Chapter 12: Radiation Heat Transfer

Radiation differs from *Conduction* and *Convection* heat t transfer mechanisms, in the sense that it does not require the presence of a material medium to occur.

Energy transfer by radiation occurs at the *speed of light* and suffers no attenuation in vacuum.

Radiation can occur between two bodies separated by a medium colder than both bodies.

According to *Maxwell theory*, energy transfer takes place via *electromagnetic waves* in radiation. Electromagnetic waves transport energy like other waves and travel at the speed of light.

Electromagnetic waves are characterized by their frequency v (Hz) and wavelength λ (μm), where:

$$\lambda = c / v$$

where c is the speed of light in that medium; in a vacuum $c_0 = 2.99 \times 10^8$ m / s. Note that the frequency and wavelength are inversely proportional.

The speed of light in a medium is related to the speed of light in a vacuum,

$$c = c_0 / n$$

where n is the index of refraction of the medium, n = 1 for air and n = 1.5 for water.

Note that the frequency of an electromagnetic wave depends only on the source and is independent of the medium.

The frequency of an electromagnetic wave can range from a few cycles to millions of cycles and higher per second.

Einstein postulated another theory for electromagnetic radiation. Based on this theory, electromagnetic radiation is the propagation of a collection of discrete packets of energy called *photons*. In this view, each photon of frequency v is considered to have energy of

$$e = hv = hc / \lambda$$

where $h = 6.625 \times 10^{-34}$ *J.s* is the *Planck's* constant.

Note that in Einstein's theory h and c are constants, thus the energy of a photon is inversely proportional to its wavelength. Therefore, shorter wavelength radiation possesses more powerful photon energies (X-ray and gamma rays are highly destructive).

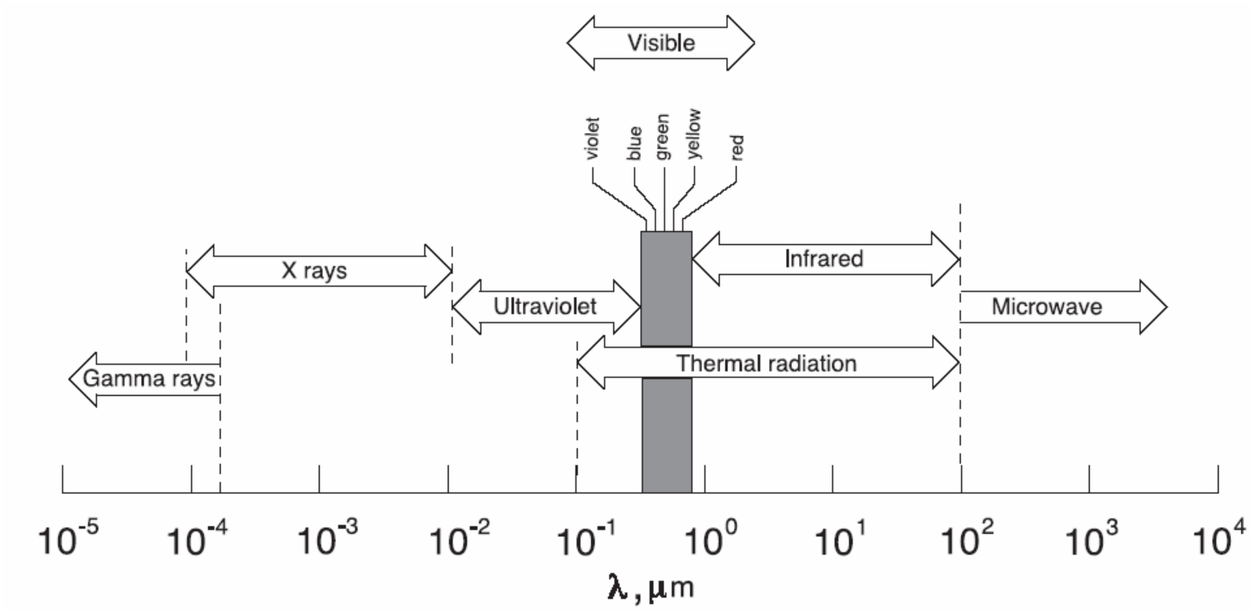


Fig. 12-1: Electromagnetic spectrum.

Electromagnetic radiation covers a wide range of wavelength, from 10^{-10} μm for cosmic rays to 10^{10} μm for electrical power waves.

As shown in Fig. 12-1, thermal radiation wave is a narrow band on the electromagnetic wave spectrum.

Thermal radiation emission is a direct result of vibrational and rotational motions of molecules, atoms, and electrons of a substance. Temperature is a measure of these activities. Thus, the rate of thermal radiation emission increases with increasing temperature.

What we call light is the visible portion of the electromagnetic spectrum which lies within the thermal radiation band.

Thermal radiation is a volumetric phenomenon. However, for opaque solids such as metals, radiation is considered to be a surface phenomenon, since the radiation emitted by the interior region never reach the surface.

Note that the radiation characteristics of surfaces can be changed completely by applying thin layers of coatings on them.

Blackbody Radiation

A blackbody is defined as a perfect emitter and absorber of radiation. At a specified temperature and wavelength, no surface can emit more energy than a blackbody.

A blackbody is a *diffuse emitter* which means it emits radiation uniformly in all direction. Also a blackbody absorbs all incident radiation regardless of wavelength and direction.

The radiation energy emitted by a blackbody per unit time and per unit surface area can be determined from the *Stefan-Boltzmann* Law:

$$E_b = \sigma T^4 \quad \left(W/m^2\right)$$
$$where$$
$$\sigma = 5.67 \times 10^{-8} \frac{W}{m^2 K^4}$$

where T is the absolute temperature of the surface in K and E_b is called the blackbody emissive power.

A large cavity with a small opening closely resembles a blackbody.

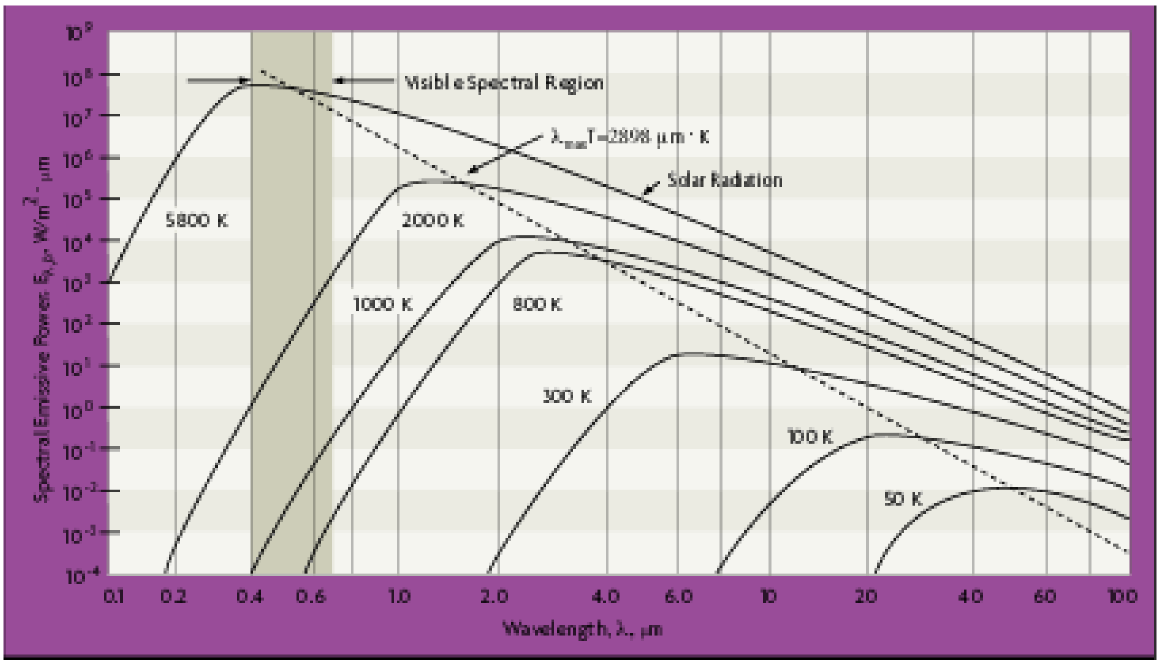


Fig. 12-2: Variation of blackbody emissive power with wavelength

Spectral blackbody emissive power is the amount of radiation energy emitted by a blackbody at an absolute temperature T per unit time, per unit surface area, and per unit wavelength.

$$E_{b\lambda}(T) = \frac{C_1}{\lambda^5\left[\exp(C_2/\lambda T)-1\right]} \quad \left(\frac{W}{m^2.\mu m}\right)$$
$$C_1 = 2\pi h c_0^2 = 3.742 \times 10^8 \left(W.\mu m^4/m^2\right)$$
$$C_2 = hc_0/k = 1.439 \times 10^4 \left(\mu m.K\right)$$
$$k = 1.3805 \times 10^{-23} \left(J/K\right) \text{Boltzmann's constant}$$

This is called *Plank's distribution law* and is valid for a surface in a vacuum or gas. For other mediums, it needs to be modified by replacing C_1 by C_1/n^2, where n is the index of refraction of the medium, The wavelength at which the peak emissive power occurs for a given temperature can be obtained from Wien's displacement law:

$$(\lambda T)_{\max\ power} = 2897.8 \quad \mu m.K$$

It can be shown that integration of the spectral blackbody emissive power $E_{b\lambda}$ over the entire wavelength spectrum gives the total blackbody emissive power E_b:

$$E_b(T) = \int_0^\infty E_{b\lambda}(T)d\lambda = \sigma T^4 \quad (W/m^2)$$

The Stefan-Boltzmann law gives the total radiation emitted by a blackbody at all wavelengths from 0 to infinity. But, we are often interested in the amount of radiation emitted over some wavelength band.

To avoid numerical integration of the Planck's equation, a non-dimensional quantity f_λ is defined which is called the *blackbody radiation function* as

$$f_\lambda(T) = \frac{\int_0^\lambda E_{b\lambda}(T)d\lambda}{\sigma T^4}$$

The function f_λ represents the fraction of radiation emitted from a blackbody at temperature T in the wavelength band from 0 to λ. Table 12-2 in Cengel book lists f_λ as a function of λT.

Therefore, one can write:

$$f_{\lambda_1-\lambda 2}(T) = f_{\lambda 2}(T) - f_{\lambda 1}(T)$$
$$f_{\lambda-\infty}(T) = 1 - f_\lambda(T)$$

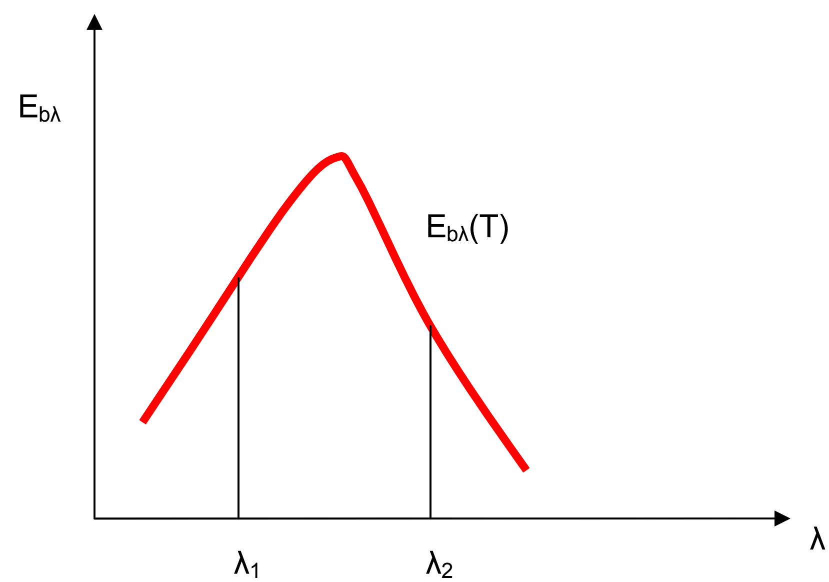


Fig. 12-3: Fraction of radiation emitted in the wavelength between λ_1 and λ_2

Example 12-1

The temperature of the filament of a light bulb is 2500 K. Assuming the filament to be a blackbody, determine the fraction of the radiant energy emitted by the filament that falls in the visible range. Also determine the wavelength at which the emission of radiation from the filament peaks.

Solution

The visible range of the electromagnetic spectrum extends from 0.4 to 0.76 micro meter. Using Table 12-2:

$$\lambda_1 T = 0.4\mu m(2500K) = 1000\mu m.K \rightarrow f_{\lambda 1} = 0.000321$$
$$\lambda_2 T = 0.76\mu m(2500K) = 1900\mu m.K \rightarrow f_{\lambda 2} = 0.053035$$
$$f_{\lambda 2} - f_{\lambda 1} = 0.05271$$

which means only about 5% of the radiation emitted by the filament of the light bulb falls in the visible range. The remaining 95% appears in the infrared region or the “invisible light”.

Radiation Properties

A blackbody can serve as a convenient reference in describing the emission and absorption characteristics of real surfaces.

Emissivity

The emissivity of a surface is defined as the ratio of the radiation emitted by the surface to the radiation emitted by a blackbody at the same temperature. Thus,

$$0 \le \varepsilon \le 1$$

Emissivity is a measure of how closely a surface approximate a blackbody, $\varepsilon_{blackbody} = 1$.

The emissivity of a surface is not a constant; it is a function of temperature of the surface and wavelength and the direction of the emitted radiation, ε = ε (T, λ, θ) where θ is the angle between the direction and the normal of the surface.

The *total emissivity* of a surface is the average emissivity of a surface over all direction and wavelengths:

$$\varepsilon(T) = \frac{E(T)}{E_b(T)} = \frac{E(T)}{\sigma\, T^4} \rightarrow E(T) = \varepsilon(T)\,\sigma\, T^4$$

Spectral emissivity is defined in a similar manner:

$$\varepsilon_\lambda(T) = \frac{E_\lambda(T)}{E_{b\lambda}(T)}$$

where $E_\lambda(T)$ is the spectral emissive power of the real surface. As shown, the radiation emission from a real surface differs from the Planck’s distribution.

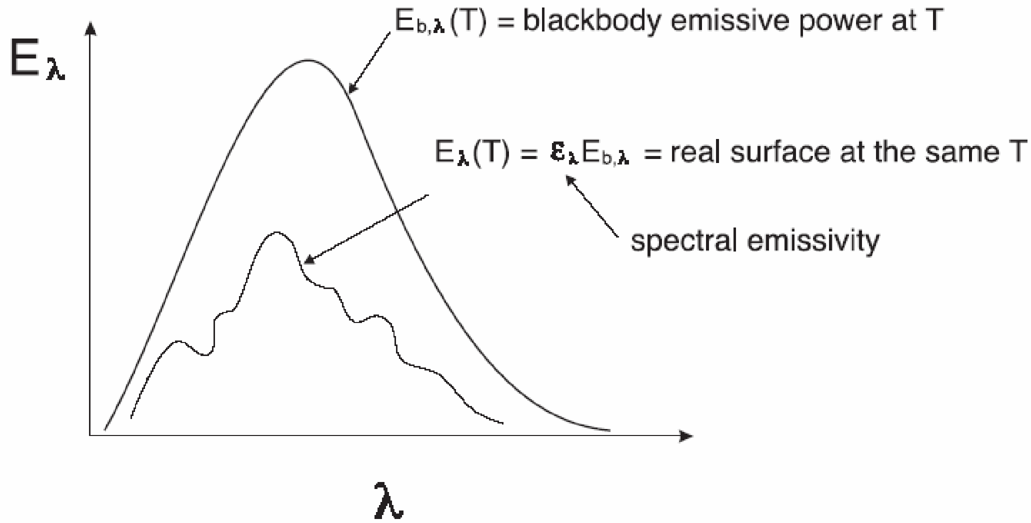


Fig. 12-4: Comparison of the emissive power of a real surface and a blackbody.

To make the radiation calculations easier, we define the following approximations:

<u>Diffuse surface</u>: is a surface which its properties are independent of direction.

<u>Gray surface</u>: is a surface which its properties are independent from wavelength.

Therefore, the emissivity of a gray, diffuse surface is the total hemispherical (or simply the total) emissivity of that surface.

A gray surface should emit as much as radiation as the real surface it represents at the same temperature:

$$\varepsilon(T) = \frac{\int_0^\infty \varepsilon_\lambda(T) E_{b\lambda}(T) d\lambda}{\sigma T^4}$$

Emissivity is a strong function of temperature, see Fig. 12-20 Cengel book.

Absorptivity, Reflectivity, and Transmissivity

The radiation energy incident on a surface per unit area per unit time is called *irradiation*, G.

<u>Absorptivity</u> α: is the fraction of irradiation absorbed by the surface.

<u>Reflectivity</u> ρ: is the fraction of irradiation reflected by the surface.

<u>Transmissivity</u> τ: is the fraction of irradiation transmitted through the surface.

<u>Radiosity</u> J: total radiation energy streaming from a surface, per unit area per unit time. It is the summation of the reflected and the emitted radiation.

$$\text{absorptivity}: \quad \alpha = \frac{\text{absorbed radiation}}{\text{incident radiation}} = \frac{G_{abs}}{G} \qquad 0 \le \alpha \le 1$$

$$\text{reflectivity}: \quad \rho = \frac{\text{reflected radiation}}{\text{incident radiation}} = \frac{G_{ref}}{G} \qquad 0 \le \rho \le 1$$

$$\text{transmissivity}: \quad \tau = \frac{\text{transmitted radiation}}{\text{incident radiation}} = \frac{G_{tr}}{G} \qquad 0 \le \tau \le 1$$

Applying the first law of thermodynamics, the sum of the absorbed, reflected, and the transmitted radiation radiations must be equal to the incident radiation:

$$G_{abs} + G_{ref} + G_{tr} = G$$

Divide by G:

$$\alpha + \rho + \tau = 1$$

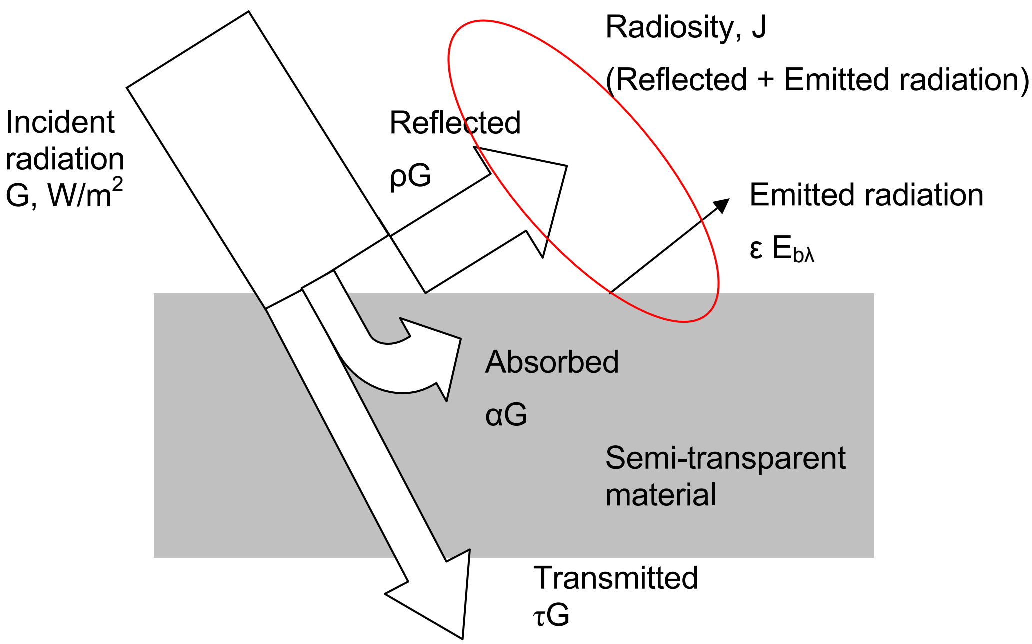


Fig. 12-5: The absorption, reflection, and transmission of irradiation by a semi-transparent material.

For opaque surfaces $\tau = 0$ and thus: $\alpha + \rho = 1$. The above definitions are for total hemi-spherical properties (over all direction and all frequencies). We can also define these properties in terms of their spectral counterparts:

$$
\begin{aligned}
&G_\lambda = \rho_\lambda G_\lambda + \tau_\lambda G_\lambda + \alpha_\lambda G_\lambda \\
&where \\
&\rho_\lambda = \rho_\lambda(T,\lambda) \qquad \text{spectral reflectivity} \\
&\alpha_\lambda = \alpha_\lambda(T,\lambda) \qquad \text{spectral absorptivity} \\
&\tau_\lambda = \tau_\lambda(T,\lambda) \qquad \text{spectral transmissivity} \\
&\text{thus} \\
&1 = \rho_\lambda + \tau_\lambda + \alpha_\lambda
\end{aligned}
$$

Note that the *absorptivity* α is almost *independent of surface temperature* and it strongly depends on the temperature of the *source* at which the incident radiation is originating. For example α of the concrete roof is about 0.6 for solar radiation (source temperature 5762 K) and 0.9 for radiation originating from the surroundings (source temperature 300 K).

Kirchhoff's Law

Consider an isothermal cavity and a surface at the same temperature T. At the steady state (equilibrium) thermal condition

$$G_{abs} = \alpha\, G = \alpha\, \sigma\, T^4$$

and radiation emitted

$$E_{emit} = \varepsilon\, \sigma\, T^4$$

Since the small body is in thermal equilibrium, $G_{abs} = E_{emit}$

$$\varepsilon(T) = \alpha(T)$$

The total hemispherical emissivity of a surface at temperature T is equal to its total hemi-spherical absorptivity for radiation coming from a blackbody at the same temperature T. This is called the *Kirchhoff's law*.

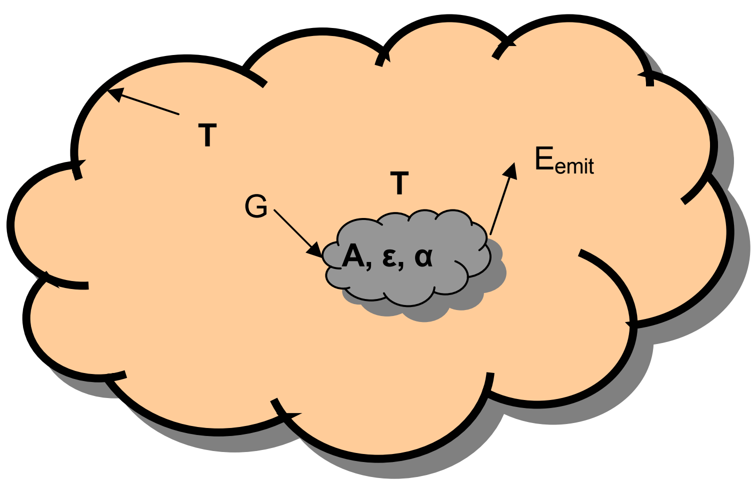


Fig. 12-6: Small body contained in a large isothermal cavity.

The Kirchhoff's law can be written in the spectral form:

$$\varepsilon_\lambda(T) = \alpha_\lambda(T)$$

and in the spectral directional form

$$\varepsilon_{\lambda,\theta}(T) = \alpha_{\lambda,\theta}(T)$$

The Kirchhoff's law makes the radiation analysis easier ($\varepsilon = \alpha$), especially for opaque surfaces where $\rho = 1 - \alpha$.

Note that Kirchhoff's law cannot be used when there is a large temperature difference (more than 100 K) between the surface and the source temperature.

Solar Radiation

The solar energy reaching the edge of the earth's atmosphere is called the *solar constant*:

$$G_s = 1353 \text{ W / m}^2$$

Owing to the ellipticity of the earth's orbit, the actual solar constant changes throughout the year within +/- 3.4%. This variation is relatively small; thus G_s is assumed to be a constant.

The effective surface temperature of the sun can be estimated from the solar constant (by treating the sun as a blackbody).

The solar radiation undergoes considerable attenuation as it passes through the atmosphere as a result of absorption and scattering:

- Absorption by the oxygen occurs in a narrow band about $\lambda = 0.76$ μm.
- The ozone layer absorbs ultraviolet radiation at wavelengths below $\lambda = 0.3$ μm almost completely and radiation in the range of 0.3 – 0.4 μm considerably.
- Absorption in the infrared region is dominated by water vapor and carbon dioxide. Dust/pollutant particles also absorb radiation at various wavelengths.
- As a result the solar radiation reaching the earth's surface is about 950 W/m^2 on a clear day and much less on a cloudy day, in the wavelength band 0.3 to 2.5 μm.

Scattering and reflection by air molecules (and other particles) are other mechanisms that attenuate the solar radiation. Oxygen and nitrogen molecules scatter radiation at short wavelengths (corresponding to violet and blue colors). That is the reason the sky seems blue!

The gas molecules (mostly CO_2 and H_2O) and the suspended particles in the atmosphere emit radiation as well as absorbing it. It is convenient to consider the atmosphere (sky) as a *blackbody* at some lower temperature. This fictitious temperature is called the effective sky temperature T_{sky}.

$$G_{sky} = \sigma T^4_{sky}$$

$$T_{sky} = 230 \text{ K for cold clear sky}$$

$$T_{sky} = 285 \text{ K for warm cloudy sky}$$

Using Kirchhoff's law we can write α = ε since the temperature of the sky is on the order of the room temperature.

The View Factor

Radiation heat transfer between surfaces depends on the orientation of the surfaces relative to each other as well as their radiation properties and temperatures.

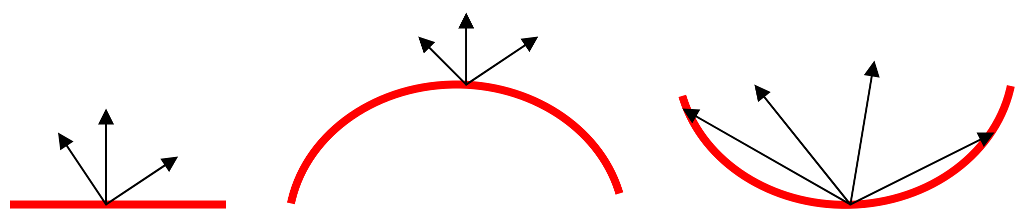
View factor (or shape factor) is a purely geometrical parameter that accounts for the effects of orientation on radiation between surfaces.

In view factor calculations, we assume uniform radiation in all directions throughout the surface, i.e., surfaces are *isothermal* and *diffuse*. Also the medium between two surfaces *does not absorb, emit, or scatter radiation*.

$F_{i\rightarrow j}$ *or* F_{ij} = the fraction of the radiation leaving surface *i* that strikes surface *j* directly.

Note the following:

- The view factor ranges between zero and one.
- $F_{ij} = 0$ indicates that two surfaces do not see each other directly. $F_{ij} = 1$ indicates that the surface j completely surrounds surface i.
- The radiation that strikes a surface does not need to be absorbed by that surface.
- F_{ii} is the fraction of radiation leaving surface *i* that strikes itself directly. $F_{ii} = 0$ for plane or convex surfaces, and $F_{ii} \neq 0$ for concave surfaces.



Plane surface, $F_{ii} = 0$

Convex surface, $F_{ii} = 0$

Concave surface, $F_{ii} \neq 0$

Fig. 12-7: View factor between surface and itself.

Calculating view factors between surfaces are usually very complex and difficult to perform. View factors for selected geometries are given in Table 12-4 and !2-5 and Figs. 12-41 to 12-44 in Cengel book.

View Factor Relations

Radiation analysis of an enclosure consisting of N surfaces requires the calculations of N^2 view factors. However, all of these calculations are not necessary. Once a sufficient number of view factors are available, the rest of them can be found using the following relations for view factors.

The Reciprocity Rule

The view factor F_{ij} is not equal to F_{ji} unless the areas of the two surfaces are equal. It can be shown that:

$$A_i F_{ij} = A_j F_{ji}$$

The Summation Rule

In radiation analysis, we usually form an enclosure. The conservation of energy principle requires that the entire radiation leaving any surface *i* of an enclosure be intercepted by the surfaces of enclosure. Therefore,

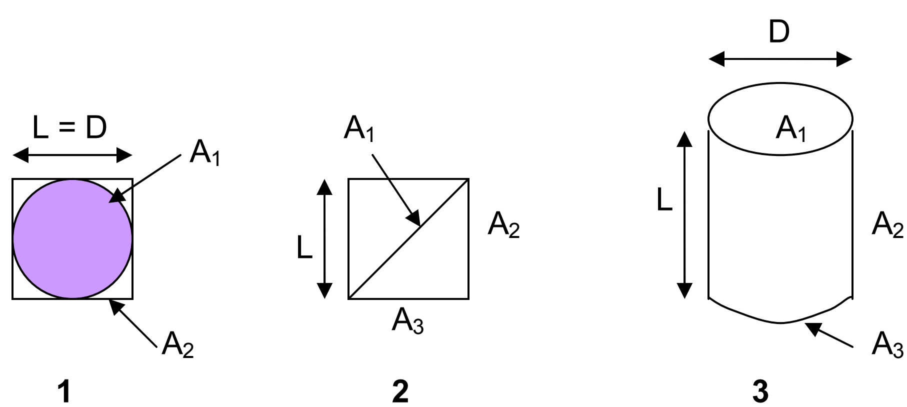
$$\sum_{j=1}^{N} F_{ij} = 1$$

The summation rule can be applied to each surface of an enclosure by varying i from 1 to N (number of surfaces). Thus the summation rule gives N equations. Also reciprocity rule gives 0.5 N (N-1) additional equations. Therefore, the total number of view factors that need to be evaluated directly for an N-surface enclosure becomes

$$N^2 - \left[N + \frac{1}{2}N(N-1)\right] = \frac{1}{2}N(N-1)$$

Example 12-2

Determine the view factors F_{12} and F_{21} for the following geometries:



1) Sphere of diameter D inside a cubical box of length L = D.

2) Diagonal partition within a long square duct.

3) End and side of a circular tube of equal length and diameter, L = D.

Assumptions:

Diffuse surfaces.

Solution:

1) sphere within a cube:

By inspection, $F_{12} = 1$

By reciprocity and summation:

$$F_{21} = \frac{A_1}{A_2} F_{12} = \frac{\pi D^2}{6L^2} \times 1 = \frac{\pi}{6}$$

$$F_{21} + F_{22} = 1 \rightarrow F_{22} = 1 - \frac{\pi}{6}$$

2) Partition within a square duct:

From summation rule, $F_{11} + F_{12} + F_{13} = 1$ where $F_{11} = 0$

By symmetry $F_{12} = F_{13}$

Thus, $F_{12} = 0.5$.

From reciprocity:

$$F_{21} = \frac{A_1}{A_2} F_{12} = \frac{\sqrt{2}L}{L} \times 0.5 = 0.71$$

3) Circular tube: from Fig. 12-43, with $r_2 / L = 0.5$ and $L / r_1 = 2$, $F_{13} \approx 0.17$.

From summation rule,

$F_{11} + F_{12} + F_{13} = 1$ with $F_{11} = 0$, $F_{12} = 1 - F_{13} = 0.83$

From reciprocity,

$$F_{21} = \frac{A_1}{A_2} F_{12} = \frac{\pi D^2 / 4}{\pi D L} \times 0.83 = 0.21$$

The Superposition Rule

The view factor from a surface i to a surface j is equal to the sum of the view factors from surface i to the parts of surface j.

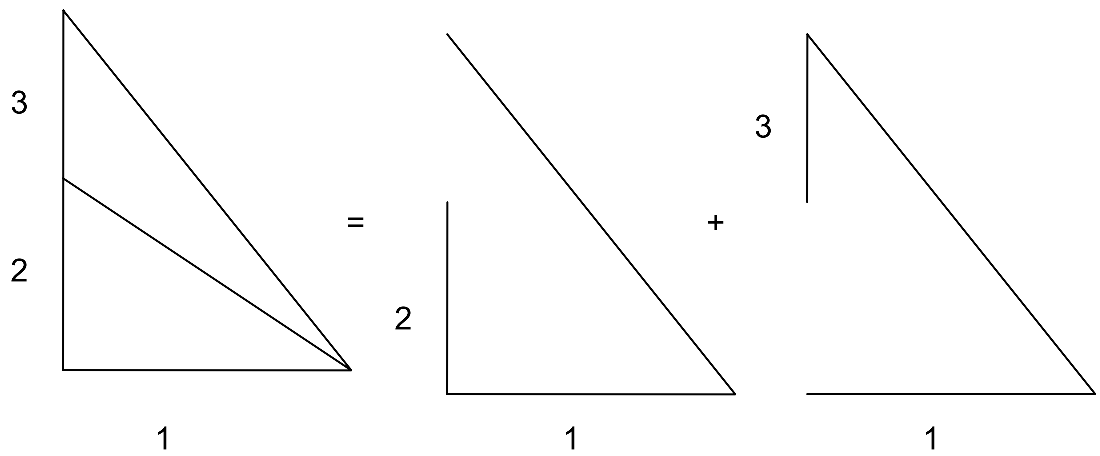


Fig. 12-8: The superposition rule for view factors.

$$F_{1\rightarrow(2,3)} = F_{1\rightarrow 2} + F_{1\rightarrow 3}$$

The Symmetry Rule

Two (or more) surfaces that possess symmetry about a third surface will have identical view factors from that surface.

Example: 12-3

Find the view factor from the base of a pyramid to each of its four sides. The base is a square and its side surfaces are isosceles triangles.

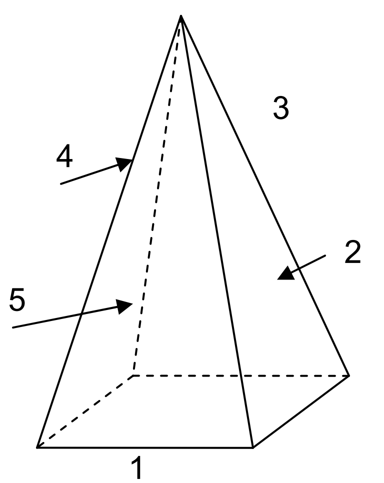
From symmetry rule, we have:

$$F_{12} = F_{13} = F_{14} = F_{15}$$

Also, the summation rule yields:

$$F_{11} + F_{12} + F_{13} + F_{14} + F_{15} = 1$$

Since, $F_{11} = 0$ (flat surface), we find; $F_{12} = F_{13} = F_{14} = F_{15} = 0.25$



Pyramid in example 12-2.

The Crossed-Strings Method

Geometries such as channels and ducts that are very long in one direction can be considered two-dimensional (since radiation through end surfaces can be neglected). The view factor between their surfaces can be determined by cross-string method developed by H. C. Hottel, as follows:

$$F_{i\rightarrow j} = \frac{\sum(\text{crossed strings}) - \sum(\text{uncrossed strings})}{2\times(\text{string on surface i})}$$

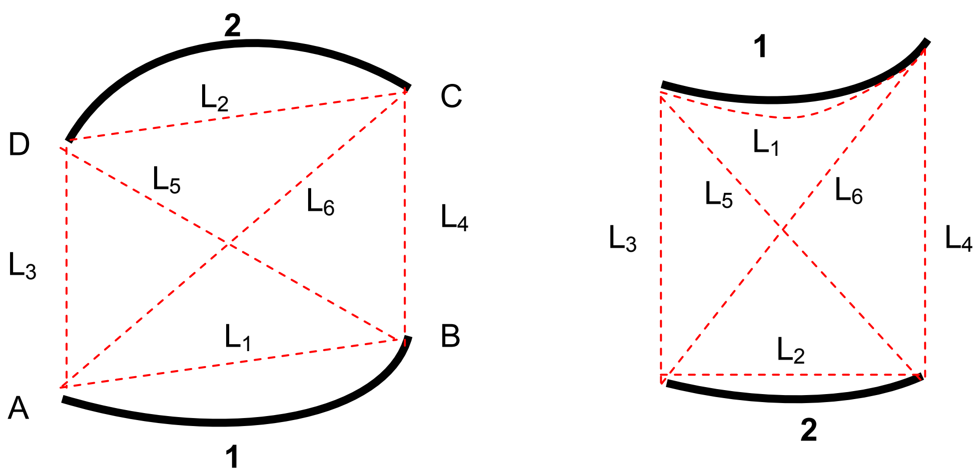


Fig. 12-9: Cross-string method.

$$F_{12} = \frac{(L_5 + L_6) - (L_3 + L_4)}{2L_1}$$

Note that the surfaces do not need to be flat.

Radiation Heat Transfer

The analysis of radiation exchange between surfaces is complicated because of reflection. This can be simplified when surfaces are assumed to be black surfaces.

The net radiation between two surfaces can be expressed as

$$\dot{Q}_{12} = \begin{pmatrix}\text{radiation leaving surface 1}\\ \text{that directly stikes surface 2}\end{pmatrix} - \begin{pmatrix}\text{radiation leaving surface 2}\\ \text{that directly stikes surface 1}\end{pmatrix}$$

$$\dot{Q}_{12} = A_1 F_{12} E_{b1} - A_2 F_{21} E_{b2} \quad (W)$$

Applying reciprocity $A_1 F_{12} = A_2 F_{21}$ yields

$$\dot{Q}_{12} = A_1 F_{12} \sigma (T_1^4 - T_2^4) \quad (W)$$

Consider an enclosure consisting of N black surfaces maintained at specified temperatures. For each surface *i*, we can write

$$\dot{Q}_i = \sum_{j=1}^{N} \dot{Q}_{ij} = \sum_{j=1}^{N} A_i F_{ij} \sigma \left(T_i^4 - T_j^4 \right) \qquad (W)$$

Using the sign convention, a negative heat transfer rate indicates that the radiation heat transfer is to surface *i* (heat gain).

Now, we can extend this analysis to non-black surfaces. It is common to assume that the surfaces are **opaque**, **diffuse**, and **gray**. Also, surfaces are considered to be **isothermal**. Also the fluid inside the cavity is not participating in the radiation.

Radiosity J is the total radiation energy streaming from a surface, per unit area per unit time. It is the summation of the reflected and the emitted radiation.

For a surface i that is gray and opaque ($\varepsilon_i = \alpha_i$ and $\alpha_i + \rho_i = 1$), the Radiosity can be expressed as

$$J_i = \varepsilon_i E_{bi} + \rho_i G_i$$
$$J_i = \varepsilon_i E_{bi} + (1 - \varepsilon_i) G_i \qquad (W/m^2)$$
$$J_i = \varepsilon_i E_{bi} = \sigma T_i^4 \qquad (\text{for a blackbody})$$

Note that the radiosity of a blackbody is equal to its emissive power.

Using an energy balance, the net rate of radiation heat transfer from a surface i of surface area A_i can be expressed as

$$\dot{Q}_i = A_i (J_i - G_i) \qquad (W)$$
$$\dot{Q}_i = A_i \left(J_i - \frac{J_i - \varepsilon_i E_{bi}}{1 - \varepsilon_i} \right) = \frac{A_i \varepsilon_i}{1 - \varepsilon_i} (E_{bi} - J_i)$$

In electrical analogy to Ohm's law, a thermal resistance can be defined as

$$\dot{Q}_i = \frac{E_{bi} - J_i}{R_i}$$
$$R_i = \frac{1 - \varepsilon_i}{A_i \varepsilon_i}$$

where R_i is called the *surface resistance* to radiation.

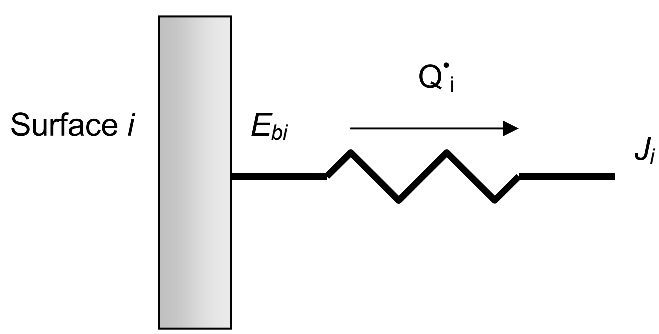


Fig. 12-10: Surface resistance to radiation.

Note that the surface resistance to radiation for a blackbody is zero.

For *insulated* or *adiabatic* surfaces, the net heat transfer through them is zero. In this cases, the surface is called *reradiating surface*. There is no net heat transfer to a reradiating surface.

Net Radiation between Two Surfaces

Consider two diffuse, gray, and opaque surfaces of arbitrary shape maintained at uniform temperatures. The net rate of radiation heat transfer from surface i to surface j can be expressed

$$\dot{Q}_{ij} = A_i J_i F_{ij} - A_j J_j F_{ji} \qquad (W)$$

$$\text{Applying reciprocity}$$

$$\dot{Q}_{ij} = A_i F_{ij} \left(J_i - J_j \right) \qquad (W)$$

In analogy with Ohm's law, a resistance can be defined as

$$\dot{Q}_{ij} = \frac{J_i - J_j}{R_{ij}}$$

$$R_{ij} = \frac{1}{A_i F_{ij}}$$

where R_{ij} is called the *space resistance* to radiation.

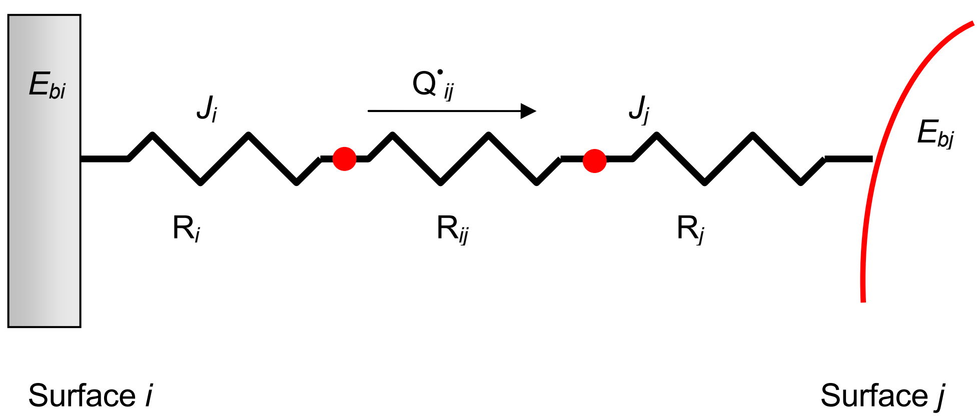


Fig. 12-11: Electrical network, surface and space resistances.

In an N-surface enclosure, the conservation of energy principle requires that the net heat transfer from surface i to be equal to the sum of the net heat transfers from i to each of the N surfaces of the enclosure.

$$\dot{Q}_i = \sum_{j=1}^{N} \dot{Q}_{ij} = \sum_{j=1}^{N} \frac{J_i - J_j}{R_{ij}} \qquad (W)$$

We have already derived a relationship for the net radiation from a surface

$$\dot{Q}_i = \frac{E_{bi} - J_i}{R_i} \qquad (W)$$

Combining these two relationships gives:

$$\frac{E_{bi} - J_i}{R_i} = \sum_{j=1}^{N} \frac{J_i - J_j}{R_{ij}} \qquad (W)$$

Method of Solving Radiation Problem

In radiation problems, either the temperature or the net rate of heat transfer must be given for each of the surfaces to obtain a unique solution for the unknown surface temperature and heat transfer rates.

We use the *network method* which is based on the electrical network analogy. The following steps should be taken:

1. Form an enclosure; consider fictitious surface(s) for openings, room, etc.
2. Draw a surface resistance associated with each surface of the enclosure
3. Connect the surface resistances with space resistances
4. Solve the radiations problem (radiosities) by treating it as an electrical network problem.

Note that this method is not practical for enclosures with more than 4 surfaces.

Example 12-4: Hot Plates in Room

Two parallel plates 0.5 by 1.0 m are spaced 0.5 m apart. One plate is maintained at 1000°C and the other at 500°C. The emissivities of the plates are 0.2 and 0.5, respectively. The plates are located in a very large room, the walls of which are maintained at 27°C. The plates exchange heat with each other and with the room, but only the plate surfaces facing each other are to be considered in the analysis. Find the net heat transfer rate to each plate and the room; neglect other modes of heat transfer, i.e., conduction and convection.

Assumptions:

Diffuse, gray, and opaque surfaces and steady-state heat transfer.

Solution:

This is a three-body problem, the two plates and room. The radiation network is shown below.

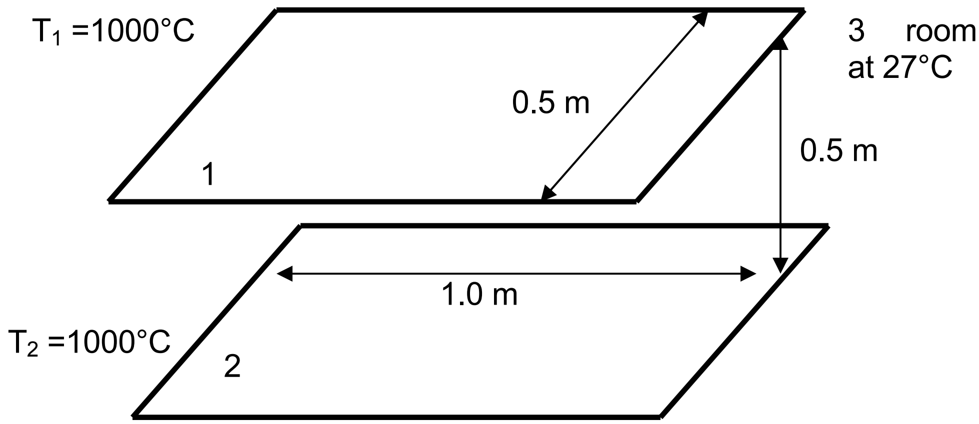


Fig. 12-12: Schematic for Problem 12-4

where,

T_1 = 1000°C = 1273 K $\qquad$ $A_1 = A_2$ = 0.5 m2

T_2 = 500°C = 773 K $\qquad$ T_3 = 27°C = 300 K

ε_1 = 0.2 $\qquad$ ε_2 = 0.5

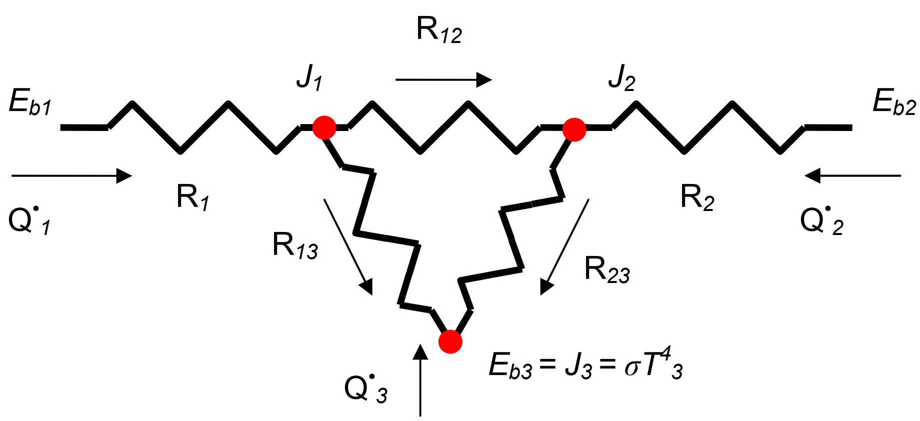


Fig. 12-12: Thermal network for Problem 12-4.

We can assume that the room is a blackbody, since its surface resistance is negligible:

$$R_3 = \frac{1-\varepsilon_3}{A_3\varepsilon_3} \approx 0 \qquad A_3 >>$$

From Fig. 12-41 in Cengel book, the shape factor F12 = 0.285

Using reciprocity and $A_1 = A_2$, $F_{12} = F_{21} = 0.285$

Applying summation rule

$$F_{11} + F_{12} + F_{13} = 1$$

Since $F_{11} = 0$ (flat plate), $F_{13} = 1 - 0.285 = 0.715$

Finally, from symmetry $F_{23} = F_{13} = 0.715$

The surface resistances are

$$R_1 = \frac{1-\varepsilon_1}{A_1\varepsilon_1} = \frac{1-0.2}{(0.2)(0.5m^2)} = 8.0$$

$$R_2 = \frac{1-\varepsilon_2}{A_2\varepsilon_2} = \frac{1-0.5}{(0.5)(0.5m^2)} = 2.0$$

Space resistances are

$$R_{12} = \frac{1}{A_1F_{12}} = \frac{1}{(0.285)(0.5m^2)} = 7.018$$

$$R_{23} = \frac{1}{A_2F_{23}} = \frac{1}{(0.715)(0.5m^2)} = 2.797$$

$$R_{13} = \frac{1}{A_1F_{13}} = \frac{1}{(0.175)(0.5m^2)} = 2.797$$

We need to find the radiosity for surface 1 and 2 only, since surface 3 is a blackbody, $J_3 = E_{b3} = \sigma T^4_3$

For node J_1:

$$\frac{E_{b1}-J_1}{R_1} + \frac{J_2-J_1}{R_{12}} + \frac{J_3-J_1}{R_{13}} = 0$$

For node J_2:

$$\frac{E_{b2}-J_2}{R_2} + \frac{J_1-J_2}{R_{12}} + \frac{J_3-J_2}{R_{23}} = 0$$

where

$$E_{b1} = \sigma T_1^4 = 148.87 \quad kW/m^2$$

$$E_{b2} = \sigma T_2^4 = 20.241 \quad kW/m^2$$

$$J_3 = E_{b3} = \sigma T_3^4 = 0.4592 \quad kW/m^2$$

Substituting values and solving two equations, one finds:

$J_1 = 33.469$ kW/m2 and $J_2 = 15.054$ kW/m2

The total heat loss by plate 1 is:

$$\dot{Q}_1 = \frac{E_{b1}-J_1}{R_1} = 14.425 \quad kW$$

$$\dot{Q}_2 = \frac{E_{b2}-J_2}{R_2} = 2.594 \quad kW$$

The total radiation received by the room is

$$\dot{Q}_3 = \frac{J_1 - J_3}{R_{13}} + \frac{J_2 - J_3}{R_{23}} = 17.020 \quad kW$$

Note that from an energy balance, we must have:

$$\dot{Q}_3 = \dot{Q}_1 + \dot{Q}_2$$