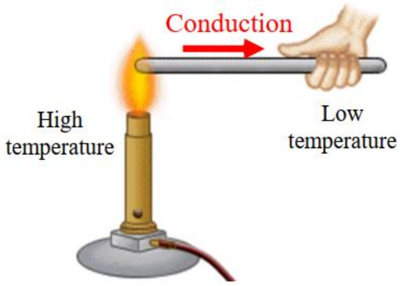
Chapter 12 Heat Conduction and Thermal Expansion

Curriculum Specification	**Remarks**		
	Before	**After**	**Revision**
12.1 Heat Conduction			
a) Define heat conduction. (C1, C2)			
b) Solve problem related to rate of heat transfer, $\frac{dQ}{dt} = -kA\frac{dT}{dx}$ through cross-sectional area (Maximum two insulated objects in series). (C3, C4)			
c) Discuss graphs of temperature-distance, *T*-*x* for heat conduction through insulated and non-insulated rods. (C1, C2)			
12.2 Thermal Expansion			
a) Define coefficient of linear, area and volume thermal expansion. (C1, C2)			
b) Solve problems related to thermal expansion of linear, area and volume (include expansion of liquid in a container: $\Delta L = \alpha L_o \Delta T$, $\beta = 2\alpha$, $\gamma = 3\alpha$ (C3, C4)			

12.1 Heat Conduction

- Heat conduction is a **process** whereby heat is transferred through a solid from a region of high temperature to a region of lower temperature.
- When the rod is in steady condition, the rate of heat flows (dQ/dt) is constant along the rod.



$$\frac{dQ}{dt} = -kA\left(\frac{dT}{dx}\right)$$

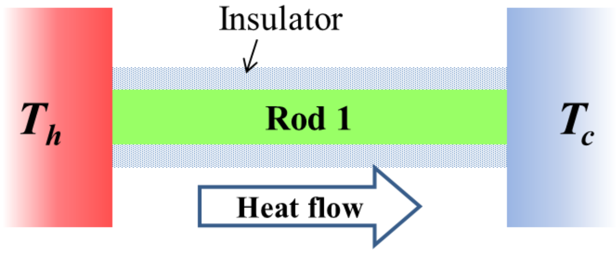
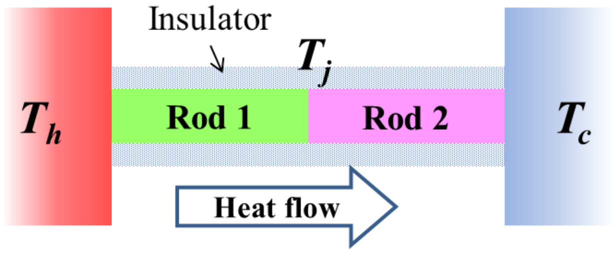
The temperature change, ***dT*** is the same in the **Kelvin and Celsius** scales.

– ve sign in the equation shows that heat always flow in the **direction** of decreasing temperature.

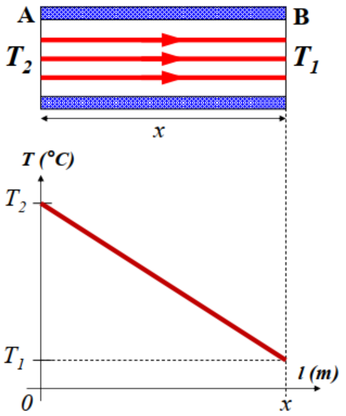
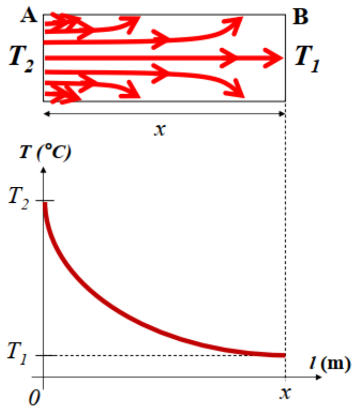
Thermal conductivity, *k*:
- The characteristic of heat conducting ability of a material.
- Indicator of how fast a material able to conduct heat. **Good conductors will have higher values of *k* compared to poor conductors.**

Temperature gradient, $\frac{dT}{dx}$:
- It is the temperature difference per unit length.

Example

One Rod	**Two Joined Rods**
	
$\frac{dQ}{dt} = -kA\left(\frac{dT}{dx}\right)$	$\left(\frac{dQ}{dt}\right)_1 = \left(\frac{dQ}{dt}\right)_2$
$\frac{dQ}{dt} = -kA\left(\frac{T_c - T_h}{dx}\right)$	$\left(-kA\frac{\boldsymbol{T_j} - T_h}{dx}\right)_1 = \left(-kA\frac{T_c - \boldsymbol{T_j}}{dx}\right)_2$

<u>Graph of Temperature-Distance, *T-x*</u>

<table>
<tr><th>Insulated</th><th>Non-insulated</th></tr>
<tr><th colspan="2">One Rod</th></tr>
<tr><td>
</td><td>
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- No heat loss through the side surface of the rod because is covered with insulator.
- All the heat flow from the hot end of A to the cold end of B.
- Temperature varies linearly with distance along the rod. Decreasing of temperature is uniform.

</td><td>

- Heat is lost to the surrounding from the sides of the rod
- Temperature – distance graph is a curve. The decreasing of temperature is not uniform.
- Loss of heat at A > loss of heat at B.

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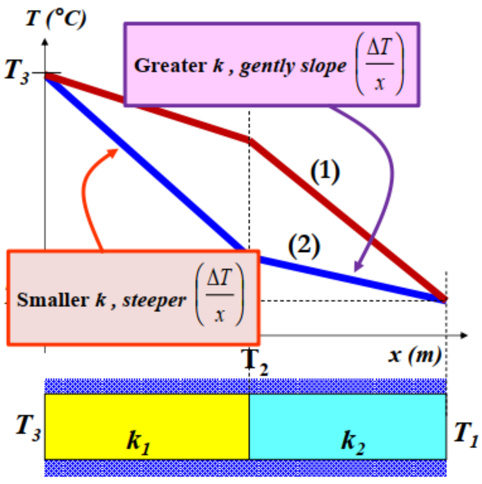
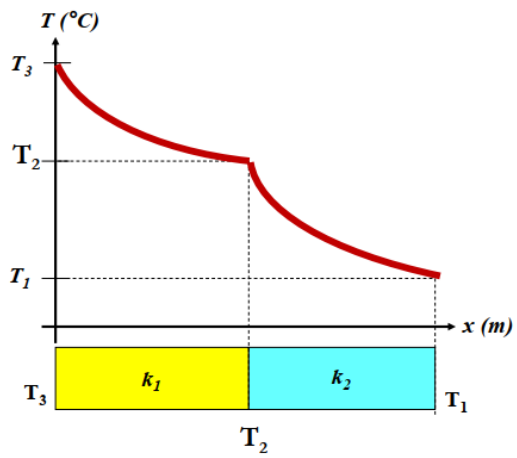
<tr><th colspan="2">Two Joined Rods</th></tr>
<tr><td>

If $k_1 > k_2 \rightarrow$ Line (1)
If $k_1 < k_2 \rightarrow$ Line (2)</td><td>
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- Since rods are insulated, no heat loss to surrounding.
- Temperature varies linearly with distance (straight line graph).
- Values of *k* are different, thus the temperature gradients are different for both rods.

</td><td>

- Heat loss to surrounding.
- Temperature decreases non uniformly with distance (curve line graph).
- Values of *k* are different, thus the curves are different for both rods.

</td></tr>
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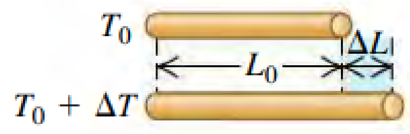
12.2 Thermal Expansion

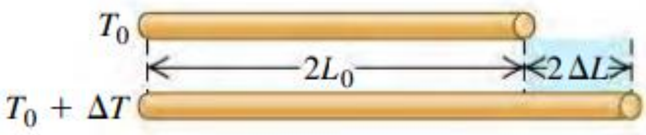
- Most materials expand when heated and contract when cooled. **Thermal expansion** is a consequence of the change in the dimensions of a body accompanying a change in temperature.
- 3 types of expansion: **Linear** expansion. **area** expansion, **volume** expansion
- In solid, all types of thermal expansion are occurred.
- In liquid and gas, only volume expansion is occurred.
- At the **same temperature**, the **gas expands greater than liquid and solid**.

Linear Expansion

- Suppose a rod of material has a length L_o at some initial temperature T_o. When the temperature changes by ΔT, the length changes by ΔL.

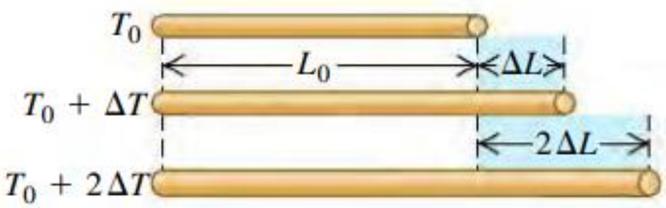


- If the initial length is doubled, ΔL will also become doubled.



$$\boldsymbol{\Delta L \propto L_o}$$

- When the temperature changes by $2\Delta T$, the length changes by $2\Delta L$.



$$\boldsymbol{\Delta L \propto \Delta T}$$

- Thus, we can conclude that $\boldsymbol{\Delta L \propto L_o \Delta T}$. In equation:

$$\Delta L = \alpha L_o \Delta T$$

- Since $\Delta L = L - L_o$, the equation can also be written as

$$L = L_o(1 + \alpha \Delta T)$$

Coefficient of linear expansion, *α*:

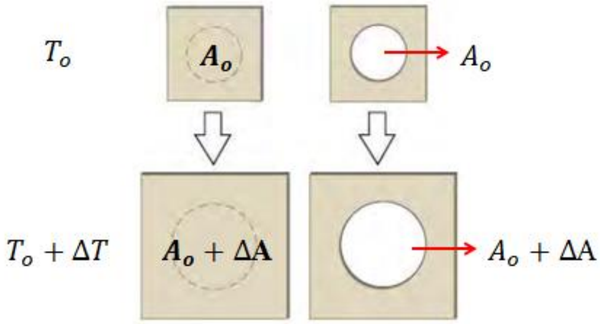
- The fractional change in length per degree change in temperature.

$$\alpha = \frac{\Delta L}{L_o \Delta T}$$

- Unit: K^{-1} or $(°C)^{-1}$

Area Expansion

- Suppose a plate of material has a area/hole A_o at some initial temperature T_o.



- When the temperature changes by ΔT, the area/hole changes by ΔA.
- Thus, we can conclude that $\boldsymbol{\Delta A \propto A_o \Delta T}$. In equation:

$$\boxed{\Delta A = \beta A_o \Delta T}$$

- Since $\Delta A = A - A_o$, the equation can also be written as

$$\boxed{A = A_o(1 + \beta \Delta T)}$$

Coefficient of area expansion, β:

- The fractional change in area per degree change in temperature.

$$\boxed{\beta = \frac{\Delta A}{A_o \Delta T}}$$

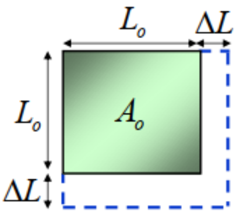
- Unit: K^{-1} or $(°C)^{-1}$
- The coefficient of area expansion is twice as much as the coefficient of linear expansion.

$$\boxed{\beta = 2\alpha}$$

Additional Knowledge

Proof of $\beta = 2\alpha$

Consider a square plate with side length, l_0 is heated and expands uniformly as shown below



$$A_o = L_o^2 \text{ and } A = L^2 \text{ where } L = L_o + \Delta L$$

$$A = (L_o + \Delta L)^2$$

$$A = L_o^2 + 2L_o\Delta L + (\Delta L)^2$$

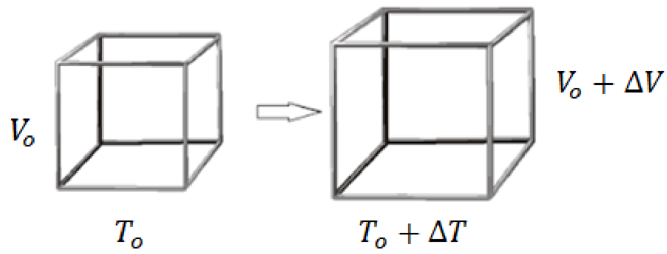
Since ΔL is small, then $(\Delta L)^2 \simeq 0$

Thus we get, $A = L_o^2\left(1 + 2\dfrac{\Delta L}{L_o}\right)$ where $L_o^2 = A_o$ and $\dfrac{\Delta L}{L_o} = \alpha \Delta T$

$A = A_0(1 + 2\alpha \Delta T)$ compare with $A = A_0(1 + \beta \Delta T)$

Volume Expansion

- Suppose a cube of material has a volume V_o at some initial temperature T_o.



- When the temperature changes by ΔT, the volume changes by ΔV
- Thus, we can conclude that $\boldsymbol{\Delta V \propto V_o \Delta T}$. In equation:

$$\boxed{\Delta V = \gamma V_o \Delta T}$$

- Since $\Delta V = V - V_o$, the equation can also be written as

$$\boxed{V = V_o(1 + \gamma \Delta T)}$$

Coefficient of volume expansion, *γ*:

- The fractional change in volume per degree change in temperature.

$$\boxed{\gamma = \frac{\Delta V}{V_o \Delta T}}$$

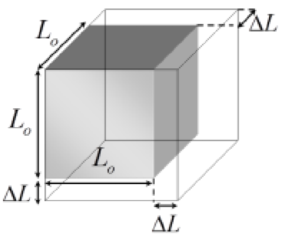
- Unit: K^{-1} or $(°C)^{-1}$
- The coefficient of volume expansion is three times as much as the coefficient of linear expansion.

$$\boxed{\gamma = 3\alpha}$$

Additional Knowledge

Proof of $\gamma = 3\alpha$

Consider a metal cube with side length, L_o is heated and expands uniformly as shown



$$V_o = L_o^3 \text{ and } V = L^3 \text{ where } L = L_o + \Delta L$$

$$V = (L_o + \Delta L)^3$$

$$V = L_o^3 + 3L_o^2 \Delta L + 3L_o \Delta L^2 + (\Delta L)^3$$

Since ΔL is small, then $(\Delta L)^2 \simeq 0$ and $(\Delta L)^3 \simeq 0$

Thus we get, $V = L_o^2\left(1 + 3\dfrac{\Delta L}{L_o}\right)$ where $L_o^3 = V_o$ and $\dfrac{\Delta L}{L_o} = \alpha \Delta T$

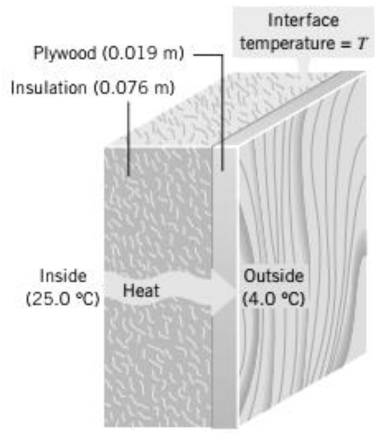
$V = V_o(1 + 3\alpha \Delta T)$ compare with $V = V_o(1 + \gamma \Delta T)$

Expansion of Liquid in a Container

- Thermal expansion of liquid depends on the expansion of the container that it fills.
- The liquid in a container overflow or not when temperature rises depends on the change in volume of both the liquid and the container.
- When temperature increases, both the liquid & container expand.
- If they were to expand by same amount, there would be no overflow.
- However, $\gamma_{liquid} > \gamma_{solid}$, thus liquid expands much more than the container and liquid will spill out from the container.
- Overflow volume is given by

$$\Delta V_{\text{overflow}} = \Delta V_{\text{liquid}} - \Delta V_{\text{container}}$$

Exercise

	Heat Conduction
1.	A total of 215 W heat flows by conduction from the blood capillaries under the skin to the body's surface area. It is given that the surface area of the body and the temperature gradient between the blood capillaries and the skin are 1.3 m^2 and –826.97 K m^{-1} respectively. Determine the thermal conductivity of the human tissue. **(Answer: 0.20 W m^{-1} °C^{-1})**
2.	When excessive heat is produced within the body, it must be transferred to the skin and dispersed if the temperature at the body interior is to be maintained at the normal value of 37.0 °C. One possible mechanism for transfer is conduction through body fat. Suppose that heat travels through 0.030 m of fat in reaching the skin, which has a total surface area of 1.7 m^2 and a temperature of 34.0 °C. Find the amount of heat that reaches the skin in half an hour. **(Answer: 6.1 × 10^4 J)**
3.	A rod 1.30 m long consists of a 0.800 m length of aluminium joined end to end to a 0.50 m length of brass. The free end of the aluminium section is maintained at 150.0 °C and the free end of the brass piece is maintained at 20.0 °C. No heat is lost through the sides of the rod. At steady state, Calculate the temperature of the point where the two metals are joined. Given k of aluminium = 205 J s^{-1} m^{-1} °C^{-1} and k of brass = 109 J s^{-1} m^{-1} °C^{-1}. **(Answer: 90.2 °C)**
4.	One wall of a house consists of 0.019 m thick plywood backed by 0.076 m thick insulation, as shown Figure. The temperature at the inside surface is 25.0 °C, while the temperature at the outside surface is 4.0 °C, both being constant. The thermal conductivities of the insulation and the plywood are, respectively, 0.03 and 0.08 J s^{-1} m^{-1} °C^{-1}, and the area of the wall is 35 m^2. Find the heat conducted through the wall in one hour a) with the insulation. b) without the insulation. **(Answer: 9.5×10^5 J; 110×10^5 J)** 

	Thermal Expansion
1.	The steel bed of a suspension bridge is 200 m long at 20 °C. If the extremes of temperature to which it might be exposed are −30 °C and +40 °C, how much will it contract and expand? **(Answer: 4.8×10^{-2} m; −12.0×10^{-2} m)**
2.	A sheet of copper has an area of 600 cm^2 when the temperature is 10°C. Find the area of this sheet when the temperature is 60°C. Given α = 1.7 × 10^{-5} K^{-1}. **(Answer: 601.02 cm^2)**
3.	A metal sphere with radius of 9.0 cm at 30.0 °C is heated until the temperature of 100.0 °C. Determine the percentage of change in density for that sphere. Given density, $\rho = m/V$ and $\gamma_{metal\ sphere}$ = 5.1 × 10^{-5} °C^{-1}. **(Answer: 0.36 %)**
4.	A 1000 cm^3 glass thermos is filed with mercury at room temperature 20 °C. When it is heated at temperature 100°C, 8 cm^3 of mercury spilt out. If the coefficient of volume expansion of mercury is 18.2×10^{-5} °C^{-1}, determine the coefficient of volume expansion of glass. **(Answer: 8.2×10^{-5} °C)**