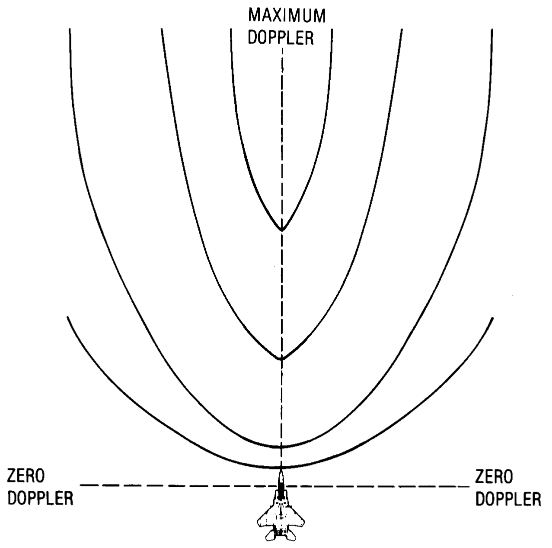
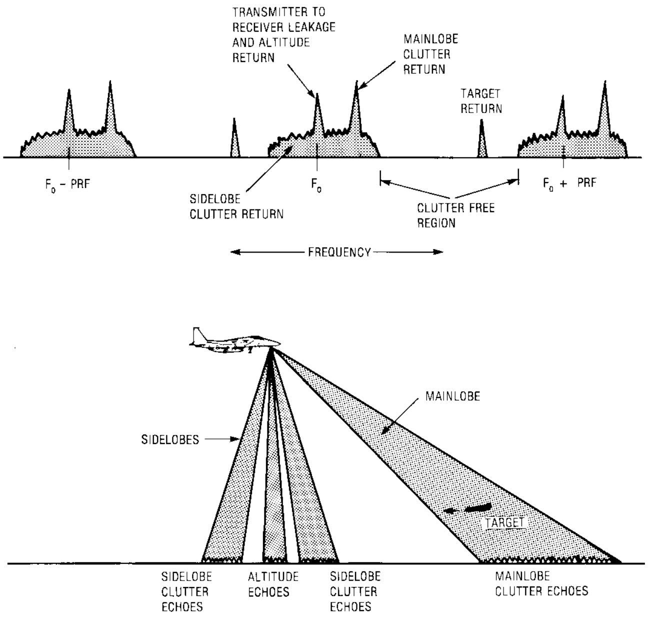
Chapter 14
MTI and Pulsed Doppler Radar



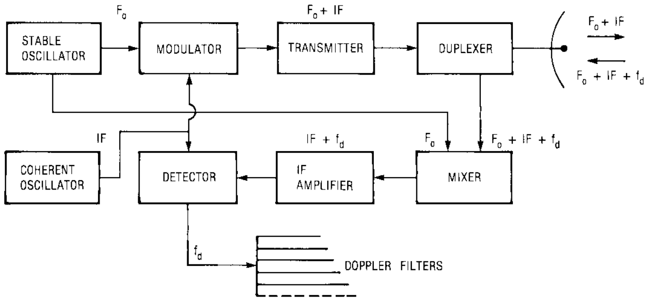
Curves of equal doppler for an airborne radar.



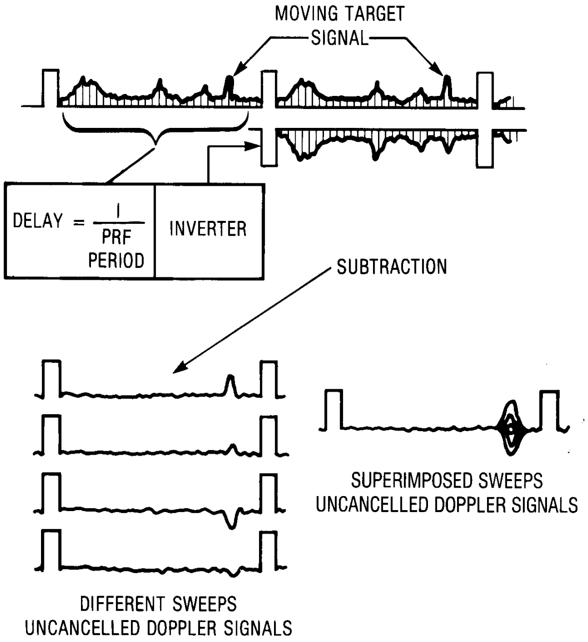
Airborne target/clutter doppler returns.

MTI and Pulsed Doppler Radar

- Moving Target Indication (MTI) radar: A delay line canceller filter to isolate moving targets from nonmoving background
  - Ambiguous velocity
  - Unambiguous range
- Pulsed Doppler radar: Doppler data are extracted by the use of range gates and Doppler filters.
  - Unambiguous velocity
  - Unambiguous or ambiguous range



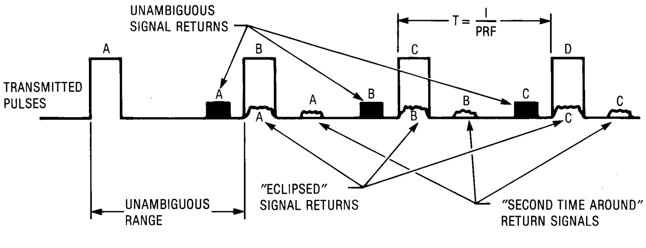
Pulsed doppler radar block diagram.



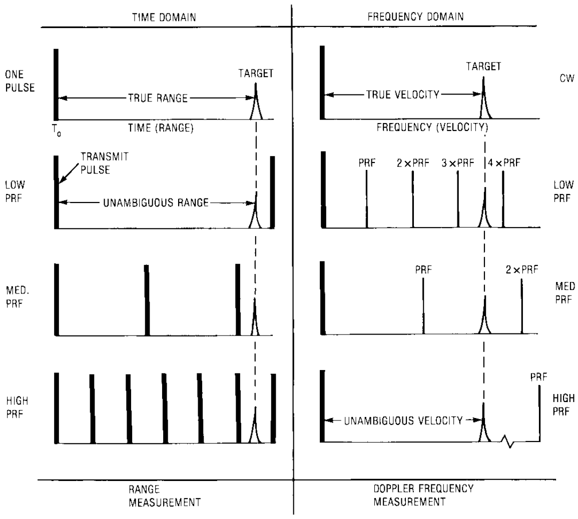
Delay line type of moving target indication.

Pulsed Radar

- High-PRF: unambiguous Doppler frequency, highly ambiguous range
  - solve TX-RX coupling problem of CW system
  - Range blind during TX time periods
- Low-PRF: unambiguous range, highly Doppler frequency
  - circumvents the TX-RX coupling
  - Introduce Doppler blind zones (ground clutter)
- Medium-PRF: ambiguous Doppler frequency, ambiguous range
  - circumvents the TX-RX coupling
  - Improve noise-limited detection relative to low-PRF waveform
  - Minimize the number of introduced blind zones relative to low-PRF system.



Signal returns in range measurements.



Pulsed Radar Parameters

- Range: range is obtained from transmit-to-receive pulse delay T

  $$2R = cT \Rightarrow R = cT/2$$

  - $1\mu s \rightarrow 150m$, $1ns \rightarrow 15cm$
- Range Resolution: Pulse width must be shorter than the propagation time from target 1 to target 2 and back

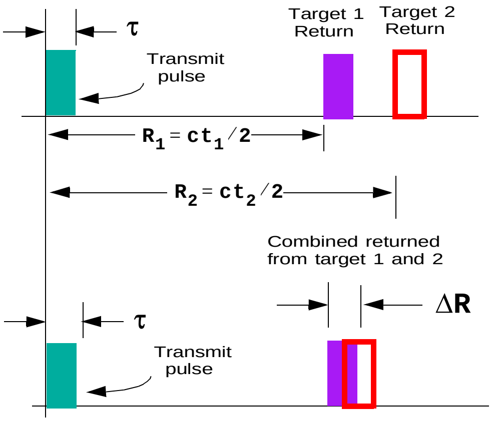
  $$2(R_2 - R_1) = ct \Rightarrow t = 2(R_2 - R_1)/c$$

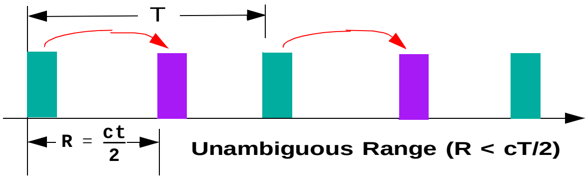
  $$\Delta R = c\tau/2$$

- Unambiguous range

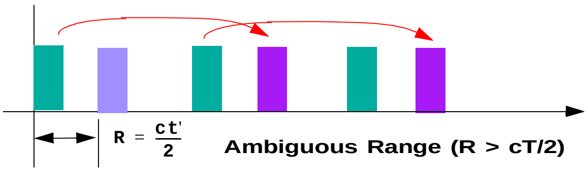
  $R_{unamb} = cT/2$, **T: pulse repetition interval (PRI)**

  - There are ways to get around this by using a staggered PRI (Multi-PRF)



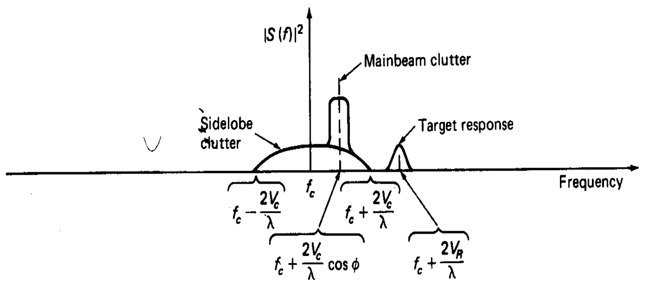


Unambiguous Range (R < cT/2)

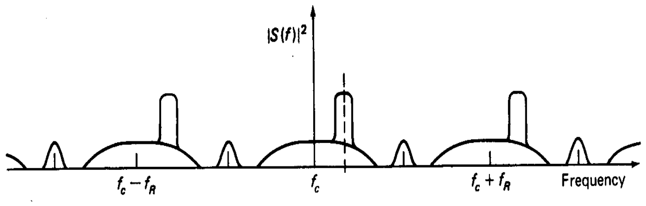


Ambiguous Range (R > cT/2)

Pulsed Doppler Power Spectrum



(a) CW radar received expected power spectrum



(b) Pulsed Doppler received expected power spectrum

Figure 14-1. Sample received expected power spectrums for a CW and a pulsed Doppler radar.

$$f_d = \frac{2V_c}{\lambda}\cos\phi$$

ϕ: **angle between the platform velocityand the line of sight (LOS)**

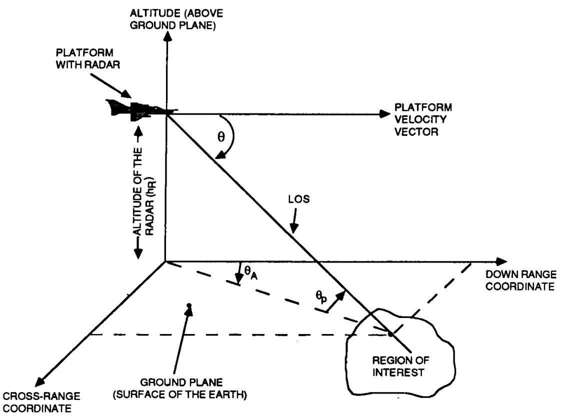


Figure 14-2. Radar/terrain geometry.

Pulsed Radar

- Noncoherent Pulsed Radar
  - No reference signal
- Coherent Pulsed Radar
  - TX phase is reserved
- MTI Radar
  - detection of moving target by suppressing fixed targets

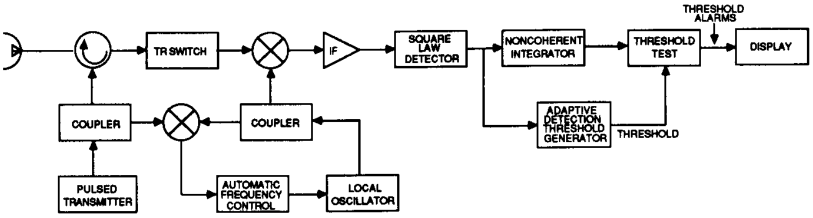


Figure 14-23. Block diagram of noncoherent pulsed search radar.

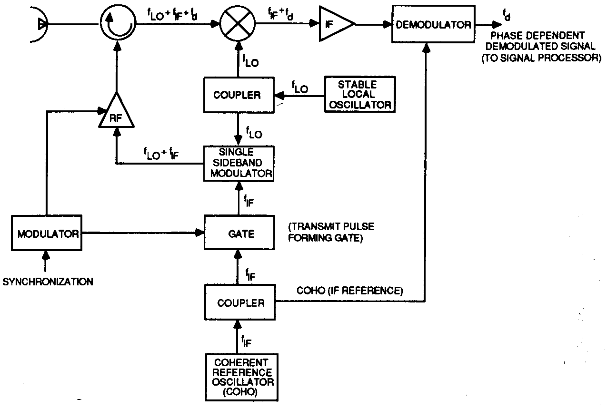


Figure 14-27. Simplified diagram of coherent pulsed radar.

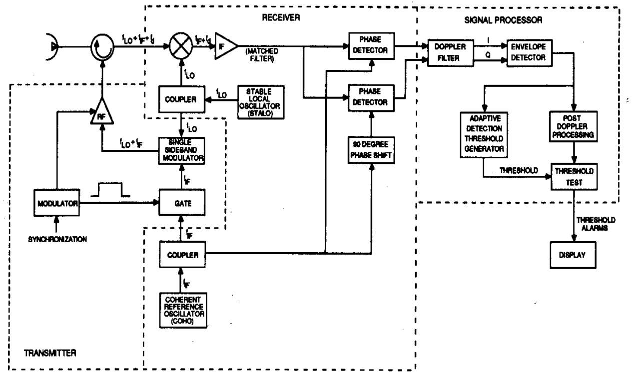
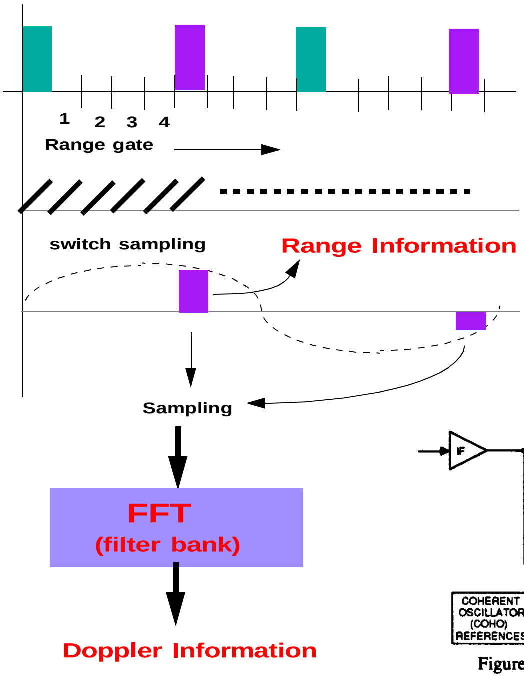


Figure 14-28. Pulsed coherent MTI search radar.

Pulsed Doppler Radar



1 2 3 4

Range gate

switch sampling

Range Information

Sampling

FFT (filter bank)

Doppler Information

Analog

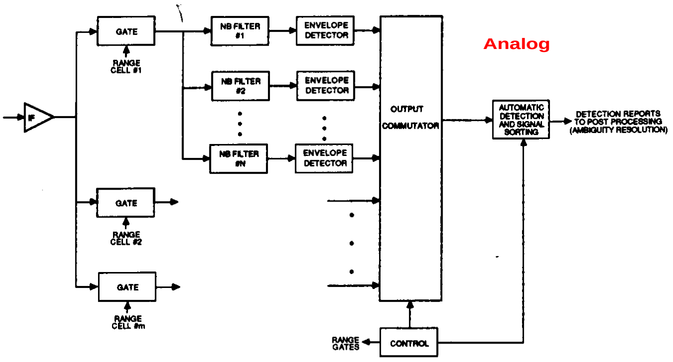


Figure 14-44. Block diagram of a sample signal processor for pulsed Doppler radar (analog).

Digital

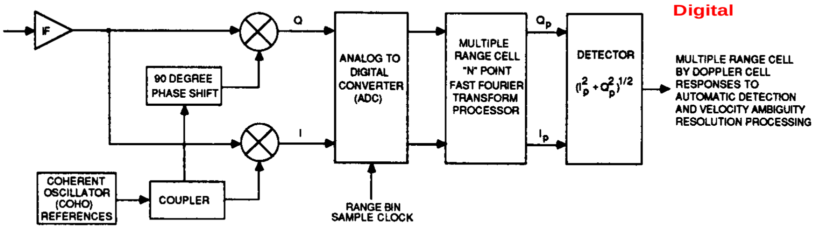
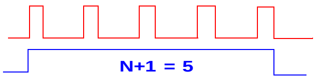


Figure 14-45. Block diagram of a sample signal processor for pulsed Doppler radar (digital).

Power Spectrum Density (Pulsed)

- As the antenna scans, the beam dwell time is finite.
- **T_i**: interpulse period; τ_p : pulse period

$$s(t) = \sum_{m=-N/2}^{N/2} s_1(t - mT_i)$$



where

$$s(t) = \text{rect}\,(t/\tau_p)$$
$$\text{rect}\,(t/\tau_p) = 1.0 \text{ for } -\tau_p/2 \le t \le \tau_p/2$$
$$= 0.0 \text{ elsewhere}$$
$$s_1(t - mT_i) = s_1(t) \text{ replicated at time centers } mT_i$$

Since the amplitude spectrum $S(f)$ is defined as the Fourier transform of the time waveform, $s(t)$, it can be shown[15] that

$$S(f) = S_1(f) \sum_{m=-N/2}^{N/2} \exp(-j2\pi fnt_i)$$
$$= S_1(f)\,\frac{\sin((N+1)\pi f T_i)}{\sin(\pi f T_i)}$$

Since the transform* of $s_1(t)$ is given as

$$S_1(f) = \tau_p \text{ sinc } (f\tau_p)$$

the transform of the train of $N + 1$ pulses is then given as

$$S(f) = \tau_p \text{ sinc } (f\tau_p)\,\frac{\sin((N+1)\pi f T_i)}{\sin(\pi f T_i)}$$

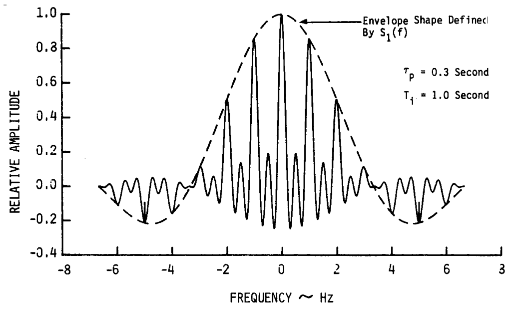


Figure 14-16. Amplitude spectrum of a finite number of rectangular pulses [$(N + 1) = 5$].

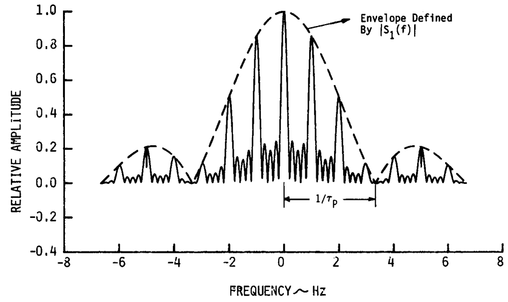


Figure 14-17. Magnitude of the amplitude spectrum for a finite number of rectangular pulses [$(N + 1) = 5$].

Power Spectrum Density (Pulsed) 2

- A five-burst waveform: the return from a scatter at a slant range $\mathbf{R_T}$.
  - $\mathbf{R_T} \pm \mathbf{R_{AMB}}$ contains four pulse samples
  - $\mathbf{R_T} + \mathbf{R_{AMB}}$ contains only one pulse sample
- The shape of both the spectrum and ambiguity function for τ is important → determine performance of MTI and pulsed Doppler radars.

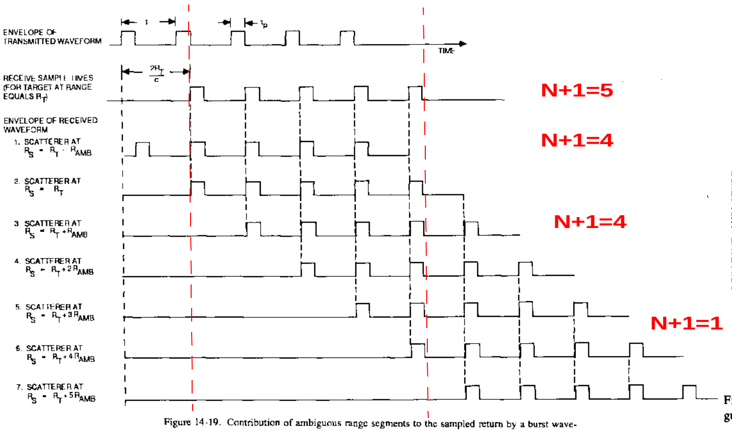


Figure 14-19. Contribution of ambiguous range segments to the sampled return by a burst waveform of five pulses.

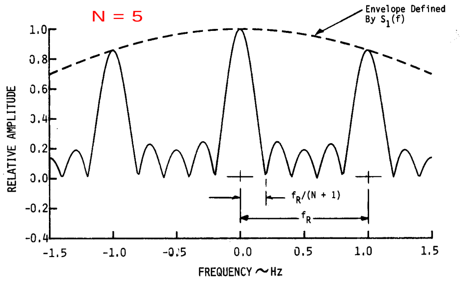


Figure 14-18. Expanded plot of the magnitude of the amplitude spectrum for a finite number $[(N + 1) = 5]$ of rectangular pulses.

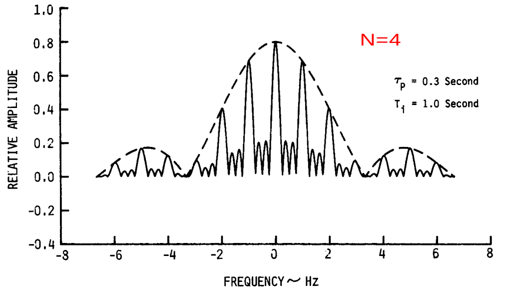
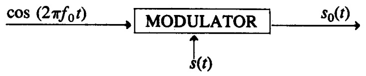


Figure 14-20. Magnitude of the amplitude spectrum for a finite number $[(N + 1) = 4]$ of rectangular pulses.

Power Spectrum Density (Pulsed RF)

If the modulation function, $s(t)$, is used to modulate a carrier term in the radar transmitter as represented in the following sketch



so that

$$s_0(t) = s(t)\cos(2\pi f_0 t)$$

it can be shown[17] that

$$S_0(f) = S(f) * [\tfrac{1}{2}(\delta(f - f_0) + \delta(f + f_0))]$$

where * denotes convolution and $\delta(f)$ is the delta function as defined in Chapter 13, or that

$$S_0(f) = \frac{\tau_p}{2}\left[\operatorname{sinc}((f - f_0)\tau_p)\,\frac{\sin((N+1)\pi(f - f_0)T_i)}{\sin(\pi(f - f_0)T_i)} + \operatorname{sinc}((f + f_0)\tau_p)\,\frac{\sin((N+1)\pi(f + f_0)T_i)}{\sin(\pi(f + f_0)T_i)}\right]$$

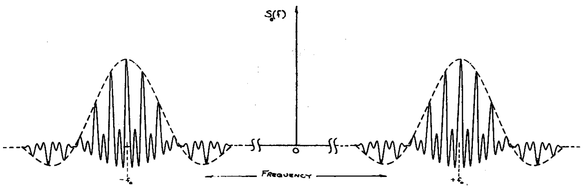
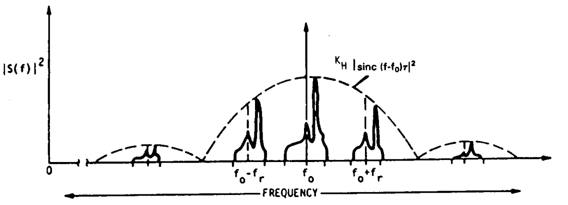
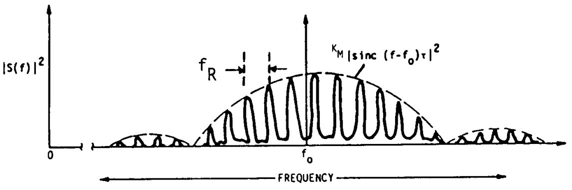


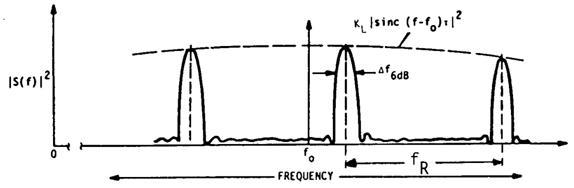
Figure 14-21. Voltage spectrum of a five-pulse burst RF waveform.



a) HIGH PRF



b) MEDIUM PRF



c) LOW PRF (IN VICINITY OF THE CENTRAL LINE ONLY)

Figure 14-22. Sample received spectra.

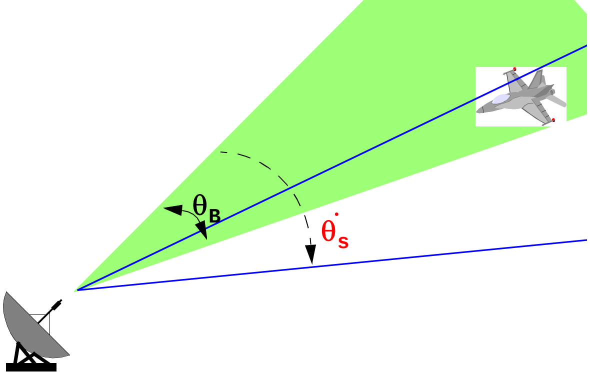
Radar Equation for Pulsed Radar

- Until now, we have not said a great deal about filtering of the return signal except to say that matched filtering is desirable and $\mathbf{B_{IF} = 1/\tau}$ for most pulse radars.
- In some cases, we need a better idea about bandwidth for estimation S/N ratio.
- Recall that we previously developed a radar equation of the form

$$R_{max} = \left[\frac{P_t G^2 \lambda^2 \sigma}{(4\pi)^3 kTBFL(S_o/N_o)_{min}}\right]^{1/4}$$

previously we consider this to be a single pulse.

- Integration of pulses: Depending on scan rate & PRF, we may receive more than 1 pulse from a target. We can use that our advantage
  - $n_B = (\theta_B/\dot{\theta}_s) \cdot f_p$: number of pulses for integration during dwelling time.
  - θ_B: beamwidth, $\dot{\theta}_s$: antenna scan rate
  - f_p: PRF



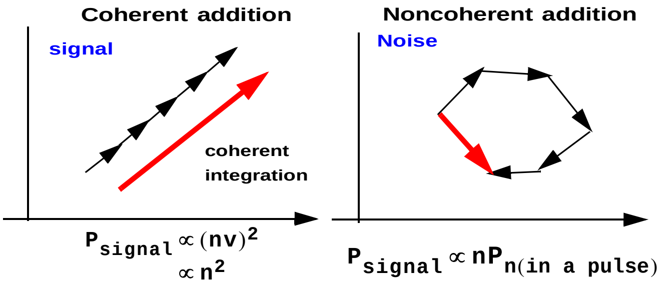
- An example: If $\dot{\theta}_s = 5\text{rpm}$ $\theta_B = 1.5°$, PRF=300Hz

$$\dot{\theta}_s = 5\text{rpm}(\text{round/per min.})$$
$$= 5 \times 360(1/60) = 30°/\text{sec}$$
$$n_B = \left(\frac{1.5}{30}\right) \times 300 = 15 \text{ pulses}$$

Pulse Integration

- Two techniques
  - Predetection Integration
  - Postdetection Integration
- Predetection Integration is coherent but somewhat more difficult to implement than postdetection
- Postdetection is incoherent but some improvement in (S_o/N_o) can be obtained.

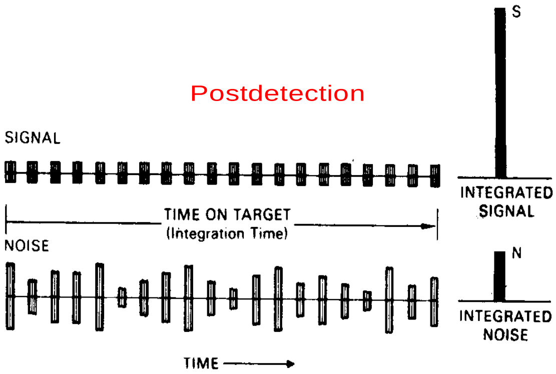
Predetection



$$P_{signal} \propto (nv)^2 \propto n^2$$

$$P_{signal} \propto nP_{n(\text{in a pulse})}$$

Postdetection



The improvement in signal-to-noise ratio is ultimately limited only by the time-on-target, provided target echoes remain correlated.

Pulse Integration

- Recall we were developing alternate expression for the radar system equation

$$R_{max} = \left[\frac{P_t G^2\lambda^2\sigma nE_i(n)}{(4\pi)^3 kTBFL(S_o/N_o)_{min}}\right]^{1/4}$$

  - $(S_o/N_o)_{min}$: single pulse S/N required for prespecification P_{FA} .
  - $E_i(n)$:efficiency factor; n: # of pulses integrated
  - Note that for a pulse radar P_t is a peak power, we can also express in terms of average power $P_{avg} = P_t(\tau/T) = \tau P_t f_p$, where τ : Pulse width, T : PRI; τ/T : duty cycle.
  - $P_t = P_{avg}/\tau f_p$; $E\tau = P_{avg}/f_p$: Energy per pulse

$$R_{max} = \left[\frac{P_{av} G^2\lambda^2\sigma nE_i(n)}{(4\pi)^3 kT(B\tau)FL(S_o/N_o)_{min} f_p}\right]^{1/4} = \left[\frac{E\tau G^2\lambda^2\sigma nE_i(n)}{(4\pi)^3 kT(B\tau)FL(S_o/N_o)_{min}}\right]^{1/4}$$

- We can express $(S/N)_{min} = (S_o/N_o)_{min}/(nE_i(n))$
  - for ideal predetection $E_i(n) = 1$; $n^{-1/2} \le E_i(n) \le 1$
  - $I_i(n) = nE_i(n)$: effective # pulses integrated
- Example: $P_{FA} = 10^{-12}$ $P_D = 0.9$, find $(S_o/N_o)_{min} \cong 15.8dB$. If 1000 pulses are integrated (postdetection square law)
- $$(S/N)_{min} = (S_o/N_o)_{min}/(nE_i(n))$$

$$= 15.8 - 10\log(130) = -5.34dB$$

$$R_{max} = \left[\frac{P_t G^2\lambda^2\sigma}{(4\pi)^3 kTBFL(S/N)_{min}}\right]^{1/4} = \left[\frac{P_t G^2\lambda^2\sigma nE_i(n)}{(4\pi)^3 kTBFL(S_o/N_o)_{min}}\right]^{1/4}$$

Noncoherent Pulsed Radar

- Noncoherent Pulsed Radar
  - No reference signal used by the receiver is phase coherent to the output phase of the transmitter.
  - A free-running pulsed transmitter
  - Automatic Frequency Control (AFC) → Local OSC is made to track the transmitter frequency
  - IF signal is bandpass filtered and amplified by IF amplifier
  - Square law (noncoherent) detection → noncoherent integrated signal processor → CFAR
- Problems encountered in detecting small-RCS in expected background clutter environments
  - WB Filter → high noise
  - Radar designer is left with only a few techniques to minimize the performance limits imposed by return from background clutter.
  - constrain parameters: operating frequency, maximum permitted antenna dimension dimensions...

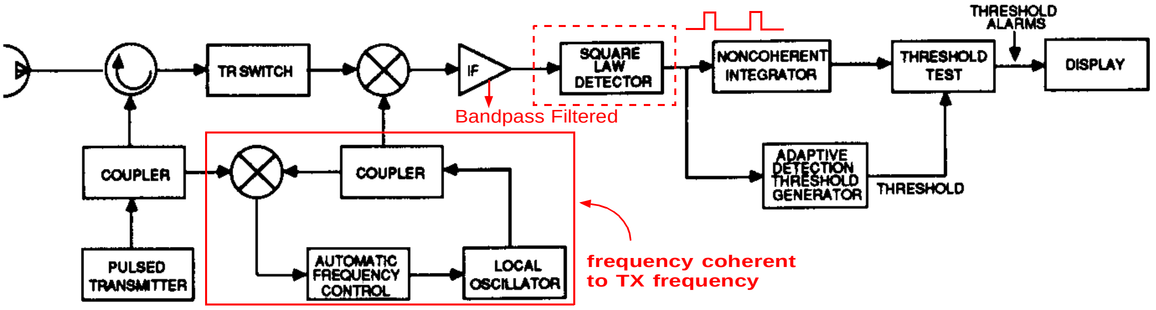
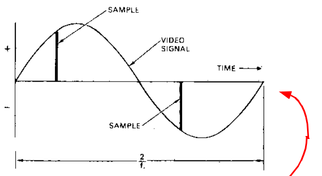


Figure 14-23. Block diagram of noncoherent pulsed search radar.

Coherent Pulsed Radar

- The phase of TX waveform is preserved is a reference signal → the receiver for signal demodulation
- The use of STRALO and COHO reference signals to store the phase of the later signal processing identifies the radar.
- Relative complexity between coherent and noncoherent systems
  - If it were not for performance, noncoherent configuration would be used extensively used in search radar applications.
- Advantage of coherent detection: exploitation of different Doppler shift to isolate desired target responses from large dominating (in amplitude) background returns
  - Relative motion between desired target and its background
- Some techniques through which noncoherent radar can be used to accomplish Doppler shift-aided detection of targets



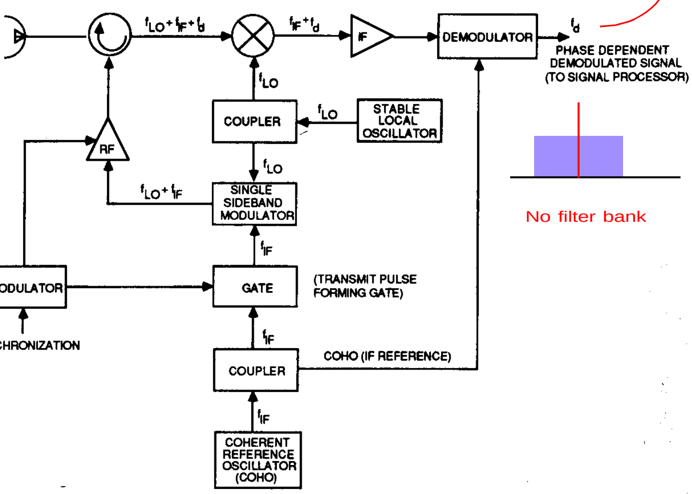


Figure 14-27. Simplified diagram of coherent pulsed radar.

Pulsed Coherent MTI

- The detection of moving targets are improved by suppression of fixed targets.
- This is expanded to incorporate Doppler processing as one possible form of MTI implementation
- is deified as one uses simple band reject to reject the return from fixed targets → Enhanced detection of the moving target
- Doppler filter
  - A relative narrow bandwidth clutter is rejected.
  - A broad passband (unknown Doppler shift)
  - Post-Doppler processing stage → Noncoherent integration
  - Rejection notches in the passband should be placed in frequency about the response that are to be rejected and should be as wide as required to achieve the desired clutter cancellation.

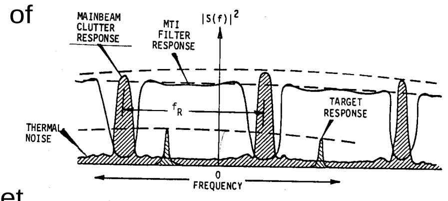


Figure 14-29. Sample MTI filter frequency response.

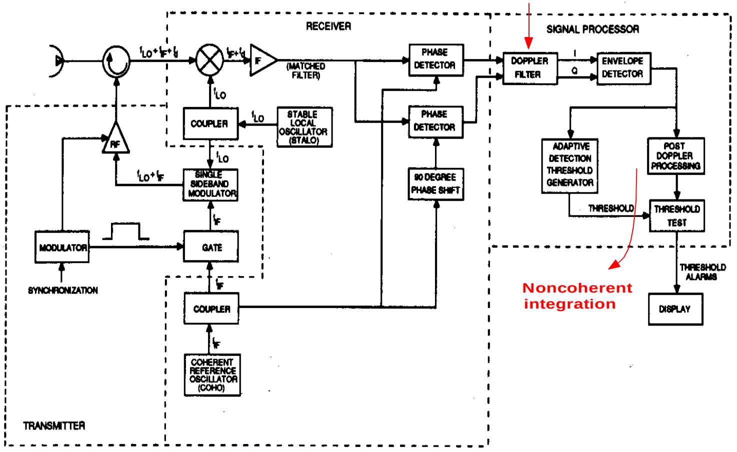


Figure 14-28. Pulsed coherent MTI search radar.

MTI filter (Delay line canceller)

Two techniques are available for realizing MTI filter

- Delay line canceller
  - Digital filter (Multipulse canceller)
  - A real-time delay is equal to the PRI
  - Digital implementations can provide the desired passband with the flexibility of passband programmability → The preferred choice
- Range gate and filter

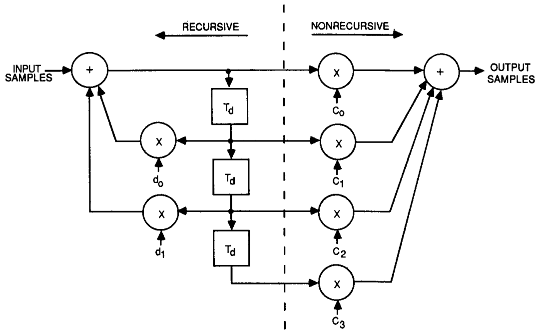


Figure 14-30. Generalized digital filter (multipulse canceller).

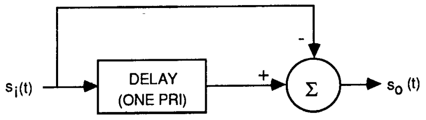
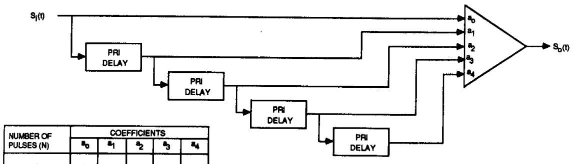


Figure 14-38. Simple two-pulse canceller (single delay).



NUMBER OF PULSES (N)	COEFFICIENTS				
	a_0	a_1	a_2	a_3	a_4
2	1	-1	0	0	0
3	1	-2	1	0	0
4	1	-3	3	-1	0
5	1	-4	6	-4	1

Figure 14-42. *N* pulse canceller.

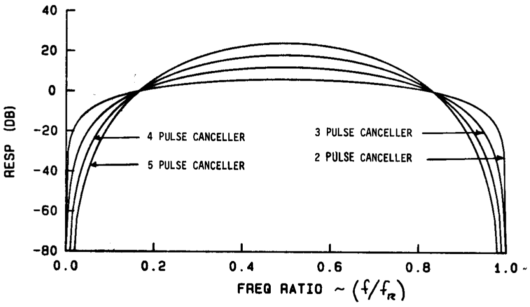


Figure 14-43. Sample passband for simple feed-forward, multipulse cancellers.

Range gate and filter

- output of phase detector (I or Q) is provided as input to N sample-and-hold circuits
- Each sample-and-hold circuit receives a sample gate → a lumped constant (active) bandpass filter
- f_L : lower corner frequency. f_H : higher corner frequency. f_R : PRF
- Noncoherent MTI
  - A-scope range video presentations contain "butterflies" at the slant range of the moving target.
  - Each butterfly is created by the fluctuating amplitude of the sum of the return from both the background and the target

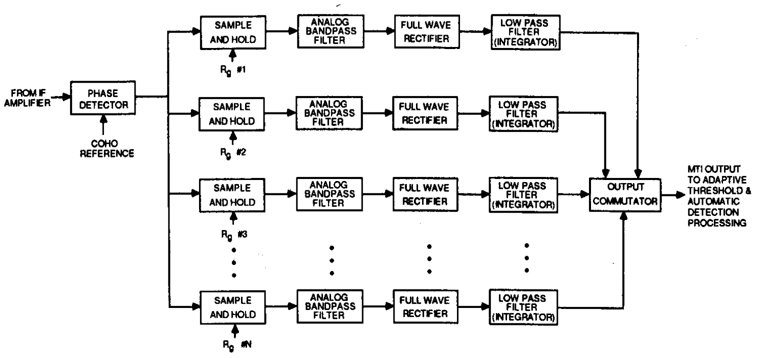


Figure 14-31. Range gate and filter MTI processor.

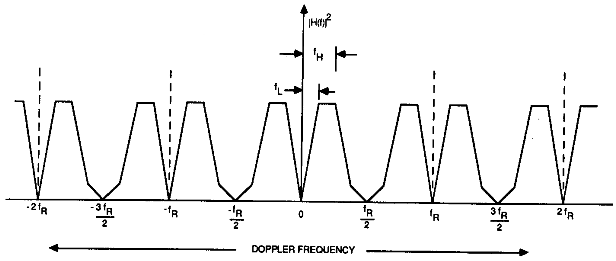
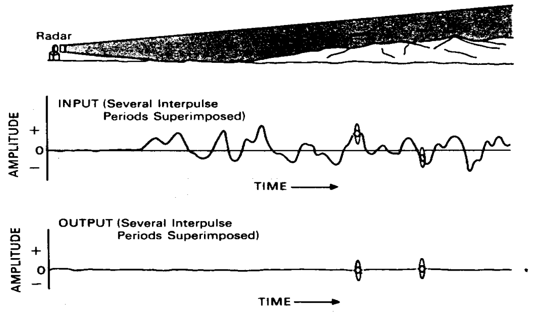


Figure 14-32. Sample passband for a range gate and filter MTI.

Noncoherent MTI

- Noncoherent MTI
  - A-scope range video presentations contain “butterflies” at the slant range of the moving target.
  - Each butterfly is created by the fluctuating amplitude of the sum of the return from both the background and the target



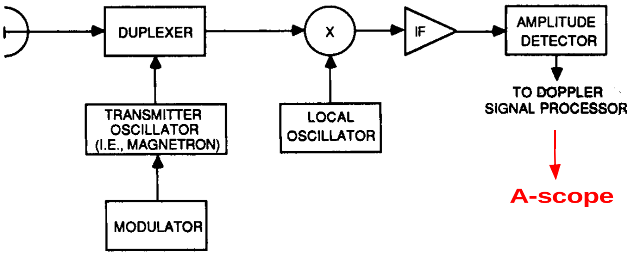


Figure 14-33. Simplified diagram of noncoherent MTI radar.

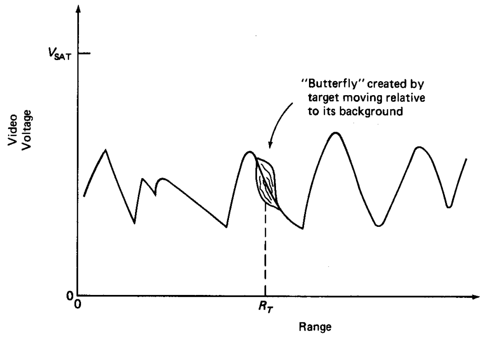


Figure 14-34. A-scope presentation of video from a monochromatic, noncoherent pulsed radar.

Pulse Doppler Radar (Analog)

A pulsed Doppler radar exploits Doppler shift to obtain velocity information from a pulsed waveform

- High-PRF pulsed Doppler airbone radar
- Medium-PRF pulsed Doppler airbone radar

Two approaches

- Analog filtering
- Digital filtering

Analog filtering

- processing at IF
- Output of IF amplifier is gated
- gated output is then filtered by a bank of contiguous narrowband (NB) filters
- Automatic detection and signal-sorting.

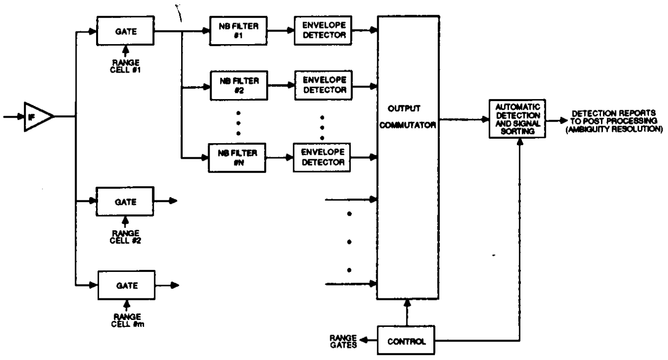
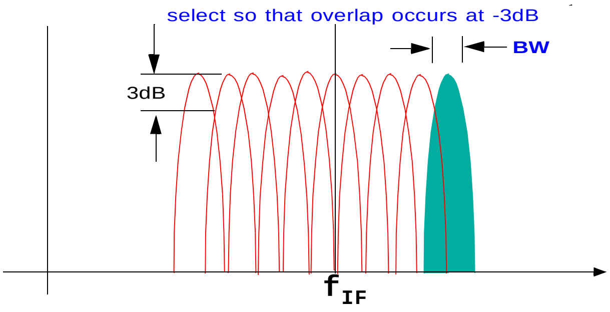
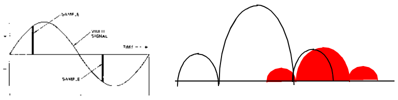


Figure 14-44. Block diagram of a sample signal processor for pulsed Doppler radar (analog).



Pulse Doppler Radar (Digital)

- Digital filtering (FFT)
  - In-phase (I) and quadrature (Q) video
  - Analog-to-digital conversion (A/D)
  - N-point fast Fourier transform (FFT)
  - N contiguous filter passbands (span from zero to PRF)
  - The number of range cells (m) and Doppler cells (N) increases → the amount of hardware grows dramatically.
  - Modern digital technology leads to hardware efficient realizations
- N-point FFT which produces a finite impulse response filter is the desired matched-filter for a finite sequence (step scan algorithms)
- NB filter (infinite impulse response) in the analog approach can be only an approximation of the desired matched-filter.
- selection of digital approach for most modern pulsed Doppler radar designs.
- Spectral leakage
  - uniformly weighted set of N samples → $\mathrm{Sin}(nx)/\sin(x)$
  - Other Doppler shift → High sidelobes in the same range cell
  - aperture weighting is used to minimize the spectral leakage.



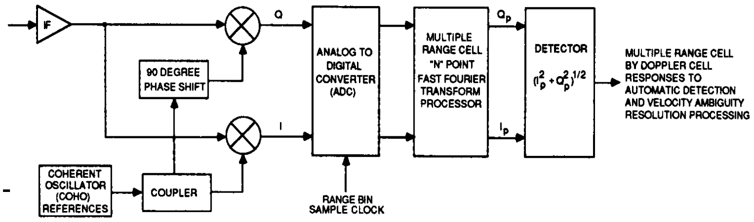


Figure 14-45. Block diagram of a sample signal processor for pulsed Doppler radar (digital).