LECTURE EIGHT

Saving, Capital Accumulation and Output (continued)

Chapter 11 of Macroeconomics,
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Dynamics of Capital and Output

- Our two main relations are:

$$\frac{Y_t}{N} = f\left(\frac{K_t}{N}\right)$$

$$\frac{K_{t+1}}{N} - \frac{K_t}{N} = s\frac{Y_t}{N} - \delta\frac{K_t}{N}$$

- **First relation:**
  Capital determines output.

- **Second relation:**
  Output determines capital accumulation

Combining the two relations, we can study the behavior of output and capital over time.

Dynamics of Capital and Output

$$\frac{Y_t}{N} = f\left(\frac{K_t}{N}\right) \qquad \frac{K_{t+1}}{N} - \frac{K_t}{N} = s\frac{Y_t}{N} - \delta\frac{K_t}{N}$$

From our main relations above, we express output per worker (Y_t/N) in terms of capital per worker to derive the equation below:

$$\frac{K_{t+1}}{N} - \frac{K_t}{N} = sf\left(\frac{K_t}{N}\right) - \delta\frac{K_t}{N}$$

Change in capital from year t to year $t+1$	=	Investment during year t	−	Depreciation during year t

Dynamics of Capital and Output

$$\frac{K_{t+1}}{N} - \frac{K_t}{N} = sf\left(\frac{K_t}{N}\right) - \delta\frac{K_t}{N}$$

Change in capital from year t to year $t + 1$ = Investment during year t – Depreciation during year t

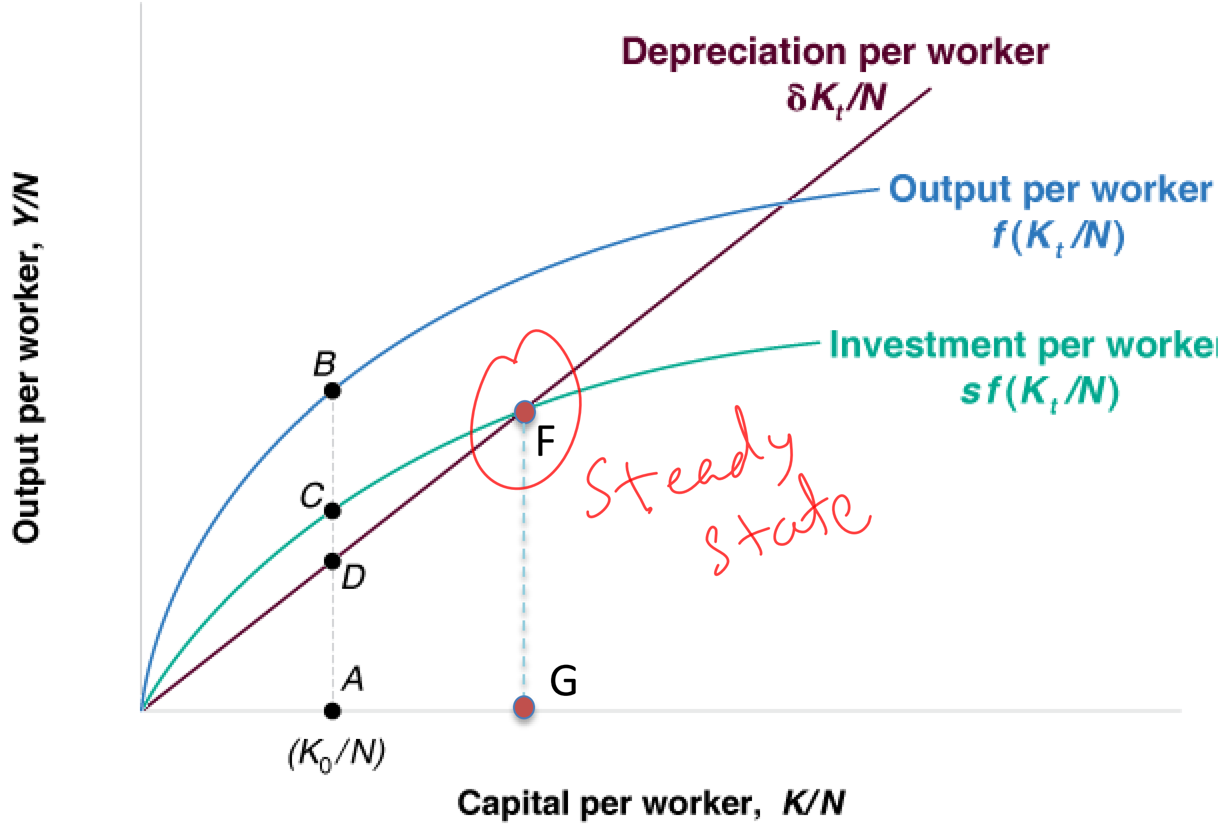
This relation describes what happens to capital per worker. The change in capital per worker from this year to next year depends on the difference between two terms:

- If investment per worker exceeds depreciation per worker, the change in capital per worker is positive: Capital per worker increases.
- If investment per worker is less than depreciation per worker, the change in capital per worker is negative: Capital per worker decreases.

Dynamics of Capital and Output

Graph 1

At K_0/N, capital per worker is low, investment exceeds depreciation, thus, capital per worker and output per worker tend to increase over time.

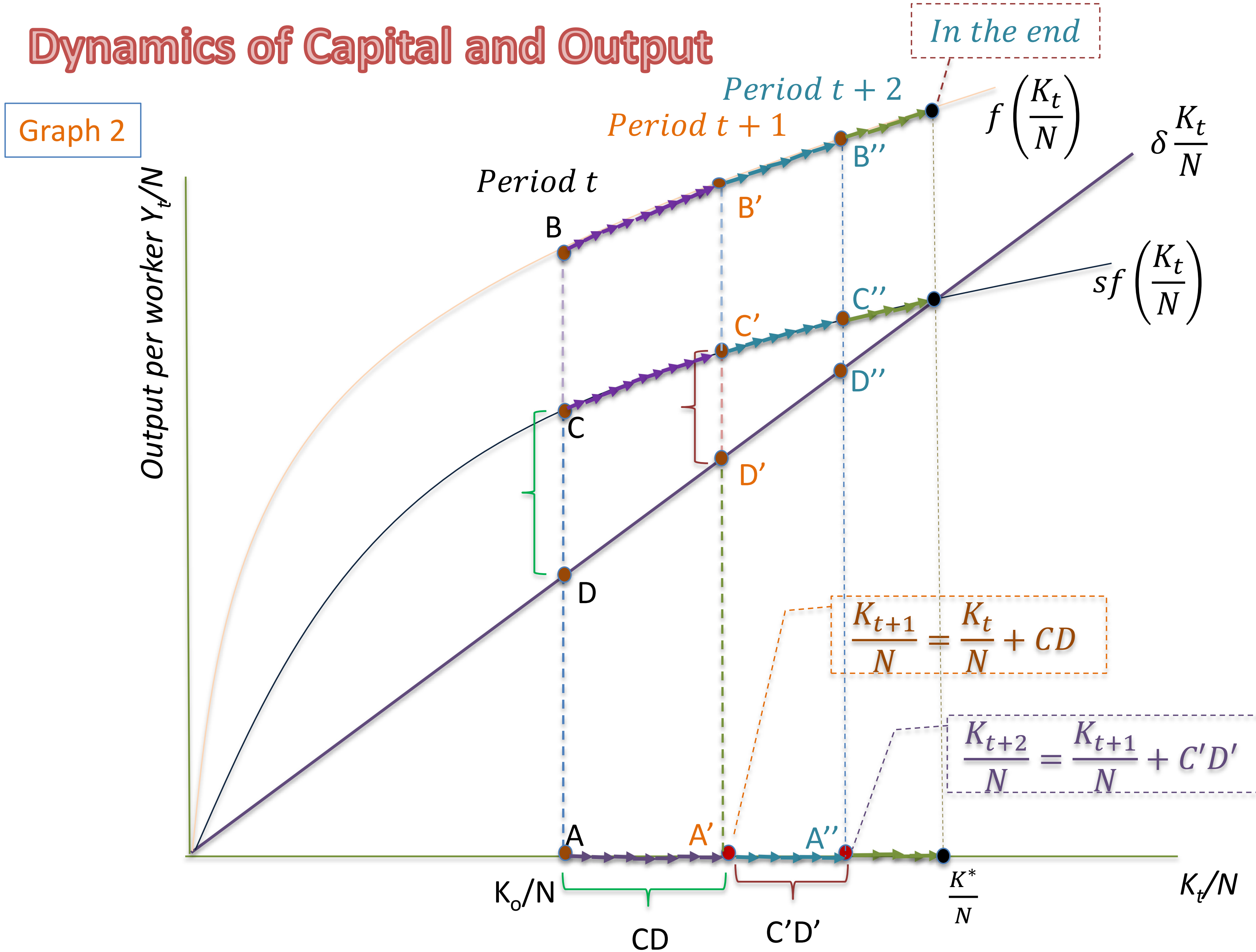


$$\frac{K_{t+1}}{N} - \frac{K_t}{N} = sf\left(\frac{K_t}{N}\right) - \delta \frac{K_t}{N}$$

Graph 1 explained

- The highest curve is the graph of Y_t/N against K_t/N or $\frac{Y_t}{N} = f\left(\frac{K_t}{N}\right)$. The second curve is $sf\left(\frac{K_t}{N}\right)$ which is lower because 0<s<0.
- The straight line shows that the higher the amount of $\frac{K_t}{N}$, the larger the depreciated amount.
- So when $sf\left(\frac{K_t}{N}\right) = \delta\frac{K_t}{N}$, that is when the curve $sf\left(\frac{K_t}{N}\right)$ and straight line, $\delta\frac{K_t}{N}$ meet at point F. At this point, the capital accumulation is equal to zero. All investment is spent on making up for the depreciated amount of capital. Therefore, FG is the amount of investment which is also the amount of depreciated capital.
- Suppose that the economy is not at point F but is having capital per capita to be $\frac{K_0}{N}$. So amount of output produced is AB, the amount of saving (or investment is equal to AC); but since the amount of depreciated capital is equal to AD so take AC subtract AD, we have the amount of capital being added on to the next period is equal to CD.

Dynamics of Capital and Output



Graph 2 explained (1)

- Suppose that the economy initially has K_o/N (or K_t/N) at time t.
- In this period, the economy invests (or saves) AC amount of output into capital for time t+1. But AD amount is for making up for the amount of capital per capita depreciated. So, the amount of capital accumulation (or the *increase* in capital per capita) is CD.
- Thus, CD amount of capital would be added on to the capital per capita stock. That is why the amount of capital per capita in the next period (t+1) is $\frac{K_t}{N}$+ CD.
- Note that, to be precise, we have: $\frac{K_{t+1}}{N} = \frac{K_t}{N} - \delta \frac{K_t}{N} + AC$; but we know: $\frac{K_t}{N} - \delta \frac{K_t}{N} + AC = \frac{K_t}{N} - \delta \frac{K_t}{N} + AD + CD$; note here that $AD = \delta K_t$ (see grh. 2). Therefore: $\frac{K_t}{N} - \delta \frac{K_t}{N} + AC = \boldsymbol{\frac{K_t}{N} + CD = \frac{K_{t+1}}{N}}$

Graph 2 explained (2)

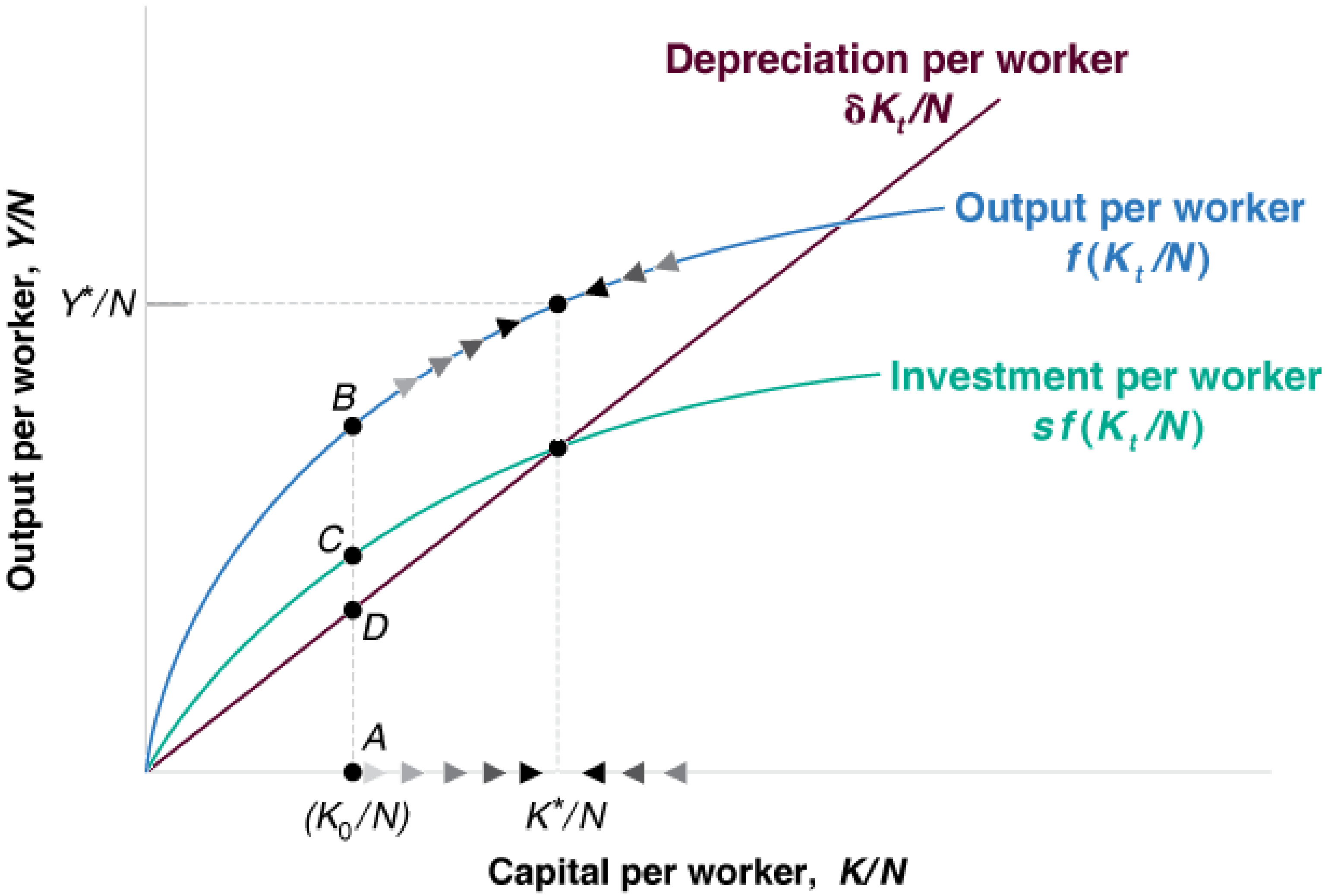
- Since in period t+1 we have $\frac{K_{t+1}}{N}$, the economy will produce at $A'B'$ and save an amount equal to $A'C'$. The depreciated amount of capital is now $A'D'$; so the *increase* in the capital per capita for period t+2 is equal to $C'D'$
- Thus, the amount of capital per capita in the *next* period ($\frac{K_{t+2}}{N}$) is: $\frac{K_{t+2}}{N} = \frac{K_{t+1}}{N} + C'D' = \frac{K_t}{N} + CD + C'D'$
- In period t+2, we can see that the capital per capita stock is higher, $\frac{K_{t+2}}{N}$; so the economy will produce higher output per capital which is equal to $A''B''$, the amount of capital added on to the capital stock in period t+3 would be $C''D''$.
- This process would be repeated for many periods until the capital per capita reach K*/N, at which the investment is just enough to cover for the depreciation of capital.

Dynamics of Capital and Output

Graph 3

Figure 11 - 2

Capital and Output Dynamics



Further comments on Dynamics of Capital and Output

- At K*/N, output per worker and capital per worker remain constant at their long-run equilibrium levels.
  - When capital and output are low, investment exceeds depreciation, and capital increases. (as shown earlier)
  - When capital and output are high, investment is less than depreciation, and capital decreases. (Homework)
- Investment per worker increases with capital per worker, but by less and less as capital per worker increases.
- Depreciation per worker increases in proportion to capital per worker.

Steady-State Capital and Output

$$\frac{K_{t+1}}{N} - \frac{K_t}{N} = sf\left(\frac{K_t}{N}\right) - \delta\frac{K_t}{N}$$

- The state in which output per worker and capital per worker are no longer changing is called the steady state of the economy. In steady state, the left side of the equation above equals zero, then:

$$sf\left(\frac{K^*}{N}\right) = \delta\frac{K^*}{N}$$

Given the steady state of capital per worker (K^*/N), the steady-state value of output per worker (Y^*/N), is given by the production function:

$$\frac{Y^*}{N} = f\left(\frac{K^*}{N}\right)$$

Implications of Alternative Saving Rates

The Saving Rate and Output

- Three observations about the effects of the saving rate on the growth rate of output per worker are:

1. *The saving rate has no effect on the long run growth rate of output per worker, which is equal to zero.*

2. Nonetheless, *the saving rate determines the level of output per worker in the long run.* Other things equal, countries with a higher saving rate will achieve higher output per worker in the long run.

3. An increase in the saving rate will lead to higher growth of output per worker for some time, but not forever.

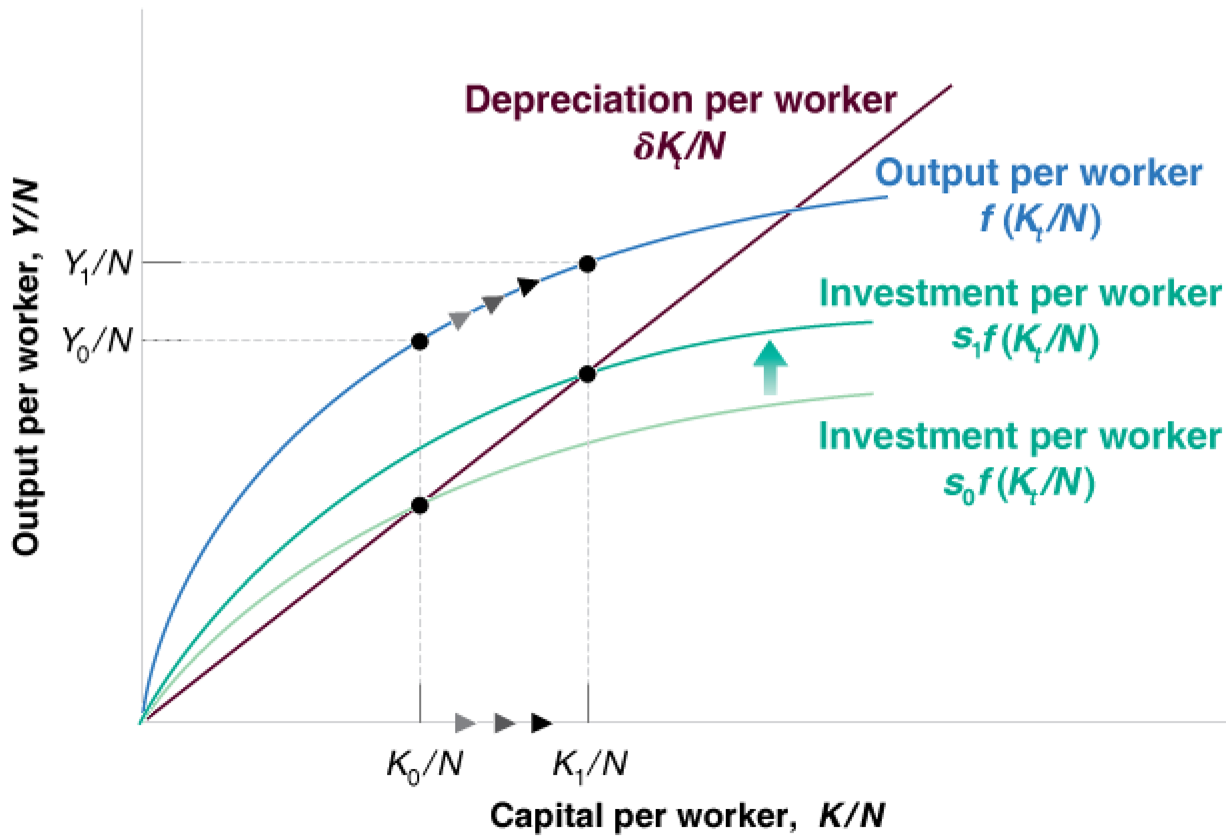
The Saving Rate and Output

Graph 4

Figure 11 - 3

The Effects of Different Saving Rates

A country with a higher saving rate achieves a higher **steady- state** level of output per worker.



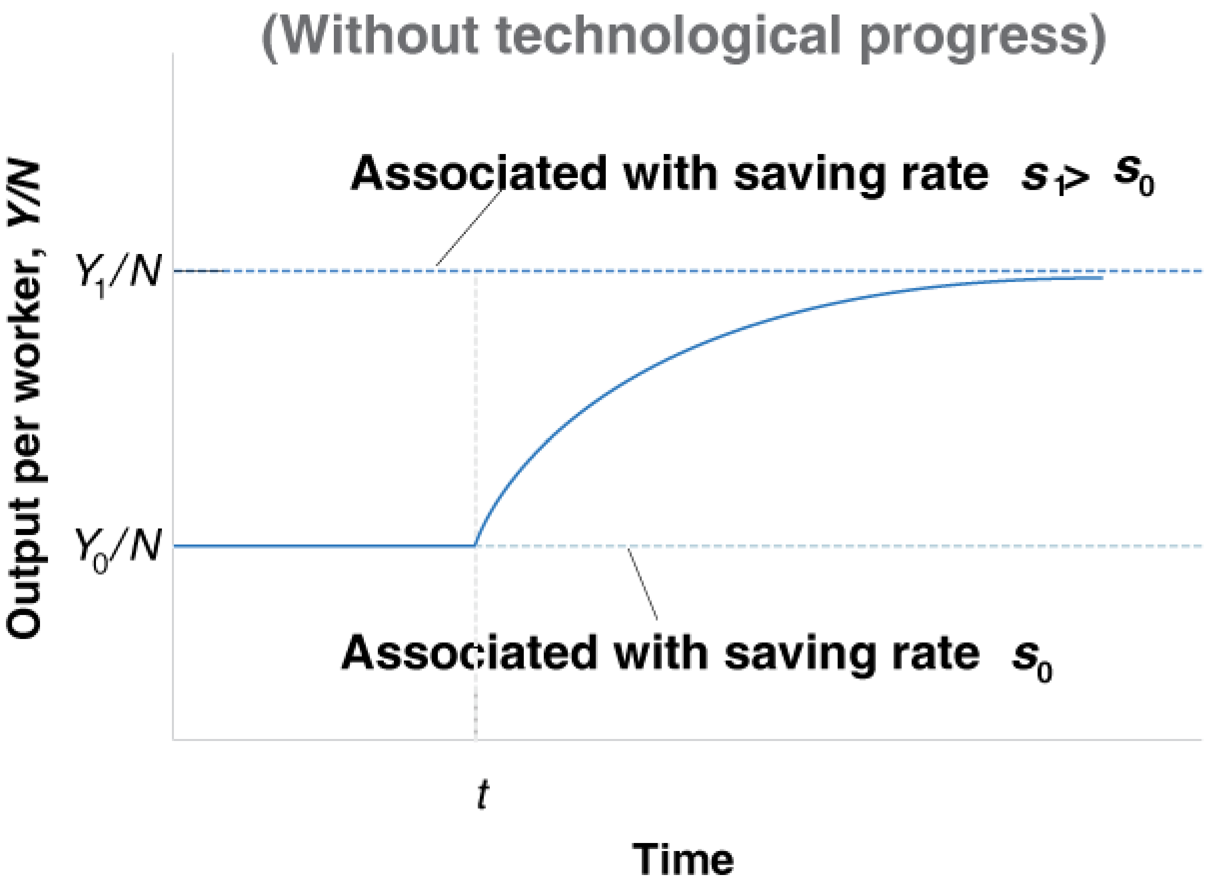
The Saving Rate and Output

Graph 5

Figure 11 - 4

The Effects of an Increase in the Saving Rate on Output per Worker

An increase in the saving rate leads to a period of higher growth until output reaches its new, higher steady-state level.



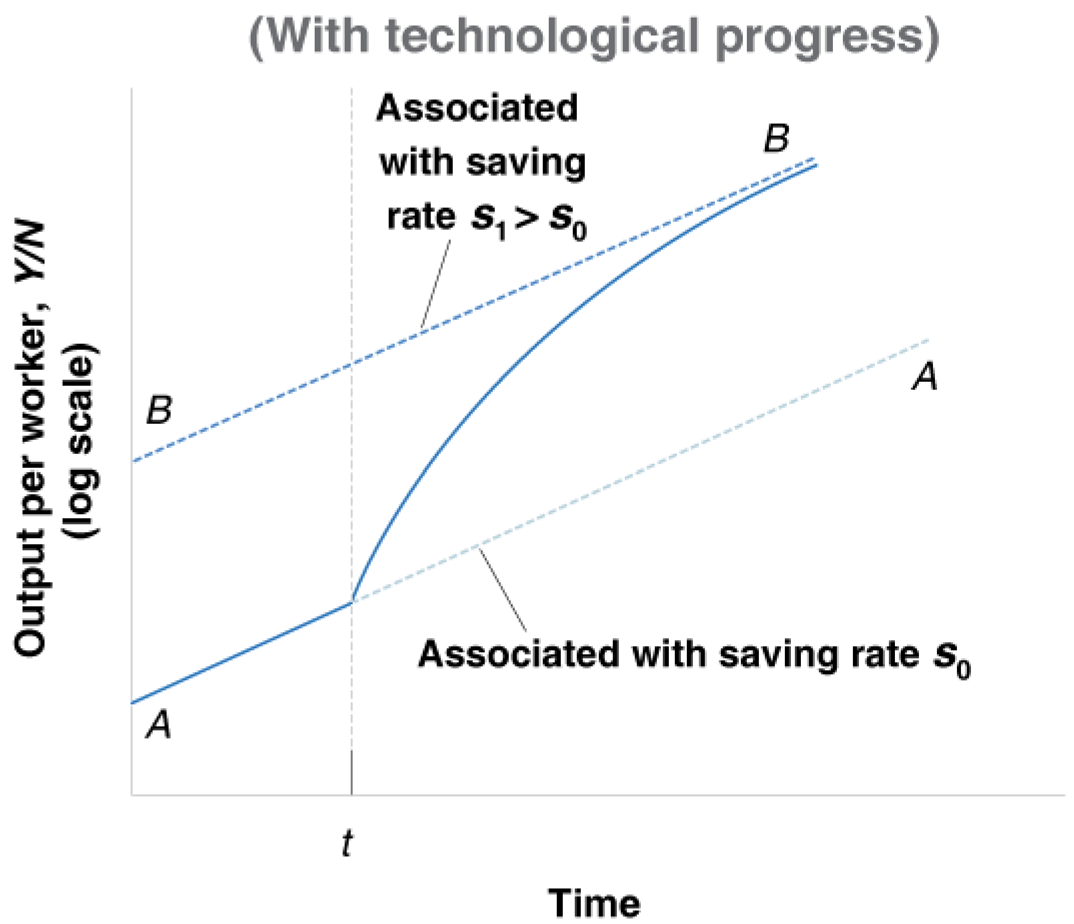
The Saving Rate and Output

Graph 6

Figure 11 - 5

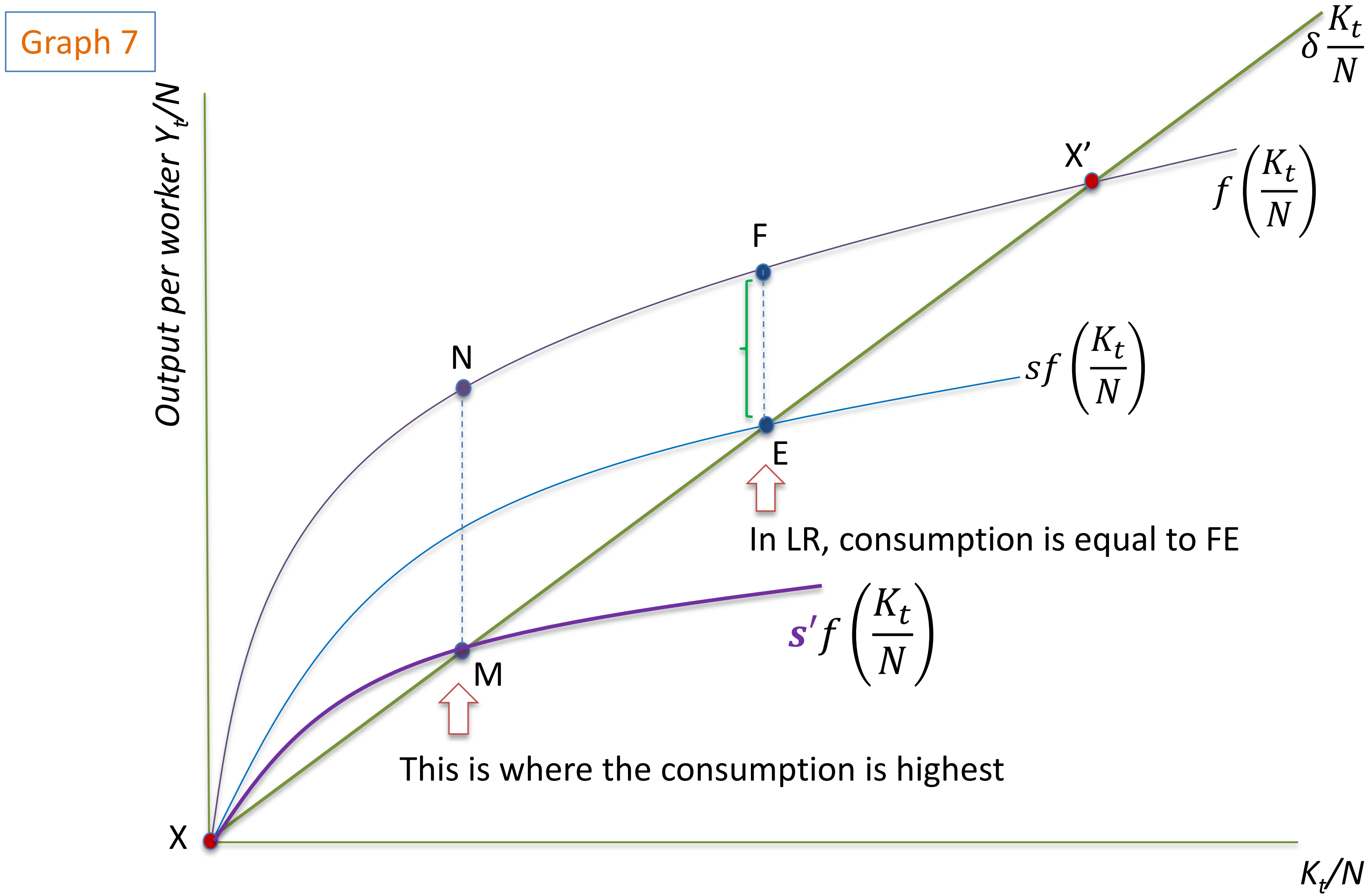
The Effects of an Increase in the Saving Rate on Output per Worker in an Economy with Technological Progress

An increase in the saving rate leads to a period of higher growth until output reaches a new, higher path.



The Saving Rate and Consumption

Dynamics of Capital and Output



The Saving Rate and Consumption

An increase in the saving rate always leads to an increase in the level of *output* per worker. But output is not the same as consumption. To see why, consider what happens for two **extreme values** of the saving rate:

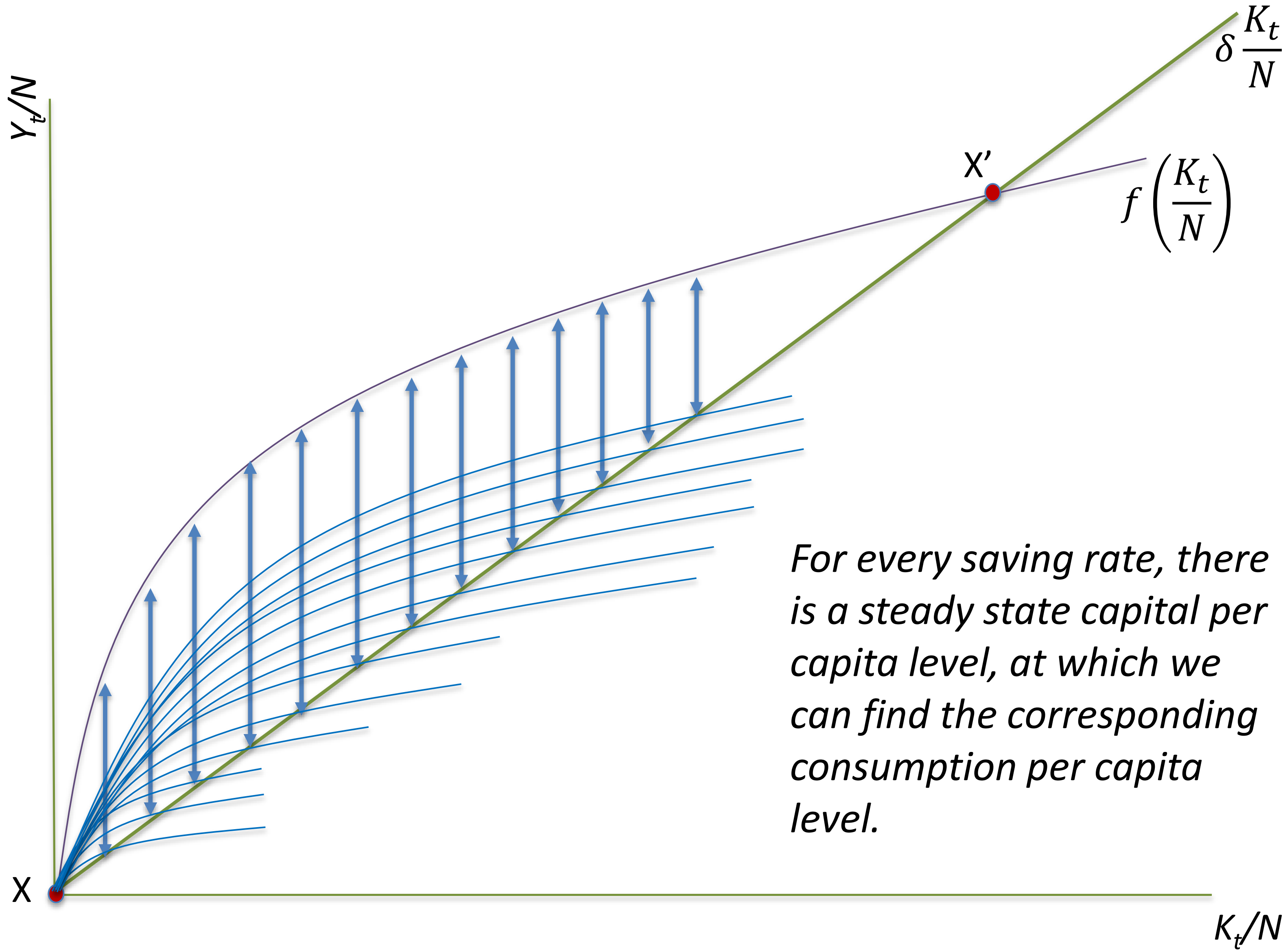
- An economy in which the saving rate is (and has always been) 0 is an economy in which capital is equal to zero. In this case, output is also equal to zero, and so is consumption. A saving rate equal to zero implies zero consumption in the long run. (Point X on graph 7)
- Now consider an economy in which the saving rate is equal to one: People save all their income. The level of capital, and thus output, in this economy will be very high. But because people save all their income, consumption is equal to zero. A saving rate equal to one also implies zero consumption in the long run. (Point X' on graph 7)

The Saving Rate and Consumption

- The level of capital associated with the value of the saving rate that yields the highest level of consumption in steady state is known as the golden-rule level of capital.

Dynamics of Capital and Output

Graph 8



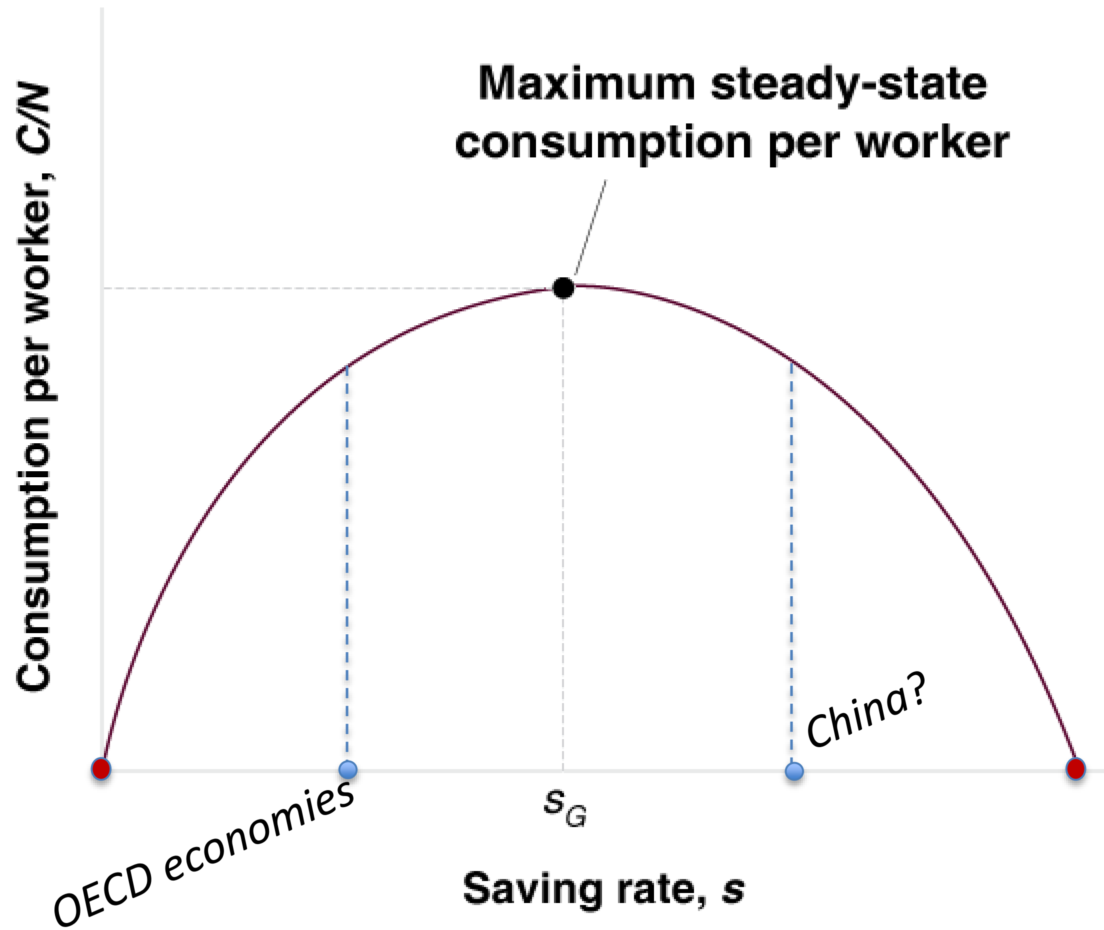
For every saving rate, there is a steady state capital per capita level, at which we can find the corresponding consumption per capita level.

The Saving Rate and Consumption

Figure 11 - 6

The Effects of the Saving Rate on Steady- State Consumption per Worker

An increase in the saving rate leads to an increase and then to a decrease in steady-state consumption per worker.



The Saving Rate and Consumption

Government can affect the saving rates in various way:

- Vary public saving: if public saving is positive, overall saving would be higher and vice versa.
- Vary taxes: e.g. tax breaks to provide more incentive to save

- Government should care about the consumption per worker, not the output per worker because consumption per worker reflects the welfare of the people.
- The level of capital per capita that yields highest level of consumption per capita in *steady state* is known as the **golden-rule level of capital.**

Some discussions on Saving Rate and Consumption

- If the capital per capita is below the golden rule level, there should be an increase in the saving rate in the economy.

- But immediately after the increase in saving rate, there would be lower consumption in the short run. (can you show it?)

- However, in the long run, there would be higher consumption per capita. **<u>So there is a problem:</u>**

  - Should the governments care about the future generations more or the current generation more?

  - Because by increasing saving rate (through policies) the OECD governments can help the future generations but hurt the current generation.

  - But the politics is about votes, rights to vote belong to the current generation, not the future generations → it is unlikely that the governments will ask the current generations to make large sacrifices → capital level would be still below the golden rule level.

Social Security, Saving, & Capital Accumulation in US

FOCUS

- One way to run a social security system is the **fully-funded system**, where workers are taxed, their contributions invested in *financial assets*, and **when workers retire**, they receive the principal plus the interest payments on their investments.

- Another way to run a social security system is the **pay-as-you-go system**, where the taxes that workers pay are the benefits that ***current*** retirees receive.

Getting a Sense of Magnitudes

- Assume the production function is:

$$Y = \sqrt{K}\sqrt{N}$$

Output per worker is: $\frac{Y}{N} = \frac{\sqrt{K}\sqrt{N}}{N} = \frac{\sqrt{K}}{\sqrt{N}} = \sqrt{\frac{K}{N}}$

Output per worker, as it relates to capital per worker is:

$$f\left(\frac{K_t}{N}\right) = \sqrt{\frac{K_t}{N}}$$

Given our second relation, $\frac{K_{t+1}}{N} - \frac{K_t}{N} = sf\left(\frac{K_t}{N}\right) - \delta\frac{K_t}{N}$

Then, $\frac{K_{t+1}}{N} - \frac{K_t}{N} = s\sqrt{\frac{K_t}{N}} - \delta\frac{K_t}{N}$

The Effects of the Saving Rate on Steady-State Output

$$\frac{Y^*}{N} = \sqrt{\frac{K^*}{N}} = \sqrt{\left(\frac{s}{\delta}\right)^2} = \frac{s}{\delta}$$

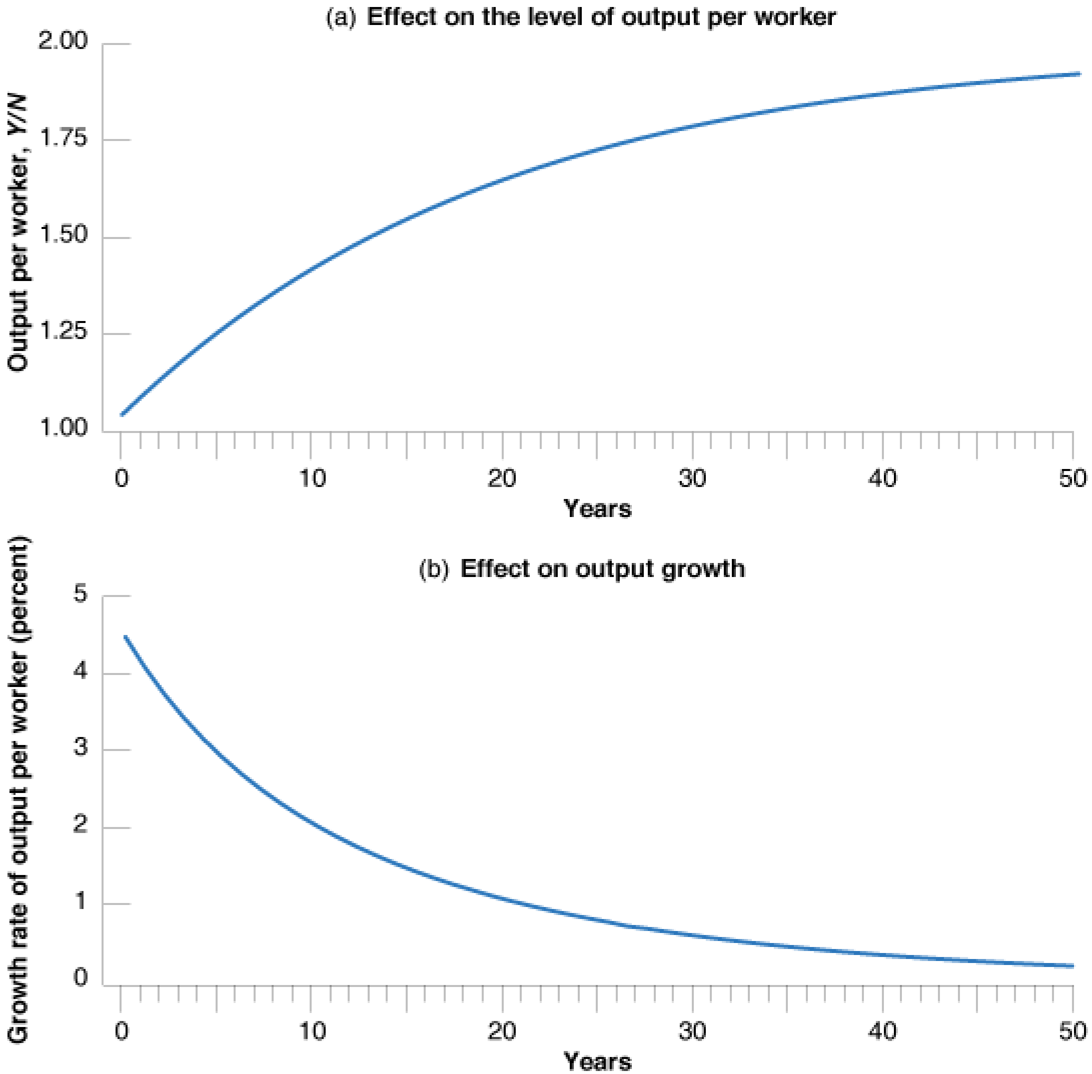
- Steady-state output per worker is equal to the ratio of the saving rate to the depreciation rate. *(result applicable only for this particular production function)*
- A higher saving rate and a lower depreciation rate both lead to higher steady-state capital per worker and higher steady-state output per worker.

The Dynamic Effects of an Increase in the Saving Rate

Figure 11 - 7

The Dynamic Effects of an Increase in the Saving Rate from 10% to 20% on the Level and the Growth Rate of Output per Worker

It takes a long time for output to adjust to its new, higher level after an increase in the saving rate. Put another way, an increase in the saving rate leads to a long period of higher growth.



The U.S. Saving Rate and the Golden Rule

- In steady state, consumption per worker is equal to output per worker minus depreciation per worker.

$$\frac{C}{N} = \frac{Y}{N} - \delta\frac{K}{N}$$

Knowing that: $\frac{K^*}{N} = \left(\frac{s}{\delta}\right)^2$ and $\frac{Y^*}{N} = \sqrt{\frac{K^*}{N}} = \sqrt{\left(\frac{s}{\delta}\right)^2} = \frac{s}{\delta}$

then: $\frac{C}{N} = \frac{s}{\delta} - \delta\left(\frac{s}{\delta}\right)^2 = \frac{s(1-s)}{\delta}$

These equations are used to derive Table 11-1 in the next slide.

The U.S. Saving Rate and the Golden Rule

Table 11-1 **The Saving Rate and the Steady-state Levels of Capital, Output, and Consumption per Worker**

Saving Rate, *s*	Capital per Worker, (K/N)	Output per Worker, (Y/N)	Consumption per Worker, (C/N)
0.0	0.0	0.0	0.0
0.1	1.0	1.0	0.9
0.2	4.0	2.0	1.6
0.3	9.0	3.0	2.1
0.4	16.0	4.0	2.4
0.5	25.0	5.0	2.5
0.6	36.0	6.0	2.4
–	–	–	–
1.0	100.0	10.0	0.0

Physical versus Human Capital

Physical versus Human Capital

- The set of skills of the workers in the economy is called human capital.
- An economy with many highly skilled workers is likely to be much more productive than an economy in which most workers cannot read or write.
- The conclusions drawn about physical capital accumulation remain valid after the introduction of human capital in the analysis.

Extending the Production Function

- When the level of output per workers depends on both the level of physical capital per worker, *K/N*, and the level of human capital per worker, *H/N*, the production function may be written as:

$$\frac{Y}{N} = f\left(\frac{K}{N}, \frac{H}{N}\right)$$
$$(+,\ +)$$

An increase in capital per worker or the average skill of workers leads to an increase in output per worker.

Extending the Production Function

- *For example, a production function with human capital, capital and labour can look like this:*

$$Y = K^{\alpha} H^{\beta} N^{1-\alpha-\beta}$$

$$\Rightarrow \frac{Y}{N} = \left(\frac{K}{N}\right)^{\alpha} \left(\frac{H}{N}\right)^{\beta} \left(\frac{N}{N}\right)^{1-\alpha-\beta}$$

$$\Rightarrow \frac{Y}{N} = \left(\frac{K}{N}\right)^{\alpha} \left(\frac{H}{N}\right)^{\beta} = \mathrm{f}\left(\frac{K}{N}, \frac{H}{N}\right)$$

Extending the Production Function

- A measure of human capital may be constructed as follows:

- Suppose an economy has 100 workers, half of them unskilled and half of them skilled. The relative wage of skilled workers is twice that of unskilled workers.

- Then:

$$H = [(50 \times 1) + (50 \times 2)] = 150 \Rightarrow \frac{H}{N} = \frac{150}{100} = 1.5$$

Human Capital, Physical Capital, and Output

- An increase in how much society “saves” in the form of human capital—through education and on-the-job-training—increases steady-state human capital per worker, which leads to an increase in output per worker.

- In the long run, output per worker depends not only on how much society saves but also *how much it spends on education.*

Human Capital, Physical Capital, & Output

- In the United States, spending on education comprises about 6.5% of GDP, compared to 16% investment in physical capital. This comparison:
  - Accounts for the fact that education is partly consumption.
  - Does not account for the opportunity cost of education.
  - Does not account for the opportunity cost of on-the-job-training.
  - Considers gross, not net investment. Depreciation of human capital is slower than that of physical capital.

Endogenous Growth

- A recent study has concluded that output per worker depends roughly equally on the amount of physical capital and the amount of human capital in the economy.
- Models that generate steady growth even without technological progress are called models of endogenous growth, where growth depends on variables such as the saving rate and the rate of spending on education.
  - Output per worker depends on the level of both physical capital per worker and human capital per worker.
  - Is technological progress unrelated to the level of human capital in the economy? Can't a better-educated labor force lead to a higher rate of technological progress? These questions take us to the topic of the next chapter: the sources and the effects of technological progress.