CHAPTER : 14 OSCILLATIONS

1 marks:

1. The length of a second's pendulum on the surface of earth is 1 m. What will be the length of a second's pendulum on the surface of moon?
   Ans : T = 2π $\sqrt{l/g}$, T remains same in both cases. g $\propto l$.
   g_m = 1/6 g_e , so l_m = 1/6 l_e. Length reduces to 1/ 6 th

2. The total energy of a particle executing SHM is E. What is the K.E when the displacement is equal to one-half of the amplitude?
   Ans : K.E = ½ m $\omega^2(A^2 - x^2)$, x = ½ A , K.E = ¾ E

3. Can a simple pendulum vibrate at the centre of Earth?
   Ans : No, because at the centre of earth, g = 0

4. When is the potential and kinetic energies of a harmonic oscillator becomes maximum?
   Ans : P.E is maximum at extreme position and K.E is maximum at mean position

5. A simple pendulum is described by a = -16 x ; where a is acceleration , x is displacement in metres. What is the time period?
   Ans : a = -16 x = $-\omega^2$ x , T = 2π/ ω = 2π/4 = π/2 s
6. What is the direction of restoring force on the oscillator performing S.H.M.?
   Answer: It is directed towards the mean position.
7. How does the magnitude of restoring force on an oscillator vary with displacement in a simple harmonic motion?
   Answer:The magnitude of restoring force on an oscillator is proportional to the displacement of oscillator from the mean position, in a simple harmonic motion?
8. Define the amplitude of oscillation.
   Answer:It is the maximum displacement of the oscillator from the mean position
9. How are the frequency and the time period of an oscillator related?
   Answer:Frequency is the inverse of the time period
10. Write the relationship between acceleration and displacement of a particle in S.H.M?
    Answer a =$-w^2y$
11. What is the phase difference between the displacement and acceleration of a particle executing S.H.M?
    Answer 180 degree

12. A simple pendulum is attached to the roof of a lift. If time period of oscillation, when the lift is stationary is T, Find frequency of oscillation when the lift falls freely.

Answer:Explanation: In a freely falling lift,

g=0

v=1/2π×√(g/l)

=1/2π×√ (0/l) =0.

13. There is a simple pendulum hanging from the ceiling of a lift. When the lift is standstill, the time period of the pendulum is T. If the resultant acceleration becomes g/4, find new time period of the pendulum.

Answer:Time period of the pendulum is given by.

$T=2\times\sqrt{(l/g)}$

$T^{`}=2\pi\times\sqrt{((l/g)/4)}$

$T^{`}=2T$.

14. What is time period of a pendulum hanged in a satellite? (T is the time period on earth)

Answer:

In a satellite, g= 0

$T=2\pi\sqrt{(l/g)}$

$=2\pi\sqrt{(l/0)} \quad =\infty$

15. A particle executing simple harmonic motion of amplitude 5cm has maximum speed of 31.4 cm/s. Find the frequency of its oscillation. 1

Answer: Explanation: $v_{max} = 2\pi vA$

$31.4 = 2\times 3.14v\times 5$

$v=1Hz$

16. A hollow spherical pendulum is filled with mercury has time period T. If mercury is thrown out completely, then the new time period 1

Answer: Position of centre of gravity remains unaffected when mercury is thrown out. Hence effective length and time period remain same.

17.Write displacement equation respecting the following condition obtained in SHM.

Amplitude = 0.01m

Frequency = 600Hz

Initial phase = $\frac{\pi}{6}$

Ans. Y = A Sin $(2\pi v t + \phi_o)$

= 0.01 Sin $(1200\pi t + \frac{\pi}{6})$

18. What is the condition to be satisfied by a mathematical relation between time and displacement to describe a periodic motion?

Ans. A periodic motion repeats after a definite time interval T. So,

$$y(t) = y(t+T) = y(t+2T) \; etc.$$

2 Marks :

1. Obtain an expression for the time period of the horizontal oscillations of a massless loaded spring.
2. Why is restoring force necessary for a body to execute S.H.M?
   Ans : A body in SHM oscillates about its mean position . At the mean position, it possess KE with which it moves to extreme position. Then the body return to mean position only if it is acted upon by a restoring force.

3. Define the time period and frequency of an oscillator and write there SI units.
   Answer: Time taken in one oscillation is called time period. Its SI unit is s.
   The number of oscillations in one second is called frequency. Its SI unit is Hz.
4. Define simple harmonic oscillation and give an example.
   Answer: In this motion, a particle moves to and fro repeatedly about a mean position under a restoring forceswhich directly proportional to displacement of particle from mean position and directed towards the mean position.
   Example – a vibrating string, simple pendulum.
5. Human heart beats 72 times per minute, on an average. Calculate the beat frequency and period.
   Answer: Frequency = N/t=72/60=1.2 Hz
   Period = t/N =60/72 second
6. . show that in S.H.M. the acceleration of the particle is directly proportional to the displacement at the given instant.

   Hint: displacement relationship

   Then its differential for velocity and then for acceleration.

7. A second's pendulum is mounted in a rocket. If the rocket moves up with uniform acceleration a. justify the change in the period of oscillation. 2
   Answer: aExplanation:
   When at rest, $T=2\pi\sqrt{(l/g)}$
   When the rocket moves up with uniform acceleration a,
   $T'=2\pi\sqrt{(l/(g+a))}$
   Clearly,
   $T > T'$
   The period of oscillation will be decrease

8. The composition of two simple harmonic motions of equal periods at right angle to each other and with a phase difference of π. What is the nature of displacement of the particle along which they are oscillating.
   Answer: Let $x = a\sin\omega t$
   $y = b\sin(\omega t+\pi) = -b\sin\omega t$
   $x/a = -y/b$, $y=-(b/a)\,x$
   This is the equation of a straight line

9.Does the function $y = \sin^2\omega t$ represent a periodic or a S.H.M? What is period of motion?

Ans. Displacement y = $\sin^2\omega t$

Velocity v = $\frac{dy}{dt}$ = 2sin ωt × cosωt × ω

v = ω sin 2ωt

Acceleration a = $\frac{dv}{dt}$ = ω×cos 2ωt × 2ω

a = 2 ω^2cos 2ωt.

As the acceleration is not proportional to displacement y, the given function does not represent SHM. It represents a periodic motion of angular frequency 2ω.

∴ Time Period T = $\frac{2\pi}{Angular\ freq.} = \frac{2\pi}{2\omega} = \frac{\pi}{\omega}$

10.A simple Harmonic Motion is represented by $\frac{d^2x}{dt^2}$+αx = 0. What is its time period?

Ans.$\frac{d^2x}{dt^2} = -$ αx Or a = $-$ αx

$$T = 2\pi\sqrt{\frac{x}{a}} = 2\pi\sqrt{\frac{x}{\alpha x}} = \frac{2\pi}{\sqrt{\alpha}}$$

$$\mathbf{T = \frac{2\pi}{\sqrt{\alpha}}}$$

11.The Length of a simple pendulum executing SHM is increased by 2.1%. What is the percentage increase in the time period of the pendulum of increased length?

Ans. Time Period, T = $2\pi\sqrt{\frac{l}{g}}$ i.e. T $\propto\sqrt{l}$.

The percentage increase in time period is given by

$\frac{\Delta T}{T} \times 100 = \frac{1}{2}\frac{\Delta l}{l}$ × 100 (for small variation)

$= \frac{1}{2}$ × 2.1%

= 1.05%

3 Marks :

1. (i) Show that for small oscillations, the motion of a simple pendulum is simple harmonic.
   (ii) Obtain an expression for its Time Period.
2. You have a light spring, a meter scale and known mass. How will you find the time period of oscillation of mass without the use of a clock?
   Ans : Suspended the known mass from one end of spring whose another end is connected to a rigid celling. Note the extension l in the spring with the help of meter scale. If k is the spring constant of the spring, then in equilibrium position,

   Kl=mg or m/k= l/g

Timeperiod of the loaded spring,

$T = 2\pi\sqrt{m/k} = 2\pi\sqrt{l/g}$

Knowing, the value of l, time period of oscillation T can be determined.

3. What is a spring factor ? Find its value in case of two springs connected in
   (i) Series
   (ii) Parallel

   Ans : Spring factor or force constant, k=F/y

   (i) For two springs connected in series , effective spring factor k is given by

   $1/k=1\backslash k_1+1/k_2$ or $k=k_1 k_2/k_1 + k_2$ ∴ $T=2\pi\sqrt{m(k_1+k_2)/k1k2}$

   (ii)For two springs of springs factors k_1 and k_2 connected in parallel, effective spring factor,

   $K=k_1+ k_2$ ∴ $T=2\pi\sqrt{m}/k_{1+}k_2$

4. Write the displacement relation for SHM and Differentiate between the phase and phase constant.
   Answer: displacement relation for SHM x (t) = A cos (ωt + φ)
   The time varying quantity, (ωt+ φ)is called the phase of the motion. It describes thestate of motion at a given time.
   The constant φ iscalled the phase constant (or phase angle).It describes the initial state of motion.
5. Deduce an expression for the velocity of a Particle executing S.H.M. When is the particle velocity maximum and when minimum?

   Hint: derivation V max at mean position

   V min at extreme positions of displacement

6. Show that the acceleration of a particle in SHM is proportional to its displacement from the mean position. Hence write the expression for the time period of SHM.

   Ans. Diagram and explanation

7. What is a periodic function? Prove that the function sin ωt+ cosωt is periodic.

   Answer: it is a function whose value is same after a certain time, say T
   For example cos(ωt +ωT) = cosωt if ωT =2 π
   function sin ωt + cosωt
   $=\sqrt{2}[\left(\frac{1}{\sqrt{2}}\right)\sin \omega t + \left(\frac{1}{\sqrt{2}}\right)\cos\omega t]$
   $=\sqrt{2}$ [sin 45 sin ωt + cos 45 cosωt]
   $=\sqrt{2}\cos(\omega t-45)$
   As it is a cosine function hence it is periodic

8. The displacement in SHM is given by sin ωt + cosωt. What is the amplitude of oscillation?

X(t) = sin ωt + cosωt
$=\sqrt{2}\ [\left(\frac{1}{\sqrt{2}}\right)\sin \omega t + \left(\frac{1}{\sqrt{2}}\right)\cos \omega t]$
$=\sqrt{2}$ [sin 45 sin ωt + cos 45 cosωt]
$=\sqrt{2}$cos(ωt-45)
= A cos (ωt + φ)
Hence its amplitude A=$\sqrt{2}$ units

9. show that of a particle is moving in S.H.M. its velocity at a distance ✓3/2 of its amplitude from the central position is half its velocity in central position.

Answer: y = ✓3/2A

v=w✓(A^2 - y^2)= wA/2 =Vmax/2

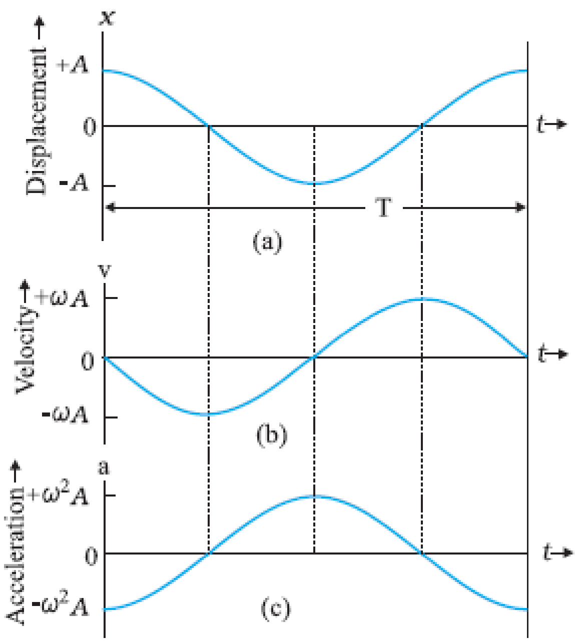
10. Draw displacement-time, velocity-time and acceleration-time graphs for a particle executing simple harmonic motion.

Ans. If a particle executing SHM its displacement equation can be

x(t) = A cosωt

v (t) = - ωA sinωt

a(t) = - ω^2A cosωt



11.The equation of a plane progressive wave is, $y = 10Sin2\pi(t - 0.005x)$where y & x are in cm & t in second. Calculate the amplitude, frequency, wavelength & velocity of the wave.

Ans. Given, y = $10Sin2\pi(t - 0.005x)$…………..

Standard equation for harmonic wave is, $y = A\ Sin2\pi(\frac{t}{T} - \frac{x}{\lambda})$…………….

Comparing eqn (1) & (2), $A = 10,\ \frac{1}{t} = 1,\ \frac{1}{\lambda} = 0.005$

(i) Amplitude A = 10cm

(ii) Frequency $\nu = \frac{1}{T} = 1Hz$

(iii) Wavelength $\lambda = \frac{1}{0.005} = 200cm$

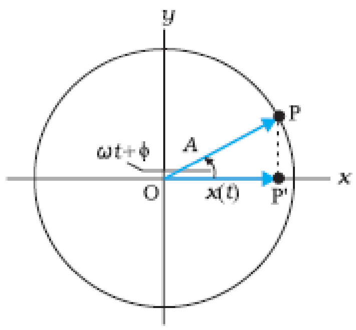
(iv) Velocity v = $\nu\ \lambda$ = 1 x 200 = 200cm/s

5 Marks :

1. (i) Derive the expression for the total energy of a particle executing S.H.M .
   (ii) Show graphically the variation of P.E and K.E with time in S.H.M
   (iii) What is the frequency of these energies with respect to the frequency of the particle executing S.H.M?

2. Show that simple harmonic motion may be regarded as the projection of uniform circular motion along a diameter of the circle. Hence derive an expression for the displacement, velocity and acceleration of a particle in SHM.

Ans. Relation between SHM and uniform circular motion and diagram



the motion of areference particle P executing a uniform circular motion with (constant) angular speed ω in a reference circle. The radius *A* of the circle is the magnitude of the particle's position vector. At any time *t*, the angular position of the particle is ω*t* + φ, where φ is its angular position at *t* = 0. The projection of particle P on the *x*-axis is a point P′ , which we can take as a second particle. The projection of the position vector of particle P on the *x*-axis gives the location *x*(*t*) of P′ . Thus we have,

$x(t) = A\cos(\omega t + \varphi)$

Simple harmonic motion is the projection of uniform circular motion on a diameter of the circle in which the latter motion takes place.

Derivation expression for displacement

$x(t) = A\cos(\omega t + \varphi)$

Derivation expression for velocity

$$v(t) = -\omega A \sin(\omega t + \varphi)$$

Derivation expression for acceleration
$a(t) = -\omega^2 A \cos(\omega t + \varphi)$

Numericals:- Level 1

1.The pan attached to a spring balance has a mass of 1kg . A weight of 2kg when placed on the pan stretches the spring by 10 cm .What is the frequency with which the empty pan will oscillate?

Ans : F =2kgwt =2X 9.8 N =19.6 N

Displacement, y=10cm=0.1 m

∴ Force constant, k=F/y=19.6/0.1=196N/m

For the empty pan, m=1kg

Hence the frequency of oscillation of the empty pan will be

$\nu = 1/2\pi\sqrt{k/m} = 1/2\pi \times 14 = 7/\pi$ Hz

2. . In a pendulum, the amplitude is 0.05 m and a period of 2 s. Compute the maximum velocity.

Ans. $V_{max} = \omega A = \frac{2\pi}{T} A$
= 3.142 X 0.05
= 0.1571 ms^{-1}

3. An object of mass 0.2kg executes simple harmonic motion along the x-axis with a frequency of (25/π)Hz. At the position x = 0.04, the object has kinetic energy of 0.5J and potential energy 0.4J.Find the amplitude of oscillation. 2

Answer:
Explanation: Total energy.
$E = 2\pi^2 m v^2 A^2$
$0.5+0.4 = 2\pi^2 \times 0.2 \times (25/\pi)^2 A^2$
$A^2 = 0.9/(0.4 \times 25^2)$
$A = 3/(2 \times 25) = 3/50$ m
=6cm

4. A spring of force constant 800N/m has an extension of 5cm.Calculate the work done in extending it from 5cm to 15cm. 3

Answer:
Explanation:

At $x_1 = 5$ cm,

$U_1=1/2\times k(x_1)^2$

$=1/2\times 800\times 0.05^2$

$=1J$

At $x_2=15$cm,

$U_2=1/2\times k(x_2)^2=1/2\times 800\times 0.15^2$

$=9J$

The work done is given by,

$W=U_2-U_1$

$=9-1$

$=8J.$

5. A particle executes simple harmonic oscillation. Its amplitude is a. The period of oscillation is T. Calculate the minimum time taken by the particle to travel half of the amplitude from the equilibrium position? 2

Answer: Explanation:

$y= a \sin(\omega t)$

$a/2= a \sin(2\pi t/T)$

$½ =\sin(2\pi t/T)$

$2\pi t/T=\pi/6$

$t=T/12.$

6. If the length of a simple pendulum is increased by 2%, find new time period of simple pendulum

Answer: a 2

Explanation: $T\propto\sqrt{l}$

Percentage increase in time period,

$\Delta T/T\times 100=1/2\times\Delta l/l\times 100$

$=1/2\times 2\%$

$=1\%$

7.A simple Harmonic motion has an amplitude A and time period T. What is the time taken to travel from x = A to x = A/2.

Ans. Displacement from mean position = $A - {}^{A}\!/_{2} = {}^{A}\!/_{2}$.

When the motion starts from the positive extreme position, $y = A \cos\omega t$.

$$\therefore \frac{A}{2} = A \cos\frac{2\pi}{T}t.$$

$\cos\frac{2\pi}{T}t = ½ = \cos\frac{\pi}{3}$

$$\text{or} \quad \frac{2\pi}{T}t = \frac{\pi}{3}$$

$$\therefore t = {}^{T}\!/_{6}$$

Level 2

1.A linear harmonic oscillator of force constant 2 x 10_6 N /m and amplitude 0.01 m has a total mechanical energy of 160 J .What Is the maximum K.E. and minimum P.E.?

Ans : Here, maximum K.E. =max. PE =1/2kr ^{2}

=1/2 x (2 x 10 ^{6}) x $(0.01)^2$=100 J

In **SHM**, P.E. is minimum at mean position and K.E. is maximum at mean position. But the total mechanical energy in **SHM** is constant .Therefore, at mean position,

Total mechanical energy is = max. K.E. + minimum P.E.

160= 100 + minimum P.E.So, minimum P.E. =160-100=60 J

2. The periodic time of a body executing SHM is 2s. After how much time interval from t = 0, will its
displacement be half of its amplitude?

Ans. $y = A \sin\omega t = A\sin\frac{2\pi}{T} t$

$A/2 = A\sin\frac{2\pi}{T} t$

$\sin\pi t = 1/2 = \sin\pi/6$

t = 1/6 s

3. Two simple pendulum whose lengths are 100cm and 121cm are suspended side by side. Their bobs are pulled together and then released. After how many minimum oscillations of the longer pendulum, will the two be in phase again? 3

Explanation:

$T \propto \sqrt{l}$

For the two pendulums in same phase

$nT_1=(n+1)\ T_2$

$n\sqrt{(l_1)}=(n+1)\ \sqrt{(l_2)}$

$n\sqrt{121}=(n+1)\sqrt{100}$

$n\times 11=(n+1)10$

n=10

Ans. Displacement from mean position = $A - A/2 = A/2$.

When the motion starts from the positive extreme position, y = A cosωt.

$\therefore\ \frac{A}{2} = A\cos\frac{2\pi}{T}t.$

$\cos\frac{2\pi}{T}t$ = ½ = $\cos\frac{\pi}{3}$

or $\frac{2\pi}{T}t = \frac{\pi}{3}$

$\therefore$ t = $T/6$

4.The equation of a plane progressive wave is, $y = 10Sin2\pi(t - 0.005x)$where y & x are in cm & t in second. Calculate the amplitude, frequency, wavelength & velocity of the wave.

Ans. Given, y = $10Sin2\pi(t - 0.005x)$…………..

Standard equation for harmonic wave is, $y = A\ Sin2\pi(\frac{t}{T} - \frac{x}{\lambda})$…………..

Comparing eqn (1) & (2), $A = 10,\ \frac{1}{t} = 1,\ \frac{1}{\lambda} = 0.005$

(v) Amplitude A = 10cm

(vi) Frequency $\nu = \frac{1}{T} = 1Hz$

(vii) Wavelength $\lambda = \frac{1}{0.005} = 200cm$

(viii) Velocity v = $\nu\ \lambda$ = 1 x 200 = 200cm/s

5.Write displacement equation respecting the following condition obtained in SHM.

Amplitude = 0.01m

Frequency = 600Hz

Initial phase = $\frac{\pi}{6}$

Ans. Y = A Sin ($2\pi\nu t + \phi_o$)

= 0.01 Sin ($1200\pi t + \frac{\pi}{6}$)

6. The amplitude of oscillations of two similar pendulums similar in all respect are 2cm & 5cm respectively. Find the ratio of their energies of oscillations.

Ans.$\frac{E_1}{E_2} = (\frac{A_1}{A_2})^2 = \left(\frac{2}{3}\right)^2 = 4:25$

Level 3

1.A simple pendulum has time period T_1. The point of suspension is now moved upward according to the relation y = Kt^2 (K = 1 m/s^2), where y is the vertical displacement. The time period becomes T_2. What is the ratio T_1^2/T_2^2? g = 10 m/s^2

Ans : $T_1 = 2\pi \sqrt{l/g}$

$Y = Kt^2$, $\frac{dy}{dx}$ = v = 2Kt, (upward velocity)

Upward acceleration, a = 2 K= 2 m/s^2

$T_2 = 2\pi \sqrt{\frac{l}{g+a}}$, $T_1^2/T_2^2 = 6/5$

2.A simple pendulum consisting of an inextensible length 'l' and mass 'm' is oscillating in a stationary lift. The lift then accelerates upwards with a constant acceleration of 4.5 m/s^2. Write expression for the time period of simple pendulum in two cases. Does the time period increase, decrease or remain the same, when lift is accelerated upwards?

Ans. When the lift is stationary, $T = 2\pi\sqrt{\frac{l}{g}}$

When the lift accelerates upwards with an acceleration of 4.5 m/s^2

$T' = 2\pi\sqrt{\frac{l}{g+45}}$

Therefore, the time period decreases when the lift accelerates upwards.

3.A simple Harmonic motion has an amplitude A and time period T. What is the time taken to travel from x = A to x = A/2.

HOT

1.A mass M is suspended from a spring of negligible mass . The spring is pulled a little and then released so that the mass executes **SHM** of time period T.If the mass is increased by m,the time period becomes 5T/3. What is the ratio m/M ?

Ans : $T = 2\pi \sqrt{m/k}$ with m + M, Time period becomes , $5T/3 = 2\pi \sqrt{m + M/k}$

m/M = 16/9

2.A body oscillates with SHM according to the equation (in SI units),

$x = 5 \cos [2\pi \text{ rad s}^{-1}\, t + \pi/4]$.

At t = 1.5 s, calculate the (a) displacement,(b) speed and (c) acceleration of the body.

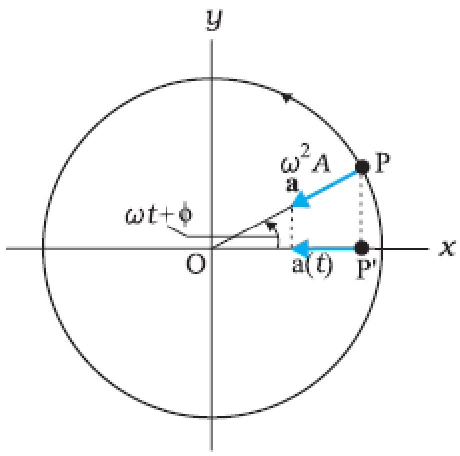
Ans. The angular frequency ω of the body = 2π s^{-1} and its time period T = 1 s.

At t = 1.5 s

(a) displacement $= (5.0 \text{ m}) \cos [(2\pi \text{ s}^{-1}) \times 1.5 \text{ s} + \pi/4]$
$= (5.0 \text{ m}) \cos [(3\pi + \pi/4)]$
$= -5.0 \times 0.707 \text{ m}$
$= -3.535 \text{ m}$

(b) speed $= -(5.0 \text{ m})(2\pi \text{ s}^{-1}) \sin [(2\pi \text{ s}^{-1}) \times 1.5 \text{ s} + \pi/4]$
$= -(5.0 \text{ m})(2\pi \text{ s}^{-1}) \sin [(3\pi + \pi/4)]$
$= 10\pi \times 0.707 \text{ m s}^{-1}$
$= 22 \text{ m s}^{-1}$

(c) acceleration $= -(2\pi \text{ s}^{-1})^2 \times$ displacement
$= -(2\pi \text{ s}^{-1})^2 \times (-3.535 \text{ m})$
$= 140 \text{ m s}^{-2}$



$a(t) = -\omega^2 A \cos(\omega t + \varphi)$
$= -\omega^2 x(t)$

This equation expresses the acceleration of a particle executing SHM. It show that the acceleration of a particle in SHM is proportional to its displacement from the mean position and acts in the opposite direction of the displacement.

(a) A light wave is reflected from a mirror. The incident & reflected wave superimpose to form stationary waves. But no nodes & antinodes are seen, why?

(b) A standing wave is represented by y=2ASinKxCoswt.If one of the component wave is $y_1 = ASin(\omega t - Kx)$, what is the equation of the second component wave?

Ans. (a) As is known, the distance between two successive nodes or two successive antinodes is$\frac{\lambda}{2}$. The wavelength of visible light is of the order of $10^{-7} m$. As such as a small distance cannot be detected by the eye or by a ordinary optical instrument. Therefore, nodes and antinodes are not seen.

(b) As, $2SinACosB = Sin(A + B) + Sin(A - B)$

$$y = 2ASinKxCos\omega t$$

$$= ASin(Kx + \omega t) + ASin(Kx - \omega t)$$

According to superposition principle,

$$y = y_1 + y_2$$

$$and\ y_1 = ASin(\omega t - Kx) = -ASin(Kx - \omega t)$$

$$y_2 = y - y_1 = 2ASinKxCos\omega t + ASin(Kx - \omega t)$$
$$= ASin(Kx + \omega t) + 2ASin(Kx - \omega t)$$
$$= ASin(Kx + \omega t) - 2ASin(\omega t - Kx)$$

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