

Chapter 13 non calculus

Speed of Wave Pulses

In our introduction to special relativity in Chapter 1, we showed how to measure the speed of a wave pulse on a stretched Slinky™ using the setup shown in Figure (1-7). We also pointed out that we can derive formulas like $v = \sqrt{T/\mu}$ for the speed of the pulse down the Slinky. And we showed how we could use the formula to tell whether or not we were moving relative to the Slinky. If, for example, we got a speed greater than $\sqrt{T/\mu}$ that would imply that we were moving by the Slinky toward the pulse as shown in Figure (1-8a).

This of course is a clumsy way to detect our motion relative to the Slinky. The obvious way is to simply look at the Slinky itself. But we were preparing for an example where we could see the wave, but could not see the medium through which it was moving. The wave was a light wave, and the formula $v = \sqrt{1/\mu_0\epsilon_0}$ for its speed was a result of Maxwell's theory of light. Since light moves through empty space, any measurement of the speed of a light pulse past us, that allowed us to detect our motion relative to empty space, would violate the principle of relativity.

Einstein's special theory of relativity was explicitly designed so that any measurement of the speed of a light in a vacuum would always give the same answer $v = \sqrt{1/\mu_0\epsilon_0}$ and therefore tells us nothing about our motion through empty space.

In Chapter One we promised to derive the formula for the speed of a wave pulse on a Slinky. This chapter is where we keep that promise. In fact, we will derive two formulas, because there are two different kinds of wave pulses that can travel down the Slinky. One is called a **transverse wave** where the coils in the pulse move sideways. The other, a **compressional pulse**, is where the coils squeeze together.

The compressional wave on a Slinky is a good model for a **sound wave**, thus our derivation of the speed of that wave should apply to the speed of sound waves in other elastic media that obey Hooke's law. We will test the formula not only with a Slinky, but also with a stretched steel guitar string and a steel rod. The formula work equally well in each case.

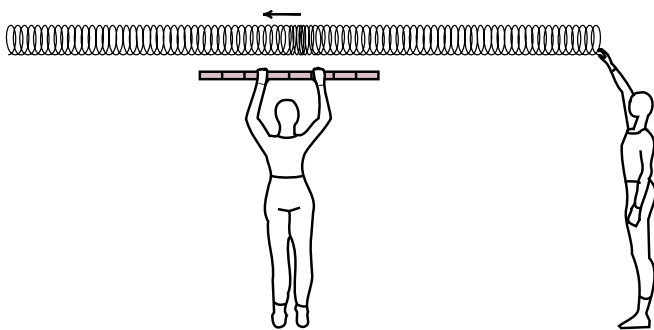


Figure 1-7
Experiment to measure the speed of a wave pulse on a suspended Slinky. Here the instructor holds the meter stick at rest.

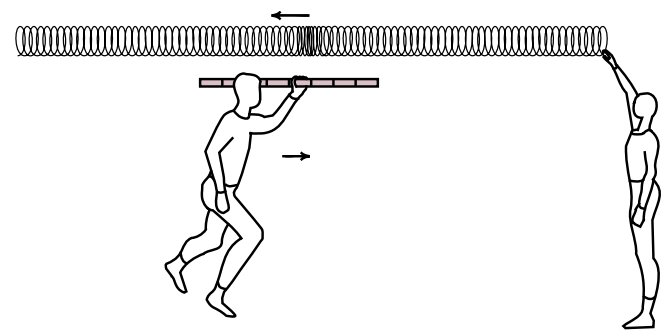


Figure 1-8a
Bill runs toward the source of the pulse while measuring its speed.

SLINKY WAVE PULSES

As we mentioned in the introduction, we can send two kinds of wave pulses down our horizontally suspended Slinky. If you click on Figure (1), you get a movie showing these two waves. The movie first shows the stretched Slinky suspended by threads attached to every fourth coil. The other ends of the threads are each attached to washers which slide along a horizontal pipe, allowing us to change the length of the suspended Slinky. Figure (2) shows how we store the Slinky, threads, and washers, when not in use.

If the CD with the movie is not available, you can see the structure of the pulses in video frames of Figures (3) and (4). Figure (3) shows a fairly long transverse pulse traveling most of the way down the Slinky. The frames are 1/10 of a second apart, and the wave was created by flicking the end of the Slinky side-ways.

In Figure (4) we see the much shorter compressional pulse which was created by pulling the end of the Slinky back a few inches and letting go. We could

make much more accurate timing measurements on the short compressional pulse than the long transverse one. To the accuracy we could time the transverse wave, we found that both pulses took the same time to go down the Slinky, bounce off the other end, and come back.

We will study another example, namely a stretched steel guitar string, where the transverse and compressional waves travel at very different speeds. When we study the formulas for these wave speeds, we will easily see why the wave speeds on the Slinky are essentially the same.

Another feature is that the farther we stretch the Slinky, the faster the compressional wave travels. This leads to a surprising prediction which we will discuss shortly.

Figure 2
Slinky and washers in storage.

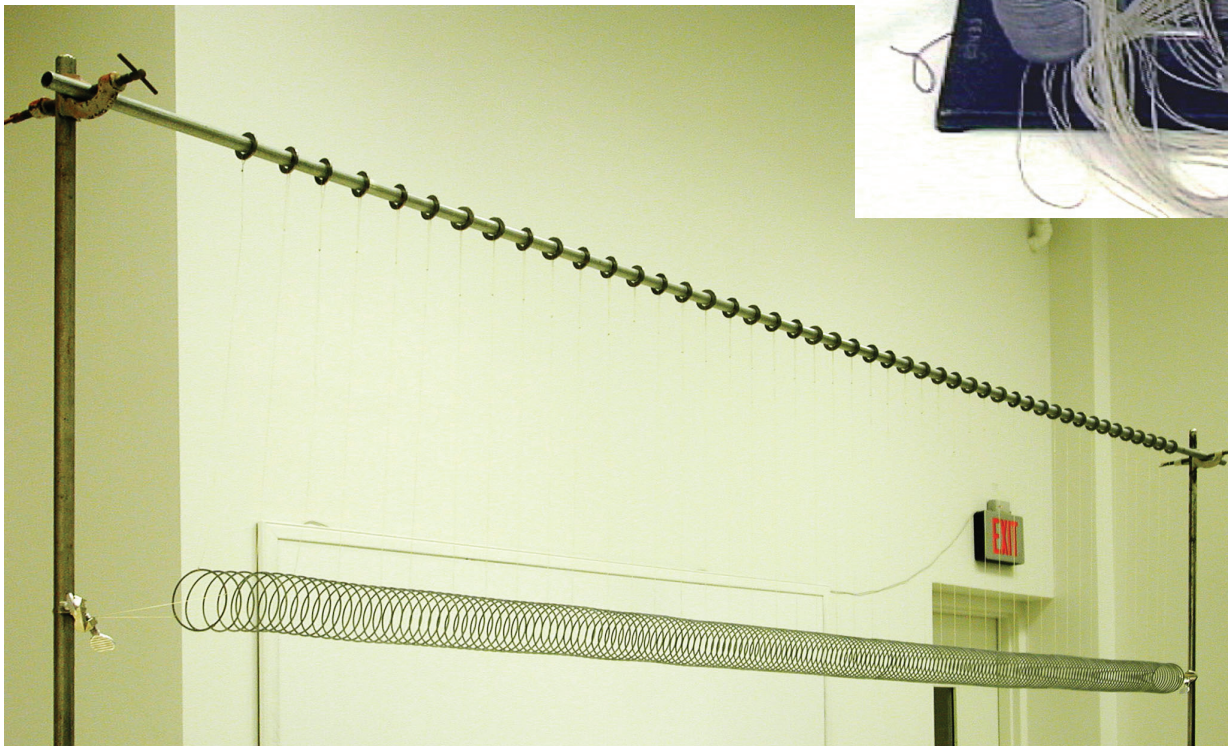
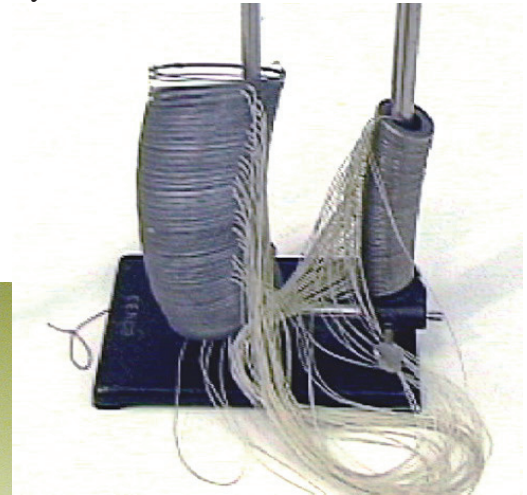


Figure 1
Stretched horizontal Slinky™ for demonstration of both transverse and compressional wave pulses.

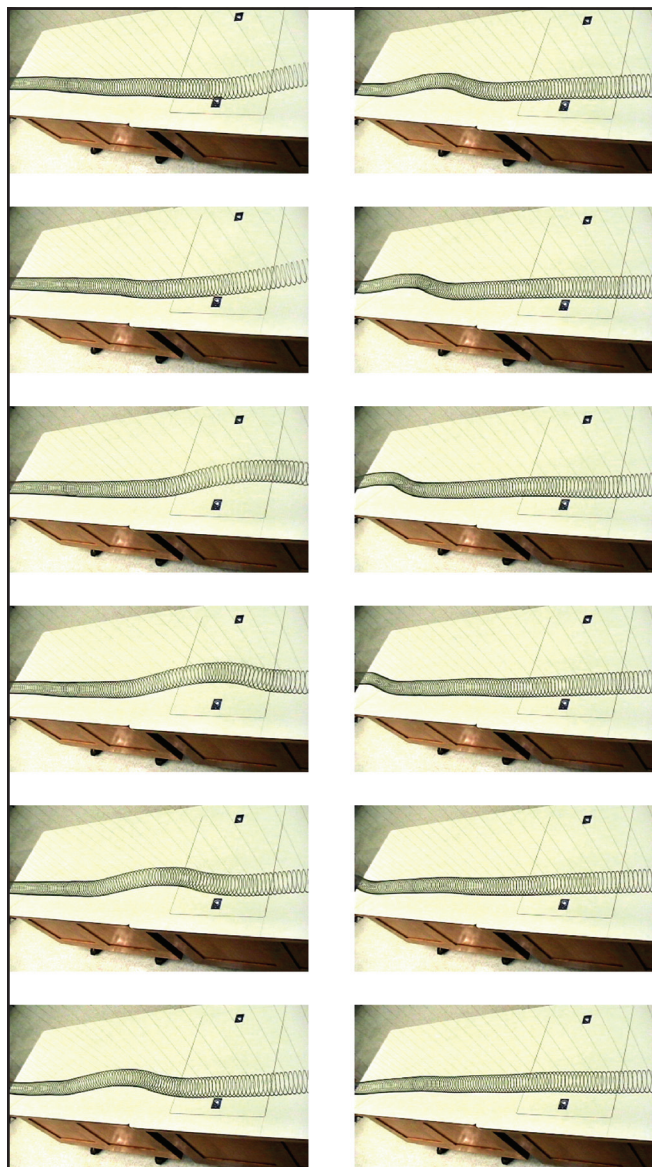


Figure 3
Transverse (sideways) wave pulse. These frames show a fairly long transverse pulse traveling most of the way down the Slinky. The frames are 1/10 of a second apart, and the wave was created by flicking the end of the Slinky sideways.

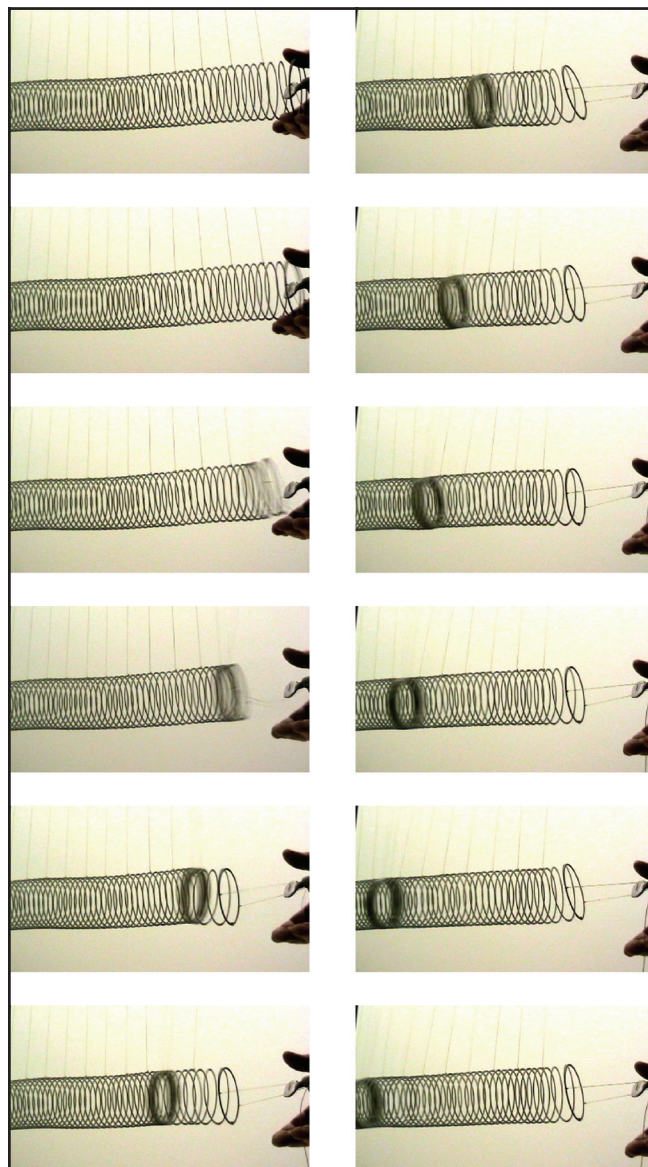


Figure 4
Compressional wave pulse. We see the much shorter compressional pulse which was created by pulling the end of the Slinky back a few inches and letting go.

HOOKE'S LAW FOR THE SLINKY

Our formulas for the speed of waves on the Slinky are derived from Hooke's law in the form [see Chapter 9, page 9-8]

$$F = T = K(L - L_0) \quad \text{also (9-14)} \quad (1)$$

where F is the force required to stretch the Slinky to a length L . It is also the tension T in the stretched Slinky. In Equation (1), L_0 is the unstretched length and K is the spring constant.

We graphed the force required to stretch the Slinky to four different lengths and got the result shown in Figure (5). From this graph we got a spring constant

$$K = \frac{F}{L - L_0} = .266 \frac{\text{newtons}}{\text{meter}} \quad (1a)$$

and the unstretched length turned out to be

$$L_0 = -.071 \text{ meters} \quad (1b)$$

The negative value of L_0 simply means that the Slinky coil was wound so tightly that if the coils had no thickness, some force would be required to stretch the Slinky to zero length.

In making the measurements for Figure (5), at each length L we had to make sure that the threads supporting the Slinky were straight up. We did not do this at first, and started to get peculiar results because the threads were pulling back on the coils.

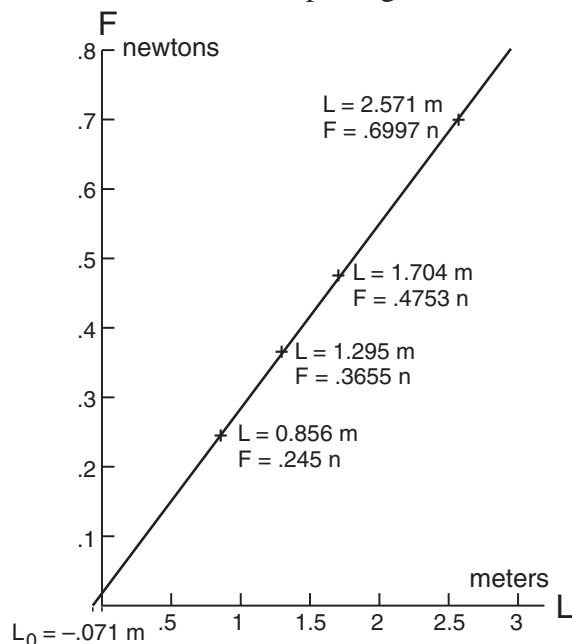


Figure 5
Measuring the force required to stretch the Slinky.

Exercise 1

Using the experimental values for one of the four points in Figure (5), and the Hooke's law Equation (1), calculate the Slinky's spring constant K . Then do the same for another point and compare your two values for K . (Remember L_0 is negative.)

WAVE SPEED FORMULAS

Using Hooke's law, we will derive the following formulas for the speeds v_T for transverse waves and v_C for compressional waves. The formulas are

$$v_T = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{K(L - L_0)}{\mu}} \quad \text{transverse wave} \quad (2)$$

$$v_C = \sqrt{\frac{KL}{\mu}} \quad \text{compressional wave} \quad (3)$$

where μ is the mass per unit length. Our Slinky had a mass

$$M = .510 \text{ kg} \quad (4)$$

thus μ is given by

$$\mu = \frac{M}{L} = \frac{.510 \text{ kg}}{L \text{ meter}} \quad (5)$$

and we see that μ changes as we stretch the Slinky.

Shortly we will derive the formula $v_T = \sqrt{T/\mu}$ for the speed of the transverse wave. There is a trick that makes this derivation quite easy to do. It is a derivation that is found in almost all introductory physics texts.

As far as we could find, the formula, $v_C = \sqrt{KL/\mu}$ for compressional waves is correctly derived only in relatively advanced textbooks on wave motion. We did not even provide a good derivation of the formula in our calculus based version of this text. We finally worked out a fairly simple and accurate non calculus derivation of v_C that we discuss after deriving v_T .

Before we go into these derivations, let us look at some of the consequences of these formulas. The special feature of a Slinky is that you can stretch it to great lengths L while its unstretched length L_0 is very small. (If you did Exercise 1, you would have seen that including L_0 made almost no difference.) If you neglect L_0 , the formulas for v_T and v_C are the same, which is why the two wave speeds are nearly equal.

Here is the second feature which came to us as quite a surprise. First express the formula $v_C = \sqrt{KL/\mu}$ in terms of the Slinky mass M rather than its mass per unit length μ . With $\mu = M/L$ we get

$$v_C = \sqrt{\frac{KL}{M/L}} = \sqrt{\frac{KL^2}{M}} = L\sqrt{\frac{K}{M}} \quad (6)$$

Now calculate the total time T_C it takes a compressional pulse to go down and back, a total distance of $2L$. That time is

$$T_C = \frac{2L}{v_C} \frac{m}{m/sec} = \frac{2L}{v_C} \text{ sec} \quad (7)$$

Using Equation (6) for v_C gives

$$T_C = \frac{2L}{v_C} = \frac{2L}{L\sqrt{K/M}}$$

The factors of L cancel and we are left with

$$\boxed{T_C = 2\sqrt{\frac{M}{K}}} \text{ time for compressional pulse to go down and back} \quad (8)$$

Exercise 2

The mass of our Slinky was $m=.510$ kg. Calculate the predicted length of time a compressional pulse should take to go down and back.

The cancellation of L in Equation (8) implies that the time the compressional wave takes to go down and back does not depend on how far we stretch the Slinky. If we stretch the Slinky farther, the pulse speeds up to compensate for the extra distance it has to travel.

When we first derived the formula $v_C = \sqrt{KL/\mu}$ and saw this prediction, we did not believe it, and wondered if we had derived the correct formula. We took a video camera into the lab and photographed the compressional pulse traveling down and back for four different distances. The distances traveled, and the times required, expressed in number N of video frames and in seconds are

L meters	N frames	T_C sec
.86 m	85	2.83 s
1.29 m	85	2.83 s
1.76 m	86	2.87 s
2.60 m	86	2.87 s

(9)

We stretched the Slinky from .86 meters to 2.60 meters, changing its length by a factor of 3, and we changed the measured time by only one video frame of 1/30 sec. And our experimental error was at least \pm one video frame. That was a really successful theoretical prediction.

The four Slinky lengths we used are seen in the video frames of Figure (6). If you click on any of these pictures you get to see a QuickTime™ movie of the motion of the compressional pulses. (The movie itself is compressed, showing only every other frame for a time spacing of 1/15 sec.)

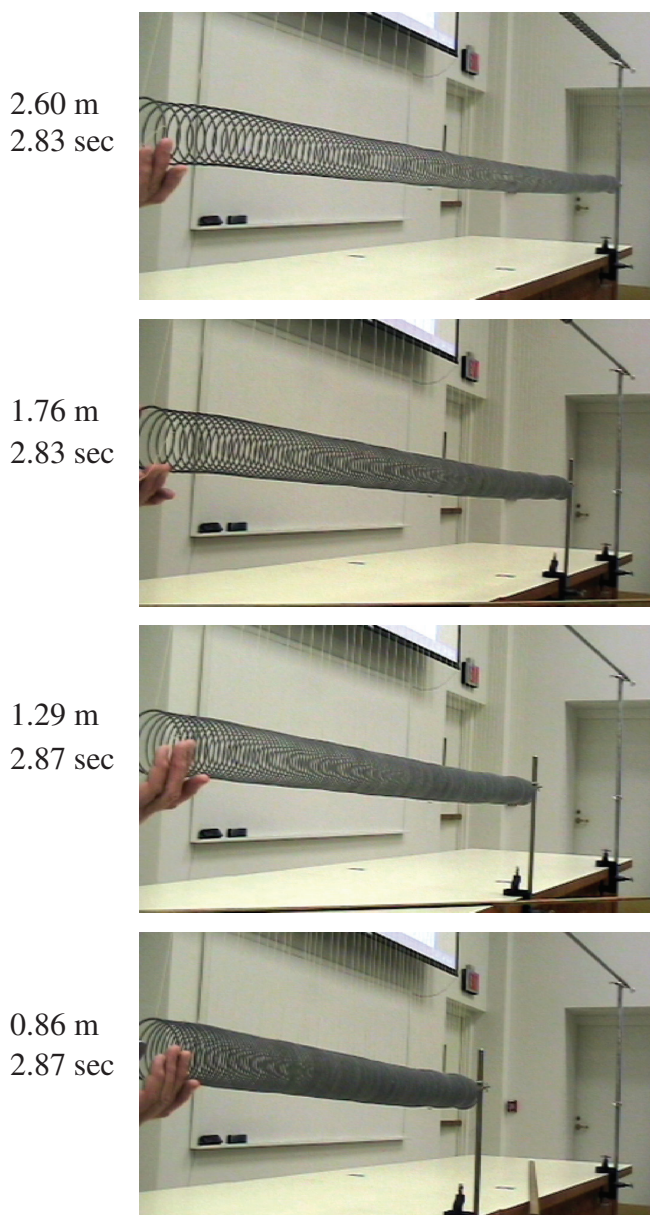


Figure 6 (movie)
Frames from movie on compressional waves in Slinky.

TRANSVERSE WAVE SPEED DERIVATION

We are now ready to begin deriving our wave speed formulas. Both formulas result from combining Hooke’s force law with Newton’s second law. Because the transverse wave speed formula $v_T = \sqrt{T/\mu}$ has a much shorter derivation, we include that fully in the main text. In the next section we will outline the derivation of the formula $v_C = \sqrt{KL/\mu}$ and put the details in an appendix.

In a lecture, you get a better demonstration of a transverse wave pulse using a stretched rubber rope than the stretched Slinky. That is because you can flip the end of the rope up and down to make a more visible vertical pulse, and you can use a longer rope to see the pulse travel farther. But with either the rope or Slinky you get the same formula $v_T = \sqrt{T/\mu}$.

To calculate the transverse wave speed, consider a wave pulse moving down a rope at velocity \vec{v} as shown in Figure (7a). To analyze the pulse, imagine that you are running along with the pulse at the same velocity \vec{v} . From your point of view, shown in Figure (7b), the pulse is at rest and the rope is moving back through the pulse at a speed v .

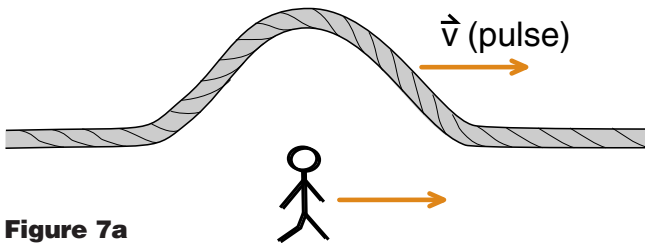


Figure 7a
Wave pulse, and an observer, moving to the right at a speed v .

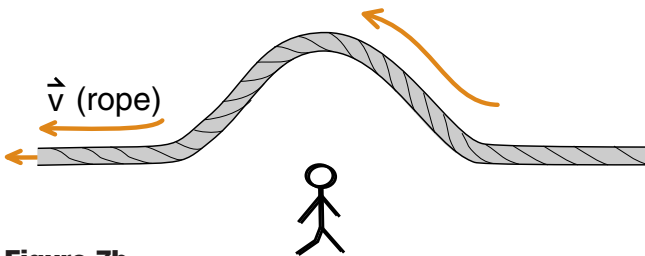


Figure 7b
From the moving observer’s point of view, the pulse is stationary and the rope is moving through the pulse at a speed v .

Now look at the top of the wave pulse. For any reasonably shaped pulse, its top will be circular, fitting around a circle of radius r as shown in Figure (7c). This radius r is also called the *radius of curvature* of the rope at the top of the pulse.

Finally consider a short piece of rope of length ℓ at the top of the pulse as shown in Figure (7d). If this piece of rope subtends an angle 2θ on the circle, as shown, then $\ell = 2r\theta$ and the mass m of this section of rope is

$$m = \mu\ell = \mu 2r\theta \quad \begin{matrix} \text{mass of short} \\ \text{section of rope} \end{matrix} \quad (10)$$

where μ is the mass per unit length of the rope.

The net force on this piece of rope is caused by the tension T in the rope. As seen in Figure (7d), the ends of the piece of rope point down at an angle θ . Thus the tension at each end has a downward component $T \sin(\theta)$ for a total downward force F_y of magnitude

$$F_y = 2T \sin(\theta) \approx 2T\theta \quad \begin{matrix} \text{downward} \\ \text{component of} \\ \text{tension force} \end{matrix} \quad (11)$$

If we keep the angle θ small and just look at a very small section of the rope, we can approximate $\sin(\theta)$ by θ , which we did in Equation (11).

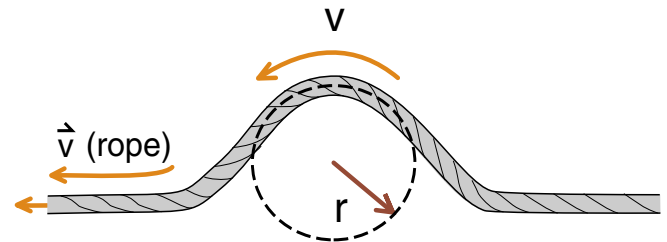


Figure 7c
Assume that the top of the pulse fits over a circle of radius r .

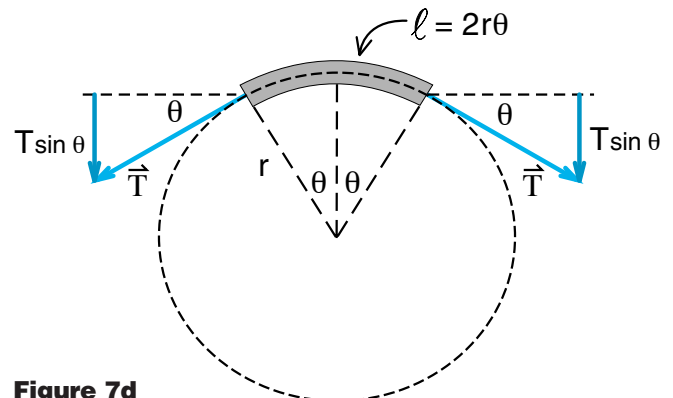


Figure 7d
The ends of the rope point down at an angle θ , giving a net restoring force $F_y = 2T \sin \theta$.

The final step is to note that this section of rope is moving at a speed v around a circle of radius r . Thus we know its acceleration; it is accelerating downward, **toward the center of the circle**, with a magnitude v^2/r .

$$a_y = \frac{v^2}{r} \quad \begin{array}{l} \text{downward} \\ \text{acceleration of} \\ \text{section of rope} \end{array} \quad (12)$$

Applying Newton's second law to the downward component of the motion of the section of rope, we get using Equations (1), (2) and (3)

$$F_y = ma_y$$

$$2T\theta = (\mu 2r\theta) \frac{v^2}{r} \quad (13)$$

Both the variables r and θ cancel, and we are left with

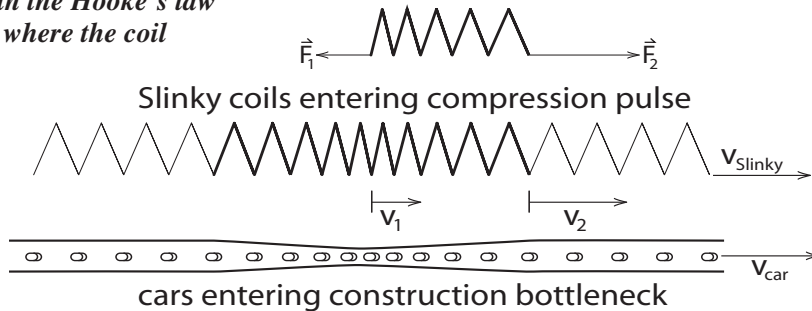
$$T = \mu v^2$$

$v = \sqrt{\frac{T}{\mu}}$

speed of a wave pulse on a rope with tension T , mass per unit length μ (14)

A result we stated back in Chapter 1.

Figure 8
The point of view that the Slinky is moving through a stationary pulse resembles the situation of cars moving through a construction bottleneck. Cars slow down as they enter the bottleneck and speed up as they leave. Coils slow down as they enter the pulse, and accelerate as they leave. This acceleration is caused by the difference ($F_1 - F_2$) in the Hooke's law force. (F_1 is weaker where the coil spacing is less.)



SPEED OF COMPRESSIONAL WAVES

For the derivation of the speed v_C of a compressional pulse in a Slinky, we also walk along with the pulse so that we see a stationary pulse with the Slinky coils moving through it as shown in Figure (8). Inside the pulse the Slinky coils are closer together. To see that the coils must also be moving slower inside the pulse than outside it, we thought of an analogy to the flow of cars in a highway construction bottleneck.

As you approach such a construction site, you have to slow down, and endure bumper to bumper traffic. When you get past the bottleneck, you can speed up (accelerate), and then resume driving at your normal speed. The same thing happens to the Slinky coils. As they approach the pulse, they have to slow down and come closer together. Once through the pulse they speed up (accelerate) and resume their normal spacing and speed.

As shown in Figure (8), the acceleration out of the pulse is caused by the difference in the Hooke's law force on the closely spaced coils inside the pulse and the more stretched apart coils outside the pulse. In the appendix we relate the acceleration of the coils out of the pulse to this net Hooke's law force to obtain the formula $v_C = \sqrt{KL/\mu}$. (The dependence on L_0 cancels when we calculate the difference in the Hooke's law forces.)

SPEED OF SOUND IN STEEL

Watching the compressional pulse bounce back and forth in the stretched Slinky gave us an idea for an experiment that might allow us to measure the speed of a sound pulse in steel. We found a section of steel electrical conduit pipe about as long as our stretched Slinky, 10 ft or 3.048 meters long. The idea was that if we hit one end of the pipe with a hammer, a compressional pulse should travel down the pipe and bounce back and forth like the Slinky pulse. If we taped a microphone to the other end of the pipe, it might be able to detect (hear) the bouncing sound pulse.

We did a bit of checking to see if this would be a reasonable experiment. Textbooks give a value of about 5,000 meters per second for the speed of sound in steel. That means that a sound pulse traveling down and back, a total distance of 6 meters, should take a time of approximately

$$t = \frac{6 \text{ meters}}{5000 \text{ meters/sec}} = 1.2 \times 10^{-3} \text{ sec} \quad (15)$$

Using the name millisecond (ms) for 10^{-3} seconds, we have a prediction that the bounce time should be 1.2 ms. That is too short a time to observe directly, but turns out to be an easy time to measure using an oscilloscope.

About MacScope II

Before we discuss the details of our experiment, we need to say something about the oscilloscope we will be using. We have written the computer program *MacScope II* that turns any USB equipped computer into an easy-to-use, dual beam, self triggered storage oscilloscope that handles the audio frequency range. Do not worry about all these names, we mention them for those who are familiar with oscilloscopes. The rest of the discussion is also for those who have not worked with an oscilloscope.

The MacScope program is on the *Physics2000 CD*, and the latest version is available, as a free download, from the website www.physics2000.com. The latest version of the instruction manual is also freely available at the same website.

A main use of MacScope is to study sound waves. In such an application you plug a microphone either into your computer if it has a microphone input, or into a Griffin iMic™ audio adaptor which sends the audio signal into the computer via the USB port. Figure (9) shows an \$8 Wal-Mart microphone plugged into a \$40 Griffin iMic which is then plugged into the USB port of our portable computer. This setup makes a fairly handy portable oscilloscope.

The microphone converts the pressure vibrations of a sound wave into a voltage signal which MacScope plots on the computer screen. The time scales that MacScope can display range from about 1/10,000 of a second (.1 milliseconds) up to about .1 second (100 milliseconds), which allows it to plot frequencies in the range of 20 cycles per second (Hz) up to 20,000 cycles per second. This is the frequency range the ear can hear, and an oscilloscope that works in this range is called an *audio oscilloscope*.

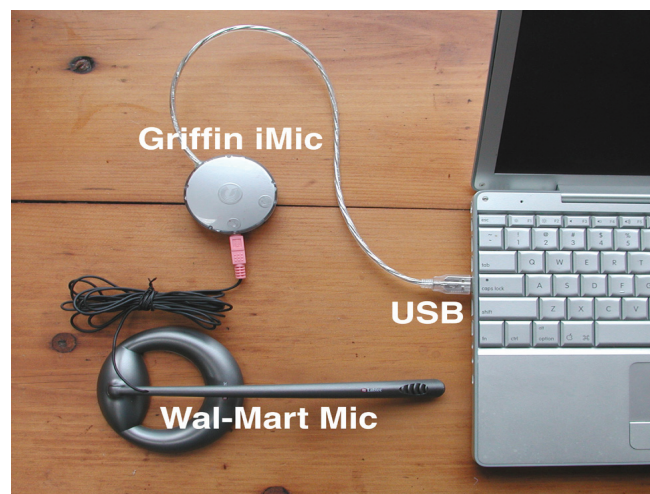


Figure 9
This is the oscilloscope setup we will use in a number of experiments. We have an \$8 Wal-Mart microphone plugged into a \$40 Griffin iMic which is then plugged into the USB port of our portable computer. The computer is running the MacScope II oscilloscope program (works on Mac & Windows) which is available without charge at www.physics2000.com.

The Experiment

As shown in Figure (10), we taped the Wal-Mart microphone to one end of our steel pipe and adjusted the MacScope sound input following the steps in the instruction manual. We then hit the other end of the pipe with a hammer, and after several tries adjusting the strength of our hammer blow, we got curves like Figure (11).

The point of Figure (11) is that we most likely do see a series of cycles of a pulse bouncing back and forth down the pipe. We get a better feeling for the data by looking at it at several different time scales. In Figure (10a) which spans 20 milliseconds, we see 16 cycles representing 16 round trips of the sound pulse.

In Figure (10b) we expanded the scale to a range of 10 ms and selected 8 cycles. When we select data, the selected curve thickens and a data window appears telling us the length of our selection. We see that 8 cycles took 9.48 ms for a time

$$t = \frac{9.48 \text{ ms}}{8 \text{ cycle}} = 1.19 \frac{\text{milliseconds}}{\text{cycle}}$$

Since the length of the pipe is 3.048 meters (10 ft), the round trip is 6.096 meters and the speed of the pulse is

$$v_c = \frac{6.096 \text{ meters}}{1.19 \times 10^{-3} \text{ sec}} = 5120 \frac{\text{meters}}{\text{sec}} \quad (16)$$

which is in the range of the usual textbook value for the speed of sound in steel.

In Figure (10c) we expanded the time scale to 5 ms to get a better look at the shape of our sound pulse. It is more complex than the compact compressional pulse we saw on the Slinky. We will shortly describe another experiment designed to prove that this complex pulse was really a sound pulse bouncing back and forth.

In Figure (10d), we set the time scale back to 50 ms so that we can see the overall shape of the series of pulses. In this view we see that the pulses have fairly well died out by 40 ms, or some 33 round trips.



Figure 10
Microphone taped to steel pipe.

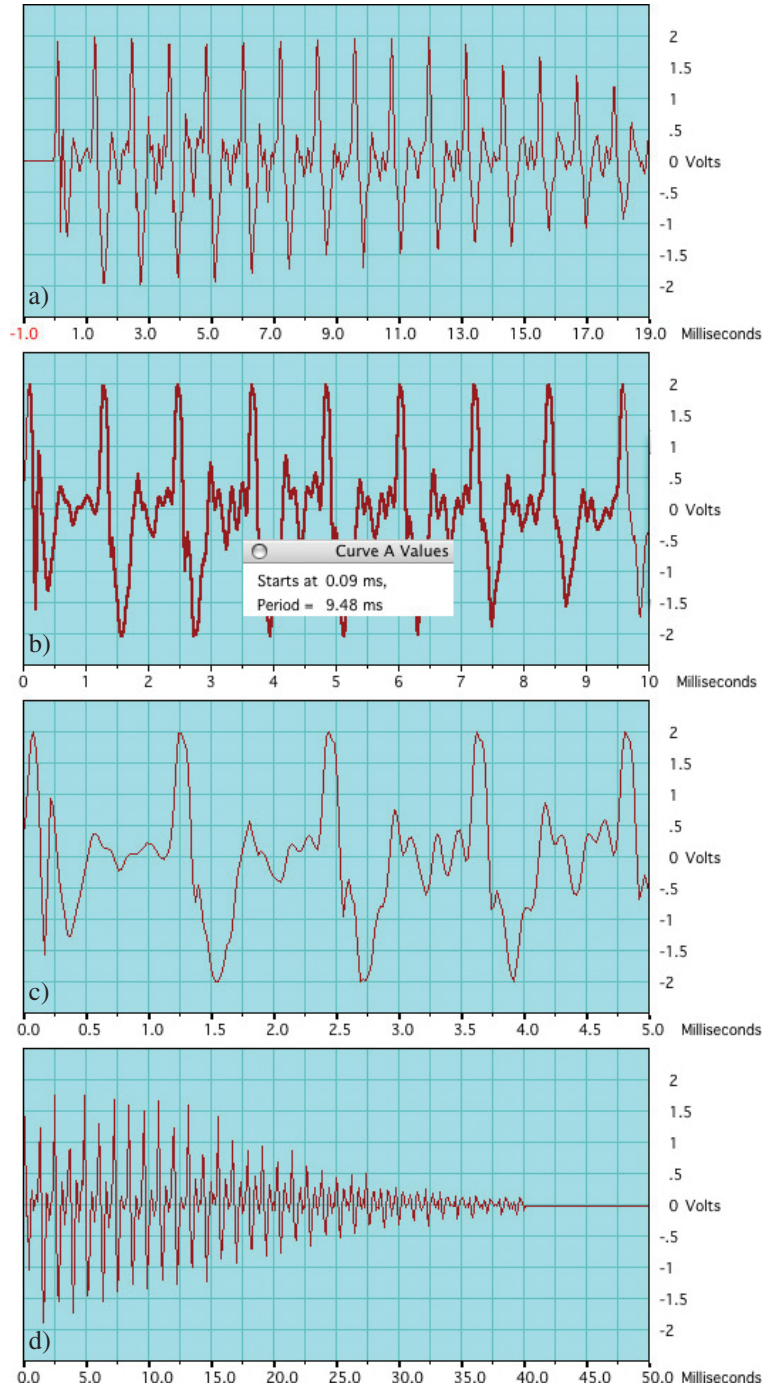


Figure 11
Sound recorded at the far end of the pipe. Looking at the data on different time scales gives a more complete picture. In (11b), we see that 8 round trips took 9.48 milliseconds, for a time of 1.19 milliseconds per trip.

Triggering

In Figure (11a), we moved the time scale back to -1 ms to show that there was no pulse before time $t = 0$. You might wonder how we managed to start recording the pulse at precisely the time $t = 0$. We did it by setting the scope to *Stop on Trigger*.

To make a trigger work, we first looked at the kind of data that was coming in before we hit the pipe. This data we call “noise” and a sample of such noise is shown in Figure (12).

Looking at the noise in Figure (12) we see that (except at the end) it never gets above 5 or 10 millivolts (1 millivolt = 10^{-3} volts). The spikes in Figure (13) go up to 2 volts, or 2000 millivolts, much higher than the noise.

To start recording spikes at $t = 0$, we set the *trigger level* at 25 millivolts (25mv) and told MacScope to *Stop on Trigger* as shown in Figure (13). What MacScope does is to look at the data but ignore it until the voltage rises to the trigger level of 25 mv. Once the voltage reaches the trigger level, MacScope records 5 more windows of data, starting at $t = 0$, and saves one window of data before $t = 0$. The data before $t = 0$ is called *pre trigger data*. Back in Figure (11a) we showed 1 ms of pre trigger data which looked flat at that voltage scale.

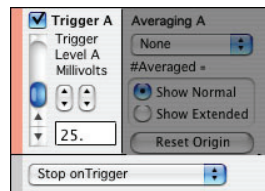


Figure 13
Trigger control.

Figure (12) is actually the pre trigger data for Figure (11), with the voltage scale magnified so that you can see the structure of the noise. Notice that at the very end of this pre trigger data, at $t = 0$, the voltage has risen to the trigger level of 25 millivolts (mV).

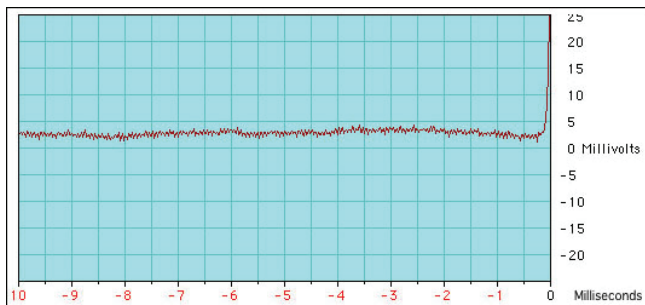


Figure 12
Pretrigger data for Figure (11). MacScope sets $t = 0$ when the voltage first reaches the trigger level.

The Time Delay

The complex structure of the steel pipe sound pulse bothered us at first. We wondered why it was so complex. Why were there smaller trailing pulses following the big pulses?

Then we remembered an important feature of mechanical switches—they always *bounce*. The bouncing of a typical electronic switch occurs over the range of milliseconds to microseconds. Circuit designers have ways of recording only the first bounce and ignoring the rest. Such a switch is called *debounced*.

We suspect that the smaller trailing pulses in Figure (11) are due to the hammer bouncing on the end of the pipe when we hit the pipe. But our suspicions do not prove that we are actually looking at a sound pulse bouncing back and forth.

A proof would be as follows. Tape a microphone to both ends of the pipe, hit one end, and show that no sound reaches the other end until a travel time of .60 milliseconds that sound should take to travel 3 meters. If we see this time delay, we know for sure that we are looking at a sound pulse.

MacScope can handle two microphones simultaneously because of the computer’s stereo input. One microphone goes into the left input, the other into the right input, and MacScope plots the two inputs in separate windows. Using somewhat old fashioned names, this makes MacScope a *dual beam* oscilloscope, and the two windows traditionally have the name *Curve A* and *Curve B*. (In the old oscilloscopes, the two curves were drawn in the same window, and you had to figure out which was which.)

We had one problem using two microphones. The Wal-Mart microphones are powered by the computer. One result is that having the computer power both microphones inevitably mixed up the signals from the two microphones. We found no way to disentangle the signals.

We fixed the problem by purchasing two battery powered mono lapel microphones from Radio Shack. Since the microphones cost \$25 each, this doubled the cost of our experiment.

Figure (14) shows the results of our two microphone experiment. We triggered both Curve A and Curve B at 25 millivolts and got the two curves shown. The upper curve (A) is from the microphone on the end we hit with a hammer. Curve B shows the pulse reaching the microphone at the far end. Here we clearly see the expected 0.60 millisecond time delay. (When both curves are triggered, the triggering of one curve sets $t = 0$ for both curves.) During this .6 ms delay the sound pulse was traveling down the pipe.

Exercise 3

Our steel pipe was 3.048 meters long and had a mass of 1.968 kg. Assuming that the pipe was behaving like a Hooke's law spring when it was hit by the hammer, find the spring constant K of the pipe. How does the pipe's spring constant compare with the one we found for the Slinky?

In Exercise (3) we asked you to find the spring constant for our steel pipe. This calculation assumes that the pipe is acting like a Hooke's law spring, at least for the kind of compression we can get by hitting the pipe with a hammer.

From our point of view the pipe does not appear to behave like a spring. It looks rigid. There is no easy way to stretch the pipe to measure its spring constant. But if it is acting like a spring, we can find its spring constant K by measuring the speed v_C of a compressional wave and using our formula $v_C = \sqrt{KL/\mu}$.

At the end of this chapter we will do an experiment where we measure the speed of a compressional wave in a stretched steel guitar string. The advantage of using the guitar string is that we can stretch it and directly measure its spring constant K .

We find that the guitar string compressional wave obeys our Hooke's law formula $v_C = \sqrt{KL/\mu}$, and this speed turns out to also be very close to the 5,000 meters per second that we got for the steel pipe. The matching of compressional wave speeds in the pipe and guitar string is good evidence that both are Hooke's law springs.

We put off our study of the steel guitar string until the end of the chapter because our measurement of the speed of the sound pulse depends upon our understanding of the nature of *standing waves*, a subject we will discuss shortly.

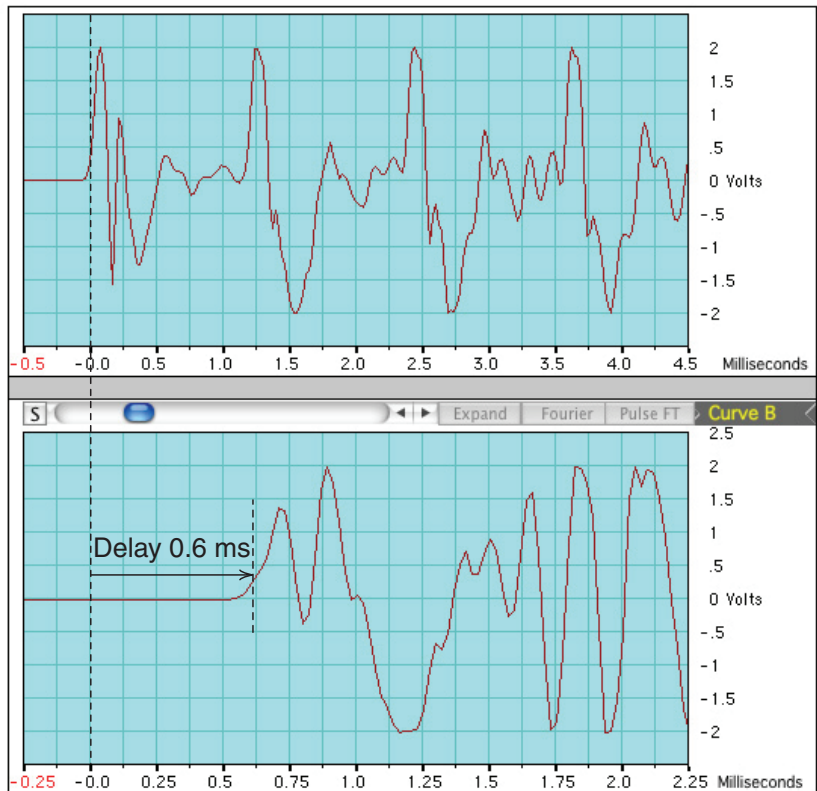
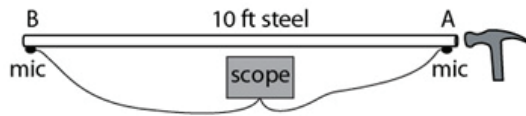


Figure 14
 The signal from the microphone at the far end, Curve B, shows the 0.6 millisecond delay as the sound travels down the pipe.

SPEED OF SOUND IN AIR

So far, our study of the speed of sound, i.e., compressional waves, has been in steel, shaped either as a Slinky or as a pipe. And the formula $v_C = \sqrt{KL/\mu}$ applied equally well for both set-ups.

A calculation that turned out to be trickier than expected was the prediction of the speed of a sound pulse in air. We can picture a sound pulse in a tube of air as a short region where the air molecules are compressed as illustrated in Figure (15). If we put a loudspeaker at the end of the air column and apply a voltage pulse to the speaker, the speaker cone will jump forward and strike the air column in much the same way that our hammer struck the steel pipe. As a result, a compressional pulse will travel down the air column as it did in the Slinky and the pipe.

We might try to directly measure the spring constant of the air column by sealing one end of the tube, putting a piston in the other end, and pushing on the piston to measure the elasticity of the air column as shown in Figure (16). From such measurements we could get a Hooke's law spring constant for the column of air, and then use our formula $v_C = \sqrt{KL/\mu}$ to predict the speed of sound pulses in air.

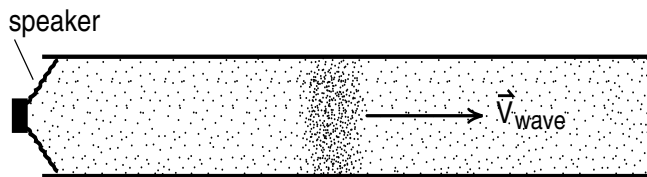


Figure 15
Compressional wave pulse traveling down through a column of air.

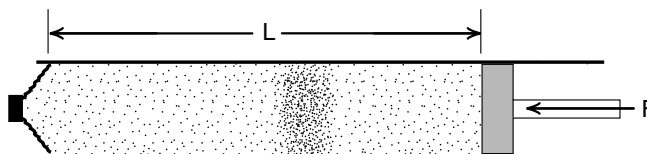


Figure 16
Measuring the spring constant of a column of air.

If you did this experiment you would get a slightly wrong answer, about 10% too low. The error would be caused by the way you measured the spring constant K .

There are two ways to measure the air column's spring constant, and they give different results. The two ways are 1) to push the piston in rapidly, and 2) push the piston in slowly. When you compress a gas rapidly, the gas heats up, which increases the pressure of the gas and makes the air column stiffer. If you push the piston in slowly there is time for the excess heat to flow out of the gas, the temperature of the gas does not rise, and the air column is less stiff. As a result you get a noticeably lower spring constant when you push the piston in slowly.

[The process where we compress slowly and keep the temperature constant is called an *isothermal* compression, and when we compress rapidly and do not let heat escape is called an *adiabatic* compression.]

Issac Newton was the first to calculate the speed of sound in air, but got a result that was about 10% too low. It was century and a quarter later that Laplace recognized the problem with Newton's calculation. Newton used the spring constant for a constant temperature (isothermal) compression. But a sound pulse travels so fast (about 300 meters per second), that there is not enough time for the heat to leak out of the pulse and one should use the fast compression (adiabatic) spring constant. When you do this you get the observed speed of sound.

SOUND SPEED IN OTHER MATERIALS

All introductory physics texts show a list of sound speeds for various materials. We will do the same. Except for the values of the speed of sound in steel, which we measured using MacScope, the other values are taken from textbooks.

Substance	Speed of Sound in meters/sec
Gases (at atmospheric pressure)	
Air at 0° C	331
Air at 20° C	343
Helium at 20° C	965
Hydrogen at 20° C	1284
Liquids	
Water at 0° C	1402
Water at 20° C	1482
Sea water at 20° C	1522
Solids	
Aluminum	6420
Steel	5100
Granite	6000
Nuclear matter	near c

Table 1

You will notice from Table 1 that the speed of sound is much greater in the light gases such as hydrogen and helium than it is in heavier gases like the oxygen and nitrogen of air. The main reason for this is that if we have two similar sized columns of gas at the same temperature and pressure, they will, quite surprisingly, have the same number of gas molecules. Thus the column with the lighter molecules will have less mass per unit length, and thus a greater speed $v_C = \sqrt{KL/\mu}$. (The spring constants K will be about the same.)

EARTHQUAKES

The main difference between a liquid and a solid is that in a liquid the molecules can slide past each other, while in a solid the molecules are held in place by molecular forces. These forces which prevent molecules from sliding past each other can also supply a transverse restoring force allowing a solid to transmit both transverse and compressional waves. An earthquake, for example, is a sudden disruption of the earth that produces both transverse waves called *S waves* and compressional (longitudinal) waves called *P waves*. These waves can easily be detected using a device called a seismograph which monitors the vibration of the earth. It turns out that the S and P waves from an earthquake travel at different speeds, and will thus arrive at a seismometer at different times. By measuring the difference in arrival time and knowing the speed of the waves, you can determine how far away the earthquake was.

Exercise 4

The typical speed of a transverse S wave through the earth is about 4.5 kilometers per second, while the compressional P wave travels nearly twice as fast, about 8.0 kilometers per second. On your seismograph, you detect two sharp pulses indicating the occurrence of an earthquake. The first pulse is from the P wave, the second from the S wave. The pulses arrive three minutes apart. How far away did the earthquake occur?

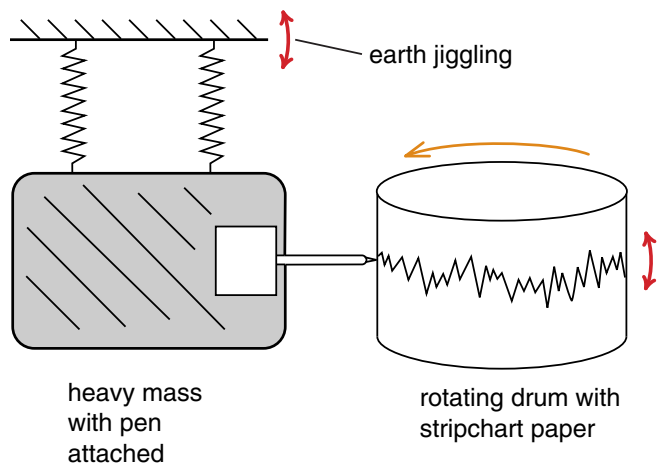


Figure 17
Sketch of a simple seismograph for detecting earthquake waves. When the earth shakes, the mass tends to remain at rest, thus the pen records the relative motion of the stationary mass and shaking earth.

APPENDIX DERIVATION OF THE SPEED OF THE COMPRESSIONAL WAVE IN A SLINKY

In the Slinky movie, we create a compressional pulse by pulling back on the end of the Slinky and letting go. The striking feature of the movie is that the pulse took the same length of time to go down and back, independent of the length L of the stretched Slinky. Here we derive the formula that predicted this result, namely

$$v_{\text{pulse}} = \sqrt{\frac{KL}{\mu}} \quad (3 \text{ repeated})$$

Figure (18) is a sketch of a compressional Slinky pulse. The coils drawn as bold lines are part of the compressional pulse, where the lighter lines represent the uncompressed part of the Slinky. We are assuming that the pulse is moving to the left at a speed v_{pulse} .

The analysis is simplified if we take the point of view that we are walking along with the pulse, so that we see a stationary pulse with the Slinky moving to the right, through the pulse, as shown in Figure (19). The uncompressed part of the Slinky is moving to the right at a speed v_{Slinky} which is opposite to v_{me} of Figure (18).

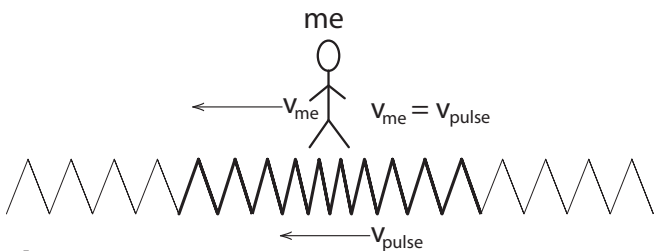


Figure 18
Compressional pulse in a Slinky moving to the left at a speed v_{pulse} . I am walking along with the pulse.

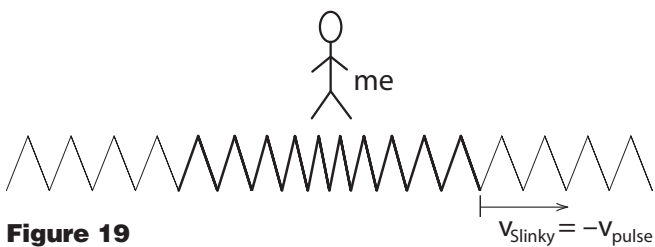


Figure 19
I see a stationary pulse with the Slinky coils moving through it.

What may seem surprising is that the more closely spaced coils inside the pulse are traveling a bit slower than the coils outside the pulse. But this is actually a familiar situation. If you are driving down a highway and come to a construction site, the cars slow down and come closer together. After you get through the construction site, the cars speed up and their spacing increases. In an aerial view of the highway, the compression of the car spacing at the construction site would look much like our compressional pulse on the Slinky, as shown in Figure (20).

As the cars approach the construction site they have to slow down, i.e., decelerate. Once through the site, they have to accelerate to get back up to speed. By analogy, the Slinky coils entering the pulse have to slow down, and then accelerate again once through the pulse. It is the Hooke's law spring forces that cause the coils to decelerate and then accelerate.

Our analysis will focus on the coils leaving the pulse, the coils that have to speed up from the slower speed v_1 at the center of the pulse to the higher speed v_2 of the coils outside. We will calculate the acceleration of these coils, the spring forces causing this acceleration, and then use Newton's second law to find the formula for the speed of the pulse.

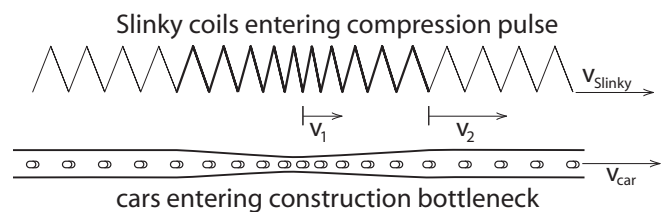


Figure 20
Slinky coils passing through the compressional pulse behave like cars driving through a construction bottleneck. They slow down and get closer together as they enter the pulse or bottleneck, and speed up when they leave.

Conservation of Coils

We can relate the spacing d_1 and d_2 between coils to their respective speeds v_1 and v_2 by using the idea that coils do not get lost as they pass through the pulse. If we calculate the number of coils per second passing a point along the Slinky, that number must be the same all the way along. For example, if there were fewer coils per second leaving the pulse than the number per second passing by the center of the pulse, that would imply that not all the coils made it out.

The use of dimensions tells us how to calculate the number of coils per second. The coil spacings d_1 and d_2 have dimensions

$$(d_1, d_2) \frac{\text{meters}}{\text{coil}} \quad (17)$$

while the velocities v_1 and v_2 have the usual dimensions meters/second. The ratios v_1/d_1 and v_2/d_2 have dimensions

$$\frac{v_1 \text{ meters/second}}{d_1 \text{ meters/coil}} \quad (18)$$

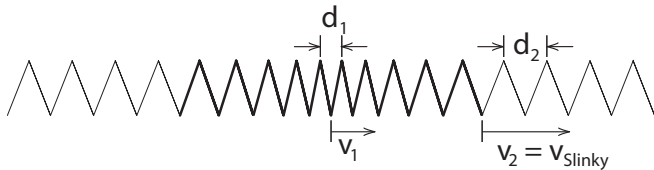
The meters cancel and we are left with

$$\frac{v_1 \text{ coils}}{d_1 \text{ second}} \quad (19)$$

as our formula for the number of coils per second passing by us. Conservation of coils requires

$$\frac{v_1}{d_1} = \frac{v_2}{d_2} \quad (20)$$

In Figure (21) you can see that both d and v increase as the coil comes out of the pulse, with the result the ratio d/v remains constant.



$$\frac{v_1}{d_1} = \frac{v_2}{d_2} \text{ conservation of coils}$$

Figure 21

In order that no coils get lost, the ratio of the coil speed to the coil spacing has to remain constant throughout the pulse.

Later on we will use Equation (20) in a slightly different form which we get as follows

$$v_1 = d_1 \frac{v_2}{d_2}$$

$$\boxed{\frac{v_1}{v_2} = \frac{d_1}{d_2}} \quad (20a)$$

Equation (20a) will allow us to express changes in velocity v in terms of changes in coil spacings d .

Acceleration of the Coils

As we mentioned, coils leaving the pulse have to accelerate from the lower speed v_1 to the higher speed v_2 . They have to do this in the time t it takes a coil to move from the center to the edge of the pulse. If the acceleration A is constant during this time, the constant acceleration formulas give

$$v_2 = v_1 + At$$

$$A = \frac{v_2 - v_1}{t} \quad (21)$$

Let the half width of the pulse be W as shown in Figure (22), and let $v_{av} = (v_1 + v_2)/2$ be the average velocity of a coil leaving the pulse. Then the time t to leave the pulse is

$$t = \frac{W \text{ meters}}{v_{av} \text{ meters/sec}} = \frac{W}{v_{av}} \text{ sec}$$

Our formula for the coil's acceleration while leaving becomes

$$A = \frac{1}{t}(v_2 - v_1) = \frac{v_{av}}{W}(v_2 - v_1) \quad (22)$$

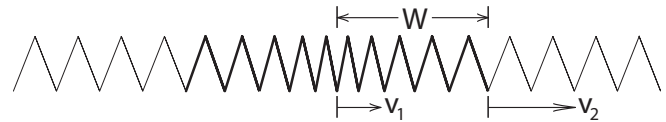


Figure 22

Calculating the acceleration of the coils leaving the pulse.

Weak Pulse

Equation (22) is an exact result if the coil exits with constant acceleration. However we get a simpler formula if we assume that the pulse is weak, meaning that the velocity v_1 at the center of the pulse is not much less than the outside velocity v_2 . That means that the average velocity v_{av} is not much different from v_2 , and we can replace v_{av} by v_2 in Equation (22), to get

$$A = \frac{v_{av}(v_2 - v_1)}{W} \approx \frac{v_2(v_2 - v_1)}{W} = \frac{v_2^2}{W} \left(1 - \frac{v_1}{v_2}\right) \quad (23)$$

We will find it convenient to use our formula for the conservation of coils to express the ratio v_1/v_2 in terms of the spacings d_1 and d_2 .

$$\frac{v_1}{v_2} = \frac{d_1}{d_2} \quad (20a \text{ repeated})$$

Thus

$$A = \frac{v_2^2}{W} \left(1 - \frac{d_1}{d_2}\right) \quad (24)$$

Mass of Exiting Coils

We will assume that the acceleration of the exiting coils is caused by the Hooke's law spring force acting on them. We are going to calculate the net spring force on the entire section of the exiting coils (all the coils within the width W), and then equate this net force to the total mass M of the exiting coils times their acceleration A .

To calculate the mass M of the exiting coils, we will again assume that the pulse is weak, and that the mass per unit length inside the pulse is not much greater than it is in the rest of the Slinky. With this assumption, the mass M of a length W of the Slinky is simply

$$M \text{ kg} = \mu \frac{\text{kg}}{\text{meter}} \times W \text{ meters} = \mu W \text{ kg} \quad (25)$$

MA for Exiting Coils

The product MA for the exiting coils, using Equations (24) and (25), becomes

$$MA = \mu W \times \frac{v_2^2}{W} \left(1 - \frac{d_1}{d_2}\right)$$

The half width W of the pulse cancels and the equation for MA becomes

$$MA = \mu v_2^2 \left(1 - \frac{d_1}{d_2}\right) \quad (26)$$

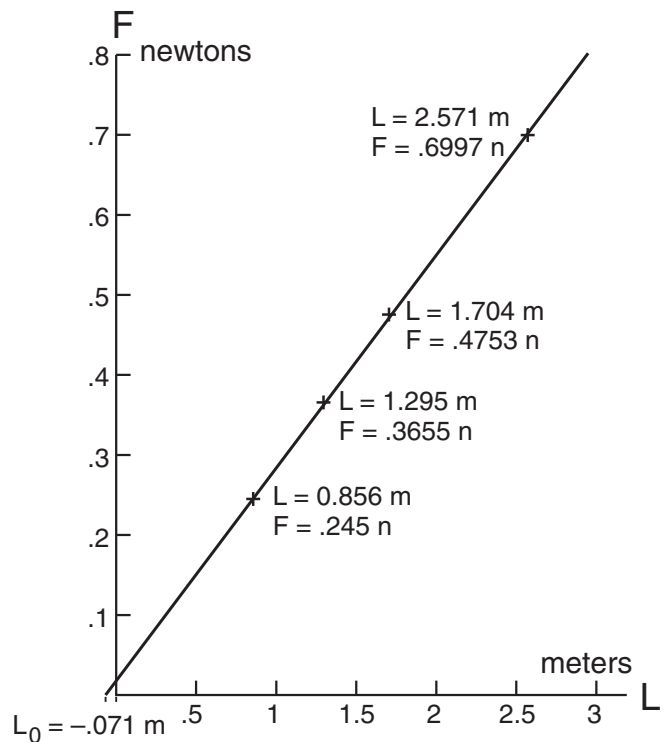


Figure 5 (repeated)
Measuring the force required to stretch the Slinky.

Hooke's Law for the Suspended Slinky

Before we used Hooke's law, we wanted to make sure that it actually applied to the suspended Slinky. After all, there are a lot of threads attached to the Slinky and the threads might have some effect on the motion of the pulse. (The threads do affect your measurements if you are not careful to make sure that the threads are straight up and down.)

In Figure (23) we show the results of the measurements of the forces F required to stretch the Slinky four different lengths L . The result is that all the points lie along a straight line that intersects the zero force axis at a distance

$$L_0 = -6.2 \text{ cm } (-.062 \text{ m}) \tag{27}$$

The formula for this line is

$$F = K(L - L_0) \quad \text{Hooke's law} \tag{28}$$

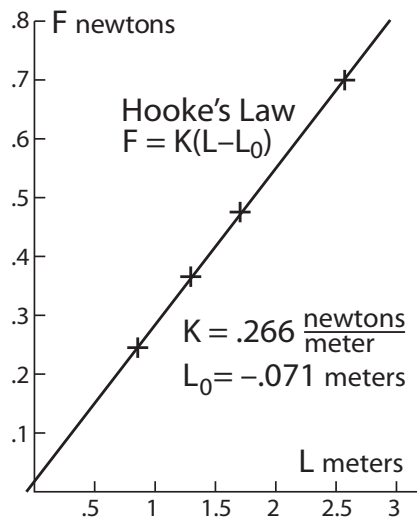
which is Hooke's law. The spring constant K for this particular Slinky is

$$K = .266 \frac{\text{newtons}}{\text{meter}} \tag{29}$$

which we obtain by solving Equation (8) for K and using numerical values for one of the points on the line. The summary of these results is shown in Figure (23).

The minus sign for the unstretched length L_0 simply means that the Slinky was wound in such a way that some force is required even to begin to stretch it.

Figure 23
Demonstration that our suspended Slinky obeys Hooke's law. The mass of the Slinky is .510 kg.



Net Spring Force

We can now use Hooke's law to calculate the net spring force acting on the section of coils emerging from the pulse. These are the coils in the section of length W , subject to the left directed force F_1 acting in the middle of the pulse, and the right directed force F_2 at the edge of the pulse, as shown in Figure (24).

The spring force F_1 is not as strong as the spring force F_2 because the coils at the center are not stretched as far apart as the coils at the edge. It is the difference in the strengths of these forces, $(F_2 - F_1)$ that gives rise to the net right directed force that causes the acceleration A that we just calculated.

Here we note that F_1 is the force required to stretch the spring so that all the coils have a spacing d_1 , while F_2 is the force required to stretch the spring so that the coil spacing is d_2 . With the spacings d_1 and d_2 , the total lengths L_1 and L_2 of the spring are

$$L_1 = Nd_1; \quad L_2 = Nd_2$$

where N is the number of coils in the Slinky.

Using Hooke's law to express the spring forces F_1 and F_2 in terms of the spring lengths L_1 and L_2 , we have

$$F_1 = K(L_1 - L_0)$$

$$F_2 = K(L_2 - L_0)$$

When we calculate the difference $(F_2 - F_1)$ in the spring forces, we get

$$\begin{aligned} (F_2 - F_1) &= K(L_2 - L_0) - K(L_1 - L_0) \\ &= K(L_2 - L_1) - KL_0 + KL_0 \end{aligned}$$

$$(F_2 - F_1) = K(L_2 - L_1)$$

The important feature of this calculation is that the KL_0 terms cancel, and we see that the net force $(F_2 - F_1)$ acting on our section of the Slinky does not depend on the spring's unstretched length L_0 .

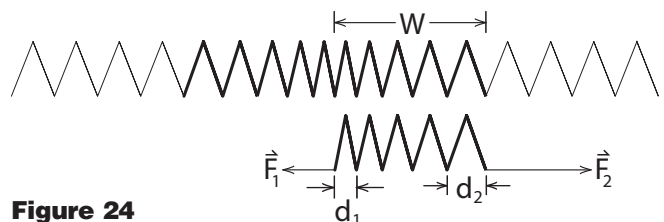


Figure 24

Because the coils are more stretched apart at the edge of the pulse than at the center, the spring tension F_2 is greater at the edge than the tension F_1 at the center. The net force on this section of Slinky is $(F_2 - F_1)$.

In order to relate our net force $F_{\text{net}} = (F_2 - F_1)$ to our formula for MA in Equation (26) we will rewrite Equation (30) in the form

$$F_{\text{net}} = (F_2 - F_1) = KL_2 \left(1 - \frac{L_1}{L_2}\right) \quad (30a)$$

Our next step will be to express this net force in terms of the coil spacings d_1 and d_2 rather than the total lengths L_1 and L_2 .

In Figure (25) we see that if there are N coils in the Slinky and we simply stretch the Slinky to a length L_1 , the distance L_1 and d_1 will be related by

$$L_1 = Nd_1; \quad d_1 = \frac{L_1}{N} \quad (31a)$$

Likewise if we pull the Slinky out to a greater distance L_2 we have

$$L_2 = Nd_2; \quad d_2 = \frac{L_2}{N} \quad (31b)$$

Since our formula for MA involved the ratio d_1/d_2 , we can use Equations (31) to express this ratio in terms of the spring lengths L

$$\frac{d_1}{d_2} = \frac{L_1/N}{L_2/N} = \frac{L_1}{L_2} \quad (32)$$

Using Equation (32) in Equation (30a) gives

$$F_{\text{net}} = KL_2 \left(1 - \frac{d_1}{d_2}\right) \quad (33)$$

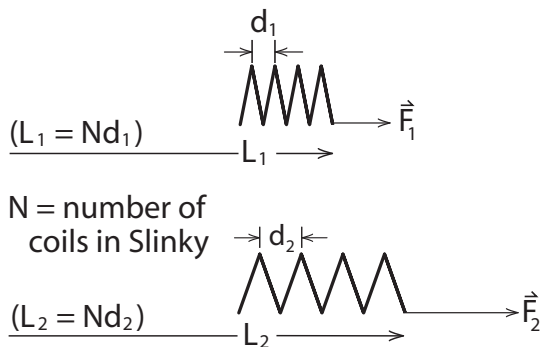


Figure 25
Relationship between the spring length and the coil spacing.

Next, note that $L_2 = Nd_2$ is essentially the length L of the Slinky, since all the coils in the Slinky except the few in the pulse have a separation d_2 . Thus our formula for F_{net} becomes

$$F_{\text{net}} = KL \left(1 - \frac{d_1}{d_2}\right) \quad (34)$$

Applying Newton's Second Law

To apply Newton's second law, we equate F_{net} from Equation (34) to MA from Equation (26) to get

$$F_{\text{net}} = MA$$

$$KL \left(1 - \frac{d_1}{d_2}\right) = \mu v_2^2 \left(1 - \frac{d_1}{d_2}\right)$$

The factors $(1 - d_1/d_2)$ cancel and we are left with

$$KL = \mu v_2^2 \quad (35)$$

Remembering that v_2 , the speed of the Slinky through the pulse, has the same magnitude as the pulse speed v_{pulse} , we get

$$KL = \mu v_{\text{pulse}}^2 \quad (36)$$

Solving for v_{pulse} gives us our final result

$$v_{\text{pulse}} = \sqrt{\frac{KL}{\mu}} \quad (37)$$

speed of compressional pulse in stretched Slinky

CHAPTER 13 REVIEW

We began the text with a description of how to measure the speed of a wave pulse on a stretched Slinky. We also mentioned that there was a formula for the speed of the wave pulse, a formula that we would derive later in the text.

This chapter is where we kept that promise, but it turns out that there are two kinds of wave pulses we can create, with a separate formula for each one. One pulse, where the coils move sideways, is called a transverse pulse whose speed is given by the formula

$$v_{\text{transverse}} = \sqrt{\frac{k(L - L_0)}{\mu}} \quad (2)$$

where k is the spring constant, L the length of the Slinky, L_0 its unstretched length, and μ its mass per unit length.

The other kind of pulse, which we called a compressional pulse where the coils either squeeze together or pull apart, has a different speed given by the formula

$$v_{\text{compressional}} = \sqrt{\frac{kL}{\mu}} \quad (3)$$

A sound wave in air is also a compressional wave.

CHAPTER EXERCISES

Exercise 1 On page 4

Calculate the Slinky's spring constant K .

Exercise 2 On page 5

Calculate the predicted length of time a compressional pulse should take to go down and back.

Exercise 3 On page 11

Calculate the spring constant of the steel pipe. How does that compare with the spring constant we found for the Slinky?

Exercise 4 On page 13

On your seismograph, you detect two sharp pulses indicating the occurrence of an earthquake. The first pulse is from the P wave, the second from the S wave. The pulses arrive three minutes apart. How far away did the earthquake occur?

REVIEW QUESTIONS

Here are some questions about these two kinds of waves you should be able to answer without looking back at the text. (But look if you have to).

5) To within experimental error, the compressional pulse and the transverse pulse on the stretched Slinky traveled at the same speed. Explain why.

6) The formulas for the speed of the compressional pulse can be used to predict that the time a pulse takes to travel down the length L of the Slinky and back, does not depend on how far we have stretched the Slinky. To within experimental error we got the same travel time for $L = .86$ meters and for $L = 2.60$ meters, three times as far.

a) Explain physically how this could happen.

b) Use the formula for $v_{\text{compressional}}$ to show that the trip down and back should not depend on L .

7) We measured the speed of a compressional pulse in a steel pipe by hitting the pipe with a hammer and observing a compressional pulse bounce back and forth between the ends of the pipe, just as the pulses on the Slinky bounced back and forth. Using an oscilloscope, we measured the time the pulse took to go down and back the 10.48 meter length of the pipe, and we got a time of 1.19 milliseconds. The mass of the pipe was 1.968 kg. Without looking back at the text, calculate the spring constant K of the steel pipe.

