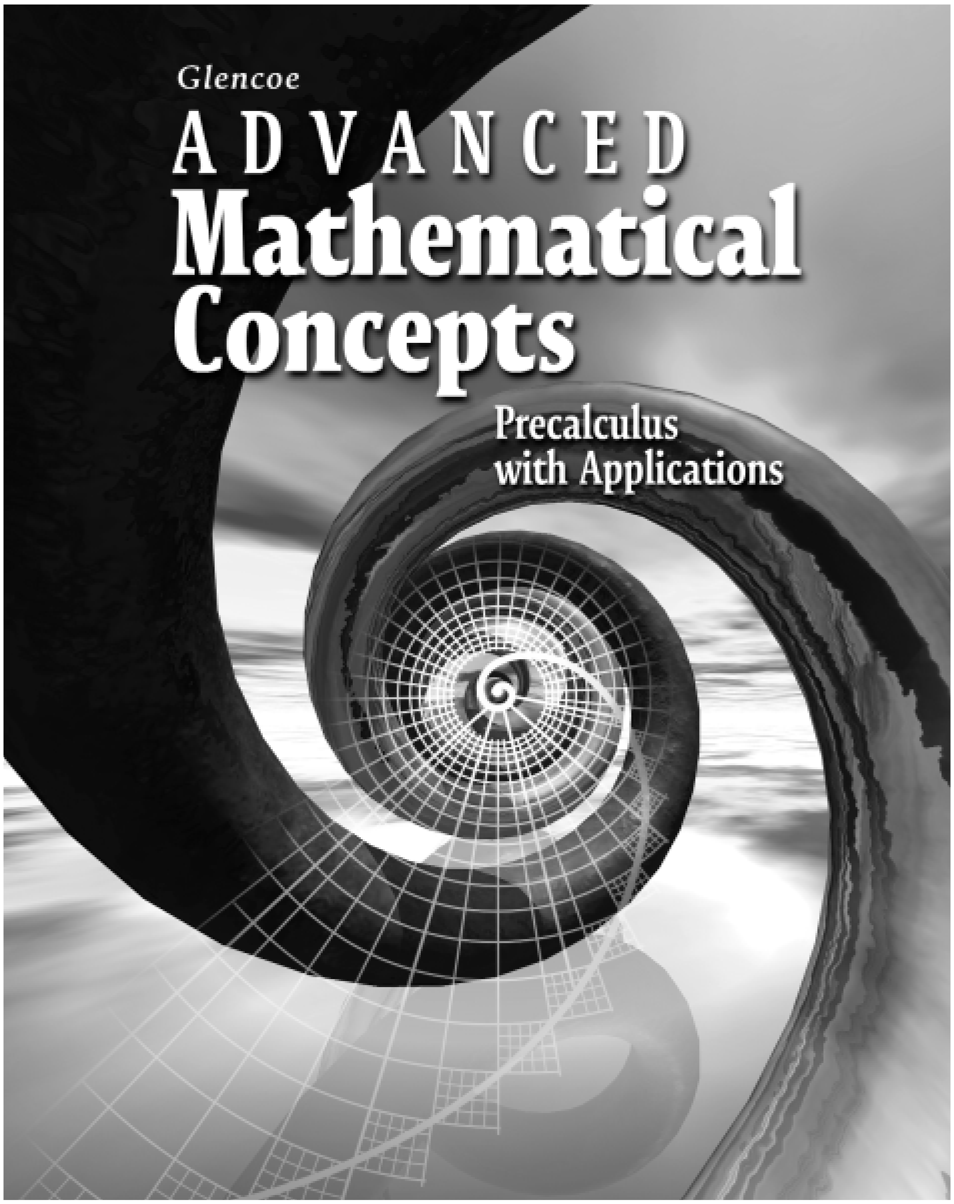
# Chapter 12
# Resource Masters





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**StudentWorks™** This CD-ROM includes the entire Student Edition along with the Study Guide, Practice, and Enrichment masters.

**TeacherWorks™** All of the materials found in this booklet are included for viewing and printing in the *Advanced Mathematical Concepts TeacherWorks* CD-ROM.

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*Advanced Mathematical Concepts*
*Chapter 12 Resource Masters*

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# Contents

# A Teacher's Guide to Using the Chapter 12 Resource Masters

The ***Fast File*** Chapter Resource system allows you to conveniently file the resources you use most often. The *Chapter 12 Resource Masters* include the core materials needed for Chapter 12. These materials include worksheets, extensions, and assessment options. The answers for these pages appear at the back of this booklet.

All of the materials found in this booklet are included for viewing and printing in the *Advanced Mathematical Concepts TeacherWorks* CD-ROM.

**Vocabulary Builder** Pages vii-ix include a student study tool that presents the key vocabulary terms from the chapter. Students are to record definitions and/or examples for each term. You may suggest that students highlight or star the terms with which they are not familiar.

*When to Use* Give these pages to students before beginning Lesson 12-1. Remind them to add definitions and examples as they complete each lesson.

**Study Guide** There is one Study Guide master for each lesson.

*When to Use* Use these masters as reteaching activities for students who need additional reinforcement. These pages can also be used in conjunction with the Student Edition as an instructional tool for those students who have been absent.

**Practice** There is one master for each lesson. These problems more closely follow the structure of the Practice section of the Student Edition exercises. These exercises are of average difficulty.

*When to Use* These provide additional practice options or may be used as homework for second day teaching of the lesson.

**Enrichment** There is one master for each lesson. These activities may extend the concepts in the lesson, offer a historical or multicultural look at the concepts, or widen students' perspectives on the mathematics they are learning. These are not written exclusively for honors students, but are accessible for use with all levels of students.

*When to Use* These may be used as extra credit, short-term projects, or as activities for days when class periods are shortened.

## Assessment Options

The assessment section of the *Chapter 12 Resources Masters* offers a wide range of assessment tools for intermediate and final assessment. The following lists describe each assessment master and its intended use.

## Chapter Assessments

### *Chapter Tests*

- *Forms 1A, 1B, and 1C* Form 1 tests contain multiple-choice questions. Form 1A is intended for use with honors-level students, Form 1B is intended for use with average-level students, and Form 1C is intended for use with basic-level students. These tests are similar in format to offer comparable testing situations.
- *Forms 2A, 2B, and 2C* Form 2 tests are composed of free-response questions. Form 2A is intended for use with honors-level students, Form 2B is intended for use with average-level students, and Form 2C is intended for use with basic-level students. These tests are similar in format to offer comparable testing situations.

  All of the above tests include a challenging Bonus question.
- The **Extended Response Assessment** includes performance assessment tasks that are suitable for all students. A scoring rubric is included for evaluation guidelines. Sample answers are provided for assessment.

## Intermediate Assessment

- A **Mid-Chapter Test** provides an option to assess the first half of the chapter. It is composed of free-response questions.
- Four free-response **quizzes** are included to offer assessment at appropriate intervals in the chapter.

## Continuing Assessment

- The **SAT and ACT Practice** offers continuing review of concepts in various formats, which may appear on standardized tests that they may encounter. This practice includes multiple-choice, quantitative-comparison, and grid-in questions. Bubble-in and grid-in answer sections are provided on the master.
- The **Cumulative Review** provides students an opportunity to reinforce and retain skills as they proceed through their study of advanced mathematics. It can also be used as a test. The master includes free-response questions.

## Answers

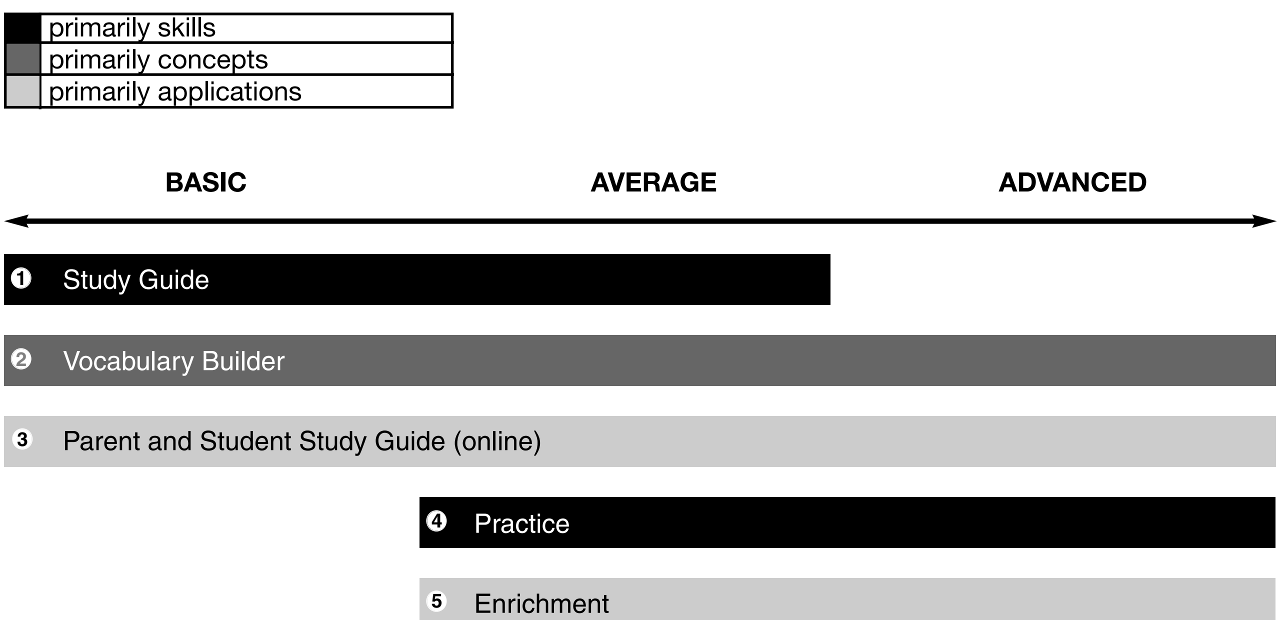
- Page A1 is an answer sheet for the SAT and ACT Practice questions that appear in the Student Edition on page 835. Page A2 is an answer sheet for the SAT and ACT Practice master. These improve students' familiarity with the answer formats they may encounter in test taking.
- The answers for the lesson-by-lesson masters are provided as reduced pages with answers appearing in red.
- Full-size answer keys are provided for the assessment options in this booklet.

# Chapter 12 Leveled Worksheets

Glencoe's **leveled worksheets** are helpful for meeting the needs of every student in a variety of ways. These worksheets, many of which are found in the **FAST FILE Chapter Resource Masters**, are shown in the chart below.

- **Study Guide** masters provide worked-out examples as well as practice problems.
- Each chapter's **Vocabulary Builder** master provides students the opportunity to write out key concepts and definitions in their own words.
- **Practice** masters provide average-level problems for students who are moving at a regular pace.
- **Enrichment** masters offer students the opportunity to extend their learning.

**Five Different Options to Meet the Needs of Every Student in a Variety of Ways**



Chapter 12

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# Reading to Learn Mathematics

## *Vocabulary Builder*

**This is an alphabetical list of the key vocabulary terms you will learn in Chapter 12. As you study the chapter, complete each term's definition or description. Remember to add the page number where you found the term.**

| Vocabulary Term | Found on Page | Definition/Description/Example |
|---|---|---|
| arithmetic mean | | |
| arithmetic sequence | | |
| arithmetic series | | |
| Binomial Theorem | | |
| common difference | | |
| common ratio | | |
| comparison test | | |
| convergent series | | |
| divergent series | | |
| escaping point | | |

*(continued on the next page)*

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# Reading to Learn Mathematics

## *Vocabulary Builder* *(continued)*

| Vocabulary Term | Found on Page | Definition/Description/Example |
|---|---|---|
| Euler's Formula | | |
| exponential series | | |
| Fibonacci sequence | | |
| fractal geometry | | |
| geometric mean | | |
| geometric sequence | | |
| geometric series | | |
| index of summation | | |
| infinite sequence | | |
| infinite series | | |
| limit | | |

*(continued on the next page)*

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# Reading to Learn Mathematics

## *Vocabulary Builder* *(continued)*

| Vocabulary Term | Found on Page | Definition/Description/Example |
|---|---|---|
| mathematical induction | | |
| *n* factorial | | |
| *n*th partial sum | | |
| orbit | | |
| Pascal's Triangle | | |
| prisoner point | | |
| ratio test | | |
| recursive formula | | |
| sequence | | |
| sigma notation | | |
| term | | |
| trigonometric series | | |

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# Study Guide

## Arithmetic Sequences and Series

A **sequence** is a function whose domain is the set of natural numbers. The **terms** of a sequence are the range elements of the function. The difference between successive terms of an **arithmetic sequence** is a constant called the **common difference**, denoted as $d$. An **arithmetic series** is the indicated sum of the terms of an arithmetic sequence.

*Example 1* **a. Find the next four terms in the arithmetic sequence −7, −5, −3, . . . .**
**b. Find the 38th term of this sequence.**

**a.** Find the common difference.
$a_2 - a_1 = -5 - (-7)$ or 2

The common difference is 2. Add 2 to the third term to get the fourth term, and so on.
$a_4 = -3 + 2$ or $-1$ $\quad a_5 = -1 + 2$ or 1
$a_6 = 1 + 2$ or 3 $\quad a_7 = 3 + 2$ or 5

The next four terms are −1, 1, 3, and 5.

**b.** Use the formula for the $n$th term of an arithmetic sequence.
$a_n = a_1 + (n - 1)d$
$a_{38} = -7 + (38 - 1)2 \quad n = 38, a_1 = -7, d = 2$
$a_{38} = 67$

*Example 2* **Write an arithmetic sequence that has three arithmetic means between 3.2 and 4.4.**

The sequence will have the form 3.2, _?_, _?_, _?_, 4.4.

First, find the common difference.
$a_n = a_1 + (n - 1)d$
$4.4 = 3.2 + (5 - 1)d \quad n = 5, a_5 = 4.4, a_1 = 3.2$
$4.4 = 3.2 + 4d$
$d = 0.3$

Then, determine the arithmetic means.

| $a_2$ | $a_3$ | $a_4$ |
|---|---|---|
| 3.2 + 0.3 = 3.5 | 3.5 + 0.3 = 3.8 | 3.8 + 0.3 = 4.1 |

The sequence is 3.2, 3.5, 3.8, 4.1, 4.4.

*Example 3* **Find the sum of the first 50 terms in the series 11 + 14 + 17 + · · · + 158.**

$S_n = \frac{n}{2}(a_1 + a_n)$
$S_{50} = \frac{50}{2}(11 + 158) \quad n = 50, a_1 = 11, a_{50} = 158$
$= 4225$

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12-1

# Practice

## Arithmetic Sequences and Series

***Find the next four terms in each arithmetic sequence.***

**1.** $-1.1, 0.6, 2.3, \ldots$ **2.** $16, 13, 10, \ldots$ **3.** $p, p + 2, p + 4, \ldots$

***For exercises 4–12, assume that each sequence or series is arithmetic.***

**4.** Find the 24th term in the sequence for which $a_1 = -27$ and $d = 3$.

**5.** Find $n$ for the sequence for which $a_n = 27$, $a_1 = -12$, and $d = 3$.

**6.** Find $d$ for the sequence for which $a_1 = -12$ and $a_{23} = 32$.

**7.** What is the first term in the sequence for which $d = -3$ and $a_6 = 5$?

**8.** What is the first term in the sequence for which $d = -\frac{1}{3}$ and $a_7 = -3$?

**9.** Find the 6th term in the sequence $-3 + \sqrt{2}, 0, 3 - \sqrt{2}, \ldots$.

**10.** Find the 45th term in the sequence $-17, -11, -5, \ldots$.

**11.** Write a sequence that has three arithmetic means between 35 and 45.

**12.** Write a sequence that has two arithmetic means between $-7$ and 2.75.

**13.** Find the sum of the first 13 terms in the series $-5 + 1 + 7 + \cdots + 67$.

**14.** Find the sum of the first 62 terms in the series $-23 - 21.5 - 20 - \cdots$.

**15.** ***Auditorium Design*** Wakefield Auditorium has 26 rows, and the first row has 22 seats. The number of seats in each row increases by 4 as you move toward the back of the auditorium. What is the seating capacity of this auditorium?

 

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# Enrichment

## Quadratic Formulas for Sequences

An ordinary arithmetic sequence is formed using a rule such as $bn + c$. The first term is $c$, $b$ is called the common difference, and $n$ takes on the values 0, 1, 2, 3, and so on. The value of term $n + 1$ equals $b(n + 1) + c$ or $bn + b + c$. So, the value of a term is a function of the term number.

Some sequences use quadratic functions. A method called *finite differences* can be used to find the values of the terms. Notice what happens when you subtract twice as shown in this table.

| $n$ | $an^2 + bn + c$ | | |
|---|---|---|---|
| 0 | $c$ | | |
| | | $a + b$ | |
| 1 | $a + b + c$ | | $2a$ |
| | | $3a + b$ | |
| 2 | $4a + 2b + c$ | | $2a$ |
| | | $5a + b$ | |
| 3 | $9a + 3b + c$ | | $2a$ |
| | | $7a + b$ | |
| 4 | $16a + 4b + c$ | | |

A sequence that yields a common difference after two subtractions can be generated by a quadratic expression. For example, the sequence 1, 5, 12, 22, 35, . . . gives a common difference of 3 after two subtractions. Using the table above, you write and solve three equations to find the general rule. The equations are $1 = c$, $5 = a + b + c$, and $12 = 4a + 2b + c$.

***Solve each problem.***

**1.** Refer to the sequence in the example above. Solve the system of equations for $a$, $b$, and $c$ and then find the quadratic expression for the sequence. Then write the next three terms.

**2.** The number of line segments connecting $n$ points forms the sequence 0, 0, 1, 3, 6, 10, . . . , in which $n$ is the number of points and the term value is the number of line segments. What is the common difference after the second subtraction? Find a quadratic expression for the term value.

**3.** The maximum number of regions formed by $n$ chords in a circle forms the sequence 1, 2, 4, 7, 11, 16, . . . (A chord is a line segment joining any two points on a circle.) Draw circles to illustrate the first four terms of the sequence. Then find a quadratic expression for the term value.

12-2

# Study Guide

## Geometric Sequences and Series

A **geometric sequence** is a sequence in which each term after the first, $a_1$, is the product of the preceding term and the **common ratio**, $r$. The terms between two nonconsecutive terms of a geometric sequence are called **geometric means.** The indicated sum of the terms of a geometric sequence is a **geometric series.**

***Example 1*** **Find the 7th term of the geometric sequence 157, −47.1, 14.13, . . . .**

First, find the common ratio.
$a_2 \div a_1 = -47.1 \div 157$ or $-0.3$
The common ratio is $-0.3$.

Then, use the formula for the $n$th term of a geometric sequence.
$a_n = a_1 r^{n-1}$
$a_7 = 157(-0.3)^6$    *$n = 7$, $a_1 = 157$, $r = -0.3$*
$a_7 = 0.114453$

The 7th term is 0.114453.

***Example 2*** **Write a sequence that has two geometric means between 6 and 162.**

The sequence will have the form 6, __?__, __?__, 162.

First, find the common ratio.
$a_n = a_1 r^{n-1}$
$162 = 6r^3$    *$a_4 = 162$, $a_1 = 6$, $n = 4$*
$27 = r^3$    *Divide each side by 6.*
$3 = r$    *Take the cube root of each side.*

Then, determine the geometric sequence.
$a_2 = 6 \cdot 3$ or 18    $a_3 = 18 \cdot 3$ or 54

The sequence is 6, 18, 54, 162.

***Example 3*** **Find the sum of the first twelve terms of the geometric series $12 - 12\sqrt{2} + 24 - 24\sqrt{2} + \cdots$.**

First, find the common ratio.
$a_2 \div a_1 = -12\sqrt{2} \div 12$ or $-\sqrt{2}$
The common ratio is $-\sqrt{2}$.

$$S_n = \frac{a_1 - a_1 r^n}{1 - r}$$

$$S_{12} = \frac{12 - 12\left(-\sqrt{2}\right)^{12}}{1 - \left(-\sqrt{2}\right)} \qquad n = 12,\ a_1 = 12,\ r = -\sqrt{2}$$

$$S_{12} = 756\left(1 - \sqrt{2}\right) \qquad \textit{Simplify.}$$

The sum of the first twelve terms of the series is $756\left(1 - \sqrt{2}\right)$.

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# 12-2 Practice

## Geometric Sequences and Series

***Determine the common ratio and find the next three terms of each geometric sequence.***

**1.** $-1, 2, -4, \ldots$ **2.** $-4, -3, -\frac{9}{4}, \ldots$ **3.** $12, -18, 27, \ldots$

***For exercises 4–9, assume that each sequence or series is geometric.***

**4.** Find the fifth term of the sequence $20, 0.2, 0.002, \ldots$.

**5.** Find the ninth term of the sequence $\sqrt{3}, -3, 3\sqrt{3}, \ldots$.

**6.** If $r = 2$ and $a_4 = 28$, find the first term of the sequence.

**7.** Find the first three terms of the sequence for which $a_4 = 8.4$ and $r = 4$.

**8.** Find the first three terms of the sequence for which $a_6 = \frac{1}{32}$ and $r = \frac{1}{2}$.

**9.** Write a sequence that has two geometric means between 2 and 0.25.

**10.** Write a sequence that has three geometric means between $-32$ and $-2$.

**11.** Find the sum of the first eight terms of the series $\frac{3}{4} + \frac{9}{20} + \frac{27}{100} + \cdots$.

**12.** Find the sum of the first 10 terms of the series $-3 + 12 - 48 + \cdots$.

**13.** ***Population Growth*** A city of 100,000 people is growing at a rate of 5.2% per year. Assuming this growth rate remains constant, estimate the population of the city 5 years from now.

12-2

# Enrichment

## Sequences as Functions

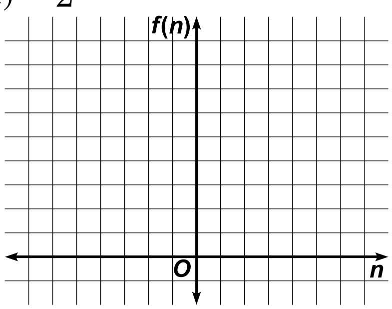
A **geometric sequence** can be defined as a function whose domain is the set of positive integers.

$$\begin{array}{cccccc} n = & 1 & 2 & 3 & 4 & \ldots \\ & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ f(n) = & ar^{1-1} & ar^{2-1} & ar^{3-1} & ar^{4-1} & \ldots \end{array}$$

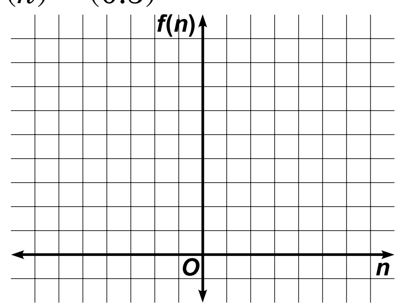
In the exercises, you will have the opportunity to explore geometric sequences from a function and graphing point of view.

***Graph each geometric sequence for n = 1, 2, 3 and 4.***

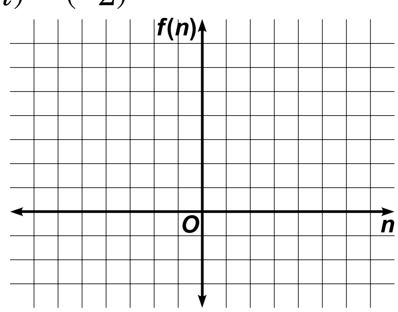
**1.** $f(n) = 2^n$



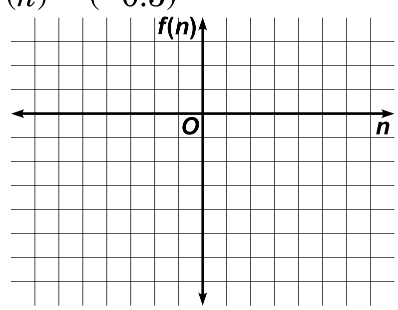

**2.** $f(n) = (0.5)^n$




**3.** $f(n) = (-2)^n$




**4.** $f(n) = (-0.5)^n$

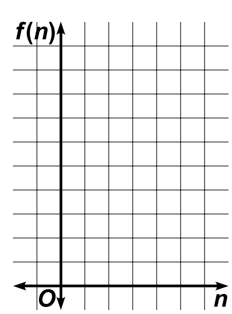



**5.** Describe how the graph of a geometric sequence depends on the common ratio.

**6.** Let $f(n) = 2^n$, where $n$ is a positive integer.

**a.** Show graphically that for any $M$ the graph of $f(n)$ rises above and stays above the horizontal line $y = M$.

**b.** Show algebraically that for any $M$, there is a positive integer $N$ such that $2^n > M$ for all $n > N$.



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# Study Guide

## Infinite Sequences and Series

An **infinite sequence** is one that has infinitely many terms. An **infinite series** is the indicated sum of the terms of an infinite sequence.

***Example 1*** **Find** $\lim\limits_{n\to\infty} \frac{4n^2 - n + 3}{n^2 + 1}$.

Divide each term in the numerator and the denominator by the highest power of $n$ to produce an equivalent expression. In this case, $n^2$ is the highest power.

$$\lim \frac{4n^2 - n + 3}{n^2 + 1} = \lim \frac{\frac{4n^2}{n^2} - \frac{n}{n^2} + \frac{3}{n^2}}{\frac{n^2}{n^2} + \frac{1}{n^2}}$$

$$= \lim_{n\to\infty} \frac{4 - \frac{1}{n} + \frac{3}{n^2}}{1 + \frac{1}{n^2}}$$ *Simplify.*

$$= \frac{\lim\limits_{n\to\infty} 4 - \lim\limits_{n\to\infty} \frac{1}{n} + \lim\limits_{n\to\infty} 3 \cdot \lim\limits_{n\to\infty} \frac{1}{n^2}}{\lim\limits_{n\to\infty} 1 + \lim\limits_{n\to\infty} \frac{1}{n^2}}$$ *Apply limit theorems.*

$$= \frac{4 - 0 + 3 \cdot 0}{1 + 0} \text{ or } 4$$ $\lim\limits_{n\to\infty} 4 = 4, \lim\limits_{n\to\infty} \frac{1}{n} = 0, \lim\limits_{n\to\infty} 3 = 3, \lim\limits_{n\to\infty} \frac{1}{n^2} = 0, \lim\limits_{n\to\infty} 1 = 1$

Thus, the limit is 4.

***Example 2*** **Find the sum of the series** $\frac{3}{2} - \frac{3}{8} + \frac{3}{32} - \cdots$.

In the series $a_1 = \frac{3}{2}$ and $r = -\frac{1}{4}$.

Since $|r| < 1$, $S = \frac{a_1}{1 - r}$.

$$S = \frac{a_1}{1 - r} = \frac{\frac{3}{2}}{1 - \left(-\frac{1}{4}\right)}$$ $a_1 = \frac{3}{2}$ *and* $r = -\frac{1}{4}$

$$= \frac{12}{10} \text{ or } 1\frac{1}{5}$$

The sum of the series is $1\frac{1}{5}$.

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# Practice

## Infinite Sequence and Series

***Find each limit, or state that the limit does not exist and explain your reasoning.***

**1.** $\lim\limits_{n\to\infty} \frac{n^2-1}{n^2+1}$

**2.** $\lim\limits_{n\to\infty} \frac{4n^2-5n}{3n^2+4}$

**3.** $\lim\limits_{n\to\infty} \frac{5n^2+1}{6n}$

**4.** $\lim\limits_{n\to\infty} \frac{(n-1)(3n+1)}{5n^2}$

**5.** $\lim\limits_{n\to\infty} \frac{3n-(-1)^n}{4n^2}$

**6.** $\lim\limits_{n\to\infty} \frac{n^3+1}{n^2}$

***Write each repeating decimal as a fraction.***

**7.** $0.\overline{75}$

**8.** $0.\overline{592}$

***Find the sum of each infinite series, or state that the sum does not exist and explain your reasoning.***

**9.** $\frac{2}{5}+\frac{6}{25}+\frac{18}{125}+\cdots$

**10.** $\frac{3}{4}+\frac{15}{8}+\frac{75}{16}+\cdots$

**11.** ***Physics*** A tennis ball is dropped from a height of 55 feet and bounces $\frac{3}{5}$ of the distance after each fall.

**a.** Find the first seven terms of the infinite series representing the vertical distances traveled by the ball.

**b.** What is the total vertical distance the ball travels before coming to rest?

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# Enrichment

## Solving Equations Using Sequences

You can use sequences to solve many equations. For example, consider $x^2 + x - 1 = 0$. You can proceed as follows.

$$x^2 + x - 1 = 0$$

$$x(x + 1) = 1$$

$$x = \frac{1}{1 + x}$$

Next, define the sequence: $a_1 = 0$ and $a_n = \frac{1}{1 + a_{n-1}}$.

The limit of the sequence is a solution to the original equation.

**1.** Let $a_1 = 0$ and $a_n = \frac{1}{1 + a_{n-1}}$.

**a.** Write the first five terms of the sequence. Do not simplify.

**b.** Write decimals for the first five terms of the sequence.

**c.** Use a calculator to compute $a_6$, $a_7$, $a_8$, and $a_9$. Compare $a_9$ with the positive solution of $x^2 + x - 1 = 0$ found by using the quadratic formula.

**2.** Use the method described above to find a root of $3x^2 - 2x - 3 = 0$.

**3.** Write a BASIC program using the procedure outlined above to find a root of the equation $3x^2 - 2x - 3 = 0$. In the program, let $a_1 = 0$ and $a_n = \frac{3}{3a_{n-1} - 2}$. Run the program. Compare the time it takes to run the program to the time it takes to evaluate the terms of the sequence by using a calculator.

 

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# Study Guide

## Convergent and Divergent Series

If an infinite series has a sum, or limit, the series is **convergent.** If a series is not convergent, it is **divergent.** When a series is neither arithmetic nor geometric and all the terms are positive, you can use the **ratio test** or the **comparison test** to determine whether the series is convergent or divergent.

| | |
|---|---|
| **Ratio Test** | Let $a_n$ and $a_{n+1}$ represent two consecutive terms of a series of positive terms. Suppose $\lim\limits_{n\to\infty} \frac{a_{n+1}}{a_n}$ exists and $r = \lim\limits_{n\to\infty} \frac{a_{n+1}}{a_n}$. The series is convergent if $r < 1$ and divergent if $r > 1$. If $r = 1$, the test provides no information. |

| | |
|---|---|
| **Comparison Test** | • A series of positive terms is convergent if, for $n > 1$, each term of the series is equal to or less than the value of the corresponding term of some convergent series of positive terms.<br>• A series of positive terms is divergent if, for $n > 1$, each term of the series is equal to or greater than the value of the corresponding term of some divergent series of positive terms. |

***Example 1*** **Use the ratio test to determine whether the series $\frac{1 \cdot 2}{2^1} + \frac{2 \cdot 3}{2^2} + \frac{3 \cdot 4}{2^3} + \frac{4 \cdot 5}{2^4} + \cdots$ is convergent or divergent.**

First, find the $n$th term. Then use the ratio test.

$a_n = \frac{n(n+1)}{2^n}$ $\qquad a_{n+1} = \frac{(n+1)(n+2)}{2^{n+1}}$

$$r = \lim_{n\to\infty} \frac{\frac{(n+1)(n+2)}{2^{n+1}}}{\frac{n(n+1)}{2^n}}$$

$$r = \lim_{n\to\infty} \frac{(n+1)(n+2)}{2^{n+1}} \cdot \frac{2^n}{n(n+1)}$$ *Multiply by the reciprocal of the divisor.*

$$r = \lim_{n\to\infty} \frac{n+2}{2n}$$ $\frac{2^n}{2^{n+1}} = \frac{1}{2}$

$$r = \lim_{n\to\infty} \frac{1 + \frac{2}{n}}{2}$$ *Divide by the highest power of n and apply limit theorems.*

$$r = \frac{1}{2}$$ *Since $r < 1$, the series is convergent.*

***Example 2*** **Use the comparison test to determine whether the series $\frac{1}{4^2} + \frac{1}{7^2} + \frac{1}{10^2} + \frac{1}{13^2} + \cdots$ is convergent or divergent.**

The general term of the series is $\frac{1}{(3n+1)^2}$. The general term of the convergent series $1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \cdots$ is $\frac{1}{n^2}$. Since $\frac{1}{(3n+1)^2} < \frac{1}{n^2}$ for all $n \geq 1$, the series $\frac{1}{4^2} + \frac{1}{7^2} + \frac{1}{10^2} + \frac{1}{13^2} + \cdots$ is also convergent.

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12-4

# Practice

## Convergent and Divergent Series

***Use the ratio test to determine whether each series is convergent or divergent.***

**1.** $\frac{1}{2} + \frac{2^2}{2^2} + \frac{3^2}{2^3} + \frac{4^2}{2^4} + \cdots$

**2.** $0.006 + 0.06 + 0.6 + \cdots$

**3.** $\frac{4}{1 \cdot 2 \cdot 3} + \frac{8}{1 \cdot 2 \cdot 3 \cdot 4} + \frac{16}{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5} + \cdots$

**4.** $5 + \frac{5}{3^3} + \frac{5}{5^3} + \frac{5}{7^3} + \cdots$

***Use the comparison test to determine whether each series is convergent or divergent.***

**5.** $2 + \frac{2}{2^3} + \frac{2}{3^3} + \frac{2}{4^3} + \cdots$

**6.** $\frac{5}{2} + 1 + \frac{5}{8} + \frac{5}{11} + \cdots$

**7.** ***Ecology*** A landfill is leaking a toxic chemical. Six months after the leak was detected, the chemical had spread 1250 meters from the landfill. After one year, the chemical had spread 500 meters more, and by the end of 18 months, it had reached an additional 200 meters.

**a.** If this pattern continues, how far will the chemical spread from the landfill after 3 years?

**b.** Will the chemical ever reach the grounds of a hospital located 2500 meters away from the landfill? Explain.

12-4

# Enrichment

## Alternating Series

The series below is called an alternating series.

$$1 - 1 + 1 - 1 + \cdots$$

The reason is that the signs of the terms alternate. An interesting question is whether the series converges. In the exercises, you will have an opportunity to explore this series and others like it.

**1.** Consider $1 - 1 + 1 - 1 + \cdots$ .

**a.** Write an argument that suggests that the sum is 1.

**b.** Write an argument that suggests that the sum is 0.

**c.** Write an argument that suggests that there is no sum. (*Hint*: Consider the sequence of partial sums.)

If the series formed by taking the absolute values of the terms of a given series is convergent, then the given series is said to be **absolutely convergent**. It can be shown that any absolutely convergent series is convergent.

**2.** Make up an alternating series, other than a geometric series with negative common ratio, that has a sum. Justify your answer.

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# Study Guide

## Sigma Notation and the *n*th Term

A series may be written using **sigma notation.**

$$\text{maximum value of } n \rightarrow \quad \sum_{n=1}^{k} a_n \leftarrow \text{expression for general term}$$

*starting value of n* → $n=1$; ↑ *index of summation*

***Example 1*** **Write each expression in expanded form and then find the sum.**

**a.** $\sum_{n=1}^{5} (n + 2)$

First, write the expression in expanded form.

$$\sum_{n=1}^{5} (n + 2) = (1 + 2) + (2 + 2) + (3 + 2) + (4 + 2) + (5 + 2)$$

Then, find the sum by simplifying the expanded form. $3 + 4 + 5 + 6 + 7 = 25$

**b.** $\sum_{m=1}^{\infty} 2\left(\frac{1}{4}\right)^m$

$$\sum_{m=1}^{\infty} 2\left(\frac{1}{4}\right)^m = 2\left(\frac{1}{4}\right)^1 + 2\left(\frac{1}{4}\right)^2 + 2\left(\frac{1}{4}\right)^3 + \cdots$$

$$= \frac{1}{2} + \frac{1}{8} + \frac{1}{32} + \cdots$$

This is an infinite series. Use the formula $S = \frac{a_1}{1 - r}$.

$S = \frac{\frac{1}{2}}{1 - \frac{1}{4}}$ $\quad a_1 = \frac{1}{2}, r = \frac{1}{4}$

$S = \frac{2}{3}$

***Example 2*** **Express the series $26 + 37 + 50 + 65 + \cdots + 170$ using sigma notation.**

Notice that each term is one more than a perfect square. Thus, the $n$th term of the series is $n^2 + 1$. Since $5^2 + 1 = 26$ and $13^2 + 1 = 170$, the index of summation goes from $n = 5$ to $n = 13$.

Therefore, $26 + 37 + 50 + 65 + \cdots + 170 = \sum_{n=5}^{13} (n^2 + 1)$.

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12-5

# Practice

## Sigma Notation and the *n*th Term

***Write each expression in expanded form and then find the sum.***

**1.** $\sum_{n=3}^{5}(n^2 - 2^n)$

**2.** $\sum_{q=1}^{4}\frac{2}{q}$

**3.** $\sum_{t=1}^{5}t(t-1)$

**4.** $\sum_{t=0}^{3}(2t-3)$

**5.** $\sum_{c=2}^{5}(c-2)^2$

**6.** $\sum_{i=1}^{\infty}10\left(\frac{1}{2}\right)^i$

***Express each series using sigma notation.***

**7.** $3 + 6 + 9 + 12 + 15$

**8.** $6 + 24 + 120 + \cdots + 40{,}320$

**9.** $\frac{1}{1} + \frac{1}{4} + \frac{1}{9} + \cdots + \frac{1}{100}$

**10.** $24 + 19 + 14 + \cdots + (-1)$

**11.** ***Savings*** Kathryn started saving quarters in a jar. She began by putting two quarters in the jar the first day and then she increased the number of quarters she put in the jar by one additional quarter each successive day.

**a.** Use sigma notation to represent the total number of quarters Kathryn had after 30 days.

**b.** Find the sum represented in part **a**.

 

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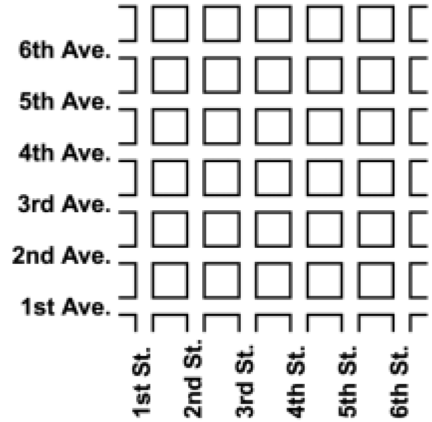
12-5

# Enrichment

## Street Networks: Finding All Possible Routes




A section of a city is laid out in square blocks. Going north from the intersection of 1st Avenue and 1st Street, the avenues are 1st, 2nd, 3rd, and so on. Going east, the streets are numbered in the same way.

Factorials can be used to find the number, $r(e, n)$, of different routes between two intersections.

$$r(e, n) = \frac{[(e-1) + (n-1)]!}{(e-1)!\,(n-1)!}$$

The number of streets going east is $e$; the number of avenues going north is $n$.

The following problems examine the possible routes from one location to another. Assume that you never use a route that is unnecessarily long. Assume that $e \geq 1$ and $n \geq 1$.

***Solve each problem.***

**1.** List all the possible routes from 1st Street and 1st Avenue to 4th Street and 3rd Avenue. Use ordered pairs to show the routes, with street numbers first and avenue numbers second. Each route must start at (1, 1) and end at (4, 3).

**2.** Use the formula to compute the number of routes from (1, 1) to (4, 3). There are 4 streets going east and 3 avenues going north.

**3.** Find the number of routes from 1st Street and 1st Avenue to 7th Street and 6th Avenue.

12-6

# Study Guide

## The Binomial Theorem

Two ways to expand a binomial are to use either **Pascal's triangle** or the **Binomial Theorem.** The Binomial Theorem states that if $n$ is a positive integer, then the following is true.

$$(x + y)^n = x^n + nx^{n-1}y + \frac{n(n-1)}{1 \cdot 2}x^{n-2}y^2 + \frac{n(n-1)(n-2)}{1 \cdot 2 \cdot 3}x^{n-3}y^3 + \cdots + y^n$$

To find individual terms of an expansion, use this form of the Binomial Theorem:

$$(x + y)^n = \sum_{r=0}^{n} \frac{n!}{r!(n-r)!}x^{n-r}y^r.$$

***Example 1*** **Use Pascal's triangle to expand $(x + 2y)^5$.**

First, write the series without the coefficients. The expression should have 5 + 1, or 6, terms, with the first term being $x^5$ and the last term being $y^5$. The exponents of $x$ should decrease from 5 to 0 while the exponents of $y$ should increase from 0 to 5. The sum of the exponents of each term should be 5.

$x^5 + x^4y + x^3y^2 + x^2y^3 + xy^4 + y^5$ *$x^0 = 1$ and $y^0 = 1$*

Replace each $y$ with $2y$.

$x^5 + x^4(2y) + x^3(2y)^2 + x^2(2y)^3 + x(2y)^4 + (2y)^5$

Then, use the numbers in the sixth row of Pascal's triangle as the coefficients of the terms, and simplify each term.

$$\begin{aligned}(x + 2y)^5 &= \overset{1}{\downarrow}x^5 + \overset{5}{\downarrow}5x^4(2y) + \overset{10}{\downarrow}10x^3(2y)^2 + \overset{10}{\downarrow}10x^2(2y)^3 + \overset{5}{\downarrow}5x(2y)^4 + \overset{1}{\downarrow}(2y)^5\\ &= x^5 + 10x^4y + 40x^3y^2 + 80x^2y^3 + 80xy^4 + 32y^5\end{aligned}$$

***Example 2*** **Find the fourth term of $(5a + 2b)^6$.**

$$(5a + 2b)^6 = \sum_{r=0}^{6} \frac{6!}{r!(6-r)!}(5a)^{6-r}(2b)^r$$

To find the fourth term, evaluate the general term for $r = 3$.
*Since r increases from 0 to n, r is one less than the number of the term.*

$$\begin{aligned}\frac{6!}{r!(6-r)!}(5a)^{6-r}(2b)^r &= \frac{6!}{3!(6-3)!}(5a)^{6-3}(2b)^3\\ &= \frac{6 \cdot 5 \cdot 4 \cdot 3!}{3!3!}(5a)^3(2b)^3\\ &= 20{,}000a^3b^3\end{aligned}$$

The fourth term of $(5a + 2b)^6$ is $20{,}000a^3b^3$.

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# Practice

## The Binomial Theorem

***Use Pascal's triangle to expand each binomial.***

**1.** $(r + 3)^5$

**2.** $(3a - b)^4$

***Use the Binomial Theorem to expand each binomial.***

**3.** $(x - 5)^4$

**4.** $(3x + 2y)^4$

**5.** $(a - \sqrt{2})^5$

**6.** $(2p - 3q)^6$

***Find the designated term of each binomial expansion.***

**7.** 4th term of $(2n - 3m)^4$

**8.** 5th term of $(4a + 2b)^8$

**9.** 6th term of $(3p + q)^9$

**10.** 3rd term of $(a - 2\sqrt{3})^6$

**11.** A varsity volleyball team needs nine members. Of these nine members, at least five must be seniors. How many of the possible groups of juniors and seniors have at least five seniors?

12-6

# Enrichment

## Patterns in Pascal's Triangle

You have learned that the coefficients in the expansion of $(x + y)^n$ yield a number pyramid called **Pascal's triangle**.

| | |
|---|---|
| Row 1 $\longrightarrow$ | 1 |
| Row 2 $\longrightarrow$ | 1 1 |
| Row 3 $\longrightarrow$ | 1 2 1 |
| Row 4 $\longrightarrow$ | 1 3 3 1 |
| Row 5 $\longrightarrow$ | 1 4 6 4 1 |
| Row 6 $\longrightarrow$ | 1 5 10 10 5 1 |
| Row 7 $\longrightarrow$ | 1 6 15 20 15 6 1 |

As many rows can be added to the bottom of the pyramid as you need.

This activity explores some of the interesting properties of this famous number pyramid.

**1.** Pick a row of Pascal's triangle.

**a.** What is the sum of all the numbers in all the rows above the row you picked?

**b.** What is the sum of all the numbers in the row you picked?

**c.** How are your answers for parts **a** and **b** related?

**d.** Repeat parts **a** through **c** for at least three more rows of Pascal's triangle. What generalization seems to be true?

**e.** See if you can prove your generalization.

**2.** Pick any row of Pascal's triangle that comes after the first.

**a.** Starting at the left end of the row, find the sum of the odd numbered terms.

**b.** In the same row, find the sum of the even numbered terms.

**c.** How do the sums in parts a and b compare?

**d.** Repeat parts **a** through **c** for at least three other rows of Pascal's triangle. What generalization seems to be true?

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# Study Guide

## Special Sequences and Series

The value of $e^x$ can be approximated by using the **exponential series.** The **trigonometric series** can be used to approximate values of the trigonometric functions. **Euler's formula** can be used to write the exponential form of a complex number and to find a complex number that is the natural logarithm of a negative number.

***Example 1*** **Use the first five terms of the trigonometric series to approximate the value of sin $\frac{\pi}{6}$ to four decimal places.**

$\sin x \approx x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \frac{x^9}{9!}$

Let $x = \frac{\pi}{6}$, or about 0.5236.

$\sin \frac{\pi}{6} \approx 0.5236 - \frac{(0.5236)^3}{3!} + \frac{(0.5236)^5}{5!} - \frac{(0.5236)^7}{7!} + \frac{(0.5236)^9}{9!}$

$\sin \frac{\pi}{6} \approx 0.5236 - 0.02392 + 0.00033 - 0.000002 + 0.000000008$

$\sin \frac{\pi}{6} \approx 0.5000$ *Compare this result to the actual value, 0.5.*

***Example 2*** **Write $4 - 4i$ in exponential form.**

Write the polar form of $4 - 4\boldsymbol{i}$.
Recall that $a + b\boldsymbol{i} = r(\cos\theta + \boldsymbol{i}\sin\theta)$, where
$r = \sqrt{a^2 + b^2}$ and $\theta = \text{Arctan}\,\frac{b}{a}$ when $a > 0$.

$r = \sqrt{4^2 + (-4)^2}$ or $4\sqrt{2}$, and *$a = 4$ and $b = -4$*

$\theta = \text{Arctan}\,\frac{-4}{4}$ or $-\frac{\pi}{4}$

$$4 - 4\boldsymbol{i} = 4\sqrt{2}\left[\cos\left(-\frac{\pi}{4}\right) + \boldsymbol{i}\sin\left(-\frac{\pi}{4}\right)\right]$$
$$= 4\sqrt{2}e^{-\boldsymbol{i}\frac{\pi}{4}}$$

Thus, the exponential form of $4 - 4\boldsymbol{i}$ is $4\sqrt{2}e^{-\boldsymbol{i}\frac{\pi}{4}}$.

***Example 3*** **Evaluate ln(−12.4).**

$\ln(-12.4) = \ln(-1) + \ln(12.4)$
$\approx \boldsymbol{i}\pi + 2.5177$ *Use a calculator to compute ln(12.4).*

Thus, $\ln(-12.4) \approx \boldsymbol{i}\pi + 2.5177$. *The logarithm is a complex number.*

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# Practice

## Special Sequences and Series

***Find each value to four decimal places.***

**1.** $\ln(-5)$ **2.** $\ln(-5.7)$ **3.** $\ln(-1000)$

***Use the first five terms of the exponential series and a calculator to approximate each value to the nearest hundredth.***

**4.** $e^{0.5}$ **5.** $e^{1.2}$

**6.** $e^{2.7}$ **7.** $e^{0.9}$

***Use the first five terms of the trigonometric series to approximate the value of each function to four decimal places. Then, compare the approximation to the actual value.***

**8.** $\sin \frac{5\pi}{6}$ **9.** $\cos \frac{3\pi}{4}$

***Write each complex number in exponential form.***

**10.** $13\left(\cos \frac{\pi}{3} + \boldsymbol{i} \sin \frac{\pi}{3}\right)$ **11.** $5 + 5\boldsymbol{i}$

**12.** $1 - \sqrt{3}\boldsymbol{i}$ **13.** $-7 + 7\sqrt{3}\boldsymbol{i}$

**14.** ***Savings*** Derika deposited \$500 in a savings account with a 4.5% interest rate compounded continuously. (*Hint:* The formula for continuously compounded interest is $A = Pe^{rt}$.)

**a.** Approximate Derika's savings account balance after 12 years using the first four terms of the exponential series.

**b.** How long will it take for Derika's deposit to double, provided she does not deposit any additional funds into her account?

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# 12-7 Enrichment

## Power Series

A **power series** is a series of the form

$$a_0 + a_1x + a_2x^2 + a_3x^3 + \cdots$$

where each $a_i$ is a real number. Many functions can be represented by power series. For instance, the function $f(x) = e^x$ can be represented by the series

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots .$$

***Use a graphing calculator or computer to graph the functions in Exercises 1–4.***

**1.** $f_2(x) = 1 + x$

**2.** $f_3(x) = 1 + x + \frac{x^2}{2!}$

**3.** $f_4(x) = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!}$

**4.** $f_5(x) = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!}$

**5.** Write a statement that relates the sequence of graphs suggested by Exercises 1–4 and the function $y = e^x$.

**6.** The series $1 + x^2 + x^4 + x^6 + \cdots$ is a power series for which each $a_i = 1$. The series is also a geometric series with first term 1 and common ratio $x^2$.

**a.** Find the function that this power series represents.

**b.** For what values of $x$ does the series give the values of the function in part **a**?

**7.** Find a power series representation for the function $f(x) = \frac{3}{1 + x^2}$.

12-8

# Study Guide

## Sequences and Iteration

Each output of composing a function with itself is called an *iterate.* To iterate a function $f(x)$, find the function value $f(x_0)$ of the initial value $x_0$. The second iterate is the value of the function performed on the output, and so on.

The function $f(z) = z^2 + c$, where $c$ and $z$ are complex numbers, is central to the study of **fractal geometry.** This type of geometry can be used to describe things such as coastlines, clouds, and mountain ranges.

***Example 1*** **Find the first four iterates of the function $f(x) = 4x + 1$ if the initial value is $-1$.**

$x_0 = -1$
$x_1 = 4(-1) + 1$ or $-3$
$x_2 = 4(-3) + 1$ or $-11$
$x_3 = 4(-11) + 1$ or $-43$
$x_4 = 4(-43) + 1$ or $-171$

The first four iterates are $-3$, $-11$, $-43$, and $-171$.

***Example 2*** **Find the first three iterates of the function $f(z) = 3z - i$ if the initial value is $1 + 2i$.**

$z_0 = 1 + 2\boldsymbol{i}$
$z_1 = 3(1 + 2\boldsymbol{i}) - \boldsymbol{i}$ or $3 + 5\boldsymbol{i}$
$z_2 = 3(3 + 5\boldsymbol{i}) - \boldsymbol{i}$ or $9 + 14\boldsymbol{i}$
$z_3 = 3(9 + 14\boldsymbol{i}) - \boldsymbol{i}$ or $27 + 41\boldsymbol{i}$

The first three iterates are $3 + 5\boldsymbol{i}$, $9 + 14\boldsymbol{i}$, and $27 + 41\boldsymbol{i}$.

***Example 3*** **Find the first three iterates of the function $f(z) = z^2 + c$, where $c = 2 - i$ and $z_0 = 1 + i$.**

$$\begin{aligned} z_1 &= (1 + \boldsymbol{i})^2 + 2 - \boldsymbol{i}. \\ &= 1 + \boldsymbol{i} + \boldsymbol{i} + \boldsymbol{i}^2 + 2 - \boldsymbol{i} \\ &= 1 + \boldsymbol{i} + \boldsymbol{i} + (-1) + 2 - \boldsymbol{i} \qquad \boldsymbol{i}^2 = -1 \\ &= 2 + \boldsymbol{i} \end{aligned}$$

$$\begin{aligned} z_2 &= (2 + \boldsymbol{i})^2 + 2 - \boldsymbol{i} \\ &= 4 + 2\boldsymbol{i} + 2\boldsymbol{i} + \boldsymbol{i}^2 + 2 - \boldsymbol{i} \\ &= 4 + 2\boldsymbol{i} + 2\boldsymbol{i} + (-1) + 2 - \boldsymbol{i} \\ &= 5 + 3\boldsymbol{i} \end{aligned}$$

$$\begin{aligned} z_3 &= (5 + 3\boldsymbol{i})^2 + 2 - \boldsymbol{i} \\ &= 25 + 15\boldsymbol{i} + 15\boldsymbol{i} + 9\boldsymbol{i}^2 + 2 - \boldsymbol{i} \\ &= 25 + 15\boldsymbol{i} + 15\boldsymbol{i} + 9(-1) + 2 - \boldsymbol{i} \\ &= 18 + 29\boldsymbol{i} \end{aligned}$$

The first three iterates are $2 + \boldsymbol{i}$, $5 + 3\boldsymbol{i}$, and $18 + 29\boldsymbol{i}$.

12-8

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# Practice

## Sequences and Iteration

***Find the first four iterates of each function using the given initial value. If necessary, round your answers to the nearest hundredth.***

**1.** $f(x) = x^2 + 4; x_0 = 1$

**2.** $f(x) = 3x + 5; x_0 = -1$

**3.** $f(x) = x^2 - 2; x_0 = -2$

**4.** $f(x) = x(2.5 - x); x_0 = 3$

***Find the first three iterates of the function $f(z) = 2z - (3 + i)$ for each initial value.***

**5.** $z_0 = \boldsymbol{i}$

**6.** $z_0 = 3 - \boldsymbol{i}$

**7.** $z_0 = 0.5 + \boldsymbol{i}$

**8.** $z_0 = -2 - 5\boldsymbol{i}$

***Find the first three iterates of the function $f(z) = z^2 + c$ for each given value of c and each initial value.***

**9.** $c = 1 - 2\boldsymbol{i}; z_0 = 0$

**10.** $c = \boldsymbol{i}; z_0 = \boldsymbol{i}$

**11.** $c = 1 + \boldsymbol{i}; z_0 = -1$

**12.** $c = 2 - 3\boldsymbol{i}; z_0 = 1 + \boldsymbol{i}$

**13.** ***Banking*** Mai deposited \$1000 in a savings account. The annual yield on the account is 5.2%. Find the balance of Mai's account after each of the first 3 years.

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# Enrichment

## Depreciation

To run a business, a company purchases assets such as equipment or buildings. For tax purposes, the company distributes the cost of these assets as a business expense over the course of a number of years. Since assets depreciate (lose some of their market value) as they get older, companies must be able to figure the depreciation expense they are allowed to take when they file their income taxes.

Depreciation expense is a function of these three values:

1. **asset cost**, or the amount the company paid for the asset;
2. **estimated useful life**, or the number of years the company can expect to use the asset;
3. **residual or trade-in value**, or the expected cash value of the asset at the end of its useful life.

In any given year, the **book value** of an asset is equal to the asset cost minus the accumulated depreciation. This value represents the unused amount of asset cost that the company may depreciate in future years. The useful life of the asset is over once its book value is equal to its residual value.

There are several methods of determining the amount of depreciation in a given year. In the **declining-balance method**, the depreciation expense allowed each year is equal to the book value of the asset at the beginning of the year times the depreciation rate. Since the depreciation expense for any year is dependent upon the depreciation expense for the previous year, the process of determining the depreciation expense for a year is an iteration.

***The table below shows the first two iterates of the depreciation schedule for a $2500 computer with a residual value of $500 if the depreciation rate is 40%.***

| End of Year | Asset Cost | Depreciation Expense | Book Value at End of Year |
|---|---|---|---|
| 1 | $2500 | $1000<br>(40% of $2500) | $1500<br>($2500 - $1000) |
| 2 | $2500 | $600<br>(40% of $1500) | $900<br>($1500 - $600) |

**1.** Find the next two iterates for the depreciation expense function.

**2.** Find the next two iterates for the end-of-year book value function.

**3.** Explain the depreciation expense for year 5.

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# Study Guide

## Mathematical Induction

A method of proof called **mathematical induction** can be used to prove certain conjectures and formulas. The following example demonstrates the steps used in proving a summation formula by mathematical induction.

***Example*** **Prove that the sum of the first $n$ positive even integers is $n(n + 1)$.**

Here $S_n$ is defined as $2 + 4 + 6 + \cdots + 2n = n(n + 1)$.

**1.** First, verify that $S_n$ is valid for the first possible case, $n = 1$. Since the first positive even integer is 2 and $1(1 + 1) = 2$, the formula is valid for $n = 1$.

**2.** Then, assume that $S_n$ is valid for $n = k$.

$S_k \Rightarrow 2 + 4 + 6 + \cdots + 2k = k(k + 1)$. *Replace n with k.*

Next, prove that $S_n$ is also valid for $n = k + 1$.

$S_{k+1} \Rightarrow 2 + 4 + 6 + \cdots + 2k + 2(k + 1)$

$= k(k + 1) + 2(k + 1)$ *Add 2(k + 1) to both sides.*

We can simplify the right side by adding $k(k + 1) + 2(k + 1)$.

$S_{k+1} \Rightarrow 2 + 4 + 6 + \cdots + 2k + 2(k + 1)$

$= (k + 1)(k + 2)$ *(k + 1) is a common factor.*

If $k + 1$ is substituted into the original formula $(n(n + 1))$, the same result is obtained.

$(k + 1)[(k + 1) + 1]$ or $(k + 1)(k + 2)$

Thus, if the formula is valid for $n = k$, it is also valid for $n = k + 1$. Since $S_n$ is valid for $n = 1$, it is also valid for $n = 2$, $n = 3$, and so on. That is, the formula for the sum of the first $n$ positive even integers holds.

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12-9

# Practice

## Mathematical Induction

***Use mathematical induction to prove that each proposition is valid for all positive integral values of n.***

**1.** $\frac{1}{3} + \frac{2}{3} + \frac{3}{3} + \cdots + \frac{n}{3} = \frac{n(n+1)}{6}$

**2.** $5^n + 3$ is divisible by 4.

 

12-9

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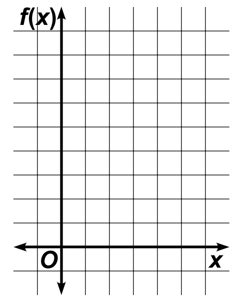
# Enrichment

## Conjectures and Mathematical Induction

Frequently, the pattern in a set of numbers is not immediately evident. Once you make a conjecture about a pattern, you can use mathematical induction to prove your conjecture.

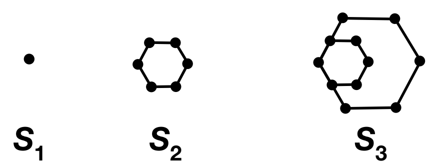



**1. a.** Graph $f(x) = x^2$ and $g(x) = 2^x$ on the axes shown at the right.

**b.** Write a conjecture that compares $n^2$ and $2^n$, where $n$ is a positive integer.

**c.** Use mathematical induction to prove your response from part b.

**2.** Refer to the diagrams at the right.




**a.** How many dots would there be in the fourth diagram $S_4$ in the sequence?

**b.** Describe a method that you can use to determine the number of dots in the fifth diagram $S_5$ based on the number of dots in the fourth diagram, $S_4$. Verify your answer by constructing the fifth diagram.

**c.** Find a formula that can be used to compute the number of dots in the $n$th diagram of this sequence. Use mathematical induction to prove your formula is correct.

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Chapter 12

# Chapter 12 Test, Form 1A

***Write the letter for the correct answer in the blank at the right of each problem.***

**1.** Find the 15th term in the arithmetic sequence 14, 10.5, 7, . . . . **1.** ________

**A.** $-21$ **B.** $-63$ **C.** $63$ **D.** $-35$

**2.** Find the sum of the first 36 terms in the arithmetic series $-0.2 + 0.3 + 0.8 + \cdots$. **2.** ________

**A.** 318.6 **B.** 332.2 **C.** 307.8 **D.** 315

**3.** Find the sixth term in the geometric sequence $\sqrt{3}y^3, -3y^5, 3\sqrt{3}y^7, \ldots$. **3.** ________

**A.** $-27y^{13}$ **B.** $9\sqrt{3}y^{13}$ **C.** $27y^{13}$ **D.** $-9\sqrt{3}y^{13}$

**4.** Find the sum of the first five terms in the geometric series $-\frac{3}{2} + 1 - \frac{2}{3} + \cdots$. **4.** ________

**A.** $\frac{55}{54}$ **B.** $-\frac{55}{54}$ **C.** $\frac{55}{27}$ **D.** $-\frac{55}{27}$

**5.** Find three geometric means between $-\sqrt{2}$ and $-4\sqrt{2}$. **5.** ________

**A.** $2, -2\sqrt{2}, 4$ **B.** $-2, 2\sqrt{2}, -4$

**C.** $2, 2\sqrt{2}, 4$ **D.** A or C

**6.** Find $\lim_{n\to\infty}\left[1 + \frac{(-1)^n}{n}\right]$. **6.** ________

**A.** 1 **B.** 0 **C.** $-1$ **D.** does not exist

**7.** Find the sum of $\sqrt{27} + \sqrt{9} + \sqrt{3} + \cdots$. **7.** ________

**A.** $\frac{1}{2}(9 + 9\sqrt{3})$ **B.** $9 + 9\sqrt{3}$ **C.** $\frac{1}{2}(9 - 9\sqrt{3})$ **D.** does not exist

**8.** Write $3.\overline{123}$ as a fraction. **8.** ________

**A.** $\frac{1040}{3333}$ **B.** $\frac{1040}{333}$ **C.** $\frac{1040}{33}$ **D.** $\frac{3333}{1040}$

**9.** Which of the following series is convergent? **9.** ________

**A.** $\sqrt{3} + 3 + 3\sqrt{3} + \cdots$ **B.** $6\sqrt{2} + 12 + 12\sqrt{2} + \cdots$

**C.** $6\sqrt{2} + 6 + 3\sqrt{2} + \cdots$ **D.** $6\sqrt{2} - 12 + 12\sqrt{2} - \cdots$

**10.** Which of the following series is divergent? **10.** ________

**A.** $1 + 3\left(\frac{1}{4}\right) + 9\left(\frac{1}{4}\right)^2 + 27\left(\frac{1}{4}\right)^3 + \cdots$ **B.** $1 + 3\left(\frac{1}{5}\right) + 9\left(\frac{1}{5}\right)^2 + 27\left(\frac{1}{5}\right)^3 + \cdots$

**C.** $1 + 3\left(\frac{1}{7}\right) + 9\left(\frac{1}{7}\right)^2 + 27\left(\frac{1}{7}\right)^3 + \cdots$ **D.** $1 + 3\left(\frac{1}{2}\right) + 9\left(\frac{1}{2}\right)^2 + 27\left(\frac{1}{2}\right)^3 + \cdots$

**11.** Write $\sum_{k=0}^{3}\left(-\frac{1}{2}\right)^k$ in expanded form and then find the sum. **11.** ________

**A.** $-\frac{1}{2} - \frac{1}{4} - \frac{1}{8}; -\frac{7}{8}$ **B.** $1 - \frac{1}{2} - \frac{1}{4} - \frac{1}{8}; \frac{1}{8}$

**C.** $-\frac{1}{2} + \frac{1}{4} - \frac{1}{8}; \frac{3}{8}$ **D.** $1 - \frac{1}{2} + \frac{1}{4} - \frac{1}{8}; \frac{5}{8}$

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# Chapter 12 Test, Form 1A (continued)

**12.** Express the series $-27 + 9 - 3 + 1 - \cdots$ using sigma notation. **12.** ________

**A.** $\sum_{k=0}^{\infty} -3k$ **B.** $\sum_{k=0}^{3} -27\left(-\frac{1}{3}\right)^k$

**C.** $\sum_{k=0}^{\infty} -27\left(-\frac{1}{3}\right)^k$ **D.** $\sum_{k=0}^{\infty} 27\left(-\frac{1}{3}\right)^k$

**13.** The expression $81p^4 + 108p^3r^3 + 54p^2r^6 + 12pr^9 + r^{12}$ is the expansion of which binomial? **13.** ________

**A.** $(p + 3r^3)^4$ **B.** $(3p + r^3)^4$ **C.** $(3p^3 + r)^4$ **D.** $(3p + 3r^3)^4$

**14.** Find the fifth term in the expansion of $(3x^2 - \sqrt{y})^6$. **14.** ________

**A.** $135x^4y^2$ **B.** $45x^4y^2$ **C.** $-135x^4y^2$ **D.** $-45x^4y^2$

**15.** Use the first five terms of the trigonometric series to find the value of $\sin \frac{\pi}{12}$ to four decimal places. **15.** ________

**A.** 0.2618 **B.** 0.2588 **C.** 0.7071 **D.** 0.2648

**16.** Find $\ln(-91.48)$. **16.** ________

**A.** 4.5161 **B.** $\boldsymbol{i}\pi - 4.5161$ **C.** $\boldsymbol{i}\pi + 4.5161$ **D.** $-4.5161$

**17.** Write $3 - \sqrt{3}\boldsymbol{i}$ in exponential form. **17.** ________

**A.** $9e^{\boldsymbol{i}\frac{11\pi}{6}}$ **B.** $9e^{\boldsymbol{i}\frac{5\pi}{3}}$ **C.** $2\sqrt{3}e^{\boldsymbol{i}\frac{11\pi}{6}}$ **D.** $2\sqrt{3}e^{\boldsymbol{i}\frac{5\pi}{3}}$

**18.** Find the first three iterates of the function $f(z) = -z + \boldsymbol{i}$ for $z_0 = 2 + 3\boldsymbol{i}$. **18.** ________

**A.** $2 - 2\boldsymbol{i}, 2 + 3\boldsymbol{i}, 2 - 2\boldsymbol{i}$ **B.** $-2 - 2\boldsymbol{i}, 2 + 3\boldsymbol{i}, -2 - 2\boldsymbol{i}$

**C.** $2 + 3\boldsymbol{i}, -2 - 2\boldsymbol{i}, 2 + 3\boldsymbol{i}$ **D.** $-2 - 2\boldsymbol{i}, 2 - 3\boldsymbol{i}, -2 - 2\boldsymbol{i}$

**19.** Find the first three iterates of the function $f(z) = z^2 - c$ for $c = 1 - 2\boldsymbol{i}$ and $z_0 = 1 + \boldsymbol{i}$. **19.** ________

**A.** $-1 + 4\boldsymbol{i}, -15 - 8\boldsymbol{i}, 219 + 194\boldsymbol{i}$

**B.** $-1 + 4\boldsymbol{i}, -16 - 6\boldsymbol{i}, 220 + 192\boldsymbol{i}$

**C.** $-1 + 4\boldsymbol{i}, -16 - 6\boldsymbol{i}, 221 + 194\boldsymbol{i}$

**D.** $-1 + 4\boldsymbol{i}, -16 - 6\boldsymbol{i}, 219 + 194\boldsymbol{i}$

**20.** Suppose in a proof of the summation formula $1 + 5 + 25 + \cdots + 5^{n-1} = \frac{1}{4}(5^n - 1)$ by mathematical induction, you show the formula valid for $n = 1$ and assume that it is valid for $n = k$. What is the next equation in the induction step of this proof? **20.** ________

**A.** $1 + 5 + 25 + \cdots + 5^{k-1} + 5^{k+1-1} = \frac{1}{4}(5^k - 1) + \frac{1}{4}(5^{k+1} - 1)$

**B.** $1 + 5 + 25 + \cdots + 5^k + 5^{k+1} = \frac{1}{4}(5^k - 1) + 5^{k+1-1}$

**C.** $1 + 5 + 25 + \cdots + 5^{k-1} + 5^{k+1-1} = \frac{1}{4}(5^{k+1} - 1) + 5^{k+1-1}$

**D.** $1 + 5 + 25 + \cdots + 5^{k-1} + 5^{k+1-1} = \frac{1}{4}(5^k - 1) + 5^{k+1-1}$

**Bonus** Solve $\sum_{n=0}^{6}(3n - 2x) = 7$ for $x$. **Bonus:** ________

**A.** $\frac{11}{2}$ **B.** 8 **C.** 4 **D.** $\frac{19}{3}$

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# Chapter 12 Test, Form 1B

***Write the letter for the correct answer in the blank at the right of each problem.***

**1.** Find the 27th term in the arithmetic sequence $-8, 1, 10, \ldots$. **1.** ________
**A.** 174 **B.** 242 **C.** 235 **D.** 226

**2.** Find the sum of the first 20 terms in the arithmetic series $14 + 3 - 8 - \cdots$. **2.** ________
**A.** $-195$ **B.** $-1810$ **C.** 195 **D.** 1810

**3.** Find the sixth term in the geometric sequence $11, -44, 176, \ldots$. **3.** ________
**A.** 11,264 **B.** $-11{,}264$ **C.** 45,056 **D.** $-45{,}056$

**4.** Find the sum of the first five terms in the geometric series $2 - \frac{4}{3} + \frac{8}{9} - \cdots$. **4.** ________
**A.** $\frac{81}{55}$ **B.** $\frac{13}{27}$ **C.** $\frac{110}{81}$ **D.** $\frac{275}{81}$

**5.** Find three geometric means between $-\frac{2}{3}$ and $-54$. **5.** ________
**A.** 2, 6, 18 **B.** $-2, 6, -18$ **C.** $2, -6, 18$ **D.** A or C

**6.** Find $\lim \frac{4n^3 + 7n^2}{5n^3 - 7n^2 + 3}$. **6.** ________
**A.** $\frac{5}{4}$ **B.** 0 **C.** $\frac{4}{5}$ **D.** does not exist

**7.** Find the sum of $\frac{11}{5} - \frac{33}{55} + \frac{99}{605} - \cdots$. **7.** ________
**A.** $\frac{121}{70}$ **B.** $-\frac{121}{70}$ **C.** $\frac{22}{7}$ **D.** does not exist

**8.** Write $0.\overline{123}$ as a fraction. **8.** ________
**A.** $\frac{41}{33}$ **B.** $\frac{41}{3333}$ **C.** $\frac{41}{333}$ **D.** $\frac{333}{41}$

**9.** Which of the following series is convergent? **9.** ________
**A.** $7.5 + 1.5 + 0.3 + \cdots$ **B.** $1.2 - 3.6 + 10.8 - \cdots$
**C.** $1.2 + 3.6 + 10.8 + \cdots$ **D.** $-2.5 + 2.5 - 2.5 + \cdots$

**10.** Which of the following series is divergent? **10.** ________
**A.** $\frac{1}{3^2} + \frac{1}{6^2} + \frac{1}{9^2} + \cdots$ **B.** $\frac{2^2}{3} + \frac{2^4}{6} + \frac{2^6}{9} + \cdots$
**C.** $\frac{1 \cdot 2}{3^1} + \frac{2 \cdot 3}{3^2} + \frac{3 \cdot 4}{3^3} + \cdots$ **D.** $\frac{0.5^2}{3} + \frac{0.5^4}{6} + \frac{0.5^6}{9} + \cdots$

**11.** Write $\sum_{k=2}^{4} 5\left(\frac{2}{3}\right)^k$ in expanded form and then find the sum. **11.** ________
**A.** $5\left(\frac{2}{3}\right)^2 + \left(\frac{2}{3}\right)^2 + \left(\frac{2}{3}\right)^2; \frac{28}{9}$ **B.** $\left(\frac{5 \cdot 2}{3}\right)^2 + \left(\frac{5 \cdot 2}{3}\right)^3 + \left(\frac{5 \cdot 2}{3}\right)^4; \frac{15{,}700}{81}$
**C.** $5\left(\frac{2}{3}\right)^1 + 5\left(\frac{2}{3}\right)^2 + 5\left(\frac{2}{3}\right)^3; \frac{190}{27}$ **D.** $5\left(\frac{2}{3}\right)^2 + 5\left(\frac{2}{3}\right)^3 + 5\left(\frac{2}{3}\right)^4; \frac{380}{81}$

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# Chapter 12 Test, Form 1B (continued)

**12.** Express the series $0.7 + 0.007 + 0.00007 + \cdots$ using sigma notation. **12.** ________

**A.** $\sum_{k=1}^{\infty} 0.7(10)^{k-1}$ **B.** $\sum_{k=1}^{\infty} 7(10)^{1-2k}$ **C.** $\sum_{k=1}^{\infty} 7(10)^{1-k}$ **D.** $\sum_{k=1}^{\infty} 0.7(10)^{-k}$

**13.** The expression $243c^5 + 810c^4d + 1080c^3d^2 + 720c^2d^3 + 240cd^4 + 32d^5$ is the expansion of which binomial? **13.** ________

**A.** $(3c + d)^5$ **B.** $(c + 2d)^5$ **C.** $(2c + 3d)^5$ **D.** $(3c + 2d)^5$

**14.** Find the third term in the expansion of $(3x - y)^6$. **14.** ________

**A.** $1215x^4y^2$ **B.** $1215x^2y^4$ **C.** $-1215x^2y^4$ **D.** $-1215x^4y^2$

**15.** Use the first five terms of the exponential series $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots$ to approximate $e^{3.9}$. **15.** ________

**A.** 39.40 **B.** 24.01 **C.** 32.03 **D.** 90.11

**16.** Find $\ln(-102)$. **16.** ________

**A.** 4.6250 **B.** $\boldsymbol{i}\pi + 4.6250$ **C.** $\boldsymbol{i}\pi - 4.6250$ **D.** $-4.6250$

**17.** Write $15\sqrt{3} - 15\boldsymbol{i}$ in exponential form. **17.** ________

**A.** $30e^{\boldsymbol{i}\frac{11\pi}{6}}$ **B.** $30e^{\boldsymbol{i}\frac{5\pi}{6}}$ **C.** $30e^{\boldsymbol{i}\frac{7\pi}{6}}$ **D.** $15e^{\boldsymbol{i}\frac{11\pi}{6}}$

**18.** Find the first three iterates of the function $f(z) = -2z$ for $z_0 = 1 - 3\boldsymbol{i}$. **18.** ________

**A.** $2 - 6\boldsymbol{i}, 4 - 12\boldsymbol{i}, 8 - 24\boldsymbol{i}$ **B.** $-2 + 6\boldsymbol{i}, 4 - 12\boldsymbol{i}, -8 + 24\boldsymbol{i}$
**C.** $-2 + 6\boldsymbol{i}, 4 + 12\boldsymbol{i}, 8 + 24\boldsymbol{i}$ **D.** $2 - 6\boldsymbol{i}, -4 + 12\boldsymbol{i}, 8 - 24\boldsymbol{i}$

**19.** Find the first three iterates of the function $f(z) = z^2 - c$ for $c = \boldsymbol{i}$ and $z_0 = 1$. **19.** ________

**A.** $1 - \boldsymbol{i}, 2 - 3\boldsymbol{i}, -5 - 13\boldsymbol{i}$ **B.** $1 - \boldsymbol{i}, -3\boldsymbol{i}, -9 - \boldsymbol{i}$
**C.** $1 - \boldsymbol{i}, -3\boldsymbol{i}, 9 + \boldsymbol{i}$ **D.** $1 - \boldsymbol{i}, -2\boldsymbol{i}, -4 - \boldsymbol{i}$

**20.** Suppose in a proof of the summation formula $7 + 9 + 11 + \cdots + 2n + 5 = n(n + 6)$ by mathematical induction, you show the formula valid for $n = 1$ and assume that it is valid for $n = k$. What is the next equation in the induction step of this proof? **20.** ________

**A.** $7 + 9 + 11 + \cdots + 2k + 5 + 2(k + 1) + 5 = k(k + 6) + (k + 1)(k + 1 + 6)$
**B.** $7 + 9 + 11 + \cdots + 2(k + 1) + 5 = k(k + 6)$
**C.** $7 + 9 + 11 + \cdots + 2k + 5 = k(k + 6)$
**D.** $7 + 9 + 11 + \cdots + 2k + 5 + 2(k + 1) + 5 = k(k + 6) + 2(k + 1) + 5$

**Bonus** If $a_1, a_2, a_3, \ldots, a_n$ is an arithmetic sequence, where $a_n \neq 0$, then $\frac{1}{a_1}, \frac{1}{a_2}, \frac{1}{a_3}, \ldots, \frac{1}{a_n}$ is a harmonic sequence. Find one harmonic mean between $\frac{1}{2}$ and $\frac{1}{8}$. **Bonus:** ________

**A.** $\frac{1}{4}$ **B.** $\frac{1}{5}$ **C.** $\frac{1}{6}$ **D.** $\frac{5}{16}$

Chapter 12

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# Chapter 12 Test, Form 1C

***Write the letter for the correct answer in the blank at the right of each problem.***

**1.** Find the 21st term in the arithmetic sequence 9, 3, −3, . . . . **1.** ________

**A.** −111 **B.** −129 **C.** −117 **D.** −126

**2.** Find the sum of the first 20 terms in the arithmetic series $-6 - 12 - 18 - \cdots$. **2.** ________

**A.** −2520 **B.** −1266 **C.** −1140 **D.** −1260

**3.** Find the 10th term in the geometric sequence −2, 6, −18, . . . . **3.** ________

**A.** 118,098 **B.** −118,098 **C.** 39,366 **D.** −39,366

**4.** Find the sum of the first eight terms in the geometric series $-4 + 8 - 16 + \cdots$. **4.** ________

**A.** −342 **B.** −1020 **C.** −340 **D.** 340

**5.** Find one geometric mean between 2 and 32. **5.** ________

**A.** −16 **B.** 8 **C.** 12 **D.** 4

**6.** Find $\lim_{n \to \infty} \frac{n^3 - 5}{n^2}$. **6.** ________

**A.** $\frac{3}{2}$ **B.** −5 **C.** $\frac{8}{5}$ **D.** does not exist

**7.** Find the sum of $16 - 4 + 1 - \cdots$. **7.** ________

**A.** 64 **B.** $\frac{64}{5}$ **C.** 20 **D.** does not exist

**8.** Write $0.\overline{8}$ as a fraction. **8.** ________

**A.** $\frac{88}{999}$ **B.** $\frac{8}{9}$ **C.** $\frac{8}{99}$ **D.** $\frac{9}{8}$

**9.** Which of the following series is convergent? **9.** ________

**A.** $8 + 8.8 + 9.68 + \cdots$ **B.** $8 + 6 + 4 + \cdots$

**C.** $8 + 2.4 + 0.72 + \cdots$ **D.** $8 - 8 + 8 - \cdots$

**10.** Which of the following series is divergent? **10.** ________

**A.** $1 + \frac{1}{2^2} + \frac{1}{2^4} + \frac{1}{2^6} + \cdots$ **B.** $1 + \frac{1}{3^2} + \frac{1}{3^4} + \frac{1}{3^6} + \cdots$

**C.** $1 - \frac{1}{2^2} + \frac{1}{2^4} + \frac{1}{2^6} + \cdots$ **D.** $1 + \left(\frac{3}{2}\right)^2 + \left(\frac{3}{2}\right)^4 + \left(\frac{3}{2}\right)^6 + \cdots$

**11.** Write $\sum_{k=1}^{4} 3^{k-1}$ in expanded form and then find the sum. **11.** ________

**A.** $1 + 3 + 9 + 27; 40$ **B.** $1 + \frac{1}{3} + \frac{1}{9} + \frac{1}{27}; \frac{40}{27}$

**C.** $3 + 9 + 27 + 81; 120$ **D.** $0 + 2 + 8 + 26; 36$

 

# Chapter 12 Test, Form 1C (continued)

**12.** Express the series $5 + 9 + 13 + \cdots + 101$ using sigma notation. **12.** ________

**A.** $\sum_{k=1}^{\infty}(4k + 1)$ **B.** $\sum_{k=1}^{25}(4k + 1)$ **C.** $\sum_{k=1}^{25}(4k - 1)$ **D.** $\sum_{k=1}^{24}(4k + 1)$

**13.** The expression $32x^5 + 80x^4 + 80x^3 + 40x^2 + 10x + 1$ is the expansion of which binomial? **13.** ________

**A.** $(2x + 1)^5$ **B.** $(x + 2)^5$ **C.** $(2x + 2)^5$ **D.** $(2x - 1)^5$

**14.** Find the fourth term in the expansion of $(3x + y)^7$. **14.** ________

**A.** $105x^4y^3$ **B.** $420x^4y^3$ **C.** $1701x^4y^3$ **D.** $2835x^4y^3$

**15.** Use the first five terms of the exponential series $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots$ to approximate $e^5$. **15.** ________

**A.** 65.375 **B.** 148.41 **C.** 48.41 **D.** 76.25

**16.** Find $\ln(-21)$. **16.** ________

**A.** 3.0445 **B.** $\boldsymbol{i}\pi - 3.0445$ **C.** $\boldsymbol{i}\pi + 3.0445$ **D.** $-3.0445$

**17.** Write $1 - \boldsymbol{i}$ in exponential form. **17.** ________

**A.** $\sqrt{2}e^{\boldsymbol{i}\frac{\pi}{4}}$ **B.** $\sqrt{2}e^{\boldsymbol{i}\frac{7\pi}{4}}$ **C.** $e^{\boldsymbol{i}\frac{7\pi}{4}}$ **D.** $e^{\boldsymbol{i}\frac{\pi}{4}}$

**18.** Find the first three iterates of the function $f(z) = z + \boldsymbol{i}$ for $z_0 = 1$. **18.** ________

**A.** $1, 1 + \boldsymbol{i}, 1 + 2\boldsymbol{i}$ **B.** $1 + \boldsymbol{i}, 2 + 2\boldsymbol{i}, 3 + 3\boldsymbol{i}$
**C.** $1 + \boldsymbol{i}, 1 + 2\boldsymbol{i}, 1 + 3\boldsymbol{i}$ **D.** $1 + \boldsymbol{i}, 1 + \boldsymbol{i}, 1 + \boldsymbol{i}$

**19.** Find the first three iterates of the function $f(z) = z^2 - c$ for $c = \boldsymbol{i}$ and $z_0 = \boldsymbol{i}$. **19.** ________

**A.** $-1 - \boldsymbol{i}, \boldsymbol{i}, -1-\boldsymbol{i}$ **B.** $-1 + \boldsymbol{i}, 3\boldsymbol{i}, -9$
**C.** $1 - \boldsymbol{i}, -3\boldsymbol{i}, 9 + \boldsymbol{i}$ **D.** $-1 + \boldsymbol{i}, 2\boldsymbol{i}, -4 + \boldsymbol{i}$

**20.** Suppose in a proof of the summation formula $1 + 5 + 9 + \cdots + 4n - 3 = n(2n - 1)$ by mathematical induction, you show the formula valid for $n = 1$ and assume that it is valid for $n = k$. What is the next equation in the induction step of this proof? **20.** ________

**A.** $1 + 5 + 9 + \cdots + 4k - 3 + 4(k + 1) - 3 = k(2k - 1) + 4(k + 1) - 3$
**B.** $1 + 5 + 9 + \cdots + 4k - 3 = k(2k - 1) + 4(k + 1) - 3$
**C.** $1 + 5 + 9 + \cdots + 4k - 3 = k(2k - 1)$
**D.** $1 + 5 + 9 + \cdots + 4k - 3 + 4(k + 1) - 3 = k(2k - 1) + (k + 1)[2(k + 1) - 1]$

**Bonus** If $a_1, a_2, a_3, \ldots, a_n$ is an arithmetic sequence, where $a_n \neq 0$, then $\frac{1}{a_1}, \frac{1}{a_2}, \frac{1}{a_3}, \ldots, \frac{1}{a_n}$ is a harmonic sequence. Find one harmonic mean between 2 and 3. **Bonus:** ________

**A.** $\frac{2}{5}$ **B.** $\frac{5}{2}$ **C.** $\frac{5}{12}$ **D.** $\frac{12}{5}$

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# Chapter 12 Test, Form 2A

**1.** Find $d$ for the arithmetic sequence in which $a_1 = 14$ and $a_{28} = 32$. **1.** ________________

**2.** Find the 15th term in the arithmetic sequence $11\frac{4}{5}, 10\frac{2}{5}, 9, 7\frac{3}{5}, \ldots$. **2.** ________________

**3.** Find the sum of the first 27 terms in the arithmetic series $35.5 + 34.3 + 33.1 + 31.9 + \cdots$. **3.** ________________

**4.** Find the ninth term in the geometric sequence 25, 10, 4, . . . . **4.** ________________

**5.** Find the sum of the first eight terms in the geometric series $\frac{1}{5} + 2 + 20 + \cdots$. **5.** ________________

**6.** Form a sequence that has three geometric means between 6 and 54. **6.** ________________

**7.** Find $\lim_{n\to\infty} \frac{13n^4 + 5n^2}{9n^3 - 5n^2 + 4}$ or state that the limit does not exist. **7.** ________________

**8.** Find the sum of the series $6\sqrt{2} + 6 + 3\sqrt{2} + 3 + \cdots$ or state that the sum does not exist. **8.** ________________

**9.** Write $0.0\overline{64}$ as a fraction. **9.** ________________

***Determine whether each series is convergent or divergent.***

**10.** $\frac{1}{2\sqrt{2} \cdot 1^3} + \frac{1}{2\sqrt{2} \cdot 2^3} + \frac{1}{2\sqrt{2} \cdot 3^3} + \cdots$ **10.** ________________

**11.** $\frac{2^1}{1} + \frac{2^2}{2} + \frac{2^3}{3} + \cdots$ **11.** ________________

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# Chapter 12 Test, Form 2A (continued)

**12.** Write $\sum_{k=2}^{7} 27\left(-\frac{1}{3}\right)^{k-2}$ in expanded form and then find the sum. **12.** ________________

**13.** Express the series $\frac{3 \cdot 9}{10} + \frac{3 \cdot 11}{12} + \frac{3 \cdot 13}{14} + \cdots + \frac{3 \cdot 23}{24}$ using sigma notation. **13.** ________________

**14.** Use the Binomial Theorem to expand $(1 + \sqrt{3})^5$. **14.** ________________

**15.** Find the fifth term in the expansion of $(3x^3 + 2y^2)^5$. **15.** ________________

**16.** Use the first five terms of the exponential series to approximate $e^{2.7}$. **16.** ________________

**17.** Find ln $(-12.7)$ to four decimal places. **17.** ________________

**18.** Find the first three iterates of the function $f(z) = 3z + 1$ for $z_0 = 2 + \boldsymbol{i}$. **18.** ________________

**19.** Find the first three iterates of the function $f(z) = z^2 + c$ for $c = 1 + \boldsymbol{i}$ and $z_0 = 2\boldsymbol{i}$. **19.** ________________

**20.** Use mathematical induction to prove that $1 + 5 + 25 + \cdots + 5^{n-1} = \frac{1}{4}(5^n - 1)$. Write your proof on a separate piece of paper. **20.** ________________

**Bonus** If $f(z) = z^2 + z + c$ is iterated with an initial value of $3 - 4\boldsymbol{i}$ and $z_1 = 4 - 11\boldsymbol{i}$, find $c$. **Bonus:** ________________

NAME ______________ DATE ______________ PERIOD ______________

Chapter 12

# Chapter 12 Test, Form 2B

**1.** Find $d$ for the arithmetic sequence in which $a_1 = 6$ and $a_{13} = -42$. **1.** ______________

**2.** Find the 40th term in the arithmetic sequence $7, 4.4, 1.8, -0.8, \ldots$. **2.** ______________

**3.** Find the sum of the first 30 terms in the arithmetic series $10 + 6 + 2 - 2 - \cdots$. **3.** ______________

**4.** Find the ninth term in the geometric sequence $\frac{1}{27}, \frac{1}{9}, \frac{1}{3}, \ldots$. **4.** ______________

**5.** Find the sum of the first eight terms in the geometric series $64 - 32 + 16 - 8 + \cdots$. **5.** ______________

**6.** Form a sequence that has three geometric means between $-4$ and $-324$. **6.** ______________

**7.** Find $\lim \frac{2n - 1}{n^3}$ or state that the limit does not exist. **7.** ______________

**8.** Find the sum of the series $12 + 8 + \frac{16}{3} + \cdots$ or state that the sum does not exist. **8.** ______________

**9.** Write $8.\overline{18}$ as a fraction. **9.** ______________

***Determine whether each series is convergent or divergent.***

**10.** $\frac{4}{1} + \frac{7}{1 \cdot 2} + \frac{10}{1 \cdot 2 \cdot 3} + \cdots$ **10.** ______________

**11.** $\frac{3^1}{1} + \frac{3^2}{2} + \frac{3^3}{3} + \cdots$ **11.** ______________

NAME ______________________ DATE ______________ PERIOD ________

# Chapter 12 Test, Form 2B (continued)

**12.** Write $\sum_{k=0}^{3}(k+1)(k+2)$ in expanded form and then find the sum. **12.** ______________

**13.** Express the series $\frac{1 \cdot 0}{2} + \frac{2 \cdot 1}{3} + \frac{3 \cdot 2}{4} + \cdots + \frac{10 \cdot 9}{11}$ using sigma notation. **13.** ______________

**14.** Use the Binomial Theorem to expand $(2p - 3q)^4$. **14.** ______________

**15.** Find the fifth term in the expansion of $(4x + 2y)^7$. **15.** ______________

**16.** Use the first five terms of the cosine series $\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!} - \cdots$ to approximate the value of $\cos \frac{\pi}{4}$ to four decimal places. **16.** ______________

**17.** Find $\ln(-13.4)$ to four decimal places. **17.** ______________

**18.** Find the first three iterates of the function $f(z) = 0.5z$ for $z_0 = 4 - 2\boldsymbol{i}$. **18.** ______________

**19.** Find the first three iterates of the function $f(z) = z^2 + c$ for $c = 2\boldsymbol{i}$ and $z_0 = 1$. **19.** ______________

**20.** Use mathematical induction to prove that $7 + 9 + 11 + \cdots + (2n + 5) = n(n + 6)$. Write your proof on a separate piece of paper. **20.** ______________

**Bonus** Find the sum of the coefficients of the expansion of $(x + y)^7$. **Bonus:** ______________

NAME ______________________ DATE ______________ PERIOD _________

# Chapter 12 Test, Form 2C

**1.** Find $d$ for the arithmetic sequence in which $a_1 = 5$ and $a_{12} = 38$. **1.** ______________

**2.** Find the 31st term in the arithmetic sequence 9.3, 9, 8.7, 8.4, . . . . **2.** ______________

**3.** Find the sum of the first 23 terms in the arithmetic series $6 + 11 + 16 + 21 + \cdots$. **3.** ______________

**4.** Find the fifth term in the geometric sequence $-10, -40, -160, \ldots$. **4.** ______________

**5.** Find the sum of the first 10 terms in the geometric series $3 - 6 + 12 - 24 + \cdots$. **5.** ______________

**6.** Form a sequence that has two geometric means between 9 and $\frac{1}{3}$. **6.** ______________

**7.** Find $\lim\limits_{n \to \infty} \frac{2n^2}{n^2 + 1}$ or state that the limit does not exist. **7.** ______________

**8.** Find the sum of the series $\frac{1}{12} + \frac{1}{2} + 3 + \cdots$ or state that the sum does not exist. **8.** ______________

**9.** Write $0.\overline{53}$ as a fraction. **9.** ______________

***Determine whether each series is convergent or divergent.***

**10.** $\frac{1}{2^0} + \frac{1}{2^2} + \frac{1}{2^4} + \cdots$ **10.** ______________

**11.** $\frac{2^1}{1} + \frac{2^2}{2} + \frac{2^3}{3} + \cdots$ **11.** ______________

NAME ______________________ DATE ______________ PERIOD ________

# Chapter 12 Test, Form 2C (continued)

**12.** Write $\sum_{k=4}^{7} 3k$ in expanded form and then find the sum. **12.** ______________

**13.** Express the series $\frac{1 \cdot 2}{2} + \frac{2 \cdot 3}{4} + \frac{3 \cdot 4}{6} + \cdots + \frac{8 \cdot 9}{16}$ using sigma notation. **13.** ______________

**14.** Use the Binomial Theorem to expand $(2p + 1)^4$. **14.** ______________

**15.** Find the fourth term in the expansion of $(2x - 3y)^4$. **15.** ______________

**16.** Use the first five terms of the sine series $\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \frac{x^9}{9!} - \cdots$ to find the value of $\sin \frac{\pi}{5}$ to four decimal places. **16.** ______________

**17.** Find $\ln(-58)$ to four decimal places. **17.** ______________

**18.** Find the first three iterates of the function $f(z) = 2z$ for $z_0 = 1 + 4\boldsymbol{i}$. **18.** ______________

**19.** Find the first three iterates of the function $f(z) = z^2 + c$ for $c = \boldsymbol{i}$ and $z_0 = 1$. **19.** ______________

**20.** Use mathematical induction to prove that $1 + 5 + 9 + \cdots + 4n - 3 = n(2n - 1)$. Write your proof on a separate piece of paper. **20.** ______________

**Bonus** Find the sum of the coefficients of the expansion of $(x + y)^5$. **Bonus:** ______________

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Chapter 12

# Chapter 12 Open-Ended Assessment

***Instructions: Demonstrate your knowledge by giving a clear, concise solution to each problem. Be sure to include all relevant drawings and justify your answers. You may show your solution in more than one way or investigate beyond the requirements of the problem.***

**1. a.** Write a word problem that involves an arithmetic sequence. Write the sequence and solve the problem. Tell what the answer represents.

**b.** Find the common difference and write the $n$th term of the arithmetic sequence in part **a**.

**c.** Find the sum of the first 12 terms of the arithmetic sequence in part **a**. Explain in your own words why the formula for the sum of the first $n$ terms of an arithmetic series works.

**d.** Does the related arithmetic series converge? Why or why not?

**2. a.** Write a word problem that involves a geometric sequence. Write the sequence and solve the problem. Tell what the answer represents.

**b.** Find the common ratio and write the $n$th term of the geometric sequence in part **a**.

**c.** Find the sum of the first 11 terms of the sequence in part **a**.

**d.** Describe in your own words a test to determine whether a geometric series converges. Does the geometric series in part **a** converge?

**3. a.** Explain in your own words how to use mathematical induction to prove that a statement is true for all positive integers.

**b.** Use mathematical induction to prove that the sum of the first $n$ terms of a geometric series is given by the formula $S_n = \frac{a_1 - a_1 r^n}{1 - r}$, where $r \neq 1$.

**4.** Find the fourth term in the expansion of $\left(\frac{\sqrt{x}}{y^2} - \frac{y}{\sqrt{x}}\right)^6$.

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Chapter 12

# Chapter 12 Mid-Chapter Test (Lessons 12-1 through 12-4)

**1.** Find the 20th term in the arithmetic sequence 15, 21, 27, . . . . **1.** ________________

**2.** Find the sum of the first 25 terms in the arithmetic series $11 + 14 + 17 + 20 + \cdots$. **2.** ________________

**3.** Find the 12th term in the geometric sequence $2^{-4}, 2^{-3}, 2^{-2}, \ldots$. **3.** ________________

**4.** Find the sum of the first 10 terms in the geometric series $2 - 6 + 18 - 54 + \cdots$. **4.** ________________

**5.** Write a sequence that has two geometric means between 64 and $-8$. **5.** ________________

**6.** Find $\lim_{n\to\infty} \frac{n^2 + 1}{2n^2 - 3n}$ or state that the limit does not exist. **6.** ________________

**7.** Find the sum of the series $\frac{1}{8} + \frac{1}{4} + \frac{1}{2} + \cdots$ or state that the sum does not exist. **7.** ________________

**8.** Write $0.\overline{63}$ as a fraction. **8.** ________________

***Determine whether each series is convergent or divergent.***

**9.** $5 + \frac{5^2}{1 \cdot 2} + \frac{5^3}{1 \cdot 2 \cdot 3} + \frac{5^4}{1 \cdot 2 \cdot 3 \cdot 4} + \cdots$ **9.** ________________

**10.** $\frac{2^2}{2} + \frac{2^3}{3} + \frac{2^4}{4} + \frac{2^5}{5} + \cdots$ **10.** ________________

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Chapter 12

# Chapter 12, Quiz A (Lessons 12-1 and 12-2)

**1.** Find the 11th term in the arithmetic sequence $\sqrt{3} + \sqrt{5}, 0, -\sqrt{3} - \sqrt{5}, \ldots.$ **1.** ______________

**2.** Find $n$ for the sequence for which $a_n = 19$, $a_1 = -13$, and $d = 2$. **2.** ______________

**3.** Find the sum of the first 17 terms in the arithmetic series $4.5 + 4.7 + 4.9 + \cdots$. **3.** ______________

**4.** Find the fifth term in the geometric sequence for which $a_3 = \sqrt{5}$ and $r = 3$. **4.** ______________

**5.** Find the sum of the first six terms in the geometric series $1 + 1.5 + 2.25 + \cdots$. **5.** ______________

**6.** Write a sequence that has one geometric mean between $\frac{1}{3}$ and $\frac{5}{27}$. **6.** ______________

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Chapter 12

# Chapter 12, Quiz B (Lessons 12-3 and 12-4)

***Find each limit, or state that the limit does not exist.***

**1.** $\lim_{n\to\infty} \frac{3n^4}{2n^2 + 5}$ **2.** $\lim_{n\to\infty} \frac{(2n + 1)(n - 2)}{2n^2}$ **3.** $\lim_{n\to\infty} \frac{n + 1}{n^2 - 4}$

**1.** ______________
**2.** ______________
**3.** ______________

***Find the sum of each series, or state that the sum does not exist.***

**4.** $\frac{1}{2} - \frac{1}{4} + \frac{1}{8} - \frac{1}{16} + \cdots$ **5.** $-\frac{3}{5} + 1 - \frac{5}{3} + \cdots$

**4.** ______________
**5.** ______________

**6.** Write the repeating decimal $0.\overline{45}$ as a fraction. **6.** ______________

***Determine whether each series is convergent or divergent.***

**7.** $0.002 + 0.02 + 0.2 + \cdots$ **7.** ______________

**8.** $\frac{5}{8} + \frac{5}{9} + \frac{5}{10} + \frac{5}{11} + \cdots$ **8.** ______________

**9.** $\frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \frac{1}{3 \cdot 4} + \frac{1}{4 \cdot 5} + \cdots$ **9.** ______________

**10.** $\frac{2}{1 \cdot 2} + \frac{3}{2 \cdot 3} + \frac{4}{3 \cdot 4} + \cdots$ **10.** ______________

NAME ______________ DATE ______________ PERIOD ______________

Chapter 12

# Chapter 12, Quiz C (Lessons 12-5 and 12-6)

**1.** Write $\sum_{n=2}^{4}\left(2^{n-1}+\frac{1}{2}\right)$ in expanded form and then find the sum. **1.** ______________

**2.** Express the series $\frac{16}{81}+\frac{8}{27}+\frac{4}{9}+\cdots$ using sigma notation. **2.** ______________

**3.** Express the series $1 \cdot 2 + 3 \cdot 4 + 5 \cdot 6 + \cdots + 199 \cdot 200$ using sigma notation. **3.** ______________

**4.** Use the Binomial Theorem to expand $(3a - d)^4$. **4.** ______________

**5.** Use the first five terms of the exponential series $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots$ to approximate $e^{4.1}$ to the nearest hundredth. **5.** ______________

**6.** Use the first five terms of the trigonometric series $\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \frac{x^9}{9!} - \cdots$ to approximate $\sin \frac{\pi}{3}$ to four decimal places. **6.** ______________

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Chapter 12

# Chapter 12, Quiz D (Lessons 12-8 and 12-9)

**1.** Find the first four iterates of the function $f(x) = \frac{1}{10}x + 1$ for $x_0 = 1$. **1.** ______________

**2.** Find the first three iterates of the function $f(z) = 2z + \boldsymbol{i}$ for $z_0 = 3 - \boldsymbol{i}$. **2.** ______________

**3.** Find the first three iterates of the function $f(z) = z^2 + c$ for $c = -1 + 2\boldsymbol{i}$ and $z_0 = \boldsymbol{i}$. **3.** ______________

**4.** Use mathematical induction to prove that $1 + 3 + 5 + \cdots + (2n - 1) = n^2$. Write your proof on a separate piece of paper. **4.** ______________

**5.** Use mathematical induction to prove that $5 + 11 + 17 + \cdots + (6n - 1) = n(3n + 2)$. Write your proof on a separate piece of paper. **5.** ______________

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# Chapter 12 SAT and ACT Practice

***After working each problem, record the correct answer on the answer sheet provided or use your own paper.***

## Multiple Choice

**1.** In a basket of 80 apples, exactly 4 are rotten. What percent of the apples are not rotten?

**A** 4%
**B** 5%
**C** 20%
**D** 95%
**E** 96%

**2.** Which grade had the largest percent increase in the number of students from 1999 to 2000?

| Grade | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|
| 1999 | 60 | 55 | 65 | 62 | 60 |
| 2000 | 80 | 62 | 72 | 72 | 70 |

**A** 8
**B** 9
**C** 10
**D** 11
**E** 12

**3.** Find the length of a chord of a circle if the chord is 6 units from the center and the length of the radius is 10 units.

**A** 4
**B** 8
**C** 16
**D** $2\sqrt{34}$
**E** $4\sqrt{34}$

**4.** A chord of length 16 is 4 units from the center of a circle. Find the diameter.

**A** $2\sqrt{5}$
**B** $4\sqrt{5}$
**C** $8\sqrt{5}$
**D** $4\sqrt{3}$
**E** $8\sqrt{3}$

**5.** If $x + y = 4$ and $2x - y = 5$, then $x + 2y =$

**A** 1
**B** 2
**C** 4
**D** 5
**E** 6

**6.** If $\frac{15k}{3kx + 36} = 1$ and $x = 4$, then $k =$

**A** 2
**B** 3
**C** 4
**D** 8
**E** 12

**7.** If 12% of a class of 25 students do not have pets, how many students in the class do have pets?

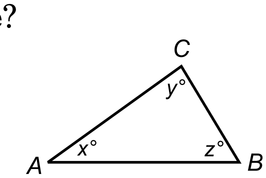
**A** 3
**B** 12
**C** 13
**D** 20
**E** 22

**8.** In a senior class there are 400 boys and 500 girls. If 60% of the boys and 50% of the girls live within 1 mile of school, what percent of the seniors do not live within 1 mile of school?

**A** about 45.6%
**B** about 54.4%
**C** about 55.5%
**D** about 44.4%
**E** about 61.1%

**9.** In $\triangle ABC$ below, if $BC < BA$, which of the following is true?

**A** $x > y$
**B** $y < z$
**C** $y > x$
**D** $y = x$
**E** $z < x$



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# Chapter 12 SAT and ACT Practice (continued)

**10.** Which is the measure of each angle of a regular polygon with $r$ sides?

**A** $\frac{1}{r}(360°)$
**B** $(r - 2)180°$
**C** $360°$
**D** $\frac{1}{r}(r - 2)180°$
**E** $60°$

**11.** A tractor is originally priced at \$7000. The price is reduced by 20% and then raised by 5%. What is the net reduction in price?

**A** \$5950 **B** \$5880
**C** \$1400 **D** \$1120
**E** \$1050

**12.** The original price of a camera provided a profit of 30% above the dealer's cost. The dealer sets a new price of \$195, a 25% increase above the original price. What is the dealer's cost?

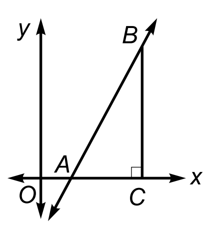
**A** \$243.75 **B** \$202.80
**C** \$156.00 **D** \$120.00
**E** None of these

**13.** Segments of the lines $x = 4$, $x = 9$, $y = -5$, and $y = 4$ form a rectangle. What is the area of this rectangle in square units?

**A** 6 **B** 10
**C** 18 **D** 20
**E** 45

**14.** In the figure below, the coordinates of $A$ are (4, 0) and of $C$ are (15, 0). Find the area of $\triangle ABC$ if the equation of $\overleftrightarrow{AB}$ is $2x - y = 8$.

**A** 100 units$^2$
**B** 121 units$^2$
**C** 132 units$^2$
**D** 144 units$^2$
**E** 169 units$^2$




**15.** In 1998, Bob earned \$2800. In 1999, his earnings increased by 15%. In 2000, his earnings decreased by 15% from his earnings in 1999. What were his earnings in 2000?

**A** \$2800.00 **B** \$2380.98
**C** \$2381.15 **D** \$2737.00
**E** None of these

**16.** Londa is paid a 15% commission on all sales, plus \$8.50 per hour. One week, her sales were \$6821.29. How many hours did she work to earn \$1371.69?

**A** 68.3 **B** 41.0
**C** 120.4 **D** 28.2
**E** None of these

**17–18. Quantitative Comparison**

**A** if the quantity in Column A is greater
**B** if the quantity in Column B is greater
**C** if the two quantities are equal
**D** if the relationship cannot be determined from the information given

| | **Column A** | **Column B** |
|---|---|---|
| **17.** | $x > 0$ | |
| | $x$ plus an increase of 75% of $x$ | $0.75x$ |
| **18.** | One day, 90% of the girls and 80% of the boys were present in class. | |
| | Number of boys absent | Number of girls absent |

**19. Grid-In** How many dollars must be invested at a simple-interest rate of 7.2% to earn \$1440 in interest in 5 years?

**20. Grid-In** Seventy-two is 150% of what number?

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Chapter 12

# Chapter 12 Cumulative Review (Chapters 1-12)

**1.** Solve the system by using a matrix equation.
$2x + 3y = 11$
$y = 12 + x$

**1.** ________________

**2.** Determine whether $f(x) = \frac{x + 5}{x^2 - 3x}$ is continuous at $x = 3$. Justify your response using the continuity test.

**2.** ________________

**3.** Determine the binomial factors of $2x^3 + x^2 - 13x + 6$.

**3.** ________________

**4.** Write an equation of the sine function with amplitude 1, period $\frac{2\pi}{3}$, phase shift $\frac{\pi}{15}$, and vertical shift 2.

**4.** ________________

**5.** Find the distance between the parallel lines $2x + 5y = 10$ and $2x + 5y = -5$.

**5.** ________________

**6.** A 300-newton force and a 500-newton force act on the same object. The angle between the forces measures 95°. Find the magnitude and direction of the resultant force.

**6.** ________________

**7.** Find the product $4\left(\cos \frac{2\pi}{3} + \boldsymbol{i} \sin \frac{2\pi}{3}\right) \cdot 3\left(\cos \frac{5\pi}{6} + \boldsymbol{i} \sin \frac{5\pi}{6}\right)$. Then write the result in rectangular form.

**7.** ________________

**8.** Write the equation of the ellipse $6x^2 + 9y^2 = 54$ after a rotation of 45° about the origin.

**8.** ________________

**9.** If $1500 is invested in an account bearing 8.5% interest compounded continuously, find the balance of the account after 18 months.

**9.** ________________

**10.** Express the series $\frac{3}{3} - \frac{6}{5} + \frac{9}{7} - \frac{12}{9} + \cdots - \frac{30}{21}$ using sigma notation.

**10.** ________________

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# Trigonometry Semester Test

***Write the letter for the correct answer in the blank at the right of each problem.***

**1.** Solve $2^{x+1} = 17.6$. Round your answer to the nearest hundredth. **1.** ________
**A.** 1.99 **B.** 3.14 **C.** $-2.45$ **D.** $-3.84$

**2.** Simplify $\boldsymbol{i}^{29} - \boldsymbol{i}^{20}$. **2.** ________
**A.** 0 **B.** $-2$ **C.** $-1 + \boldsymbol{i}$ **D.** $-2\boldsymbol{i}$

**3.** Find the third iterate of the function $f(x) = 2x + 1$, if the initial value is $x_0 = 3$. **3.** ________
**A.** 15 **B.** 31 **C.** 7 **D.** 12

**4.** Find the nineteenth term in the arithmetic sequence 10, 7, 4, 1, .... **4.** ________
**A.** 102 **B.** 0 **C.** $-47$ **D.** $-44$

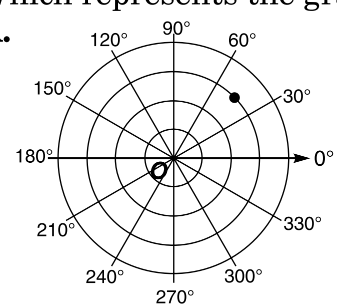
**5.** If $\tan\theta = -\frac{1}{4}$ and $\theta$ has its terminal side in Quadrant II, find the exact value of $\sin\theta$. **5.** ________
**A.** $-4$ **B.** $-\sqrt{17}$ **C.** 2 **D.** $\frac{\sqrt{17}}{17}$

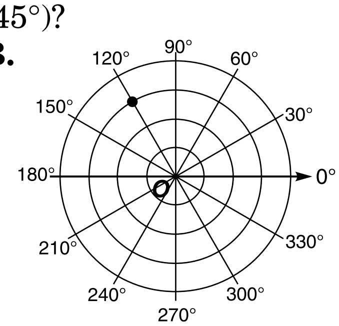
**6.** Which expression is equivalent to $\tan\left(\frac{\pi}{2} - \theta\right)$? **6.** ________
**A.** $\sin\theta$ **B.** $\cot\theta$ **C.** $-\cos\theta$ **D.** $\sec\theta$

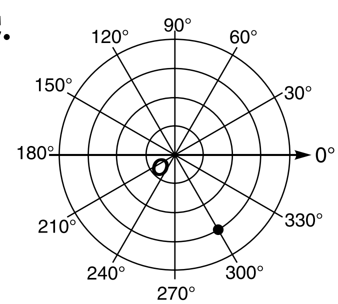
**7.** Find the ordered pair that represents the vector from $A(1, -2)$ to $B(2, 3)$. **7.** ________
**A.** $\langle 2, 3\rangle$ **B.** $\langle 1, 2\rangle$ **C.** $\langle 1, 5\rangle$ **D.** $\langle 3, 1\rangle$

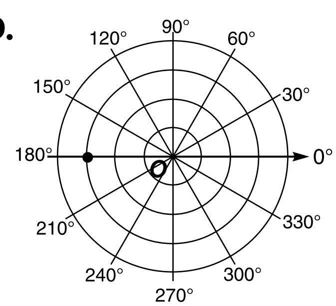
**8.** Which series is divergent? **8.** ________
**A.** $1 + \frac{1}{2^3} + \frac{1}{3^3} + \frac{1}{4^3} + \cdots$ **B.** $1 + \frac{1}{2} + \frac{1}{3} + \cdots$
**C.** $1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \cdots$ **D.** $2 + \frac{2}{3} + \frac{2}{9} + \frac{2}{27} + \cdots$

**9.** Which represents the graph of $(3, 45°)$? **9.** ________

**A.**



**B.**



**C.**



**D.**


# Trigonometry Semester Test (continued)

**10.** Express $x^{\frac{2}{9}}y^{\frac{1}{3}}$ using radicals. **10.** ________

**A.** $\sqrt[3]{x^2y}$ **B.** $\sqrt[9]{x^2y}$ **C.** $\sqrt[9]{x^2y^3}$ $\sqrt[3]{x^3y}$

**11.** Which expression is equivalent to $\sin^2\theta$ for all values of $\theta$? **11.** ________

**A.** $\sin\left(\frac{\pi}{2} - \theta\right)$ **B.** 1 **C.** $2\sin\theta\cos\theta$ **D.** $1 - \cos^2\theta$

**12.** Find the inner product of $\vec{\mathbf{v}}$ and $\vec{\mathbf{w}}$ if $\vec{\mathbf{v}} = \langle 1, 2, 0\rangle$ and $\vec{\mathbf{w}} = \langle 3, -2, 1\rangle$. **12.** ________

**A.** 3 **B.** 2 **C.** $-1$ **D.** 1

**13.** Simplify $(1 + \boldsymbol{i})(-2 + 2\boldsymbol{i})$. **13.** ________

**A.** $3 + 2\boldsymbol{i}$ **B.** $2 + \boldsymbol{i}$ **C.** $-2 + 3\boldsymbol{i}$ **D.** $-4$

**14.** Which equation is a trigonometric identity? **14.** ________

**A.** $\sin 2\theta = \sin\theta\cos\theta$ **B.** $\tan\theta = \frac{\cos\theta}{\sin\theta}$

**C.** $\tan 2\theta = \frac{2\tan\theta}{1 - \tan^2\theta}$ **D.** $\cos 2\theta = 4\cos^2\theta - 1$

**15.** Write $\overrightarrow{MN}$ as the sum of unit vectors for $M(-11, 6, -7)$ and $N(4, -3, -15)$. **15.** ________

**A.** $-7\vec{\mathbf{i}} + 3\vec{\mathbf{j}} - 22\vec{\mathbf{k}}$ **B.** $15\vec{\mathbf{i}} - 9\vec{\mathbf{j}} - 8\vec{\mathbf{k}}$

**C.** $-15\vec{\mathbf{i}} + 9\vec{\mathbf{j}} + 8\vec{\mathbf{k}}$ **D.** $7\vec{\mathbf{i}} - 9\vec{\mathbf{j}} + 8\vec{\mathbf{k}}$

**16.** Express $\sqrt[5]{x^{25}y^2}$ using rational exponents. **16.** ________

**A.** $x^5y^{\frac{5}{2}}$ **B.** $x^2y^2$ **C.** $x^5y^{\frac{2}{5}}$ **D.** $x^{\frac{1}{2}}y^{\frac{2}{5}}$

**17.** Express $(x^2y^3)^{\frac{2}{3}}$ using radicals. **17.** ________

**A.** $\sqrt[2]{x^4y^6}$ **B.** $\sqrt[2]{x^6y^9}$ **C.** $\sqrt[3]{x^2y^3}$ **D.** $\sqrt[3]{x^4y^6}$

**18.** Which polar equation represents a rose? **18.** ________

**A.** $r = 3\theta$ **B.** $r = 3 + 3\sin\theta$

**C.** $r = 3\cos 2\theta$ **D.** $r^2 = 4\cos 2\theta$

**19.** Use a sum or difference identity to find the exact value of $\cos 105°$. **19.** ________

**A.** $\frac{\sqrt{2}}{4}$ **B.** $\frac{\sqrt{2} + \sqrt{6}}{4}$ **C.** $\frac{\sqrt{2} - \sqrt{6}}{4}$ **D.** $\frac{\sqrt{2} - \sqrt{3}}{2}$

NAME ____________________ DATE ____________ PERIOD ________

# Trigonometry Semester Test (continued)

**20.** Write the equation of the line $2x + 3y = 6$ in parametric form. **20.** ________

**A.** $x = t; y = -\frac{2}{3}t + 2$ **B.** $x = 2t; y = 2t + 3$

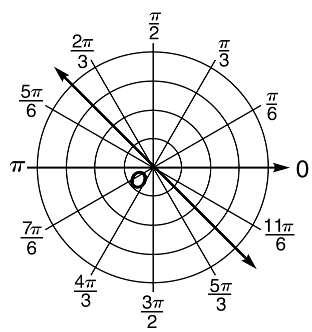
**C.** $x = t; y = -\frac{3}{2}t + 2$ **D.** $x = 3t; y = t + 2$

**21.** Find the sum of the geometric series $\frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \cdots$. **21.** ________

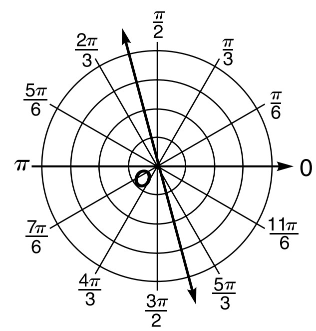
**A.** $\frac{2}{7}$ **B.** $\frac{1}{2}$ **C.** $\frac{2}{3}$ **D.** 1

**22.** Which is the graph of the equation $\theta = \frac{3\pi}{4}$? **22.** ________

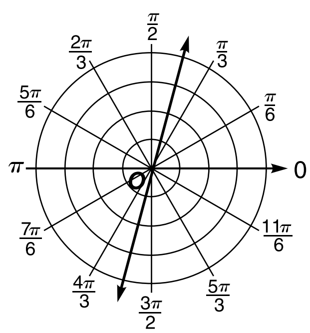
**A.**



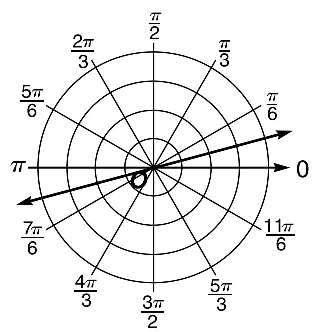

**B.**




**C.**




**D.**




**23.** Solve $\log_6 x = 2$. **23.** ________

**A.** 3 **B.** 36 **C.** 12 **D.** 4

**24.** Express $0.3 + 0.03 + 0.003 + \cdots$ using sigma notation. **24.** ________

**A.** $\sum_{k=1}^{\infty} 3 \cdot 10^{-k}$ **B.** $\sum_{k=1}^{\infty} 0.3 \cdot 10^{-k}$

**C.** $\sum_{k=0}^{\infty} 3 \cdot 10^{-k}$ **D.** $\sum_{k=1}^{\infty} 3 \cdot 10^{-2k}$

**25.** Andre kicks a soccer ball with an initial velocity of 48 feet per second at an angle of 17° with the horizontal. After 0.35 second, what is the height of the ball? **25.** ________

**A.** 16.07 ft **B.** 4.91 ft **C.** 14.11 ft **D.** 2.95 ft

# Trigonometry Semester Test (continued)

**26.** Find $\vec{\mathbf{v}} \times \vec{\mathbf{w}}$ if $\vec{\mathbf{v}} = \langle 2, -4, 6 \rangle$ and $\vec{\mathbf{w}} = \langle 2, 1, -1 \rangle$. **26.** ________________

**27.** Write an equation in slope-intercept form of the line whose parametric equations are $x = 4t + 3$ and $y = -2t - 7$. **27.** ________________

**28.** Use a calculator to find antiln $(-0.23)$ to the nearest hundredth. **28.** ________________

**29.** Find the first four terms of the geometric sequence for which $a_9 = 6561$ and $r = 3$. **29.** ________________

**30.** Find the polar coordinates of the point with rectangular coordinates (2, 2). **30.** ________________

**31.** Find the rectangular coordinates of the point with polar coordinates $\left(\sqrt{2}, \frac{3\pi}{4}\right)$. **31.** ________________

**32.** Express cos 840° as a trigonometric function of an angle in Quadrant II. **32.** ________________

**33.** Solve $2 \sin^2 x - \sin x = 0$ for principle values of $x$. Express in degrees. **33.** ________________

**34.** Find the distance between the point $P(1, 0)$ and the line with equation $2x + 3y = -2$. **34.** ________________

**35.** If $\cos \theta = \frac{3}{4}$ and $\theta$ has its terminal side in Quadrant I, find the exact value of $\sin \theta$. **35.** ________________

**36.** Express $2 + 4 + 6 + 8 + 10$ using sigma notation. **36.** ________________

**37.** Find $\lim\limits_{n \to \infty} \frac{(3n + 2)(n - 5)}{n^2}$. **37.** ________________

**38.** Use a calculator to evaluate $3\sqrt[3]{\pi}$ to the nearest ten-thousandth. **38.** ________________

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# Trigonometry Semester Test (continued)

**39.** Evaluate $\left(\frac{1}{9}\right)^{-\frac{1}{2}}$. **39.** ______________

**40.** Use the Binomial Theorem to find the fourth term in the expansion of $(2c - 3d)^8$. **40.** ______________

**41.** Find the ordered triple that represents the vector from $C(2, 0, 1)$ to $D(6, 4, 3)$. **41.** ______________

**42.** Find the distance between the lines with equations $3x + 2y + 1 = 0$ and $3x + 2y + 4 = 0$. **42.** ______________

**43.** Find an ordered pair to represent $\vec{\mathbf{u}}$ in $\vec{\mathbf{u}} = 2\vec{\mathbf{w}} + \vec{\mathbf{v}}$ if $\vec{\mathbf{w}} = \langle 2, 3\rangle$ and $\vec{\mathbf{v}} = \langle 5, 5\rangle$. **43.** ______________

**44.** Write the rectangular equation $y = 1$ in polar form. **44.** ______________

**45.** Write the polar equation $\theta = \frac{\pi}{2}$ in rectangular form. **45.** ______________

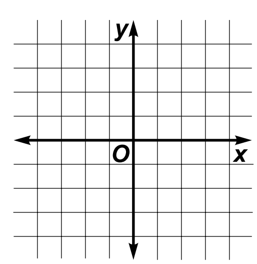
**46.** If $\overrightarrow{CD}$ is a vector from $C(1, 2, -1)$ to $D(2, 3, 2)$, find the magnitude of $\overrightarrow{CD}$. **46.** ______________

**47.** Are the vectors $\langle 2, -2, 1\rangle$ and $\langle 3, 2, -2\rangle$ perpendicular? Write *yes* or *no.* **47.** ______________

**48.** Solve $235 = 4e^{0.35t}$. Round your answer to the nearest hundredth. **48.** ______________

**49.** Write $0.\overline{63}$ as a fraction. **49.** ______________

**50.** Graph the equation $y = 3^{x-1}$. **50.**



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NAME ______________________ DATE ______________ PERIOD ________

# Trigonometry Final Test

***Write the letter for the correct answer in the blank at the right of each problem.***

**1.** Given $f(x) = 2x + 4$ and $g(x) = x^3 + x + 2$, find $(f - g)(x)$. **1.** ________

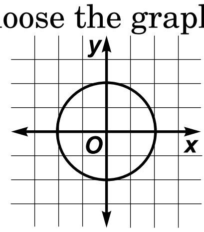
**A.** $x^3 + x + 2$ **B.** $x^3 + 3x + 6$

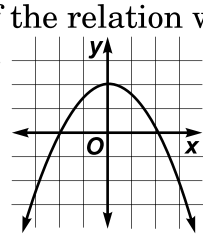
**C.** $-x^3 - 3x + 2$ **D.** $-x^3 + x + 2$

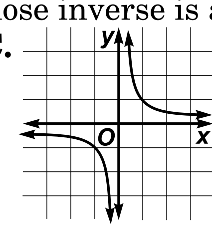
**2.** Describe the graph of $f(x) = \frac{x^2 - 25}{x + 5}$. **2.** ________

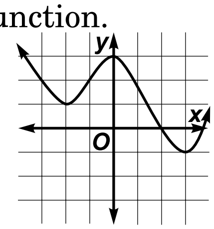
**A.** The graph has infinite discontinuity.
**B.** The graph has jump discontinuity.
**C.** The graph has point discontinuity.
**D.** The graph is continuous.

**3.** Choose the graph of the relation whose inverse is a function. **3.** ________

**A.**


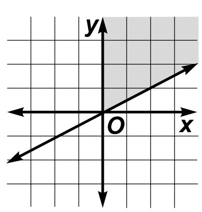

**B.**


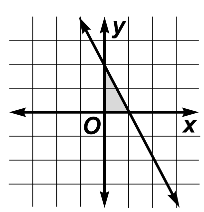

**C.**


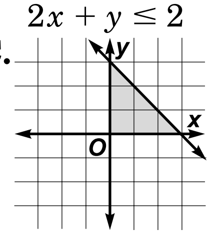

**D.**


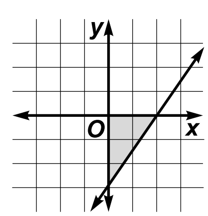

**4.** Which is the graph of the system? **4.** ________

$x \geq 0$
$y \geq 0$
$2x + y \leq 2$

**A.**



**B.**



**C.**



**D.**



**5.** For which measures does $\triangle ABC$ have no solution? **5.** ________

**A.** $A = 60°, a = 8, b = 8$
**B.** $A = 70°, b = 10, c = 10$
**C.** $A = 45°, a = 1, b = 20$
**D.** $A = 60°, B = 30°, c = 2$

# Trigonometry Final Test (continued)

**6.** Choose the amplitude, period, and phase shift of the function $y = \frac{1}{3}\cos(2x - \pi)$. **6.** ________

**A.** $\frac{1}{3}, \pi, \frac{\pi}{2}$ **B.** $\frac{1}{3}, \pi, \pi$ **C.** $3, \pi, \frac{\pi}{2}$ **D.** $\frac{1}{3}, \pi, \frac{\pi}{4}$

**7.** Given $f(x) = x + 1$ and $g(x) = x^2 + 2$, find $[g \circ f](x)$. **7.** ________

**A.** $x^2 + 2x + 3$ **B.** $x^2 + 2x + 1$ **C.** $x^2 + 3$ **D.** $x^3 + x^2 + 2x + 2$

**8.** Find one positive angle and one negative angle that are coterminal with the angle $\frac{3\pi}{4}$. **8.** ________

**A.** $\frac{5\pi}{4}, -\frac{11\pi}{4}$ **B.** $\frac{2\pi}{3}, -\frac{5\pi}{4}$ **C.** $\frac{11\pi}{4}, -\frac{\pi}{4}$ **D.** $\frac{11\pi}{4}, -\frac{5\pi}{4}$

**9.** Simplify $\frac{1 - \cos^2\theta}{\sin\theta}$. **9.** ________

**A.** $\frac{\cos\theta}{\sin\theta}$ **B.** $\tan\theta$ **C.** $\sin\theta$ **D.** $\cos\theta$

**10.** Find an ordered triple to represent $\vec{\mathbf{u}}$ in $\vec{\mathbf{u}} = 2\vec{\mathbf{v}} + 3\vec{\mathbf{w}}$ if $\vec{\mathbf{v}} = \langle -1, 0, 2\rangle$ and $\vec{\mathbf{w}} = \langle 2, 3, 1\rangle$. **10.** ________

**A.** $\langle 2, 3, 1\rangle$ **B.** $\langle 4, 3, 2\rangle$ **C.** $\langle 3, 1, -2\rangle$ **D.** $\langle 4, 9, 7\rangle$

**11.** Write $\log_7 49 = 2$ in exponential form. **11.** ________

**A.** $2^7 = 49$ **B.** $7^2 = 49$

**C.** $7^{\frac{1}{2}} = 49$ **D.** $49^{\frac{1}{2}} = 7$

**12.** Write parametric equations of $-4x + 5y = 10$. **12.** ________

**A.** $x = t; y = \frac{4}{5}t + 2$ **B.** $x = t; y = 4t + 10$

**C.** $x = t; y = -4t + 5$ **D.** $x = t; y = \frac{5}{4}t + 2$

**13.** Describe the transformation that relates the graph of $y = 100x^3$ to the parent graph $y = x^3$. **13.** ________

**A.** The parent graph is reflected over the line $y = x$.
**B.** The parent graph is stretched horizontally.
**C.** The parent graph is stretched vertically.
**D.** The parent graph is translated horizontally right 1 unit.

**14.** Identify the polar form of the linear equation $\sqrt{3}x + y = 4$. **14.** ________

**A.** $4 = r\cos(\theta - 30°)$ **B.** $2 = r\cos(\theta - 30°)$

**C.** $3 = r\sin(\theta + 70°)$ **D.** $2 = r\cos(\theta - 42°)$

# Trigonometry Final Test (continued)

**15.** Which function is an even function? **15.** ________

**A.** $y = x^4 + x^2$ **B.** $y = x^3$
**C.** $y = x^4 + x^5$ **D.** $y = 2x^4 + 3x$

**16.** Find the area of $\triangle ABC$ to the nearest tenth if $B = 30°$, $C = 70°$, and $a = 10$. **16.** ________

**A.** 15.2 units$^2$ **B.** 30.3 units$^2$
**C.** 19.2 units$^2$ **D.** 23.9 units$^2$

**17.** The line with equation $9x + y = 2$ is perpendicular to the line that passes through **17.** ________

**A.** (0, 0) and (1, 3). **B.** (1, 2) and (10, 3).
**C.** (2, −5) and (3, 4). **D.** (10, 4) and (1, 5).

**18.** Find the 17th term of the arithmetic sequence $\frac{1}{3}, 1, \frac{5}{3}, \ldots$. **18.** ________

**A.** 9 **B.** $\frac{17}{3}$ **C.** 11 **D.** 7

**19.** Solve $2x^2 + 4x + 1 = 0$ by using the Quadratic Formula. **19.** ________

**A.** $1 \pm \frac{\sqrt{2}}{2}$ **B.** $-1 \pm \frac{\sqrt{2}}{2}$ **C.** 2, 4 **D.** $-1 \pm \sqrt{6}$

**20.** What are the dimensions of matrix $AB$ if $A$ is a $2 \times 3$ matrix and $B$ is a $3 \times 7$ matrix? **20.** ________

**A.** $7 \times 2$ **B.** $3 \times 3$ **C.** $3 \times 2$ **D.** $2 \times 7$

**21.** Find the polar coordinates of the point whose rectangular coordinates are $(-1, 1)$. **21.** ________

**A.** $\left(\sqrt{2}, \frac{\pi}{4}\right)$ **B.** $\left(\sqrt{2}, -\frac{3\pi}{4}\right)$

**C.** $\left(\sqrt{2}, -\frac{\pi}{4}\right)$ **D.** $\left(\sqrt{2}, \frac{3\pi}{4}\right)$

**22.** Find the fifteenth term of the geometric sequence $\frac{5}{3}, \frac{5}{6}, \frac{5}{12}, \ldots$. **22.** ________

**A.** $\frac{5}{196{,}608}$ **B.** $\frac{5}{49{,}152}$ **C.** $\frac{5}{98{,}304}$ **D.** $\frac{1}{2}$

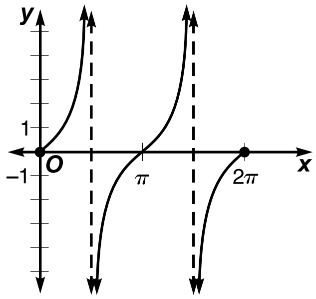
**23.** Find the magnitude of $\overrightarrow{AB}$ from $A(2, 4, 0)$ to $B(1, 2, 2)$. **23.** ________

**A.** $\sqrt{7}$ **B.** $2\sqrt{3}$ **C.** 3 **D.** $\sqrt{5}$

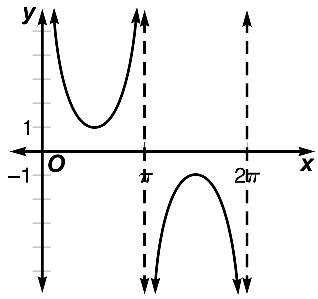
# Trigonometry Final Test (continued)

**24.** Choose the graph of $y = \csc x$ on the interval $0 \le x \le 2\pi$. **24.** ________

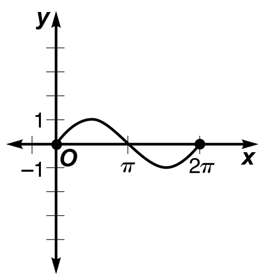
**A.**



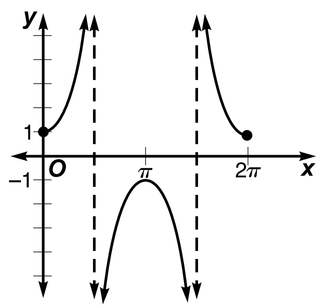

**B.**




**C.**




**D.**




**25.** Without graphing, describe the end behavior of $y = -x^4 + 2x^3 + x^2 + 1$. **25.** ________

**A.** $y \to \infty$ as $x \to \infty$; $y \to \infty$ as $x \to -\infty$
**B.** $y \to -\infty$ as $x \to \infty$; $y \to -\infty$ as $x \to -\infty$
**C.** $y \to -\infty$ as $x \to \infty$; $y \to \infty$ as $x \to -\infty$
**D.** $y \to \infty$ as $x \to \infty$; $y \to -\infty$ as $x \to -\infty$

**26.** Which expression is equivalent to $\sin(90° - \theta)$? **26.** ________

**A.** $\sin \theta$ **B.** $\cos \theta$ **C.** $\tan \theta$ **D.** $\sec \theta$

**27.** Describe the transformation that relates the graph of $y = -\sqrt{x + 3}$ to the parent graph $y = \sqrt{x}$. **27.** ________

**A.** The parent graph is reflected over the $x$-axis and translated left 3 units.
**B.** The parent graph is compressed vertically and translated right 3 units.
**C.** The parent graph is reflected over the $x$-axis and translated up 3 units.
**D.** The parent graph is reflected over the $y$-axis and translated left 3 units.

**28.** Express cos 854° as a function of an angle in Quandrant II. **28.** ________

**A.** tan 134° **B.** sin 64° **C.** cos 134° **D.** −cos 134°

**29.** Express $\sqrt[3]{27a^9b^2}$ by using rational exponents.

**A.** $3a^3b^{\frac{3}{2}}$ **B.** $9a^6b^{\frac{2}{3}}$ **C.** $27a^3b^{\frac{2}{3}}$ **D.** $3a^3b^{\frac{2}{3}}$ **29.** ________

**30.** Identify the equation of the tangent function with period $2\pi$, phase shift $\frac{\pi}{2}$, and vertical shift $-5$. **30.** ________

**A.** $y = \tan(x + \pi) - 5$ **B.** $y = \tan\left(\frac{1}{2}x - \frac{\pi}{4}\right) - 5$

**C.** $y = -5 \tan\left(\frac{1}{2}x + \frac{\pi}{4}\right)$ **D.** $y = \tan\left(2x - \frac{\pi}{2}\right)$

# Trigonometry Final Test (continued)

**31.** Given $f(x) = x^3 - 2x$, find $f(-2)$. **31.** ______________

**32.** Find the area of a sector if the central angle measures 35° and the radius of the circle is 45 centimeters. Round to the nearest tenth. **32.** ______________

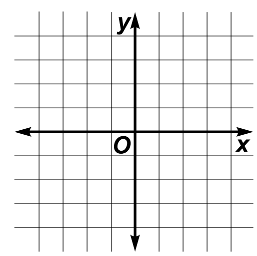
**33.** Find $A + B$ if $A = \begin{bmatrix} 1 & 3 \\ -2 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & -1 \\ 2 & 4 \end{bmatrix}$. **33.** ______________

**34.** Solve the system of equations algebraically. **34.** ______________
$3x + 2y = 3$
$-2x + 5y = 17$

**35.** Find the inverse of $y = (x + 1)^3$. **35.** ______________

**36.** State the domain and range of the relation $\{(-2, 5), (0, 3), (4, 5), (9, -3)\}$. Then state whether the relation is a function. **36.** ______________

**37.** Graph $y < -|x - 1| + 2$. **37.**




**38.** Find the sum of the infinite geometric series $35 + 7 + 1.4 + \cdots$. **38.** ______________

**39.** Find $AB$ if $A = \begin{bmatrix} 3 & -2 \\ 1 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 2 \\ -1 & 3 \end{bmatrix}$. **39.** ______________

**40.** Simplify $\sin\theta \csc\theta (\sin^2\theta + \cos^2\theta)$. **40.** ______________

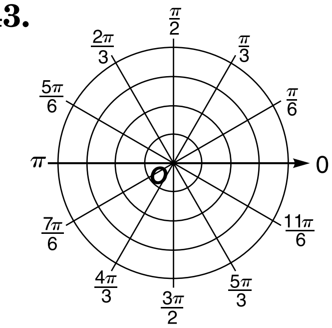
**41.** A football is kicked with an initial velocity of 38 feet per second at an angle of 32° to the horizontal. How far has the football traveled horizontally after 0.25 second? Round your answer to the nearest tenth. **41.** ______________

**42.** Find the sum of the first 23 terms of the arithmetic series $-10 + -3 + 4 + \cdots$. **42.** ______________

# Trigonometry Final Test (continued)

**43.** Graph the polar equation $\theta = \frac{\pi}{3}$.

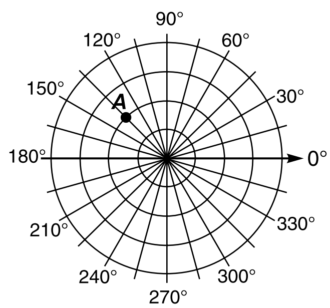
**43.**




**44.** During one year, the cost of tuition, room, and board at a state university increased 6%. If the cost continues to increase at a rate of 6% per year, how long will it take for the cost of tuition, room, and board to triple? Round your answer to the nearest year.

**44.** ____________________

**45.** Name four different pairs of polar coordinates that represent the point $A$.




**45.** ____________________

**46.** Evaluate $\log_7 15$ to four decimal places.

**46.** ____________________

**47.** Use a calculator to approximate the value of $\sec(-137°)$ to four decimal places.

**47.** ____________________

**48.** Find the sum of the first 13 terms of the geometric series $2 + 6 + 18 + \cdots$.

**48.** ____________________

**49.** Find the exact value of $\sin 105°$.

**49.** ____________________

**50.** Find the multiplicative inverse of $\begin{bmatrix} 5 & 2 \\ 2 & 1 \end{bmatrix}$.

**50.** ____________________

**51.** Find the value of $\begin{vmatrix} 2 & -1 \\ 3 & 4 \end{vmatrix}$.

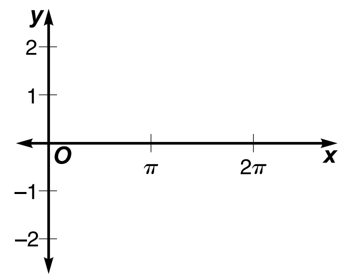
**51.** ____________________

NAME ______________________ DATE ______________ PERIOD ________

# Trigonometry Final Test (continued)

**52.** The vector $\vec{\mathbf{u}}$ has a magnitude of 4.5 centimeters and a direction of 56°. Find the magnitude of its horizontal component. Round to the nearest hundredth. **52.** ________________

**53.** Graph the equation $y = 2 \sin (2x + \pi)$. **53.**




**54.** Write $3^{-2} = \frac{1}{9}$ in logarithmic form. **54.** ________________

**55.** Use a sum or difference identity to find the value of $\cos 285°$. **55.** ________________

**56.** Solve $e^{\frac{1}{x}} = 5$. Round your answer to the nearest hundredth. **56.** ________________

**57.** Write $\overrightarrow{MN}$ as a sum of unit vectors for $M(2, 1, 0)$ and $N(5, -3, 2)$. **57.** ________________

**58.** Express $3 + 5 + 9 + 17 + \cdots + 513$ using sigma notation. **58.** ________________

**59.** Express $2 - 3\boldsymbol{i}$ in polar form. Express your answer in radians to the nearest hundredth. **59.** ________________

**60.** Find the third iterate of the function $f(z) = 2z + 1$, if the initial value is $z_0 = 1 + \boldsymbol{i}$. **60.** ________________

**61.** Find the rectangular coordinates of the point whose polar coordinates are $\left(2\sqrt{2}, \frac{5\pi}{4}\right)$. **61.** ________________

**62.** Find the discriminant of $w^2 + 4w + 5 = 0$ and describe the nature of the roots of the equation. **62.** ________________

**63.** Find the value of $\mathrm{Cos}^{-1}\left(\frac{1}{2}\right)$ in degrees. **63.** ________________

**64.** State the degree and leading coefficient of the polynomial function $f(x) = -6x^3 + 2x^4 + 3x^5 + 2$. **64.** ________________

# Trigonometry Final Test (continued)

**65.** Write an equation in slope-intercept form of the line with the given parametric equations. $x = -3t + 5$
$y = 1 - t$

**65.** ________________

**66.** Solve the system of equations.
$x - 5y + z = 1$
$2x + y + 2z = 2$
$x - 3y - 4z = 6$

**66.** ________________

**67.** Use the Remainder Theorem to find the remainder when $2x^3 + x^2 + 2x$ is divided by $x - 1$.

**67.** ________________

**68.** Find the value of $\cos\left[\text{Sin}^{-1}\left(\frac{\sqrt{3}}{2}\right)\right]$.

**68.** ________________

**69.** Solve $\log_5 x - \log_5 9 = \log_5 4$.

**69.** ________________

**70.** Find the rational zero(s) of $f(x) = 3x^3 - 2x^2 - 6x + 4$.

**70.** ________________

**71.** Find the number of possible positive real zeros of $f(x) = 3x^4 - 2x^3 + 3x^2 - x - 3$.

**71.** ________________

**72.** List all possible rational zeros of $f(x) = 3x^3 + bx^2 + cx + 2$, where $b$ and $c$ are integers.

**72.** ________________

**73.** Given a central angle of 87°, find the length of its intercepted arc in a circle of radius 19 centimeters. Round to the nearest tenth.

**73.** ________________

**74.** Solve $\triangle ABC$ if $A = 27.2°$, $B = 76.1°$, and $a = 31.2$. Round to the nearest tenth.

**74.** ________________

**75.** Solve $\triangle ABC$ if $A = 40°$, $b = 10.2$, and $c = 5.7$. Round to the nearest tenth.

**75.** ________________

NAME ______________________________ DATE _______________ PERIOD ________

# SAT and ACT Practice Answer Sheet

## (10 Questions)

1 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ

2 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ

3 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ

4 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ

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10

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| 4 | 4 | 4 | 4 |
| 5 | 5 | 5 | 5 |
| 6 | 6 | 6 | 6 |
| 7 | 7 | 7 | 7 |
| 8 | 8 | 8 | 8 |
| 9 | 9 | 9 | 9 |

NAME ______________________ DATE ____________ PERIOD ________

# SAT and ACT Practice Answer Sheet

## (20 Questions)

1 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ

2 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ

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6 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ

7 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ

8 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ

9 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ

10 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ

11 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ

12 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ

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19

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20

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| 9 | 9 | 9 | 9 |

12-1

NAME ______________ DATE ________ PERIOD ______

# Practice

## Arithmetic Sequences and Series

***Find the next four terms in each arithmetic sequence.***

**1.** $-1.1, 0.6, 2.3, \ldots$
**4.0, 5.7, 7.4, 9.1**

**2.** $16, 13, 10, \ldots$
**7, 4, 1, −2**

**3.** $p, p + 2, p + 4, \ldots$
**$p + 6, p + 8, p + 10, p + 12$**

***For exercises 4–12, assume that each sequence or series is arithmetic.***

**4.** Find the 24th term in the sequence for which $a_1 = -27$ and $d = 3$.
**42**

**5.** Find $n$ for the sequence for which $a_n = 27$, $a_1 = -12$, and $d = 3$.
**14**

**6.** Find $d$ for the sequence for which $a_1 = -12$ and $a_{23} = 32$.
**2**

**7.** What is the first term in the sequence for which $d = -3$ and $a_6 = 5$?
**20**

**8.** What is the first term in the sequence for which $d = -\frac{1}{3}$ and $a_7 = -3$?
**−1**

**9.** Find the 6th term in the sequence $-3 + \sqrt{2}, 0, 3 - \sqrt{2}, \ldots$.
**$12 - 4\sqrt{2}$**

**10.** Find the 45th term in the sequence $-17, -11, -5, \ldots$.
**247**

**11.** Write a sequence that has three arithmetic means between 35 and 45.
**35, 37.5, 40, 42.5, 45**

**12.** Write a sequence that has two arithmetic means between −7 and 2.75.
**−7, −3.75, −0.5, 2.75**

**13.** Find the sum of the first 13 terms in the series $-5 + 1 + 7 + \cdots + 67$.
**403**

**14.** Find the sum of the first 62 terms in the series $-23 - 21.5 - 20 - \cdots$.
**1410.5**

**15.** ***Auditorium Design*** Wakefield Auditorium has 26 rows, and the first row has 22 seats. The number of seats in each row increases by 4 as you move toward the back of the auditorium. What is the seating capacity of this auditorium?
**1872**



12-1

NAME ______________ DATE ________ PERIOD ______

# Enrichment

## Quadratic Formulas for Sequences

An ordinary arithmetic sequence is formed using a rule such as $bn + c$. The first term is $c$, $b$ is called the common difference, and $n$ takes on the values 0, 1, 2, 3, and so on. The value of term $n + 1$ equals $b(n + 1) + c$ or $bn + b + c$. So, the value of a term is a function of the term number.

Some sequences use quadratic functions. A method called *finite differences* can be used to find the values of the terms. Notice what happens when you subtract twice as shown in this table.

| $n$ | $an^2 + bn + c$ | | |
|---|---|---|---|
| 0 | $c$ | | |
| 1 | $a + b + c$ | $a + b$ | $2a$ |
| 2 | $4a + 2b + c$ | $3a + b$ | $2a$ |
| 3 | $9a + 3b + c$ | $5a + b$ | $2a$ |
| 4 | $16a + 4b + c$ | $7a + b$ | |

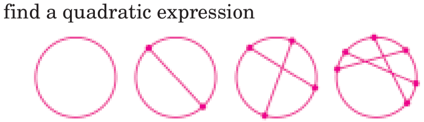
A sequence that yields a common difference after two subtractions can be generated by a quadratic expression. For example, the sequence 1, 5, 12, 22, 35, . . . gives a common difference of 3 after two subtractions. Using the table above, you write and solve three equations to find the general rule. The equations are $1 = c$, $5 = a + b + c$, and $12 = 4a + 2b + c$.

***Solve each problem.***

**1.** Refer to the sequence in the example above. Solve the system of equations for $a$, $b$, and $c$ and then find the quadratic expression for the sequence. Then write the next three terms.
**$\frac{3}{2}n^2 + \frac{5}{2}n + 1$; 51, 70, 92**

**2.** The number of line segments connecting $n$ points forms the sequence 0, 0, 1, 3, 6, 10, . . . , in which $n$ is the number of points and the term value is the number of line segments. What is the common difference after the second subtraction? Find a quadratic expression for the term value.
**1; $\frac{1}{2}n^2 - \frac{1}{2}n$**

**3.** The maximum number of regions formed by $n$ chords in a circle forms the sequence 1, 2, 4, 7, 11, 16, . . . (A chord is a line segment joining any two points on a circle.) Draw circles to illustrate the first four terms of the sequence. Then find a quadratic expression for the term value.
**$\frac{1}{2}n^2 + \frac{1}{2}n + 1$**



# 12-2 Practice

NAME ______________ DATE ________ PERIOD ______

## Geometric Sequences and Series

***Determine the common ratio and find the next three terms of each geometric sequence.***

**1.** $-1, 2, -4, \ldots$
**$-2$; 8, $-16$, 32**

**2.** $-4, -3, -\frac{9}{4}, \ldots$
**$\frac{3}{4}$; $-\frac{27}{16}$, $-\frac{81}{64}$, $-\frac{243}{256}$**

**3.** $12, -18, 27, \ldots$
**$-\frac{3}{2}$; $-\frac{81}{2}$, $\frac{243}{4}$, $-\frac{729}{8}$**

***For exercises 4–9, assume that each sequence or series is geometric.***

**4.** Find the fifth term of the sequence 20, 0.2, 0.002, . . . .
**0.0000002**

**5.** Find the ninth term of the sequence $\sqrt{3}, -3, 3\sqrt{3}, \ldots$.
**$81\sqrt{3}$**

**6.** If $r = 2$ and $a_4 = 28$, find the first term of the sequence.
**$\frac{7}{2}$**

**7.** Find the first three terms of the sequence for which $a_4 = 8.4$ and $r = 4$.
**0.13125, 0.525, 2.1**

**8.** Find the first three terms of the sequence for which $a_6 = \frac{1}{32}$ and $r = \frac{1}{2}$.
**1, $\frac{1}{2}$, $\frac{1}{4}$**

**9.** Write a sequence that has two geometric means between 2 and 0.25.
**2, 1, 0.5, 0.25**

**10.** Write a sequence that has three geometric means between $-32$ and $-2$.
**$-32, \pm 16, -8, \pm 4, -2$**

**11.** Find the sum of the first eight terms of the series $\frac{3}{4} + \frac{9}{20} + \frac{27}{100} + \cdots$.
**about 1.84351**

**12.** Find the sum of the first 10 terms of the series $-3 + 12 - 48 + \cdots$.
**629,145**

**13.** ***Population Growth*** A city of 100,000 people is growing at a rate of 5.2% per year. Assuming this growth rate remains constant, estimate the population of the city 5 years from now.
**about 128,848**



# 12-2 Enrichment

NAME ______________ DATE ________ PERIOD ______

## Sequences as Functions

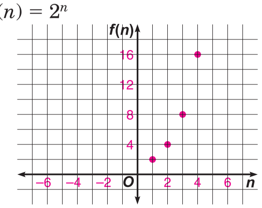
A **geometric sequence** can be defined as a function whose domain is the set of positive integers.

$$\begin{array}{cccccc} n = & 1 & 2 & 3 & 4 & \ldots \\ & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ f(n) = & ar^{1-1} & ar^{2-1} & ar^{3-1} & ar^{4-1} & \ldots \end{array}$$

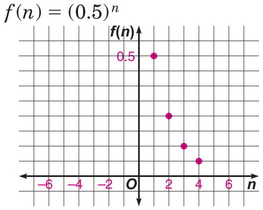
In the exercises, you will have the opportunity to explore geometric sequences from a function and graphing point of view.

***Graph each geometric sequence for n = 1, 2, 3 and 4.***

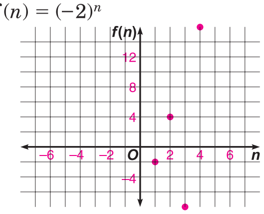
**1.** $f(n) = 2^n$



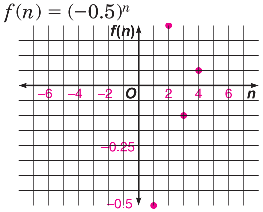

**2.** $f(n) = (0.5)^n$




**3.** $f(n) = (-2)^n$




**4.** $f(n) = (-0.5)^n$

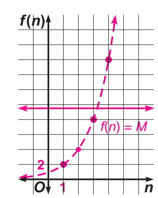



**5.** Describe how the graph of a geometric sequence depends on the common ratio.
**$r > 1$: graph rises to the right.**
**$r < -1$: graph rises and falls and high and low points move away from the $n$-axis.**
**$0 < r < 1$: graph falls to the right.**
**$-1 < r < 0$: graph rises and falls and high and low points approach the $n$-axis.**

**6.** Let $f(n) = 2^n$, where $n$ is a positive integer.

**a.** Show graphically that for any $M$ the graph of $f(n)$ rises above and stays above the horizontal line $y = M$.

**b.** Show algebraically that for any $M$, there is a positive integer $N$ such that $2^n > M$ for all $n > N$.
**Choose a positive integer $N > \log_2 M$. Then $2^N > 2^{\log_2 M} = M$.**



NAME ________________ DATE ________ PERIOD ______

## 12-3 Practice

### Infinite Sequence and Series

***Find each limit, or state that the limit does not exist and explain your reasoning.***

**1.** $\lim_{n\to\infty} \frac{n^2-1}{n^2+1}$
**1**

**2.** $\lim_{n\to\infty} \frac{4n^2-5n}{3n^2+4}$
$\frac{4}{3}$

**3.** $\lim_{n\to\infty} \frac{5n^2+1}{6n}$
**does not exist; dividing by $n^2$, we find $\lim_{n\to\infty} \frac{5+\frac{1}{n^2}}{\frac{6}{n}}$, which simplifies to $\frac{5+0}{6\cdot 0} = \frac{5}{0}$ as $n$ approaches infinity. Since this fraction is undefined, the sequence has no limit.**

**4.** $\lim_{n\to\infty} \frac{(n-1)(3n+1)}{5n^2}$
$\frac{3}{5}$

**5.** $\lim_{n\to\infty} \frac{3n-(-1)^n}{4n^2}$
**0**

**6.** $\lim_{n\to\infty} \frac{n^3+1}{n^2}$
**does not exist; $\lim_{n\to\infty} \frac{n^3+1}{n^2} = \lim_{n\to\infty} \left(n + \frac{1}{n^2}\right)$. $\lim_{n\to\infty} \frac{1}{n^2} = 0$, but as $n$ approaches infinity, $n$ becomes increasingly large, so the sequence has no limit.**

***Write each repeating decimal as a fraction.***

**7.** $0.\overline{75}$
$\frac{25}{33}$

**8.** $0.\overline{592}$
$\frac{16}{27}$

***Find the sum of each infinite series, or state that the sum does not exist and explain your reasoning.***

**9.** $\frac{2}{5} + \frac{6}{25} + \frac{18}{125} + \cdots$
**1**

**10.** $\frac{3}{4} + \frac{15}{8} + \frac{75}{16} + \cdots$
**does not exist; since $r = \frac{5}{2}$, $|r| > 1$ and the sequence increases without limit.**

**11.** ***Physics*** A tennis ball is dropped from a height of 55 feet and bounces $\frac{3}{5}$ of the distance after each fall.

**a.** Find the first seven terms of the infinite series representing the vertical distances traveled by the ball. **55, 33, 33, 19.8, 19.8, 11.88, 11.88**

**b.** What is the total vertical distance the ball travels before coming to rest? **220 ft**



NAME ________________ DATE ________ PERIOD ______

## 12-3 Enrichment

### Solving Equations Using Sequences

You can use sequences to solve many equations. For example, consider $x^2 + x - 1 = 0$. You can proceed as follows.

$$x^2 + x - 1 = 0$$
$$x(x+1) = 1$$
$$x = \frac{1}{1+x}$$

Next, define the sequence: $a_1 = 0$ and $a_n = \frac{1}{1+a_{n-1}}$.

The limit of the sequence is a solution to the original equation.

**1.** Let $a_1 = 0$ and $a_n = \frac{1}{1+a_{n-1}}$.

**a.** Write the first five terms of the sequence. Do not simplify.

$$0, \frac{1}{1+0}, \frac{1}{1+1}, \frac{1}{1+\frac{1}{1+1}}, \frac{1}{1+\frac{1}{1+\frac{1}{1+1}}}$$

**b.** Write decimals for the first five terms of the sequence.
**0, 1, 0.5, 0.6667, 0.6**

**c.** Use a calculator to compute $a_6$, $a_7$, $a_8$, and $a_9$. Compare $a_9$ with the positive solution of $x^2 + x - 1 = 0$ found by using the quadratic formula.
**0.625, 0.6154, 0.6190, 0.6176; solution by quadratic formula: 0.6180**

**2.** Use the method described above to find a root of $3x^2 - 2x - 3 = 0$.
**−0.7208**

**3.** Write a BASIC program using the procedure outlined above to find a root of the equation $3x^2 - 2x - 3 = 0$. In the program, let $a_1 = 0$ and $a_n = \frac{3}{3a_{n-1}-2}$. Run the program. Compare the time it takes to run the program to the time it takes to evaluate the terms of the sequence by using a calculator.

```
10 DIM A[100]
20 A[1] = 0
30 FOR I = 2 TO 100
40 A[I] = 3/(3*A[I − 1] − 2)
50 NEXT I
60 PRINT A[100]
```

NAME ______________ DATE ______________ PERIOD ______

12-4

## Practice

### Convergent and Divergent Series

***Use the ratio test to determine whether each series is convergent or divergent.***

1. $\frac{1}{2} + \frac{2^2}{2^2} + \frac{3^2}{2^3} + \frac{4^2}{2^4} + \cdots$
   **convergent**

2. $0.006 + 0.06 + 0.6 + \cdots$
   **divergent**

3. $\frac{4}{1 \cdot 2 \cdot 3} + \frac{8}{1 \cdot 2 \cdot 3 \cdot 4} + \frac{16}{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5} + \cdots$
   **convergent**

4. $5 + \frac{5}{3^3} + \frac{5}{5^3} + \frac{5}{7^3} + \cdots$
   **divergent**

***Use the comparison test to determine whether each series is convergent or divergent.***

5. $2 + \frac{2}{2^3} + \frac{2}{3^3} + \frac{2}{4^3} + \cdots$
   **convergent**

6. $\frac{5}{2} + 1 + \frac{5}{8} + \frac{5}{11} + \cdots$
   **divergent**

7. ***Ecology*** A landfill is leaking a toxic chemical. Six months after the leak was detected, the chemical had spread 1250 meters from the landfill. After one year, the chemical had spread 500 meters more, and by the end of 18 months, it had reached an additional 200 meters.
   a. If this pattern continues, how far will the chemical spread from the landfill after 3 years?
      **about 2075 m**
   b. Will the chemical ever reach the grounds of a hospital located 2500 meters away from the landfill? Explain.
      **No, the sum of the infinite series modeling this situation is about 2083. Thus the chemical will spread no more than about 2083 meters.**



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12-4

## Enrichment

### Alternating Series

The series below is called an alternating series.

$$1 - 1 + 1 - 1 + \cdots$$

The reason is that the signs of the terms alternate. An interesting question is whether the series converges. In the exercises, you will have an opportunity to explore this series and others like it.

1. Consider $1 - 1 + 1 - 1 + \cdots$.
   a. Write an argument that suggests that the sum is 1.
      $$\begin{aligned} 1 - 1 + 1 - 1 + \cdots &= 1 + (-1 + 1) + (-1 + 1) + \cdots \\ &= 1 + 0 + 0 + \cdots \\ &= 1 \end{aligned}$$
   b. Write an argument that suggests that the sum is 0.
      $$\begin{aligned} 1 - 1 + 1 - 1 + \cdots &= (1 - 1) + (1 - 1) + (1 - 1) + \cdots \\ &= 0 + 0 + 0 + \cdots \\ &= 0 \end{aligned}$$
   c. Write an argument that suggests that there is no sum. (*Hint*: Consider the sequence of partial sums.)
      **Let $S_n$ be the $n$th partial sum. Then**
      $$S_n = \begin{cases} 1 \text{ if } n \text{ is odd.} \\ 0 \text{ if } n \text{ is even.} \end{cases}$$
      **Since 1, 0, 1, 0, ... has no limit, the original series has no sum.**

If the series formed by taking the absolute values of the terms of a given series is convergent, then the given series is said to be **absolutely convergent**. It can be shown that any absolutely convergent series is convergent.

2. Make up an alternating series, other than a geometric series with negative common ratio, that has a sum. Justify your answer.
   **Sample answer:**
   $\frac{1}{1} - \frac{1}{4} + \frac{1}{9} - \frac{1}{16} + \cdots$ **is convergent because the absolute value of each term is less than or equal to the corresponding term in a $p$-series with $p = 2$.**

NAME ______________ DATE ________ PERIOD ____

12-5

## Practice

### Sigma Notation and the *n*th Term

***Write each expression in expanded form and then find the sum.***

1. $\sum_{n=3}^{5}(n^2 - 2^n)$
 $(3^2 - 2^3) + (4^2 - 2^4) + (5^2 - 2^5); -6$

2. $\sum_{q=1}^{4}\frac{2}{q}$
 $\frac{2}{1} + \frac{2}{2} + \frac{2}{3} + \frac{2}{4}; \frac{25}{6}$

3. $\sum_{t=1}^{5}t(t-1)$
 $1(1-1) + 2(2-1) + 3(3-1) + 4(4-1) + 5(5-1); 40$

4. $\sum_{t=0}^{3}(2t-3)$
 $[2(0)-3] + [2(1)-3] + [2(2)-3] + [2(3)-3]; 0$

5. $\sum_{c=2}^{5}(c-2)^2$
 $(2-2)^2 + (3-2)^2 + (4-2)^2 + (5-2)^2; 14$

6. $\sum_{i=1}^{\infty}10\left(\frac{1}{2}\right)^i$
 $10\left(\frac{1}{2}\right)^1 + 10\left(\frac{1}{2}\right)^2 + 10\left(\frac{1}{2}\right)^3 + \cdots + 10\left(\frac{1}{2}\right)^\infty; 10$

***Express each series using sigma notation.***

7. $3 + 6 + 9 + 12 + 15$
 $\sum_{n=1}^{5}3n$

8. $6 + 24 + 120 + \cdots + 40{,}320$
 $\sum_{n=3}^{8}n!$

9. $\frac{1}{1} + \frac{1}{4} + \frac{1}{9} + \cdots + \frac{1}{100}$
 $\sum_{n=1}^{10}\frac{1}{n^2}$

10. $24 + 19 + 14 + \cdots + (-1)$
 $\sum_{n=0}^{5}(24 - 5n)$

11. ***Savings*** Kathryn started saving quarters in a jar. She began by putting two quarters in the jar the first day and then she increased the number of quarters she put in the jar by one additional quarter each successive day.

 a. Use sigma notation to represent the total number of quarters Kathryn had after 30 days. $\sum_{n=1}^{30}(n+1)$

 b. Find the sum represented in part **a**.
 **495 quarters**

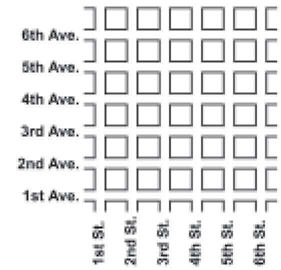


NAME ______________ DATE ________ PERIOD ____

12-5

## Enrichment

### Street Networks: Finding All Possible Routes




A section of a city is laid out in square blocks. Going north from the intersection of 1st Avenue and 1st Street, the avenues are 1st, 2nd, 3rd, and so on. Going east, the streets are numbered in the same way.

Factorials can be used to find the number, $r(e, n)$, of different routes between two intersections.

$$r(e, n) = \frac{[(e-1) + (n-1)]!}{(e-1)!\,(n-1)!}$$

The number of streets going east is $e$; the number of avenues going north is $n$.

The following problems examine the possible routes from one location to another. Assume that you never use a route that is unnecessarily long. Assume that $e \geq 1$ and $n \geq 1$.

***Solve each problem.***

1. List all the possible routes from 1st Street and 1st Avenue to 4th Street and 3rd Avenue. Use ordered pairs to show the routes, with street numbers first and avenue numbers second. Each route must start at (1, 1) and end at (4, 3).
 (1, 1) → (2, 1) → (3, 1) → (4, 1) → (4, 2) → (4, 3)
 (1, 1) → (2, 1) → (3, 1) → (3, 2) → (4, 2) → (4, 3)
 (1, 1) → (2, 1) → (3, 1) → (3, 2) → (3, 3) → (4, 3)
 (1, 1) → (2, 1) → (2, 2) → (3, 2) → (4, 2) → (4, 3)
 (1, 1) → (2, 1) → (2, 2) → (3, 2) → (3, 3) → (4, 3)
 (1, 1) → (2, 1) → (2, 2) → (2, 3) → (3, 3) → (4, 3)
 (1, 1) → (1, 2) → (2, 2) → (3, 2) → (4, 2) → (4, 3)
 (1, 1) → (1, 2) → (2, 2) → (3, 2) → (3, 3) → (4, 3)
 (1, 1) → (1, 2) → (2, 2) → (2, 3) → (3, 3) → (4, 3)
 (1, 1) → (1, 2) → (1, 3) → (2, 3) → (3, 3) → (4, 3)

2. Use the formula to compute the number of routes from (1, 1) to (4, 3). There are 4 streets going east and 3 avenues going north.
 $\frac{(3+2)!}{3!2!} = 10$

3. Find the number of routes from 1st Street and 1st Avenue to 7th Street and 6th Avenue.
 $\frac{(6+5)!}{6!5!} = 462$

NAME ______________ DATE ________ PERIOD ______

12-6

# Practice

## The Binomial Theorem

***Use Pascal's triangle to expand each binomial.***

1. $(r + 3)^5$
$r^5 + 15r^4 + 90r^3 + 270r^2 + 405r + 243$

2. $(3a - b)^4$
$81a^4 - 108a^3b + 54a^2b^2 - 12ab^3 + b^4$

***Use the Binomial Theorem to expand each binomial.***

3. $(x - 5)^4$
$x^4 - 20x^3 + 150x^2 - 500x + 625$

4. $(3x + 2y)^4$
$81x^4 + 216x^3y + 216x^2y^2 + 96xy^3 + 16y^4$

5. $(a - \sqrt{2})^5$
$a^5 - 5\sqrt{2}a^4 + 20a^3 - 20\sqrt{2}a^2 + 20a - 4\sqrt{2}$

6. $(2p - 3q)^6$
$64p^6 - 576p^5q + 2160p^4q^2 - 4320p^3q^3 + 4860p^2q^4 - 2916pq^5 + 729q^6$

***Find the designated term of each binomial expansion.***

7. 4th term of $(2n - 3m)^4$
$-216nm^3$

8. 5th term of $(4a + 2b)^8$
$286{,}720a^4b^4$

9. 6th term of $(3p + q)^9$
$10{,}206p^4q^5$

10. 3rd term of $(a - 2\sqrt{3})^6$
$180a^4$

11. A varsity volleyball team needs nine members. Of these nine members, at least five must be seniors. How many of the possible groups of juniors and seniors have at least five seniors?
**256**



NAME ______________ DATE ________ PERIOD ______

12-6

# Enrichment

## Patterns in Pascal's Triangle

You have learned that the coefficients in the expansion of $(x + y)^n$ yield a number pyramid called **Pascal's triangle**.

| | | |
|---|---|---|
| Row 1 | ⟶ | 1 |
| Row 2 | ⟶ | 1 1 |
| Row 3 | ⟶ | 1 2 1 |
| Row 4 | ⟶ | 1 3 3 1 |
| Row 5 | ⟶ | 1 4 6 4 1 |
| Row 6 | ⟶ | 1 5 10 10 5 1 |
| Row 7 | ⟶ | 1 6 15 20 15 6 1 |

As many rows can be added to the bottom of the pyramid as you need.

This activity explores some of the interesting properties of this famous number pyramid.

1. Pick a row of Pascal's triangle.
   **a.** What is the sum of all the numbers in all the rows above the row you picked?
   **See students' work.**
   **b.** What is the sum of all the numbers in the row you picked?
   **See students' work.**
   **c.** How are your answers for parts **a** and **b** related?
   **The answer for part b is 1 more than the answer for part a.**
   **d.** Repeat parts **a** through **c** for at least three more rows of Pascal's triangle. What generalization seems to be true?
   **It appears that the sum of the numbers in any row is 1 more than the sum of the numbers in all of the rows above it.**
   **e.** See if you can prove your generalization.
   **Sum of numbers in row $n = 2^{n-1}$; The sum of the numbers in the rows above row $n$ is $2^0 + 2^1 + 2^2 + \ldots + 2^{n-2}$, which, by the formula for the sum of a geometric series, is $2^{n-1} - 1$.**
2. Pick any row of Pascal's triangle that comes after the first.
   **a.** Starting at the left end of the row, find the sum of the odd numbered terms.
   **See students' work.**
   **b.** In the same row, find the sum of the even numbered terms.
   **See students' work.**
   **c.** How do the sums in parts a and b compare?
   **The sums are equal.**
   **d.** Repeat parts **a** through **c** for at least three other rows of Pascal's triangle. What generalization seems to be true?
   **In any row of Pascal's triangle after the first, the sum of the odd numbered terms is equal to the sum of the even numbered terms.**

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12-7

# Practice

## Special Sequences and Series

***Find each value to four decimal places.***

**1.** $\ln(-5)$ **$i\pi + 1.6094$**

**2.** $\ln(-5.7)$ **$i\pi + 1.7405$**

**3.** $\ln(-1000)$ **$i\pi + 6.9078$**

***Use the first five terms of the exponential series and a calculator to approximate each value to the nearest hundredth.***

**4.** $e^{0.5}$ **1.65**

**5.** $e^{1.2}$ **3.29**

**6.** $e^{2.7}$ **12.84**

**7.** $e^{0.9}$ **2.45**

***Use the first five terms of the trigonometric series to approximate the value of each function to four decimal places. Then, compare the approximation to the actual value.***

**8.** $\sin \frac{5\pi}{6}$ **0.5009; 0.5**

**9.** $\cos \frac{3\pi}{4}$ **−0.7057; −0.7071**

***Write each complex number in exponential form.***

**10.** $13\left(\cos \frac{\pi}{3} + \boldsymbol{i} \sin \frac{\pi}{3}\right)$ **$13e^{i\frac{\pi}{3}}$**

**11.** $5 + 5\boldsymbol{i}$ **$5\sqrt{2}e^{i\frac{\pi}{4}}$**

**12.** $1 - \sqrt{3}\boldsymbol{i}$ **$2e^{i\frac{\pi}{3}}$**

**13.** $-7 + 7\sqrt{3}\boldsymbol{i}$ **$14e^{i\frac{5\pi}{3}}$**

**14.** ***Savings*** Derika deposited \$500 in a savings account with a 4.5% interest rate compounded continuously. (*Hint:* The formula for continuously compounded interest is $A = Pe^{rt}$.)

**a.** Approximate Derika's savings account balance after 12 years using the first four terms of the exponential series.
**approximately \$856.02**

**b.** How long will it take for Derika's deposit to double, provided she does not deposit any additional funds into her account?
**about 15.4 years**



NAME ______________________ DATE ____________ PERIOD ________

12-7

# Enrichment

## Power Series

A **power series** is a series of the form

$$a_0 + a_1x + a_2x^2 + a_3x^3 + \cdots$$

where each $a_i$ is a real number. Many functions can be represented by power series. For instance, the function $f(x) = e^x$ can be represented by the series

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots.$$

**Check students' graphs.**

***Use a graphing calculator or computer to graph the functions in Exercises 1–4.***

**1.** $f_2(x) = 1 + x$

**2.** $f_3(x) = 1 + x + \frac{x^2}{2!}$

**3.** $f_4(x) = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!}$

**4.** $f_5(x) = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!}$

**5.** Write a statement that relates the sequence of graphs suggested by Exercises 1–4 and the function $y = e^x$.
**The functions defined by the partial sums converge to $y = e^x$. That is, as $n$ increases, the graphs of $f_n$ come into closer coincidence with the graph of $y = e^x$ for more and more values of $x$.**

**6.** The series $1 + x^2 + x^4 + x^6 + \cdots$ is a power series for which each $a_i = 1$. The series is also a geometric series with first term 1 and common ratio $x^2$.

**a.** Find the function that this power series represents. **$y = \frac{1}{1 - x^2}$**

**b.** For what values of $x$ does the series give the values of the function in part **a**? **$-1 < x < 1$**

**7.** Find a power series representation for the function $f(x) = \frac{3}{1 + x^2}$.
**$3 - 3x^2 + 3x^4 - 3x^6 + \cdots$**

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12-8

# Practice

## Sequences and Iteration

***Find the first four iterates of each function using the given initial value. If necessary, round your answers to the nearest hundredth.***

**1.** $f(x) = x^2 + 4; x_0 = 1$
**5, 29, 845, 714,029**

**2.** $f(x) = 3x + 5; x_0 = -1$
**2, 11, 38, 119**

**3.** $f(x) = x^2 - 2; x_0 = -2$
**2, 2, 2, 2**

**4.** $f(x) = x(2.5 - x); x_0 = 3$
**−1.5, −6, −51, −2728.5**

***Find the first three iterates of the function $f(z) = 2z - (3 + i)$ for each initial value.***

**5.** $z_0 = i$
**$-3 + i$**
**$-9 + i$**
**$-21 + i$**

**6.** $z_0 = 3 - i$
**$3 - 3i$**
**$3 - 7i$**
**$3 - 15i$**

**7.** $z_0 = 0.5 + i$
**$-2 + i$**
**$-7 + i$**
**$-17 + i$**

**8.** $z_0 = -2 - 5i$
**$-7 - 11i$**
**$-17 - 23i$**
**$-37 - 47i$**

***Find the first three iterates of the function $f(z) = z^2 + c$ for each given value of c and each initial value.***

**9.** $c = 1 - 2i; z_0 = 0$
**$1 - 2i$**
**$-2 - 6i$**
**$-31 + 22i$**

**10.** $c = i; z_0 = i$
**$-1 + i$**
**$-i$**
**$-1 + i$**

**11.** $c = 1 + i; z_0 = -1$
**$2 + i$**
**$4 + 5i$**
**$-8 + 41i$**

**12.** $c = 2 - 3i; z_0 = 1 + i$
**$2 - i$**
**$5 - 7i$**
**$-22 - 73i$**

**13.** ***Banking*** Mai deposited \$1000 in a savings account. The annual yield on the account is 5.2%. Find the balance of Mai's account after each of the first 3 years.
**\$1052.00, \$1106.70, \$1164.25**



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12-8

# Enrichment

## Depreciation

To run a business, a company purchases assets such as equipment or buildings. For tax purposes, the company distributes the cost of these assets as a business expense over the course of a number of years. Since assets depreciate (lose some of their market value) as they get older, companies must be able to figure the depreciation expense they are allowed to take when they file their income taxes.

Depreciation expense is a function of these three values:
1. **asset cost**, or the amount the company paid for the asset;
2. **estimated useful life**, or the number of years the company can expect to use the asset;
3. **residual or trade-in value**, or the expected cash value of the asset at the end of its useful life.

In any given year, the **book value** of an asset is equal to the asset cost minus the accumulated depreciation. This value represents the unused amount of asset cost that the company may depreciate in future years. The useful life of the asset is over once its book value is equal to its residual value.

There are several methods of determining the amount of depreciation in a given year. In the **declining-balance method**, the depreciation expense allowed each year is equal to the book value of the asset at the beginning of the year times the depreciation rate. Since the depreciation expense for any year is dependent upon the depreciation expense for the previous year, the process of determining the depreciation expense for a year is an iteration.

***The table below shows the first two iterates of the depreciation schedule for a \$2500 computer with a residual value of \$500 if the depreciation rate is 40%.***

| End of Year | Asset Cost | Depreciation Expense | Book Value at End of Year |
|---|---|---|---|
| 1 | \$2500 | \$1000 (40% of \$2500) | \$1500 (\$2500 - \$1000) |
| 2 | \$2500 | \$600 (40% of \$1500) | \$900 (\$1500 - \$600) |

1. Find the next two iterates for the depreciation expense function.
**\$360; \$40 (only \$40 remains of the asset cost before the residual value is reached)**
2. Find the next two iterates for the end-of-year book value function.
**\$540; \$500**
3. Explain the depreciation expense for year 5.
**There is no depreciation expense because the book value equals the residual value.**

12-9

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## Practice

### Mathematical Induction

***Use mathematical induction to prove that each proposition is valid for all positive integral values of n.***

**1.** $\frac{1}{3} + \frac{2}{3} + \frac{3}{3} + \cdots + \frac{n}{3} = \frac{n(n+1)}{6}$

**Step 1: Verify that the formula is valid for $n = 1$. Since $\frac{1}{3}$ is the first term in the sentence and $\frac{1(1+1)}{6} = \frac{1}{3}$, the formula is valid for $n = 1$.**

**Step 2: Assume that the formula is valid for $n = k$ and then prove that it is also valid for $n = k + 1$.**

$S_k \Rightarrow \frac{1}{3} + \frac{2}{3} + \frac{3}{3} + \cdots + \frac{k}{3} = \frac{k(k+1)}{6}$

$S_{k+1} \Rightarrow \frac{1}{3} + \frac{2}{3} + \frac{3}{3} + \cdots + \frac{k}{3} + \frac{k+1}{3} = \frac{k(k+1)}{6} + \frac{k+1}{3}$

$= \frac{k(k+1) + 2(k+1)}{6}$

$= \frac{(k+1)(k+2)}{6}$

**Apply the original formula for $n = k + 1$.**

$\frac{k+1(k+1+1)}{6}$ **or** $\frac{(k+1)(k+2)}{6}$

**Thus, if the formula is valid for $n = k$, it is also valid for $n = k + 1$. Since the formula is valid for $n = 1$, it is also valid for $n = 2$, $n = 3$, and so on. That is, the formula is valid for all positive integral values of $n$.**

**2.** $5^n + 3$ is divisible by 4.

**Step 1: Verify that $S_n$ is valid for $n = 1$.**

**$S_1 \Rightarrow 5^1 + 3 = 8$. Since 8 is divisible by 4, $S_n$ is valid for $n = 1$.**

**Step 2: Assume that $S_n$ is valid for $n = k$ and then prove that it is valid for $n = k + 1$.**

**$S_k \Rightarrow 5^k + 3 = 4r$ for some integer $r$**

**$S_{k+1} \Rightarrow 5^{k+1} + 3 = 4t$ for some integer $t$**

$5^k + 3 = 4r$

$5(5^k + 3) = 5(4r)$

$5^{k+1} + 15 = 20r$

$5^{k+1} + 3 = 20r - 12$

$5^{k+1} + 3 = 4(5r - 3)$

**Let $t = 5r - 3$, an integer. Then $5^{k+1} + 3 = 4t$.**

**Thus, if $S_k$ is valid, then $S_{k+1}$ is also valid. Since $S_n$ is valid for $n = 1$, it is also valid for $n = 2$, $n = 3$, and so on. Hence, $5^n + 3$ is divisible by 4 for all positive integral values of $n$.**



12-9

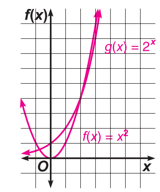
NAME ____________ DATE ________ PERIOD ________

## Enrichment

### Conjectures and Mathematical Induction

Frequently, the pattern in a set of numbers is not immediately evident. Once you make a conjecture about a pattern, you can use mathematical induction to prove your conjecture.

**1. a.** Graph $f(x) = x^2$ and $g(x) = 2^x$ on the axes shown at the right.




**b.** Write a conjecture that compares $n^2$ and $2^n$, where $n$ is a positive integer. **If $n > 4$, $n^2 < 2^n$.**

**c.** Use mathematical induction to prove your response from part b.

**$n = 5$: $5^2 = 25$, $25 < 32$, $32 = 2^5$**

**Assume the statement is true for $n = k$. Prove it is true for $n = k + 1$.**

$(k+1)^2 = k^2 + 2k + 1 < k^2 + (k-1)k + 1$ **since $2 < k - 1$**

$< k^2 + 2^k + 1 - k$ **since $k^2 < 2^k$**

$< 2^k + 2^k$ **since $1 - k < 0$**

$= 2^{k+1}$

**So the statement is true for $n > 4$.**

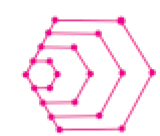
**2.** Refer to the diagrams at the right.

$S_1$ $S_2$ $S_3$

**a.** How many dots would there be in the fourth diagram $S_4$ in the sequence?

**16**

**b.** Describe a method that you can use to determine the number of dots in the fifth diagram $S_5$ based on the number of dots in the fourth diagram, $S_4$. Verify your answer by constructing the fifth diagram.



**Add 5 to the number of dots in $S_4$. $S_5$ would have 16 + 5 or 21 dots.**

**c.** Find a formula that can be used to compute the number of dots in the $n$th diagram of this sequence. Use mathematical induction to prove your formula is correct. **$S_n = 5n - 4$**

**Verify that $S_n$ is true for $n = 1$. $S_1 = 5(1) - 4$ or 1. Assume $S_n$ is true for $n = k$. Prove it is true for $n = k + 1$.**

$S_{k+1} = S_k + 5$

$S_{k+1} = 5(k+1) - 4$

$= 5k + 5 - 4$

$= (5k - 4) + 5$

$= S_k + 5$

# Chapter 12 Answer Key

**Form 1A**

| Page 537 | | Page 538 | |
|---|---|---|---|
| 1. | D | 12. | C |
| 2. | C | 13. | B |
| 3. | A | 14. | A |
| 4. | B | 15. | B |
| 5. | A | 16. | C |
| 6. | A | 17. | C |
| 7. | A | 18. | B |
| 8. | B | 19. | D |
| 9. | C | 20. | D |
| 10. | D | Bonus: | C |
| 11. | D | | |

**Form 1B**

| Page 539 | | Page 540 | |
|---|---|---|---|
| 1. | D | 12. | B |
| 2. | B | 13. | D |
| 3. | B | 14. | A |
| 4. | C | 15. | C |
| 5. | C | 16. | B |
| 6. | C | 17. | A |
| 7. | A | 18. | B |
| 8. | C | 19. | B |
| 9. | A | 20. | D |
| 10. | B | Bonus: | B |
| 11. | D | | |

# Chapter 12 Answer Key

## Form 1C

**Page 541**

1. A
2. D
3. C
4. D
5. B
6. D
7. B
8. B
9. C
10. D
11. A

**Page 542**

12. B
13. A
14. D
15. A
16. C
17. B
18. C
19. A
20. A

Bonus: C

## Form 2A

**Page 543**

1. $\frac{2}{3}$
2. $-7\frac{4}{5}$
3. 537.3
4. $\frac{256}{15{,}625}$ or 0.16384
5. 2,222,222.2
6. $6, 6\sqrt{3}, 18, 18\sqrt{3}, 54$ or $6, -6\sqrt{3}, 18, -18\sqrt{3}, 54$
7. does not exist
8. $12\sqrt{2} + 12$
9. $\frac{32}{495}$
10. convergent
11. divergent

**Page 544**

12. $27 - 9 + 3 - 1 + \frac{1}{3} - \frac{1}{9}$; $\frac{182}{9}$ or $20\frac{2}{9}$
13. $\sum_{k=5}^{12} \frac{3(2k-1)}{2k}$
14. $1 + 5\sqrt{3} + 30 + 30\sqrt{3} + 45 + 9\sqrt{3}$
15. $240x^3y^8$
16. 12.84
17. $i\pi + 2.5416$
18. $7 + 3i, 22 + 9i, 67 + 27i$
19. $-3 + i, 9 - 5i, 57 - 89i$
20. See students' work.

Bonus: $8 + 17i$

# Chapter 12 Answer Key

## Form 2B

### Page 545

1. $-4$
2. $-94.4$
3. $-1440$
4. 243
5. 42.5
6. $-4, -12, -36, -108, -324$ or $-4, 12, -36, 108, -324$
7. 0
8. 36
9. $\frac{90}{11}$
10. convergent
11. divergent

### Page 546

12. $2 + 6 + 12 + 20$; 40
13. Sample answer: $\sum_{n=1}^{10} \frac{n(n-1)}{n+1}$
14. $16p^4 - 96p^3q + 216p^2q^2 - 216pq^3 + 81q^4$
15. $35{,}840x^3y^4$
16. 0.7071
17. $i\pi + 2.5953$
18. $2 - i$, $1 - 0.5i$, $0.5 - 0.25i$
19. $1 + 2i$, $-3 + 6i$, $-27 - 34i$
20. See students' work.

Bonus: 128

## Form 2C

### Page 547

1. 3
2. 0.3
3. 1403
4. $-2560$
5. $-1023$
6. $9, 3, 1, \frac{1}{3}$
7. 2
8. does not exist
9. $\frac{53}{99}$
10. convergent
11. divergent

### Page 548

12. $12 + 15 + 18 + 21$; 66
13. $\sum_{k=1}^{8} \frac{k(k+1)}{2k}$
14. $16p^4 + 32p^3 + 24p^2 + 8p + 1$
15. $-216xy^3$
16. 0.5878
17. $i\pi + 4.0604$
18. $2 + 8i$, $4 + 16i$, $8 + 32i$
19. $1 + i$, $3i$, $-9 + i$
20. See students' work.

Bonus: 32

# Chapter 12 Answer Key

## CHAPTER 12 SCORING RUBRIC

| Level | Specific Criteria |
|---|---|
| 3 Superior | • Shows thorough understanding of the concepts *arithmetic and geometric sequences and series*, *common differences and ratios of terms*, *the binomial theorem*, and *mathematical induction*.<br>• Uses appropriate strategies to solve problems and prove a formula by mathematical induction.<br>• Computations are correct.<br>• Written explanations are exemplary.<br>• Word problems concerning arithmetic and geometric sequences are appropriate and make sense.<br>• Goes beyond requirements of some or all problems. |
| 2 Satisfactory, with Minor Flaws | • Shows understanding of the concepts *arithmetic and geometric sequences and series*, *common differences and ratios of terms*, *the binomial theorem*, and *mathematical induction*.<br>• Uses appropriate strategies to solve problems and prove a formula by mathematical induction.<br>• Computations are mostly correct.<br>• Written explanations are effective.<br>• Word problems concerning arithmetic and geometric sequences are appropriate and make sense.<br>• Satisfies all requirements of problems. |
| 1 Nearly Satisfactory, with Serious Flaws | • Shows understanding of most of the concepts *arithmetic and geometric sequences and series*, *common differences and ratios of terms*, *the binomial theorem,* and *mathematical induction*.<br>• May not use appropriate strategies to solve problems or prove a formula by mathematical induction.<br>• Computations are mostly correct.<br>• Written explanations are satisfactory.<br>• Word problems concerning arithmetic and geometric sequences are appropriate and sensible.<br>• Satisfies most requirements of problems. |
| 0 Unsatisfactory | • Shows little or no understanding of the concepts *arithmetic and geometric sequences and series*, *common differences and ratios of terms*, *the binomial theorem,* and *mathematical induction*.<br>• May not use appropriate strategies to solve problems or prove a formula by mathematical induction.<br>• Computations are incorrect.<br>• Written explanations are not satisfactory.<br>• Word problems concerning arithmetic and geometric sequences are not appropriate or sensible.<br>• Does not satisfy requirements of problems. |

# Chapter 12 Answer Key

## Open-Ended Assessment

**Page 549**

**1a.** Sample answer: Mr. Ling opened a savings account by depositing \$50. He plans to deposit \$25 more per month into the account. What is his total deposit after three months? The sequence is 50 + ($n$ − 1)25, and \$100 is his total deposit after three months.

**1b.** Sample answer: The common difference is \$25. The $n$th term is \$50 + ($n$ − 1)\$25.

**1c.** Sample answer:

$S_{12} = \frac{12}{2}(50 + 325) = 2250$

$S_{12} = 50 + 75 + 100 + 125 + 150 + 175 + 200 + 225 + 250 + 275 + 300 + 325$

$S_{12} = (50 + 325) + (75 + 300) + (100 + 275) + (125 + 250) + (150 + 225) + (175 + 200)$

Since the sums in parentheses are all equal.

$S_{12} = 6(50 + 325)$, or $\frac{12}{2}(50 + 325)$, or $\frac{n}{2}(a_1 + a_n)$

**1d.** No; arithmetic series have no limits; it is divergent.

**2a.** Sample answer: Mimi has \$60 to spend on vacation. If she spends half of her money each day, how much will she have left after the third day?

$\$60 \times \left(\frac{1}{2}\right)^3 = \$7.50$

After the third day, she has \$7.50.

**2b.** Sample answer: The common ratio is $\frac{1}{2}$. The nth term is $60\left(\frac{1}{2}\right)^{n-1}$.

**2c.** *Sample answer:*

$$S_{11} = \frac{60 - 60\left(\frac{1}{2}\right)^{11}}{1 - \frac{1}{2}} \approx 120$$

**2d.** If $r = \lim_{n\to\infty} \frac{a_{n+1}}{a_n} < 1$, the series converges; $r = \frac{1}{2}$; the series converges.

**3a.** Prove that the statement is true for $n = 1$. Then prove that if the statement is true for $n$, then it is true for $n + 1$.

**3b.** Here $S_n$ is defined as

$$a_1 + a_1r + a_1r^2 + \cdots + a_1r^{n-1} = \frac{a_1 - a_1r^n}{1 - r}$$

Step 1: Verify that the formula is valid for $n = 1$.

Since $S_1 = a_1$ and $S_1 = \frac{a_1 - a_1r^1}{1 - r}$

$= \frac{a_1(1 - r)}{1 - r}$

$= a_1$,

the formula is valid for $n = 1$.

Step 2: Assume that the formula is for $n = k$ and derive a formula for $n = k + 1$.

$S_k \Rightarrow a_1 + a_1r + a_1r^2 + \cdots + a_1r^{k-1} = \frac{a_1 - a_1r^k}{1 - r}$

$S_{k+1} \Rightarrow a_1 + a_1r + a_1r^2 + \cdots + a_1r^{k-1} + a_1r^{k+1-1}$

$$= \frac{a_1 - a_1r^k}{1 - r} + a_1r^{k+1-1}$$
$$= \frac{a_1 - a_1r^k}{1 - r} + a_1r^k$$
$$= \frac{a_1 - a_1r^k + a_1r^k - a_1r^{k+1}}{1 - r}$$
$$= \frac{a_1 - a_1r^{k+1}}{1 - r}$$

Apply the original formula for $n = k + 1$.

$S_{k+1} \Rightarrow \frac{a_1 - a_1r^{(k+1)}}{1 - r}$

The formula gives the same result as adding the ($k$ + 1) term directly. thus, if the formula is valid for $n = k$, it is also valid for $n = k + 1$. Since the formula is valid for $n = 2$, it is valid for $n = 3$, it is also valid for $n = 4$, and so on indefinitely. Thus, the formula is valid for all integral values of $n$.

**4.** From the binomial expansion, the fourth term of $\left(\frac{\sqrt{x}}{y^2} - \frac{y}{\sqrt{x}}\right)^6$ is

$\frac{6!}{3!3!}\left(\frac{\sqrt{x}}{y^2}\right)^3\left(\frac{-y}{\sqrt{x}}\right)^3 = -20 \cdot \frac{1}{y^3}$ or $-\frac{20}{y^3}$.

# Chapter 12 Answer Key

**Mid-Chapter Test**
**Page 550**

1. 129
2. 1175
3. $2^7$ or 128
4. −29,524
5. 64, −32, 16, −8
6. $\frac{1}{2}$
7. does not exist
8. $\frac{63}{99}$ or $\frac{7}{11}$
9. convergent
10. divergent

**Quiz A**
**Page 551**

1. $-9\sqrt{3} - 9\sqrt{5}$
2. 17
3. 103.7
4. $9\sqrt{5}$
5. 20.78125
6. $\frac{1}{3}, \pm\frac{\sqrt{5}}{9}, \frac{5}{27}, \ldots$

**Quiz B**
**Page 551**

1. does not exist
2. 1
3. 0
4. $\frac{1}{3}$
5. does not exist
6. $\frac{45}{99}$ or $\frac{5}{11}$
7. divergent
8. divergent
9. convergent
10. divergent

**Quiz C**
**Page 552**

1. $2\frac{1}{2} + 4\frac{1}{2} + 8\frac{1}{2}; 15\frac{1}{2}$
2. Sample answer: $\sum_{n=1}^{\infty} \frac{16}{81} \cdot \left(\frac{3}{2}\right)^{n-1}$
3. $\sum_{k=1}^{100} (2k - 1)(2k)$
4. $81a^4 - 108a^3d + 54a^2d^2 - 12ad^3 + d^4$
5. 36.77
6. 0.8660

**Quiz D**
**Page 552**

1. 1.1, 1.11, 1.111, 1.1111
2. $6 - i, 12 - i, 24 - i$
3. $-2 + 2i, -1 - 6i, -36 + 14i$
4. See students' work.
5. See students' work.

# Chapter 12 Answer Key

## SAT/ACT Practice

**Page 553**

1. D
2. A
3. C
4. C
5. D
6. E
7. E
8. A
9. C

**Page 554**

10. D
11. D
12. D
13. E
14. B
15. D
16. B
17. A
18. D
19. 4000
20. 48

## Cumulative Review

**Page 555**

1. $(-5, 7)$
2. No, $f(x)$ is undefined when $x = 3$.
3. $(x - 2)(2x - 1)(x + 3)$
4. $y = \pm\sin\left(3x - \frac{\pi}{5}\right) + 2$
5. $\frac{15\sqrt{29}}{29}$
6. 560.2 N, 32.2°
7. $12\left(\cos\frac{9\pi}{6} + i\sin\frac{9\pi}{6}\right)$; $-12i$
8. $15(x')^2 + 30x'y' + 15(y')^2 = 108$
9. $1704
10. $\sum_{n=1}^{10}\frac{(-1)^{n+1}3n}{2n+1}$

# Answer Key

**Trigonometry Semester Test Page 557**

1. B
2. C
3. B
4. D
5. D
6. B
7. C
8. B
9. A

**Page 558**

10. C
11. D
12. C
13. D
14. C
15. B
16. C
17. D
18. C
19. C

**Page 559**

20. A
21. B
22. A
23. B
24. A
25. D

# Answer Key

**Page 560**

26. $\langle -2, 14, 10 \rangle$

27. $y = -\frac{1}{2}x - \frac{11}{2}$

28. 0.79

29. 1, 3, 9, 27

30. $\left(2\sqrt{2}, \frac{\pi}{4}\right)$

31. $(-1, 1)$

32. cos 120°

33. 0°, 30°

34. $\frac{4\sqrt{13}}{13}$

35. $\frac{\sqrt{7}}{4}$

36. $\sum_{k=1}^{5} 2k$

37. 3

38. 4.3938

**Page 561**

39. 3

40. $-48{,}384c^5d^3$

41. $\langle 4, 4, 2 \rangle$

42. $\frac{3\sqrt{13}}{13}$

43. $\langle 9, 11 \rangle$

44. $r = \csc\theta$ or $1 = r\cos\left(\theta - \frac{\pi}{2}\right)$

45. $x = 0$

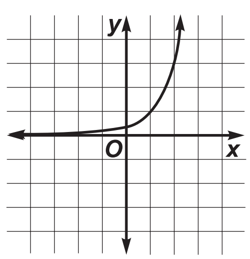
46. $\sqrt{11}$

47. yes

48. 11.64

49. $\frac{7}{11}$

50.




**Trigonometry Final Test Page 563**

1. D

2. C

3. C

4. B

5. C

# Answer Key

**Page 564**

6. A
7. A
8. D
9. C
10. D
11. B
12. A
13. C
14. B

**Page 565**

15. A
16. D
17. B
18. C
19. B
20. D
21. D
22. B
23. C

**Page 566**

24. B
25. B
26. B
27. A
28. C
29. D
30. B

# Answer Key

**Page 567**

**31.** $-4$

**32.** $618.5 \text{ cm}^2$

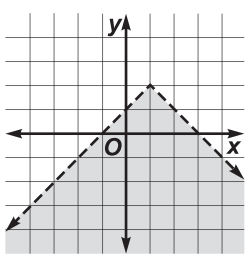
**33.** $\begin{bmatrix} 3 & 2 \\ 0 & 7 \end{bmatrix}$

**34.** $(-1, 3)$

**35.** $y = \sqrt[3]{x} - 1$

**36.** yes
$D = \{-2, 0, 4, 9\}$
$R = \{-3, 3, 5\}$

**37.**




**38.** $\frac{175}{4}$

**39.** $\begin{bmatrix} 11 & 0 \\ 3 & 2 \end{bmatrix}$

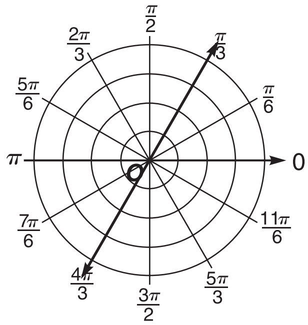
**40.** 1

**41.** 8.1 ft

**42.** 1541

**Page 568**

**43.**




**44.** 19 yrs

**45.** Sample answer:
$(2, 135°), (-2, -45°),$
$(2, -225°), (-2, 315°)$

**46.** 1.3917

**47.** $-1.3673$

**48.** 1,594,322

**49.** $\frac{\sqrt{6} + \sqrt{2}}{4}$

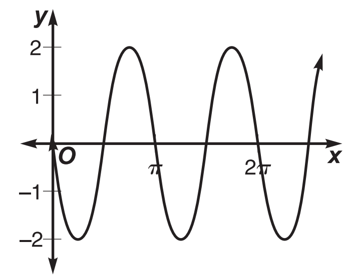
**50.** $\begin{bmatrix} 1 & -2 \\ -2 & 5 \end{bmatrix}$

**51.** 11

**Page 569**

**52.** 2.52 cm

**53.**




**54.** $\log_3 \frac{1}{9} = -2$

**55.** $\frac{\sqrt{6} - \sqrt{2}}{4}$

**56.** 0.62

**57.** $3\vec{i} - 4\vec{j} + 2\vec{k}$

**58.** $\sum_{n=1}^{9} (2^n + 1)$

**59.** $\sqrt{13}(\cos 5.30 + i \sin 5.30)$

**60.** $15 + 8i$

**61.** $(-2, -2)$

**62.** $-4$; imaginary

**63.** $60°$

**64.** 5; 3

# Answer Key

**Page 570**

65. $y = \frac{1}{3}x - \frac{2}{3}$

66. (2, 0, −1)

67. 5

68. $\frac{1}{2}$

69. 36

70. $\frac{2}{3}$

71. 3 or 1

72. $\pm\frac{1}{3}, \pm\frac{2}{3}, \pm1, \pm2$

73. 28.9 cm

74. $c = 66.4$, $C = 76.7°$, $b = 66.3$

75. $a = 6.9$, $B = 107.9°$, $C = 32.1°$

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