CHAPTER 10: Circles

Common Core State Standards
G.CO.1 – Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along line, and distance around circular arc.
G.C.1 – Prove that all circles are similar.

Mathematical Practices
4 Model with mathematics.
1 Make sense of problems and persevere in solving them.

Learning Targets

- Students will be able to identify and use parts of circles.
- Students will solve problems involving the circumference of a circle.

Section 10.1 Notes: Circles and Circumference

A __________ is the locus or set of all points in a plane equidistant from a given point called the _________ of the circle.

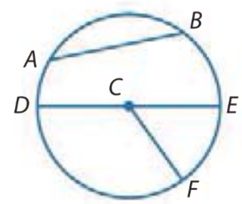
Segments that intersect a circle have special names.

KeyConcept **Special Segments in a Circle**

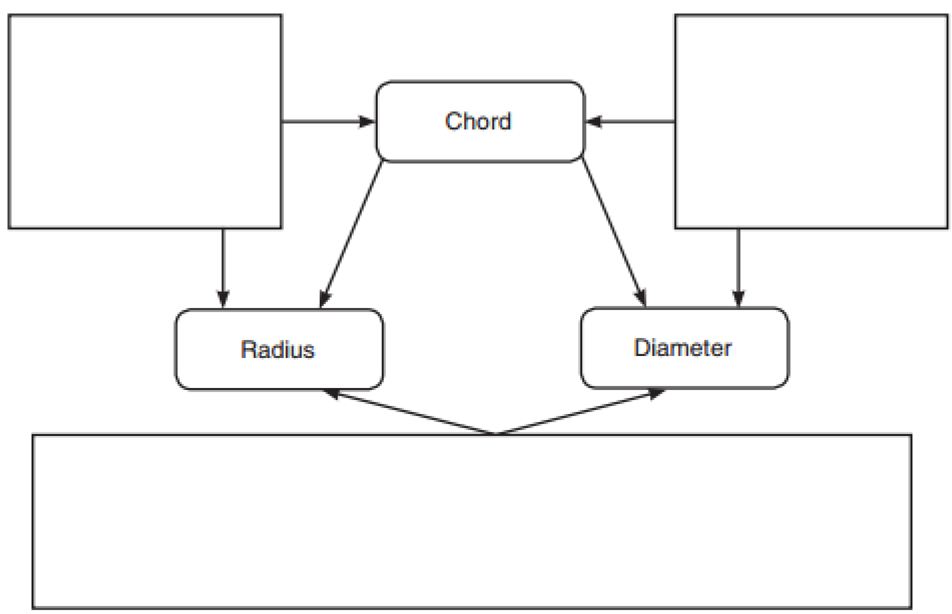
A ______ (plural radii) is a segment with endpoints at the center and on the circle.
Examples $\overline{CD}$, $\overline{CE}$, and $\overline{CF}$ are radii of ⊙C.

A ______ is a segment with endpoints on the circle.
Examples $\overline{AB}$ and $\overline{DE}$ are chords of ⊙C.

A ______ of a circle is a chord that passes through the center and is made up of collinear radii.
Example $\overline{DE}$ is a diameter of ⊙C. Diameter $\overline{DE}$ is made up of collinear radii $\overline{CD}$ and $\overline{CE}$.

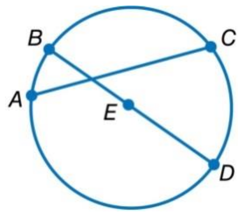


Compare and contrast the pairs of special segments of a circle in the diagram to the right.

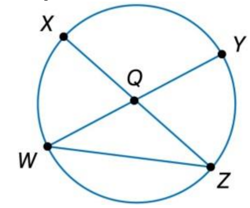


Example 1:

a) Name the circle and identify a radius.



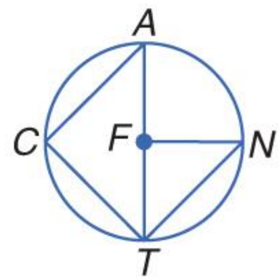
b) Identify a chord and a diameter of the circle.



For parts c and d, use the circle on the right.

c) Name the circle and identify a radius.

d) Which segment is not a chord?



KeyConcept **Radius and Diameter Relationships**

If a circle has radius *r* and diameter *d*, the following relationships are true.

Radius Formula ____________ **Diameter Formula** ____________

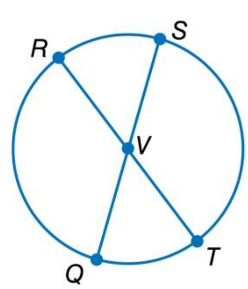
IMPORTANT

By definition, the distance from the center of a circle to any point on the circle is always the same. Therefore, all radii *r* of a circle are congruent. Since a diameter *d* is composed of two radii, all diameters of a circle are also congruent.

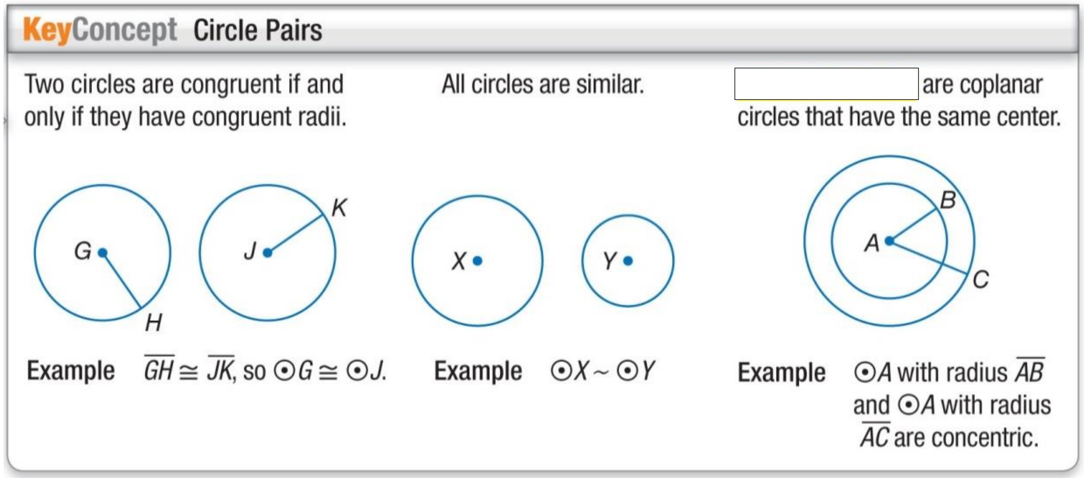
Example 2:

a) If RT = 21 cm, what is the length of $\overline{QV}$?

b) If *QS* = 26 cm, what is the length of $\overline{RV}$?



As with other figures, pairs of circles can be congruent, similar, or share the other special relationships.


KeyConcept Circle Pairs

Two circles are congruent if and only if they have congruent radii.

Example $\overline{GH} \cong \overline{JK}$, so $\odot G \cong \odot J$.

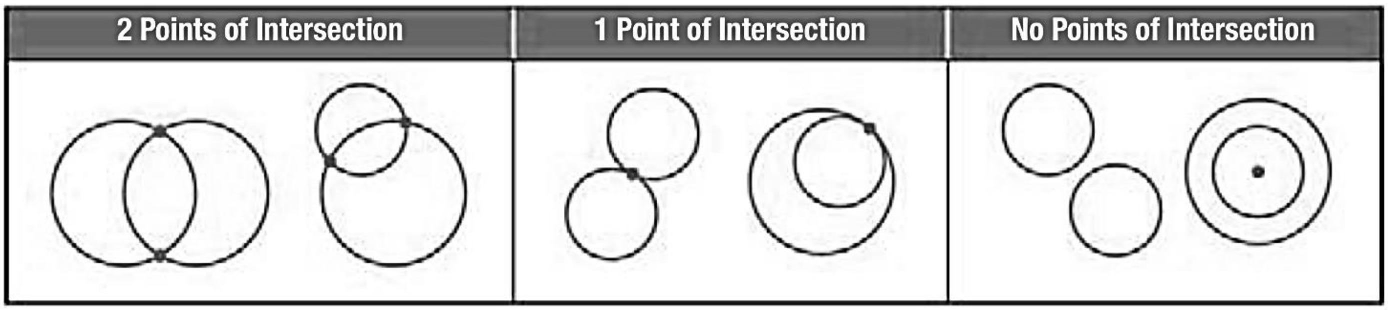
All circles are similar.

Example $\odot X \sim \odot Y$

________ are coplanar circles that have the same center.

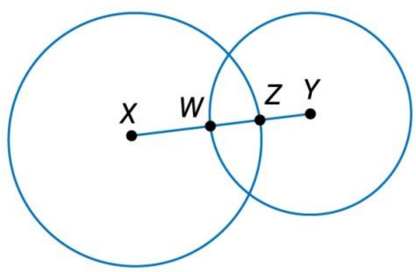
Example $\odot A$ with radius $\overline{AB}$ and $\odot A$ with radius $\overline{AC}$ are concentric.

Two circles can interset in two different ways.

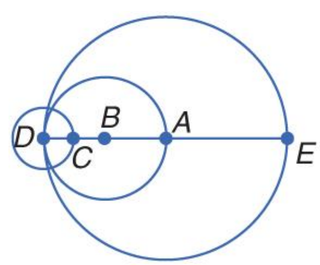


Example 3:

a) The diameter of $\odot X$ is 22 units, the diameter of $\odot Y$ is 16 units, and $WZ = 5$ units. Find XY.



b) The diameters of $\odot D$, $\odot B$, and $\odot A$ are 4 inches, 9 inches, and 18 inches. Find AC.



The ______ of a circle is the distance around the circle. By definition, the ratio $\frac{C}{d}$ is an irrational number called ______. Two formulas for circumference can be derived by using this definition.

KeyConcept Circumference

Words	
Symbols	$C = \pi d$ or $C = 2\pi r$

Example 4:

a) CROP CIRCLES A series of crop circles was discovered in Alberta, Canada, on September 4, 1999. The largest of the three circles had a radius of 30 feet. Find its circumference.

b) The Unisphere is a giant steel globe that sits in Flushing Meadows-Corona Park in Queens, New York. It has a diameter of 120 feet. Find its circumference.

Example 5:

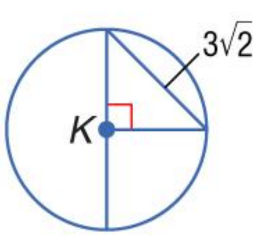
a) Find the diameter and the radius of a circle to the nearest hundredth if the circumference of the circle is 65.4 feet.

b) Find the radius of a circle to the nearest hundredth if its circumference is 16.8 meters.

A polygon is ________________ in a circle if all of its vertices lie on the circle. A circle is circumscribed about polygon if it contais all the vertices of the polygon.

- Quadrilateral *LMNP* is *inscribed* in $\odot K$.
- Circle *K* is *circumscribed about* quadrilateral *LMNP*.

c) Find the exact circumference of $\odot K$.



New Vocabulary: Write the correct term next to each definition. Use the words mentioned today to fill in the blanks.

______________________ ▶ the distance around the circle

______________________ ▶ a segment with endpoints at the center and on the circle

______________________ ▶ the locus or set of all points in a plane equidistant from a given point

______________________ ▶ two coplanar circles that have the same center

______________________ ▶ a chord which passes though the center of a circle and is made up of collinear radii

______________________ ▶ an irrational number which by definition is the ratio of the circumference of a circle to the diameter of the circle

______________________ ▶ descriptor give to a polygon which is drawn inside a circle such that all of its vertices lie on the circle

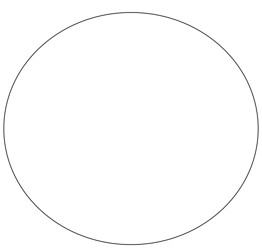
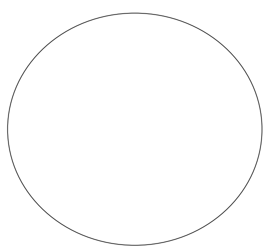
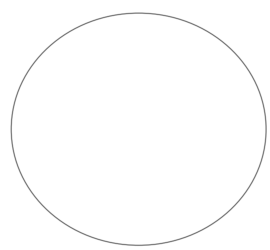
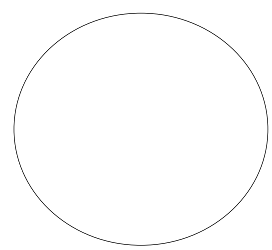
______________________ ▶ the name used to describe the given point in the definition of a circle

______________________ ▶ a segment with endpoints on the circle

______________________ ▶ descriptor given to a circle which is drawn about a polygon such that the circle contains all of the vertices of the polygon

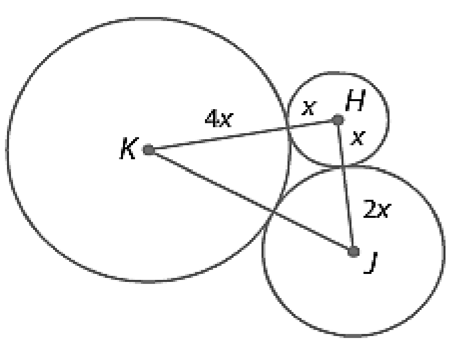
______________________ ▶ two circles with congruent radii

In the below blanks, draw a picture to represent the above vocabulary.

Center of a Circle	Radius	Diameter	Chord
			

Challenge Problem

The sum of the circumference of circles H, J, and K shown below is 56π. Find KJ.



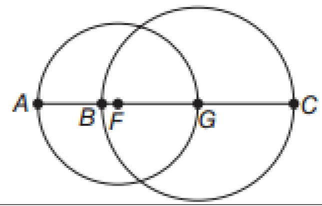
Learning Target Checklist

- ☐ I can identify and use parts of circles.
- ☐ I can solve problems involving the circumference of a circle.

Warm Up 10.2

1. Find the circumference of a circle with a radius of 7.

2. The diameters of Circle F and Circle G are 5 and 6 units, respectively. Find the measure of AB and BF.



Common Core State Standards
G.C.2 Identify and describe relationships among inscribed angles, radii, and chords.
G. C. 5 Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.

Mathematical Practices
6 Attend to precision
4 Model with mathematics

Learning Targets:

- Students will be able to identify central angles, major arcs, minor arcs, and semicircles, and find their measures.
- Students will be able to find arc lengths.

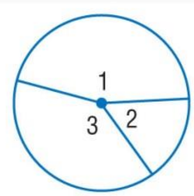
Section 10.2 Notes: Measuring Angles and Arcs

A_______________ of a circle is an angle with a vertex in the center of the circle. Its sides contain two radii of the circle.
$\angle ABC$ is a central angle of $\odot B$.

KeyConcept Sum of Central Angles

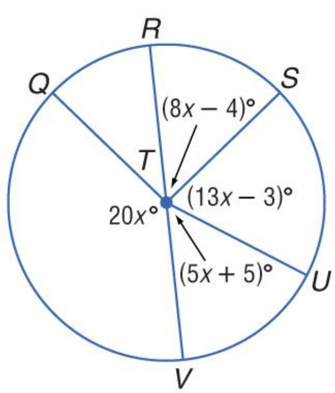
Words

Example $m\angle 1 + m\angle 2 + m\angle 3 = 360$

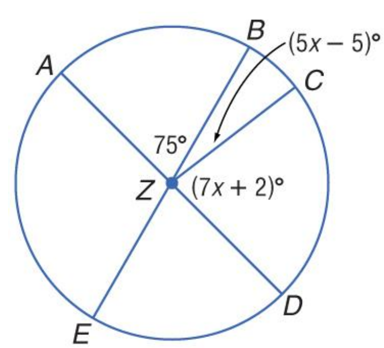


Example 1:

a) Find the value of x.

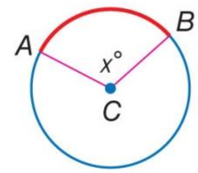
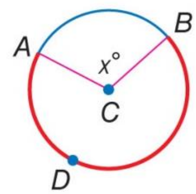
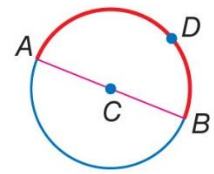


b) Find the value of x.



An ______________is a portion of a circle defined by two endpoints. __________________ separates the circle into two arcs with measures related to the measure of the central angle.

KeyConcept Arcs and Arc Measure

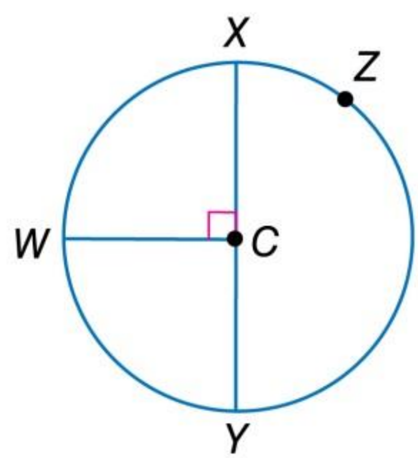
Arc	Measure
A ________ is the shortest arc connecting two endpoints on a circle.	The measure of a minor arc is less than 180 and equal to the measure of its related central angle. $m\widehat{AB} = m\angle ACB = x$ 
A ________ is the longest arc connecting two endpoints on a circle.	The measure of a major arc is greater than 180, and equal to 360 minus the measure of the minor arc with the same endpoints. $m\widehat{ADB} = 360 - m\widehat{AB} = 360 - x$ 
A ________ is an arc with endpoints that lie on a diameter.	The measure of a semicircle is 180. $m\widehat{ADB} = 180$ 

Example 2: Use the figure on the right.

a) $\overline{WC}$ is a radius of $\odot C$. Identify $\widehat{XZY}$ as a major arc, minor arc, or semicircle. Then find its measure.

b) $\overline{WC}$ is a radius of $\odot C$. Identify $\widehat{WZX}$ as a major arc, minor arc, or semicircle. Then find its measure.

c) $\overline{WC}$ is a radius of $\odot C$. Identify $\widehat{XW}$ as a major arc, minor arc, or semicircle. Then find its measure.



__________________________ are arcs in the same or congruent circles that have the same measure.

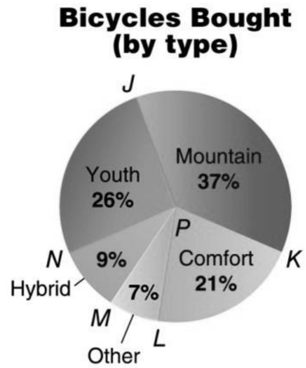
Theorem 10.1

Words In the same circle or in congruent circles, two minor arcs are congruent if and only if their central angles are congruent.

Example If $\angle 1 \cong \angle 2$, then $\widehat{FG} \cong \widehat{HJ}$.
If $\widehat{FG} \cong \widehat{HJ}$, then $\angle 1 \cong \angle 2$.

Example 3:

a) BICYCLES Refer to the circle graph. Find mKL.



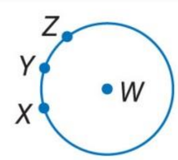
b) BICYCLES Refer to the circle graph. Find $mNJL$.

____________________ are arcs in a circle that have exactly one point in common.
In $\odot M$, HJ and JK are adjacent arcs. As with adjacent angles, you can add the measures of adjacent arcs.

Postulate 10.1 Arc Addition Postulate

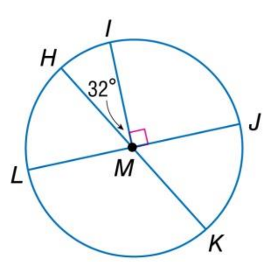
Words

Example $m\widehat{XYZ} = m\widehat{XY} + m\widehat{YZ}$

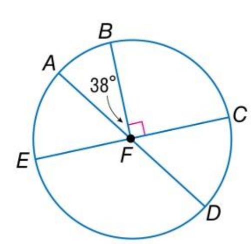


Example 4:

a) Find $mLHI$ in $\odot M$.



b) Find the $mBCD$ in $\odot F$.



New Vocabulary – Label the diagram with the terms listed at the left.

central angle ▶

arc ▶

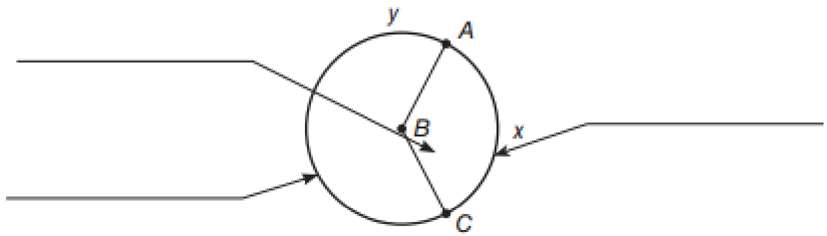
minor arc ▶

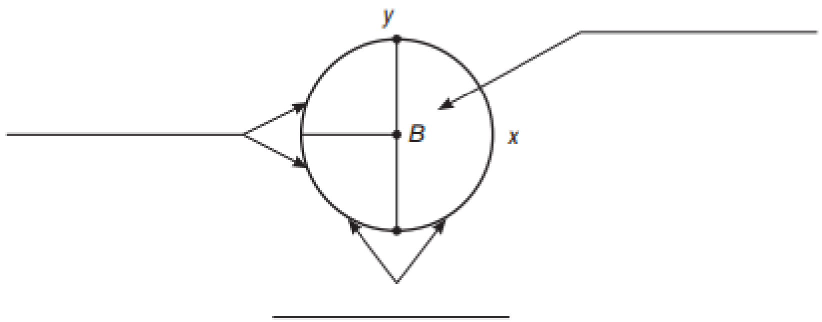
major arc ▶

semicircle ▶

congruent arcs ▶

adjacent arcs ▶





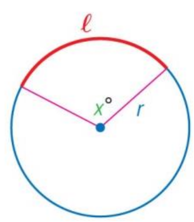
______________________ is the distance between the endpoints along an arc measured in linear units. Since an arc is a portion of a circle, its length is a fraction of the circumference.

KeyConcept Arc Length

Words The ratio of the length of an arc ℓ to the circumference of the circle is equal to the ratio of the degree measure of the arc to 360.

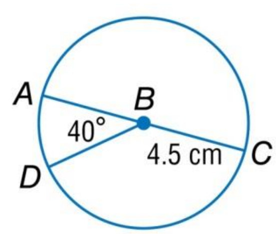
Proportion

Equation

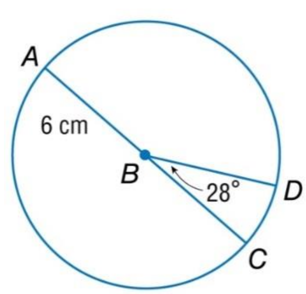


Example 5:

a) Find the length of DA. Round to the nearest hundredth.



b) Find the length of DA. Round to the nearest hundredth.



Summary of 10.2 – Angles and Arcs

Given the $m\angle EJD = 15$, find each measure in $\odot J$ in the order specified. Justify your answer.

$m\widehat{EFG} =$ ____________

____

____

____

____

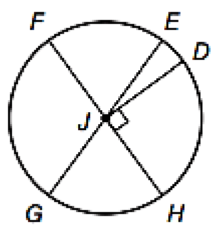
$m\widehat{ED} =$ ____________

____

____

____

____



$m\widehat{GH} =$ ____________

____

____

____

____

$m\widehat{EDH} =$ ____________

____

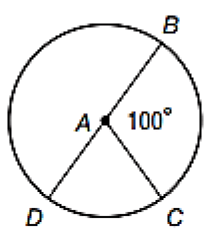
____

____

____

Summary of 10.2 – Arc Length

Find the length of $\widehat{BC}$ and $\widehat{BDC}$. The radius of $\odot A$ is 5 centimeters. What is the relationship between the two arc lengths?



$m\widehat{BC} =$ ____________

$m\widehat{BDC} =$ ____________

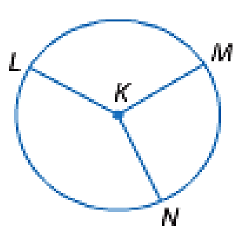
Relationship: ____________________________________

____

____

Challenge Problem

The measures of arc LM, arc MN and arc NL are in the ration 5:3:4. Find the measure of each arc.



Learning Target Checklist

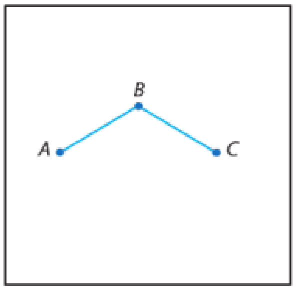
- ☐ I can identify central angles, major arcs, minor arcs, and semicircles, and find their measures.
- ☐ I can find arc lengths.

Geometry
Construction Section 10.3

Name ____________________________________

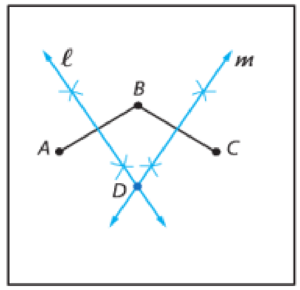
Construction Circle Through Three Noncollinear Points

Step 1



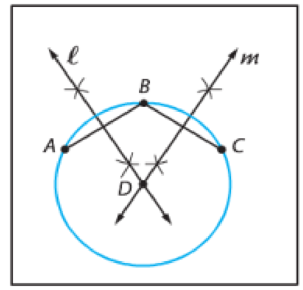
Draw three noncollinear points *A*, *B*, and *C*. Then draw segments $\overline{AB}$ and $\overline{BC}$.

Step 2



Construct the perpendicular bisectors ℓ ** and *m* of $\overline{AB}$ and $\overline{BC}$. Label the point of intersection *D*.

Step 3



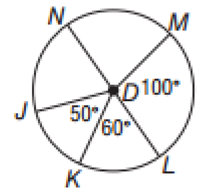
By Theorem 10.4, lines ℓ and *m* contain diameters of ⊙*D*. With the compass at point *D*, draw a circle through points *A*, *B*, and *C*. (Use a compass setting of *DA*.)

**To construct the perpendicular bisector to $\overline{AB}$, put your compass point on *A*. Open the compass setting to more than half of *AB*. Draw an arc above and below $\overline{AB}$. Using the same compass setting, put the point on *B*. Draw an arc above and below $\overline{AB}$. The compass arcs should have intersected above and below $\overline{AB}$. Draw line ℓ through these intersecting arcs.
Follow similar steps to construct the perpendicular bisector to $\overline{BC}$. Draw line *m* through a new set of intersecting arcs.

Complete this activity 2 times to create two different size circle.

Warm Up 11.3

1. Use Circle D to find the length of each arc. Round to the nearest hundredth.
   a. $\widehat{LM}$ if the radius is 5 inches.
   c. $\widehat{KL}$ if JD= 7 centimeters
   b. $\widehat{MN}$ if the diameter is 3 yards.
   $\widehat{NJK}$ if NL = 12 feet.



Common Core State Standards
G. C. 5 Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.
G.GMD.1 Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone.

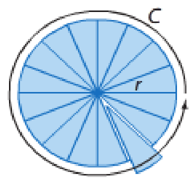
Mathematical Practices
1 Make sense of problems and persevere in solving them.
6 Attend to precision

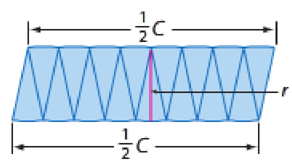
Learning Targets
- Students will be able to find areas of circles.
- Students will find areas of sectors of circles.

Section 11.3 Notes: Areas of Circles and Sectors

In Lesson 10-1, you learned that the formula for the circumference *C* of a circle with radius *r* is given by $C = 2\pi r$. You can use this formula to develop the formula for the area of a circle.

Below, a circle with radius *r* and circumference *C* has been divided into congruent pieces and then rearranged to form a figure that resembles a parallelogram.

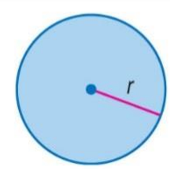




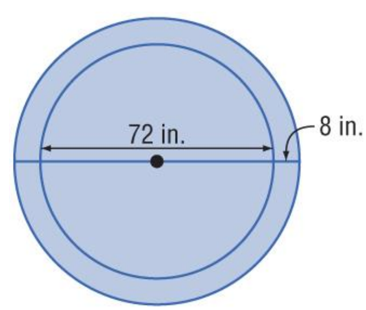
As the number of congruent pieces increases, the rearranged figure more closely approaches a parallelogram. The base of the parallelogram is $\frac{1}{2}C$ and the height is *r*, so its area is $\frac{1}{2}C \cdot r$. Since $C = 2\pi r$, the area of the parallelogram is also $\frac{1}{2}(2\pi r)r$ or πr^2.

KeyConcept Area of a Circle

Words	The area *A* of a circle is equal to π times the square of the radius *r*.
Symbols	



Example 1: An outdoor accessories company manufactures circular covers for outdoor umbrellas. If the cover is 8 inches longer than the umbrella on each side, find the area of the cover in square inches.



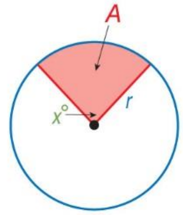
Example 2: Find the radius of a circle with an area of 58 square inches.

A slice of a circular pizza is an example of a sector of a circle. A ______________________ is a region of a circle bounded by a central angle and its intercepted major or minor arc. The formula for the area of a sector is similar to the formula for arc length.

KeyConcept Area of a Sector

The ratio of the **area A of a sector** to the **area of the whole circle, πr^2**, is equal to the ratio of the **degree measure of the intercepted arc x** to 360.

Equation: $A = \frac{x}{360} \cdot \pi r^2$



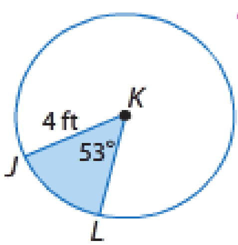
Example 3:

a) A pie has a diameter of 9 inches and is cut into 10 congruent slices. What is the area of one slice to the nearest hundredth?

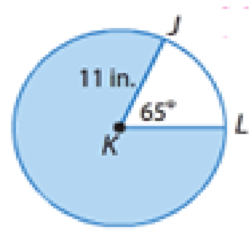
b) A pizza has a diameter of 14 inches and is cut into 8 congruent slices. What is the area of one slice to the nearest hundredth?

Area of a Sector Practice: Find the area of the shaded sector. Round to the nearest tenth.

a)

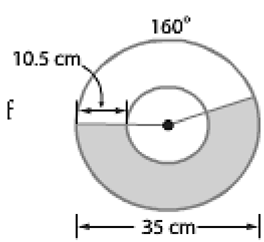


b)



Helping You Remember: A good way to remember something is to explain it to someone else. Suppose Jimmy is having trouble remembering which formula is for circumference and which is for area. How can you help him out?

Challenge Problem: Find the area of the shaded region. Round to the nearest tenth.



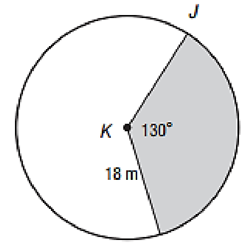
Learning Target Checklist

- ☐ Students will be able to find areas of circles.
- ☐ Students will find areas of sectors of circles.

Warm Up 10.3

1. Find the area of a circle with a diameter of 10 cm.

2. Find the area of the shaded sector. Round to the nearest tenth.



Common Core State Standards
G.C.2 Identify and describe relationships among inscribed angles, radii, and chords.
G.MG. 3 Apply geometric methods to solve problems.

Mathematical Practices
4 Model with mathematics
3 Construct viable arguments and critique the reasoning of others

Learning Targets

- Students will be able to recognize and use relationships between arcs and chords.
- Students will be able to recognize and use relationships between arcs, chords, and diameters

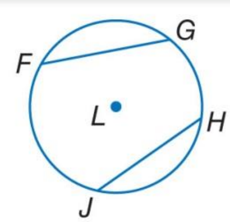
Section 10.3 Notes: Arcs and Chords

A __________ is a segment with endpoints on a circle. If a chord is not a diameter, then its endpoints divide the circle into a major and minor arc.

Theorem 10.2

Words

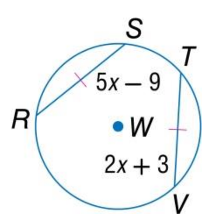
Example $\overline{FG} \cong \overline{HJ}$ if and only if $\widehat{FG} \cong \widehat{HJ}$.



Example 1:

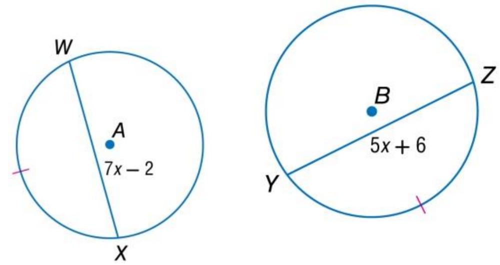
a) JEWELRY A circular piece of jade is hung from a chain by two wires around the stone. $\overline{JM} \cong \overline{KL}$ and $m\widehat{KL} = 90°$. Find $m\widehat{JM}$.

b) $\odot W$ has congruent chords $\overline{RS}$ and $\overline{TV}$. If $m\widehat{RS} = 85°$, find $m\widehat{TV}$.

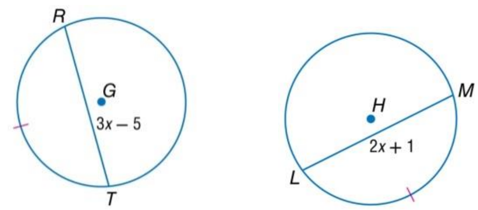


Example 2:

a) ALGEBRA In the figure, $\odot A \cong \odot B$ and $WX \cong YZ$. Find *WX*.



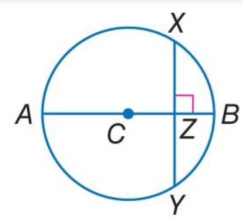
b) ALGEBRA In the figure, $\odot G \cong \odot H$ and $RT \cong LM$. Find *LM*.



Theorems

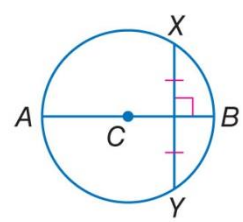
10.3 If a diameter (or radius) of a circle is perpendicular to a chord, then it bisects the chord and its arc.

Example If diameter $\overline{AB}$ is perpendicular to chord $\overline{XY}$, then $\overline{XZ} \cong \overline{ZY}$ and $\widehat{XB} \cong \widehat{BY}$.



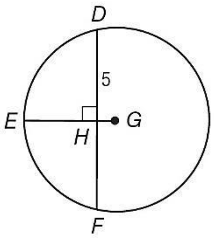
10.4 The perpendicular bisector of a chord is a diameter (or radius) of the circle.

Example If $\overline{AB}$ is a perpendicular bisector of chord $\overline{XY}$, then $\overline{AB}$ is a diameter of $\odot C$.

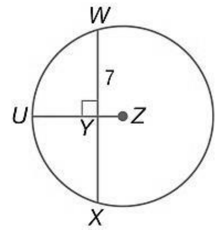


Example 3:

a) In $\odot G$, $mDEF = 150°$. Find mDE .

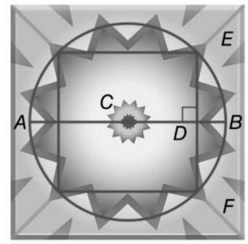


b) In $\odot Z$, $mWUX = 60°$. Find mUX .

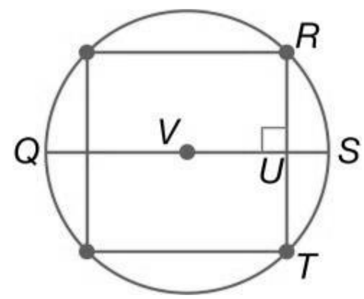


Example 4:

a) CERAMIC TILE In the ceramic stepping stone below, diameter $\overline{AB}$ is 18 inches long and chord $\overline{EF}$ is 8 inches long. Find CD.



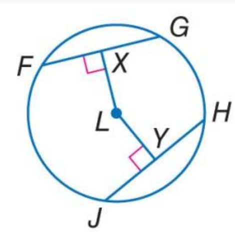
b) In the circle below, diameter $\overline{QS}$ is 14 inches long and chord $\overline{RT}$ is 10 inches long. Find VU.



Theorem 10.5

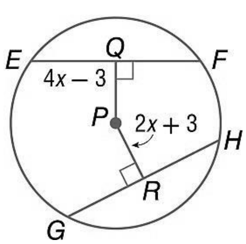
Words

Example $\overline{FG} \cong \overline{JH}$ if and only if $LX = LY$.

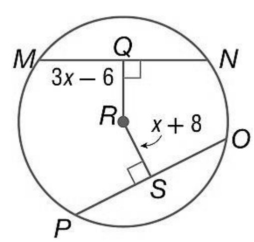


Example 5:

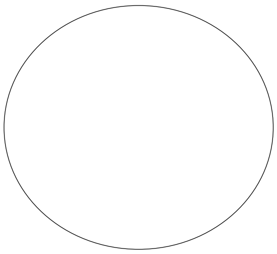
a) In $\odot P$, $EF = GH = 24$. Find PQ.



b) In $\odot R$, $MN = PO = 29$. Find RS.

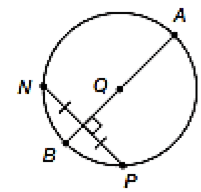


Review Vocabulary – In the circle provided, draw a chord which is not a diameter using a bolded line and label it, a diameter using a thin line and label it, and a radius using a dashed line and label it. Identify 3 arcs that were created on the circle.



ARCS -

Use the diagram below to state Theorems 10.3 and 10.4, from the student book, in your own words.



Theorem 10.3 ______________________________

____

____

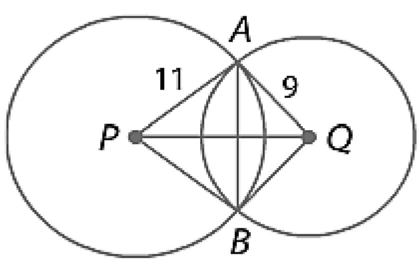
Theorem 10.4 ______________________________

____

____

Challenge Problem

The common chord AB between circle P and circle Q
Is perpendicular to the segment connecting the centers of the circles.
If AB = 10, what is the length of PQ? Explain your reasoning.

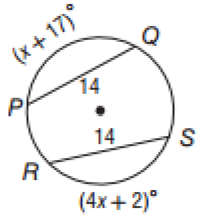


Learning Target Checklist

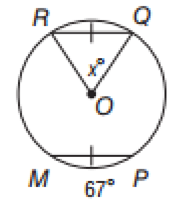
- ☐ I can recognize and use relationships between arcs and chords.
- ☐ I can recognize and use relationships between arcs, chords, and diameter.

10.4 Warm Up

1. Find the value of x.



2. Find the value of x.

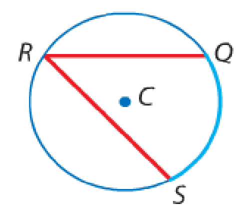


Common Core State Standards
G.C.2 Identify and describe relationships among inscribed angles, radii, and chords.
G.C.3 Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.

Mathematical Practices
7 Look for and make use of structure
3 Construct viable arguments and critique the reasoning of others.

Learning Targets
- Students will be able to find measures of inscribed angles.
- Students will be able to find measures of angles of inscribed polygons.

Section 10.4 Notes: Inscribed Angles



An ___________________ has a vertex on a circle and sides that contain chords of the circle.
In $\odot C$, $\angle QRS$ is an inscribed angle.

An ___________________ has endpoints on the sides of an inscribed angle and lies in the interior of the inscribed angle. In $\odot C$, minor arc QS is intercepted by $\angle QRS$.

There are three ways that an angle can be inscribed in a circle.

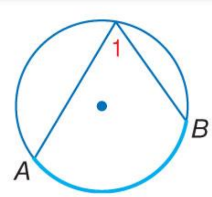
Case 1	Case 2	Case 3
Center *P* is on a side of the inscribed angle.	Center *P* is inside the inscribed angle.	The center *P* is in the exterior of the inscribed angle.

For each of these cases, the following theorem holds true.

Theorem 10.6 Inscribed Angle Theorem

Words If an angle is inscribed in a circle, then the measure of the angle equals one half the measure of its intercepted arc.

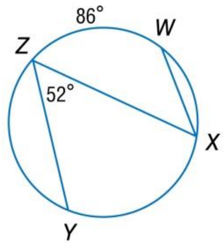
Example



Example 1:

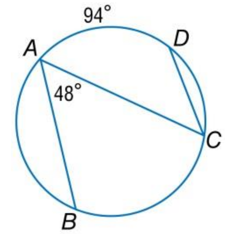
a) Find $m\angle X$.

b) Find mYX



c) Find $m\angle C$.

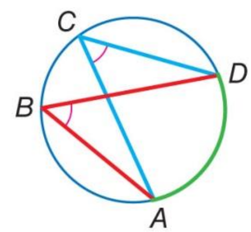
d) Find mBC



Theorem 10.7

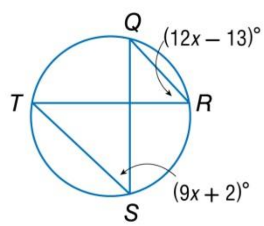
Words

Example $\angle B$ and $\angle C$ both intercept $\overset{\frown}{AD}$. So, $\angle B \cong \angle C$.

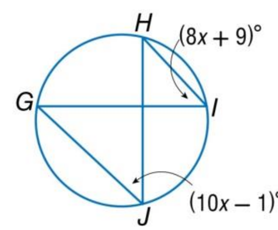


Example 2:

a) Find $m\angle R$.



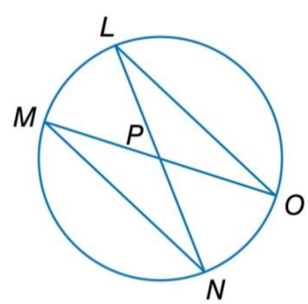
b) Find $m\angle I$.



Example 3:

a) Write a two-column proof.

Given: $LO \cong MN$
Prove: $\Delta MNP \cong \Delta LOP$

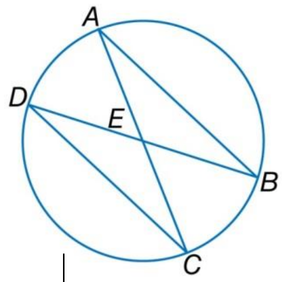


Statements	Reasons
1.	1.
2.	2.
3.	3.
4.	4.
5.	5.
6.	6.

b) Write a two-column proof.

Given: $AB \cong CD$
Prove: $\Delta ABE \cong \Delta DCE$

Select the appropriate reason that goes in the blank to complete the proof below.

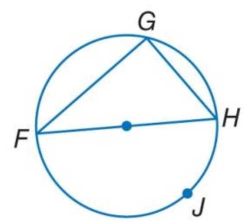


Statements	Reasons
1. $AB \cong CD$	1. Given
2. $\overline{AB} \cong \overline{CD}$	2. If minor arcs are congruent, then corresponding chords are congruent.
3. $\angle D$ intercepts BC and $\angle A$ intercepts BC.	3. Definition of intercepted arc
4. $\angle D \cong \angle A$	4. Inscribed angles of the same arc are congruent.
5. $\angle DEC \cong \angle BEA$	5. Vertical angles are congruent.
6. $\Delta ABE \cong \Delta DCE$	6. ______________________________

Theorem 10.8

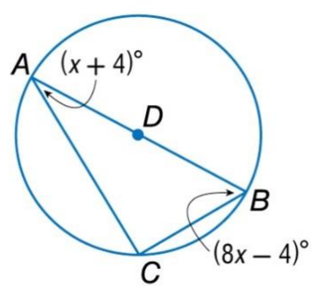
Words

Example If $\widehat{FJH}$ is a semicircle, then $m\angle G = 90$. If $m\angle G = 90$, then $\widehat{FJH}$ is a semicircle and $\overline{FH}$ is a diameter.

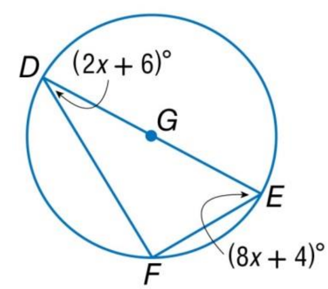


Example 4:

a) Find $m\angle B$.



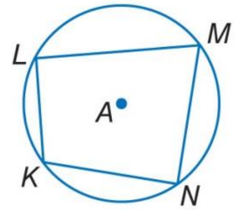
b) Find $m\angle D$.



Theorem 10.9

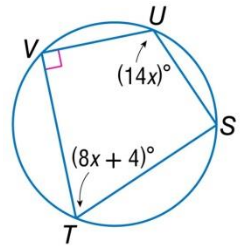
Words

Example If quadrilateral $KLMN$ is inscribed in $\odot A$, then $\angle L$ and $\angle N$ are supplementary and $\angle K$ and $\angle M$ are supplementary.

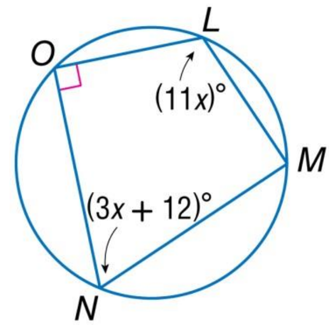


Example 5:

a) INSIGNIAS An insignia is an emblem that signifies rank, achievement, membership, and so on. The insignia shown is a quadrilateral inscribed in a circle. Find $m\angle S$ and $m\angle T$.



b) INSIGNIAS An insignia is an emblem that signifies rank, achievement, membership, and so on. The insignia shown is a quadrilateral inscribed in a circle. Find $m\angle N$.



New Vocabulary Write the definition next to each term.

inscribed angle ▶ ______________________________

____

intercepted arc ▶ ______________________________

____

Writing in math -
A 45-45-90 right triangle is inscribed in a circle. If the radius of the circle is given, explain how to find the lengths of the right triangle's legs.

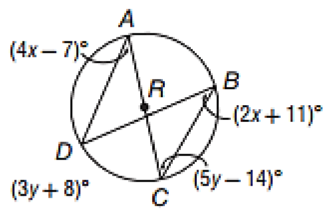
Challenge Problem
A square is inscribed in a circle. What is the ratio of the area of the circle to the area of the square?

Learning Target Checklist

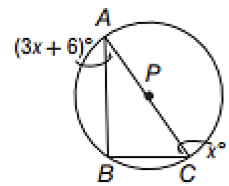
- ☐ I can find measures of inscribed angles.
- ☐ I can find measures of angles of inscribed polygons.

10.5 Warm Up

1. Find the value of x and y.



2. Find the value of x



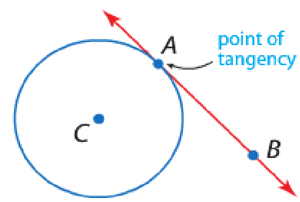
Common Core State Standards
G. CO. 12 Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.)
G.C.4 Construct a tangent line from a point outside a given circle to the circle.

Mathematical Practices
1 Make sense of problems and persevere in solving them.
2 Reason abstractly and quantitatively.

Learning Targets

- Students will be able to use properties of tangents.
- Students will be able to solve problems involving circumscribed polygons.

Section 10.5 Notes: Tangents

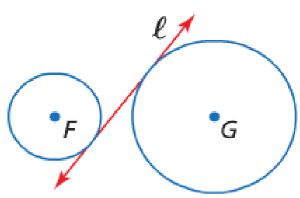


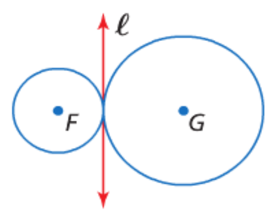
A__________________ is a line in the same plane as a circle that intersects the circle in exactly one point, called _________________________.

$\overleftrightarrow{AB}$ is tangent to $\odot C$ at point A. $\overline{AB}$ and $\overrightarrow{AB}$ are also called tangents.

A ______________________ is a line, ray, or segment that is tangent to two circles in the same plane.

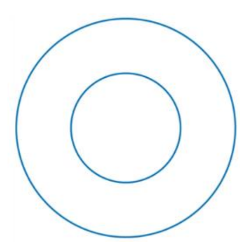
In each figure below, in *l* is a common tangent of circles *F* and *G*.



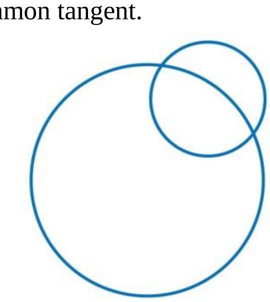


Example 1:

a) Using the figure to the right, draw the common tangents. If no common tangent exists, state no common tangent.



b) Using the figure to the right, draw the common tangents. If no common tangent exists, state no common tangent.



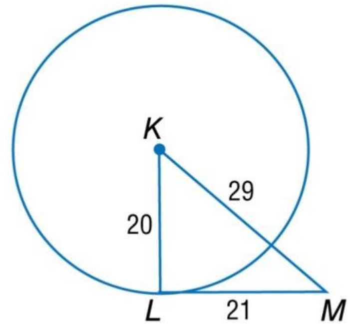
Theorem 10.10

Words In a plane, a line is tangent to a circle if and only if it is perpendicular to a radius drawn to the point of tangency.

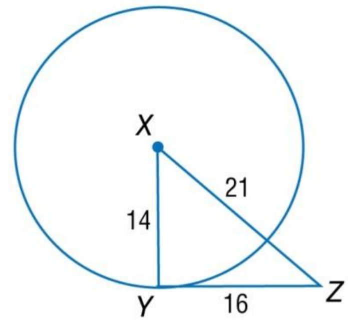
Example Line ℓ is tangent to $\odot S$ if and only if $\ell \perp \overline{ST}$.

Example 2:

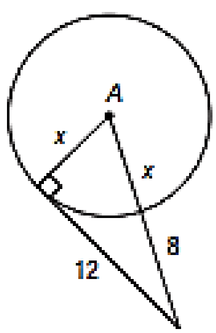
a) $\overline{KL}$ is a radius of $\odot K$. Determine whether $\overline{LM}$ is tangent to $\odot K$. Justify your answer.



b) $\overline{XY}$ is a radius of $\odot X$. Determine whether $\overline{YZ}$ is tangent to $\odot X$. Justify your answer.



Find the value of x. Assume that segments that appear to be tangent are tangent. Show your work in each box as indicated.



Identify the length of each side of the right triangle.

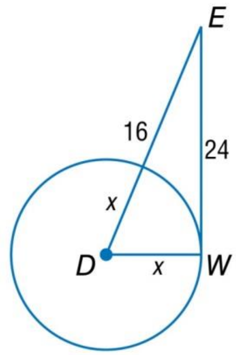
Leg 1 = ______ Leg 2 = ______

Hyp = ______

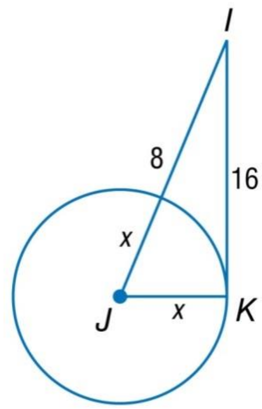
Use the Pythagorean Theorem to write an equation, then solve for x.

Example 3:

a) In the figure, $\overline{WE}$ is tangent to $\odot D$ at *W*. Find the value of *x*.



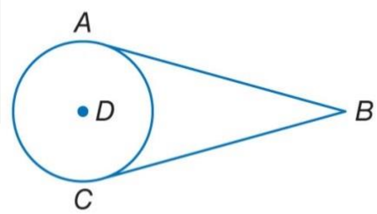
b) In the figure, $\overline{IK}$ is tangent to $\odot J$ at *K*. Find the value of *x*.



Theorem 10.11

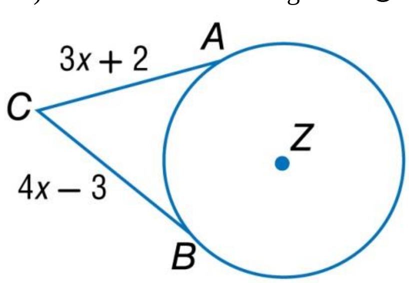
Words

Example If $\overline{AB}$ and $\overline{CB}$ are tangent to $\odot D$, then $\overline{AB} \cong \overline{CB}$.

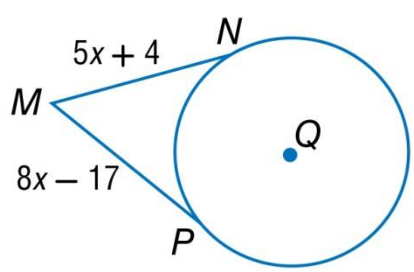


Example 4:

a) $\overline{AC}$ and $\overline{BC}$ are tangent to $\odot Z$. Find the value of *x*.

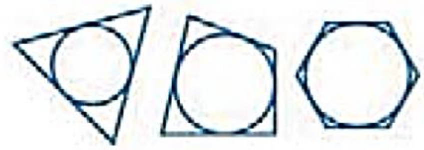


b) $\overline{MN}$ and $\overline{MP}$ are tangent to $\odot Q$. Find the value of *x*.



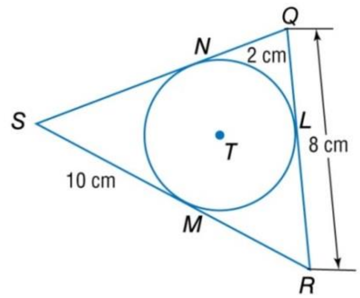
Circumscribed Polygons

A polygon is ______________________________ about a circle if every side of the polygon is tangent to the circle.

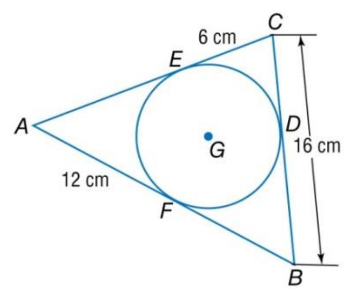
Circumscribed Polygons	Polygons Not Circumscribed
	

Example 5:

a) The round cookies are marketed in a triangular package to pique the consumer's interest. If ΔQRS is circumscribed about $\odot T$, find the perimeter of ΔQRS .

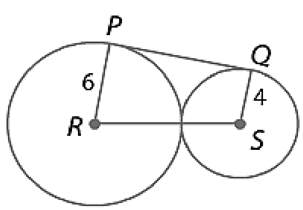


b) A bouncy ball is marketed in a triangular package to pique the consumer's interest. If ΔABC is circumscribed about $\odot G$, find the perimeter of ΔABC .



Challenge Problem

PQ is a tangent in circles R and S. Find PQ.
Explain your reasoning.



Open Ended

Draw a circumscribed triangle	Draw a inscribed triangle

Learning Target Checklist

- ☐ Students will be able to recognize and use relationships between arcs and chords.
- ☐ Students will be able to recognize and use relationships between arcs, chords, and diameter.

Geometry
Constructions Section 10.5

Name ______________________________________

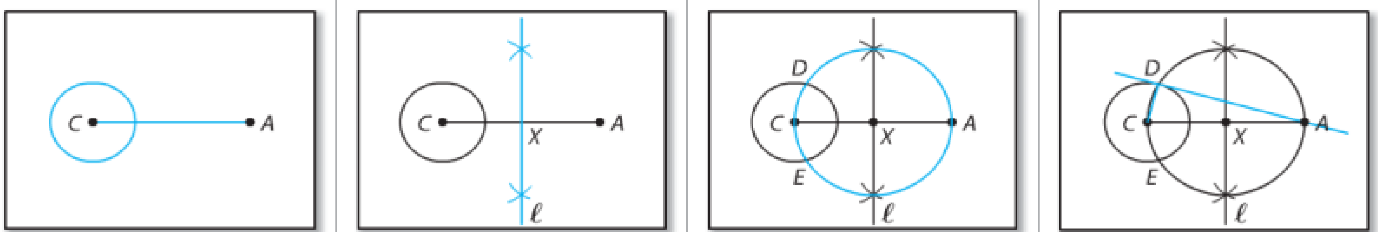
Construction Line Tangent to a Circle Through an External Point

Step 1 Use a compass to draw circle C and a point A outside of circle C. Then draw $\overline{CA}$.

Step 2 Construct line ℓ,** the perpendicular bisector of $\overline{CA}$. Label the point of intersection X.

Step 3 Construct circle X with radius $\overline{XC}$. Label the points of intersection of the two circles D and E.

Step 4 Draw $\overleftrightarrow{AD}$ and $\overline{DC}$. $\triangle ADC$ is inscribed in a semicircle. So, $\angle ADC$ is a right angle and $\overleftrightarrow{AD}$ is tangent to $\odot C$.

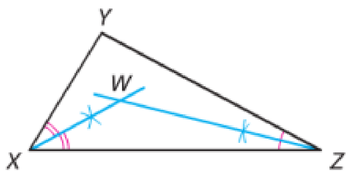


**To construct the perpendicular bisector to $\overline{AC}$, put your compass point on A. Open the compass setting to more than half of AC. Draw an arc above and below $\overline{AC}$. Using the same compass setting, put the point on C. Draw an arc above and below $\overline{AC}$. The compass arcs should have intersected above and below $\overline{AC}$. Draw line ℓ through these intersecting arcs.

Geometry Lab: Inscribed and Circumscribed Circles

Activity 1 Construct a Circle Inscribed in a Triangle

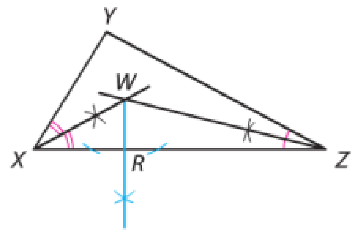
Step 1



Use

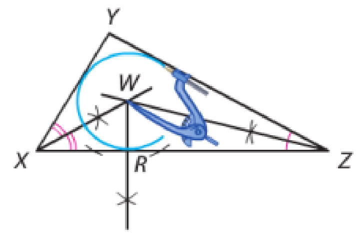
~~Draw a~~ triangle *XYZ* and construct two angle bisectors of the triangle * to locate the incenter *W*.

Step 2



Construct a segment perpendicular** to a side through the incenter. Label the intersection *R*.

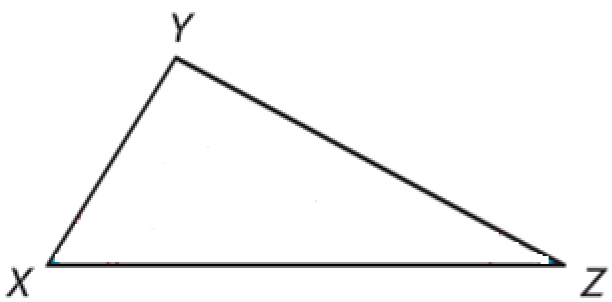
Step 3

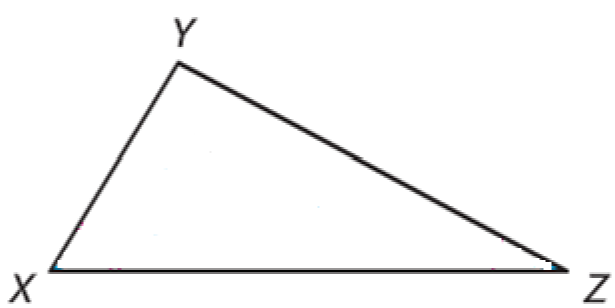


Set a compass of the length of $\overline{WR}$. Put the point of the compass on *W* and draw a circle with that radius.

* To construct an angle bisector of $\angle X$, place the compass point on *X* and make an arc on sides $\overline{XY}$and $\overline{XZ}$. Using the same setting, place the point on each of the arcs and make a third and fourth arc that intersect far away from point *X*. Draw a line from this intersection point and *X*. This is an angle bisector.
Follow similar steps to construct the angle bisector of $\angle Z$. The point where the two angle bisectors intersect is the incenter. Label it *W*.

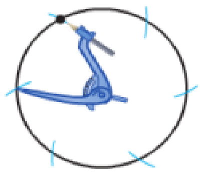
**To construct the perpendicular segment to $\overline{XZ}$, put your compass point on *W*. Open the compass setting so that you can make 2 arcs on $\overline{XZ}$. Use the same setting, place the compass point on one of the intersection points between an arc and $\overline{XZ}$ and make a third arc below $\overline{XZ}$. Use the same setting and place the compass point on the other intersection point between an arc and $\overline{XZ}$. Make a fourth arc below $\overline{XZ}$that will intersect arc three. Label this intersection point *R*. Draw $\overline{WR}$.





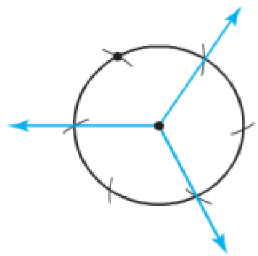
Activity 2 Construct a Triangle Circumscribed About a Circle

Step 1



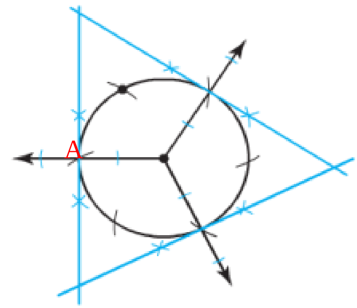
Construct a circle and draw a point.* Use the same compass setting you used to construct the circle to construct an arc on the circle from the point. Continue as shown. **

Step 2



Draw rays from the center through every other arc.

Step 3



Construct a line perpendicular to each of the rays. ***

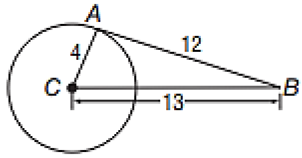
* Draw this point **on** the circle.

** Using the same compass setting that you made your circle with, place the compass point on the point you made on the circle and make an arc. Then place the compass setting on the new arc and make a second arc. Place your compass setting on the second arc and make a third arc, etc. Continue this until you have a total of 6 arcs. One arc should be going through the original point.

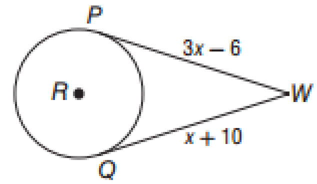
*** To construct a perpendicular line to the first ray: Use a small compass setting and place your compass point on point *A* (as in diagram above). Make two arcs on the ray; one to the left of *A* and one to the right. Next, place your compass point on the arc to the right of point *A* and open the compass setting so that it is just pass point *A*. Draw an arc above and below point *A*. Use the same setting with your compass point on the arc to the left of point *A*, and draw another arc above and below point *A*. You should now have intersecting arcs above and below point *A*. Connect these intersecting points to create a perpendicular line to your ray.
Follow similar steps for the other two rays. Draw your perpendicular lines long enough so that they will intersect to form a triangle when you are finished.

Warm Up 10.6

1. Determine whether segment AB is tangent to the circle.



2. Assume that segments PW and QW are tangent to the circle. Find the value of x.



Common Core State Standards
G.C.4 Construct a tangent line from a point outside a given circle.

Mathematical Practices
3 Construct viable arguments and critique the reasoning of others.
1 Make sense of problems and persevere in solving them.

Learning Targets

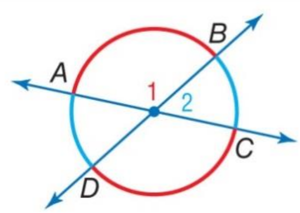
- Students will be able to find measures of angles formed by lines intersecting on or in a circle.
- Students will be able to find measures of angles formed by lines intersecting outside the circle.

Section 10.6 Notes: Secants, Tangents, and Angle Measures

A __________is a line that intersects a circle in exactly two points. Lines *j* and *k* are secants of $\odot C$.

Theorem 10.12

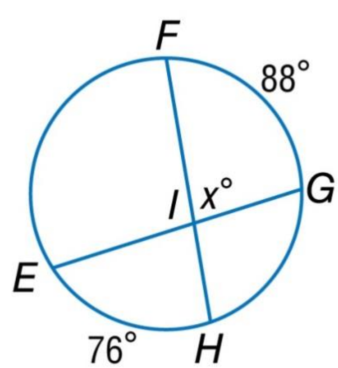
Words If two secants or chords intersect in the interior of a circle, then the measure of an angle formed is one half the *sum* of the measure of the arcs intercepted by the angle and its vertical angle.



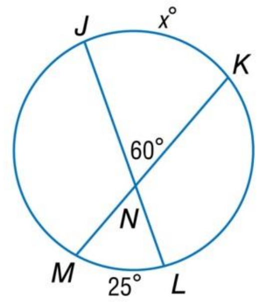
Example

Example 1:

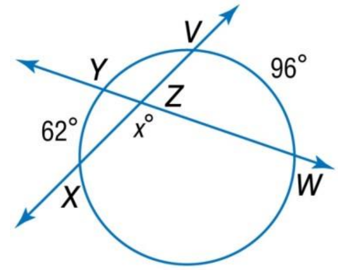
a) Find *x*.



b) Find *x*.



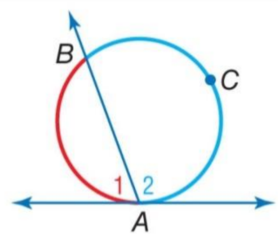
c) Find *x*.



Theorem 10.13

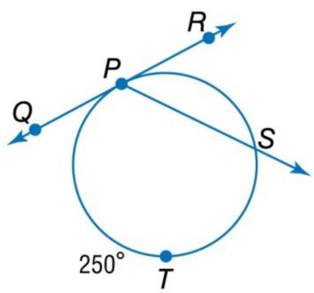
Words If a secant and a tangent intersect at the point of tangency, then the measure of each angle formed is one half the measure of its intercepted arc.

Example

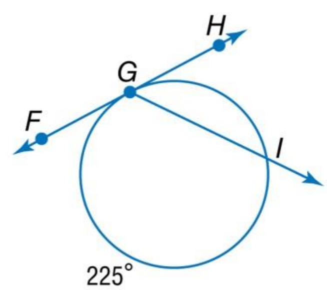


Example 2:

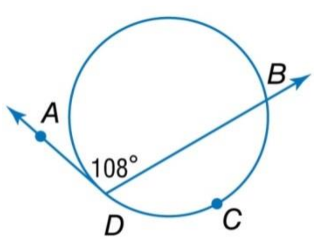
a) Find $m\angle QPS$.



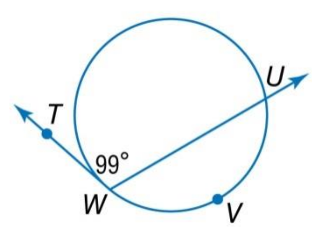
c) Find $m\angle FGI$.



b) Find $mBCD$.



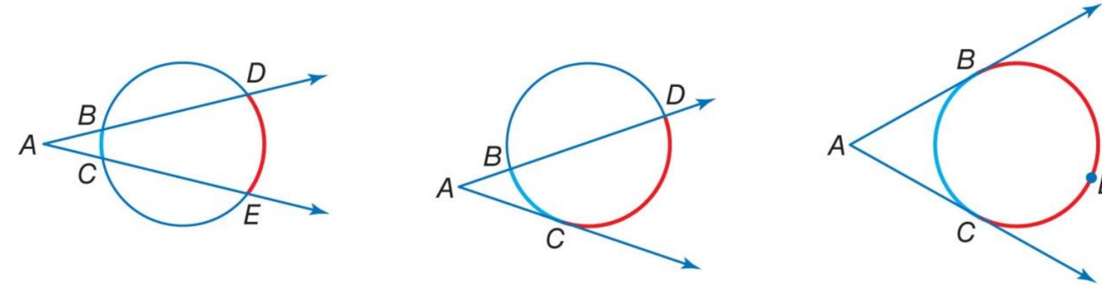
d) Find $mUVW$



Theorem 10.14

Words If two secants, a secant and a tangent, or two tangents intersect in the exterior of a circle, then the measure of the angle formed is one half the *difference* of the measures of the intercepted arcs.

Examples

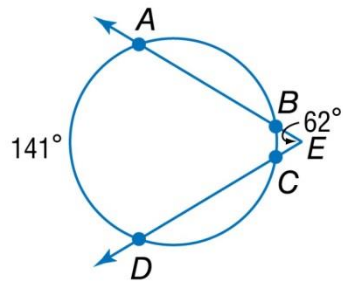


Two Secants | Secant-Tangent | Two Tangents

Example 3:

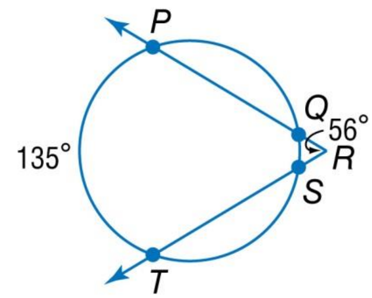
What situation do we have for a :_________________________

a) Find mBC .



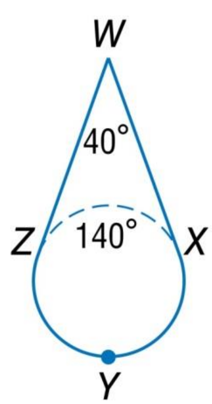
What situation do we have for c: _______________________

c) Find mQS.



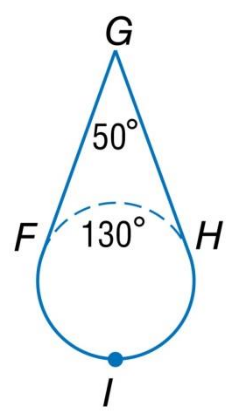
What situation do we have for b:______________________

b) Find $mXYZ$



What situation do we have for d: ________________________

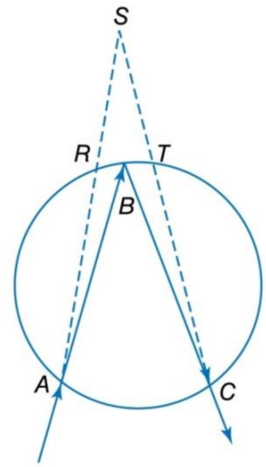
d) Find $mFIH$



Example 4:

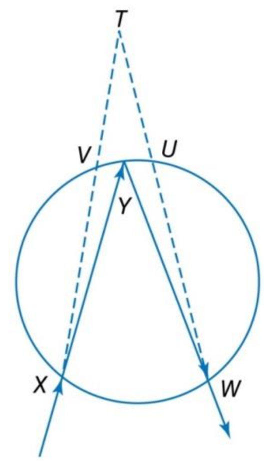
a) The diagram shows the path of a light ray as it hits a cut diamond. The ray is bent, or refracted, at points A, B, and C.

If $mAC = 96°$ and $m\angle S = 35°$, what is $mRBT$?



b) The diagram shows the path of a light ray as it hits a cut diamond. The ray is bent, or refracted, at points X, Y, and W.

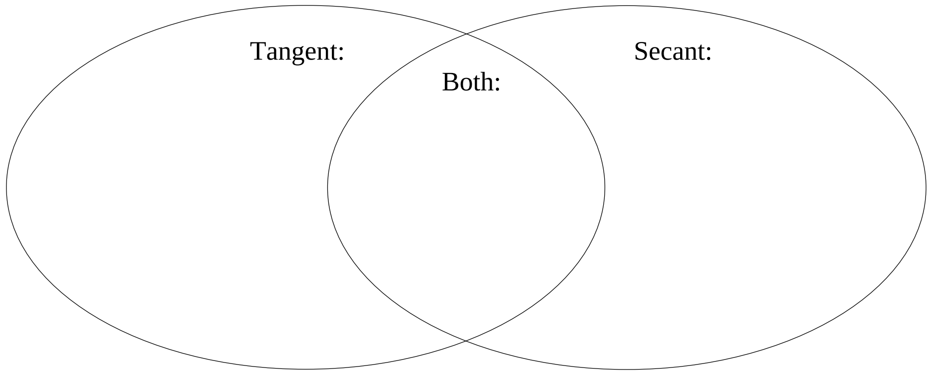
If $mXW = 100°$ and $m\angle T = 30°$, what is $mVYU$?



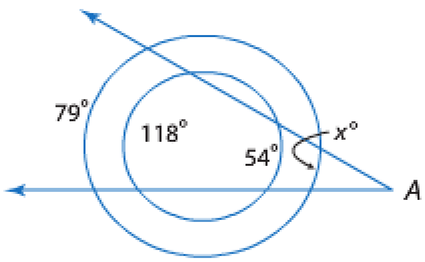
Key Concept Review:

Vertex of Angle is located…	Examples of this situation:	Angle Measurement formula:
on the circle		
Inside the circle		
outside the circle		

New Vocabulary : Compare and contrast a tangent and a secant.



Challenge Problem
The circles below are concentric. What is x?

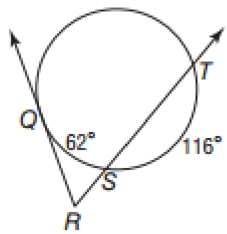


Learning Target Checklist

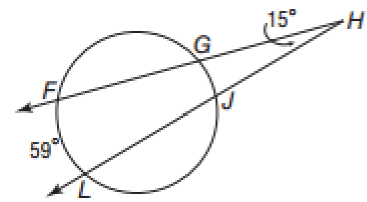
- ☐ I can find measures of angles formed by lines intersecting on or in a circle.
- ☐ I can find measures of angles formed by lines intersecting outside the circle.

Warm Up 10.7

1. Find the $m\angle R$



2. Find the $m\widehat{GJ}$



Common Core State Standards
G.C.4 Construct a tangent line from a point outside a given circle to the circle

Mathematical Practices
1 Make sense of problems and persevere in solving them
7 Look for and make use of structure

Learning Targets
- Students will be able to find measures of segments that intersect in the interior of a circle.
- Students will find measures of segments that intersect in the exterior of a circle.

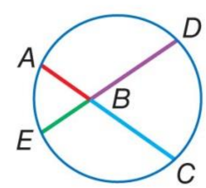
Section 10.7 Notes: Special Segments in a Circle

When two chords intersect inside a circle, each chord is divided into two segments, called ___________________.

Theorem 10.15 Segments of Chords Theorem

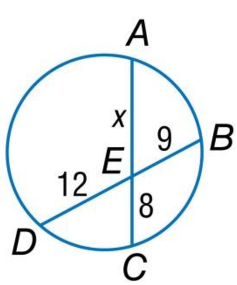
Words

Example $AB \cdot BC = DB \cdot BE$

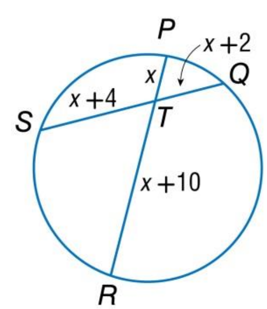


Example 1:

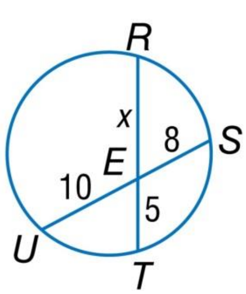
a) Find *x*.



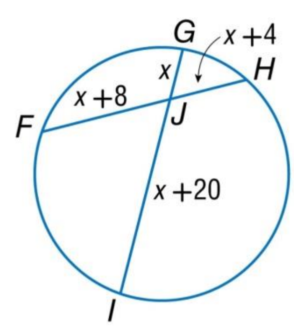
c) Find *x*.



b) Find *x*.

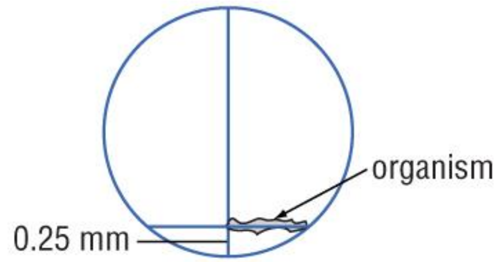


d) Find *x*.

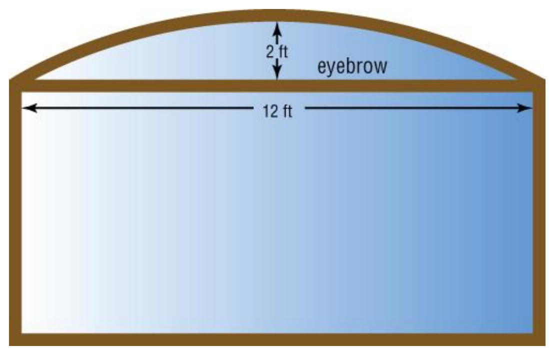


Example 2:

a) BIOLOGY Biologists often examine organisms under microscopes. The circle represents the field of view under the microscope with a diameter of 2 mm. Determine the length of the organism if it is located 0.25 mm from the bottom of the field of view. Round to the nearest hundredth.



b) ARCHITECTURE Phil is installing a new window in an addition for a client’s home. The window is a rectangle with an arched top called an eyebrow. The diagram below shows the dimensions of the window. What is the radius of the circle containing the arc if the eyebrow portion of the window is not a semicircle?

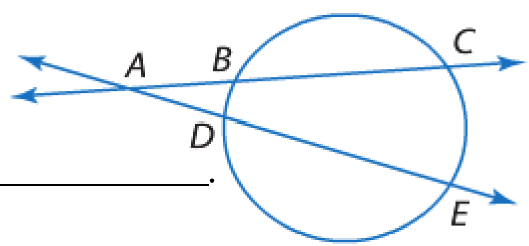


A secant segment is a segment of a ______________ that has exactly one endpoint on the circle.

In the figure, $\overline{AC}, \overline{AB}, \overline{AE}$ and $\overline{AD}$ are secant segments.

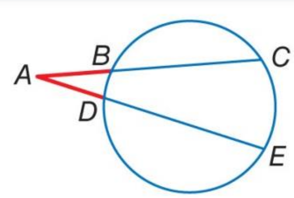
A secant segment that lies in the exterior of the circle is called an __________________________.

In the figure, $\overline{AB}$ and $\overline{AD}$ are external secant segments.



Theorem 10.16 Secant Segments Theorem

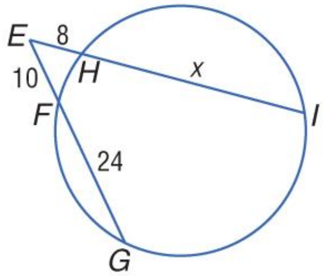
Words If two secants intersect in the exterior of a circle, then the product of the measures of one secant segment and its external secant segment is equal to the product of the measures of the other secant and its external secant segment.



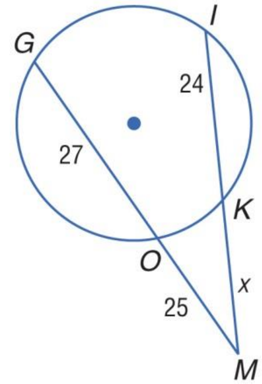
Example

Example 3:

a) Find *x*.



b) Find *x*.

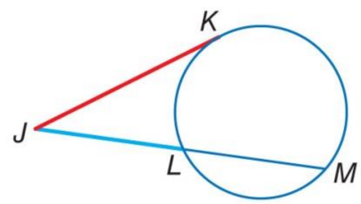


A ________________________ is a segment of a tangent with one endpoint on the circle that is both the exterior and whole segment.

Theorem 10.17

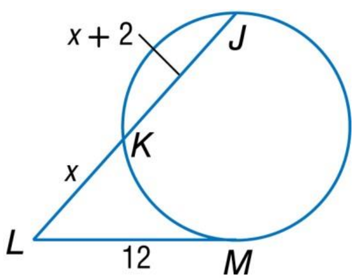
Words If a tangent and a secant intersect in the exterior of a circle, then the square of the measure of the tangent is equal to the product of the measures of the secant and its external secant segment.

Example

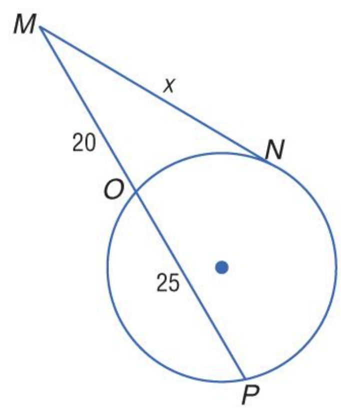


Example 4:

a) $\overline{LM}$ is tangent to the circle. Find *x*. Round to the nearest tenth.



b) Find *x*. Assume that segments that appear to be tangent are tangent.



New Vocabulary

Match each term with its definition by drawing a line to connect the two.

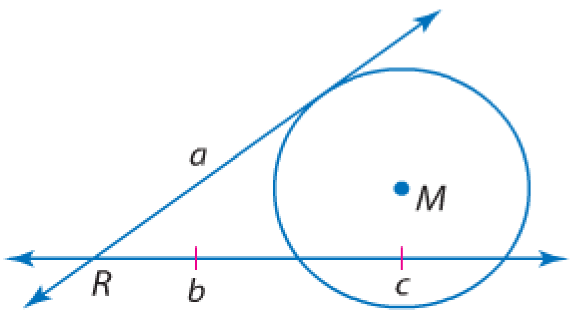
tangent segment	a segment formed when two chords intersect inside a circle
secant segment	a segment of a secant line that has exactly one endpoint on the circle
external secant segment	a segment of a tangent line that has exactly one endpoint on the circle
chord segment	a segment of a secant line that has an endpoint which lies in the exterior of the circle

Segments Intersecting Outside a Circle – Compare and contrast Theorem 10.16 with Theorem 10.15

How are they the same?	How are they different?

Challenge Problem

In the figure, a line tangent to circle M and a secant line intersect at R. Find a. Show the steps that you used.

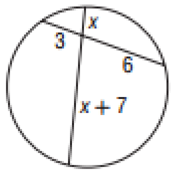


Learning Target Checklist

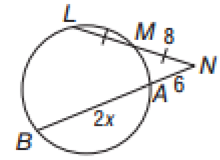
- ☐ Students will be able to find measures of segments that intersect in the interior of a circle.
- ☐ Students will find measures of segments that intersect in the exterior of a circle.

Warm Up 10.8

1. Find the value of x.



2. Find the value of x.



Common Core State Standards
G.GPE.1 Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of circle given by an equation.
G. GPE. 6 Find the point on a directed line segment between the given points that partitions the segment in a given ratio.

Mathematical Practices
2 Reason abstractly and quantitatively
7 Look for and make sure of structure

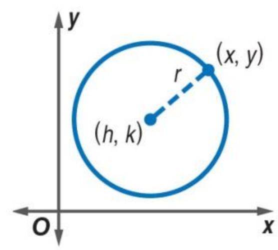
Learning Targets
- Students will be able to write the equation of a circle.
- Students will be able to graph a circle on the coordinate plane.

Section 10.8 Notes: Equations of Circles

KeyConcept Equation of a Circle in Standard Form

The standard form of the equation of a circle with center at (h, k) and radius r is ______________

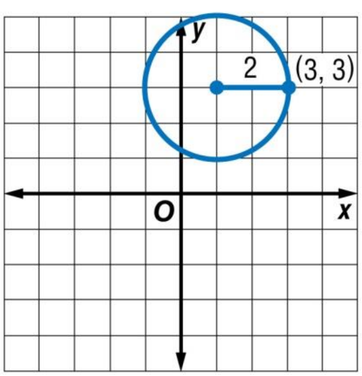
The standard form of the equation of a circle is also called the *center-radius* form.



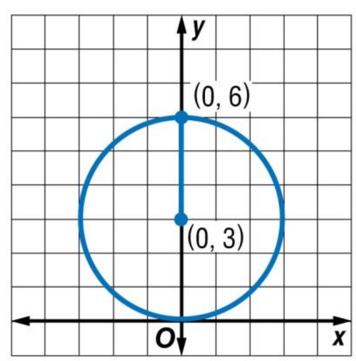
Example 1:

a) Write the equation of the circle with a center at (3, –3) and a radius of 6.

b) Write the equation of the circle graphed below.



c) Write the equation of the circle graphed below.



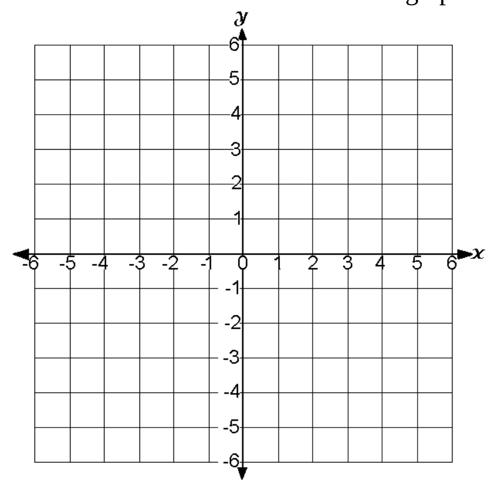
Example 2:

a) Write the equation of the circle that has its center at (–3, –2) and passes through (1, –2).

b) Write the equation of the circle that has its center at (–1, 0) and passes through (3, 0).

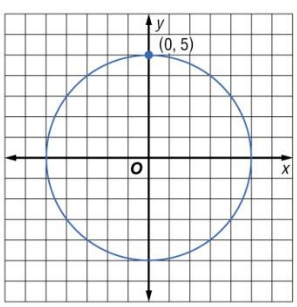
Example 3:

a) The equation of a circle is $x^2 - 4x + y^2 + 6y = -9$. State the coordinates of the center and the measure of the radius. Then graph the equation.

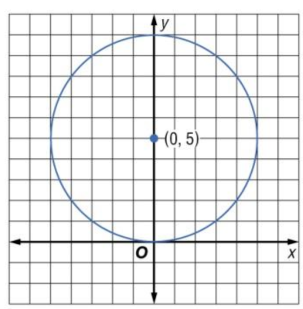


b) Which of the following is the graph of $x^2 + y^2 - 10y = 0$?

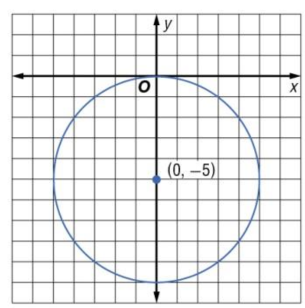
A.



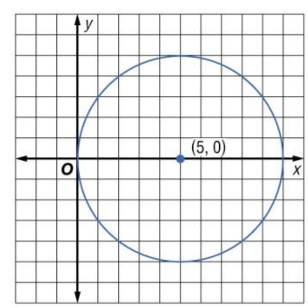
B.



C.



D.



Example 4:

a) ELECTRICITY Strategically located substations are extremely important in the transmission and distribution of a power company's electric supply. Suppose three substations are modeled by the points D(3, 6), E(–1, 1), and F(3, –4). Determine the location of a town equidistant from all three substations, and write an equation for the circle.

b) AMUSEMENT PARKS The designer of an amusement park wants to place a food court equidistant from the roller coaster located at (4, 1), the Ferris wheel located at (0, 1), and the boat ride located at (4, –3). Determine the location for the food court.

Example 5:

a) Find the point(s) of intersection between $x^2 + y^2 = 32$ and $y = x + 8$.

b) Find the points of intersection between $x^2 + y^2 = 16$ and $y = -x$

Graph the circle given by the equation $(x + 4)^2 + (y - 2)^2 = 16$.

Rewrite the equation in standard form.

____

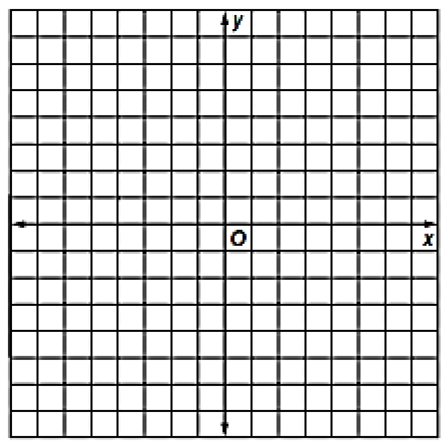
Identify the center. ________

Identify the radius. ________

Use the center and radius to identify four points on the circle.

____

____



Challenge Problem
A circle has the equation $(x - 5)^2 + (y + 7)^2 = 16$. If the center of the circle is shifted 3 units right and 9 units up, what will be the equation of the new circle? Explain your reasoning.

Learning Target Checklist

- ☐ Students will be able to write the equation of a circle.
- ☐ Students will be able to graph a circle on the coordinate plane.