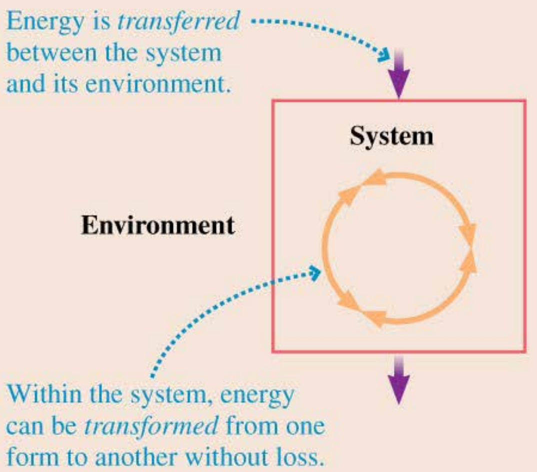
Chapter 10 Energy

Chapter Goal: To introduce the concept of energy and the basic energy model.

Chapter 10 Preview

Basic Energy Model

Energy is one of the most important concepts in physics. Chapters 10 and 11 will develop the **basic energy model**, a powerful set of ideas for solving problems in mechanics.



This chapter focuses on energy transformations within the system as one kind of energy is converted to another. Chapter 11 will explore energy transfers to and from the system. For mechanical systems, that transfer is called *work*. Part IV will expand our understanding of energy even further by incorporating the concepts of *heat* and thermodynamics.

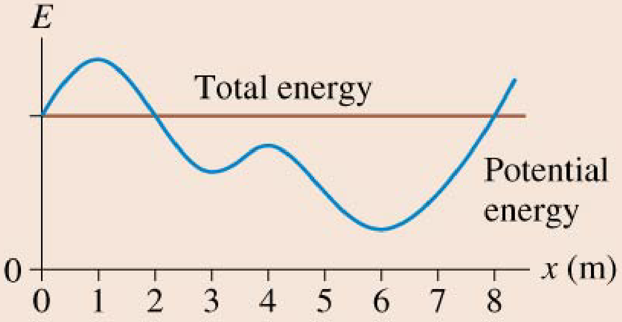
Forms of Energy

- **Kinetic energy** is energy associated with an object's motion.
- **Potential energy** is stored energy. Potential energy is associated with an object's position.
- **Thermal energy** is the energy of the random motions of atoms within an object. Thermal energy is associated with temperature.

You will learn about *gravitational potential energy,* the *elastic potential energy* of a stretched or compressed spring, and how these potential energies can be transformed into kinetic energy.

Energy Diagrams

You'll learn how to interpret an **energy diagram**, a graphical representation for understanding how the speed of a particle changes as it moves through space.



As you'll see, maxima and minima are points of unstable and stable equilibrium, respectively.

Conservation of Mechanical Energy

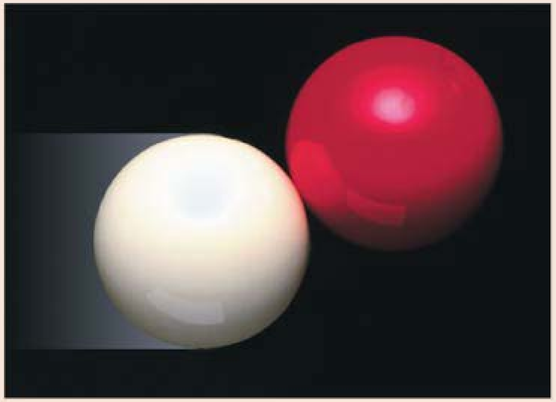
Mechanical energy, the sum of kinetic and potential energies, is *conserved* in a system that is both isolated and frictionless. As you learned with momentum, conservation means that

$$\text{final value} = \text{initial value}$$

This will be the basis for a new problem-solving strategy.

Elastic Collisions

A collision that conserves both momentum and mechanical energy is a **perfectly elastic collision.**



Collisions between two billiard balls or two steel balls come very close to being perfectly elastic.

Kinetic Energy *K*

- Kinetic energy is the energy of motion.
- All moving objects have kinetic energy.
- The more massive an object or the faster it moves, the larger its kinetic energy.

Potential Energy *U*

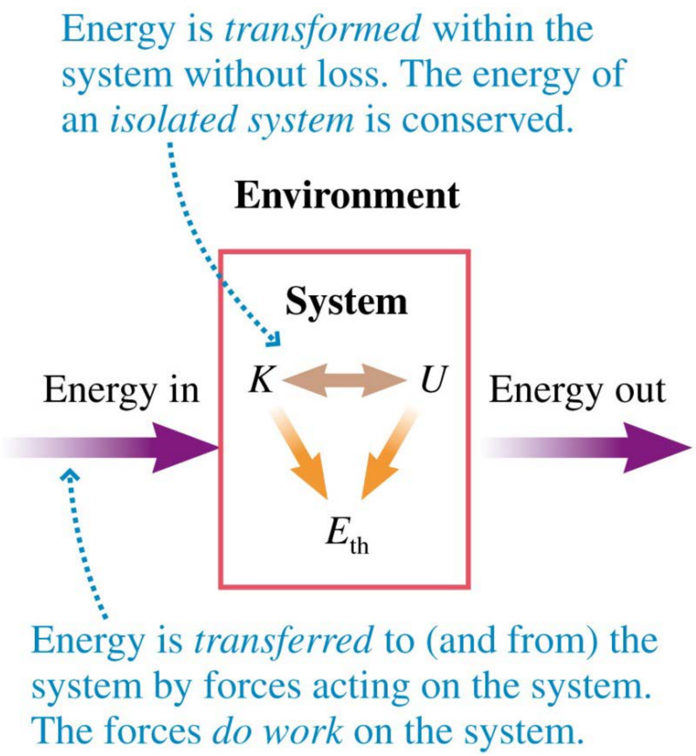
- Potential energy is stored energy associated with an object's position.
- The roller coaster's gravitational potential energy depends on its height above the ground.

Thermal Energy E_{th}

- Thermal energy is the sum of the microscopic kinetic and potential energies of all the atoms and bonds that make up the object.
- An object has more thermal energy when hot than when cold.

The Basic Energy Model

- Within a system, energy can be *transformed* from one type to another.
- The total energy of the system is not changed by these transformations.
- This is the *law of conservation of energy*.
- Energy can also be *transferred* from one system to another.



- The mechanical transfer of energy to a system via forces is called *work*.

Kinetic Energy and Gravitational Potential Energy

- The figure shows a before-and-after representation of an object in free fall.
- One of the kinematics equations from Chapter 2, with $a_y = -g$, is:

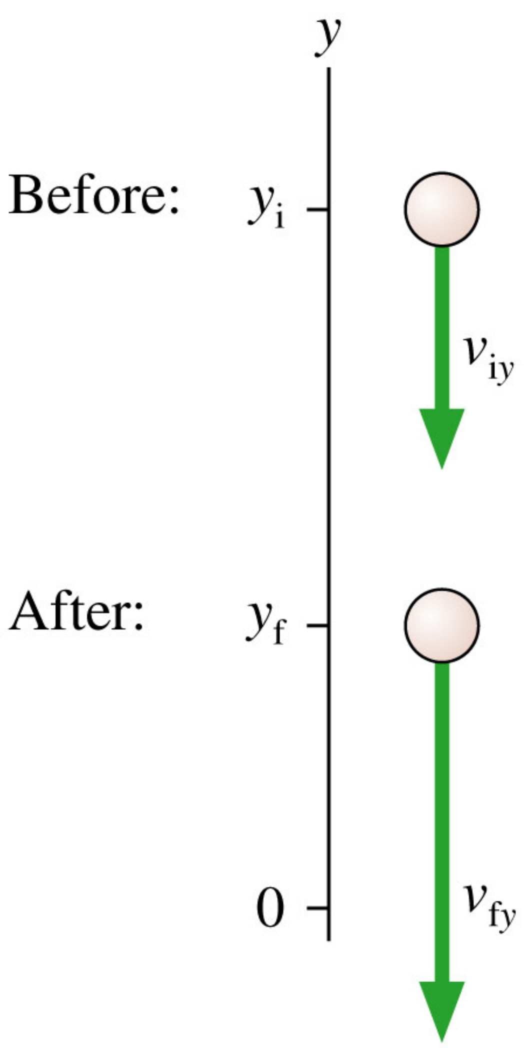
$$v_{fy}^2 = v_{iy}^2 + 2a_y\Delta y = v_{iy}^2 - 2g(y_f - y_i)$$

- Rearranging:

$$v_{fy}^2 + 2gy_f = v_{iy}^2 + 2gy_i$$

- Multiplying both sides by ½m:

$$\frac{1}{2}mv_f^2 + mgy_f = \frac{1}{2}mv_i^2 + mgy_i$$



Kinetic Energy and Gravitational Potential Energy

Define **kinetic energy** as an energy of motion:

$$K = \frac{1}{2}mv^2 \qquad \text{(kinetic energy)}$$

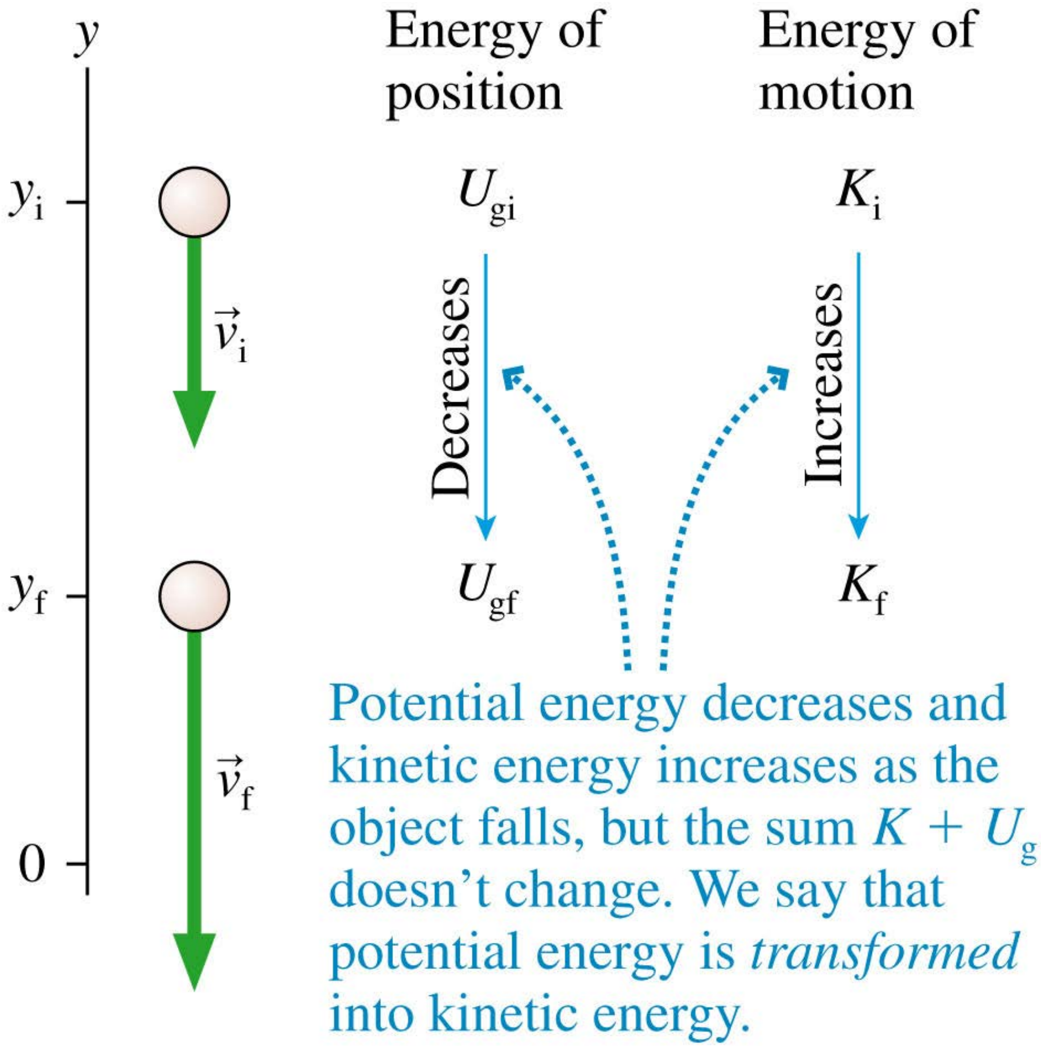
Define **gravitational potential energy** as an energy of position:

$$U_g = mgy \qquad \text{(gravitational potential energy)}$$

The sum $K + U_g$ is not changed when an object is in free fall. Its initial and final values are equal:

$$K_f + U_{gf} = K_i + U_{gi}$$

Kinetic Energy and Gravitational Potential Energy



Example 10.1 Launching a Pebble

EXAMPLE 10.1 Launching a pebble

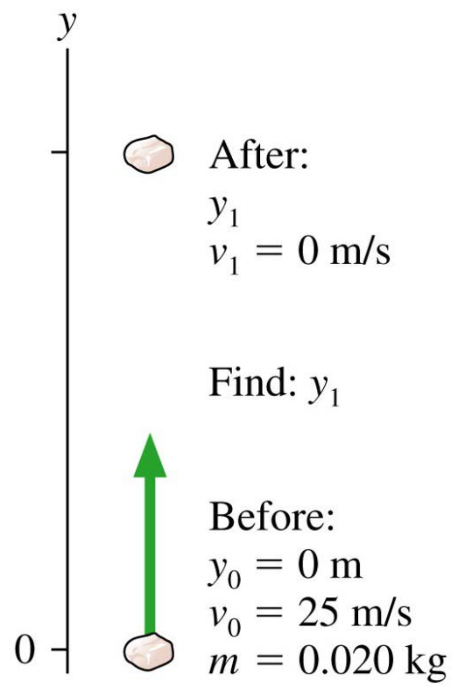
Bob uses a slingshot to shoot a 20 g pebble straight up with a speed of 25 m/s. How high does the pebble go?

MODEL This is free-fall motion, so the sum of the kinetic and gravitational potential energy does not change as the pebble rises.

Example 10.1 Launching a Pebble

EXAMPLE 10.1 **Launching a pebble**

VISUALIZE The figure below shows a before-and-after pictorial representation. The pictorial representation for energy problems is essentially the same as the pictorial representation you learned in Chapter 9 for momentum problems. We'll use numerical subscripts 0 and 1 for the initial and final points.



Example 10.1 Launching a Pebble

EXAMPLE 10.1 Launching a pebble

SOLVE Equation 10.13,

$$K_1 + U_{g1} = K_0 + U_{g0}$$

tells us that the sum $K + U_g$ is not changed by the motion. Using the definitions of K and U_g, we have

$$\frac{1}{2}mv_1^2 + mgy_1 = \frac{1}{2}mv_0^2 + mgy_0$$

Here $y_0 = 0$ m and $v_1 = 0$ m/s, so the equation simplifies to

$$mgy_1 = \frac{1}{2}mv_0^2$$

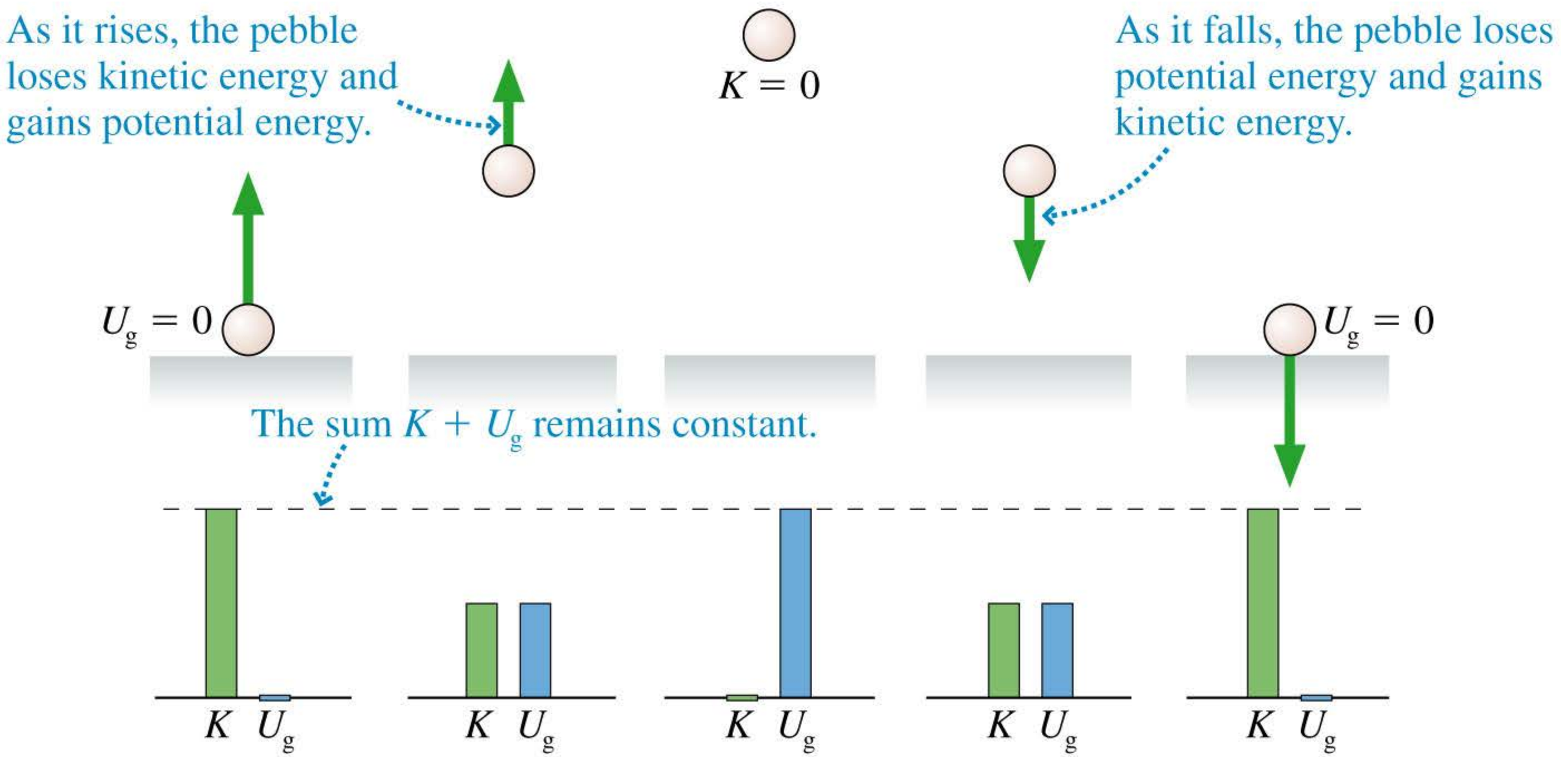
This is easily solved for the height y_1:

$$y_1 = \frac{v_0^2}{2g} = \frac{(25 \text{ m/s})^2}{2(9.80 \text{ m/s}^2)} = 32 \text{ m}$$

ASSESS Notice that the mass canceled and wasn't needed, a fact about free fall that you should remember from Chapter 2.

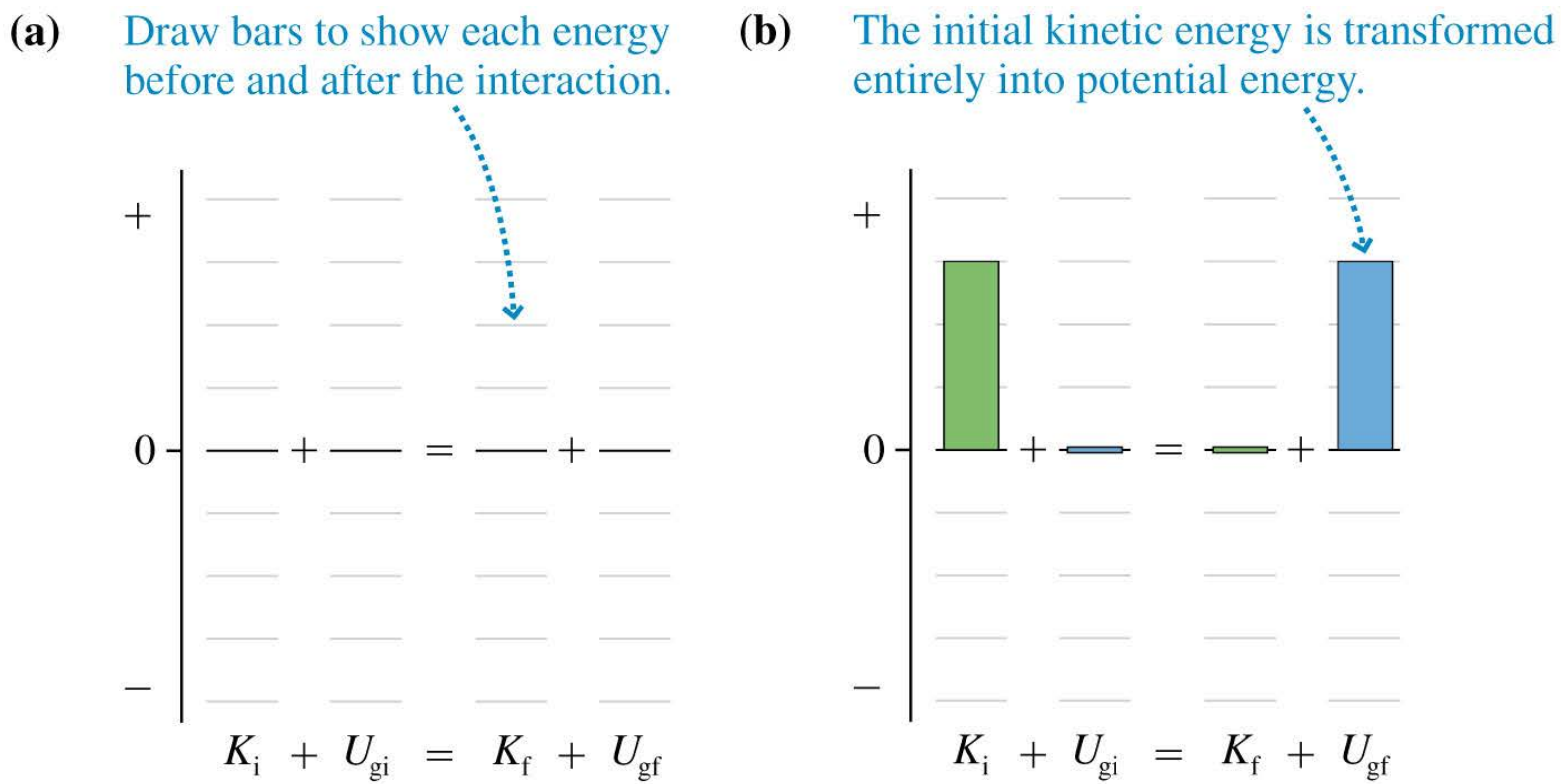
Energy Bar Charts

- A pebble is tossed up into the air.
- The simple bar charts below show how the sum of $K + U_g$ remains constant as the pebble rises and then falls.



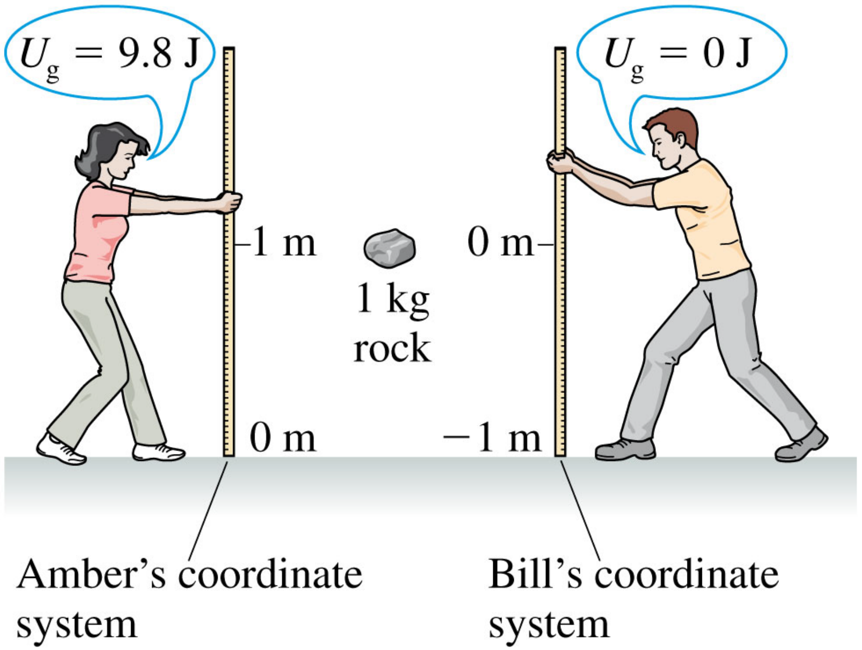
Energy Bar Charts

- The figure below shows how to make an energy bar chart suitable for problem solving.
- The chart is a graphical representation of the energy equation $K_f + U_{gf} = K_i + U_{gi}$.



The Zero of Potential Energy

- Amber and Bill use coordinate systems with different origins to determine the potential energy of a rock.
- No matter where the rock is, Amber's value of U_g will be equal to Bill's value plus 9.8 J.



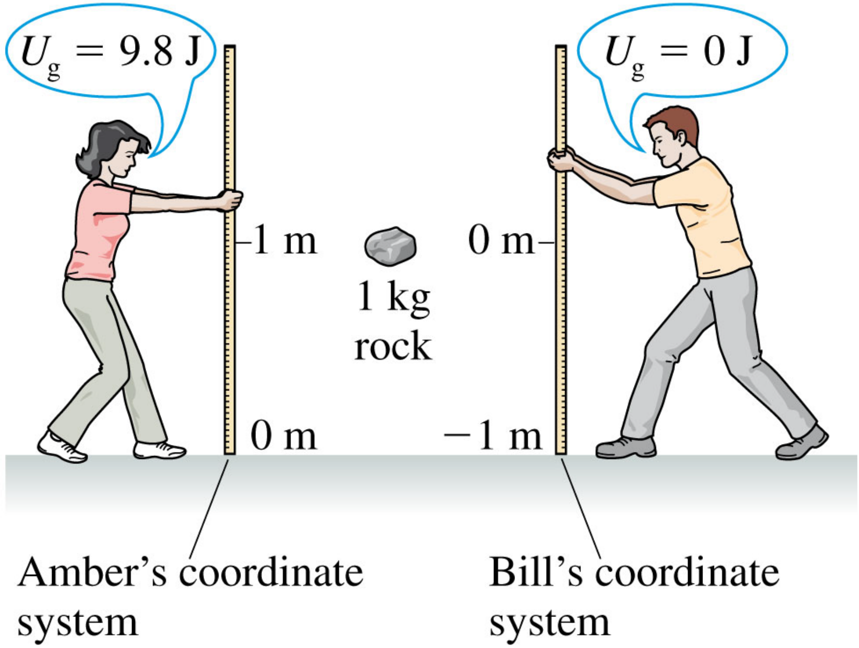
- If the rock moves, both will calculate exactly the *same* value for ΔU_g.
- In problems, only ΔU_g has physical significance, not the value of U_g itself.

Example 10.2 The Speed of a Falling Rock

EXAMPLE 10.2 **The speed of a falling rock**

The 1.0 kg rock shown below is released from rest. Use both Amber's and Bill's perspectives to calculate its speed just before it hits the ground.

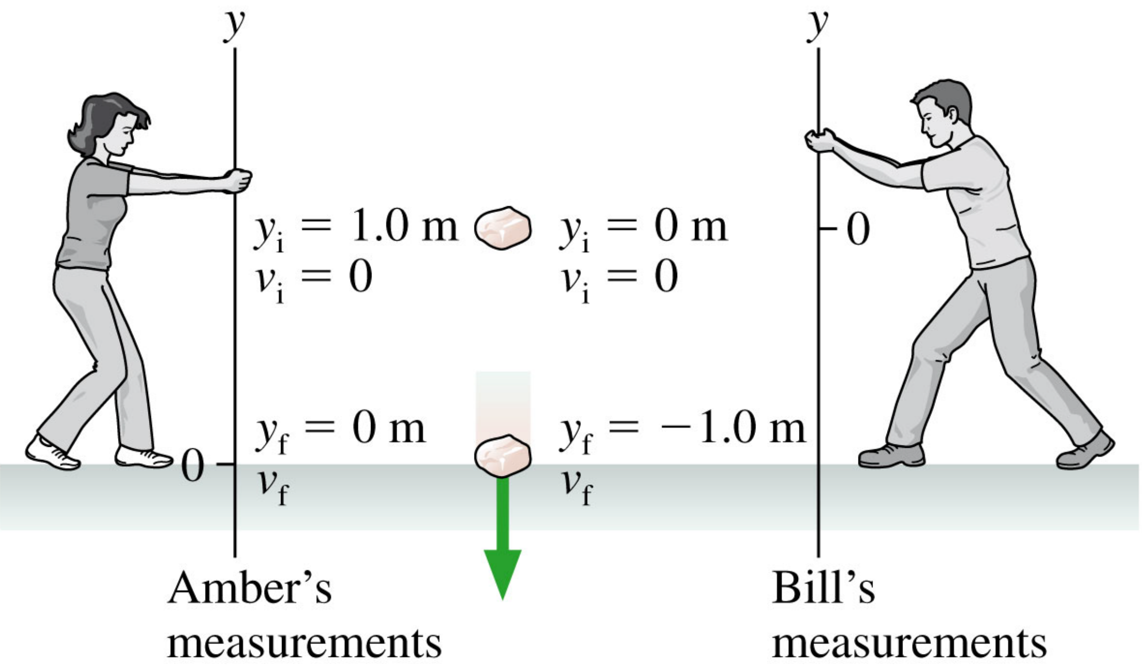
MODEL This is free-fall motion, so the sum of the kinetic and gravitational potential energy does not change as the rock falls.



Example 10.2 The Speed of a Falling Rock

EXAMPLE 10.2 The speed of a falling rock

VISUALIZE The figure below shows a before-and-after pictorial representation using both Amber's and Bill's coordinate systems.



Example 10.2 The Speed of a Falling Rock

EXAMPLE 10.2 **The speed of a falling rock**

SOLVE The energy equation is $K_f + U_{gf} = K_i + U_{gi}$. Bill and Amber both agree that $K_i = 0$ because the rock was released from rest, so we have

$$K_f = \frac{1}{2}mv_f^2 = -(U_{gf} - U_{gi}) = -\Delta U$$

According to Amber, $U_{gi} = mgy_i = 9.8$ J and $U_{gf} = mgy_f = 0$ J. Thus

$$\Delta U_{Amber} = U_{gf} - U_{gi} = -9.8 \text{ J}$$

The rock *loses* potential energy as it falls. According to Bill, $U_{gi} = mgy_i = 0$ J and $U_{gf} = mgy_f = -9.8$ J. Thus

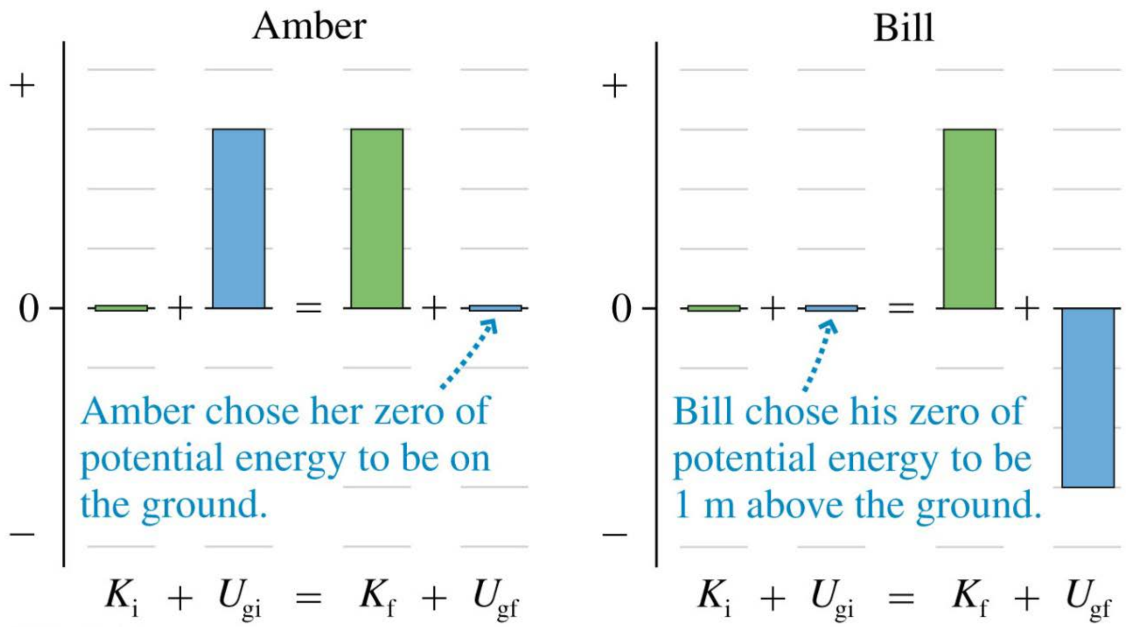
$$\Delta U_{Bill} = U_{gf} - U_{gi} = -9.8 \text{ J}$$

Bill has different values for U_{gi} and U_{gf} but the *same* value for ΔU. Thus they both agree that the rock hits the ground with speed

$$v_f = \sqrt{\frac{-2\,\Delta U}{m}} = \sqrt{\frac{-2(-9.8 \text{ J})}{1.0 \text{ kg}}} = 4.4 \text{ m/s}$$

Example 10.2 The Speed of a Falling Rock

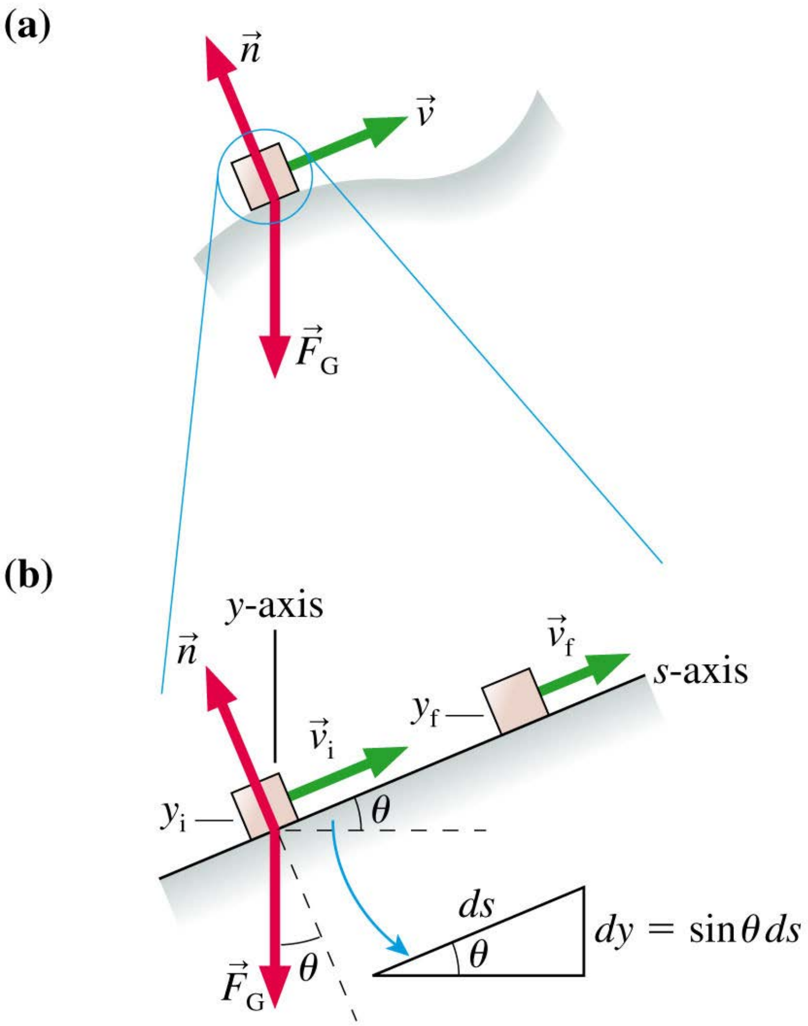
ASSESS The figure below shows energy bar charts for Amber and Bill. despite their disagreement over the value of U_g, Amber and Bill arrive at the same value for v_f and their K_f bars are the same height. **You can place the origin of your coordinate system, and thus the "zero of potential energy," wherever you choose and be assured of getting the correct answer to a problem.**



Gravitational Potential Energy on a Frictionless Surface – Slide 1 of 4

- Figure (a) shows an object of mass *m* sliding along a frictionless surface.
- Figure (b) shows a magnified segment of the surface that, over some small distance, is a straight line.
- Define an *s*-axis parallel to the direction of motion
- Newton's second law along the axis is:

$$(F_{\text{net}})_s = ma_s = m\frac{dv_s}{dt}$$



Gravitational Potential Energy on a Frictionless Surface – Slide 2 of 4

- Using the chain rule, we can write Newton's second law as:

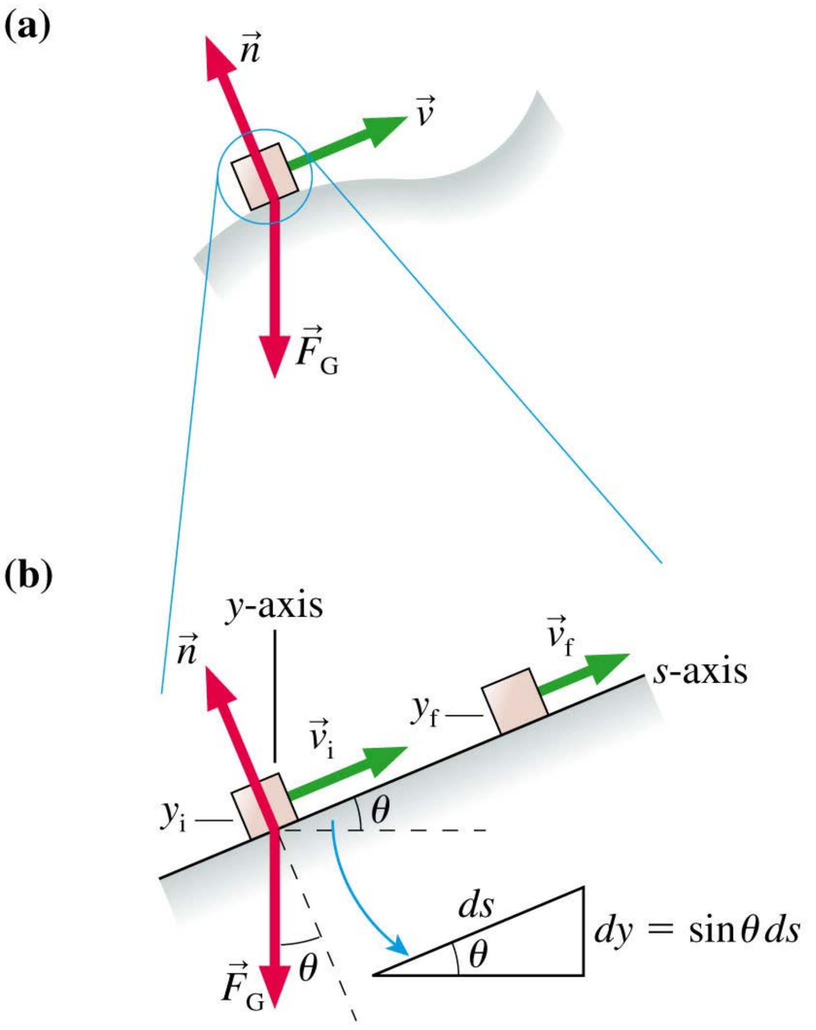
$$(F_{\text{net}})_s = m\frac{dv_s}{dt} = m\frac{dv_s}{ds}\frac{ds}{dt} = mv_s\frac{dv_s}{ds}$$

- It is clear from the diagram that the net force along s is:

$$(F_{\text{net}})_s = -F_G \sin\theta = -mg\sin\theta$$

- So Newton's second law is:

$$-mg\sin\theta = mv_s\frac{dv_s}{ds}$$



Gravitational Potential Energy on a Frictionless Surface – Slide 3 of 4

- Rearranging, we obtain:

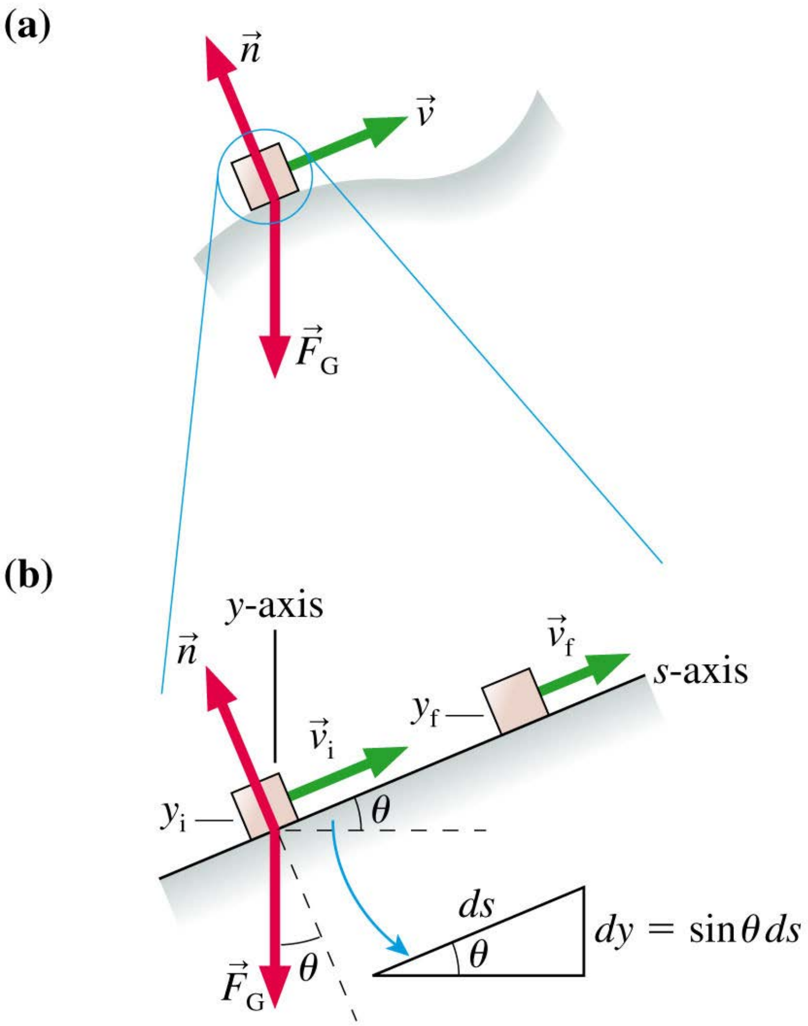
$$mv_s dv_s = -mg\sin\theta\, ds$$

- Note from the diagram that $\sin\theta\, ds = dy$, so:

$$mv_s\, dv_s = -mg\, dy$$

- Integrating this from "before" to "after":

$$\frac{1}{2}mv_f^2 + mgy_f = \frac{1}{2}mv_i^2 + mgy_i$$

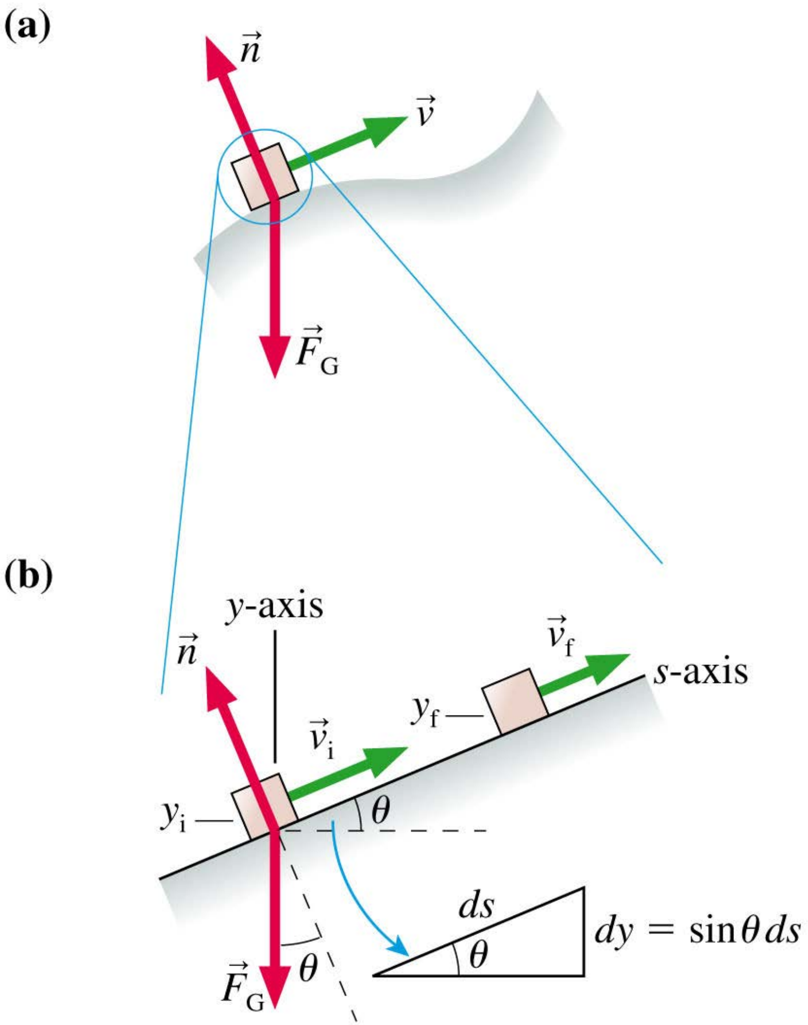


Gravitational Potential Energy on a Frictionless Surface – Slide 4 of 4

- With $K = \frac{1}{2} mv^2$ and $U_g = mgy$, we find that:

$$K_f + U_{gf} = K_i + U_{gi}$$

- The total mechanical energy for a particle moving along *any* frictionless smooth surface is conserved, regardless of the shape of the surface.



Example 10.3 The Speed of a Sled

EXAMPLE 10.3 The speed of a sled

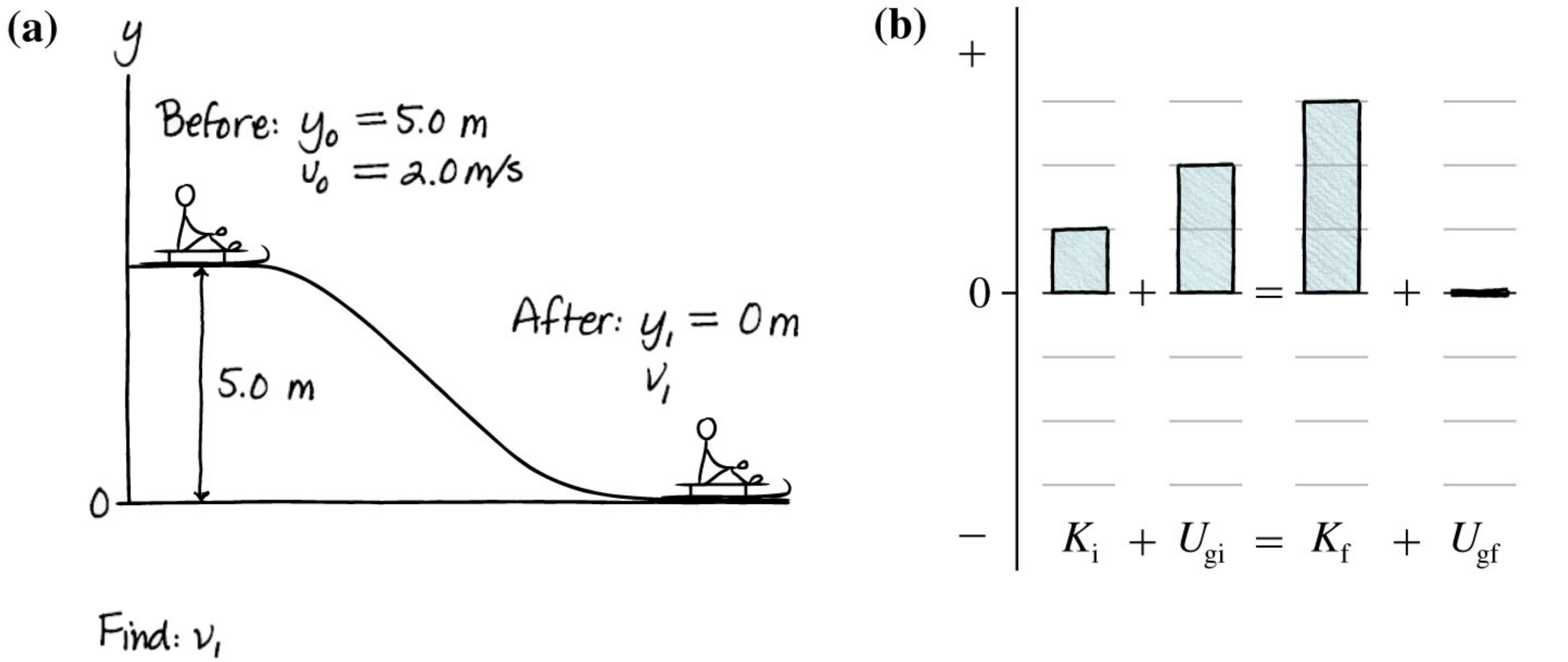
Christine runs forward with her sled at 2.0 m/s. She hops onto the sled at the top of a 5.0-m-high, very slippery slope. What is her speed at the bottom?

MODEL Model Christine and the sled as a particle. Assume the slope is frictionless. In that case, the sum of her kinetic and gravitational potential energy does not change as she slides down.

Example 10.3 The Speed of a Sled

EXAMPLE 10.3 **The speed of a sled**

VISUALIZE Figure (a) shows a before-and-after pictorial representation. We are not told the angle of the slope, or even if it is a straight slope, but the *change* in potential energy depends only on the height Christine descends and *not* on the shape of the hill. Figure (b) is an energy bar chart in which we see an initial kinetic *and* potential energy being transformed into entirely kinetic energy as she goes down the slope.



Example 10.3 The Speed of a Sled

EXAMPLE 10.3 The speed of a sled

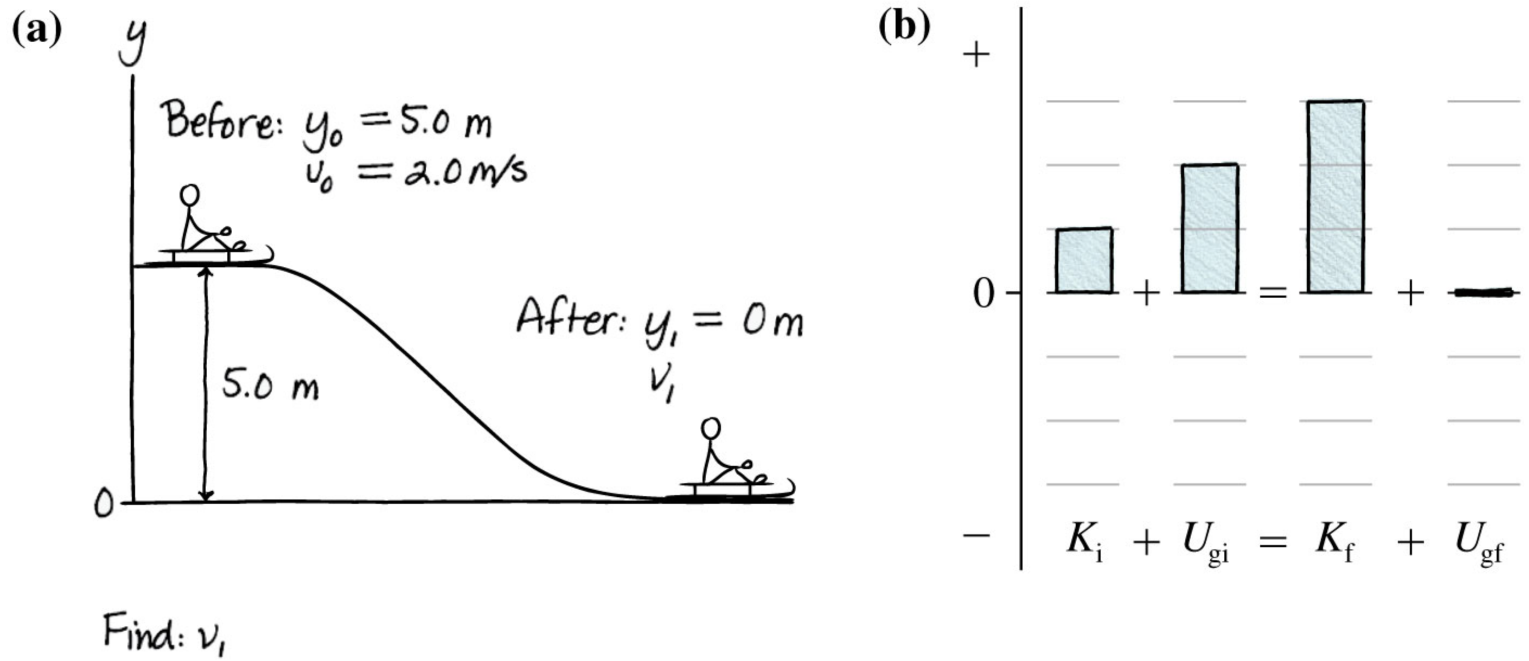
SOLVE The quantity $K + U_g$ is the same at the bottom of the hill as it was at the top. Thus

$$\frac{1}{2}mv_1^2 + mgy_1 = \frac{1}{2}mv_0^2 + mgy_0$$

This is easily solved for Christine's speed at the bottom:

$$v_1 = \sqrt{v_0^2 + 2g(y_0 - y_1)} = \sqrt{v_0^2 + 2gh} = 10 \text{ m/s}$$

ASSESS We did not need the mass of either Christine or the sled.



Problem-Solving Strategy: Conservation of Mechanical Energy

PROBLEM-SOLVING STRATEGY 10.1 **Conservation of mechanical energy**

MP

MODEL Choose a system that is isolated and has no friction or other losses of mechanical energy.

VISUALIZE Draw a before-and-after pictorial representation. Define symbols, list known values, and identify what you're trying to find.

SOLVE The mathematical representation is based on the law of conservation of mechanical energy:

$$K_f + U_f = K_i + U_i$$

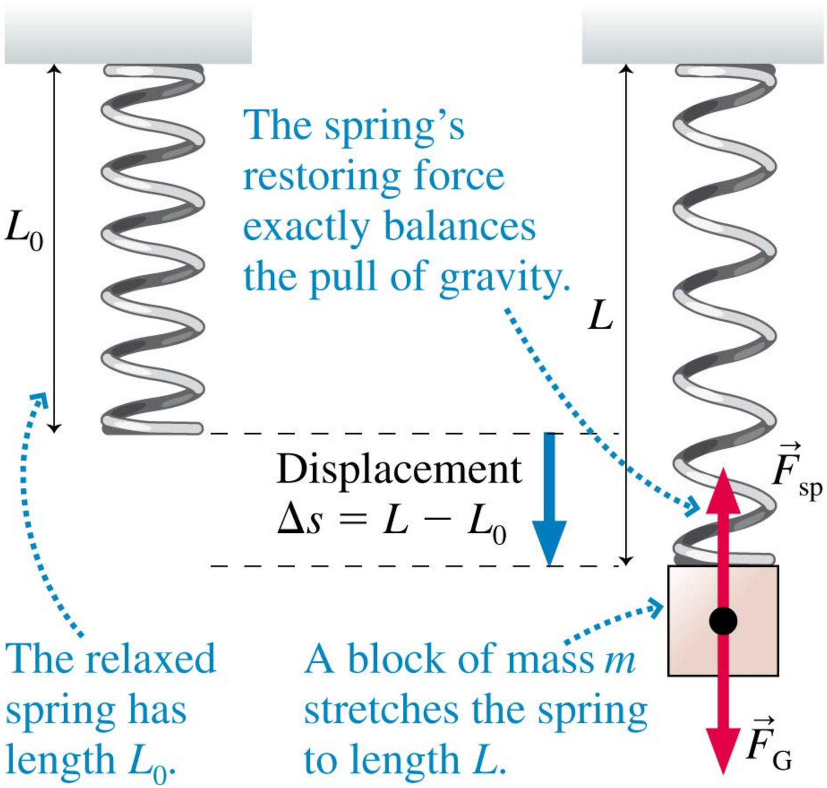
ASSESS Check that your result has the correct units, is reasonable, and answers the question.

Exercise 8

Restoring Forces and Hooke's Law

- The figure shows how a hanging mass stretches a spring of equilibrium length L_0 to a new length L.
- The mass hangs in static equilibrium, so the upward spring force balances the downward gravity force.

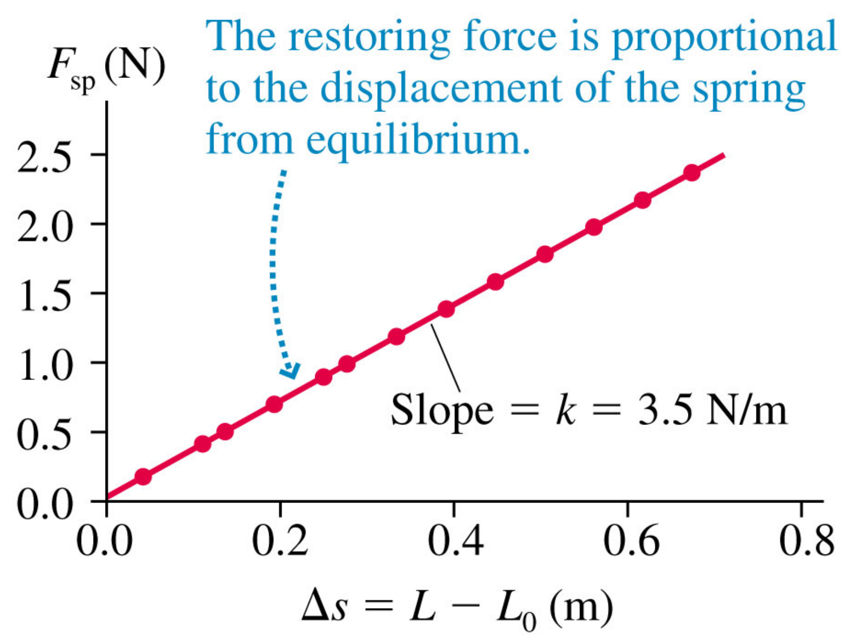
$$F_{sp} = F_G = mg$$



Restoring Forces and Hooke's Law

- The figure shows measured data for the restoring force of a real spring.
- Δs is the **displacement from equilibrium**.
- The data fall along the straight line:

$$F_{sp} = k\,\Delta s$$



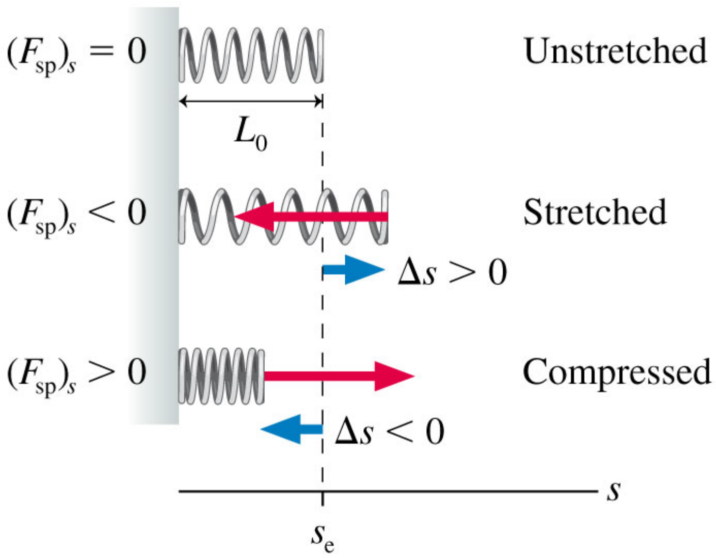
- The proportionality constant k is called the **spring constant.**
- The units of k are N/m.

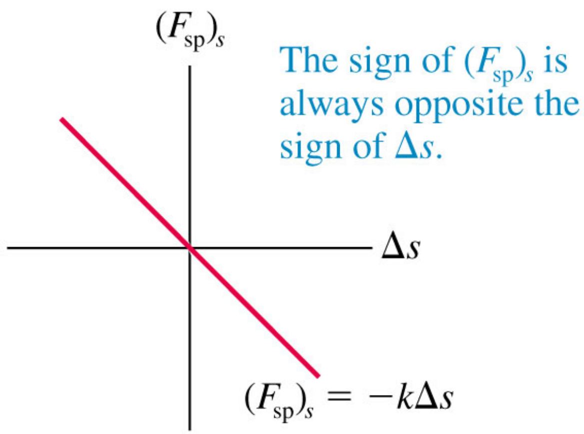
Hooke's Law

- One end of a spring is attached to a fixed wall.
- $(F_{sp})_s$ is the force produced by the free end of the spring.
- $\Delta s = s - s_e$ is the displacement from equilibrium.

$$(F_{sp})_s = -k\,\Delta s \qquad \text{(Hooke's law)}$$

- The negative sign is the mathematical indication of a *restoring* force.

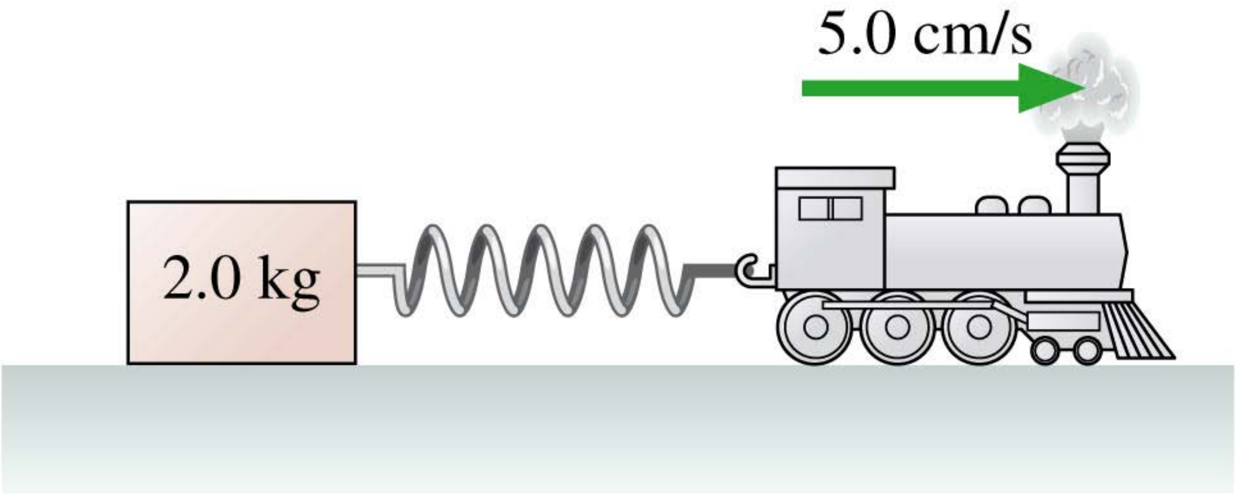




Example 10.5 Pull Until It Slips

EXAMPLE 10.5 Pull until it slips

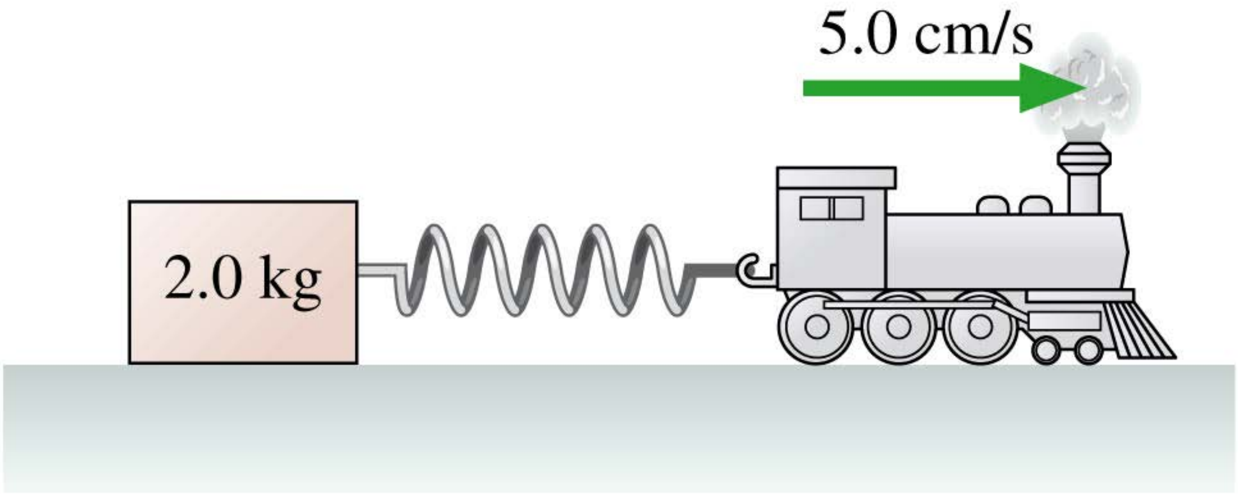
The figure shows a spring attached to a 2.0 kg block. The other end of the spring is pulled by a motorized toy train that moves forward at 5.0 cm/s. The spring constant is 50 N/m, and the coefficient of static friction between the block and the surface is 0.60. The spring is at its equilibrium length at $t = 0$ s when the train starts to move. When does the block slip?



Example 10.5 Pull Until It Slips

EXAMPLE 10.5 Pull until it slips

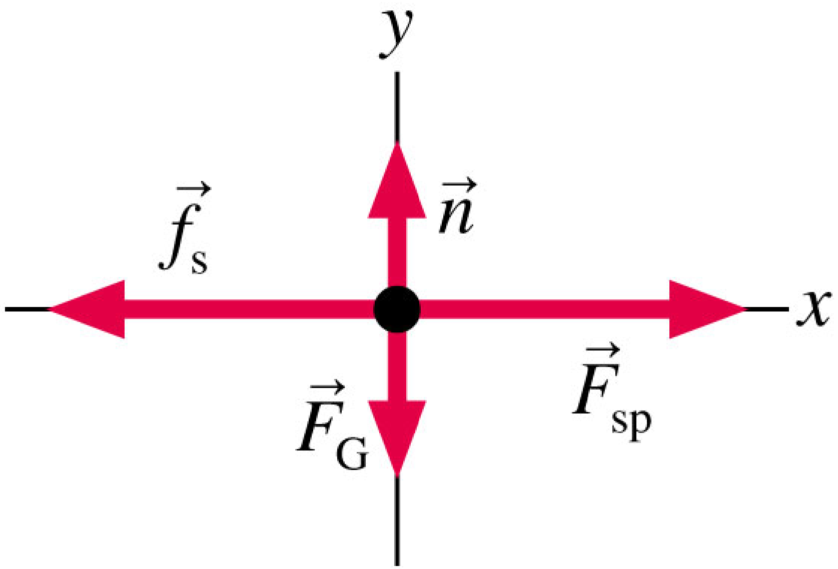
MODEL Model the block as a particle and the spring as an ideal spring obeying Hooke's law.



Example 10.5 Pull Until It Slips

EXAMPLE 10.5 **Pull until it slips**

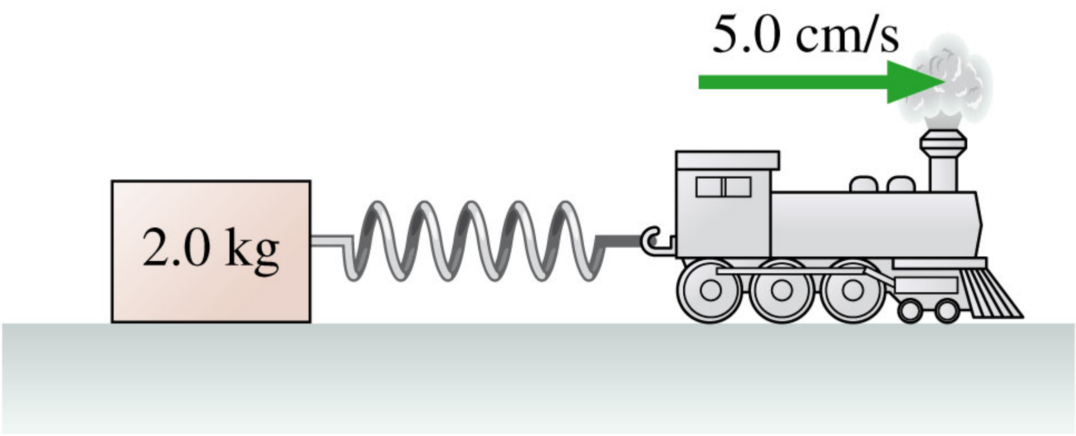
VISUALIZE The figure is a free-body diagram for the block.

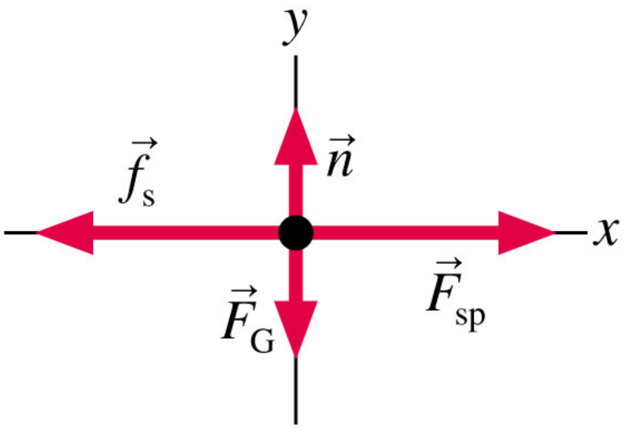


Example 10.5 Pull Until It Slips

EXAMPLE 10.5 Pull until it slips

SOLVE Recall that the tension in a massless string pulls equally at *both* ends of the string. The same is true for the spring force: It pulls (or pushes) equally at *both* ends. This is the key to solving the problem. As the right end of the spring moves, stretching the spring, the spring pulls backward on the train *and* forward on the block with equal strength. As the spring stretches, the static friction force on the block increases in magnitude to keep the block at rest.





Example 10.5 Pull Until It Slips

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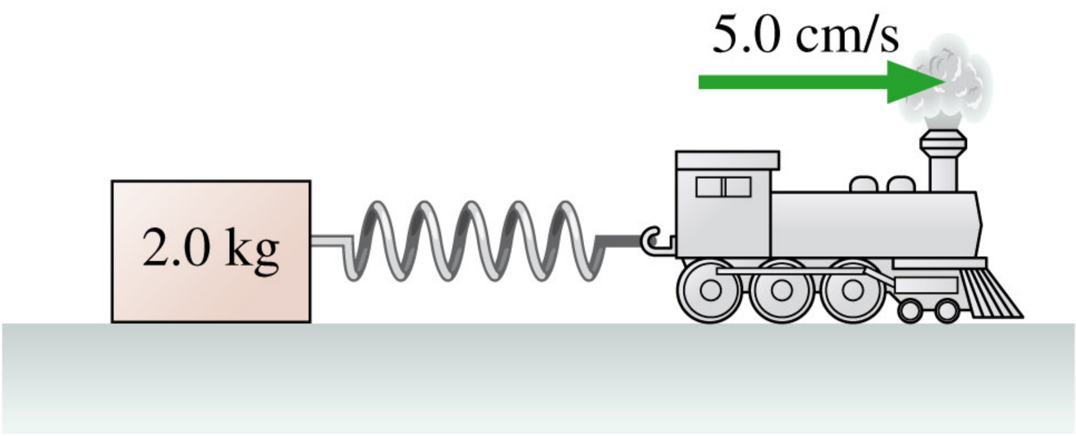
The block is in static equilibrium, so

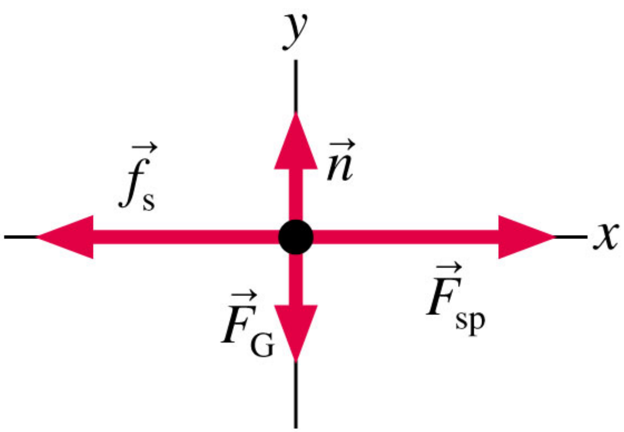
$$\sum (F_{net})_x = (F_{sp})_x + (f_s)_x = F_{sp} - f_s = 0$$

where F_{sp} is the *magnitude* of the spring force. The magnitude is $F_{sp} = k\,\Delta x$, where $\Delta x = v_x t$ is the distance the train has moved. Thus

$$f_s = F_{sp} = k\,\Delta x$$

The block slips when the static friction force reaches its maximum value $f_{s\,max} = \mu_s n = \mu_s mg$.





Example 10.5 Pull Until It Slips

EXAMPLE 10.5 **Pull until it slips**

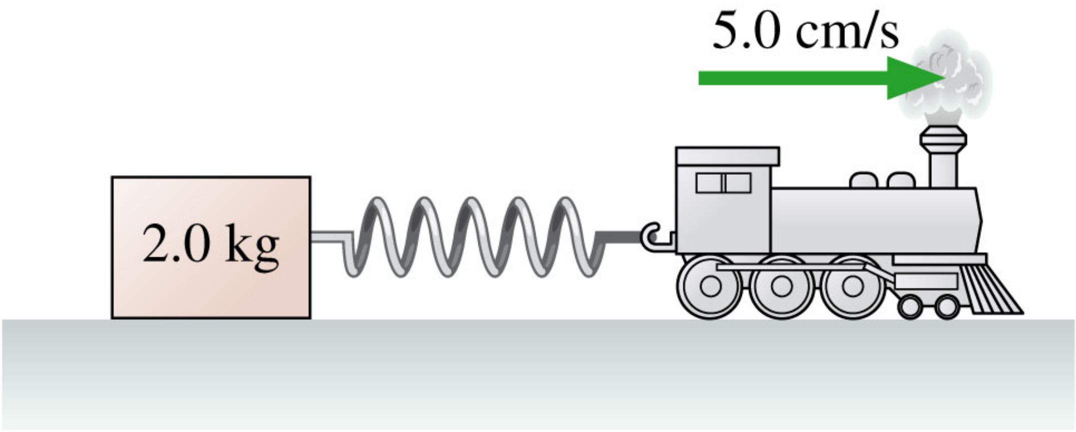
This occurs when the train has moved

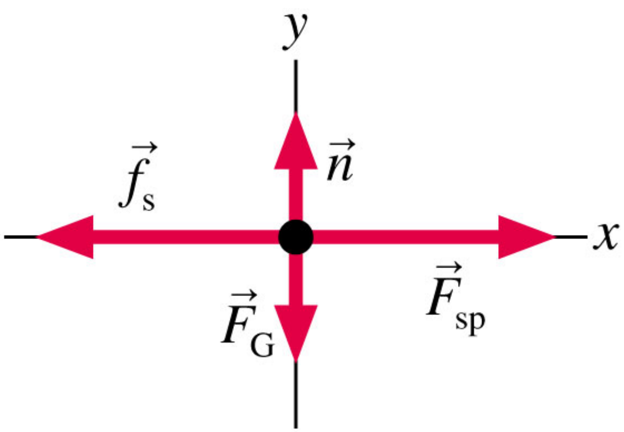
$$\Delta x = \frac{f_{s\,max}}{k} = \frac{\mu_s mg}{k} = \frac{(0.60)(2.0\text{ kg})(9.80\text{ m/s}^2)}{50\text{ N/m}}$$

$$= 0.235\text{ m} = 23.5\text{ cm}$$

The time at which the block slips is

$$t = \frac{\Delta x}{v_x} = \frac{23.5\text{ cm}}{5.0\text{ cm/s}} = 4.7\text{ s}$$





Stick-Slip Motion

- Earthquakes are an example of *stick-slip motion.*
- Tectonic plates are attempting to slide past each other, but friction causes the edges of the plates to stick together.
- Large masses of rock are somewhat elastic and can be "stretched".
- Eventually the elastic force of the deformed rocks exceeds the friction force between the plates.
- An earthquake occurs as the plates slip and lurch forward.

The slip can range from a few centimeters in a relatively small earthquake to several meters in a very large earthquake.

Elastic Potential Energy

- Springs and rubber bands store potential energy that can be transformed into kinetic energy.
- The spring force is *not* constant as an object is pushed or pulled.
- The motion of the mass is *not* constant-acceleration motion, and therefore we cannot use our old kinematics equations.
- One way to analyze motion when spring force is involved is to look at energy before and after some motion.

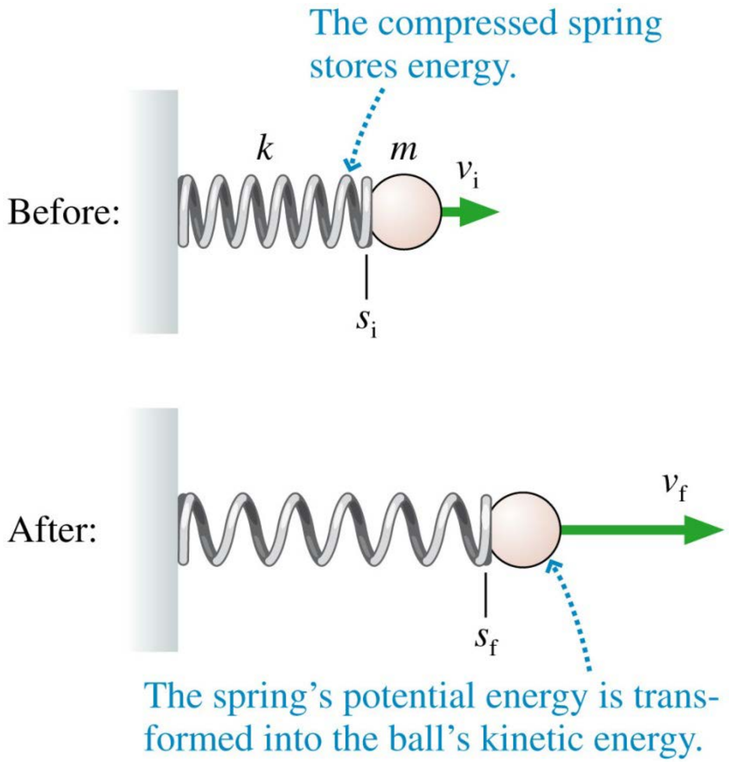
Elastic Potential Energy

- The figure shows a before-and-after situation in which a spring launches a ball.
- Integrating the net force from the spring, as given by Hooke's Law, shows that:

$$\frac{1}{2}mv_f^2 + \frac{1}{2}k(\Delta s_f)^2 = \frac{1}{2}mv_i^2 + \frac{1}{2}k(\Delta s_i)^2$$

- Here $K = ½ mv^2$ is the kinetic energy.
- We define a new quantity:

$$U_s = \frac{1}{2}k(\Delta s)^2 \qquad \text{(elastic potential energy)}$$



Elastic Potential Energy

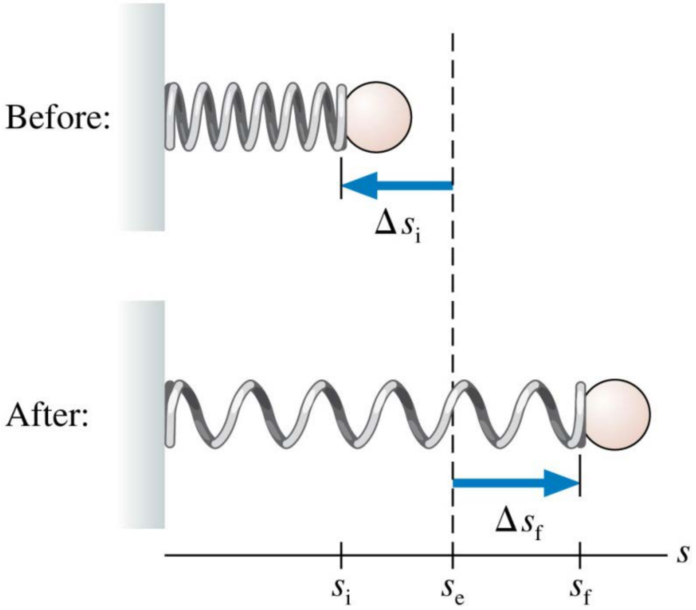
- An object moving without friction on an ideal spring obeys:

$$K_f + U_{sf} = K_i + U_{si}$$

where

$$U_s = \frac{1}{2}k(\Delta s)^2 \qquad \text{(elastic potential energy)}$$

- Because Δs is squared, U_s is positive for a spring that is either stretched or compressed.
- In the figure, U_s has a positive value both before and after the motion.



Example 10.6 A Spring-Launched Plastic Ball

EXAMPLE 10.6 A spring-launched plastic ball

A spring-loaded toy gun launches a 10 g plastic ball. The spring, with spring constant 10 N/m, is compressed by 10 cm as the ball is pushed into the barrel. When the trigger is pulled, the spring is released and shoots the ball back out. What is the ball's speed as it leaves the barrel? Assume friction is negligible.

Example 10.6 A Spring-Launched Plastic Ball

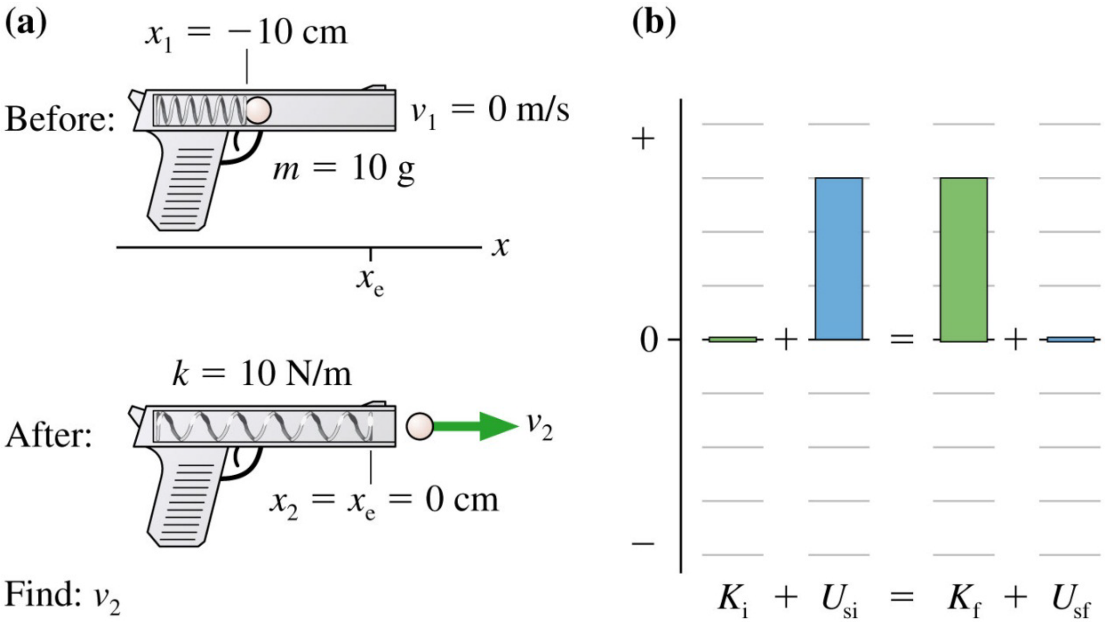
EXAMPLE 10.6 A spring-launched plastic ball

MODEL Assume an ideal spring that obeys Hooke's law. Also assume that the gun is held firmly enough to prevent recoil. There's no friction; hence the mechanical energy $K + U_s$ is conserved.

Example 10.6 A Spring-Launched Plastic Ball

EXAMPLE 10.6 A spring-launched plastic ball

VISUALIZE Figure (a) shows a before-and-after pictorial representation. We have chosen to put the origin of the coordinate system at the equilibrium position of the free end of the spring. The bar chart of figure (b) shows the potential energy stored in the compressed spring being entirely transformed into the kinetic energy of the ball.



Example 10.6 A Spring-Launched Plastic Ball

EXAMPLE 10.6 A spring-launched plastic ball

SOLVE The energy conservation equation is $K_2 + U_{s2} = K_1 + U_{s1}$. We can use the elastic potential energy of the spring, Equation 10.36, to write this as

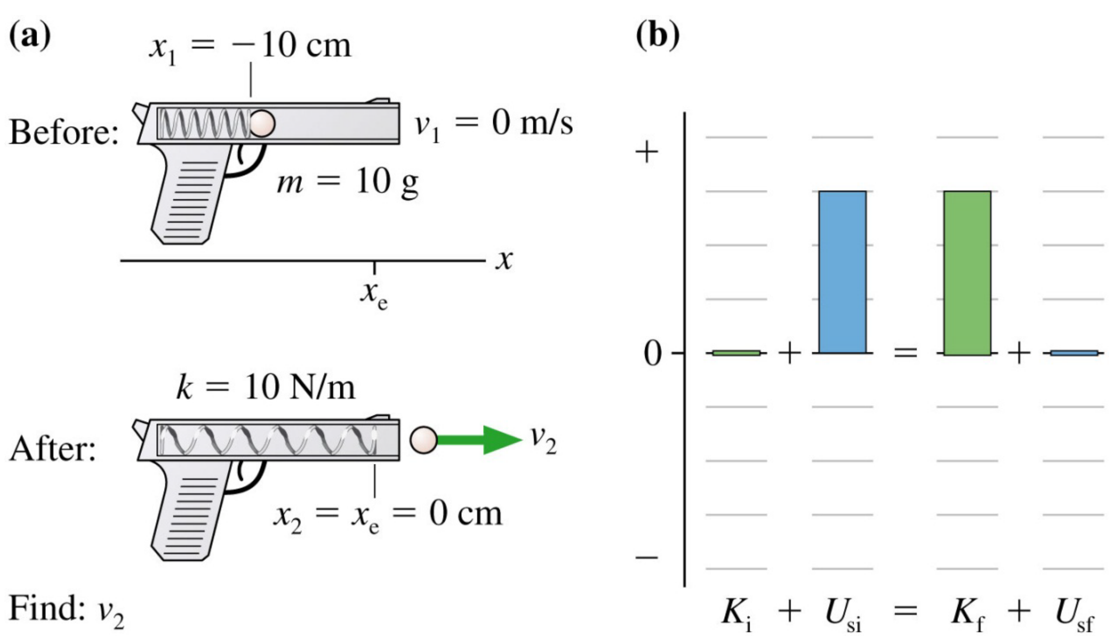
$$\frac{1}{2}mv_2^2 + \frac{1}{2}k(x_2 - x_e)^2 = \frac{1}{2}mv_1^2 + \frac{1}{2}k(x_1 - x_e)^2$$

Notice that we used x, rather than the generic s, and that we explicitly wrote out the meaning of Δx_1 and Δx_2. Using $x_2 = x_e = 0$ m and $v_1 = 0$ m/s simplifies this to

$$\frac{1}{2}mv_2^2 = \frac{1}{2}kx_1^2$$

It is now straightforward to solve for the ball's speed:

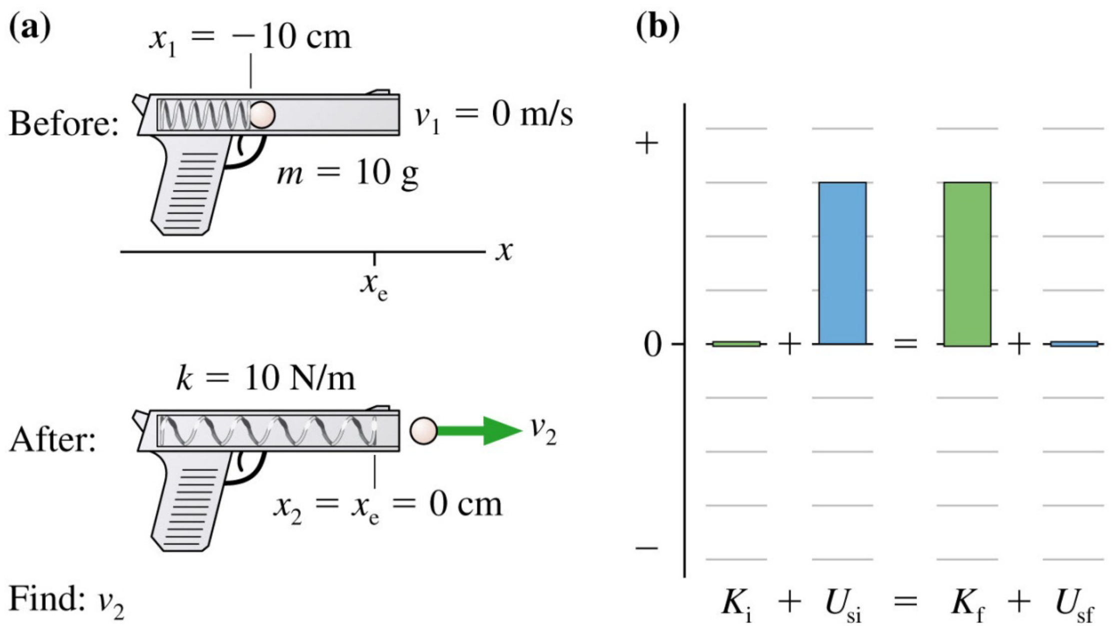
$$v_2 = \sqrt{\frac{kx_1^2}{m}} = \sqrt{\frac{(10\ \text{N/m})(-0.10\ \text{m})^2}{0.010\ \text{kg}}} = 3.2\ \text{m/s}$$



Example 10.6 A Spring-Launched Plastic Ball

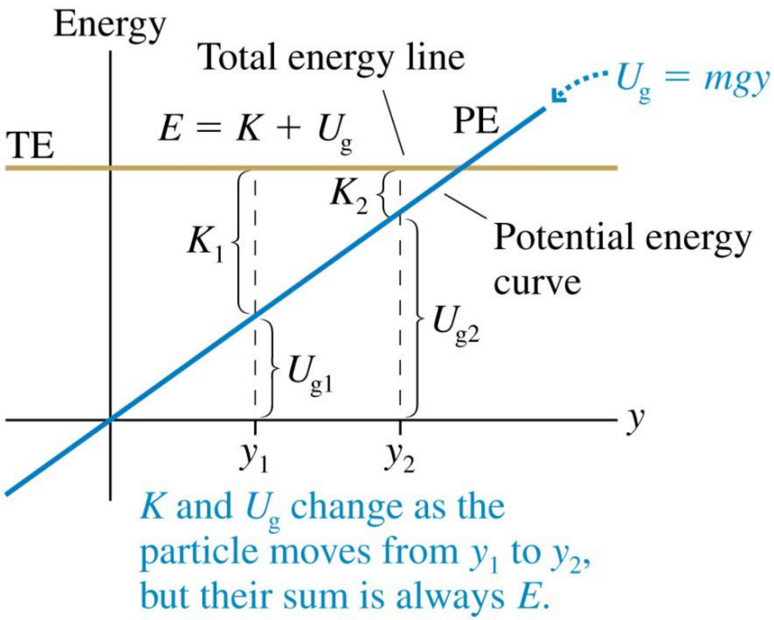
EXAMPLE 10.6 **A spring-launched plastic ball**

ASSESS This is a problem that we could *not* have solved with Newton's laws. The acceleration is not constant, and we have not learned how to handle the kinematics of nonconstant acceleration. But with conservation of energy—it's easy! The result, 3.2 m/s, seems reasonable for a toy gun.

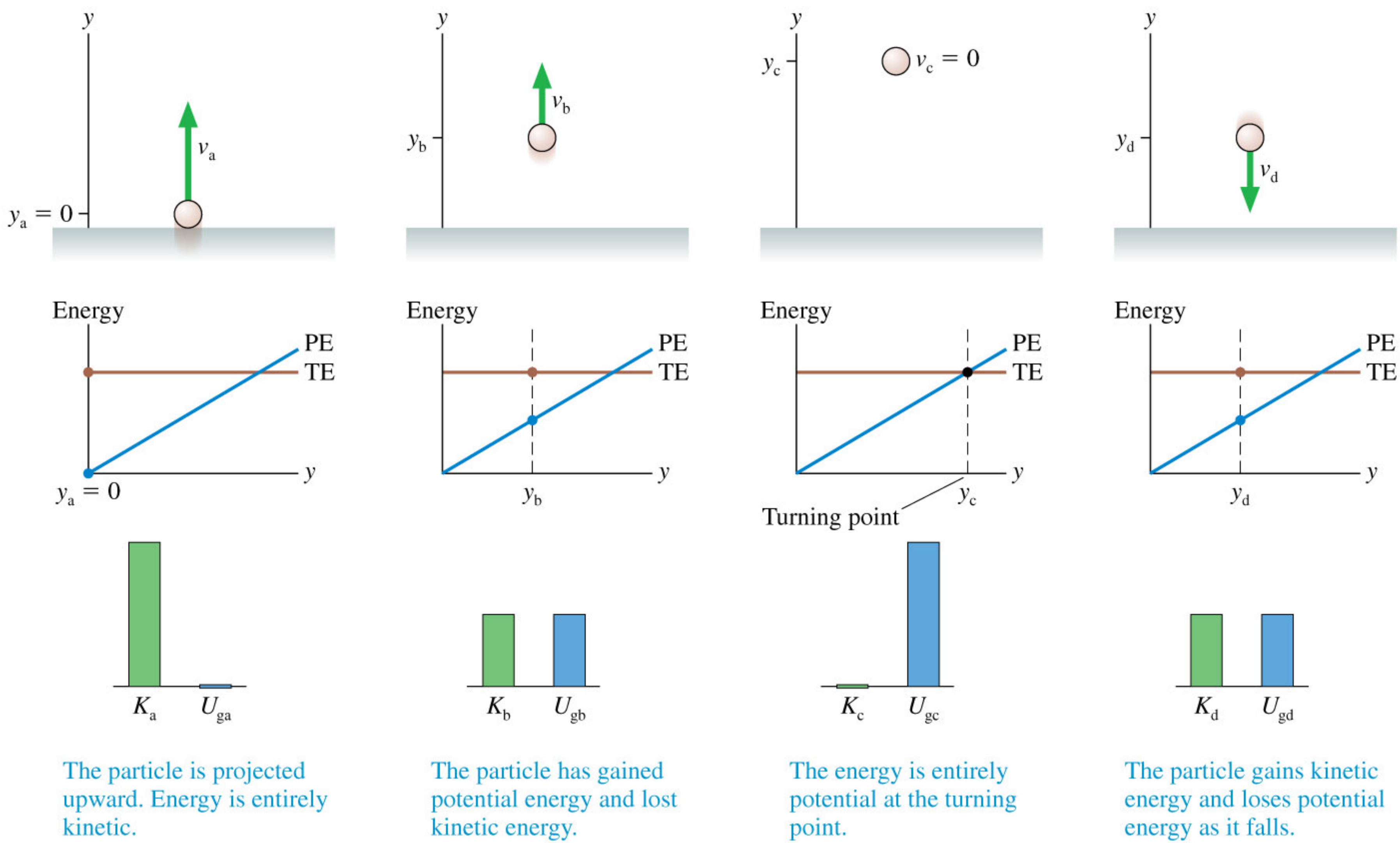


Energy Diagrams

- Potential energy is a function of position.
- Functions of position are easy to represent as graphs.
- A graph showing a system's potential energy and total energy as a function of position is called an **energy diagram**.
- Shown is the energy diagram of a particle in free fall.
- Gravitational potential energy is a straight line with slope *mg* and zero *y*-intercept.
- Total energy is a horizontal line, since mechanical energy is conserved.



A Four-Frame Movie of a Particle in Free Fall

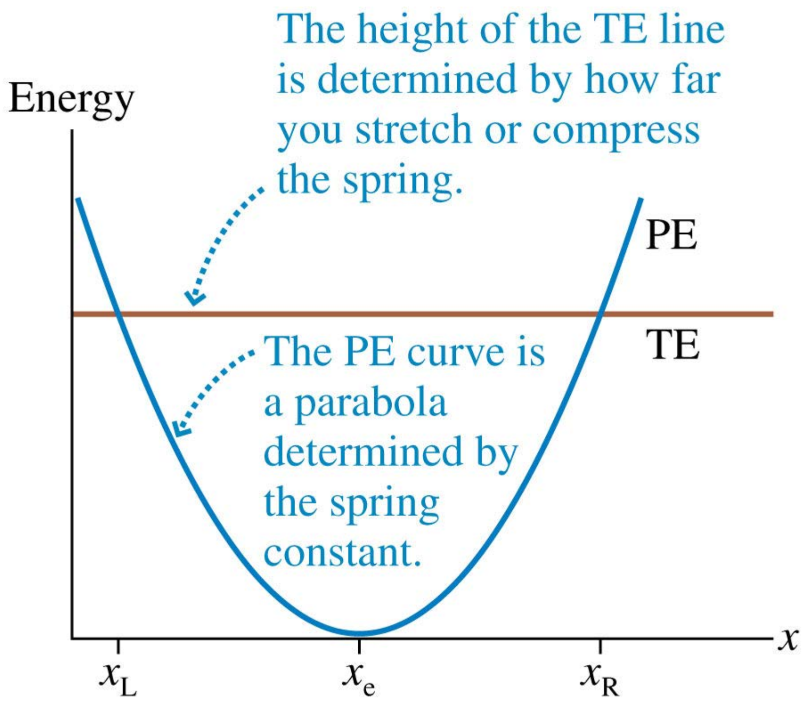


Energy Diagrams

- Shown is the energy diagram of a mass on a horizontal spring.
- The potential energy (PE) is the parabola:

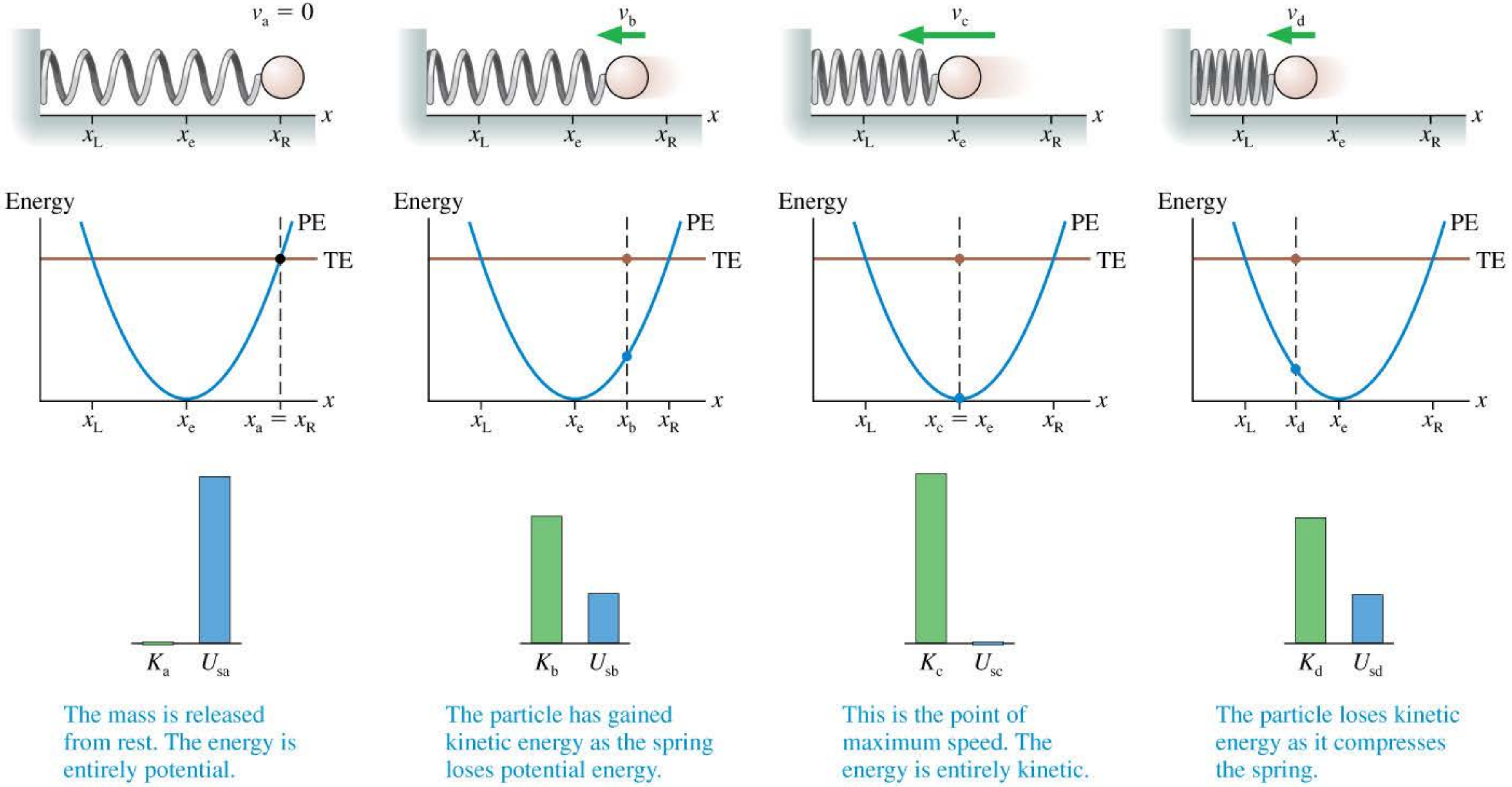
$$U_s = ½k(x - x_e)^2$$

- The PE curve is determined by the spring constant; you can't change it.



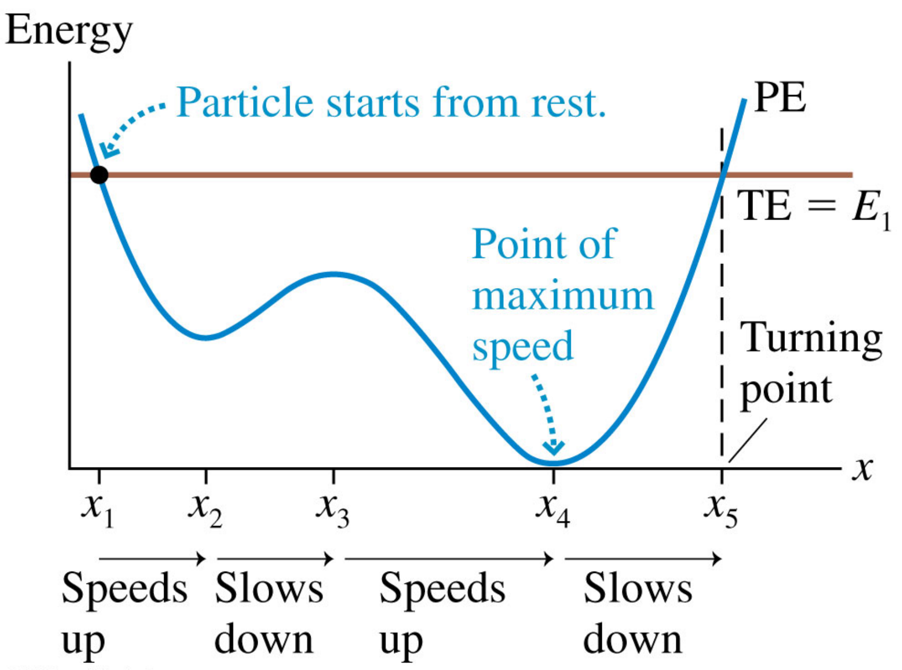
- You can set the total energy (TE) to any height you wish simply by stretching the spring to the proper length at the beginning of the motion.

A Four-Frame Movie of a Mass Oscillating on a Spring



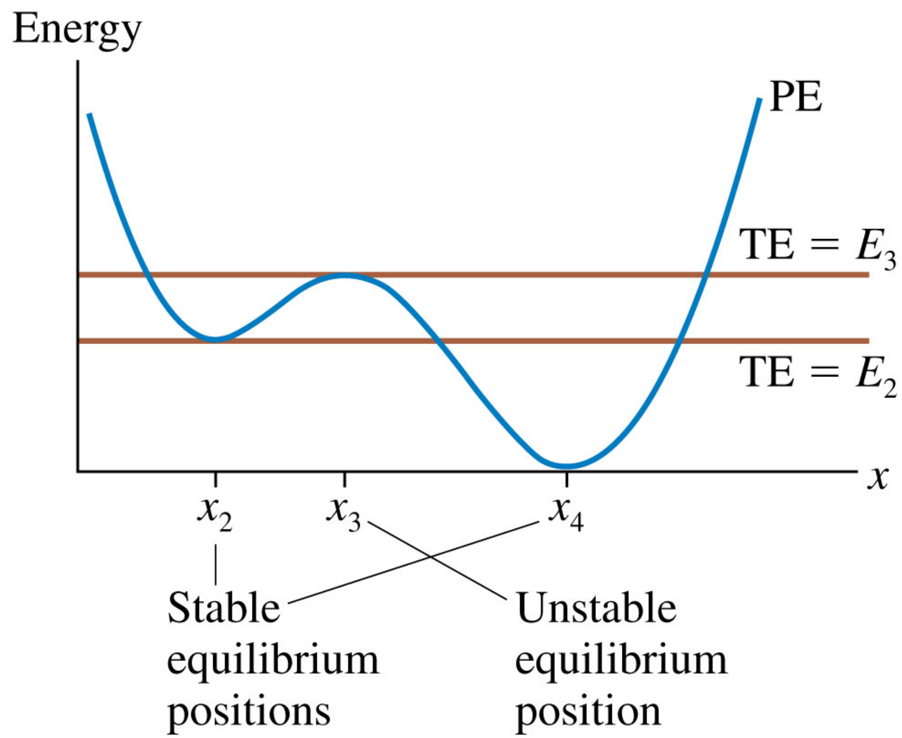
Energy Diagrams

- Shown is a more general energy diagram.
- The particle is released from rest at position x_1.
- Since K at x_1 is zero, the total energy $\mathrm{TE} = U$ at that point.
- The particle speeds up from x_1 to x_2.
- Then it slows down from x_2 to x_3.
- The particle reaches maximum speed as it passes x_4.
- When the particle reaches x_5, it turns around and reverses the motion.



Equilibrium Positions: Stable

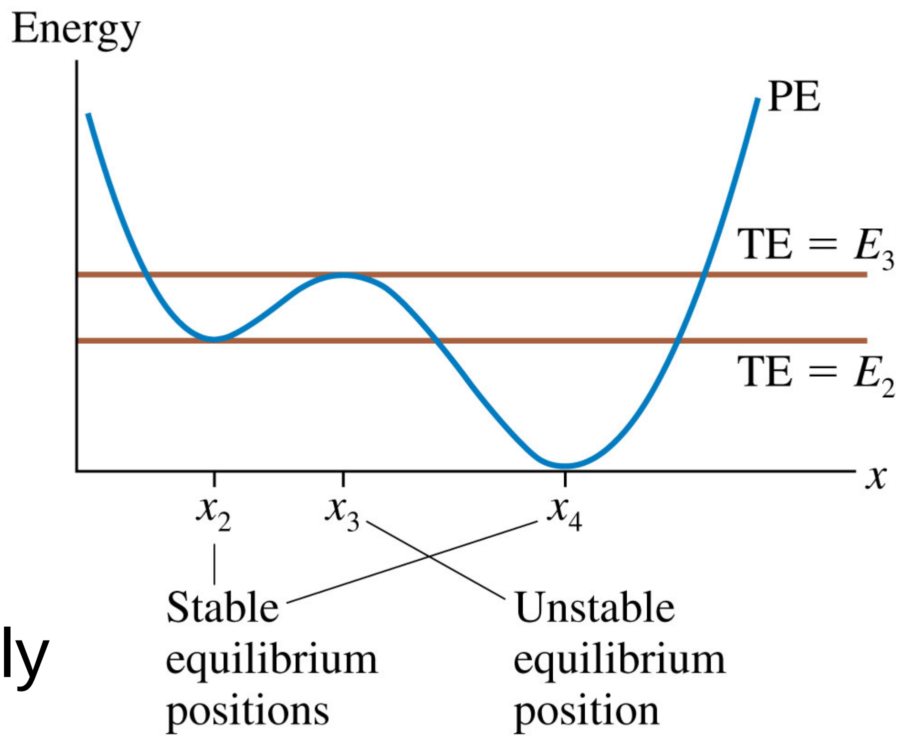
- Consider a particle with the total energy E_2 shown in the figure.
- The particle can be at rest at x_2, but it cannot move away from x_2: This is *static equilibrium*.



- If you disturb the particle, giving it a total energy slightly larger than E_2, it will oscillate very close to x_2.
- An equilibrium for which small disturbances cause small oscillations is called a point of **stable equilibrium**.

Equilibrium Positions: Unstable

- Consider a particle with the total energy E_3 shown in the figure.
- The particle can be at rest at x_3, and it does not move away from x_3: This is *static equilibrium*.
- If you disturb the particle, giving it a total energy slightly larger than E_3, it will speed up as it moves away from x_3.



- An equilibrium for which small disturbances cause the particle to move away is called a point of **unstable equilibrium**.

Tactics: Interpreting an Energy Diagram

TACTICS BOX 10.1 **Interpreting an energy diagram**

MP®

1. The distance from the axis to the PE curve is the particle's potential energy. The distance from the PE curve to the TE line is its kinetic energy. These are transformed as the position changes, causing the particle to speed up or slow down, but the sum $K + U$ doesn't change.
2. A point where the TE line crosses the PE curve is a turning point. The particle reverses direction.
3. The particle cannot be at a point where the PE curve is above the TE line.
4. The PE curve is determined by the properties of the system—mass, spring constant, and the like. You cannot change the PE curve. However, you can raise or lower the TE line simply by changing the initial conditions to give the particle more or less total energy.
5. A minimum in the PE curve is a point of stable equilibrium. A maximum in the PE curve is a point of unstable equilibrium.

Exercises 18–20

Example 10.9 Balancing a Mass on a Spring

EXAMPLE 10.9 **Balancing a mass on a spring**

A spring of length L_0 and spring constant k is standing on one end. A block of mass m is placed on the spring, compressing it. What is the length of the compressed spring?

Example 10.9 Balancing a Mass on a Spring

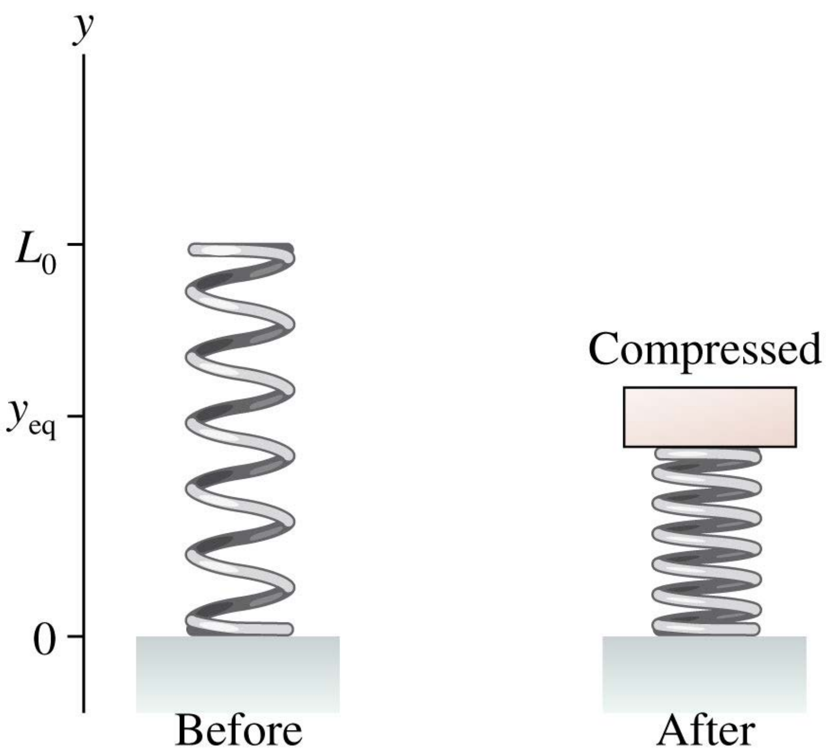
EXAMPLE 10.9 **Balancing a mass on a spring**

MODEL Assume an ideal spring obeying Hooke's law. The block + spring system has both gravitational potential energy U_g *and* elastic potential energy U_s. The block sitting on top of the spring is at a point of stable equilibrium (small disturbances cause the block to oscillate slightly around the equilibrium position), so we can solve this problem by looking at the energy diagram.

Example 10.9 Balancing a Mass on a Spring

EXAMPLE 10.9 Balancing a mass on a spring

VISUALIZE Below is a pictorial representation. We've used a coordinate system with the origin at ground level, so the equilibrium position of the uncompressed spring is $y_e = L_0$.

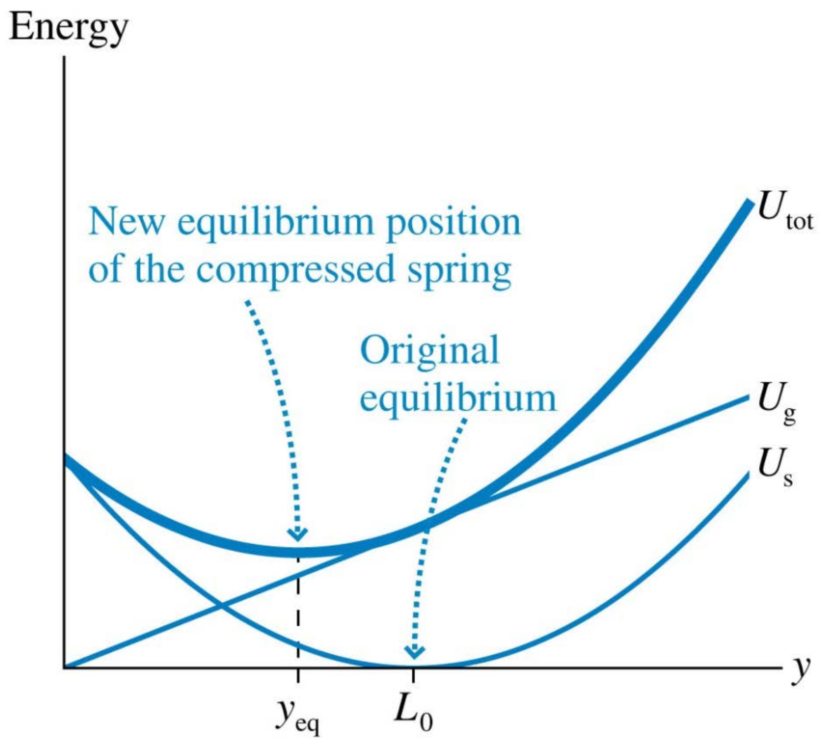


Example 10.9 Balancing a Mass on a Spring

EXAMPLE 10.9 Balancing a mass on a spring

SOLVE The figure shows the two potential energies separately and also shows the total potential energy:

$$U_{tot} = U_g + U_s = mgy + \frac{1}{2}k(y - L_0)^2$$

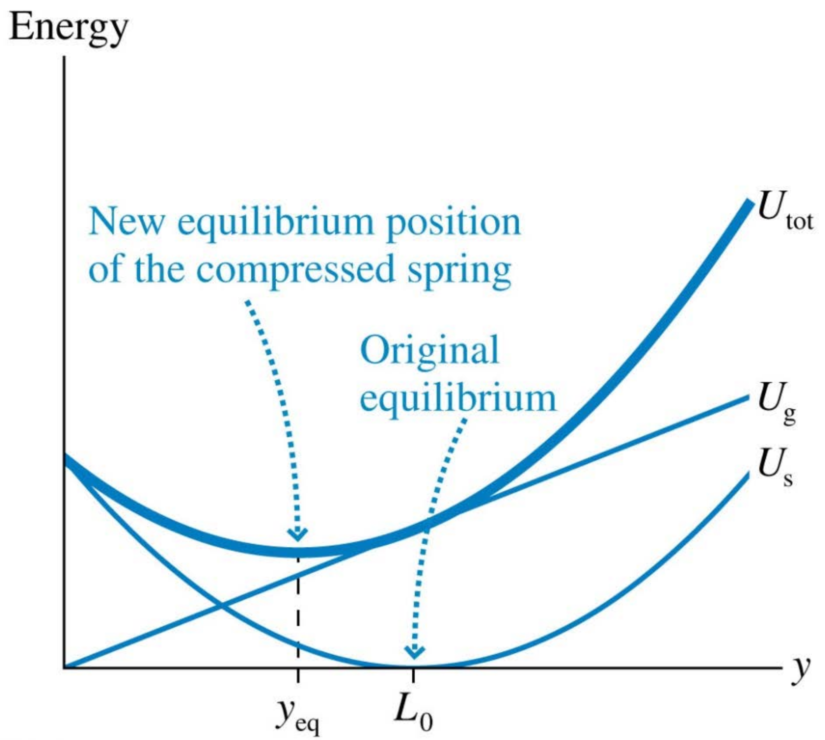


Example 10.9 Balancing a Mass on a Spring

EXAMPLE 10.9 Balancing a mass on a spring

The equilibrium position (the minimum of U_{tot}) has shifted from L_0 to a smaller value of y, closer to the ground. We can find the equilibrium by locating the position of the minimum in the PE curve. You know from calculus that the minimum of a function is at the point where the derivative (or slope) is zero. The derivative of U_{tot} is

$$\frac{dU_{tot}}{dy} = mg + k(y - L_0)$$



Example 10.9 Balancing a Mass on a Spring

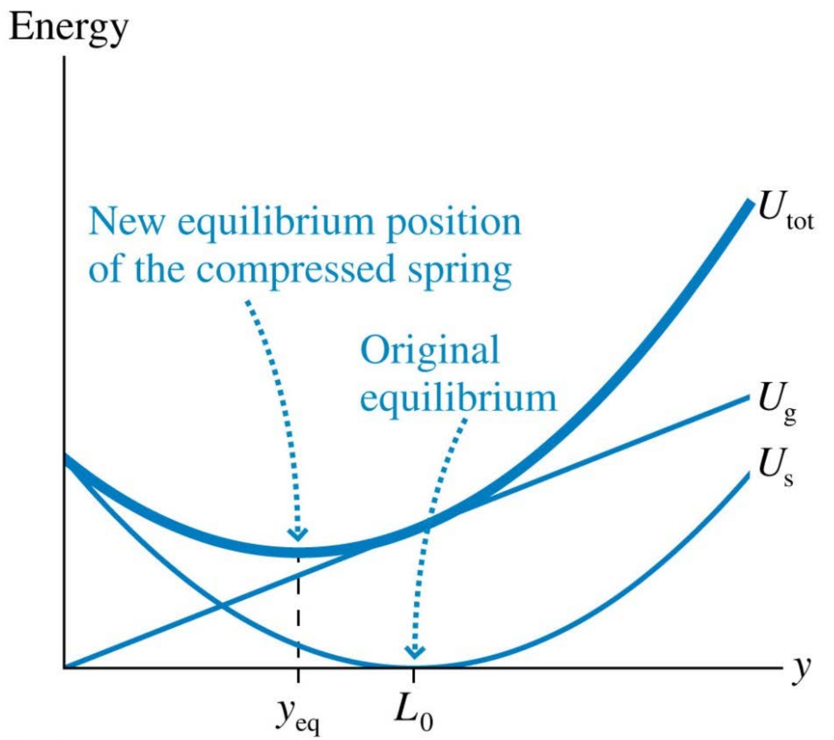
EXAMPLE 10.9 Balancing a mass on a spring

The derivative is zero at the point y_{eq}, so we can easily find

$$mg + k(y_{eq} - L_0) = 0$$

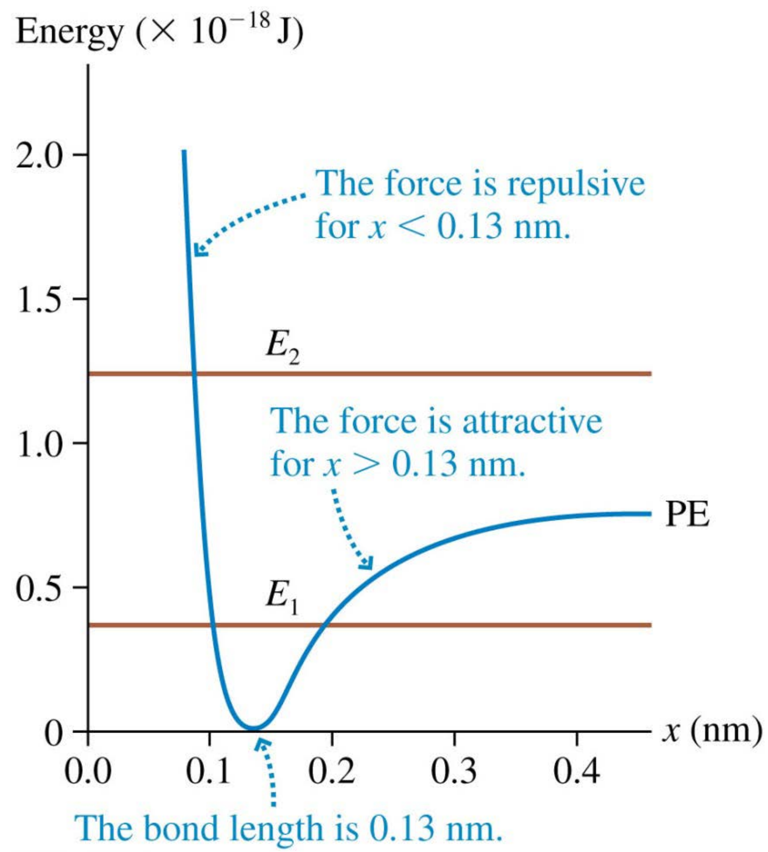
$$y_{eq} = L_0 - \frac{mg}{k}$$

The block compresses the spring by the length mg/k from its original length L_0, giving it a new equilibrium length $L_0 - mg/k$.



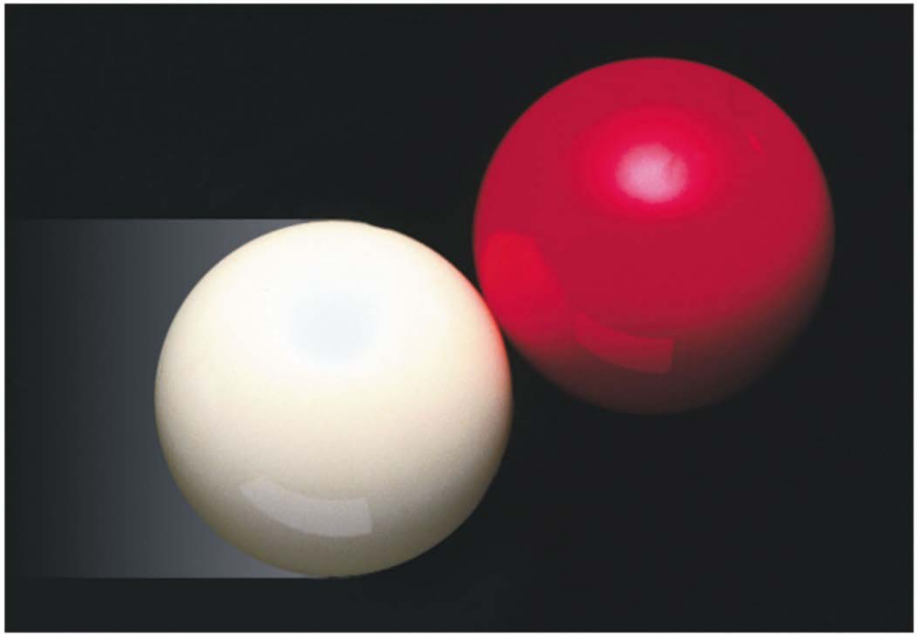
Molecular Bonds

- Shown is the energy diagram for the diatomic molecule HCl (hydrogen chloride).
- x is the distance between the hydrogen and the chlorine atoms.
- The molecule has a stable equilibrium at an atomic separation of $x_{eq} = 0.13$ nm.
- When the total energy is E_1, the molecule is oscillating, but stable.
- If the molecule's energy is raised to E_2, we have *broken the molecular bond*.



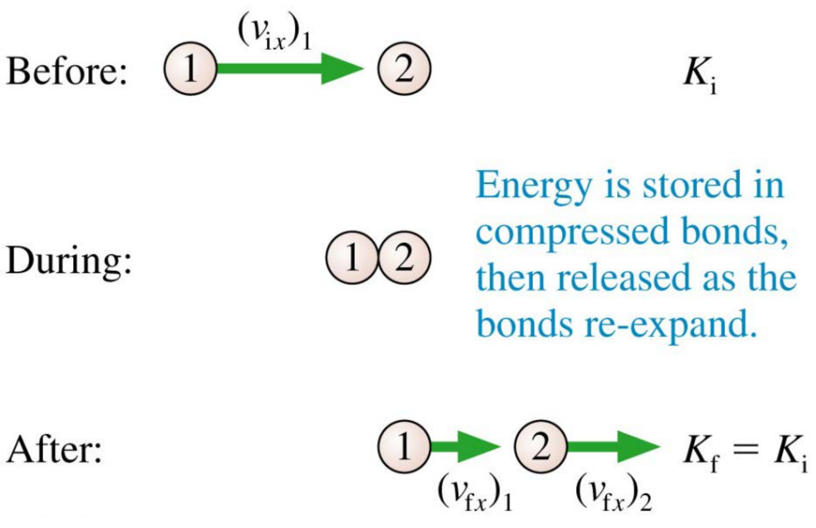
Elastic Collisions

- During an inelastic collision of two objects, some of the mechanical energy is dissipated inside the objects as thermal energy.
- A collision in which mechanical energy is conserved is called a **perfectly elastic collision**.
- Collisions between two very hard objects, such as two billiard balls or two steel balls, come close to being perfectly elastic.



A Perfectly Elastic Collision

- Consider a head-on, perfectly elastic collision of a ball of mass m_1 and initial velocity $(v_{ix})_1$, with a ball of mass m_2 initially at rest.
- The balls' velocities after the collision are $(v_{fx})_1$ and $(v_{fx})_2$.



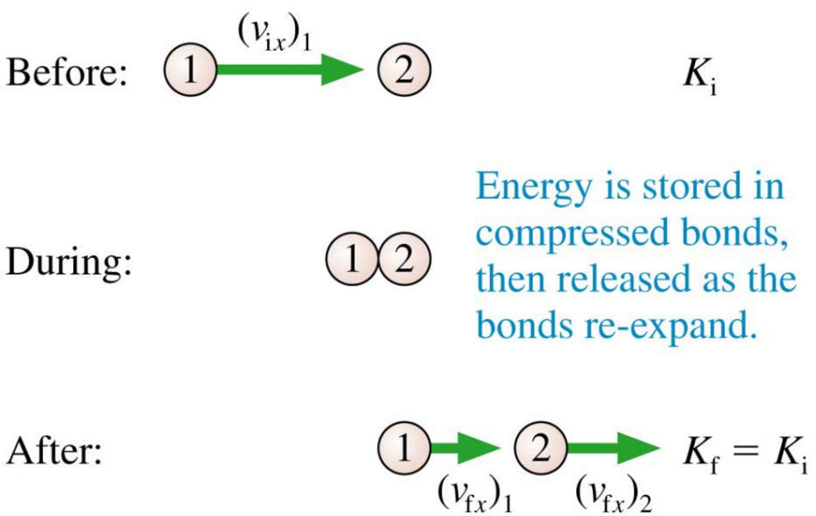
- Momentum is conserved in all isolated collisions.
- In a perfectly elastic collision in which potential energy is not changing, the kinetic energy must also be conserved.

momentum conservation: $m_1(v_{fx})_1 + m_2(v_{fx})_2 = m_1(v_{ix})_1$

energy conservation: $\frac{1}{2}m_1(v_{fx})_1^2 + \frac{1}{2}m_2(v_{fx})_2^2 = \frac{1}{2}m_1(v_{ix})_1^2$

A Perfectly Elastic Collision

- Simultaneously solving the conservation of momentum equation and the conservation of kinetic energy equations allows us to find the two unknown final velocities.
- The result is:



$$(v_{fx})_1 = \frac{m_1 - m_2}{m_1 + m_2}(v_{ix})_1$$

$$(v_{fx})_2 = \frac{2m_1}{m_1 + m_2}(v_{ix})_1$$

(perfectly elastic collision with ball 2 initially at rest)

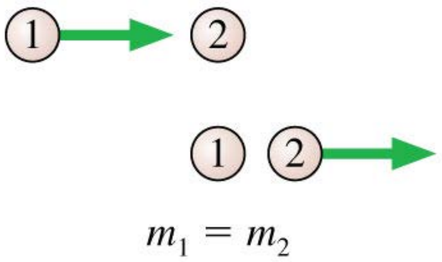
A Perfectly Elastic Collision: Special Case 1

$$(v_{fx})_1 = \frac{m_1 - m_2}{m_1 + m_2}(v_{ix})_1$$

$$(v_{fx})_2 = \frac{2m_1}{m_1 + m_2}(v_{ix})_1$$

(perfectly elastic collision with ball 2 initially at rest)

- Consider a head-on, perfectly elastic collision of a ball of mass m_1 and initial velocity $(v_{ix})_1$, with a ball of mass m_2 initially at rest.



Ball 1 stops. Ball 2 goes forward with $v_{f2} = v_{i1}$.

- Case 1: $m_1 = m_2$.
- Equations 10.42 give $v_{f1} = 0$ and $v_{f2} = v_{i1}$.
- The first ball stops and transfers all its momentum to the second ball.

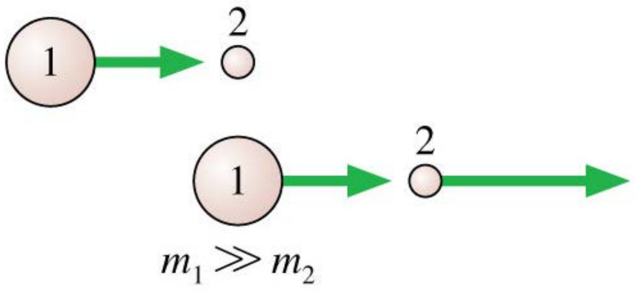
A Perfectly Elastic Collision: Special Case 2

$$(v_{\text{fx}})_1 = \frac{m_1 - m_2}{m_1 + m_2}(v_{\text{ix}})_1$$

$$(v_{\text{fx}})_2 = \frac{2m_1}{m_1 + m_2}(v_{\text{ix}})_1$$

(perfectly elastic collision with ball 2 initially at rest)

- Consider a head-on, perfectly elastic collision of a ball of mass m_1 and initial velocity $(v_{ix})_1$, with a ball of mass m_2 initially at rest.
- Case 2: $m_1 >> m_2$.
- Equations 10.42 give $v_{f1} \approx v_{i1}$ and $v_{f2} \approx 2v_{i1}$.
- The big first ball keeps going with about the same speed, and the little second ball flies off with about twice the speed of the first ball.



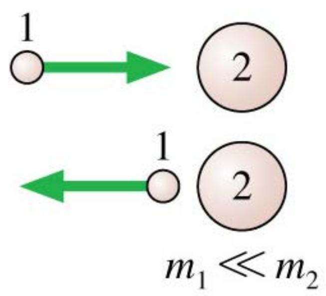
Ball 1 hardly slows down. Ball 2 is knocked forward at $v_{f2} \approx 2v_{i1}$.

A Perfectly Elastic Collision: Special Case 3

$$(v_{fx})_1 = \frac{m_1 - m_2}{m_1 + m_2}(v_{ix})_1$$
$$(v_{fx})_2 = \frac{2m_1}{m_1 + m_2}(v_{ix})_1$$

(perfectly elastic collision with ball 2 initially at rest)

- Consider a head-on, perfectly elastic collision of a ball of mass m_1 and initial velocity $(v_{ix})_1$, with a ball of mass m_2 initially at rest.
- Case 3: $m_1 << m_2$.



Ball 1 bounces off ball 2 with almost no loss of speed. Ball 2 hardly moves.

- Equations 10.42 give $v_{f1} \approx -v_{i1}$ and $v_{f2} \approx 0$.
- The little first rebounds with about the same speed, and the big second ball hardly moves at all.

Perfectly Elastic Collisions: Using Reference Frames

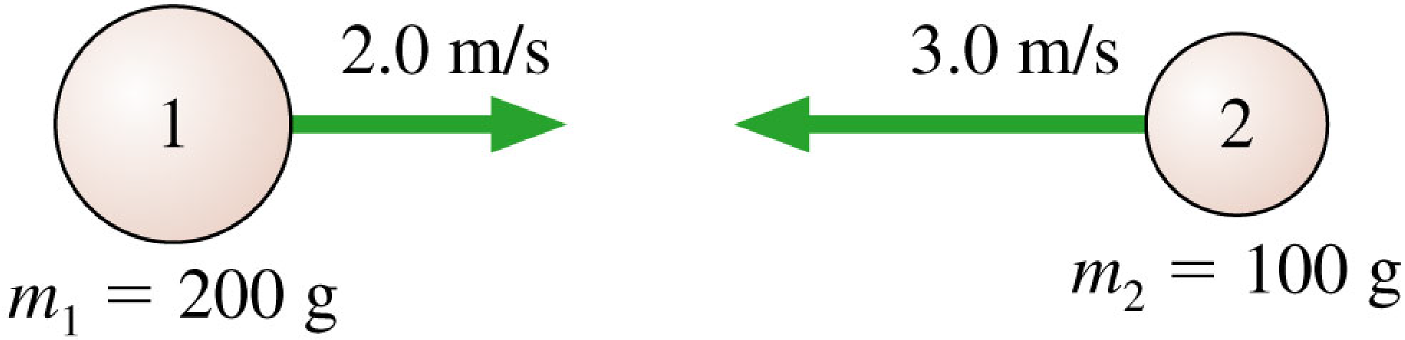
- Equations 10.42 assume ball 2 is at rest.
- What if you need to analyze a head-on collision when both balls are moving before the collision?
- You could solve the simultaneous momentum and energy equations, but there is an easier way.

TACTICS BOX 10.2 **Analyzing elastic collisions**

1. Use the Galilean transformation to transform the initial velocities of balls 1 and 2 from the "lab frame" to a reference frame in which ball 2 is at rest.
2. Use Equations 10.42 to determine the outcome of the collision in the frame where ball 2 is initially at rest.
3. Transform the final velocities back to the "lab frame."

Using Reference Frames: Quick Example

A 200 g ball moves to the right at 2.0 m/s. It has a head-on, perfectly elastic collision with a 100 g ball that is moving toward it at 3.0 m/s. What are the final velocities of both balls?

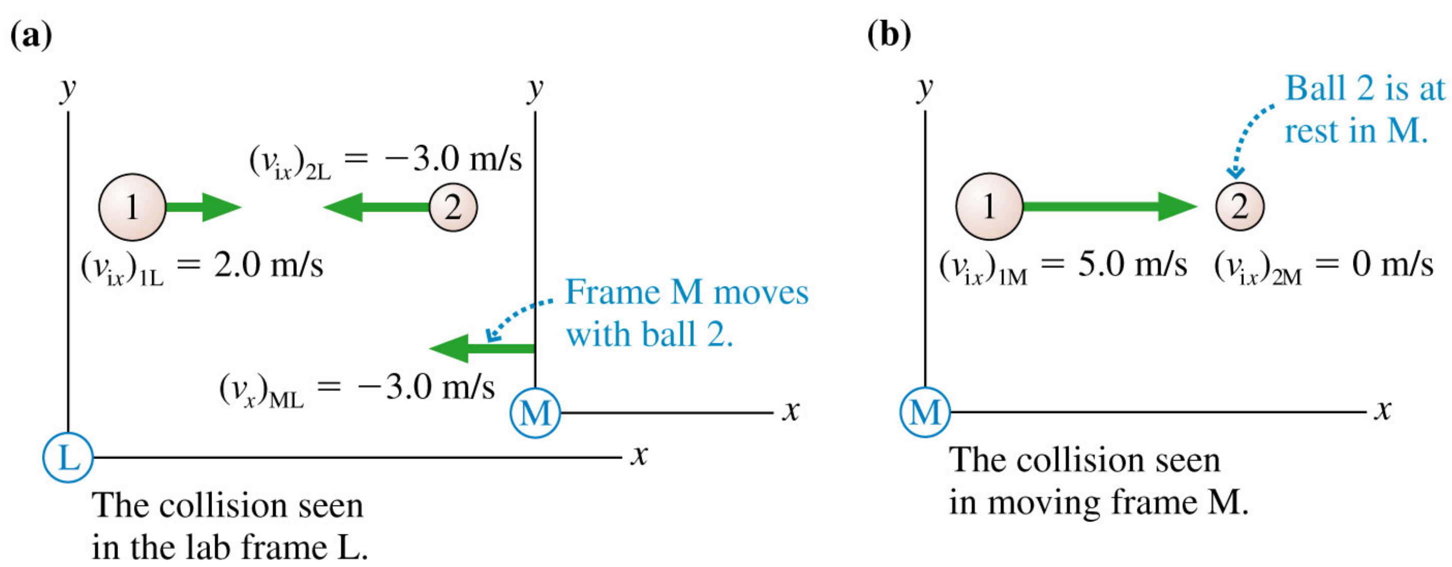


Using Reference Frames: Quick Example

- Figure (a) shows the situation just before the collision in the lab frame L.
- Figure (b) shows the situation just before the collision in the frame M that is moving along with ball 2.

$$(v_{ix})_{1M} = (v_{ix})_{1L} + (v_x)_{LM} = 2.0 \text{ m/s} + 3.0 \text{ m/s} = 5.0 \text{ m/s}$$

$$(v_{ix})_{2M} = (v_{ix})_{2L} + (v_x)_{LM} = -3.0 \text{ m/s} + 3.0 \text{ m/s} = 0 \text{ m/s}$$



Using Reference Frames: Quick Example

- We can use Equations 10.42 to find the post-collision velocities in the moving frame M:

$$(v_{fx})_{1M} = \frac{m_1 - m_2}{m_1 + m_2}(v_{ix})_{1M} = 1.7 \text{ m/s}$$

$$(v_{fx})_{2M} = \frac{2m_1}{m_1 + m_2}(v_{ix})_{1M} = 6.7 \text{ m/s}$$

- Transforming back to the lab frame L:

$$(v_{fx})_{1L} = (v_{fx})_{1M} + (v_x)_{ML} = 1.7 \text{ m/s} + (-3.0 \text{ m/s}) = -1.3 \text{ m/s}$$
$$(v_{fx})_{2L} = (v_{fx})_{2M} + (v_x)_{ML} = 6.7 \text{ m/s} + (-3.0 \text{ m/s}) = 3.7 \text{ m/s}$$

$(v_{fx})_{1L} = -1.3$ m/s $\quad (v_{fx})_{2L} = 3.7$ m/s

Chapter 10 Summary Slides

General Principles

Law of Conservation of Mechanical Energy

If a system is isolated and frictionless, then the mechanical energy $E_{mech} = K + U$ of the system is conserved. Thus

$$K_f + U_f = K_i + U_i$$

- K is the sum of the kinetic energies of all particles.
- U is the sum of all potential energies.

Solving Energy Conservation Problems

MODEL Choose an isolated system without friction or other losses of mechanical energy.

VISUALIZE Draw a before-and-after pictorial representation.

SOLVE Use the law of conservation of energy:

$$K_f + U_f = K_i + U_i$$

ASSESS Is the result reasonable?

Important Concepts

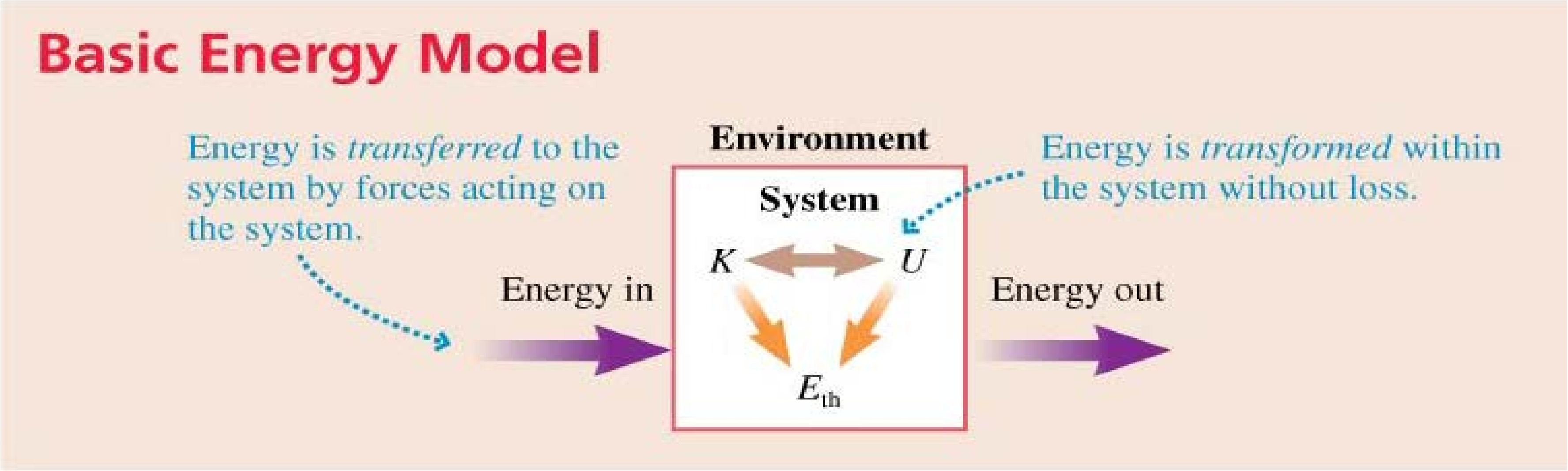
Kinetic energy is an energy of motion: $K = \frac{1}{2}mv^2$.

Potential energy is an energy of position.

- **Gravitational:** $U_g = mgy$
- **Elastic:** $U_s = \frac{1}{2}k(\Delta s)^2$

Thermal energy is due to atomic motions. Hotter objects have more thermal energy.

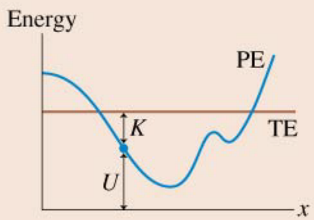
Important Concepts



Important Concepts

Energy diagrams

These diagrams show the potential-energy curve PE and the total mechanical energy line TE.



- The distance from the axis to the curve is PE.
- The distance from the curve to the TE line is KE.
- A point where the TE line crosses the PE curve is a **turning point.**
- Minima in the PE curve are points of **stable equilibrium.** Maxima are points of **unstable equilibrium.**
- Regions where PE is greater than TE are forbidden.