CHAPTER 11: RATIONAL EQUATIONS AND APPLICATIONS

Chapter Objectives

By the end of this chapter, students should be able to:
- ✓ Identify extraneous values
- ✓ Apply methods of solving rational equations to solve rational equations
- ✓ Solve applications with rational equations including revenue, distance, and work-rate problems

Contents

SECTION 11.1: RATIONAL EQUATIONS

When solving rational equations, we can solve by using the same strategy we used to solve linear equations with fractions: *clearing denominators*. However, we first need to revisit excluded values.

A. EXCLUDED VALUES REVIEW

Note
A rational expression is undefined where the denominator is zero.

MEDIA LESSON
Find excluded values of a rational expression (Duration 4:20)

View the video lesson, take notes and complete the problems below.

Find values where a rational expression is undefined.

a) $\dfrac{17}{4x-32}$

b) $\dfrac{x+19}{(x-10)(x-4)}$

c) $\dfrac{9x+10}{x^2-9x-10}$

Definition

Recall, the excluded values are values which make the expression undefined. Hence, when solving a rational equation, the solution(s) is any value(s) *except the excluded values*. If we obtain a solution that is an excluded value, we call this an **extraneous solution**.

B. SOLVE RATIONAL EQUATIONS BY CLEARING DENOMINATORS WITH THE LCD

Steps for solving rational equations with the same denominator

Step 1. Determine the excluded values of the equation.
Step 2. Clear denominators **by multiplying each term by the lowest common denominator**.
Step 3. Solve the equation.
Step 4. Verify that the solutions obtained are not an excluded value.

MEDIA LESSON
Solve rational equations by clearing the denominators-part 1 (Duration 9:15)

View the video lesson, take notes and complete the problems below.

Example: Solve the rational equations.

1) $\dfrac{x+8}{6} + 3 = \dfrac{3x+8}{8}$

6: _____
8:_____
LCD: _____

Excluded values: _____

2) $\dfrac{4}{x-2} + \dfrac{1}{2} = \dfrac{3}{4}$

LCD=_____

Excluded values: _____

YOU TRY

a) Solve for x: $\dfrac{2}{3}x - \dfrac{5}{6} = \dfrac{3}{4}$

b) Solve for x: $\dfrac{9}{x+1} - \dfrac{5}{2} = \dfrac{4}{3}$

	MEDIA LESSON
	Solve rational equations by clearing the denominators - part 2 (Duration 4:32)

View the video lesson, take notes and complete the problems below.

Example: Solve the rational equations.

1) $\dfrac{2}{5x} - 3 = \dfrac{4}{x}$ $x \neq$ ___

2) $2x - \dfrac{16}{x} = 4$ $x \neq$ ___

YOU TRY

a) Solve for x: $\dfrac{5x+5}{x+2} + 3x = \dfrac{x^2}{x+2}$

b) Solve for x: $\dfrac{x}{x+2} + \dfrac{1}{x+1} = \dfrac{5}{(x+1)(x+2)}$

C. FACTORING DENOMINATORS

Often we will need to factor denominators before finding the LCD.

	MEDIA LESSON
	Solve rational equations with factoring the denominators first- Part 1 (Duration 4:27)

View the video lesson, take notes and complete the problems below.

Example: Solve the rational equations.

1) $\dfrac{x}{x-2} + \dfrac{x}{x^2-4} = \dfrac{x+3}{x+2}$ 　　　　　　　　　Excluded values: _____

	MEDIA LESSON
	Solve rational equations with factoring the denominators first – Part 2 (Duration 4:45)

View the video lesson, take notes and complete the problems below.

Example: Solve the rational equations.

2) $\dfrac{x-3}{x+6} + \dfrac{x-2}{x-3} = \dfrac{x^2}{x^2+3x-18}$ 　　　　　Excluded values: _____

YOU TRY

a) Solve for t: $\dfrac{t}{t-1} - \dfrac{1}{t-2} = \dfrac{11}{t^2-3t+2}$

b) Solve for x: $\dfrac{2x}{x+1} + \dfrac{3x}{x+1} = \dfrac{x^2}{x^2+2x+1}$

D. SOLVING RATIONAL EQUATIONS WITH EXTRANEOUS SOLUTIONS

	MEDIA LESSON
	Solve a rational equation with no solution (Duration 5:07)

View the video lesson, take notes and complete the problems below.

Solve the rational equations.

1) $\dfrac{x}{6x-36} - 9 = \dfrac{1}{x-6}$ Excluded values: _____

	MEDIA LESSON
	Solve a rational equation with extraneous solutions (Duration 5:00)

View the video lesson, take notes and complete the problems below.

Rational equations – extraneous
Because we are working with fractions, the _____ cannot be _____.

Solve the rational equations.

2) $\dfrac{x}{x-8} - \dfrac{2}{x-4} = \dfrac{-3x+56}{x^2-12x+32}$

3) $\dfrac{x}{x-2} + \dfrac{2}{x-4} = \dfrac{4x-12}{x^2-6x+8}$

YOU TRY

a) Solve for n: $\dfrac{n}{n+5} - \dfrac{2}{n-9} = \dfrac{-11n+15}{n^2-4n-45}$

EXERCISES
Solve. Be sure to verify all solutions.

1) $3x - \dfrac{1}{2} - \dfrac{1}{x} = 0$

2) $x + \dfrac{20}{x-4} = \dfrac{5x}{x-4} - 2$

3) $x + \dfrac{6}{x-3} = \dfrac{2x}{x-3}$

4) $\dfrac{2x}{3x-4} = \dfrac{4x+5}{6x-1} - \dfrac{3}{3x-4}$

5) $\dfrac{3m}{2m-5} - \dfrac{7}{3m+1} = \dfrac{3}{2}$

6) $\dfrac{4-x}{1-x} = \dfrac{12}{3-x}$

7) $\dfrac{7}{3-x} + \dfrac{1}{2} = \dfrac{3}{4-x}$

8) $\dfrac{2}{3-x} - \dfrac{6}{8-x} = 1$

9) $x + 1 = \dfrac{4}{x+1}$

10) $\dfrac{x-4}{x-1} = \dfrac{12}{x-3} + 1$

11) $\dfrac{4x}{2x-6} - \dfrac{4}{5x-15} = \dfrac{1}{2}$

12) $\dfrac{x-1}{x-3} + \dfrac{x+2}{x+3} = \dfrac{3}{4}$

13) $\dfrac{x-2}{x+3} - \dfrac{1}{x-2} = \dfrac{1}{x^2+x-6}$

14) $\dfrac{3x-5}{5x-5} + \dfrac{5x-1}{7x-7} - \dfrac{x-4}{1-x} = 2$

15) $\dfrac{x-5}{x-9} + \dfrac{x+3}{x-3} = \dfrac{-4x^2}{x^2-12x+27}$

16) $\dfrac{2x}{x+1} - \dfrac{3}{x+5} = \dfrac{-8x^2}{x^2+6x+5}$

17) $\dfrac{2x}{x+2} + \dfrac{2}{x-4} = \dfrac{3x}{x^2-2x-8}$

18) $\dfrac{x+2}{3x-1} - \dfrac{1}{x} = \dfrac{3x-3}{3x^2-x}$

19) $\dfrac{6x+5}{2x^2-2x} + \dfrac{2}{x^2-1} = \dfrac{3x}{x^2-1}$

20) $\dfrac{x-3}{x+6} + \dfrac{x-2}{x-3} = \dfrac{x^2}{x^2+3x-18}$

SECTION 11.2: WORK-RATE PROBLEMS

Work-rate equation
If the first person does a job in **time A**, a second person does a job in **time B**, and together they can do a job in **time T** (*total*). We can use the work-rate equation: $$\underbrace{\frac{1}{A}}_{\textit{job per time A}} + \underbrace{\frac{1}{B}}_{\textit{job per time B}} = \underbrace{\frac{1}{T}}_{\textit{job per time T(team)}}$$

A. ONE UNKNOWN TIME

	MEDIA LESSON Work- rate problem (Duration 4:45)

View the video lesson, take notes and complete the problems below.

Adam does a job in 4 hours. Each hour he does _____ of the job.

Betty does a job in 12 hours. Each hour she does _____ of the job.

Together, each hour they do _____ of the job.

This means it takes them, working together, _____ hours to do the entire job.

Work Equation: _____   Use _____!

Example 1: Catherine can paint a house in 15 hours. Dan can paint it in 30 hours. How long will it take them working together?

Catherine: _____ hours

Dan: _____ hours

Team: _____ hours

Example 2: Even can clean a room in 3 hours. If his sister Faith helps, it takes them $2\frac{2}{5}$ hours. How long will it take Faith working alone?

Even: _____ hours

Faith: _____ hours

Team: _____ hours

YOU TRY

a) If worker A can do a piece of work alone in 6 days and worker B can do it alone in 4 days, how long will it take the two working together to complete the job?

	Time	Job per day
Worker A		
Worker B		
Together		

b) Adam can assemble a furniture set in 5 hours. If his sister Maria helps, they can finish it in 3 hours. How long will it take Maria to do the job alone?

	Time	Job per hour
Adam		
Maria		
Together		

QR	MEDIA LESSON
	Fill and drain problem (Duration 0:54)

View the video lesson, take notes and complete the problems below.

Example: One inlet pipe can fill an empty pool in 8 hours, and a drain can empty the pool in 12 hours. How long will it take the pipe to fill the pool if the drain is left open?

	Time	Rate
Inlet pipe		
Drain		
Together		

Fill-drain equation :

YOU TRY

a) A sink can be filled by a pipe in 5 minutes, but it takes 7 minutes to drain a full sink. If both the pipe and the drain are opened, how long will it take to fill the sink?

	Time	Fill per minute
Fill the sink		
Drain the sink		
Together		

B. TWO UNKNOWN TIMES

View the video lesson, take notes and complete the problems below.

Example: If Alfonso does a job in 30 hours less than Zoe, and they can do the job together in 8 hours, long will it take each to do the job alone?

	Time (hours)	Work rate
Alfonso		
Zoe		
Together		

YOU TRY

a) Mike takes twice as long as Rachel to complete a project. Together, they can complete a project in 10 hours. How long will it take each of them to complete a project alone?

	Time	Project/ hour
Mike		
Rachel		
Together		

b) Brittney can build a large shed in 10 days less than Cosmo. If they built it together, it would take 12 days. How long would it take each of them working alone?

	Time	Built per day
Cosmo		
Brittney		
Together		

EXERCISES

1) A tank can be filled by one pipe in 20 minutes and by another in 30 minutes. How long will it take both pipes together to fill the tank?

2) Tim can finish painting his barn in 10 hours. It takes his wife JoAnn only 8 hours to do the same job. If they work together, how long will it take them to complete the job?

3) Adan can do a piece of work in 3 days, Bernie in 4 days, and Cynthia in 6 days each working alone. How long will it take them to do it working together?

4) A carpenter and his assistant can do a piece of work in 3 days. If the carpenter himself could do the work alone in 5 days, how long would the assistant take to do the work alone?

5) A sink can be filled from the faucet in 5 minutes. It takes only 3 minutes to empty the sink when the drain is open. If the sink is full and both the faucet and the drain are open, how long will it take to empty the sink?

6) Of two inlet pipes, the smaller pipe takes 5 hours longer than the larger pipe to fill a pool. When both pipes are open, the pool is filled in 6 hours. If only the larger pipe is open, how many hours are required to fill the pool?

7) It takes John 16 minutes longer than Sally to mow the lawn. If they work together they can mow the lawn in 15 minutes. How long will it take each to mow the lawn if they work alone?

8) Bill's father can paint a room in 3 hours less than Bill can paint it. Working together they can complete the job in 2 hours. How much time would each require working alone?

9) Two workers, a trainer and a trainee, working together can do a job in 3 hours. The trainer is 3 times faster than the trainee to complete the same job. How long will it take the trainee to finish the same job?

10) The faucet alone can fill the sink in 6 minutes, while it takes 8 minutes to empty it with the drain. How long will it take to fill the sink?

11) It takes Roberto 8 hours longer than Paula to repair a transmission. If it takes them 3 hours to do the job if they work together, how long will it take each of them working alone?

12) A water tank is being filled by two inlet pipes. Pipe A can fill the tank in 4 hours, while both pipes together can fill the tank in 2 hours. How long does it take to fill the tank using only Pipe B?

13) Cheng takes 10 hours longer to pave a driveway than Sammy to do a job. Working together they can do the job in 12 hours. How long does it take each working alone?

SECTION 11.3: UNIFORM MOTION PROBLEMS

We can recall uniform motion problems in the word problems chapter. We used the formula $r \cdot t = d$ and organized the given information in a table. Now, we use the equation as:

$$t = \frac{d}{r}$$

We apply the same method in this section only the equations will be rational equations.

A. UNIFORM MOTION PROBLEMS

	MEDIA LESSON
	Uniform motion applications –part 1: set up the equation (Duration 4:09)

	MEDIA LESSON
	Uniform motion applications –part 2: solve the equation (Duration 5:26)

View the video lesson, take notes and complete the problems below.

Caitlyn went on a 56 mile trip to a soccer game. On the way back, due to road construction she had to drive 28 miles per hour slower. This made the trip take 1 hour longer. How fast did she drive to the soccer game?

Recall:

	r	t	d
To			
From			

YOU TRY

a) Greg went to a conference in a city 120 miles away. On the way back, due to road construction he had to drive 10 mph slower which resulted in the return trip taking 2 hours longer. How fast did he drive on the way to the conference?

B. UNIFORM MOTION PROBLEMS WITH STREAMS AND WINDS

Another type of uniform motion problem is where a boat is traveling in a river with the current or against the current (or an airplane is flying with the wind or against the wind). If a boat is traveling downstream, the current will push it or increase the rate by the speed of the current. If a boat is traveling upstream, the current will pull against it or decrease the rate by the speed of the current.

	MEDIA LESSON
	Uniform motion – Streams & winds – Part 1: set up the equation (Duration 4:28)

	MEDIA LESSON
	Uniform motion – Streams & winds – Part 2: solve the equation (Duration 5:55)

View the video lesson, take notes and complete the problems below.

Alicia rows a boat downstream for 182 miles. The return trip upstream took 12 hours longer. If the current flows at 3 mph, how fast does Alicia row in still water?

Recall: Uniform motion: _____

        Rational equation:_____

	r	t	d
Downstream (faster)			
Upstream (slower)			

YOU TRY

a) A man rows downstream for 30 miles then turns around and returns to his original location, the total trip took 8 hours. If the current flows at 2 miles per hour, how fast would the man row in still water?

EXERCISES

1) An athlete plans to row upstream a distance of 6 kilometers and then return to his starting point in a total time of 4 hours. If the rate of the current is 2 km/hr., how fast should he row?

2) A pilot flying at a constant rate against a headwind of 30 km/hr flew for 720 kilometers, then reversed direction and returned to his starting point. He completed the round trip in 10 hours. What was the speed of the plane?

3) Tim, an open water swimmer, is training for the Olympics. To do so, he swims in a stream that is 3m/h. Tim finds that he can swim 4 miles against the current in the same amount of time that he can swim 10 miles with the current. How fast can Tim swim with no current?

4) A plane flies against the wind 288 miles from LAX to San Jose and then returns home with the same wind. The wind speed is 60m/h. If the total flying time was 2 hours, what is the speed of the plane?

5) A salmon is swimming in a river that is flowing downstream at a speed of 2 miles per hour. The salmon can swim 12 miles upstream in the same amount of time it would take to swim 24 miles downstream. What is the speed of the salmon in still water?

SECTION 11.4: REVENUE PROBLEMS

Revenue problems are the problems where a person buys a certain number of items for a certain price per item. If we multiply the number of items by the price per item, we will get the total value. We can recall revenue problems in the word problems chapter. We used the formula AVT:

Amount · Value = Total

or   Number of items · Price = Revenue

n · p = R

	MEDIA LESSON
[QR code]	Rational equation applications – Revenue problems (Duration 9:16)

View the video lesson, take notes and complete the problems below.

Revenue table:

Number of items	Price	Revenue
n	**p**	**R**

To solve: Divide by_____

Example 1: A group of college students bought a couch for $80. However, five of them failed to pay their share so the others had to each pay $8 more. How many students were in the original group?

n	p	R

Example 2: A merchant bought several pieces of silk for $70. He sold all but two of them at a profit of $4 per piece. His total profit was $18. How many pieces did he originally purchase?

n	p	R

YOU TRY

a) A man buys several fish for $56. After three fish die, he decides to sell the rest at a profit of $5 per fish. His total profit was $4. How many fish did he buy to begin with?

b) A group of students bought a couch for their dorm at cost $96. However, 2 students failed to pay their share, so each student had to pay $4 more. How many students were in the original group?

EXERCISES

1) A merchant bought some pieces of silk for $900. He marked up $15 more per each piece and made $75 profit. Find the number of pieces purchased.

2) A group of students planned to chip in to raise a total of $100 for one of their friends' birthday party but 5 persons couldn't come so they didn't pay. This increased the share of the others by $1 each. Find the amount that each person paid after.

3) A merchant bought a number of barrels of apples for $120. He kept 2 barrels and sold the remainder at a profit of $2/barrel making a total profit of $34. How many barrels did he originally buy?

4) A dealer bought a number of sheep for $440. After 5 had died, he sold the remainder at a profit of $2 each, making a profit of $60 for the sheep. How many sheep did he originally purchase?

5) Orlando bought a number of hats at equal cost for $500. He sold all but 2 for $540 at a profit of $5 for each item. How many hats did he buy?

6) A fashion store bought a lot of suits for $750. The store sold all of them for $1,000 making a profit of $10 on each suit sold. How many suits did the store buy?

7) A group of schoolboys tried to raise for $4,500 for a local animal shelter. Five boys transferred to different schools before they had a chance to raise money, hence each remaining boy was compelled to raise $45 more. How many boys were in the original group and how much had each agreed to raise?

8) The total expenses of a camping party were $72. If there had been 3 fewer persons in the party, it would have cost each person $2 more than it did. How many people were in the party and how much did it cost each one?

CHAPTER REVIEW

KEY TERMS AND CONCEPTS	
Look for the following terms and concepts as you work through the workbook. In the space below, explain the meaning of each of these concepts and terms in your own words. Provide examples that are not identical to those in the text or in the media lesson.	
Extraneous solution	
Work-rate equation	

Solve. Be sure to verify all solutions.

1) $\dfrac{1}{x+2} - \dfrac{1}{2-x} = \dfrac{3x+8}{x^2-4}$

2) $\dfrac{x+1}{x-1} - \dfrac{x-1}{x+1} = \dfrac{5}{6}$

3) $\dfrac{3}{2x+1} + \dfrac{2x+1}{1-2x} = 1 - \dfrac{8x^2}{4x^2-1}$

4) $\dfrac{3}{x+2} + \dfrac{x-1}{x+5} = \dfrac{5x+20}{6x+24}$

5) $\dfrac{x}{x-1} - \dfrac{2}{x+1} = \dfrac{4x^2}{x^2-1}$

6) $\dfrac{x-3}{x-6} + \dfrac{x+5}{x+3} = -\dfrac{2x^2}{x^2-3x-18}$

7) $\dfrac{4x+1}{x+3} + \dfrac{5x-3}{x-1} = \dfrac{8x^2}{x^2+2x-3}$

8) $\dfrac{x^2+6}{x-1} + \dfrac{x-2}{x-1} = 2x$

9) $\dfrac{x}{x+3} - \dfrac{4}{x-2} = -\dfrac{5x^2}{x^2+x-6}$

10) $\dfrac{x}{x+1} - \dfrac{3}{x-2} = -\dfrac{x^2}{x^2-x-2}$

11) $\dfrac{x+3}{x-2} + \dfrac{x-2}{x+1} = \dfrac{9x^2}{x^2-x-2}$

Solve. Show your work to demonstrate your understanding.

12) If A can do a piece of work alone in 6 days and B can do it alone in 4 days, how long will it take the two working together to complete the job?

13) If Sam can do a certain job in 3 days, while it takes Fred 6 days to do the same job, how long will it take them, working together, to complete the job?

14) Two people working together can complete a job in 6 hours. If one of them works twice as fast as the other, how long would it take the faster person, working alone, to do the job?

15) If two people working together can do a job in 3 hours, how long will it take the slower person to do the same job if one of them is 3 times as fast as the other?

16) A train traveled 240 kilometers at a certain speed. When the engine was replaced by an improved model, the speed was increased by 20 km/hr and the travel time for the trip was decreased by 1 hour. What was the rate of each engine?

17) The rate of the current in a stream is 3 km/hr. A man rowed upstream for 3 kilometers and then returned. The round trip required 1 hour and 20 minutes. How fast was he rowing?

18) A pilot flying at a constant rate against a headwind of 50 km/hr flew for 750 kilometers, then reversed direction and returned to his starting point. He completed the round trip in 8 hours. What was the speed of the plane?

19) An athlete plans to row upstream a distance of 2 kilometers and then return to his starting point in a total time of 2 hours and 20 minutes. If the rate of the current is 2 km/hr, how fast should he row?

20) An automobile made a trip of 120 miles and then returned, the round trip occupying 7 hours. Returning, the rate was increased 10 miles an hour. Find the rate of each.

21) If a train had traveled 5 miles an hour faster, it would have needed $1\frac{1}{2}$ hours less time to travel 150 miles. Find the rate of the train.

22) A merchant bought some pieces of silk for $1800. Had he bought 3 pieces more for the same money, he would have paid $10 less for each piece. Find the number of pieces purchased.

23) A number of men subscribed a certain amount to make up a deficit of $100 but 5 men failed to pay and thus increased the share of the others by $1 each. Find the amount that each man paid.

24) By going 15 miles per hour faster, a train would have required 1 hour less to travel 180 miles. How fast did it travel?

25) The railroad distance between two towns is 240 miles. If the speed of a train was increased 4 miles an hour, the trip would take 40 minutes less. What is the usual rate of the train?

26) A traveler having 18 miles to go, calculates that his usual rate would make him one-half hour late for an appointment; he finds that in order to arrive on time he must travel at a rate one-half mile an hour faster. What is his usual rate?

27) A merchant bought a number of barrels of apples for $120. He kept two barrels and sold the remainder at a profit of $2 per barrel making a total profits of $34. How many barrels did he originally buy?

28) A dealer bought a number of sheep for $440. After 5 had died he sold the remainder at a profit of $2 each making a profit of $60 for the sheep. How many sheep did he originally purchase?

29) A man bought a number of articles at equal cost for $500. He sold all but two for $540 at a profit of $5 for each item. How many articles did he buy?