PHYSICS

FOR SCIENTISTS AND ENGINEERS **A STRATEGIC APPROACH** 4/E

Chapter 13 Lecture

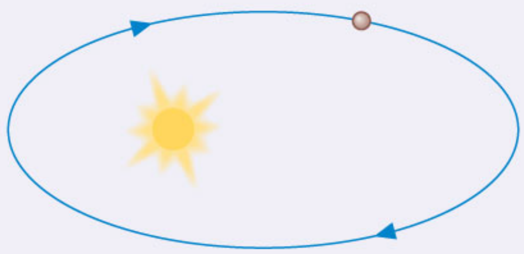
RANDALL D. KNIGHT

Chapter 13 Newton's Theory of Gravity

IN THIS CHAPTER, you will learn to understand the motion of satellites and planets.

What are Kepler's laws?

Before Galileo and the telescope, Kepler used naked-eye measurements to make three major discoveries:



- The planets move in elliptical orbits.
- Planets "sweep out" equal areas in equal times.
- For circular orbits, the square of a planet's period is proportional to the cube of the orbit's radius.

Kepler's discoveries were the first solid proof of Copernicus's assertion that earth and the other planets orbit the sun.

What is Newton's theory?

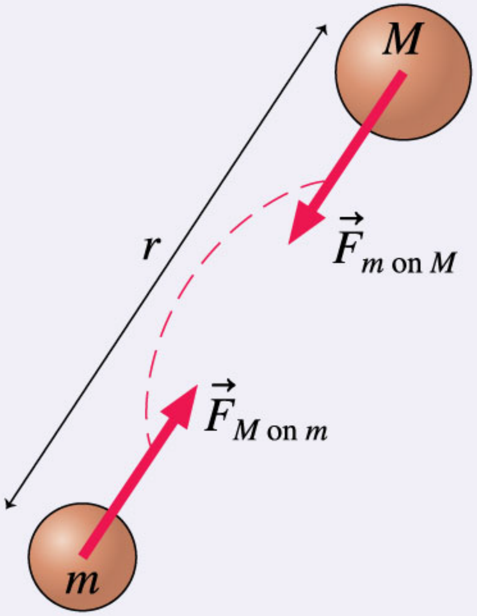
Newton proposed that *any* two masses M and m are attracted toward each other by a gravitational force of magnitude

$$F_{M\text{ on }m} = F_{m\text{ on }M} = \frac{GMm}{r^2}$$

where r is the distance between the masses and G is the gravitational constant.

- Newton's law is an inverse-square law.
- Newton's law predicts the value of g.

In addition to a specific force law for gravity, Newton's three laws of motion apply to all objects in the universe.



What is gravitational energy?

The gravitational potential energy of two masses is

$$U_G = -\frac{GMm}{r}$$

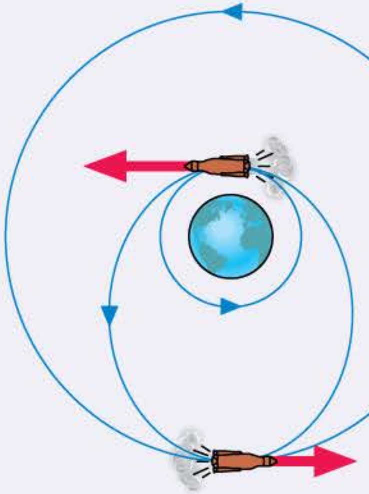
Gravitational potential energy is negative, with a zero point at infinity. The $U_G = mgy$ that you learned in Chapter 10 is a special case for objects very near the surface of a planet. Gravitationally interacting stars, planets, and satellites are always isolated systems, so mechanical energy is conserved.

« LOOKING BACK Chapter 10 Potential energy and energy conservation

What does the theory say about orbits?

Kepler's laws can be *derived* from Newton's theory of gravity:

- Orbits can be circular or elliptical.
- Orbits conserve energy and angular momentum.
- Geosynchronous orbits have the same period as the rotating planet.



« LOOKING BACK Sections 8.2–8.3 Circular motion

Why is the theory of gravity important?

Satellites, space stations, the GPS system, and future missions to planets all depend on Newton's theory of gravity. Our modern understanding of the cosmos—from stars and galaxies to the Big Bang—is based on understanding gravity. Newton's theories of motion and gravity were the beginnings of modern science.

Chapter 13 Reading Questions

Reading Question 13.1

Who discovered three laws governing planetary orbits?

A. Newton
B. Kepler
C. Faraday
D. Einstein
E. Copernicus

Reading Question 13.1

Who discovered three laws governing planetary orbits?

A. Newton
B. Kepler
C. Faraday
D. Einstein
E. Copernicus

Reading Question 13.2

What is the geometric shape of a planetary or satellite orbit?

A. Circle
B. Hyperbola
C. Sphere
D. Parabola
E. Ellipse

Reading Question 13.2

What is the geometric shape of a planetary or satellite orbit?

A. Circle
B. Hyperbola
C. Sphere
D. Parabola
✔**E. Ellipse**

Reading Question 13.3

The gravitational force between two objects of masses m_1 and m_2 that are separated by distance r is

A. Proportional to r

B. Proportional to $1/r$

C. Proportional to $1/r$

D. $(m_1 + m_2)g$

E. $(m_1 + m_2)G$

Reading Question 13.3

The gravitational force between two objects of masses m_1 and m_2 that are separated by distance r is

A. Proportional to r

B. Proportional to $1/r$

C. Proportional to $\boldsymbol{1/r}$

D. $(m_1 + m_2)g$

E. $(m_1 + m_2)G$

Reading Question 13.4

The value of g at the height of the space station's orbit is

A. $9.8\ \mathrm{m/s^2}$

B. Slightly less than $9.8\ \mathrm{m/s^2}$

C. Much less than $9.8\ \mathrm{m/s^2}$

D. Exactly zero.

Reading Question 13.4

The value of g at the height of the space station's orbit is

A. 9.8 m/s^2

✔ **B. Slightly less than 9.8 m/s^2**

C. Much less than 9.8 m/s^2

D. Exactly zero.

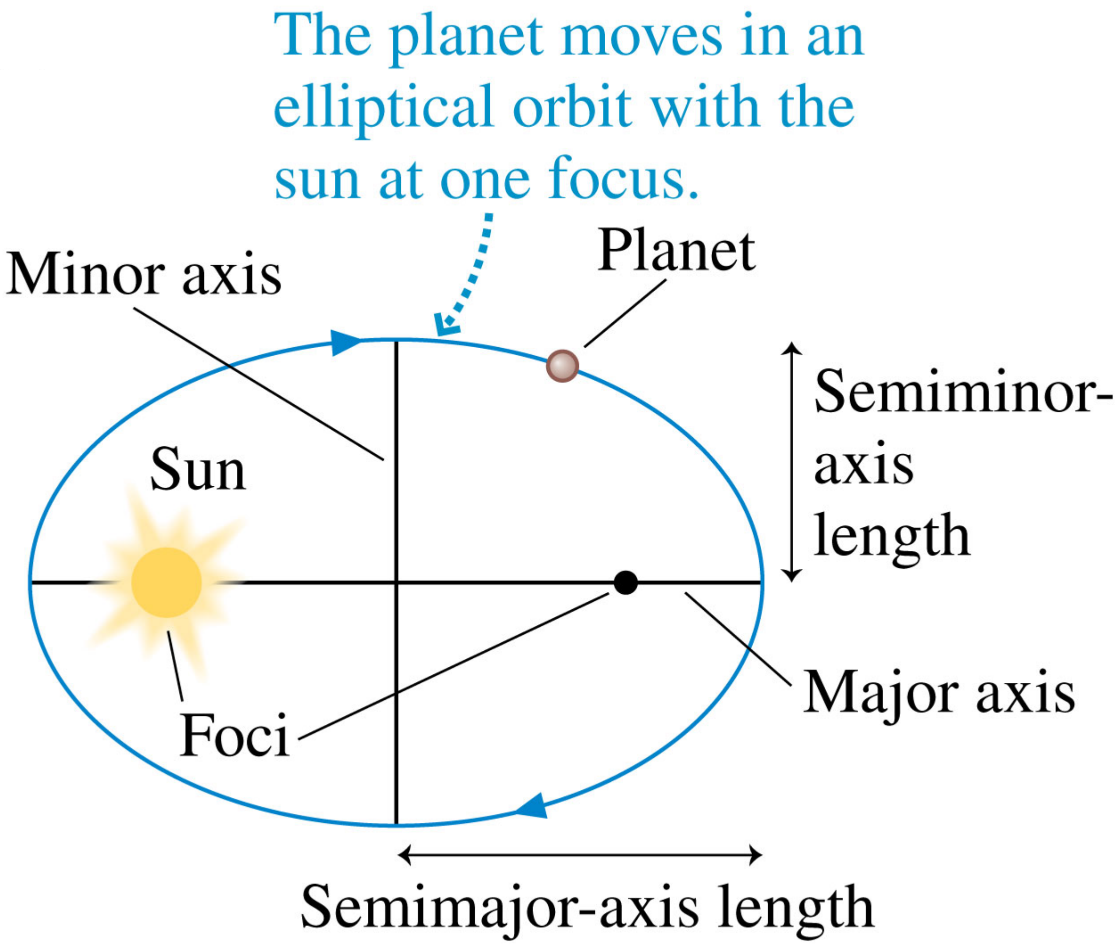
Chapter 13 Content, Examples, and QuickCheck Questions

A Little History: Kepler's Laws

- From 1570 to 1600, Tycho Brahe compiled observations of stars and planets, and noted how the planets Mercury, Venus, Mars, Jupiter, and Saturn move through the sky.
- His mathematical assistant, Johannes Kepler, analyzed Brahe's data and discovered three laws that the planets obey as they orbit the sun.

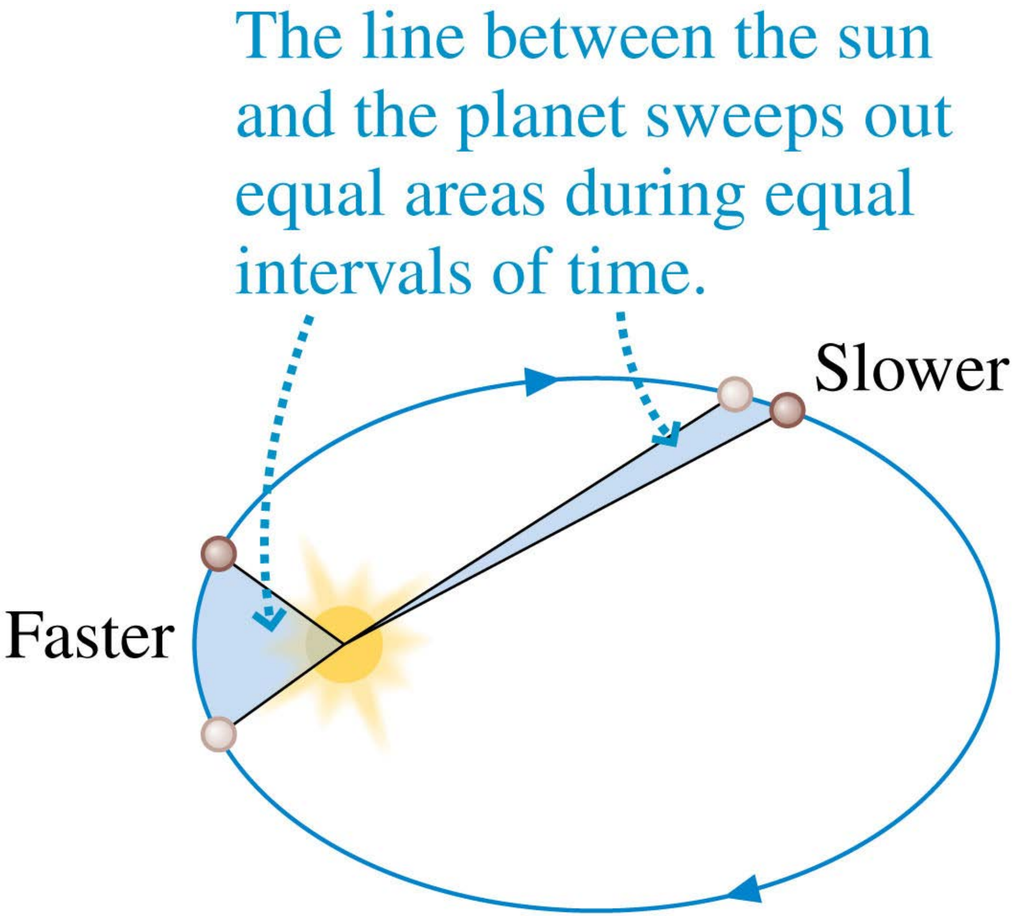
Kepler's First Law of Planetary Motion

1. Planets move in elliptical orbits, with the sun at one focus of the ellipse.

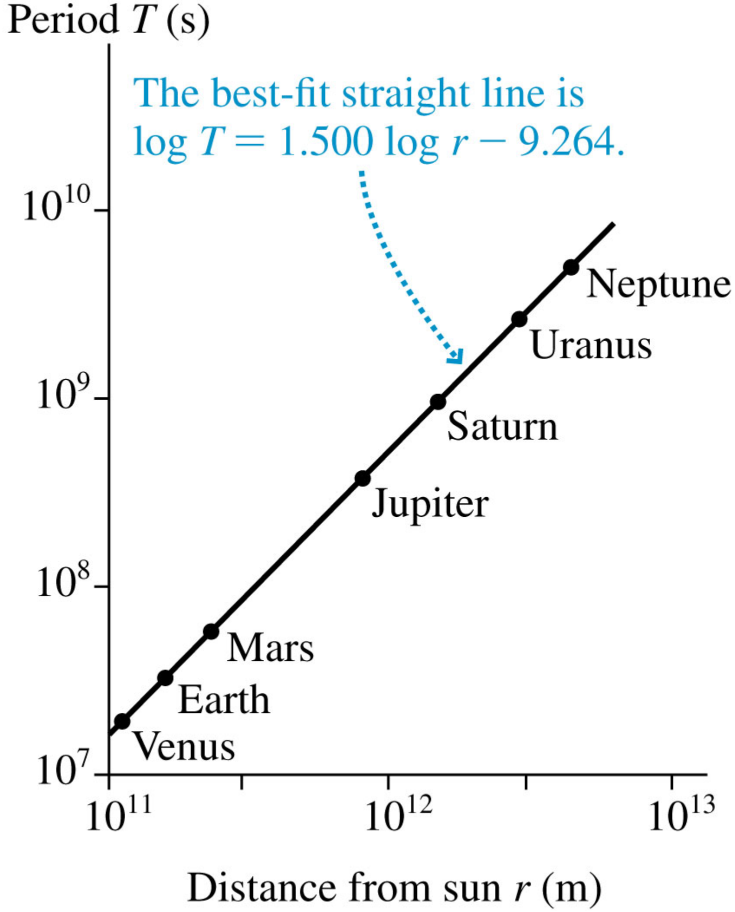


Kepler's Second Law of Planetary Motion

2. A line drawn between the sun and a planet sweeps out equal areas during equal intervals of time.

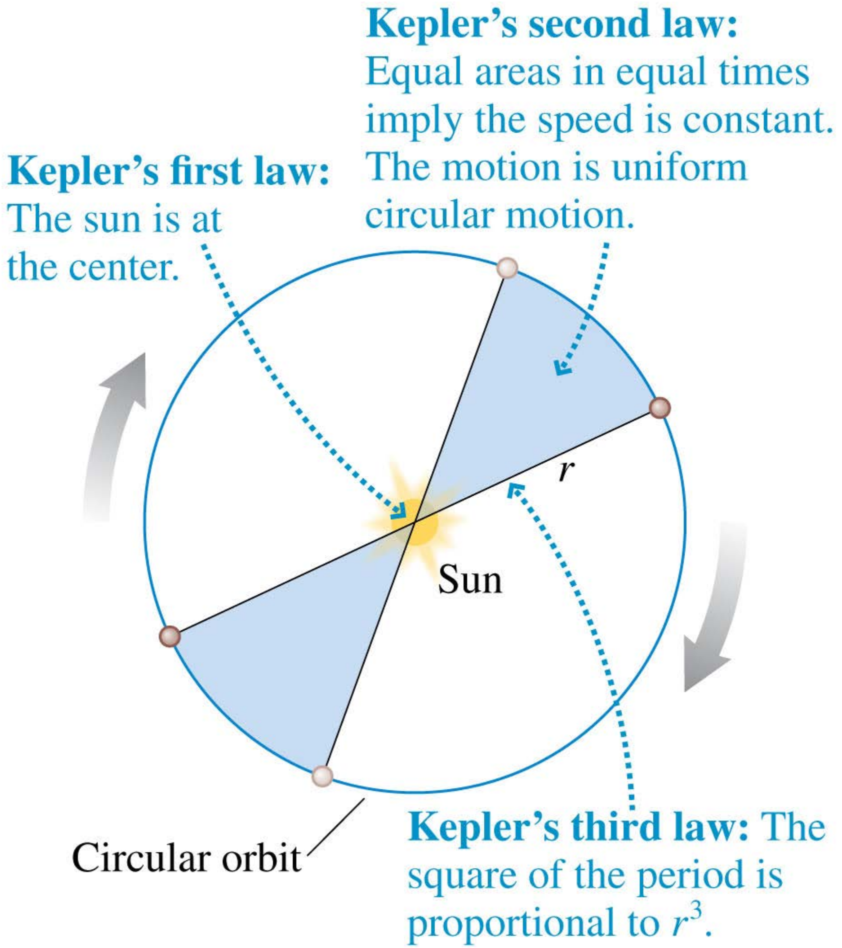


3. The square of a planet's orbital period is proportional to the cube of the semimajor-axis length.

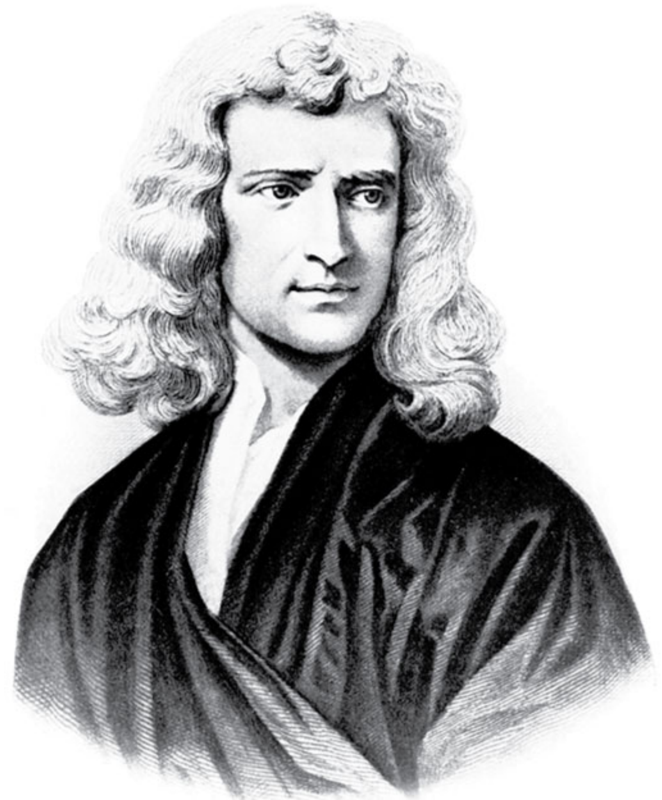


Kepler's Laws of Planetary Motion

- A circular orbit is a special case of an elliptical orbit.



Isaac Newton

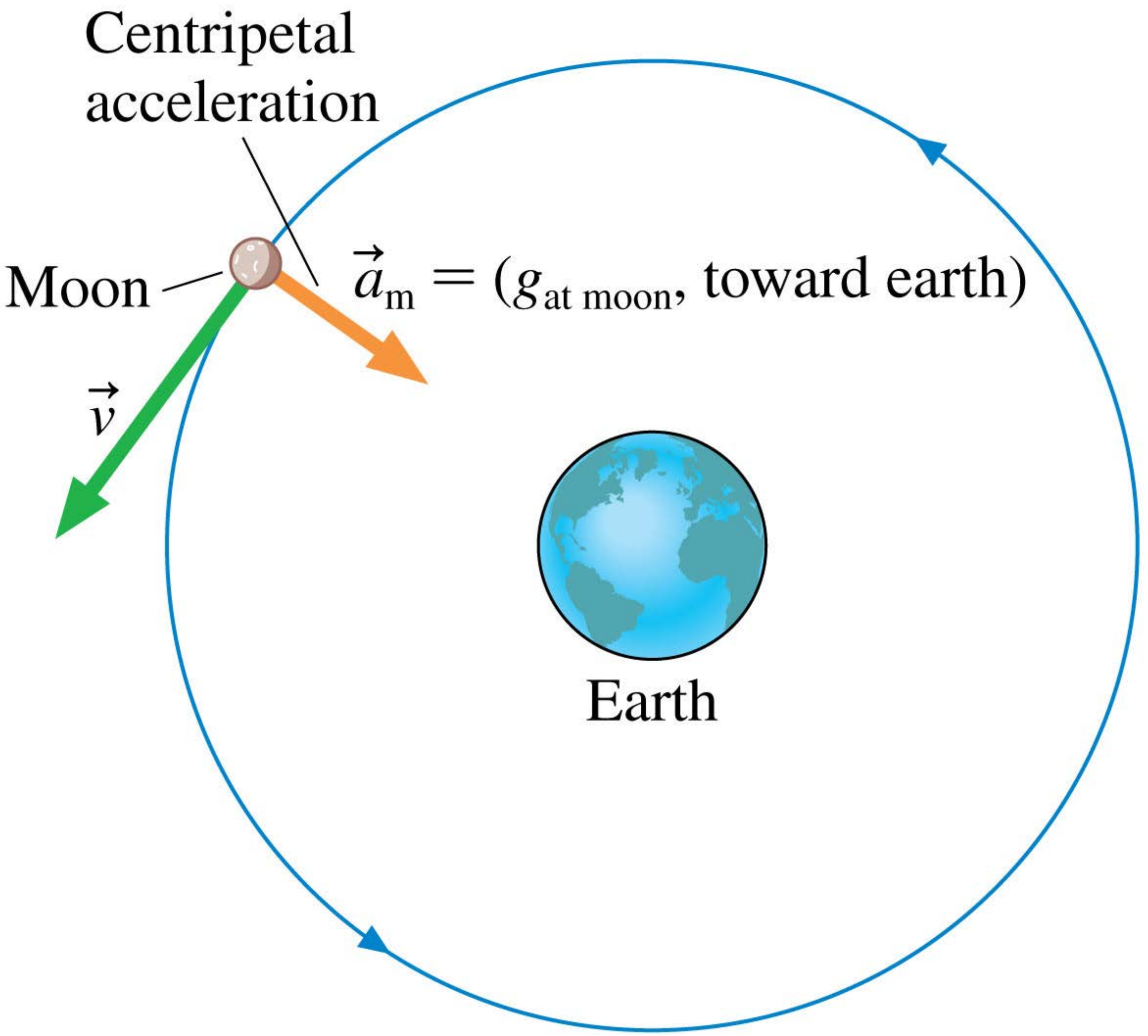


Isaac Newton, 1642–1727

- Legend has it that Newton saw an apple fall from a tree, and it occurred to him that the apple was attracted to the *center* of the earth.
- If the apple was so attracted, why not the moon?
- Newton posited that gravity is a *universal* attractive force between all objects in the universe.

Newton's Law of Gravity

- The moon is in free fall around the earth.



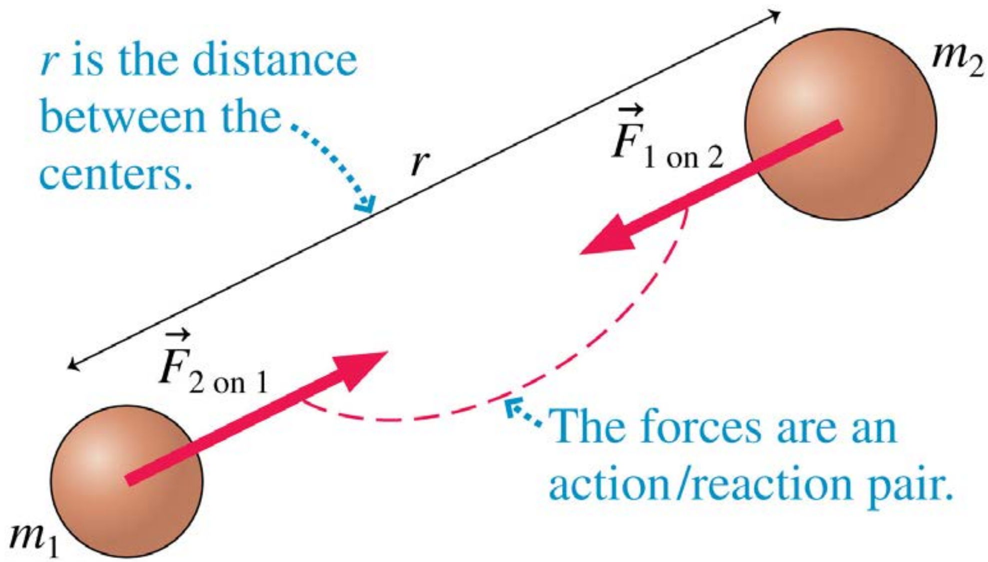
Newton's Law of Gravity

Newton's law of gravity If two objects with masses m_1 and m_2 are a distance r apart, the objects exert attractive forces on each other of magnitude

$$F_{1\text{ on }2} = F_{2\text{ on }1} = \frac{Gm_1m_2}{r^2}$$

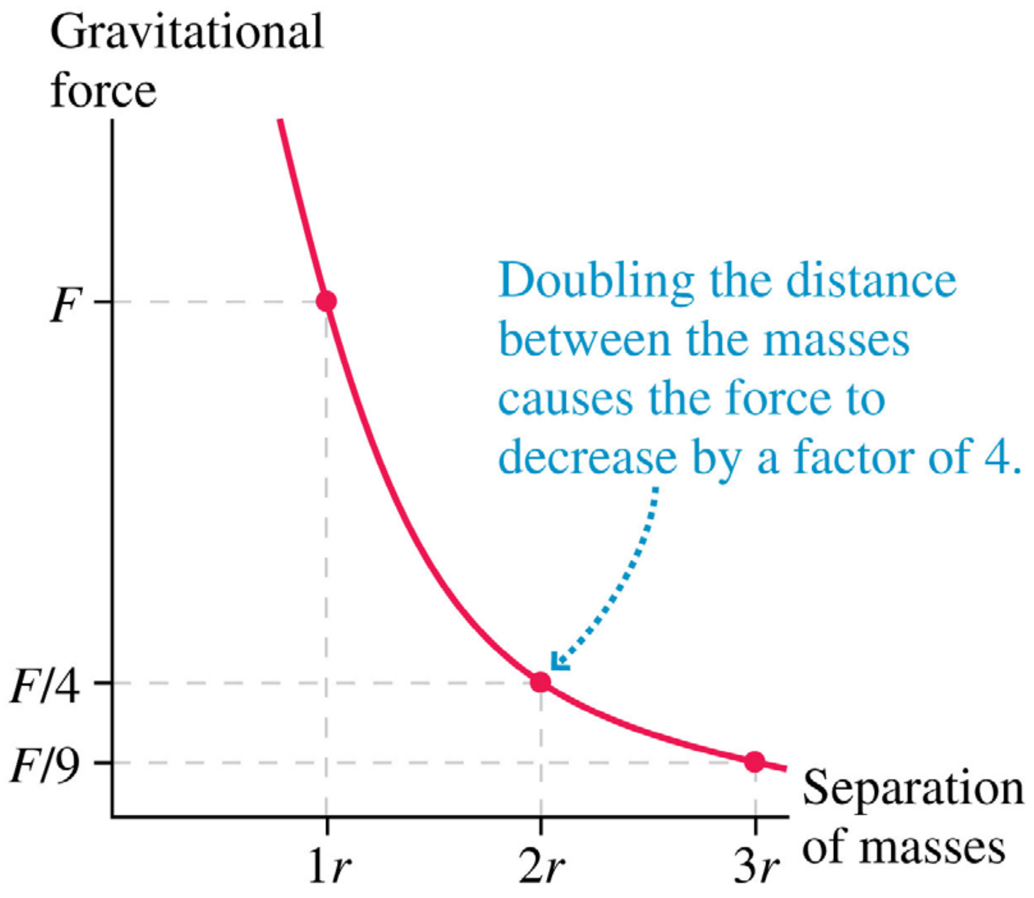
The forces are directed along the straight line joining the two objects.

- The **gravitational constant** is a *universal constant* with the value $G = 6.67 \times 10^{-11}\ \mathrm{N\ m^2/kg^2}$



Newton's Law of Gravity

- The gravitational force is an inverse-square force.



Newton's Law of Gravity

- Since G is so small, it means that the attractive force between two 1.0 kg masses, whose centers are 1.0 m apart, is $6.7 \times 10^{-11}\text{ N}$.
- This is 100 billion times weaker than the force of gravity from the earth on either of the masses!
- Although weak, gravity is a *long-range* force.
- Gravity keeps the earth orbiting the sun and the solar system orbiting the center of the Milky Way galaxy.

The dynamics of stellar motions, spanning many thousands of light years, are governed by Newton's law of gravity.

QuickCheck 13.1

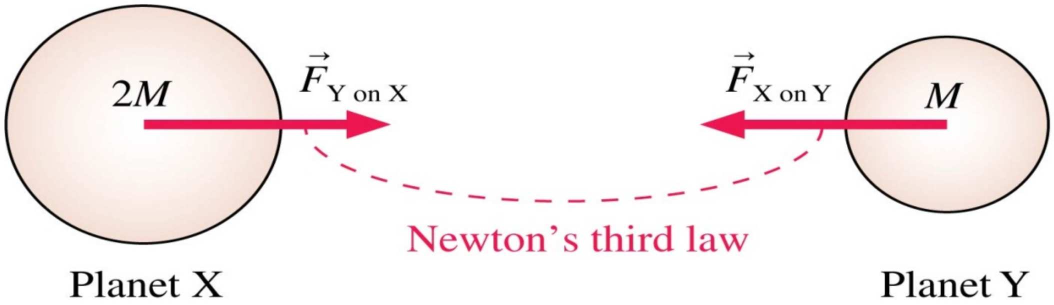
The force of Planet Y on Planet X is ___ the magnitude of $\vec{F}_{\text{X on Y}}$.

A. One quarter
B. One half
C. The same as
D. Twice
E. Four times

QuickCheck 13.1

The force of Planet Y on Planet X is ___ the magnitude of $\vec{F}_{\text{X on Y}}$.

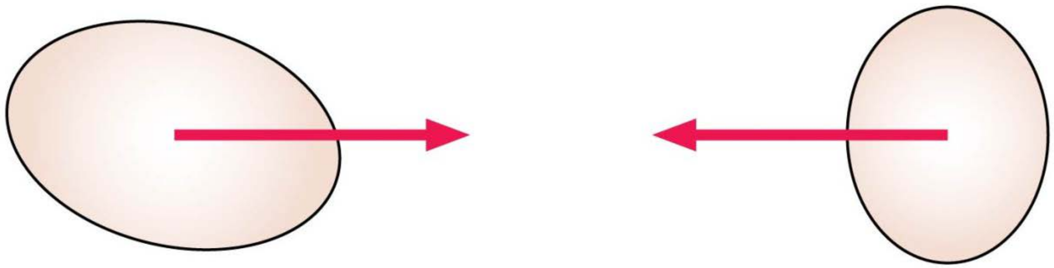
A. One quarter
B. One half
C. **The same as**
D. Twice
E. Four times



QuickCheck 13.2

The gravitational force between two asteroids is 1,000,000 N. What will the force be if the distance between the asteroids is doubled?

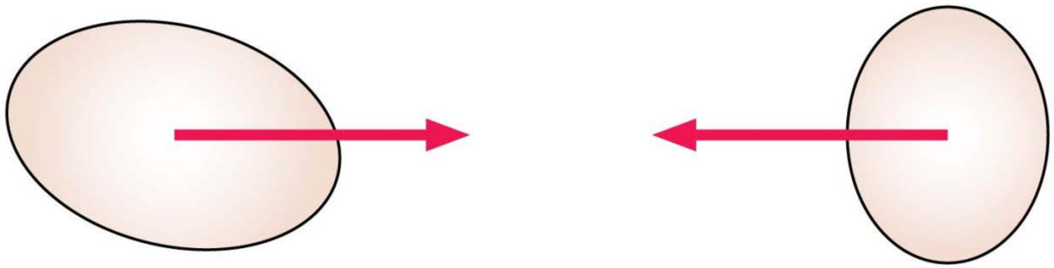
A. 250,000 N
B. 500,000 N
C. 1,000,000 N
D. 2,000,000 N
E. 4,000,000 N



QuickCheck 13.2

The gravitational force between two asteroids is 1,000,000 N. What will the force be if the distance between the asteroids is doubled?

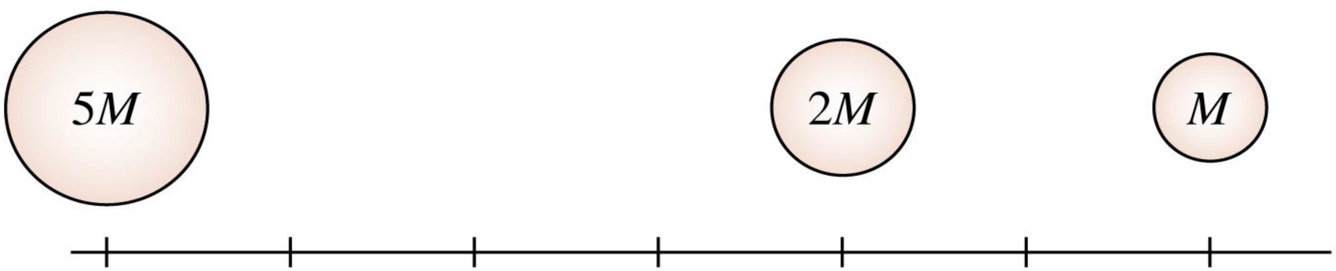
A. **250,000 N**
B. 500,000 N
C. 1,000,000 N
D. 2,000,000 N
E. 4,000,000 N



QuickCheck 13.3

Three stars are aligned in a row. The net force on the star of mass $2M$ is

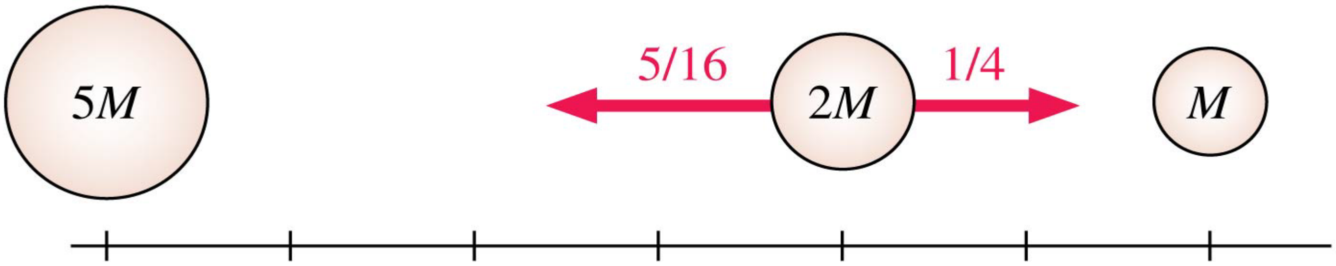
A. To the left.
B. To the right.
C. Zero.
D. Not enough information to answer.



QuickCheck 13.3

Three stars are aligned in a row. The net force on the star of mass $2M$ is

- **A. To the left.**
- B. To the right.
- C. Zero.
- D. Not enough information to answer.



The Principle of Equivalence

- The *inertial mass* of an object relates the net force acting on it to its acceleration:

$$m_{\text{inert}} = \text{inertial mass} = \frac{F}{a}$$

- The *gravitational mass* of an object appears in Newton's law of gravity and determines the strength of the gravitational attraction:

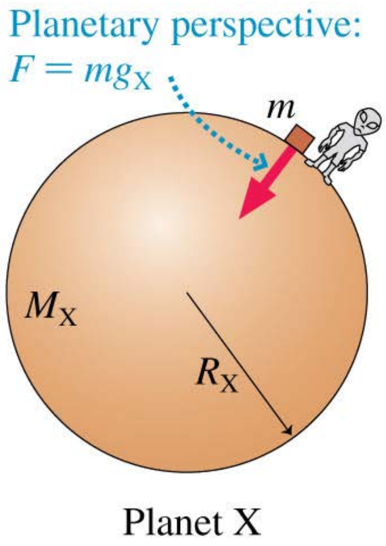
$$m_{\text{grav}} = \text{gravitational mass} = \frac{r^2 F_{M\text{ on }m}}{GM}$$

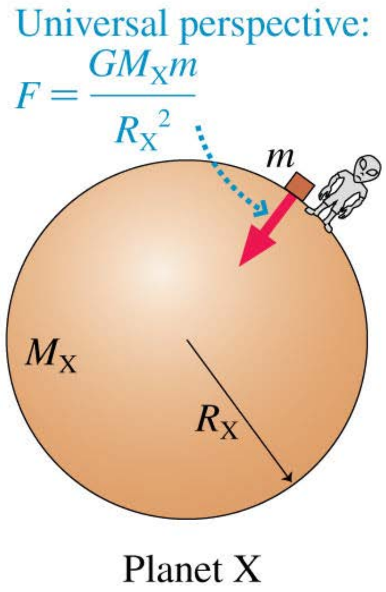
- The assertion that $m_{\text{grav}} = m_{\text{inert}}$ is called the **principle of equivalence**.

Little *g* and Big *G*

- An object of mass m sits on the surface of Planet X.
- According to an observer on the planet, the gravitational force on m should be
  $F_G = mg_{\text{surface}}$
- According to Newton's law of gravity, the gravitational force on m should be $F_{M \text{ on } m} = \dfrac{GMm}{R^2}$
- These are the same if

$$g_{\text{surface}} = \frac{GM}{R^2}$$





QuickCheck 13.4

Planet X has free-fall acceleration $8\ \mathrm{m/s^2}$ at the surface. Planet Y has twice the mass and twice the radius of Planet X. On Planet Y

A. $g = 2\ \mathrm{m/s^2}$

B. $g = 4\ \mathrm{m/s^2}$

C. $g = 8\ \mathrm{m/s^2}$

D. $g = 16\ \mathrm{m/s^2}$

E. $g = 32\ \mathrm{m/s^2}$

QuickCheck 13.4

Planet X has free-fall acceleration $8\ \mathrm{m/s^2}$ at the surface. Planet Y has twice the mass and twice the radius of Planet X. On Planet Y

A. $g = 2\ \mathrm{m/s^2}$

✓ **B. $\boldsymbol{g = 4\ \mathrm{m/s^2}}$**

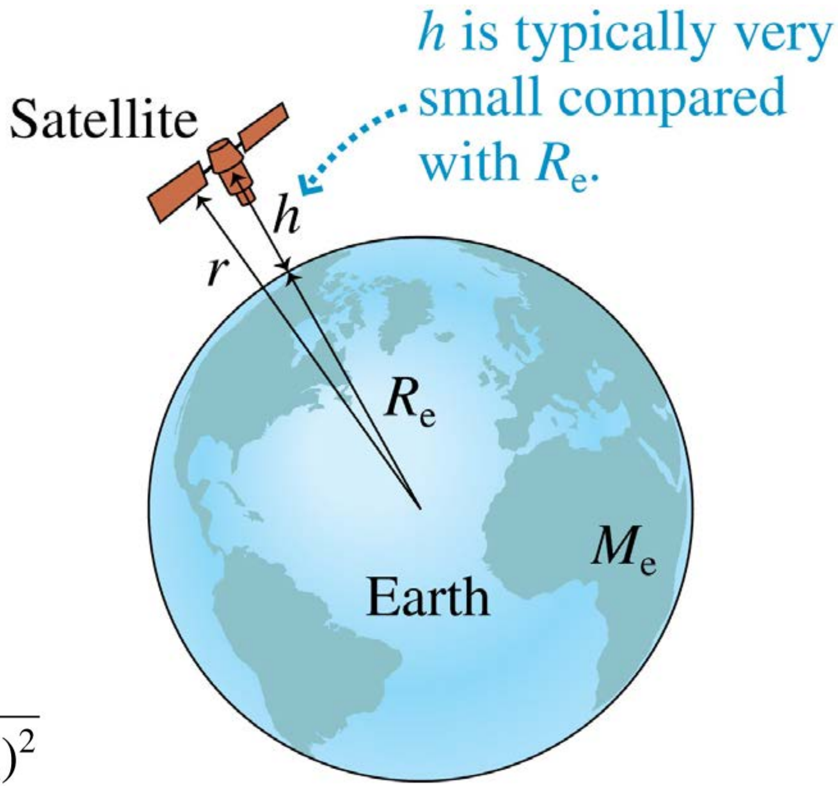
C. $g = 8\ \mathrm{m/s^2}$

D. $g = 16\ \mathrm{m/s^2}$

E. $g = 32\ \mathrm{m/s^2}$

Decrease of g with Distance

- A satellite orbits the earth at height h above the earth's surface.
- The value of g at height h above sea level is



$$g = \frac{GM_e}{(R_e + h)^2} = \frac{GM_e}{R_e^2(1 + h/R_e)^2} = \frac{g_{earth}}{(1 + h/R_e)^2}$$

where $g_{earth} = 9.83\ \text{m/s}^2$ and $R_e = 6.37 \times 10^6\ \text{m}$

Decrease of g with Distance

TABLE 13.1 Variation of g with height above the ground

Height h	Example	g (m/s^2)
0 m	ground	9.83
4500 m	Mt. Whitney	9.82
10,000 m	jet airplane	9.80
300,000 m	space station	8.90
35,900,000 m	communications satellite	0.22

QuickCheck 13.5

Astronauts on the International Space Station are weightless because

A. There's no gravity in outer space.
B. The net force on them is zero.
C. The centrifugal force balances the gravitational force.
D. g is very small, although not zero.
E. They are in free fall.

QuickCheck 13.5

Astronauts on the International Space Station are weightless because

A. There's no gravity in outer space.
B. The net force on them is zero.
C. The centrifugal force balances the gravitational force.
D. g is very small, although not zero.
E. They are in free fall.

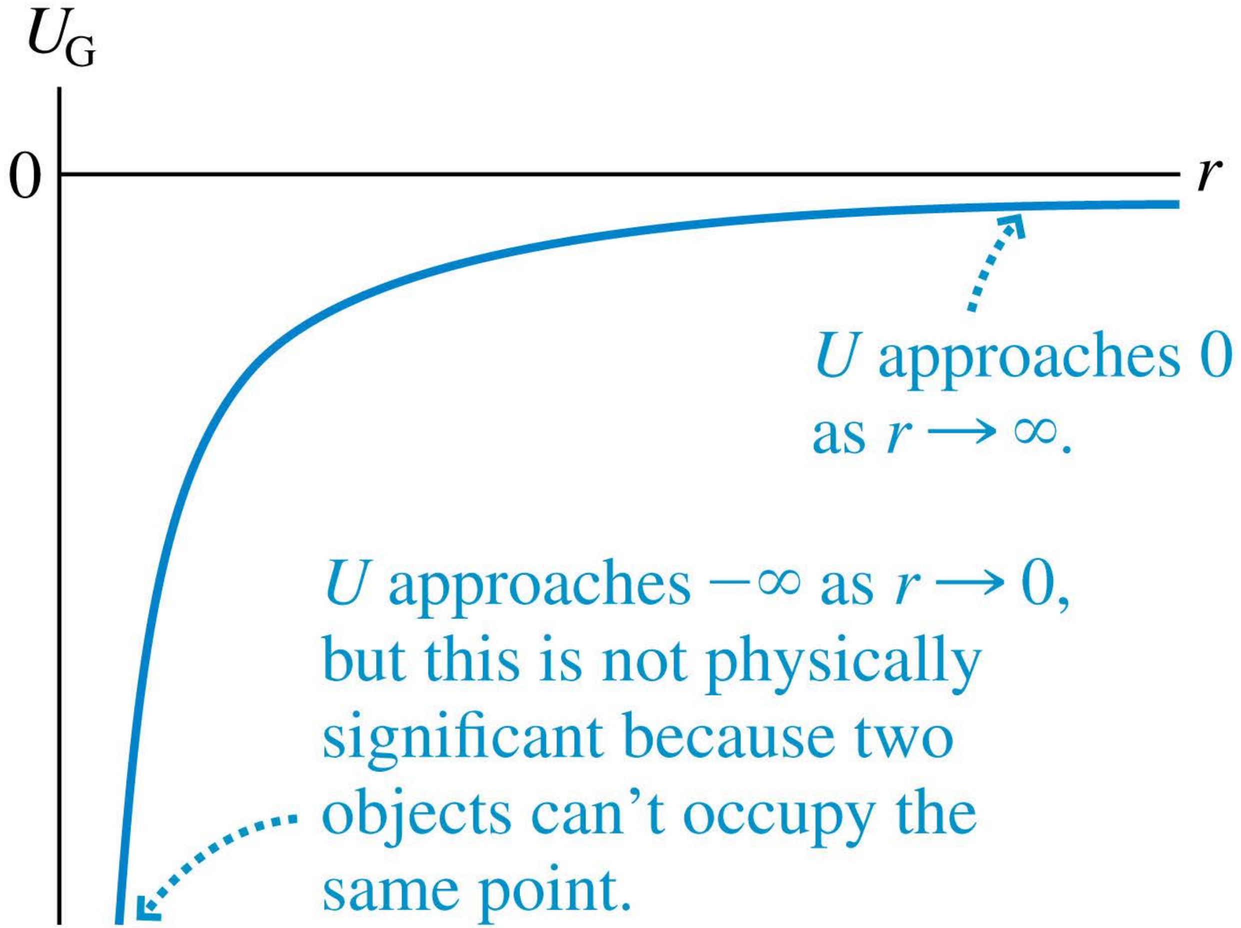
Gravitational Potential Energy

- When two isolated masses m_1 and m_2 interact over large distances, they have a gravitational potential energy of

$$U_G = -\frac{Gm_1m_2}{r}$$

where we have chosen the zero point of potential energy at $r = \infty$, where the masses will have no tendency, or potential, to move together.
Note that this equation gives the potential energy of masses m_1 and m_2 when their *centers* are separated by a distance r.

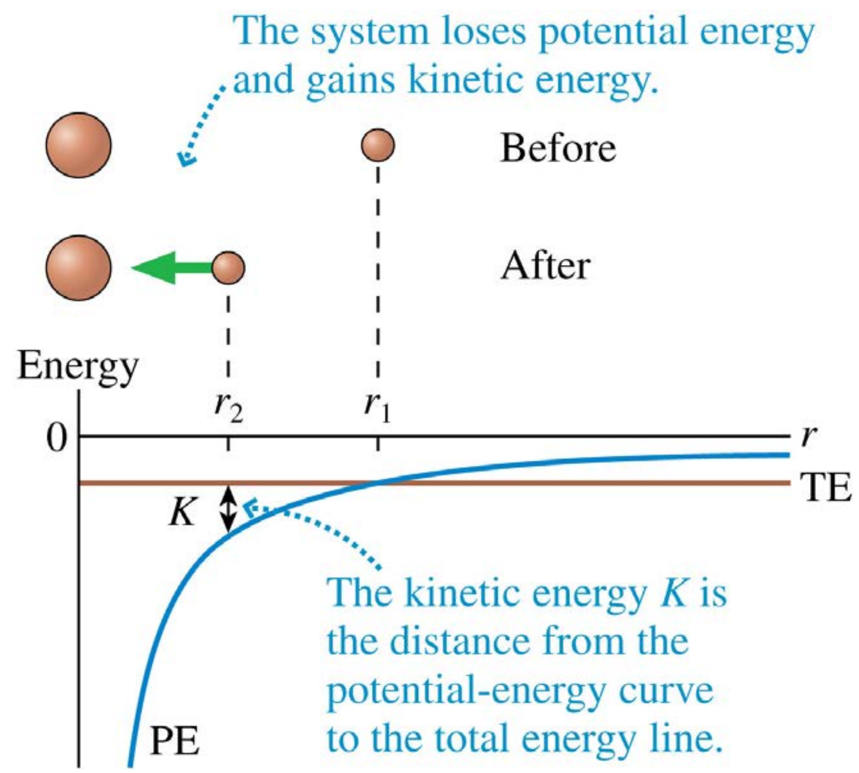
Gravitational Potential Energy



$$U_G = -\frac{Gm_1m_2}{r}$$

Gravitational Potential Energy

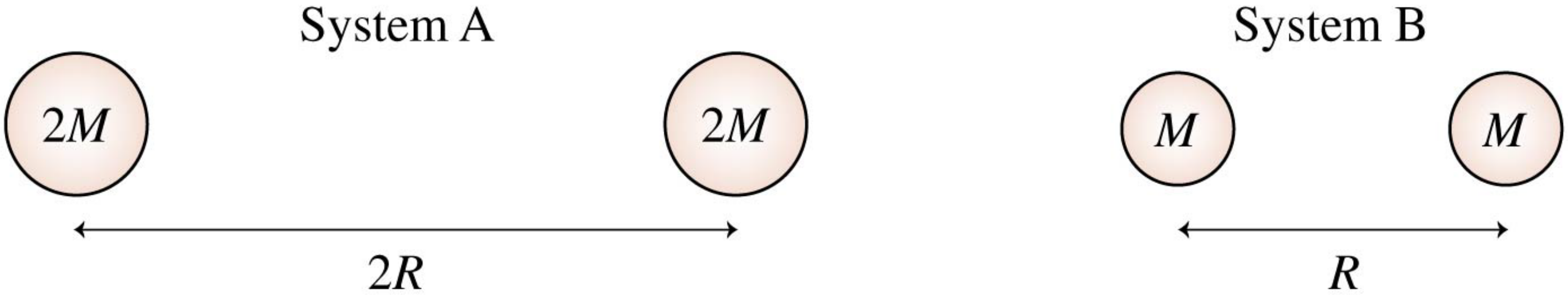
- Suppose two masses a distance r_1 apart are released from rest.
- How will the small mass move as r decreases from r_1 to r_2?
- At r_1 U is negative.
- At r_2 $|U|$ is larger and U is still negative, meaning that U has *decreased*.
- As the system *loses* potential energy, it *gains* kinetic energy while conserving E_{mech}.
- The smaller mass speeds up as it falls.



QuickCheck 13.6

Which system has more (larger absolute value) gravitational potential energy?

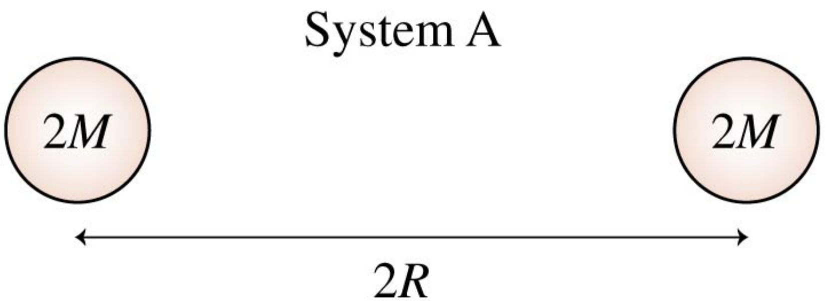
A. System A
B. System B
C. They have the same gravitational potential energy.



QuickCheck 13.6

Which system has more (larger absolute value) gravitational potential energy?

- **A. System A**
- B. System B
- C. They have the same gravitational potential energy.



$$U_A = -\frac{G(2M)(2M)}{2R} = -2\frac{GM^2}{R^2}$$

$$U_B = -\frac{G(M)(M)}{R} = -\frac{GM^2}{R^2}$$

Example 13.2 Escape Speed

EXAMPLE 13.2 | **Escape speed**

A 1000 kg rocket is fired straight away from the surface of the earth. What speed does the rocket need to "escape" from the gravitational pull of the earth and never return? Assume a nonrotating earth.

Example 13.2 Escape Speed

EXAMPLE 13.2 Escape speed

MODEL In a simple universe, consisting of only the earth and the rocket, an insufficient launch speed will cause the rocket eventually to fall back to earth. Once the rocket finally slows to a halt, gravity will ever so slowly pull it back. The only way the rocket can escape is to never stop $(v = 0)$ and thus never have a turning point! That is, the rocket must continue moving away from the earth forever. The *minimum* launch speed for escape, which is called the **escape speed,** will cause the rocket to stop $(v = 0)$ only as it reaches $r = \infty$. Now ∞, of course, is not a "place," so a statement like this means that we want the rocket's speed to approach $v = 0$ asymptotically as $r \rightarrow \infty$.

VISUALIZE FIGURE 13.13 is a before-and-after pictorial representation.

Example 13.2 Escape Speed

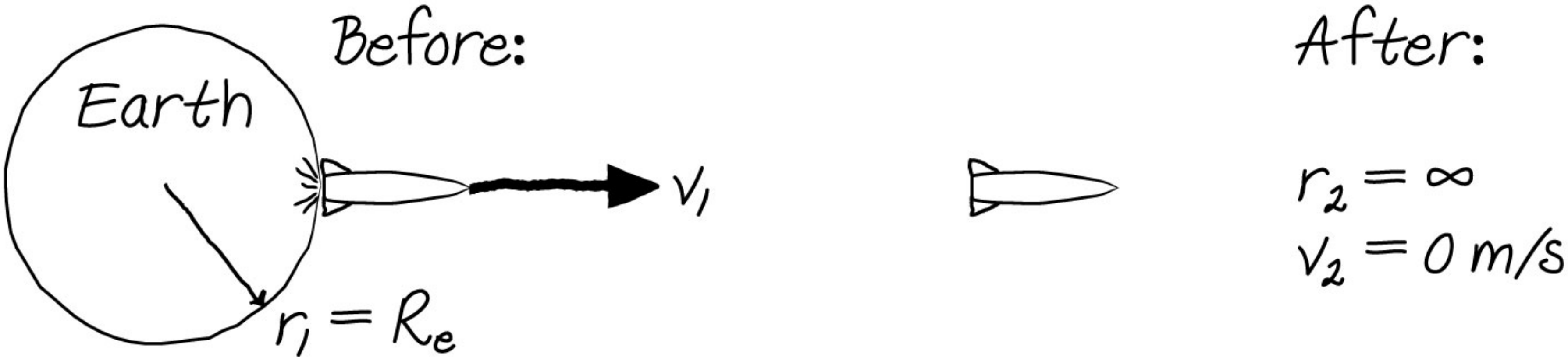
EXAMPLE 13.2 Escape speed

SOLVE Energy conservation $K_2 + U_2 = K_1 + U_1$ is

$$0 + 0 = \frac{1}{2}mv_1^{\,2} - \frac{GM_e m}{R_e}$$

where we used the fact that both the kinetic and potential energy are zero at $r = \infty$. Thus the escape speed is

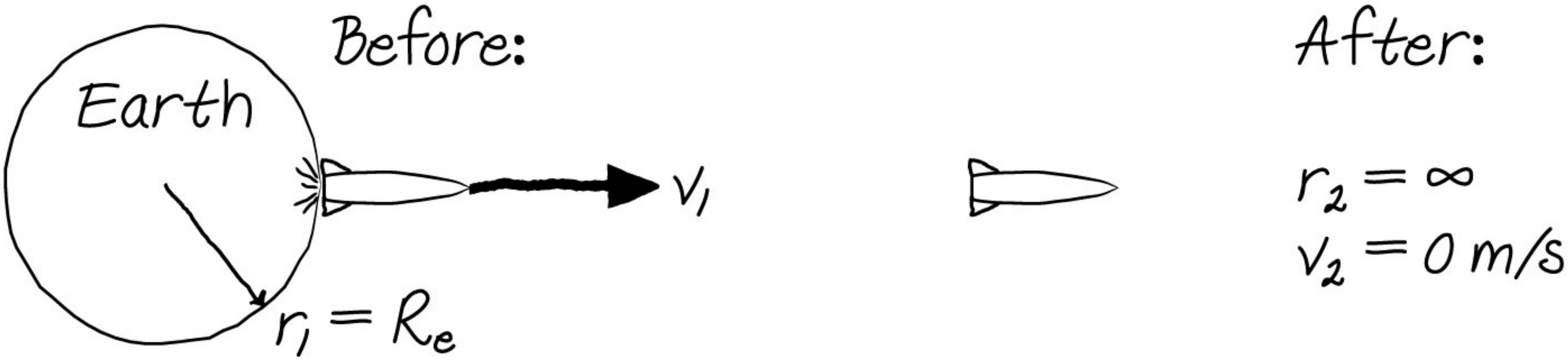
$$v_{\text{escape}} = v_1 = \sqrt{\frac{2GM_e}{R_e}} = 11{,}200 \text{ m/s} \approx 25{,}000 \text{ mph}$$



Example 13.2 Escape Speed

EXAMPLE 13.2 **Escape speed**

ASSESS It is difficult to assess the answer to a problem with which we have no direct experience, although we certainly expected the escape speed to be very large. Notice that the problem was mathematically easy; the difficulty was deciding how to interpret it. That is why—as you have now seen many times—the "physics" of a problem consists of thinking, interpreting, and modeling. We will see variations on this problem in the future, with both gravity and electricity, so you might want to review the *reasoning* involved.

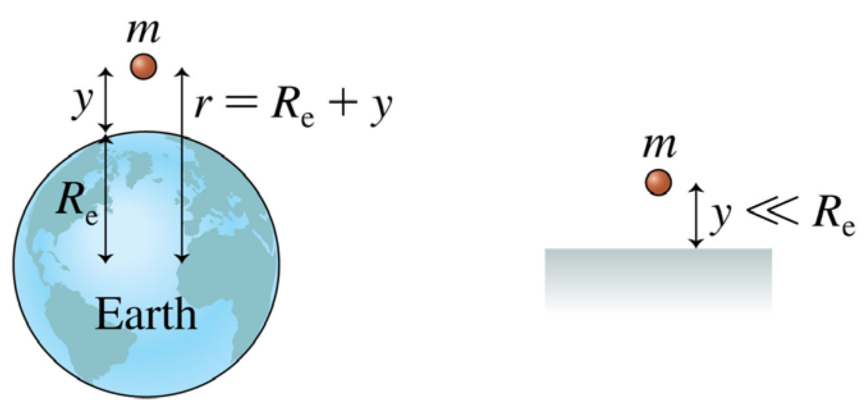


Gravitational Potential Energy

- Consider a mass a distance y above the surface of the earth.
- If y is very small compared to the radius of the earth, we can use the binomial approximation to show

$$U_G(\text{if } y \ll R_e) = U_0 + mg_{\text{earth}}y$$

- Set $U_0 = 0$ and this is our familiar formula for U_G.



For a spherical earth:

$$U_G = -\frac{GM_e m}{R_e + y}$$

We can treat the earth as flat if $y \ll R_e$:

$$U_G = mgy$$

Example 13.3 The Speed of a Satellite

EXAMPLE 13.3 **The speed of a satellite**

A less-than-successful inventor wants to launch small satellites into orbit by launching them straight up from the surface of the earth at very high speed.

a. With what speed should he launch the satellite if it is to have a speed of 500 m/s at a height of 400 km? Ignore air resistance.

b. By what percentage would your answer be in error if you used a flat-earth approximation?

MODEL Mechanical energy is conserved if we ignore drag.

Example 13.3 The Speed of a Satellite

EXAMPLE 13.3 **The speed of a satellite**

VISUALIZE **FIGURE 13.15** shows a pictorial representation.

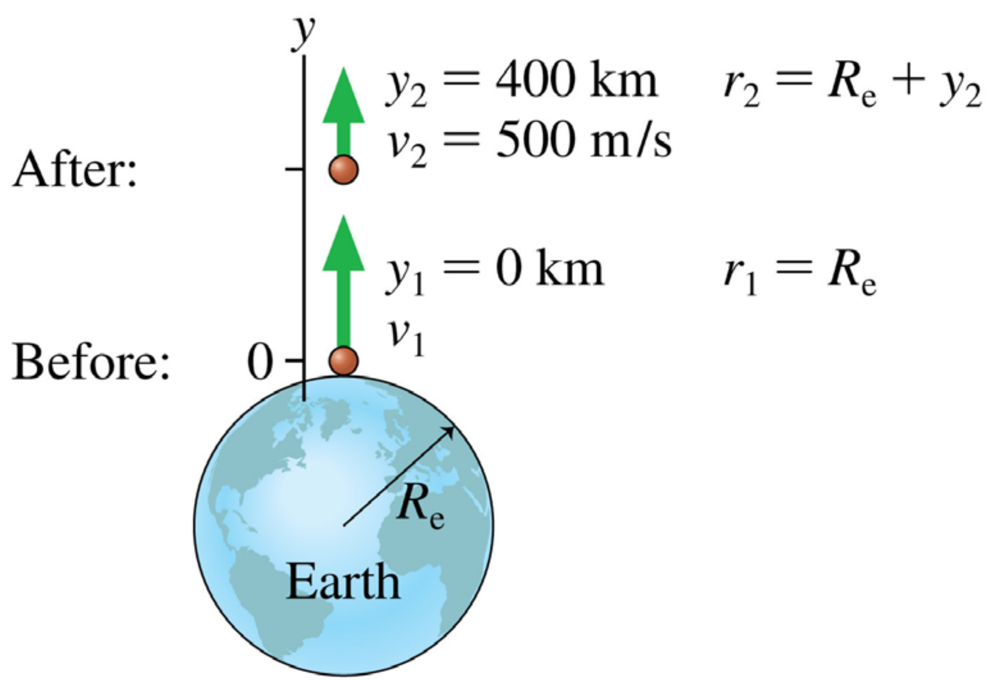
SOLVE a. Although the height is exaggerated in the figure, 400 km = 400,000 m is high enough that we cannot ignore the earth's spherical shape. The energy conservation equation $K_2 + U_2 = K_1 + U_1$ is

$$\frac{1}{2}mv_2^2 - \frac{GM_e m}{R_e + y_2} = \frac{1}{2}mv_1^2 - \frac{GM_e m}{R_e + y_1}$$

where we've written the distance between the satellite and the earth's center as $r = R_e + y$. The initial height is $y_1 = 0$. Notice that the satellite mass m cancels and is not needed. Solving for the launch speed, we have

$$v_1 = \sqrt{v_2^2 + 2GM_e\left(\frac{1}{R_e} - \frac{1}{R_e + y_2}\right)} = 2770 \text{ m/s}$$

This is about 6000 mph, much less than the escape speed.



Example 13.3 The Speed of a Satellite

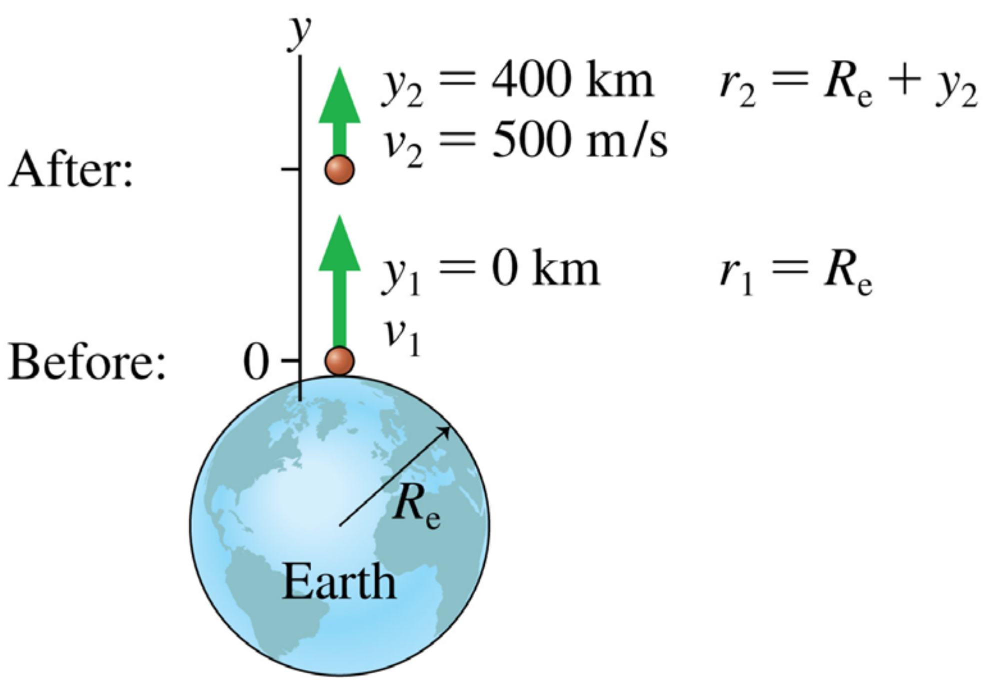
EXAMPLE 13.3 | **The speed of a satellite**

SOLVE b. The calculation is the same in the flat-earth approximation except that we use $U_G = mgy$. Thus

$$\tfrac{1}{2}mv_2^2 + mgy_2 = \tfrac{1}{2}mv_1^2 + mgy_1$$
$$v_1 = \sqrt{v_2^2 + 2gy_2} = 2840 \text{ m/s}$$

The flat-earth value of 2840 m/s is 70 m/s too big. The error, as a percentage of the correct 2770 m/s, is

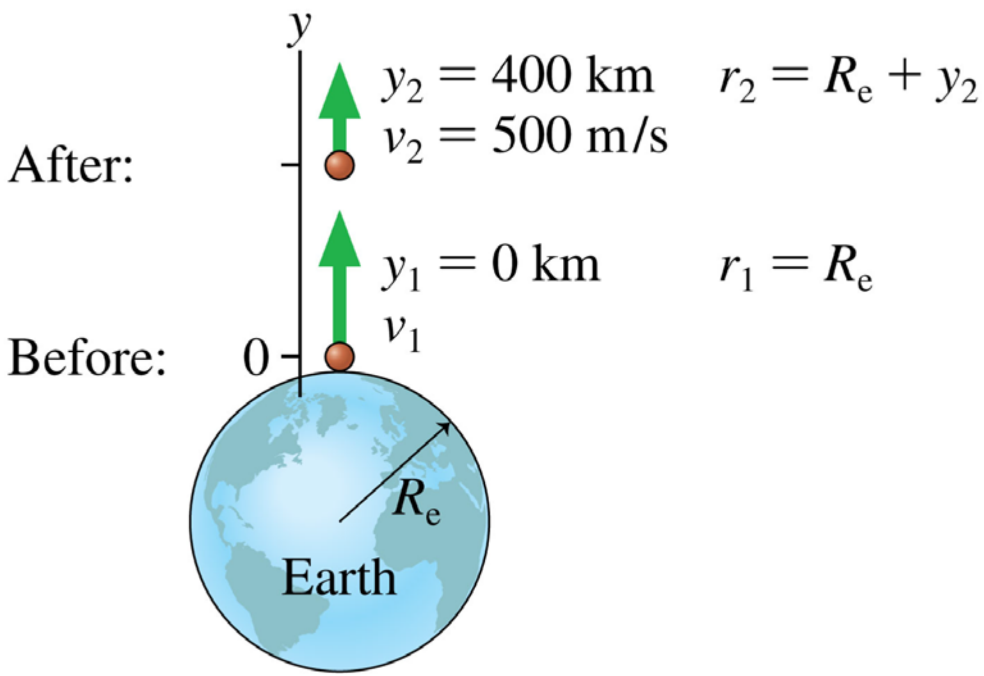
$$\text{error} = \frac{70}{2770} \times 100 = 2.5\%$$



Example 13.3 The Speed of a Satellite

EXAMPLE 13.3 | **The speed of a satellite**

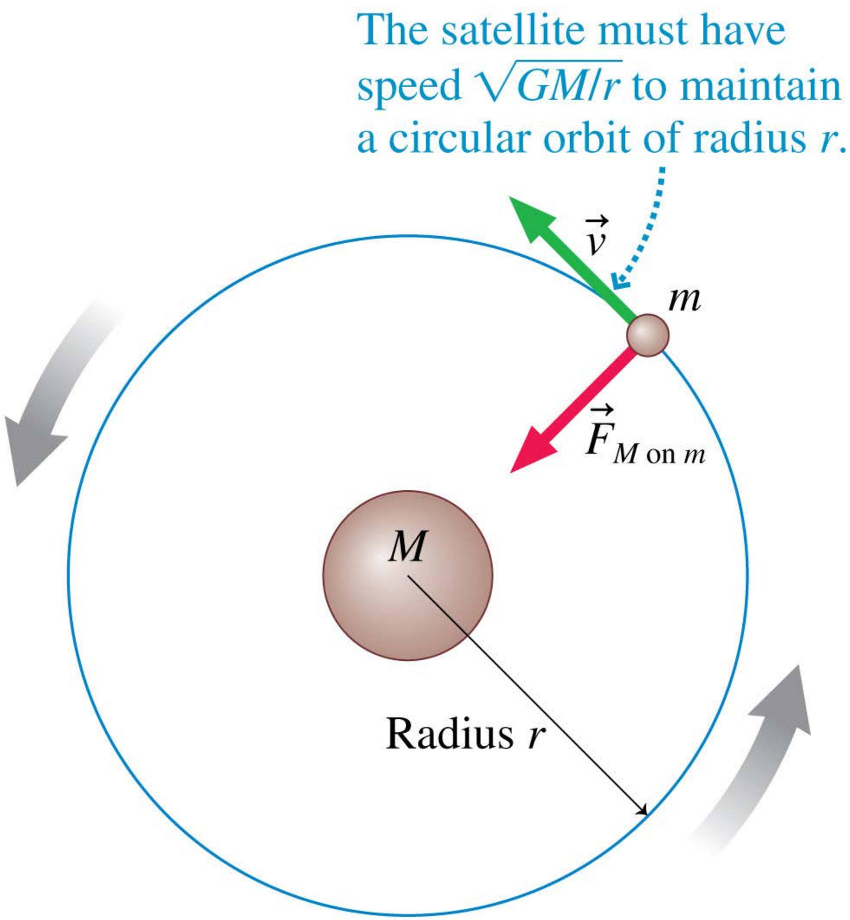
ASSESS The true speed is less than the flat-earth approximation because the force of gravity decreases with height. Launching a rocket against a decreasing force takes less effort than it would with the flat-earth force of mg at all heights.



Satellite Orbits

- A circle is a special case of an ellipse.
- If a small mass m orbits a much larger mass M, the small mass is called a satellite.
- The speed of a satellite in a circular orbit is:

$$v = \sqrt{\frac{GM}{r}}$$



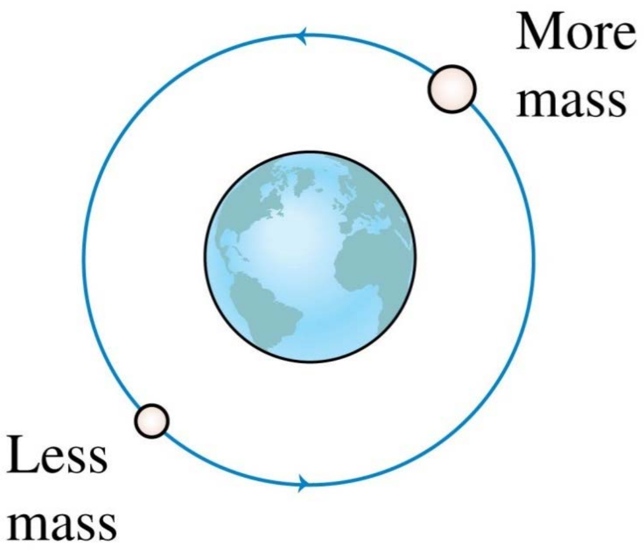
QuickCheck 13.7

Two satellites have circular orbits with the same radius. Which has a higher speed?

A. The one with more mass.

B. The one with less mass.

C. They have the same speed.



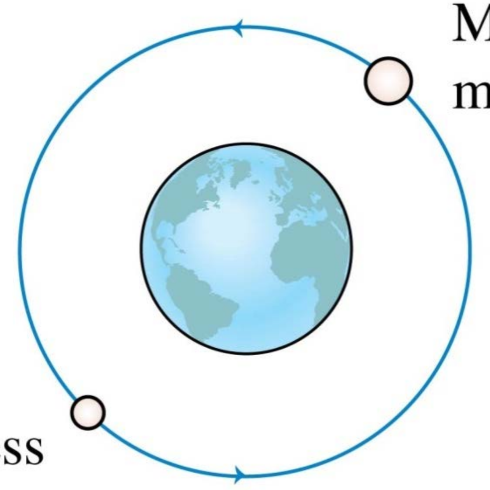
QuickCheck 13.7

Two satellites have circular orbits with the same radius. Which has a higher speed?

A. The one with more mass.

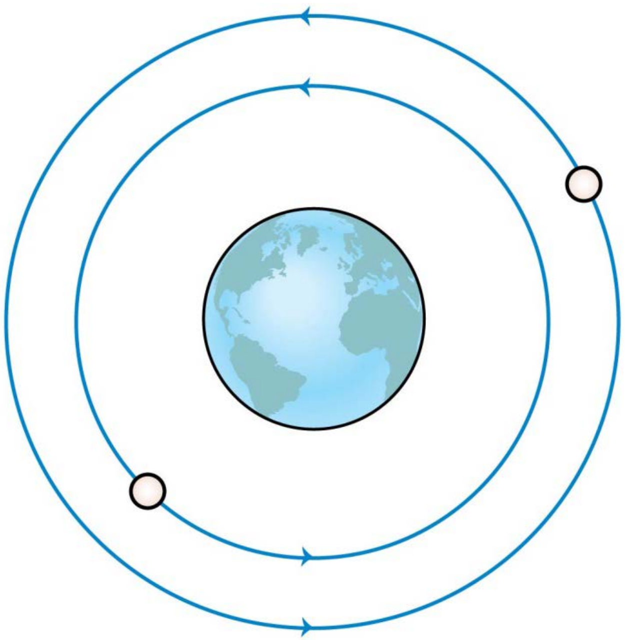
B. The one with less mass.

C. They have the same speed.



QuickCheck 13.8

Two identical satellites have different circular orbits. Which has a higher speed?



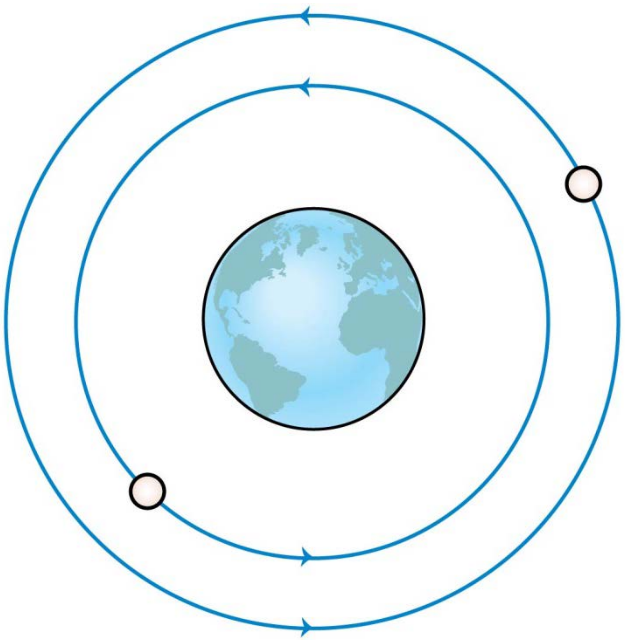
A. The one in the larger orbit.

B. The one in the smaller orbit.

C. They have the same speed.

QuickCheck 13.8

Two identical satellites have different circular orbits. Which has a higher speed?



A. The one in the larger orbit.

B. The one in the smaller orbit.

C. They have the same speed.

Example 13.4 The Speed of the Space Station

EXAMPLE 13.4 **The speed of the space station**

A small supply satellite needs to dock with the International Space Station. The ISS is in a near-circular orbit at a height of 420 km. What are the speeds of the ISS and the supply satellite in this orbit?

SOLVE Despite their different masses, the satellite and the ISS travel side by side with the same speed. They are simply in free fall together. Using $r = R_e + h$ with $h = 420\text{ km} = 4.20 \times 10^5\text{ m}$, we find the speed

$$v = \sqrt{\frac{(6.67 \times 10^{-11}\text{ N m}^2/\text{kg}^2)(5.98 \times 10^{24}\text{ kg})}{6.79 \times 10^6\text{ m}}}$$

$$= 7660\text{ m/s} \approx 17{,}000\text{ mph}$$

ASSESS The answer depends on the mass of the earth but *not* on the mass of the satellite.

Recall: Kepler's Third Law of Planetary Motion

3. The square of a planet's orbital period is proportional to the cube of the semimajor-axis length.

- The speed of a satellite in a circular orbit is

$$v = \frac{2\pi r}{T} = \sqrt{\frac{GM}{r}}$$

- Squaring both sides and solving for T^2 gives

$$T^2 = \left(\frac{4\pi^2}{GM}\right) r^3$$

- Planets are satellites of the sun, in orbits that are almost circular. The orbital radius = the semimajor-axis length for a circle.

Useful Astronomical Data

TABLE 13.2 Useful astronomical data

Planetary body	Mean distance from sun (m)	Period (years)	Mass (kg)	Mean radius (m)
Sun	—	—	1.99×10^{30}	6.96×10^{8}
Moon	3.84×10^{8}*	27.3 days	7.36×10^{22}	1.74×10^{6}
Mercury	5.79×10^{10}	0.241	3.18×10^{23}	2.43×10^{6}
Venus	1.08×10^{11}	0.615	4.88×10^{24}	6.06×10^{6}
Earth	1.50×10^{11}	1.00	5.98×10^{24}	6.37×10^{6}
Mars	2.28×10^{11}	1.88	6.42×10^{23}	3.37×10^{6}
Jupiter	7.78×10^{11}	11.9	1.90×10^{27}	6.99×10^{7}
Saturn	1.43×10^{12}	29.5	5.68×10^{26}	5.85×10^{7}
Uranus	2.87×10^{12}	84.0	8.68×10^{25}	2.33×10^{7}
Neptune	4.50×10^{12}	165	1.03×10^{26}	2.21×10^{7}

*Distance from earth.

Example 13.5 Extrasolar Planets

EXAMPLE 13.5 **Extrasolar planets**

In recent years, astronomers have discovered thousands of planets orbiting nearby stars. These are called *extrasolar planets*. Suppose a planet is observed to have a 1200 day period as it orbits a star at the same distance that Jupiter is from the sun. What is the mass of the star in solar masses? (1 *solar mass* is defined to be the mass of the sun.)

SOLVE Here "day" means earth days, as used by astronomers to measure the period. Thus the planet's period in SI units is $T = 1200 \text{ days} = 1.037 \times 10^8$ s. The orbital radius is that of Jupiter, which we can find in Table 13.2 to be $r = 7.78 \times 10^{11}$ m. Solving Equation 13.25 for the mass of the star gives

$$M = \frac{4\pi^2 r^3}{GT^2} = 2.59 \times 10^{31} \text{ kg} \times \frac{1 \text{ solar mass}}{1.99 \times 10^{30} \text{ kg}}$$

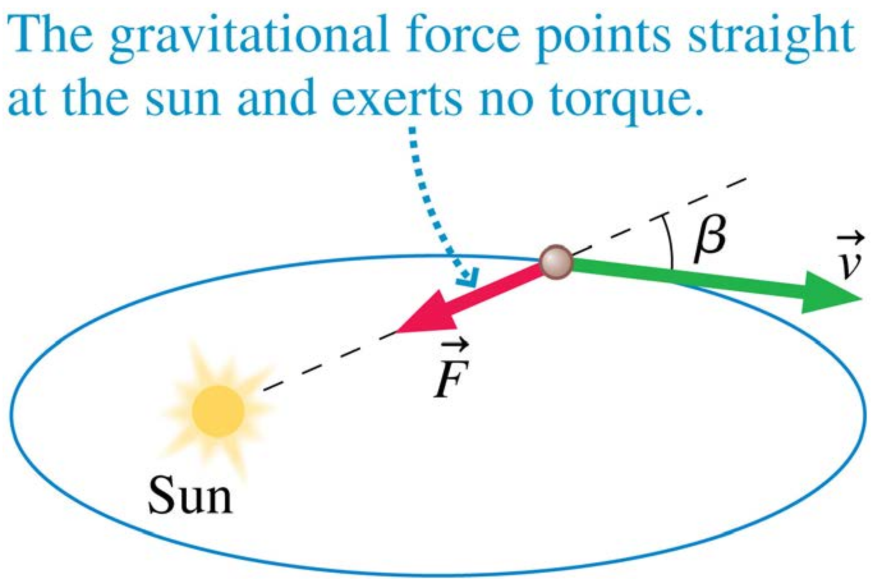
$$= 13 \text{ solar masses}$$

ASSESS This is a large, but not extraordinary, star.

Recall: Kepler's Second Law of Planetary Motion

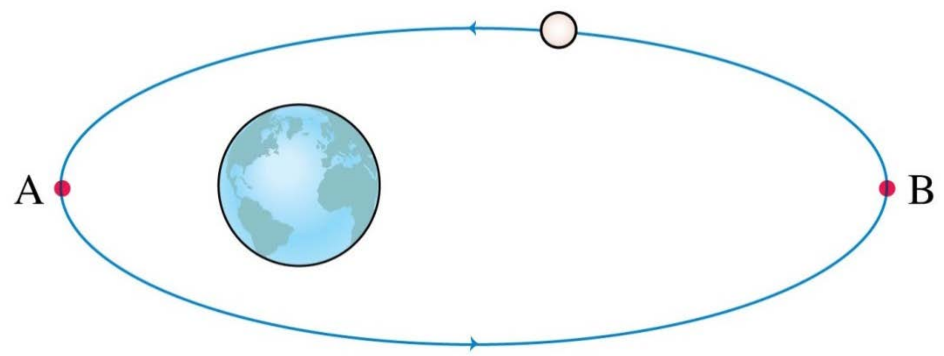
2. A line drawn between the sun and a planet sweeps out equal areas during equal intervals of time.

- The gravitational force exerts no torque, so the satellite's angular momentum, L, is conserved as it orbits.
- It can be shown that the area swept out per unit time is $L/2m$, which is constant for any satellite.



QuickCheck 13.9

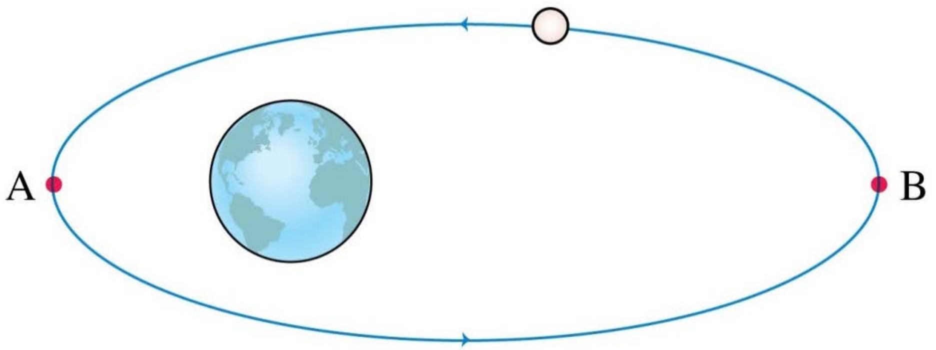
A satellite has an elliptical orbit with the earth at one focus. Which statement is true about the satellite's speed at point A and point B?



A. $v_A > v_B$
B. $v_A = v_B$
C. $v_A < v_B$

QuickCheck 13.9

A satellite has an elliptical orbit with the earth at one focus. Which statement is true about the satellite's speed at point A and point B?



A. $\boldsymbol{v_A > v_B}$

B. $v_A = v_B$

C. $v_A < v_B$

Orbital Energetics

- We know that for a satellite in a circular orbit, its speed is related to the size of its orbit by $v^2 = GM/r$. The satellite's kinetic energy is thus

$$K = \frac{1}{2}mv^2 = \frac{GMm}{2r}$$

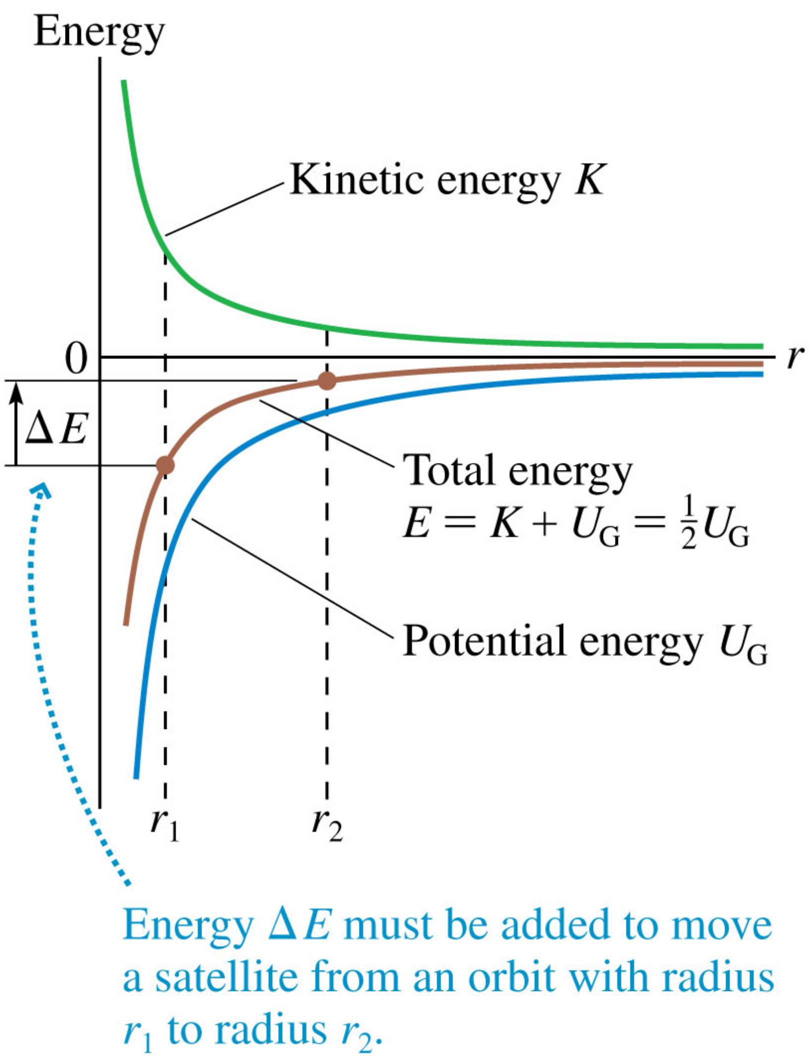
- But $-GMm/r$ is the potential energy, U_G, so

$$K = -\tfrac{1}{2}U_G$$

- If K and U do not have this relationship, then the trajectory will be elliptical rather than circular. So, the mechanical energy of a satellite in a circular orbit is always

$$E_{mech} = K + U_G = \tfrac{1}{2}U_G$$

Orbital Energetics



- The figure shows the kinetic, potential, and total energy of a satellite in a *circular orbit*.
- Notice how, for a circular orbit, $E_{mech} = \frac{1}{2} U_G$
- It requires positive energy in order to lift a satellite into a higher orbit.

Example 13.6 Raising a Satellite

EXAMPLE 13.6 **Raising a satellite**

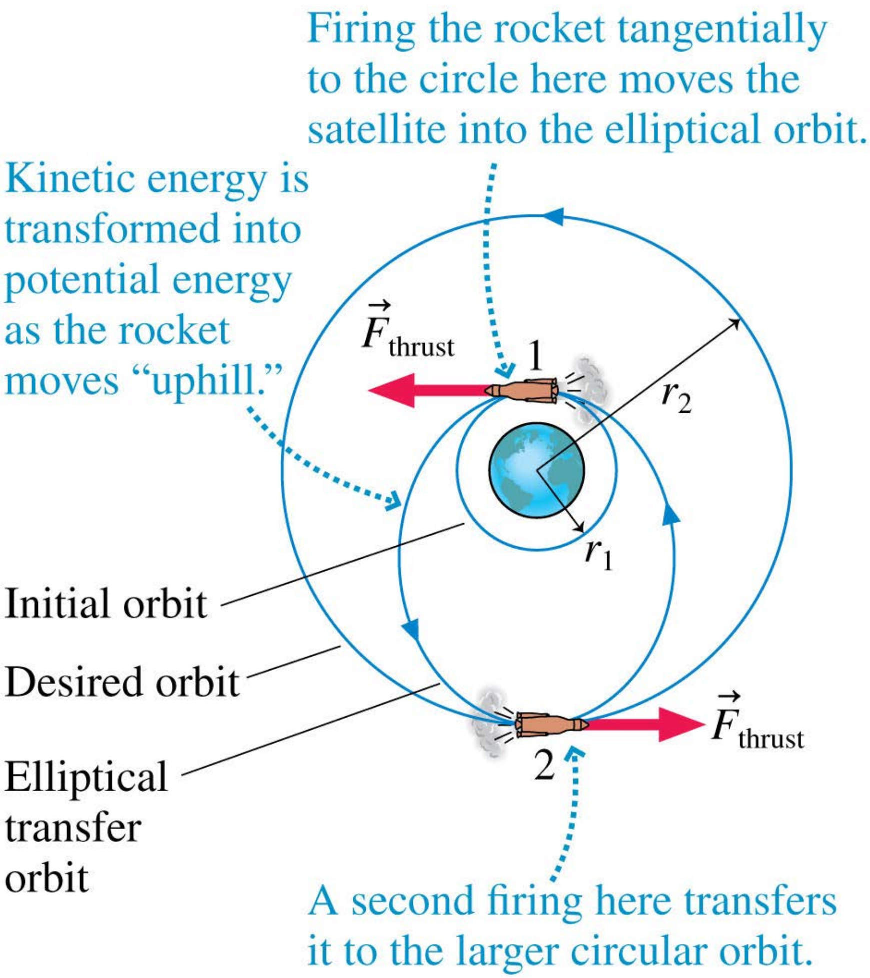
How much work must be done to boost a 1000 kg communications satellite from a low earth orbit with $h = 300$ km to a geosynchronous orbit?

SOLVE The required work is $W_{\text{ext}} = \Delta E_{\text{mech}}$, and from Equation 13.31 we see that $\Delta E_{\text{mech}} = \frac{1}{2}\Delta U_{\text{G}}$. The initial orbit has radius $r_{\text{low}} = R_{\text{e}} + h = 6.67 \times 10^6$ m. We earlier found the radius of a geosynchronous orbit to be 4.22×10^7 m. Thus

$$W_{\text{ext}} = \Delta E_{\text{mech}} = \tfrac{1}{2}\Delta U_{\text{G}} = \tfrac{1}{2}(-GM_{\text{e}}m)\left(\frac{1}{r_{\text{geo}}} - \frac{1}{r_{\text{low}}}\right) = 2.52 \times 10^{10}\ \text{J}$$

ASSESS It takes a lot of energy to boost satellites to high orbits!

Orbital Energetics



- The figure shows the steps involved to lift a satellite to a higher circular orbit.
- The first kick increases K without increasing U_G, so K is not $-\frac{1}{2}U_G$, and the orbit is elliptical.
- The satellite then slows down as r increases.
- The second kick increases K again so that $K = -\frac{1}{2}U_G$, and the orbit is circular.

Chapter 13 Summary Slides

General Principles

Newton's Theory of Gravity

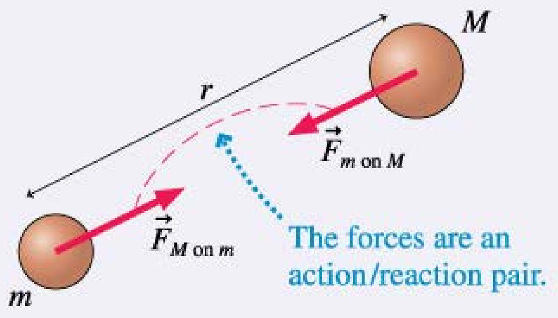
1. Two objects with masses M and m a distance r apart exert attractive **gravitational forces** on each other of magnitude

$$F_{M \text{ on } m} = F_{m \text{ on } M} = \frac{GMm}{r^2}$$

where the **gravitational constant** is $G = 6.67 \times 10^{-11}\ \text{N m}^2/\text{kg}^2$.

Gravity is an **inverse-square** force.

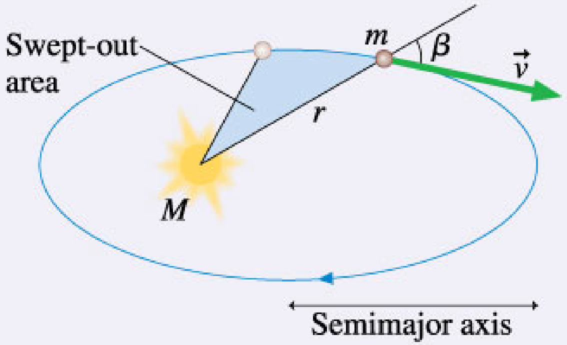
2. Gravitational mass and inertial mass are equivalent.
3. Newton's three laws of motion apply to all objects in the universe.



Important Concepts

Orbital motion of a planet (or satellite) is described by **Kepler's laws:**

1. Orbits are ellipses with the sun (or planet) at one focus.
2. A line between the sun and the planet sweeps out equal areas during equal intervals of time.
3. The square of the planet's period T is proportional to the cube of the orbit's semimajor axis.



Circular orbits are a special case of an ellipse. For a circular orbit around a mass M,

$$v = \sqrt{\frac{GM}{r}} \qquad \text{and} \qquad T^2 = \left(\frac{4\pi^2}{GM}\right)r^3$$

Important Concepts

Conservation of angular momentum

The angular momentum $L = mrv \sin \beta$ remains constant throughout the orbit. Kepler's second law is a consequence of this law.

Important Concepts

Orbital energetics

A satellite's mechanical energy $E_{\text{mech}} = K + U_G$ is conserved, where the gravitational potential energy is

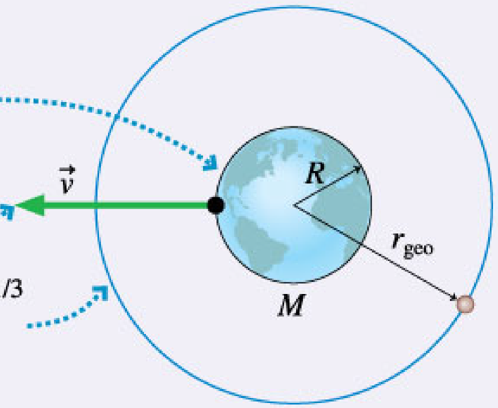
$$U_G = -\frac{GMm}{r}$$

For circular orbits, $K = -\frac{1}{2}U_G$ and $E_{\text{mech}} = \frac{1}{2}U_G$. Negative total energy is characteristic of a **bound system.**

Applications

For a planet of mass M and radius R:

- Free-fall acceleration on the surface is $g_{\text{surface}} = \dfrac{GM}{R^2}$
- **Escape speed** is $v_{\text{escape}} = \sqrt{\dfrac{2GM}{R}}$
- Radius of a **geosynchronous orbit** is $r_{\text{geo}} = \left(\dfrac{GM}{4\pi^2}T^2\right)^{1/3}$



Applications

To move a satellite to a larger orbit:

- Forward thrust to move to an elliptical transfer orbit
- A second forward thrust to move to the new circular orbit