

Then $\sin x = \dfrac{0.45}{6.10}$

$$x = \sin^{-1}(\frac{0.45}{6.10})$$

$$x \approx 4$$

The angle of the ramp is approximately 4°.

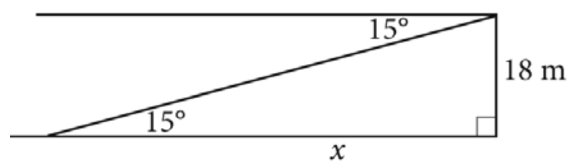
Chapter 1 Section 2 **Question 2 Page 21**

Let the horizontal distance from the bridge to the sailboat be x metres.

$$\tan 15° = \frac{18}{x}$$

$$x = \frac{18}{\tan 15°}$$

$$x \approx 67$$



The horizontal distance from the bridge to the sailboat is approximately 67 m.

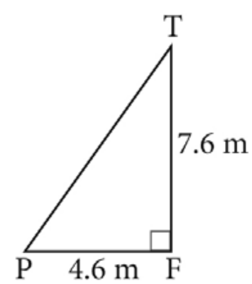
Chapter 1 Section 2 **Question 3 Page 21**

∠P is the angle of elevation from the pedestrian to the top of the flagpole.

$$\tan \text{P} = \frac{7.6}{46}$$

$$\angle \text{P} = \tan^{-1}\left(\frac{7.6}{4.6}\right)$$

$$\angle \text{P} \approx 59$$



The angle of elevation is approximately 59°.

Chapter 1 Section 2 Question 4 Page 22

Let the angle of inclination of the garage floor be *x*°.
Change all measurements to common units:
6.7 m = 670 cm

$$\tan x = \frac{9.1}{670}$$
$$x = \tan^{-1}\left(\frac{9.1}{670}\right)$$
$$x \approx 1$$

The angle of inclination of the garage floor is approximately 1°.

Chapter 1 Section 2 Question 5 Page 22

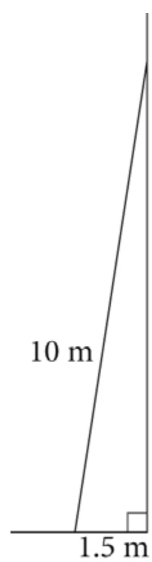
Let the supporting board be *x* feet long.

$$\frac{x}{12} = \tan 22.5°$$
$$x = 12 \times \tan 22.5°$$
$$x \approx 5$$

The supporting board is approximately 5 ft long.

Chapter 1 Section 2 Question 6 Page 22

The diagram represents the 10 m ladder leaning with its base 1.5 m from the wall.



To find the measure of ∠B,

$$\cos B = \left(\frac{1.5}{10}\right)$$
$$\angle B = \cos^{-1}\left(\frac{1.5}{10}\right)$$
$$\angle B \approx 81$$

Yes, the inspector should be concerned.
The angle at the base of the ladder is about 81°,
not between 70° and 80° as safety by-laws require.

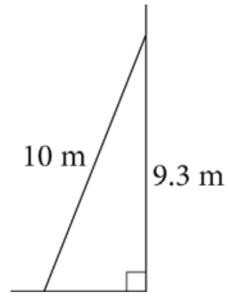
Chapter 1 Section 2 Question 7 Page 22

The diagram represents the 10 m ladder leaning with the top 9.3 m from the ground.
To find $\angle E$,

$$\sin E = \frac{9.3}{10}$$

$$\angle E = \sin^{-1}\left(\frac{9.3}{10}\right)$$

$$\angle E \approx 68°$$



The angle the ladder makes with the ground is approximately 68°.
No, the ladder is not stable according to the safety by-laws.

Chapter 1 Section 2 Question 8 Page 22

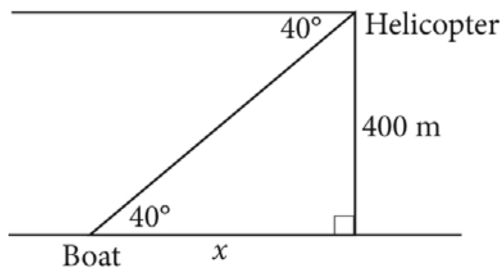
Assume the boat and helicopter are a horizontal distance of x metres apart.

Use the tangent ratio.

$$\frac{400}{x} = \tan 40°$$

$$x = \frac{400}{\tan 40°}$$

$$x \approx 477$$



The horizontal distance between the boat and the helicopter is approximately 477 m.

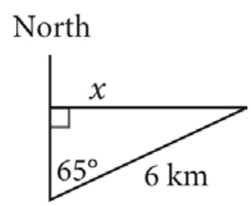
Chapter 1 Section 2 Question 9 Page 22

Let the distance off course be x kilometres after 2 h.

$$\sin 65° = \frac{x}{6}$$

$$x = 6 \times \sin 65°$$

$$x \approx 5$$



If the three team members travelled at 3 km/h at an angle of 65° east of north, they would be 5 km off the due north track, to the nearest kilometre.

Chapter 1 Section 2 Question 10 Page 22

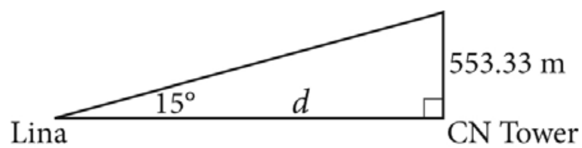
Answers may vary.

In Ontario it would seem there are no specific provincial safety standards for building skateboard ramps. Skateboard parks are designed to have a variety of ramps with different inclines and different lengths for the varying abilities of skateboarders. When the angle of the ramp is 90°, the ramp becomes a jump. The skateboarders wear safety equipment for protection (i.e., elbow and knee pads, and helmets).

Most Internet sources are of a commercial nature or give the fine details of municipal regulations. There are very few skateboard parks in Ontario for students to investigate first hand.

Chapter 1 Section 2 Question 11 Page 23

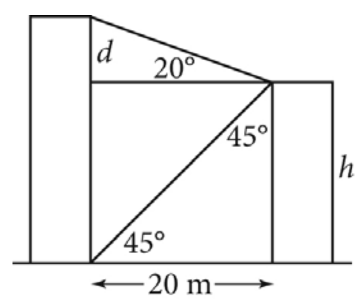
First, the situation is modelled in the diagram here.
Lina reversed the numerator and denominator in the formula for tan 15°.
Lina should have written:



$$\frac{553.33}{d} = \tan 15°$$
$$d = \frac{553.33}{\tan 15°}$$
$$d \approx 2065$$

Chapter 1 Section 2 Question 12 Page 23

The diagram represents the two apartment buildings.
The height of the smaller building is *h* metres.
The height of the taller building is equal to ($h + d$), where *d* is the additional height of the taller building, in metres.



$$\frac{h}{20} = \tan 45°$$
$$h = 20 \times \tan 45°$$
$$h = 20 \text{ m}$$

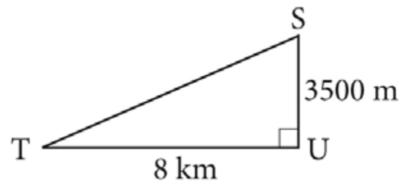
and $$\frac{d}{20} = \tan 20°$$
$$d = 20 \times \tan 20°$$
$$d \approx 7 \text{ m}$$

20 m + 7 m = 27 m

Therefore, the height of the taller building is approximately 27 m.

Chapter 1 Section 2 Question 13 Page 23

Angle T is the angle of elevation of the shuttle.



Convert all distances to kilometres:
3500 m = 3.5 km

$$\tan \text{T} = \frac{3.5}{8}$$
$$\angle\text{T} = \tan^{-1}\left(\frac{3.5}{8}\right)$$
$$\angle\text{T} \doteq 24°$$

The angle of elevation is approximately 24°.

Chapter 1 Section 2 Question 14 Page 23

The answer in the text calculates the horizontal distance travelled.

$$\frac{200}{d} = \tan 3°$$
$$d = \frac{200}{\tan 3°}$$
$$d \doteq 3816$$

The horizontal distance the pilot travelled was approximately 3816 m.

An alternative interpretation is to calculate the glide slope distance.

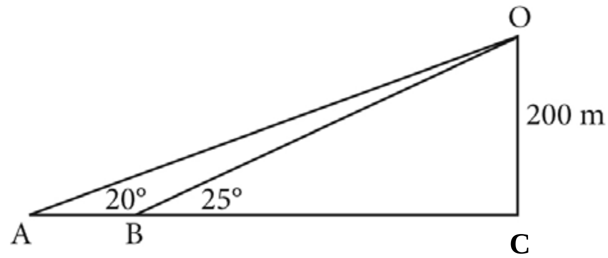
$$\frac{200}{d} = \sin 3°$$
$$d = \frac{200}{\sin 3°}$$
$$d \doteq 3821$$

The distance the pilot glided was approximately 3821 m.

Chapter 1 Section 2 **Question 15 Page 23**

The diagram represents the position of an observer, O, on a cliff and the positions of two boats at A and B. The base of the cliff is at C.

Assume that the observer and the two boats are in line (i.e., the same vertical plane) and that the water surface is calm so that the boats do not change their angles.



AC represents the distance of boat A from the base of the cliff.
BC represents the distance of boat B from the base of the cliff.
To find the distance that the two boats are apart, use AB = AC – BC.

Find AC:

$$\frac{200}{AC} = \tan 20°$$

$$AC = \frac{200}{\tan 20°}$$

$$AC \approx 549.5 \text{ m}$$

Find BC:

$$\frac{200}{BC} = \tan 25°$$

$$BC = \frac{200}{\tan 25°}$$

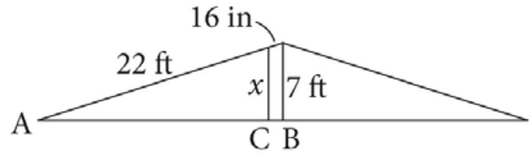
$$BC \approx 428.9 \text{ m}$$

549.5 m – 428.9 m = 121 m

Therefore, the boats are approximately 121 m apart.

Chapter 1 Section 2 **Question 16 Page 23**

The angle of the roof is ∠A.



$$\sin A = \frac{7}{22}$$
$$\angle A = \sin^{-1}\left(\frac{7}{22}\right)$$
$$\angle A \approx 19°$$

Half the width of the roof is
$$AB = 22(\cos 19°)$$
$$AB \approx 20.8 \text{ ft}$$

Let the support piece be *x* feet long.
Convert all measurements to the same units of measure:
16 in. = 1.33 ft
So,
$$\frac{x}{(20.8 - 1.33)} = \tan 19°$$
$$x = 19.47(\tan 19°)$$
$$x \approx 6.7 \text{ ft}$$

The support piece is approximately 6.7 ft long.

Chapter 1 Section 3 The Sine Law

Chapter 1 Section 3 Question 1 Page 31

a)
$$\frac{25}{\sin 35°} = \frac{b}{\sin 70°}$$
$$b = \frac{25 \times \sin 70°}{\sin 35°}$$
$$b \approx 41.0$$

Side *b* is approximately 41.0 cm.

b)
$$\frac{60}{\sin 58°} = \frac{d}{\sin 65°}$$
$$d = \frac{60 \times \sin 65°}{\sin 58°}$$
$$d \approx 64.1$$

Side *d* is approximately 64.1 m.

c) $\angle X = 180° - 55° - 47° = 78°$
$$\frac{60}{\sin 55°} = \frac{x}{\sin 78°}$$
$$x = \frac{60 \times \sin 78°}{\sin 55°}$$
$$x \approx 71.6$$

Side *x* is approximately 71.6 cm.

Chapter 1 Section 3 **Question 2 Page 31**

a) $\dfrac{12}{\sin C} = \dfrac{32}{\sin 72°}$

$\sin C = \dfrac{12 \times \sin 72°}{\sin 32°}$

$\angle C = \sin^{-1}\left(\dfrac{12 \times \sin 72°}{\sin 32°}\right)$

$\angle C \approx 20.9°$

The measure of $\angle C$ is approximately 20.9°.

b) $\dfrac{6.4}{\sin B} = \dfrac{10.2}{\sin 80°}$

$\sin B = \dfrac{6.4 \times \sin 80°}{10.2}$

$\angle B = \sin^{-1}\left(\dfrac{6.4 \times \sin 80°}{10.2}\right)$

$\angle B \approx 38.2°$

The measure of $\angle B$ is approximately 38.2°.

Chapter 1 Section 3 Question 3 Page 32

a) $\angle X = 180° - 72° - 32° = 76°$

$$\frac{12}{\sin 32°} = \frac{x}{\sin 76°} = \frac{y}{\sin 72°}$$

$$x = \frac{12 \times \sin 76°}{\sin 32°}$$

$$x \approx 22.0$$

$$\text{and } y = \frac{12 \times \sin 72°}{\sin 32°}$$

$$y \approx 21.5$$

Side x is approximately 22.0 cm, side y is approximately 21.5 cm, and $\angle X$ is 76°.

b) $$\frac{25}{\sin 83°} = \frac{15}{\sin E}$$

$$\sin E = \frac{15 \times \sin 83°}{25}$$

$$\angle E = \sin^{-1}\left(\frac{15 \times \sin 83°}{25}\right)$$

$$\angle E \approx 36.6°$$

$\angle D = 180° - 83° - 36.6° = 60.4°$

$$\frac{d}{\sin 60.4°} = \frac{25}{\sin 83°}$$

$$d = \frac{25 \times \sin 60.4°}{\sin 83°}$$

$$d \approx 21.9$$

Side d is approximately 21.9 cm, $\angle E$ is approximately 36.6°, and $\angle D$ is approximately 60.4°.

Chapter 1 Section 3 Question 4 Page 32

a) $\angle A = 180° - 39° - 79° = 62°$

$$\frac{24}{\sin 62°} = \frac{b}{\sin 39°} = \frac{c}{\sin 79°}$$

$$b = \frac{24 \times \sin 39°}{\sin 62°}$$

$$b \approx 17.1$$

$$\text{and } c = \frac{24 \times \sin 79°}{\sin 62°}$$

$$c \approx 26.7$$

The measure of $\angle A$ is 62°, side b is approximately 17.1 cm, and side c is approximately 26.7 cm.

b) $$\frac{25}{\sin 75°} = \frac{10}{\sin E}$$

$$\sin E = \frac{10 \times \sin 75°}{25}$$

$$\angle E = \sin^{-1}\left(\frac{10 \times \sin 75°}{25}\right)$$

$$\angle E \approx 22.7°$$

$$\angle F = 180° - 75° - 22.7°$$

$$\angle F = 82.3°$$

$$\frac{f}{\sin 82.3°} = \frac{25}{\sin 75°}$$

$$f = \frac{25 \times \sin 82.3°}{\sin 75°}$$

$$f \approx 25.6$$

The measure of $\angle E$ is approximately 22.7° and $\angle F$ is approximately 82.3°. The length of side f is approximately 25.6 m.

Chapter 1 Section 3 Question 5 Page 32

In ΔABC the unknown height of the tower is c.

$$\frac{50}{\sin 60°} = \frac{c}{\sin 50°}$$

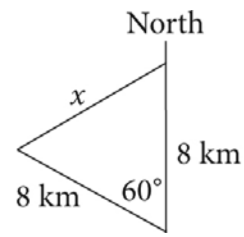
$$c = \frac{50 \times \sin 50°}{\sin 60°}$$

$$c \approx 44$$

The height of the tower is approximately 44 m.

Chapter 1 Section 3 Question 6 Page 32

The diagram models the journeys of the two pairs.
After 2 h, their distance apart is x kilometres.
Since the two sides with a common vertex are equal, the missing angles must also be equal.



$$\frac{180° - 60°}{2} = 60°$$

The triangle is equilateral.

$$\frac{x}{\sin 60°} = \frac{8}{\sin 60°}$$
$$x = \frac{8 \times \sin 60°}{\sin 60°}$$
$$x = 8$$

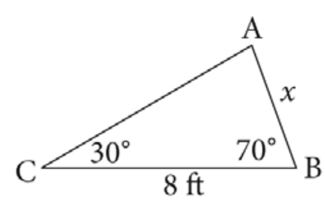
The two pairs are 8 km apart after 2 h.

Chapter 1 Section 3 Question 7 Page 32

Answers may vary. For example:
An angle that is opposite one of the two given sides must be known.

Chapter 1 Section 3 Question 8 Page 32

The diagram represents a view of the shed roof.



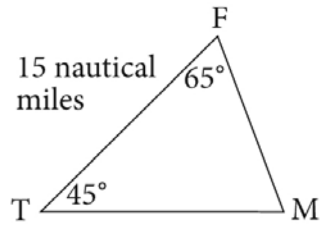
Let the length of the shorter rafter be x feet.
The angle between the rafters is
180° – 70° – 30° = 80°.

$$\frac{8}{\sin 80°} = \frac{x}{\sin 30°}$$
$$x = \frac{8 \times \sin 30°}{\sin 80°}$$
$$x \approx 4$$

The length of the shorter rafter is approximately 4 ft.

Chapter 1 Section 3 Question 9 Page 33

The islands form ΔFTM.



$\angle M = 180° - 45° - 65° = 70°$

$$\frac{15}{\sin 70°} = \frac{FM}{\sin 45°} = \frac{MT}{\sin 65°}$$
$$\frac{FM}{\sin 45°} = \frac{15}{\sin 70°}$$
$$FM = \frac{15 \times \sin 45°}{\sin 70°}$$
$$FM \doteq 11$$

The distance from Fogo to Moreton's Harbour is approximately 11 nautical miles.

$$\frac{MT}{\sin 65°} = \frac{15}{\sin 70°}$$
$$MT = \frac{15 \times \sin 65°}{\sin 70°}$$
$$MT \doteq 14$$

The distance from Twillingate to Moreton's Harbour is approximately 14 nautical miles.

Chapter 1 Section 3 Question 10 Page 33

Solutions for Achievement Checks are shown in the Teacher's Resource.

Chapter 1 Section 3 Question 11 Page 33

Let the height of the leaning tower be *x* metres.

$$\frac{5.35}{x} = \sin 5.5°$$
$$x = \frac{5.35}{\sin 5.5°}$$
$$x \doteq 55.8$$

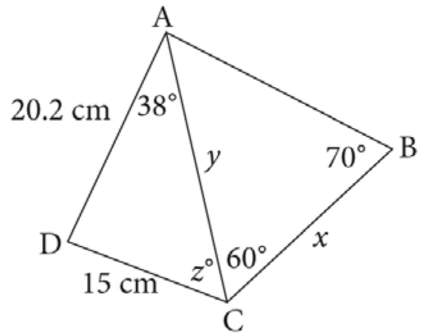
The height of the Leaning Tower of Pisa is approximately 55.8 m.

Chapter 1 Section 3 Question 12 Page 33

Answers may vary.

Many websites (e.g., Wikipedia) list the Leaning Tower of Pisa. Students may investigate and find a suitable image to cut and paste. Right click the image and paste into *The Geometer's Sketchpad®*. The angle of the Leaning Tower with the vertical should be approximately 5°.

Chapter 1 Section 3 Question 13 Page 33



In order to find the unknown length, x centimetres, first solve ΔACD.

$$\frac{20.2}{\sin z} = \frac{15.0}{\sin 38^\circ}$$

$$\sin z = \frac{20.2 \times \sin 38^\circ}{15.0}$$

$$z = \sin^{-1}\left(\frac{20.2 \times \sin 38^\circ}{15.0}\right)$$

$$z \approx 56^\circ$$

$\angle ADC = 180^\circ - 38^\circ - 56^\circ = 86^\circ$

$$\frac{y}{\sin 86^\circ} = \frac{15}{\sin 38^\circ}$$

$$y = \frac{15 \times \sin 86^\circ}{\sin 38^\circ}$$

$$y \approx 24.3 \text{ cm}$$

In ΔABC $\angle B = 70^\circ$ and $\angle C = 60^\circ$, so $\angle A = 50^\circ$.
Side y is opposite the 70° angle, so

$$\frac{24.3}{\sin 70^\circ} = \frac{x}{\sin 50^\circ}$$

$$x = \frac{24.3 \times \sin 50^\circ}{\sin 70^\circ}$$

$$x \approx 19.8 \text{ cm}$$

The length of side x is approximately 19.8 cm.

Chapter 1 Section 4 The Cosine Law

Chapter 1 Section 4 Question 1 Page 39

a)
$$b^2 = a^2 + c^2 - 2ac \cos \text{B}$$
$$b^2 = 20^2 + 25^2 - 2(20)(25) \cos 48°$$
$$b^2 = 400 + 625 - 1000 \cos 48°$$
$$b^2 = 1025 - 1000 \cos 48°$$
$$b = \sqrt{1025 - 1000 \cos 48°}$$
$$b \approx 18.9$$

Side *b* is approximately 18.9 cm.

b)
$$e^2 = d^2 + f^2 - 2df \cos \text{E}$$
$$e^2 = 60^2 + 52^2 - 2(60)(52) \cos 62°$$
$$e^2 = 3600 + 2704 - 2(60)(52) \cos 62°$$
$$e^2 = 6304 - 6240 \cos 62°$$
$$e = \sqrt{6304 - 6240 \cos 62°}$$
$$e \approx 58.1$$

Side *e* is approximately 58.1 mm.

c)
$$y^2 = x^2 + z^2 - 2xz \cos \text{Y}$$
$$y^2 = 6.5^2 + 6.0^2 - 2(6.5)(6.0) \cos 28°$$
$$y^2 = 42.25 + 36.0 - 78 \cos 28°$$
$$y^2 = 78.25 - 78 \cos 28°$$
$$y = \sqrt{78.25 - 78 \cos 28°}$$
$$y \approx 3.1$$

Side *y* is approximately 3.1 m.

Chapter 1 Section 4 Question 2 Page 39

a)

$$\begin{aligned}
a^2 &= b^2 + c^2 - 2bc\cos A \\
2bc\cos A &= b^2 + c^2 - a^2 \\
2(37)(25)\cos A &= 37^2 + 25^2 - 30^2 \\
1850\cos A &= 1369 + 625 - 900 \\
\cos A &= \frac{1094}{1850} \\
\angle A &= \cos^{-1}\left(\frac{1094}{1850}\right) \\
\angle A &\approx 53.7°
\end{aligned}$$

The measure of ∠A is approximately 53.7°.

b)

$$\begin{aligned}
d^2 &= e^2 + f^2 - 2ef\cos D \\
2ef\cos D &= e^2 + f^2 - d^2 \\
2(7)(10)\cos D &= 7^2 + 10^2 - 12^2 \\
140\cos D &= 49 + 100 - 144 \\
140\cos D &= 5 \\
\cos D &= \frac{5}{140} \\
\angle D &= \cos^{-1}\left(\frac{5}{140}\right) \\
\angle D &\approx 88.0°
\end{aligned}$$

The measure of ∠D is approximately 88.0°.

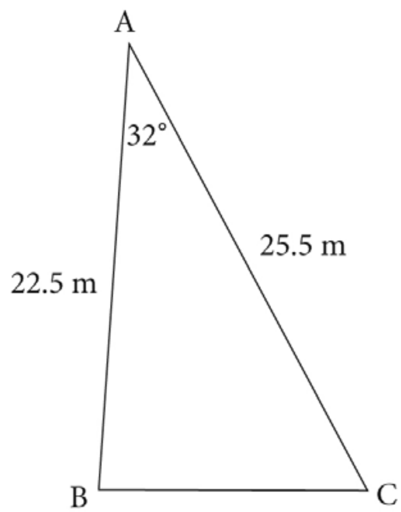
c)

$$\begin{aligned}
x^2 &= y^2 + z^2 - 2yz\cos X \\
2yz\cos X &= y^2 + z^2 - x^2 \\
2(7)(11)\cos X &= 7^2 + 11^2 - 9^2 \\
154\cos X &= 49 + 121 - 81 \\
154\cos X &= 89 \\
\cos X &= \frac{89}{154} \\
\angle X &= \cos^{-1}\left(\frac{89}{154}\right) \\
\angle X &\approx 54.7°
\end{aligned}$$

The measure of ∠X is approximately 54.7°.

Chapter 1 Section 4 **Question 3 Page 40**

Solve $\triangle ABC$.



$$a^2 = b^2 + c^2 - 2bc \cos A$$
$$a^2 = 25.5^2 + 22.5^2 - 2(25.5)(22.5) \cos 32°$$
$$a^2 = 650.25 + 506 - 1147.5 \cos 32°$$
$$a^2 = 1156.25 - 1147.5 \cos 32°$$
$$a = \sqrt{1156.25 - 1147.5 \cos 32°}$$
$$a \approx 13.5$$

Side a is approximately 13.5 m.

Use the sine law to find ∠C.

$$\frac{a}{\sin A} = \frac{c}{\sin C}$$
$$\frac{13.5}{\sin 32°} = \frac{22.5}{\sin C}$$
$$\sin C = \frac{22.5 \times \sin 32°}{13.5}$$
$$\angle C = \sin^{-1}\left(\frac{22.5 \times \sin 32°}{13.5}\right)$$
$$\angle C \approx 62°$$

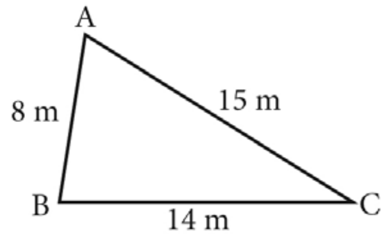
The measure of ∠C is approximately 62°.

$\angle B = 180° - 32° - 62° = 86°$

The measure of ∠B is approximately 86°.

Chapter 1 Section 4 Question 4 Page 40

Solve $\triangle$ABC.



$$a^2 = b^2 + c^2 - 2bc \cos A$$
$$2bc \cos A = b^2 + c^2 - a^2$$
$$2(15)(8) \cos A = 15^2 + 8^2 - 14^2$$
$$240 \cos A = 225 + 64 - 196$$
$$240 \cos A = 93$$
$$\cos A = \frac{93}{240}$$
$$\angle A = \cos^{-1}\left(\frac{93}{240}\right)$$
$$\angle A \approx 67°$$

The measure of $\angle$A is approximately 67°.

$$b^2 = a^2 + c^2 - 2ac \cos B$$
$$2ac \cos B = a^2 + c^2 - b^2$$
$$2(14)(8) \cos B = 14^2 + 8^2 - 15^2$$
$$224 \cos B = 196 + 64 - 225$$
$$224 \cos B = 35$$
$$\cos B = \frac{35}{224}$$
$$\angle B = \cos^{-1}\left(\frac{35}{224}\right)$$
$$\angle B \approx 81°$$

The measure of $\angle$B is approximately 81°.

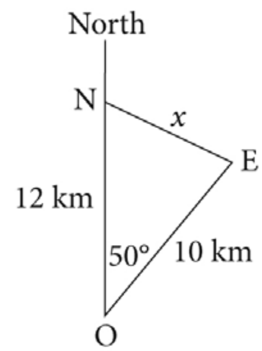
$\angle C = 180° - 81° - 67° = 32°$

The measure of $\angle$C is approximately 32°.

Chapter 1 Section 4 Question 5 Page 40

From information provided in the question, we can form ΔONE as shown, where O represents the team members' starting point and N and E the final positions of the team members.

$$\text{NE}^2 = \text{ON}^2 + \text{OE}^2 - 2(\text{ON})(\text{OE}) \cos \text{O}$$
$$\text{NE}^2 = 12^2 + 10^2 - 2(12)(10) \cos 50°$$
$$\text{NE}^2 = 144 + 100 - 240 \cos 50°$$
$$\text{NE}^2 = 244 - 240 \cos 50°$$
$$\text{NE} = \sqrt{244 - 240 \cos 50°}$$
$$\text{NE} \approx 9.5$$



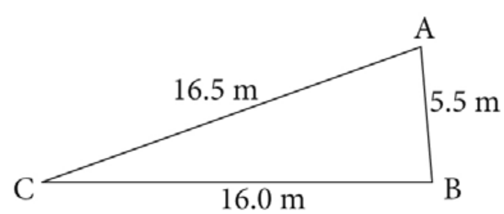
The final positions of the team members are approximately 9.5 km apart.

Chapter 1 Section 4 Question 6 Page 40

To use the cosine law you need to know:
(i) all three side lengths, OR
(ii) two sides and the enclosed angle.

Chapter 1 Section 4 Question 7 Page 40

A motocross ramp is modelled here.



The angle of inclination of the ramp is $\angle$C.

$$c^2 = a^2 + b^2 - 2ab \cos \text{C}$$
$$2ab \cos \text{C} = a^2 + b^2 - c^2$$
$$2(16)(16.5) \cos \text{C} = 16^2 + 16.5^2 - 5.5^2$$
$$528 \cos \text{C} = 256 + 272.25 - 30.25$$
$$528 \cos \text{C} = 498$$
$$\cos \text{C} = \frac{498}{528}$$
$$\angle \text{C} = \cos^{-1}\left(\frac{498}{528}\right)$$
$$\angle \text{C} \approx 19°$$

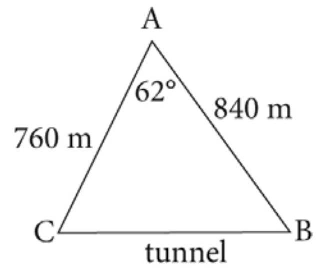
The angle of inclination of the ramp is approximately 19°.

Chapter 1 Section 4 Question 8 Page 40

The diagram represents the proposed tunnel.

Find the length of the tunnel, *a*.

$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$a^2 = 760^2 + 840^2 - 2(760)(840)\cos 62°$$
$$a^2 = 577\ 600 + 705\ 600 - 1\ 276\ 800\cos 62°$$
$$a^2 = 1\ 283\ 200 - 1\ 276\ 800\cos 62°$$
$$a = \sqrt{1\ 283\ 200 - 1\ 276\ 800\cos 62°}$$
$$a \approx 827$$



The length of the proposed tunnel is approximately 827 m.

Chapter 1 Section 4 Question 9 Page 41

Let the distance from the poultry farm to the dairy farm be *x* kilometres.

$$x^2 = 5^2 + 7^2 - 2(5)(7)\cos 62°$$
$$x^2 = 25 + 49 - 70\cos 62°$$
$$x^2 = 74 - 70\cos 62°$$
$$x = \sqrt{74 - 70\cos 62°}$$
$$x \approx 6.9$$

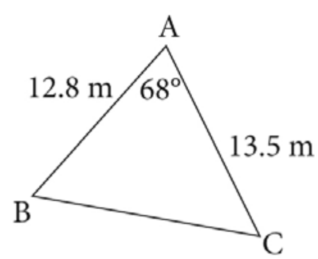
The distance from the poultry farm to the dairy farm is approximately 6.9 km.

Chapter 1 Section 4 Question 10 Page 41

ΔABC models the V-formation flight of the Canada geese, where the lead goose is at A.

Find side *a*.

$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$a^2 = 13.5^2 + 12.8^2 - 2(13.5)(12.8)\cos 68°$$
$$a^2 = 163.84 + 182.25 - 345.6\cos 68°$$
$$a^2 = 346.09 - 345.6\cos 68°$$
$$a = \sqrt{346.09 - 345.6\cos 68°}$$
$$a \approx 14.7$$



The last two geese are approximately 14.7 m apart.

Chapter 1 Section 4 Question 11 Page 41

In order to find DE = x m, first find ∠DCE.
∠DCE = ∠ACB (vertically opposite)
To find ∠ACB, first find AC (i.e., solve ΔABC).

$$\text{AC}^2 = \text{AB}^2 + \text{BC}^2 - 2(\text{AB})(\text{BC})\cos \text{B}$$
$$\text{AC}^2 = 5.6^2 + 6.3^2 - 2(5.6)(6.3)\cos 46°$$
$$\text{AC}^2 = 31.36 + 39.69 - 70.56\cos 46°$$
$$\text{AC}^2 = 71.05 - 70.56\cos 46°$$
$$\text{AC} = \sqrt{71.05 - 70.56\cos 46°}$$
$$\text{AC} \approx 4.7$$

Use the sine law to find ∠ACB.

$$\frac{4.7}{\sin 46°} = \frac{5.6}{\sin \angle\text{ACB}}$$
$$\sin \angle\text{ACB} = \frac{5.6 \times \sin 46°}{4.7}$$
$$\angle\text{ACB} = \sin^{-1}\left(\frac{5.6 \times \sin 46°}{4.7}\right)$$
$$\angle\text{ACB} \approx 59.0°$$

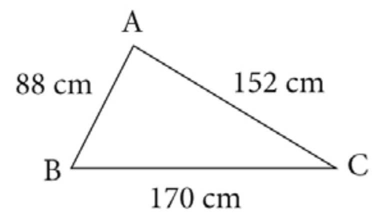
Use the cosine law to find ∠X.

$$x^2 = d^2 + e^2 - 2de\cos \text{X}$$
$$x^2 = 10.6^2 + 12.5^2 - 2(10.6)(12.5)\cos 59°$$
$$x^2 = 112.36 + 156.25 - 263\cos 59°$$
$$x^2 = 268.61 - 263\cos 59°$$
$$x = \sqrt{268.61 - 263\cos 59°}$$
$$x \approx 11.5$$

Side DE is approximately 11.5 m.

Chapter 1 Section 4 **Question 12 Page 41**

a) Solve △ABC.



$$
\begin{aligned}
a^2 &= b^2 + c^2 - 2bc\cos A \\
2bc\cos A &= b^2 + c^2 - a^2 \\
2(152)(88)\cos A &= 152^2 + 88^2 - 170^2 \\
26\,752\cos A &= 23\,104 + 7744 - 28\,900 \\
26\,752\cos A &= 1948 \\
\cos A &= \frac{1948}{26\,752} \\
\angle A &= \cos^{-1}\left(\frac{1948}{26\,752}\right) \\
\angle A &\approx 86°
\end{aligned}
$$

The measure of ∠A is approximately 86°.

$$
\begin{aligned}
b^2 &= a^2 + c^2 - 2ac\cos B \\
2ac\cos B &= a^2 + c^2 - b^2 \\
2(170)(88)\cos B &= 170^2 + 88^2 - 152^2 \\
29\,920\cos B &= 28\,900 + 7744 - 23\,104 \\
29\,920\cos B &= 13\,540 \\
\cos B &= \frac{13\,540}{29\,920} \\
\angle B &= \cos^{-1}\left(\frac{13\,540}{29\,920}\right) \\
\angle B &\approx 63°
\end{aligned}
$$

The measure of ∠B is approximately 63°.

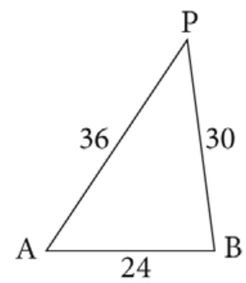
$\angle C = 180° - 86° - 63° = 31°$

The measure of ∠C is approximately 31°.

Chapter 1 Section 4 Question 13 Page 41

ΔPAB models the situation, where P is the port, and A and B are the positions of the boats after 3 h. All distances are in nautical miles.

To find ∠P,



$$p^2 = a^2 + b^2 - 2ab\cos P$$
$$2ab\cos P = a^2 + b^2 - p^2$$
$$2(36)(30)\cos P = 36^2 + 30^2 - 24^2$$
$$2160\cos P = 900 + 1296 - 576$$
$$2160\cos P = 1620$$
$$\cos P = \frac{1620}{2160}$$
$$\angle P = \cos^{-1}\left(\frac{1620}{2160}\right)$$
$$\angle P \approx 41°$$

The measure of the angle between the ships at the time they left port was approximately 41°.

Chapter 1 Section 5 Make Decisions Using Trigonometry

Chapter 1 Section 5 Question 1 Page 48

a) The figure is a right triangle. Use the primary trigonometric ratios to solve it.

b) Three sides are given in the triangle. Use the cosine law to solve it.

c) Two angles and a side are given in the triangle. Use the sine law to solve it.

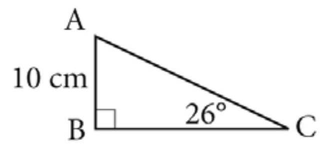
d) The figure is a right triangle. Use the primary trigonometric ratios to solve it.

e) Two sides and the enclosed angle are given in the triangle. Use the cosine law to solve it.

f) Two angles and a side are given in the triangle. Use the sine law to solve it.

Chapter 1 Section 5 **Question 2 Page 48**

a) Use the tangent ratio to find side *a*.



$$\tan 26° = \frac{10}{a}$$

$$a = \frac{10}{\tan 26°}$$

$$a \approx 20.5$$

Side *a* is approximately 20.5 cm.

To find the missing angle, subtract:

$$90° - 26° = 64°$$

The missing angle is 64°.

Use the sine ratio to find side c.

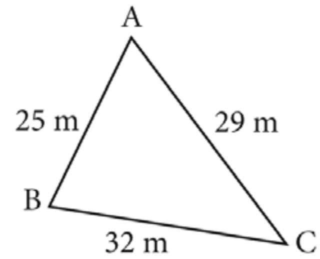
$$\sin 26° = \frac{10}{c}$$

$$c = \frac{10}{\sin 26°}$$

$$c \approx 22.8$$

Side *c* is approximately 22.8 cm.

b) Use the cosine law to find ∠A.



$$a^2 = b^2 + c^2 - 2bc \cos A$$
$$2bc \cos A = b^2 + c^2 - a^2$$
$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$
$$\cos A = \frac{29^2 + 25^2 - 32^2}{2(29)(25)}$$
$$\angle A = \cos^{-1}\left(\frac{29^2 + 25^2 - 32^2}{2(29)(25)}\right)$$
$$\angle A \approx 72.3°$$

The measure of ∠A is approximately 72.3°.

Use the cosine law to find ∠B.

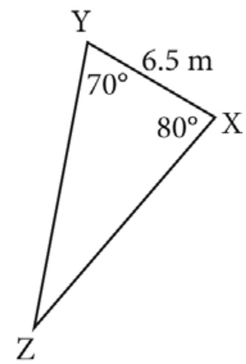
$$b^2 = a^2 + c^2 - 2ac \cos B$$
$$2ac \cos B = a^2 + c^2 - b^2$$
$$\cos B = \frac{a^2 + c^2 - b^2}{2ac}$$
$$\cos B = \frac{32^2 + 25^2 - 29^2}{2(32)(25)}$$
$$\angle B = \cos^{-1}\left(\frac{32^2 + 25^2 - 29^2}{2(32)(25)}\right)$$
$$\angle B \approx 59.7°$$

The measure of ∠B is approximately 59.7°.

Subtract to find ∠C:
$\angle C = 180° - 59.7° - 72.3° = 48°$

The measure of ∠C is approximately 48°.

c) The third angle of the triangle, ∠Z, is
$180° – 70° – 80° = 30°$



The other sides are given by *x* and *y*.
Use the sine law to solve.

$$\frac{x}{\sin 80°} = \frac{6.5}{\sin 30°} = \frac{y}{\sin 70°}$$

$$\frac{x}{\sin 80°} = \frac{6.5}{\sin 30°}$$

$$x = \frac{6.5 \times \sin 80°}{\sin 30°}$$

$$x \approx 12.8$$

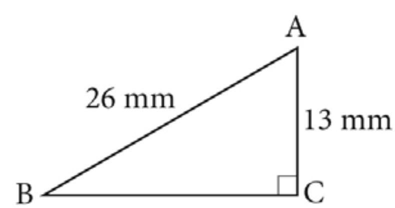
$$\frac{y}{\sin 70°} = \frac{6.5}{\sin 30°}$$

$$y = \frac{6.5 \times \sin 70°}{\sin 30°}$$

$$y \approx 12.2$$

Side *x* is approximately 12.8 m and side *y* is approximately 12.2 m.

d) To find the third side of the right triangle, use the Pythagorean theorem.



$$a^2 + b^2 = c^2$$

$$a^2 = 26^2 - 13^2$$

$$a = \sqrt{26^2 - 13^2}$$

$$a \approx 22.5$$

Side *a* is approximately 22.5 mm.

To find ∠A, use the cosine law.

$$\cos A = \frac{13}{26}$$

$$\angle A = \cos^{-1}\left(\frac{13}{26}\right)$$

$$\angle A = 60°$$

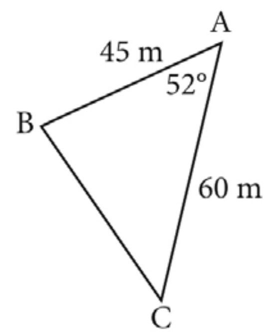
The measure of ∠A is 60°.

To find ∠B, subtract:
$90° – 60° = 30°$

The measure of ∠B is 30°.

e) Use the cosine law to find side *a*.

$$a^2 = b^2 + c^2 - 2bc \cos A$$
$$a^2 = 60^2 + 45^2 - 2(60)(45)\ \cos 52°$$
$$a = \sqrt{60^2 + 45^2 - 2(60)(45)\ \cos\ 52°}$$
$$a \approx 48.0$$



Side *a* is approximately 48.0 m.

Use the cosine law to find ∠B.

$$b^2 = a^2 + c^2 - 2ac \cos B$$
$$2ac \cos B = a^2 + c^2 - b^2$$
$$\cos B = \frac{a^2 + c^2 - b^2}{2ac}$$
$$\cos B = \frac{48^2 + 45^2 - 60^2}{2(48)(45)}$$
$$\angle B = \cos^{-1}\left(\frac{48^2 + 45^2 - 60^2}{2(48)(45)}\right)$$
$$\angle B \approx 80.3°$$

The measure of ∠B is approximately 80.3°.

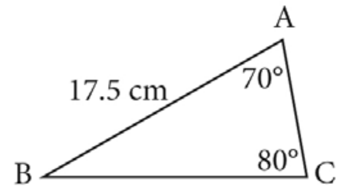
To find ∠C, subtract:
$\angle C = 180° - 52° - 80.3° = 47.7°$

The measure of ∠C is approximately 47.7°.

f) To find ∠B, subtract:

$\angle B = 180° - 70° - 80° = 30°$

The measure of ∠B is 30°.



Using the sine rule to find the sides,

$$\frac{17.5}{\sin 80°} = \frac{a}{\sin 70°} = \frac{b}{\sin 30°}$$

$$\frac{a}{\sin 70°} = \frac{17.5}{\sin 80°}$$

$$a = \frac{17.5 \times \sin 70°}{\sin 80°}$$

$$a \approx 16.7$$

$$\frac{b}{\sin 30°} = \frac{17.5}{\sin 80°}$$

$$b = \frac{17.5 \times \sin 30°}{\sin 80°}$$

$$b \approx 8.9$$

Side a is approximately 16.7 cm and side b is approximately 8.9 cm.

Chapter 1 Section 5 Question 3 Page 49

In the diagram that models the situation, the shot will be successful if the distance from the golfer to the green is less than 200 yd. This distance is given by

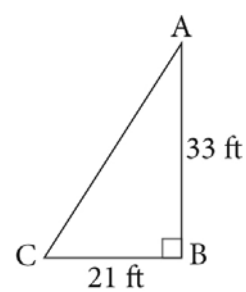
$$\frac{120}{\cos 52°} = 194.9 \text{ yd}$$

Lorie Kane can make the shot successfully if she does not make an error or if there is no wind.

Chapter 1 Section 5 Question 4 Page 49

In ΔABC, which models the golfer's situation, AB represents the height of the trees, and BC the distance of the golfer from the trees.

The golfer can hit the ball over the trees if the angle between him and the top of the trees (∠C) is less than the 60° angle at which the lob wedge will send the ball.



$$\tan C = \frac{33}{21}$$

$$\angle C = \tan^{-1}\left(\frac{33}{21}\right)$$

$$\angle C \approx 57.5°$$

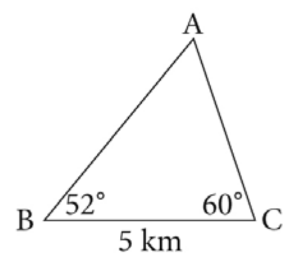
The golfer can make the shot successfully if there is no wind or error.

Chapter 1 Section 5 Question 5 Page 49

Answers may vary. For example:
Soccer: In a free kick situation, the goal keeper tries to align a defensive wall to protect the goal. The offensive player tries to kick toward an unguarded section of the goal.
Rugby or Canadian/American Football: The kicker needs to make kicks of varying lengths from different angles on the field.

Chapter 1 Section 5 Question 6 Page 49

Model the question with ΔABC as shown, where A represents the weather balloon while B and C represent the tracking stations.



To find $\angle A$, subtract:
$\angle A = 180° - 52° - 60° = 68°$

To find the missing sides, use the sine law.

$$\frac{5}{\sin 68°} = \frac{c}{\sin 60°} = \frac{b}{\sin 52°}$$

$$\frac{5}{\sin 68°} = \frac{c}{\sin 60°}$$

$$c = \frac{5 \times \sin 60°}{\sin 68°}$$

$$c \approx 4.7 \text{ km}$$

$$\frac{5}{\sin 68°} = \frac{b}{\sin 52°}$$

$$b = \frac{5 \times \sin 52°}{\sin 68°}$$

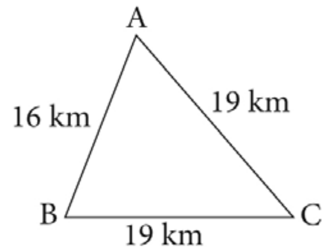
$$b \approx 4.2 \text{ km}$$

The balloon is approximately 4.2 km from the closer station.

Chapter 1 Section 5 Question 7 Page 49

Model the location of the towers with ΔABC.
Sides AC and BC are equal, so the triangle is isosceles; $\angle$A = $\angle$B.

To find $\angle$A, use the cosine law.



$$a^2 = b^2 + c^2 - 2bc \cos \text{A}$$
$$2bc \cos \text{A} = b^2 + c^2 - a^2$$
$$\cos \text{A} = \frac{b^2 + c^2 - a^2}{2bc}$$
$$\cos \text{A} = \frac{16^2 + 19^2 - 19^2}{2(16)(19)}$$
$$\angle \text{A} = \cos^{-1}\left(\frac{16^2 + 19^2 - 19^2}{2(16)(19)}\right)$$
$$\angle \text{A} \approx 65.1°$$

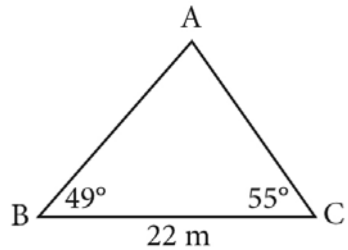
Since the triangle is isosceles, $\angle$A and $\angle$B measure approximately 65.1° each.

Subtract to find $\angle$C:
$\angle \text{C} = 180° - 2(65.1°) = 49.8°$

The measure is $\angle$C is approximately 49.8°.

Chapter 1 Section 5 Question 8 Page 50

The garden can be modelled by ΔABC, with a = 22 m, $\angle B = 49°$, and $\angle C = 55°$.



Find the perimeter of ΔABC, i.e., ($a + b + c$).

Subtract to find ∠A.
$\angle A = 180° - 49° - 55° = 76°$

Use the sine law to find the missing sides.

$$\frac{22}{\sin 76°} = \frac{b}{\sin 49°} = \frac{c}{\sin 55°}$$

$$b = \frac{22 \times \sin 49°}{\sin 76°}$$

$$b \approx 17.1 \text{ m}$$

$$c = \frac{22 \times \sin 55°}{\sin 76°}$$

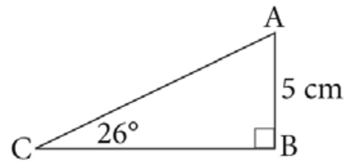
$$c \approx 18.6 \text{ m}$$

22 + 17.1 + 18.6 = 57.7

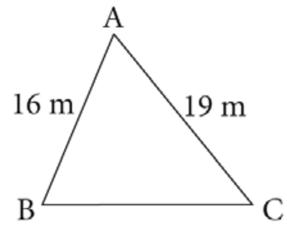
The perimeter will require approximately 57.7 m of fencing.

Chapter 1 Section 5 Question 9 Page 50

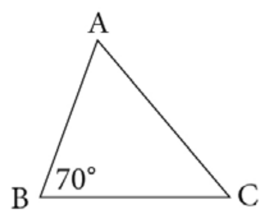
a) To solve the right triangle, one given side and one acute angle are needed, as shown here.



b) To solve the acute triangle, two given sides and one additional angle are needed.
If $\angle A$ is given, use the cosine law;
if $\angle B$ or $\angle C$ is given, use the sine law.



c) To solve the acute triangle, two sides and one given angle are needed, as shown above.
If a and c are given, use the cosine law;
if b is given with a or c, use the sine law.



Chapter 1 Section 5 Question 10 Page 50

Model the distances between the towns as shown in the text diagram, with ΔHMO where $o = 3.9$ km, $h = 4.5$ km, and $\angle M = 62°$.

a) Find m. Use the cosine law.

$$m^2 = 3.9^2 + 4.5^2 - 2(3.9)(4.5)\cos 62°$$

$$m = \sqrt{3.9^2 + 4.5^2 - 2(3.9)(4.5)\cos 62°}$$

$$m \doteq 4.4$$

The distance from Hometown to Ourtown is approximately 4.4 km.

b) The angles of the roads at Ourtown and Hometown are $\angle O$ and $\angle H$.
Use the sine law to find $\angle O$.

$$\frac{3.9}{\sin O} = \frac{4.4}{\sin 62°}$$

$$\sin O = \frac{3.9 \times \sin 62°}{4.4}$$

$$\angle O = \sin^{-1}\left(\frac{3.9 \times \sin 62°}{4.4}\right)$$

$$\angle O \doteq 51.5°$$

Subtract to find $\angle H$.

$$\angle H = 180° - 51.5° - 62° = 66.5°$$

The angle of the roads at Ourtown is approximately 51.5° and at Hometown approximately 66.5°.

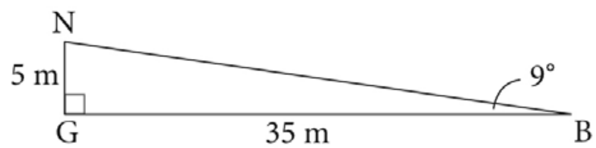
Chapter 1 Section 5 Question 11 Page 50

Solutions for Achievement Checks are shown in the Teacher's Resource.

Chapter 1 Section 5 Question 12 Page 51

From the information given in the question, set up ΔBGN to model the kick. B represents the position of David Beckham, G the position of the goalie, and N the right goal post.

Find out if the ball will land in the net or outside the post by solving this triangle.



Use the tangent ratio.

$\tan 9° = \frac{5}{35}$

The ball will be on the net side of the post at N if $35 \tan 9° < 5$.
But $35 \tan 9° = 5.5$ m, so the ball will miss the goal by 0.5 m.
Since the ball will be wide of the post, its angle of elevation does not matter.

Chapter 1 Section 5 Question 13 Page 51

To find $\angle CED$, solve $\triangle AEC$ and $\triangle CED$.
In $\triangle ABC$,

$$\frac{4.5}{AC} = \cos 18°$$

$$AC = \frac{4.5}{\cos 18°}$$

$$AC \approx 4.7 \text{ m}$$

In $\triangle AEC$,

$$\angle ECA = 68° = \angle EAC \text{ (isosceles triangle)}$$

$$\angle AEC = 180° - 2(68°) = 44°$$

$$\frac{4.7}{\sin 44°} = \frac{EC}{\sin 68°}$$

$$EC = \frac{4.7 \times \sin 68°}{\sin 44°}$$

$$EC \approx 6.3 \text{ m}$$

In $\triangle CED$,

$$CD^2 = ED^2 + EC^2 - 2(ED)(EC)\cos \angle CED$$

$$\cos \angle CED = \frac{ED^2 + EC^2 - CD^2}{2(ED)(EC)}$$

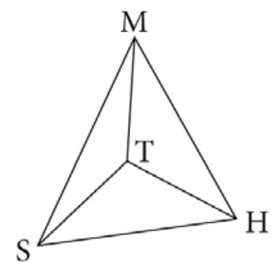
$$\cos \angle CED = \frac{4.6^2 + 6.3^2 - 5.1^2}{2(4.6)(6.3)}$$

$$\angle CED = \cos^{-1}\left(\frac{4.6^2 + 6.3^2 - 5.1^2}{2(4.6)(6.3)}\right)$$

$$\angle CED \approx 53°$$

The measure of $\angle CED$ is approximately 53°.

Chapter 1 Section 5 Question 14 Page 51



ΔMTS represents the tower, MT = 52 m, and the ranger station, S; $\angle MTS = 90°$.
ΔMTH represents the tower MT and hikers' camp H; $\angle MTH = 90°$.
ΔTSH represents the ranger station at S, the hikers' camp at H, and the base of the tower at T; $\angle STH = 60°$.

a) In ΔMTS, find TS.
$\angle MST = 2.2°$, $MT = 52$ m, and $\angle MTS = 90°$.

$$\frac{52}{TS} = \tan 2.2°$$
$$TS = \frac{52}{\tan 2.2°}$$
$$TS \approx 1353.6$$

The lookout tower is approximately 1353.6 m from the ranger station.

b) In ΔMTH, find TH. $\angle MHT = 1.5°$, $MT = 52$ m, and $\angle MTH = 90°$.

$$\frac{52}{TH} = \tan 1.5°$$
$$TH = \frac{52}{\tan 1.5°}$$
$$TH \approx 1985.8$$

The lookout tower is approximately 1985.8 m from the hikers.

c) In ΔTSH, find SH. TH = 1985.8 m, TS = 1353.6 m, and $\angle STH = 60°$.

$$SH^2 = TH^2 + TS^2 - 2(TH)(TS)\cos\angle STH$$
$$SH^2 = 1985.8^2 + 1353.6^2 - 2(1985.8)(1353.6)\cos 60°$$
$$SH = \sqrt{1985.8^2 + 1353.6^2 - 2(1985.8)(1353.6)\cos 60°}$$
$$SH \approx 1757.2$$

The hikers are approximately 1757.2 m from the ranger station.

d) Find $\angle TSH$.

$$\frac{1757.2}{\sin 60°} = \frac{1985.8}{\sin \angle TSH}$$
$$\sin \angle TSH = \frac{1985.8 \times \sin 60°}{1757.2}$$
$$\angle TSH = \sin^{-1}\left(\frac{1985.8 \times \sin 60°}{1757.2}\right)$$
$$\angle TSH \approx 78°$$

The rescue team should head from the ranger station at an angle of approximately 78° east of north (which is 12° north of east).

Chapter 1 Review

Chapter 1 Review Question 1 Page 52

a) Use the cosine ratio to find *c*.

$$\frac{31}{c} = \cos 20°$$

$$c = \frac{31}{\cos\ 20°}$$

$$c \approx 33.0$$

Side *c* is approximately 33.0 m.

Use the tangent ratio to find side *a*.

$$\frac{a}{31} = \tan\ 20°$$

$$a = 31\ \tan 20°$$

$$a \approx 11.3 \text{ m}$$

Side *a* is approximately 11.3 m.

Subtract to find ∠B.

$\angle B = 90° - 20° = 70°$

The measure of ∠B is 70°.

b) Use the sine ratio to find ∠B.

$$\sin B = \frac{19}{35}$$

$$\angle B = \sin^{-1}\left(\frac{19}{35}\right)$$

$$\angle B \approx 32.9°$$

The measure of ∠B is approximately 32.9°.

Subtract to find ∠A.

$\angle A = 90° - 32.9° = 57.1°$

The measure of ∠A is approximately 57.1°.

Use the Pythagorean theorem to find side *a*.

$$a^2 = 35^2 - 19^2$$

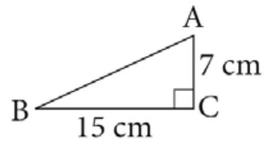
$$a = \sqrt{35^2 - 19^2}$$

$$a \approx 29.4 \text{ cm}$$

Side *a* is approximately 29.4 cm.

Chapter 1 Review **Question 2 Page 52**

Solve △ABC. Use the tangent ratio to find ∠B.



$$\tan B = \frac{7}{15}$$
$$\angle B = \tan^{-1}\left(\frac{7}{15}\right)$$
$$\angle B \approx 25.0°$$

The measure of ∠B is approximately 25.0°.

Subtract to find ∠A.
$$\angle A = 90° - 25° = 65°$$

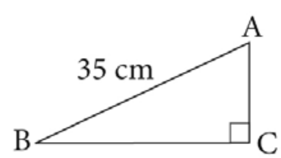
The measure of ∠A is approximately 65.0°.

Use the Pythagorean theorem to find side *c*.
$$c^2 = 7^2 + 15^2$$
$$c = \sqrt{7^2 + 15^2}$$
$$c \approx 16.6 \text{ cm}$$

Side *c* is approximately 16.6 cm.

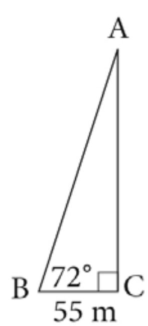
Chapter 1 Review **Question 3 Page 52**

It is not possible to solve ΔABC.
Another angle or side must be known to use the primary trigonometric ratios.



Chapter 1 Review **Question 4 Page 52**

We can model the situation with right triangle ΔABC, where the tower's shadow is represented by BC.

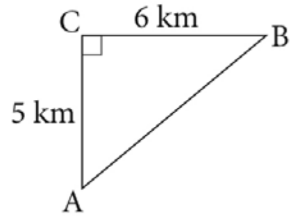


To find side *b*, use the tangent ratio.
$$\frac{b}{55} = \tan 72°$$
$$b = 55 \times \tan 72°$$
$$b \approx 169$$

The height of the tower is approximately 169 m.

Chapter 1 Review **Question 5 Page 52**

Model the person's walk with right triangle ΔABC.



To find ∠A, use the tangent ratio.

$$\tan A = \frac{6}{5}$$

$$\angle A = \tan^{-1}\left(\frac{6}{5}\right)$$

$$\angle A \approx 50.2°$$

The person stopped at 50.2° east of north.

Chapter 1 Review **Question 6 Page 52**

It is not possible to solve the triangle. In order to use the sine law, at least one angle must be known.

Chapter 1 Review **Question 7 Page 52**

In ΔABC, $\angle B = 70°$, $\angle C = 50°$, and $b = 15$ m

To find ∠A, subtract.
$\angle A = 180° - 70° - 50° = 60°$

The measure of ∠A is 60°.

Use the sine law to find the missing sides.

$$\frac{15}{\sin 70°} = \frac{a}{\sin 60°} = \frac{c}{\sin 50°}$$

$$\frac{15}{\sin 70°} = \frac{a}{\sin 60°}$$

$$a = \frac{15 \times \sin 60°}{\sin 70°}$$

$$a \approx 13.8$$

$$\frac{15}{\sin 70°} = \frac{c}{\sin 50°}$$

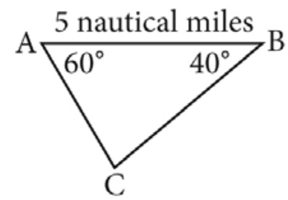
$$c = \frac{15 \times \sin 50°}{\sin 70°}$$

$$c \approx 12.2$$

Side *a* is approximately 13.8 m and side *c* is approximately 12.2 m.

Chapter 1 Review **Question 8 Page 52**

Model the situation with ΔABC.



Subtract to find ∠C.
∠C = 180° – 60° – 40° = 80°

Use the sine law to find a.

$$\frac{5}{\sin 80°} = \frac{a}{\sin 60°}$$
$$a = \frac{5 \times \sin 60°}{\sin 80°}$$
$$a \approx 4.4$$

The sailboat is approximately 4.4 nautical miles from the buoy after 45 min.

Chapter 1 Review **Question 9 Page 53**

Use the cosine law to find side d.

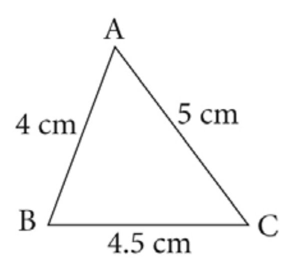
$$d^2 = e^2 + f^2 - 2ef \cos D$$
$$d^2 = 12^2 + 18^2 - 2(12)(18) \cos 58°$$
$$d = \sqrt{12^2 + 18^2 - 2(12)(18) \cos 58°}$$
$$d \approx 15.5$$

Side d is approximately 15.5 cm.

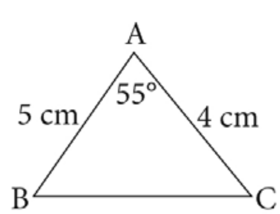
Chapter 1 Review **Question 10 Page 53**

There are two possible circumstances when you can use the cosine law.

i) Use the cosine law when you know all three sides and find the angles.



ii) If you know two sides and the enclosed angle, use the cosine law to solve the triangle.

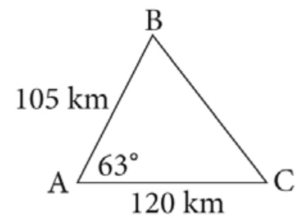


Chapter 1 Review Question 11 Page 53

Answers may vary. The triangle to be solved must have all three sides given or two sides and the enclosed angle like the examples in the solution for question 10.

Chapter 1 Review Question 12 Page 53

Model the cyclist's travels using ΔABC.



Use the cosine law to find *a*, the distance the cyclists are apart.

$$a^2 = b^2 + c^2 - 2bc \cos \text{A}$$
$$a^2 = 120^2 + 105^2 - 2(120)(105) \cos 63°$$
$$a^2 = \sqrt{120^2 + 105^2 - 2(120)(105) \cos 63°}$$
$$a \approx 118.3$$

The cyclists are approximately 118.3 km apart.

Chapter 1 Review Question 13 Page 53

In ΔKLM, use the cosine law to find ∠K.

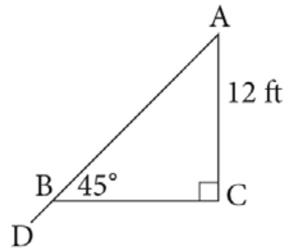
$$k^2 = l^2 + m^2 - 2lm \cos \text{K}$$
$$\cos \text{K} = \frac{l^2 + m^2 - k^2}{2lm}$$
$$\cos \text{K} = \frac{7^2 + 7^2 - 9^2}{2(7)(7)}$$
$$\angle\text{K} = \cos^{-1}\left(\frac{7^2 + 7^2 - 9^2}{2(7)(7)}\right)$$
$$\angle\text{K} \approx 80°$$

$$\angle\text{L} = \angle\text{M} = \frac{180° - 80°}{2} = 50°$$

The measure of ∠K is approximately 80° and the measures of ∠L and ∠M are approximately 50° each.

Chapter 1 Review **Question 14 Page 53**

This diagram represents the roof.
AD = 20 ft and BD = 1 ft
Will the roof rafter, AD = 20 ft, be long enough for a 1 ft overhang, BD?



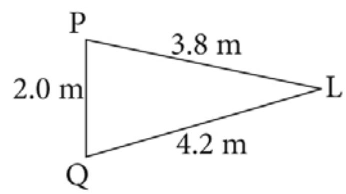
If $AB = \frac{12}{\sin 45°} = 19$ ft then the rafter is the correct length.
Calculating AB,

$$AB = \frac{12}{\sin 45°}$$
$$AB \approx 17$$

The roof rafter is too long by approximately 2 ft.

Chapter 1 Review **Question 15 Page 53**

ΔLPQ represents the situation when Leah is about to shoot the puck.
L is Leah's position and P and Q are the goal posts.



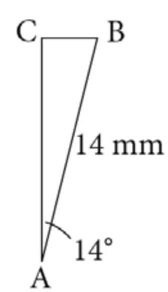
To find ∠L, use the cosine law.

$$l^2 = p^2 + q^2 - 2pq \cos L$$
$$\cos L = \frac{p^2 + q^2 - l^2}{2pq}$$
$$\cos L = \frac{4.2^2 + 3.8^2 - 2^2}{2(4.2)(3.8)}$$
$$\angle L = \cos^{-1}\left(\frac{4.2^2 + 3.8^2 - 2^2}{2(4.2)(3.8)}\right)$$
$$\angle L \approx 28.4°$$

Leah must shoot the puck within approximately a 28.4° angle to score a goal.

Chapter 1 Review **Question 16 Page 53**

The 2D drawing shows half the cone, where BC is the radius.



To find the radius, use the sine ratio.

$$\frac{BC}{14} = \sin 14°$$
$$BC = 14 \times \sin 14°$$
$$BC \approx 3.4$$

The radius of the cone is approximately 3.4 mm.

Chapter 1 Review **Question 17 Page 53**

Use the cosine law to solve ΔPQR in question 6, where p = 20 cm, q = 26 cm, and r = 18 cm.

$$p^2 = q^2 + r^2 - 2qr \cos \text{P}$$

$$\cos \text{P} = \frac{q^2 + r^2 - p^2}{2qr}$$

$$\cos \text{P} = \frac{26^2 + 18^2 - 20^2}{2(26)(18)}$$

$$\angle \text{P} = \cos^{-1}\left(\frac{26^2 + 18^2 - 20^2}{2(26)(18)}\right)$$

$$\angle \text{P} \doteq 50.1°$$

$$q^2 = p^2 + r^2 - 2pr \cos \text{Q}$$

$$\cos \text{Q} = \frac{p^2 + r^2 - q^2}{2pr}$$

$$\cos \text{Q} = \frac{20^2 + 18^2 - 26^2}{2(20)(18)}$$

$$\angle \text{Q} = \cos^{-1}\left(\frac{20^2 + 18^2 - 26^2}{2(20)(18)}\right)$$

$$\angle \text{Q} \doteq 86.2°$$

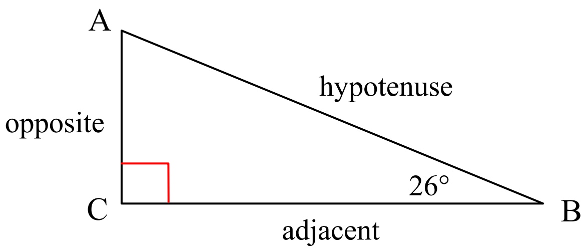
Subtract to find $\angle$R.

$$\angle \text{R} = 180° - 86.2° - 50.1° = 43.7°$$

The measures of the three angles are $\angle$P is approximately 50.1°, $\angle$Q is approximately 86.2°, and $\angle$R is approximately 43.7°.

Chapter 1 Practice Test

Chapter 1 Practice Test **Question 1 Page 54**



Chapter 1 Practice Test **Question 2 Page 54**

Given ΔABC, with $\angle C = 90°$, $b = 3.8$ m, and $a = 5.7$ m

Use the tangent ratio to find ∠B.

$$\tan B = \frac{3.8}{5.7}$$
$$\angle B = \tan^{-1}\left(\frac{3.8}{5.7}\right)$$
$$\angle B \approx 33.7°$$

The measure of ∠B is approximately 33.7°.

Subtract to find ∠C.

$$\angle C = 90° - 33.7° = 56.3°$$

The measure of ∠C is approximately 56.3°.

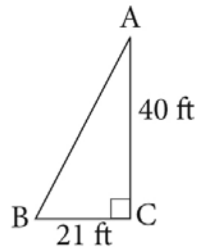
Use the Pythagorean theorem to find *c*.

$$c^2 = a^2 + b^2$$
$$c = \sqrt{a^2 + b^2}$$
$$c = \sqrt{3.8^2 + 5.7^2}$$
$$c \approx 6.9$$

Side *c* is approximately 6.9 m.

Chapter 1 Practice Test **Question 3 Page 54**

Answers may vary. For example:
Model the golfer's shot with ΔABC, where
$\angle C = 90°$, $AC = 40$ ft, and $BC = 21$ ft.
Her shot will clear the tree if



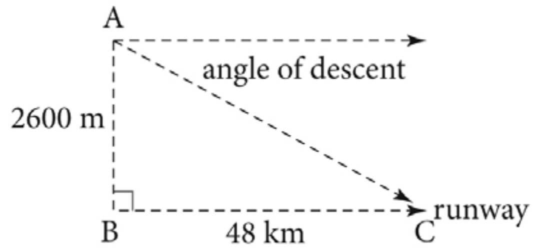
$$\angle B = \tan^{-1}\left(\frac{40}{21}\right) < 60°$$

$$\angle B = 62.3°$$

Her shot will not clear the tree.

Chapter 1 Practice Test **Question 4 Page 54**

The plane's angle of descent is equal to $\angle C$.
Use the tangent ratio to find $\angle C$.



$$\tan C = \frac{2600}{48\ 000}$$

$$\angle C = \tan^{-1}\left(\frac{2600}{48\ 000}\right)$$

$$\angle C \approx 3.1°$$

The plane’s angle of descent is approximately 3.1°.

Chapter 1 Practice Test Question 5 Page 54

Solve ΔABC, given that $c = 25$ m, $\angle A = 80°$, and $\angle B = 76°$.

Subtract to find $\angle$C.
$\angle C = 180° - 80° - 76° = 24°$

The measure of $\angle$C is 24°.

Use the sine law to find the missing sides.

$$\frac{25}{\sin 24°} = \frac{a}{\sin 50°} = \frac{b}{\sin 76°}$$

$$\frac{25}{\sin 24°} = \frac{a}{\sin 50°}$$

$$a = \frac{25 \times \sin 80°}{\sin 24°}$$

$$a \approx 60.5$$

$$\frac{25}{\sin 24°} = \frac{b}{\sin 76°}$$

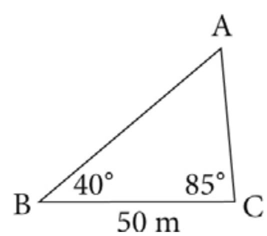
$$b = \frac{25 \times \sin 76°}{\sin 24°}$$

$$b \approx 59.6$$

Side a is approximately 60.5 m and side b is approximately 59.6 m.

Chapter 1 Practice Test Question 6 Page 54

Find the height of the tree by solving ΔABC, where AC = b represents the height of the tree, $\angle$B = 40°, and $\angle$C = 85°.



Subtract to find $\angle$A.
$\angle A = 180° - 40° - 85° = 55°$

Use the sine law to find the missing sides.

$$\frac{50}{\sin 55°} = \frac{b}{\sin 40°}$$

$$b = \frac{50 \times \sin 40°}{\sin 55°}$$

$$b \approx 39.2$$

The height of the tree is approximately 39.2 m.

Chapter 1 Practice Test **Question 7 Page 55**

Solve ΔABC, given that $\angle A = 68°$, $b = 15$ cm, and $c = 20$ cm.

Use the cosine law to find side a.

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$a^2 = 15^2 + 20^2 - 2(15)(20) \cos 68°$$

$$a = \sqrt{15^2 + 20^2 - 2(15)(20) \cos 68°}$$

$$a \approx 20$$

Side a is approximately 20 cm.

Since $a = c$, ΔABC is isosceles.

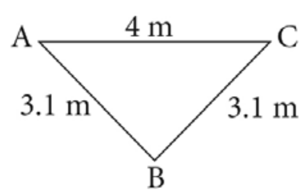
$$\angle A = \angle C = 68°$$

$$\angle B = 180° - 2(68°) = 44°$$

The measure of $\angle$A is 68° and of $\angle$B is 44°.

Chapter 1 Practice Test **Question 8 Page 55**

Model the position of the food bag by drawing ΔABC, where the food bag is tied at B. Find $\angle$B.



Use the cosine law to find $\angle$B.

$$b^2 = a^2 + c^2 - 2ac \cos B$$

$$\cos B = \frac{a^2 + c^2 - b^2}{2ac}$$

$$\cos B = \frac{3.1^2 + 3.1^2 - 4^2}{2(3.1)(3.1)}$$

$$\angle B = \cos^{-1}\left(\frac{3.1^2 + 3.1^2 - 4^2}{2(3.1)(3.1)}\right)$$

$$\angle B \approx 80.4°$$

The angle made by the food bag on the rope is approximately 80.4°.

Chapter 1 Practice Test **Question 9 Page 55**

Solve ΔABC, given a = 15.7 m, b = 14.2 m and c = 13.5 m.

Use the cosine law to find ∠A.

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\cos A = \frac{14.2^2 + 13.5^2 - 15.7^2}{2(14.2)(13.5)}$$

$$\angle A = \cos^{-1}\left(\frac{14.2^2 + 13.5^2 - 15.7^2}{2(14.2)(13.5)}\right)$$

$$\angle A \doteq 69.0°$$

The measure of ∠A is approximately 69°.

Use the cosine law to find ∠B.

$$b^2 = a^2 + c^2 - 2ac \cos B$$

$$\cos B = \frac{a^2 + c^2 - b^2}{2ac}$$

$$\cos B = \frac{15.7^2 + 13.5^2 - 14.2^2}{2(15.7)(13.5)}$$

$$\angle B = \cos^{-1}\left(\frac{15.7^2 + 13.5^2 - 14.2^2}{2(15.7)(13.5)}\right)$$

$$\angle B \doteq 57.6°$$

The measure of ∠B is approximately 57.6°.

Subtract to find ∠C.

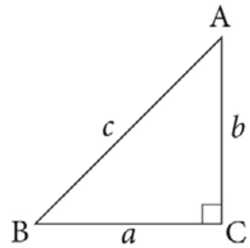
$$\angle C = 180° - 57.6° - 69° = 53.4°$$

The measure of ∠C is approximately 53.4°.

Chapter 1 Practice Test **Question 10 Page 55**

a) Answers may vary. For example:

If $\angle C = 90°$ then the sine rule is



$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin 90°}$$

$$\frac{a}{\sin A} = \frac{c}{1}$$

$$\sin A = \frac{a}{c}$$

$$\text{Similarly, } \sin B = \frac{b}{c}$$

In this case, the sine law reduces to the primary trigonometric sine ratios, which we can use to solve the triangle if either $\angle A$ or $\angle B$ and one of the sides are given.
Therefore it is more appropriate to use the primary trigonometric ratios to solve right triangles.

b) Answers may vary. For example:
The cosine law can be used to solve a right triangle; if $\angle C = 90°$.

$$\cos C = 0 = \frac{a^2 + b^2 - c^2}{2ab}$$

$$a^2 + b^2 - c^2 = 0$$

$$c^2 = a^2 + b^2$$

In this case, the cosine law is the same as the Pythagorean theorem.
Similarly for cos B and cos A: For example,

$$\cos B = \frac{a^2 + c^2 - b^2}{2ac} = \frac{a}{c}$$

$$a^2 + c^2 - b^2 = \frac{2a^2c}{c} = 2a^2$$

$$c^2 = b^2 + 2a^2 - a^2 = b^2 + a^2$$

This is the Pythagorean theorem.

To solve the right triangle, we still need to know at least one of the acute angles and one side.