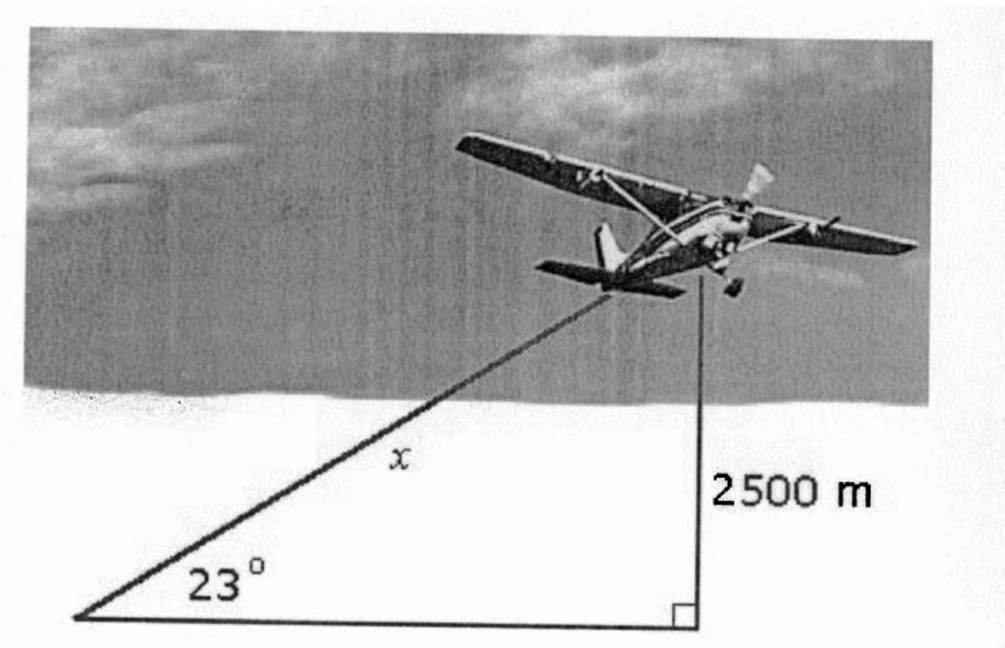
# Chapter 14 Packet
# Trigonometric Applications

**In this unit, students will be able to:**

- **Use the law of cosines to determine a missing side of a triangle**
- **Use the law of cosines to determine a missing angle of a triangle**
- **Find the area of any triangle**
- **Use the law of sines to determine a missing side of a triangle**
- **Use the law of sines to determine a missing angle of a triangle**
- **Determine the number of distinct triangles that can be made based on initial conditions (ambiguous case of the law of sines)**
- **Set up an solve force (vector) problems**




**Name:**______________________________

**Teacher:**____________________________

**Pd:** _______

# Table of Contents

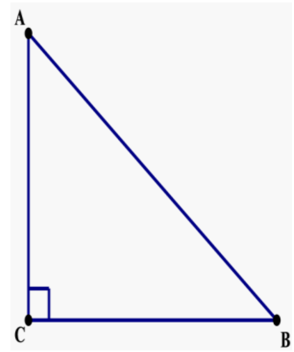


**ANSWERS TO THESE ASSIGNMENTS START ON PAGE 33 IN THIS PACKET**

# A2T: Chapter 14: Trigonometric Application
# Day 1: 14-2 and 14-3 Using Law of Cosines to find a side or an angle

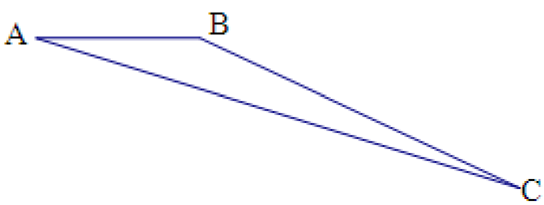
**SWBAT: use the law of cosines to solve for the missing side(s) and the missing angle(s) of a triangle**

***Warm - Up:***
Solve for the missing side of the triangle, given ∡C is a right angle BC= 5 and AB = 10, find AC.




Find the measure of ∡B.

Can we solve for the missing side of this triangle using the same methods as above? Why or why not?

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## <u>Triangle Review</u>

<u>Sum of the Degrees in a Triangle</u>:

<u>Labeling a Triangle</u>:

- lowercase letters for sides.
- UPPERCASE letters for angles.
- The same letter for a side and the opposite angle.

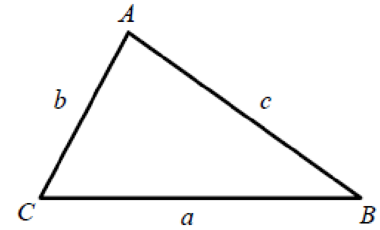
The smallest angle is across from the smallest side, ____.
The largest angle is across from the largest side, _____.

In what types of triangles can you use the Pythagorean Theorem and SOH-CAH-TOA?

An acute or obtuse triangle is called an __________ triangle

## THE LAW OF COSINES

$$c^2 = a^2 + b^2 - 2ab\cos C$$




Connection to Proofs:
Use when given:
**SSS** or **SAS**

Law of Cosines
**WORKING WITH 3 SIDES**
An Angle/Side pair must start and finish the equation. One of them will be **unknown** since it is what you are finding.

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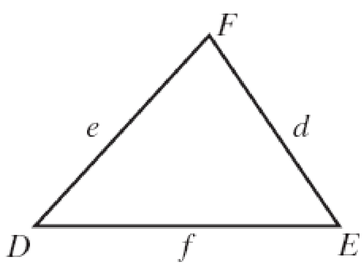
Given ΔDEF, write the law of cosines you would need to find side *d,* side *e*, side *f.*




$d^2$ =

$e^2$ =

$f^2$ =

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Concept 1: Using The Law of Cosines for determine a missing side (given SAS)

**Example 1:**

**In $\triangle ABC$, $a$ = 5, $c$ = 7, and cos $B$ = $\frac{1}{7}$. Find $b$.**

**Example 2:**

In $\triangle ABC$, $b = 12.4$, $c = 8.70$, and $m\angle A = 23$.

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Example 3: Given: c=12, b=15, $m\angle A = 84^\circ$. Find a.

## Concept 2: Using The Law of Cosines to determine missing angles (given SSS)

The Law of Cosines can be used to find a missing angle if given the lengths of all three sides of the triangle (SSS). We can do this by solving for the unknown cosine value and then using the $\cos^{-1}$ function to find the angle measure.

Example 4:

Jed is working on a stained glass project and needs to form a triangle with sides of 8, 12, and 15 inches out of lead cane to enclose the glass. To the nearest tenth of a degree, what is the largest angle he needs to create using the lead caning?

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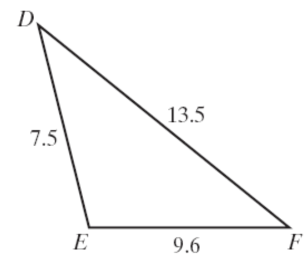
Example 5:

Find, to the nearest degree, the measure of the largest angle of $\triangle DEF$ if $DE = 7.5$, $EF = 9.6$, and $DF = 13.5$.

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Example 6:

In triangle $DEF$ below, $DE = 11$, $EF = 15$, and $DF = 20$. Determine $m\angle D$ to the nearest degree.

## Challenge:

The angles of a triangle are in a ratio of 1 : 3 : 8. The ratio of the longest side of the triangle to the next longest side is

(A) 8 : 3 (B) $\sqrt{2} : 1$

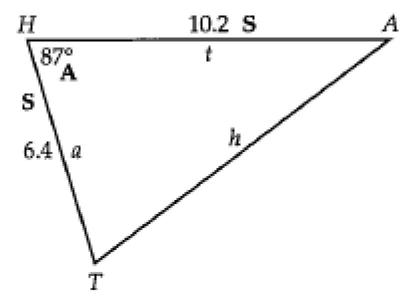
(C) $2 : \sqrt{3}$ (D) $\sqrt{6} : 2$

(E) $\sqrt{8} : \sqrt{3}$

## SUMMARY:

1. In triangle $HAT$, $a = 6.4$, $t = 10.2$, and $m\angle H = 87$. Find the length of side $h$ to the nearest tenth.

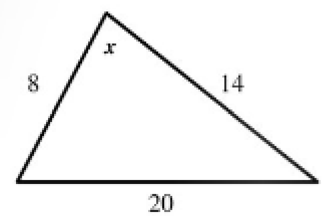
$$h^2 = a^2 + t^2 - 2at\cos H$$
$$h^2 = (6.4)^2 + (10.2)^2 - 2(6.4)(10.2)\cos 87°$$
$$h^2 = 40.96 + 104.04 - 130.56\cos 87°$$
$$h^2 = 138.1670176$$
$$h \approx 11.8$$




2. A triangle has three sides that measure 8, 14, and 20 centimeters.

(a) Find the measure of the largest angle of this triangle to the nearest *tenth* of a degree.

$$20^2 = 8^2 + 14^2 - 2(8)(14)\cos x$$
$$400 = 260 - 224\cos x$$
$$140 = -224\cos x$$
$$\cos x = -\frac{140}{224} \Rightarrow x = 128.682... \approx 128.7°$$




## Exit Ticket:

In $\Delta ABC$, if $a = 8$, $b = 5$, and $c = 9$, then $\cos A$ is

1) $\frac{7}{15}$ 3) $\frac{1}{4}$

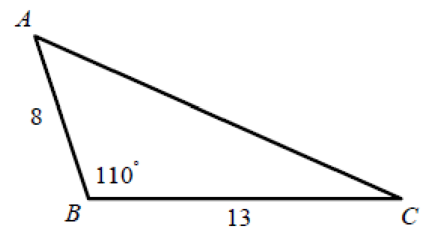
2) $-\frac{7}{15}$ 4) $-\frac{1}{4}$

## Day 2: 14-2 and 14-3 Using Law of Cosines to find a side or an angle

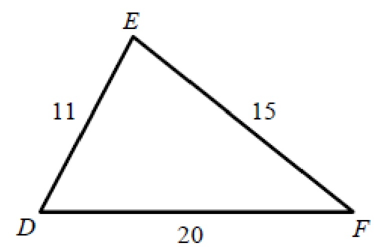
***Exercise* #1:** If $m\angle C = 90^\circ$ then the first form of the Law of Cosines results in what famous theorem? Justify your answer by substituting $90^\circ$ for $C$.

The Law of Cosines, like the Law of Sines, serves two primary purposes, namely finding missing sides and angles in a triangle. These two skills will be illustrated in *Exercises* #2 and #3, respectively.

***Exercise* #2:** In $\Delta ABC$ below, $AB = 8$, $BC = 13$, and $m\angle B = 110^\circ$. Find the length of $\overline{AC}$ to the nearest *tenth*.




***Exercise* #3:** In triangle $DEF$ below, $DE = 11$, $EF = 15$, and $DF = 20$. Determine $m\angle D$ to the nearest degree.

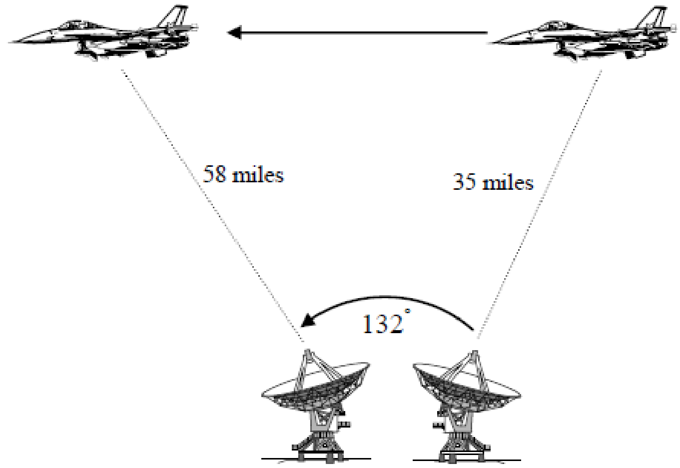



***Exercise* #4:** A triangle has three sides that measure 8, 14, and 20 centimeters.

(a) Find the measure of the largest angle of this triangle to the nearest *tenth* of a degree.

(b) Using your answer from (a), determine the area of this triangle to the nearest square centimeter.

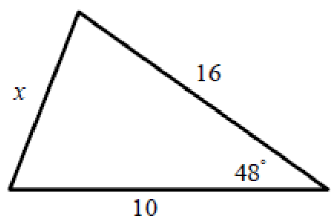
***Exercise* #5:** A satellite dish can track the speed of a plane by recording the distance to the plane at two points in time and the angle through which the dish rotates. In the picture below, a satellite measured the distance to a jet at 35 miles. After 0.25 hours (15 minutes), it measured the distance to the jet at 58 miles. If the satellite rotated through an angle that measured $132^\circ$, determine the average speed of the plane to the nearest mile per hour.



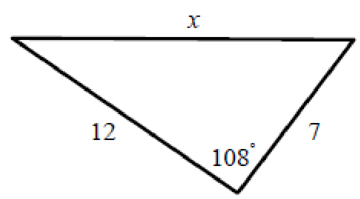

## Skills

1. In each of the triangles below, a S.A.S. scenario is given. Find the value of $x$ in each case to the nearest *tenth* using the Law of Cosines.

(a)

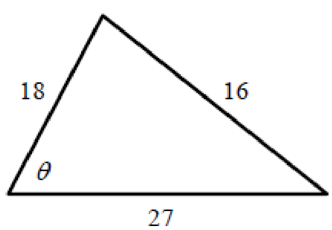



(b)

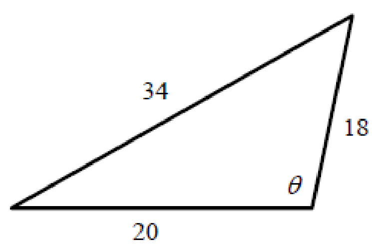



2. In each of the triangles below, a S.S.S. scenario is given. Using the Law of Cosines, find the measure of $\theta$ as marked in each triangle accurate to the nearest *degree*.

(a)




(b)



## Day 3 – A2T: 14-4 Area of a Triangle (given SAS)
## 14-5 Law of Sines (given AAS or ASA)

SWBAT: use the formula to find the area of triangles without using an altitude; use the Law of Sines to find the missing side of a triangle

## Warm – Up: QUIZ

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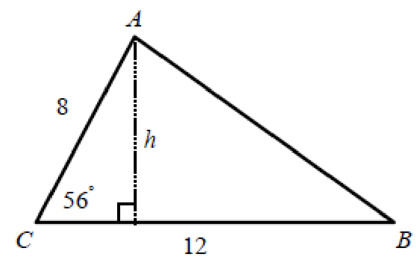
Using the sine function, we will now develop a formula for finding the area of a triangle if we know two sides and the measure of the angle included (between) the two sides.

***Exercise* #1:** In $\Delta ABC$ shown below, $AC = 8$ inches, $BC = 12$ inches and $m\angle C = 56^\circ$.

(a) Determine the height, $h$, of the triangle in terms of $\sin 56^\circ$.




(b) Determine the area of the triangle. Round your answer to the nearest square inch.

$A = \frac{1}{2}bh$

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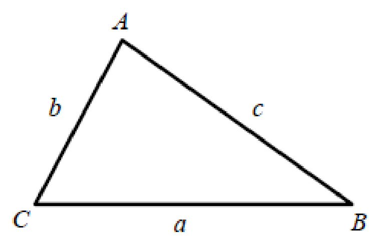
This process can be generalized with the following formula:

**THE AREA FORMULA FOR A TRIANGLE**

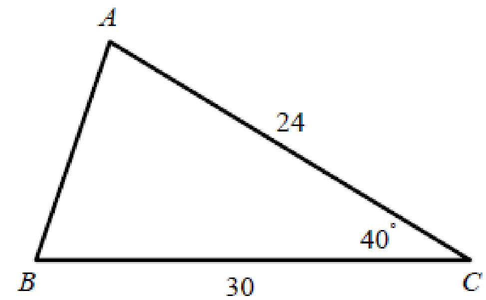


## Concept 1: Calculating the area of a triangle given SAS

**Example 1:**

In triangle $ABC$ shown below, $AC = 24$ cm, $BC = 30$ cm and $m\angle C = 40^\circ$.

(a) Determine the area of $\Delta ABC$ to the nearest square centimeter.




(b) Using your answer from part (a), determine the length of the altitude drawn from $B$ to side $\overline{AC}$.

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**Example 2:** An isosceles triangle has legs of length 12 inches and base angles that measure $32^\circ$ each. Find the area of this triangle to the nearest *tenth* of a square inch. Draw a picture to illustrate the triangle.

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**Example 3:** Which of the following represents the exact area, in square inches, of an equilateral triangle whose sides have a length of 10 inches?

(1) $50\sqrt{2}$ (3) $25\sqrt{2}$

(2) $10\sqrt{3}$ (4) $25\sqrt{3}$

________

## Concept 2: Calculating the Area of a Parallelogram

The area of a parallelogram is equal to twice the area of one of the triangles formed by one of the diagonals of the parallelogram:

**Example 4:** A garden plot is being designed in the shape of a parallelogram whose sides have lengths of 40 feet and 30 feet. If the acute angles of the parallelogram measure $45^{\circ}$, determine, to the nearest square foot, the area of the garden.

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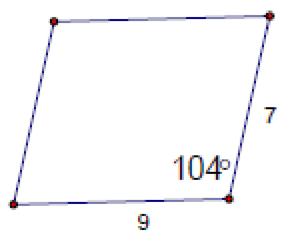
**Example 5:** Find the area of the parallelogram below to the *nearest unit*.



# A2T: 14-5 Law of Sines (given AAS or ASA)

We just learned that the area, *K*, of a (any) triangle is $k = \frac{1}{2}ab\sin C = \frac{1}{2}ac\sin B = \frac{1}{2}bc\sin A$

If we divide $\frac{1}{2}abc$ by each of these terms, we derive a new formula called the Law of Sines:

$$k = \frac{1}{2}ab\sin C = \frac{1}{2}ac\sin B = \frac{1}{2}bc\sin A$$

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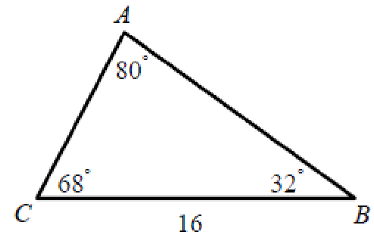
**Example 6:** In triangle $ABC$ shown below, $BC = 16$, $m\angle A = 80^\circ$, $m\angle B = 32^\circ$, and $m\angle C = 68^\circ$. Determine the lengths of $\overline{AB}$ and $\overline{AC}$ to the nearest *tenth*.

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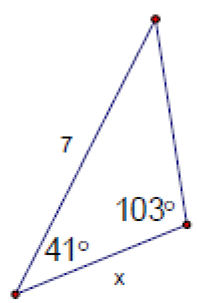
**Example 7:** Solve for x.




"Trick"

**CIRCLE The PAIRS**

You must have 1 Angle/Side Pair where you **know** the values.

**Example 8:**

In $\triangle ABC$, if $a = 24$, $m\angle A = \frac{\pi}{6}$, and $m\angle B = \frac{\pi}{2}$, find the exact value of $b$ in simplest form.

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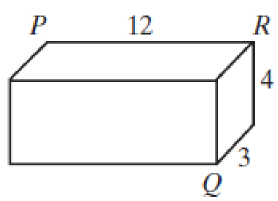
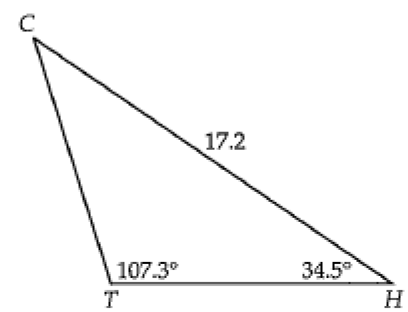
**Challenge:**




In the rectangular box pictured,
$\sin \angle RPQ$ equals

## SUMMARY:

In triangle $CTH$, $m\angle T = 107.3$, $m\angle H = 34.5$, and $CH = 17.2$ centimeters. Find the length of $\overline{CT}$ to the nearest tenth of a centimeter.




$$\frac{\sin T}{t} = \frac{\sin H}{h}$$

$$\frac{\sin 107.3°}{17.2} = \frac{\sin 34.5°}{h}$$

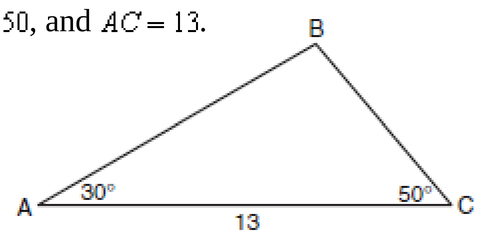
$$h = \frac{17.2 \sin 34.5°}{\sin 107.3°}$$

$$h = 10.20379898$$

$$h = 10.2 \text{ cm}$$

## Exit Ticket:

In the accompanying diagram of $\triangle ABC$, $m\angle A = 30$, $m\angle C = 50$, and $AC = 13$.




What is the length of side $\overline{AB}$ to the *nearest tenth*?

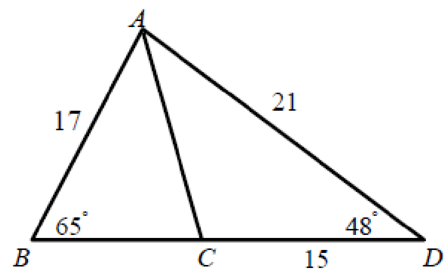
1) 6.6
2) 10.1
3) 11.5
4) 12.0

# Day 4 – A2T: 14-6 The Ambiguous Case of the Law of Sines (SSA)

**SWBAT:** determine the number of triangles possible when two sides and an angle opposite one of the sides is given.

## Warm – Up:

In $\Delta ABD$ below, $m\angle B = 65^{\circ}$, $m\angle D = 48^{\circ}$, $CD = 15$, $AD = 21$, and $AB = 17$. If points $B$, $C$, and $D$ are collinear find all possible values for $m\angle ACB$ to the nearest degree.. Hint – First use the Law of Cosines to find $AC$ then use your result along with the Law of Sines.

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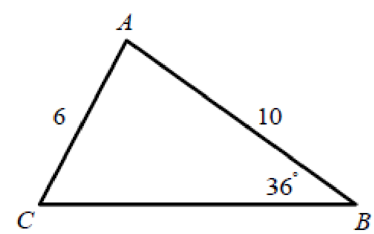
The Law of Sines is known as **ambiguous** because it does not always result in a unique angle solution for a triangle. In fact, the Law of Sines can result in zero, one or two possible angles, and hence triangles, in a given scenario. Which case exists is easily determined by simply solving the trigonometric equation that results from the Law of Sines.

***Exercise* #1:** In triangle $ABC$, which is shown below but not drawn to scale, it is known that $AC = 6$, $AB = 10$, and $m\angle B = 36^{\circ}$. Determine *all possible* values for $m\angle C$ to the nearest *tenth*.

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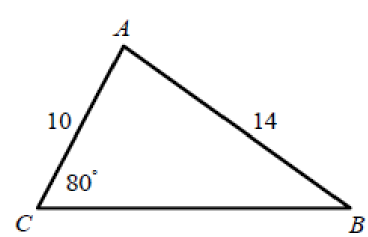
***Exercise* #2:** In triangle $ABC$, which is shown below but not drawn to scale, it is known that $AC = 10$, $AB = 14$, and $m\angle C = 80^{\circ}$.

(a) Solve an equation to find all possible values for the measure of $B$ to the nearest *tenth*.




(b) Considering the measures the angles of any triangle must sum to be $180^{\circ}$, why must we reject the obtuse solution from part (a)?

***Exercise* #3:**
How many triangles can be formed in which the shortest side measures 9 inches, the longest side measures 14 inches and the smallest angle measures $48^\circ$?

(1) 1 (3) 3

(2) 2 (4) 0

________

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***Exercise* #4:** In $\Delta DEF$, $m\angle E = 72^\circ$, $DE = 12$, and $DF = 15$. How many triangles are possible given this information?

(1) 1 (3) 3

(2) 2 (4) 0

________

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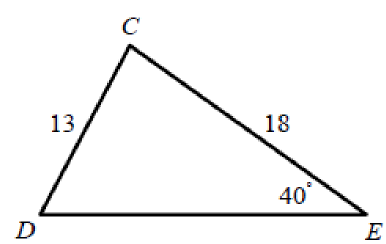
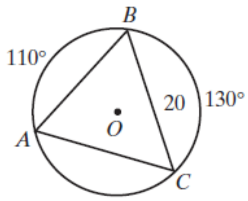
***Exercise* #5:** For $\Delta CDE$, which is shown below not drawn to scale, it is known that $CD = 13$ inches and $CE = 18$ inches. If $m\angle E = 40^\circ$, find all possible *areas* for *CDE*. Round your final answer(s) to the nearest square inch.



## Challenge:

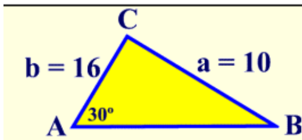



Triangle $ABC$ above is inscribed in circle $O$, $m\overset{\frown}{AB} = 110$, $m\overset{\frown}{BC} = 130$, and $BC = 20$. Find the area of $\triangle ABC$ to the *nearest integer*.

## Summary/Closure:

**Facts we need to remember:**

1. In a triangle, the sum of the interior angles is 180°.
2. No triangles can have two obtuse angles.
3. The sine function has a range of $-1 \le \sin\theta \le 1$.
4. If the $\sin\theta$ = positive decimal < 1, the $\theta$ can lie in the first quadrant (acute ∠) or in the second quadrant (obtuse ∠).




$$\frac{a}{\sin A} = \frac{b}{\sin B} \Rightarrow \frac{10}{\sin 30°} = \frac{16}{\sin B}$$

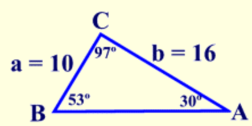
$$10(\sin B) = 16 \cdot \sin 30°$$

$$\sin B = \frac{16 \cdot (0.5)}{10} = 0.8$$

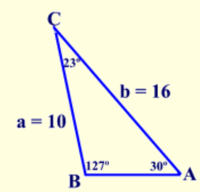
$B = \sin^{-1}(.8) = 53.13010 = 53°$.

Angles could be 30°, 53°, and 97° : sum 180°

The angle from Quadrant II could create angles 30°, 127°, and 23° : sum 180°



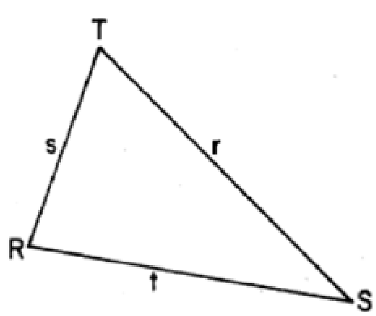

**TWO triangles** possible.




This example is the **Ambiguous Case.** The information given is the postulate SSA (or ASS, the Donkey Theorem), but the two triangles that were created are clearly not congruent. We have two triangles with two sides and the non-included angle congruent, but the triangles are not congruent to each other.

## Exit Ticket:

In triangle $RST$, what is the value of $r$ in terms of $R$, $T$, and $t$?




1) $r = \frac{tR}{T}$

2) $r = \frac{t \cdot \sin T}{\sin R}$

3) $r = \frac{\sin T}{t \cdot \sin R}$

4) $r = \frac{t \cdot \sin R}{\sin T}$

## Day 5 – A2T: Word problems – Applications of Law of Sines/Cosines

**SWBAT:** Use all triangle information to solve triangles.

**Warm – Up: QUIZ**

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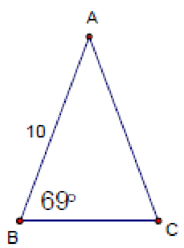
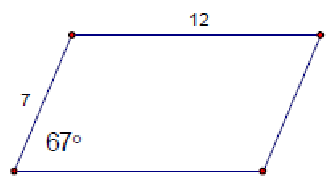
This lesson we will be looking at different situations where we can use the trig. laws and the area of a triangle formula. Below are some common shapes that arise in these problems and their most important properties. Fill in everything else that you know on each shape.

| **Isosceles Triangles** | **Parallelograms** |
|---|---|
| ➢ 2 sides congruent (called the legs)<br>➢ Base angles congruent<br><br> | ➢ Opposite Sides Parallel<br>➢ Opposite Sides Congruent<br>➢ Opposite Angles Equal<br>➢ Adjacent Angles Supplementary<br>➢ Can be cut into two congruent triangles<br><br> |

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| **Given Information** | **Formula to Use** |
|---|---|
| Two sides and the included angle (SAS) | Law of Cosines: $c^2 = a^2 + b^2 - 2ab\cos C$ |
| Three sides (SSS) | Law of Cosines: $\cos C = \dfrac{a^2 + b^2 - c^2}{2ab}$ |
| Two angles and the included side (ASA) | Law of Sines: $\dfrac{\sin A}{a} = \dfrac{\sin B}{b} = \dfrac{\sin C}{c}$ |
| Two angles and a nonincluded side (AAS) | Law of Sines: $\dfrac{\sin A}{a} = \dfrac{\sin B}{b} = \dfrac{\sin C}{c}$ |
| Two sides and a nonincluded angle (SSA) (the ambiguous case) | Law of Sines: $\dfrac{\sin A}{a} = \dfrac{\sin B}{b} = \dfrac{\sin C}{c}$ |
| Area of a triangle (SAS) | Area $= \frac{1}{2}ab\sin C$ |
| Area of a parallelogram | Area $= ab\sin C$ |

## Concept 1: SOLVING TRIANGLES

If a side and any 2 other parts of a triangle are known, it is possible to find the measures of the other sides and angles.

**1)**

| | |
|---|---|
| $A = 25°$ | $a = 11$ |
| $B =$ ________ | $b = 14$ |
| $C =$ ________ | $c =$ __________ |

Drawing:

Triangles possible: ____

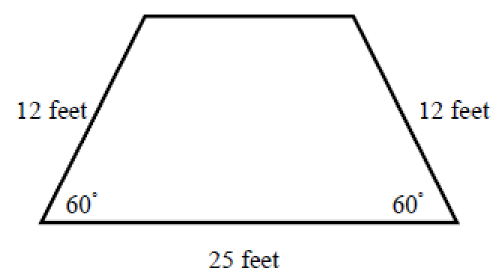
**2)**

| | |
|---|---|
| $A = 68°$ | $a =$ __________ |
| $B =$ ________ | $b = 8$ |
| $C =$ ________ | $c = 5$ |

## Concept 2: Applications of Law of Sines/Law of Cosines
## Example 1:

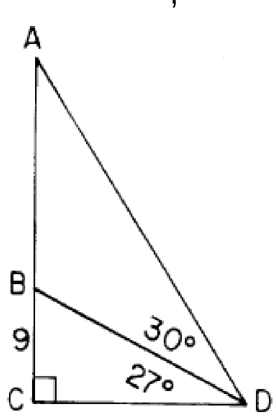
An isosceles triangle has base angles of 53.4° and a base equal to 14.7 inches. Find, to the nearest tenth of an inch, the length of the equal sides of the triangle.

***Exercise* #2:** A carpenter is building a structure in the shape of an isosceles trapezoid whose base angles measure $60^\circ$. The base of the trapezoid has a length of 25 feet, while the legs of the trapezoid have lengths of 12 feet. The carpenter would like to stabilize the trapezoid by placing support beams along the diagonals of the trapezoid at a cost of \$2.50 per linear foot. Determine the combined cost of the diagonals to the nearest dollar.

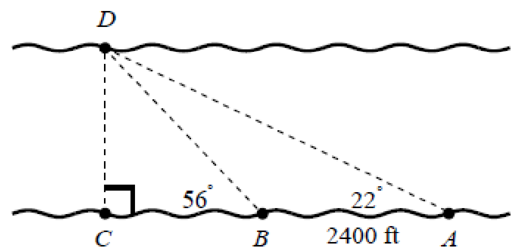



## Example 3:

In the accompanying diagram of a right triangle ACD, B lies on $\overline{AC}$, $\overline{BD}$ is drawn such that $m\angle CDB = 27$, $m\angle BDA = 30$, and $BC = 9$. Find AB to the nearest tenth.



**Example 4:** To measure the distance across a wide river surveyors use a technique of measuring angles to a fixed point on the other side of the river. In the diagram below, a survey starts a point $A$ and finds $m\angle DAC = 22^\circ$. The survey then moves 2400 feet to point $B$ and finds $m\angle DBC = 56^\circ$. Using this information, find the length of $\overline{DC}$, the distance across the river, to the nearest foot. Note, although common, this survey technique assumes the river's sides are relatively parallel and straight.



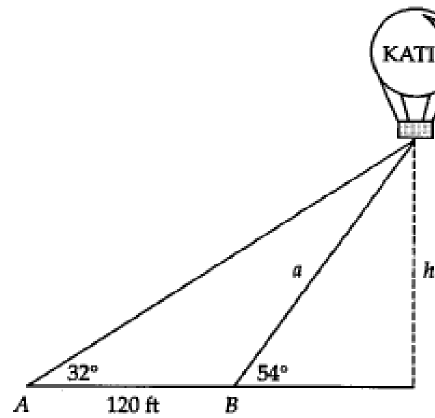

**Example 5:** A town planning board wishes to place sod on their village commons that is in the shape of a triangle whose sides have lengths of 120 feet, 165 feet, and 200 feet. If the sod costs \$0.35 per square foot, determine the cost, to the nearest dollar, for covering the commons in sod.

## SUMMARY:

Katie is out with her parents at the Long Island Fair when she sees a large balloon with her name on it. Her dad tells her the angle of elevation from where she is standing to the foot of the balloon is 32° but Katie is in too much of a hurry to get closer to the balloon to listen. She runs 120 feet toward the area where the balloon is hovering before her mom, a math teacher, catches up with her and says that the angle of elevation from where she is now to the foot of the balloon is 54°. But Katie wants to know only one thing. "I want to go up there. How high up is it?" she asks. Answer Katie's question to the nearest tenth of a foot.

**SOLUTION**

First we must find the length of the hypotenuse, $a$, between the two triangles using the Law of Sines. In order to do this, we must realize that $\angle ABK$ is 126° and $\angle AKB$ is 22°.




$$\frac{\sin 32^\circ}{a} = \frac{\sin 22^\circ}{120}$$

$$a = \frac{120 \sin 32^\circ}{\sin 22}$$

$$a = 169.75$$

Now we can use the SOH-CAH-TOA rule to find the height of the right triangle.

$$\sin 54^\circ = \frac{h}{169.75}$$

$$h = 137.3$$

We can tell Katie that the balloon is 137.3 feet up in the air.

## Exit Ticket:

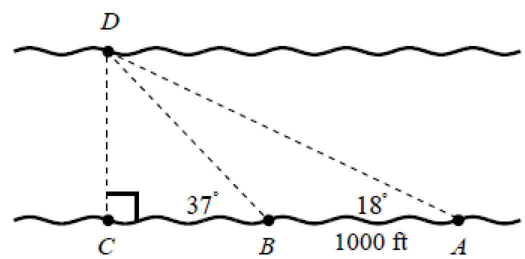
In $\triangle XYZ$, $m\angle X = 45$, $m\angle Y = 60$, and $XY = 8$. The measure of $\overline{XZ}$ is

(1) 7.2
(2) 8.9
(3) 9.8
(4) 10.3

## Trigonometric Applications
## Algebra 2 with Trigonometry - Homework

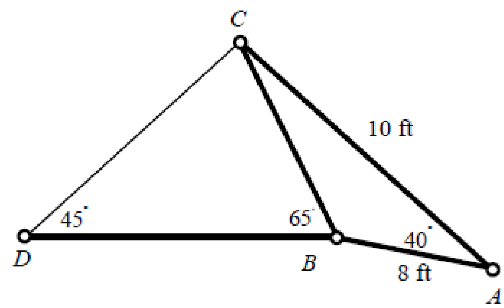
### Applications

1. Measuring Across Another River – A surveyor would like to measure the distance across a river, similar to *Exercise* #1. The surveyor spots a point $D$ across the river at an angle of $18^{\circ}$ from point $A$ as marked on the diagram. The surveyor then moves 1000 feet downstream to point $B$ and measures the angle to $D$ as $37^{\circ}$. Determine the distance across the river from point $C$ to $D$ to the nearest *foot*.




2. A portion of a barn, in the shape of an isosceles triangle, must be painted. The base of the triangle measures 30 feet long and the legs measure 20 feet each. A can of weatherproofing paint will cover 50 square feet of area. What is the minimum number of cans needed to cover this triangular portion? Justify your answer.

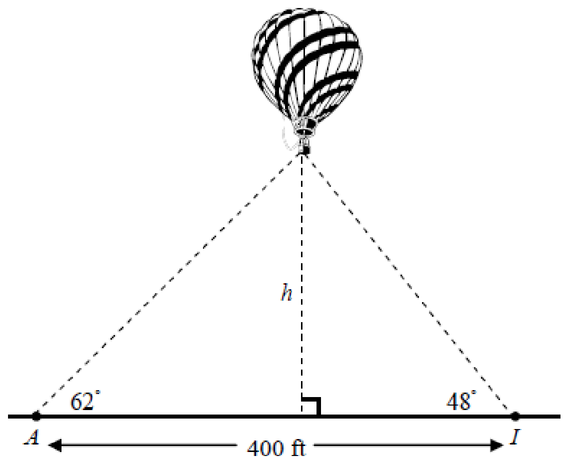
3. A crane is being created by four steel members (bold) and a cable, as shown in the diagram below. It is known that $AC = 10$ ft , $AB = 8$ ft, $m\angle A = 40^\circ$, $m\angle CBD = 65^\circ$, and $m\angle D = 45^\circ$.




(a) Determine the length of support member $\overline{BC}$ to the nearest *hundredth* of a foot.

(b) Determine the length of the cable $\overline{CD}$ to the nearest *hundredth* of a foot.

4. Isaac and Albert spot a hot air balloon flying at a low altitude. They decide to calculate its height by standing 400 feet apart on a level surface and measuring the angle of elevation to the balloon. From Albert, the balloon was at an angle of elevation of $62^\circ$ and from Isaac it was $48^\circ$. Determine the height, $h$, of the balloon above the ground to the nearest foot.

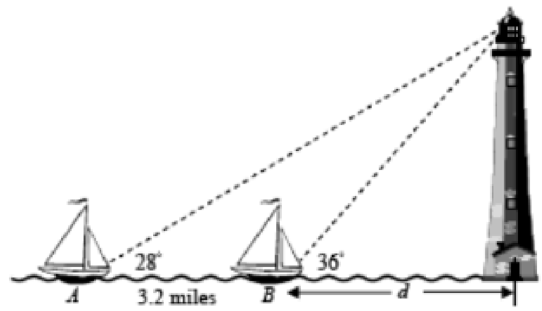


# Day 6 – A2T: Force (Vectors)

**SWBAT**: Use law of sines and law of cosines to solve forces problems

**Warm – Up:**

A boat starts at point A and finds the angle of elevation to the lighthouse measures $28^\circ$. After traveling 3.2 miles towards the light house, the angle of elevation now measures $36^\circ$. Determine the distance, d, from the boat to the lighthouse to the *nearest thousandth* of a mile. Convert your answer to the nearest *hundred feet.* (There are 5280 feet in a mile).

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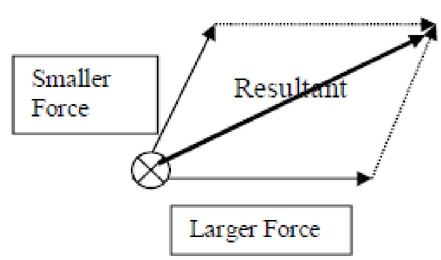
Vocabulary:

- **Force**: pressure exerted on an object, represented by a vector.
- **Vector**: A directed line segment. The length of the line segment is called the magnitude of the force, and the arrow at the end represents the direction of the force. A vector is drawn like a ray, but really isn't a ray, because a ray is infinite in length.
- **The object** is represented by a point.
- **Resultant vector**: the path the object takes after the forces are applied. It is also represented by a vector, somewhere (**not usually in the middle**) between the two forces
- The resultant force is the diagonal (between the forces) of the parallelogram that would be formed with the two forces as adjacent sides.

- The **forces picture always looks the same**!
- Think of the parallelogram as **two congruent triangles**.
- You often have to work with the **supplementary angle**, not the one you are given.
- **The resultant is always closer to the larger force**. In other words, there is a smaller angle between them.



**Concept 1: Calculating the resultant force.**

**Example 1:**

A tractor stuck in the mud is being pulled out by two trucks. One truck applies a force of 1,200 pounds, and the other truck applies a force of 1,700 pounds. The angle between the forces applied by the two trucks is 72. Find the magnitude of the resultant force, to the nearest pound.

**Example 2:**

Forces of 40 pounds and 70 pounds act on a body at an angle measure 60º. Find the magnitude of the resultant of these forces to the nearest hundredth of a pound.

**Concept 2: Calculating either the smaller or larger force.**

**Example 3:**

The measures of the angles between the resultant and two applied forces are 65° and 42°, and the magnitude of the resultant is 24 pounds. Find, to the nearest pound, the magnitude of the larger force.

**Example 4:**

A resultant force of 143 pounds is needed to move a body. Two applied forces act at angles of 35º and 47º. Find to the nearest pound the magnitude of the larger force.

**Concept 3:  Calculating the angle between 2 forces.**

**Example 5:**

Two tow trucks try to pull a car out of a ditch.  One tow truck applies a force of 1,500 pounds while the other truck applies a force of 2,000 pounds.  The resultant force is 3,000 pounds. Find the angle between the two applied forces, rounded to the nearest degree.

**Example 6:**

Two forces of 40 pounds and 55 pounds act on a body, forming an acute angle with each other.  The angle between the resultant and the 40-pound force is 22º20'.  Find, to the nearest ten minutes, the angle between the two given forces.

## SUMMARY:

Two forces act on a body forming a resultant force of 46 pounds. If the angle between the resultant and the smaller force of 19.8 pounds is 54.9°, what is the magnitude of the larger force, to the nearest tenth of a pound?

**SOLUTION**

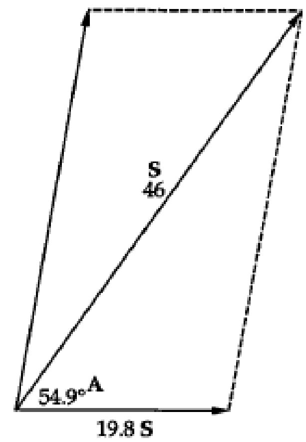
Since the given information forms a Side-Angle-Side pattern, we use the Law of Cosines to solve this problem.

***Note:*** Although we are looking for the length of the second vector, we use the opposite side of the parallelogram in our calculations.

$$c^2 = a^2 + b^2 - 2ab\cos C = 46^2 + 19.8^2 - 2(46)(19.8)\cos 54.9°$$

$$c^2 = 1{,}460.610433$$

$$c = 38.2 \text{ pounds}$$




## Exit Ticket:

We can use the **Law of Sines** to find the missing side of a triangle if we are given

_____ _____ _____ OR _____ _____ _____

We can use the **Law of Sines** to find a missing angle given 2 sides and the non-included angle.

We can use the **Law of Cosines** to find the missing side of a triangle if we are given _____ _____ _____.

We can use the **Law of Cosines** to find the missing angle of a triangle if we are given _____ _____ _____.

## Day 6 - FORCES – HOMEWORK

1. Gerardo and Bennie are pushing a box. Gerardo pushes with a force of 50 pounds in an easterly direction, and Bennie pushes with a force of 39 pounds in a northeasterly direction. The resultant force forms an angle of 32° with the 39–pound force.

   Find the angle between the 50–pound force and the 39–pound force, to the *nearest tenth of a degree.*

   Find the magnitude of the resultant force, to the *nearest pound.*

---

2. Two equal forces act on a body at an angle of 80°. If the resultant force is 100 newtons, find the value of one of the two equal forces, to the *nearest hundredth of a newton.*

3. One force of 20 pounds and one force of 15 pounds act on a body at the same point so that the resultant force is 19 pounds. Find, to the *nearest degree*, the angle between the two original forces.

---

4. *a* Two forces of 25 pounds and 38 pounds act on a body at an angle of 74.5°. Find, to the *nearest tenth of a pound*, the magnitude of the resultant force.
   *b* Using the answer found in part *a*, find the angle between the resultant and the larger force to the *nearest tenth of a degree.*

5. Two forces are applied to an object. The measure of the angle between the 30.2-pound applied force and the 50.1-pound resultant is 25°.

   *a* Find the magnitude of the second applied force to the *nearest tenth of a pound.*

   *b* Using the answer found in part *a*, find the measure of the angle between the second applied force and the resultant to the *nearest degree.*

6. Forces of 40 pounds and 70 pounds act on a body at an angle measuring 60°. Find the magnitude of the resultant of these forces to the *nearest hundredth of a pound.*

7. Two forces act on an object. The first force has a magnitude of 85 pounds and makes an angle of 31°30' with the resultant. The magnitude of the resultant is 130 pounds.

   *a* Find the magnitude of the second force to the *nearest tenth of a pound.*

   *b* Using the results from part *a*, find, to the *nearest ten minutes* or *nearest tenth of a degree*, the angle that the second force makes with the resultant.

8. Two forces act on a body at an angle of 100°. The forces are 30 pounds and 40 pounds.

   *a* Find the magnitude of the resultant force to the *nearest tenth* of a pound.

   *b* Find the angle formed by the greater of the two forces and the resultant force to the *nearest degree.*

9. Two forces act on a body at an angle of 120°. The forces are 28 pounds and 35 pounds.

   *a* Find the magnitude of the resultant force to the *nearest tenth* of a pound.

   *b* Find the angle formed by the greater of the two forces and resultant force to the *nearest degree.*

10. Two forces act on a body to product a resultant force of 70 pounds. One of the forces is 50 pounds and forms an angle of 67°40' with the resultant force. Find, to the *nearest pound*, the magnitude of the other force.

# ANSWER KEYS

# Day 1 – Law of Cosines

## pp. 555

**Developing Skills**

3. $m^2 = a^2 + r^2 - 2ar\cos M$
4. $p^2 = n^2 + o^2 - 2no\cos P$

| | | |
|---|---|---|
| **5.** $2\sqrt{7}$ | **6.** $\sqrt{37}$ | **7.** $8\sqrt{2}$ |
| **8.** 4 | **9.** $\sqrt{26}$ | **10.** $\sqrt{13}$ |
| **11.** $2\sqrt{19}$ | **12.** $9\sqrt{7}$ | **13.** $2\sqrt{10}$ |
| **14.** 5.6 | **15.** 147.0 | **16.** 4.8 |
| **17.** 98.6 | **18.** 1.7 | **19.** 7.5 |

## pp.558

**5**. -0.575

**7.** $\cos A = \frac{7}{8}$, $\cos B = \frac{11}{16}$, $\cos C = -\frac{1}{4}$

**8.** $\cos A = -\frac{1}{8}$, $\cos B = \frac{3}{4}$, $\cos C = \frac{3}{4}$

**9.** $\cos D = -\frac{17}{192}$, $\cos E = \frac{29}{48}$, $\cos F = \frac{61}{72}$

**10.** $\cos P = \frac{37}{40}$, $\cos Q = \frac{13}{20}$, $\cos R = -\frac{5}{16}$

**11.** $\cos M = \frac{11}{80}$, $\cos N = \frac{95}{256}$, $\cos P = \frac{139}{160}$

**12.** $\cos A = \frac{12}{13}$, $\cos B = \frac{5}{13}$, $\cos C = 0$

**13.** $33°, 64°, 83°$

**14.** $36°, 40°, 104°$

**15.** $42°, 51°, 87°$

**16.** $47°, 47°, 86°$

**17.** $48°, 63°, 69°$

**18.** $16°, 74°, 90°$

---

## Day 2 – Law of Cosines

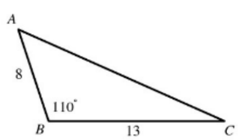
***Exercise #1:*** If $m\angle C = 90°$ then the first form of the Law of Cosines results in what famous theorem? Justify your answer by substituting $90°$ for $C$.

$$\boxed{\begin{array}{c} c^2 = a^2 + b^2 - 2ab\cos 90° \\ \text{But } \cos 90° = 0. \\ c^2 = a^2 + b^2 - 2ab(0) \end{array}} \Rightarrow \boxed{\begin{array}{c} c^2 = a^2 + b^2 \\ \text{The Pythagorean Theorem} \end{array}}$$

The Law of Cosines, like the Law of Sines, serves two primary purposes, namely finding missing sides and angles in a triangle. These two skills will be illustrated in *Exercises* #2 and #3, respectively.

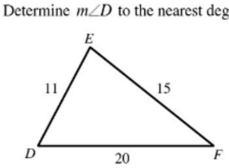
***Exercise #2:*** In $\Delta ABC$ below, $AB = 8$, $BC = 13$, and $m\angle B = 110°$. Find the length of $\overline{AC}$ to the nearest *tenth*.

$$\begin{array}{c} b^2 = 8^2 + 13^2 - 2(8)(13)\cos 110° \\ b^2 = 304.140... \\ b = \sqrt{304.140...} = 17.439... \approx 17.4 \end{array}$$




***Exercise #3:*** In triangle $DEF$ below, $DE = 11$, $EF = 15$, and $DF = 20$. Determine $m\angle D$ to the nearest degree.

$$\begin{array}{c} 15^2 = 11^2 + 20^2 - 2(11)(20)\cos D \\ 225 = 521 - 440\cos D \\ -296 = -440\cos D \Rightarrow \cos D = \frac{296}{440} \Rightarrow D_r = \cos^{-1}\left(\frac{296}{440}\right) = 48° \\ \text{Our solutions lie in Quads. I and IV} \Rightarrow \\ D = 48° \text{ or } D = 360° - 48° = \cancel{312°} \\ \text{Note that unlike sine, cosine is NOT ambiguous.} \end{array}$$



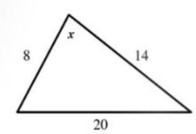

***Exercise #4:*** A triangle has three sides that measure 8, 14, and 20 centimeters.

(a) Find the measure of the largest angle of this triangle to the nearest *tenth* of a degree.

$$\begin{array}{c} 20^2 = 8^2 + 14^2 - 2(8)(14)\cos x \\ 400 = 260 - 224\cos x \\ 140 = -224\cos x \\ \cos x = -\frac{140}{224} \Rightarrow x = 128.682... \approx 128.7° \end{array}$$

$$\begin{array}{c} \text{Area} = \frac{1}{2}(8)(14)\sin 128.7° \\ = 43.704... \approx 44 \text{ cm}^2 \end{array}$$

Convenient Space Below for Diagram



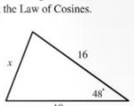

***Exercise #5:***

$$\begin{array}{c} x^2 = 58^2 + 35^2 - 2(58)(35)\cos 132° \\ x^2 = 7305.670... \\ x = \sqrt{7305.670...} = 85.473... \\ \text{Speed} = \frac{\text{Distance}}{\text{Time}} = \frac{85.473... \text{ mi}}{0.25 \text{ hrs}} = 341.892... \approx 342 \text{ mph} \end{array}$$

## Skills

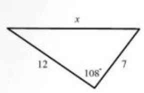
1. In each of the triangles below, a S.A.S. scenario is given. Find the value of $x$ in each case to the nearest *tenth* using the Law of Cosines.

(a)




$$\begin{array}{c} x^2 = 10^2 + 16^2 - 2(10)(16)\cos 48° \\ x^2 = 141.878... \Rightarrow x = \sqrt{141.878...} \\ x = 11.911... \approx 11.9 \end{array}$$

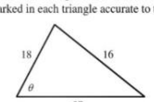
(b)




$$\begin{array}{c} x^2 = 12^2 + 7^2 - 2(12)(7)\cos 108° \\ x^2 = 244.914... \Rightarrow x = \sqrt{244.914...} \\ x = 15.649... \approx 15.6 \end{array}$$

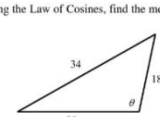
2. In each of the triangles below, a S.S.S. scenario is given. Using the Law of Cosines, find the measure of $\theta$ as marked in each triangle accurate to the nearest *degree*.

(a)




$$\begin{array}{c} 16^2 = 18^2 + 27^2 - 2(18)(27)\cos\theta \\ 256 = 1053 - 972\cos\theta \\ -797 = -972\cos\theta \Rightarrow \cos\theta = \frac{-797}{-972} \\ \theta = \cos^{-1}\left(\frac{797}{972}\right) = 34.919... \approx 35° \end{array}$$

(b)




$$\begin{array}{c} 34^2 = 20^2 + 18^2 - 2(20)(18)\cos\theta \\ 1156 = 724 - 720\cos\theta \\ 432 = -720\cos\theta \Rightarrow \cos\theta = \frac{432}{-720} \\ \theta = \cos^{-1}\left(-\frac{432}{720}\right) = 126.869... \approx 127° \end{array}$$

**3.**

$$\begin{array}{c} 29^2 = 20^2 + 21^2 - 2(20)(21)\cos L \\ 841 = 841 - 840\cos L \\ 0 = -840\cos L \Rightarrow \cos L = \frac{0}{-840} = 0 \\ L = \cos^{-1}(0) = 90° \end{array}$$

**(2)**

## Day 3 - Area of Triangles/Law of Sines

pp. 563 #3-17 eoo, pp.567 #3-19 eoo

### 14-4 Area of a Triangle *(pages 563–564)*

**Writing About Mathematics**

1. Since $\angle A$ and $\angle B$ are supplementary, $\sin A = \sin(180° - A) = \sin B$.
2. Yes. The area of the rhombus is $(PQ)(PS)(\sin P)$. Since the sides are congruent, $(PQ)(PS)(\sin P) = (PQ)^2(\sin P)$.

**Developing Skills**

3. 3 sq units
4. 30 sq units
5. 60 sq units
6. 108 sq units
7. 16.8 sq units
8. 12 sq units
9. 77.5 sq units
10. 24,338.5 sq units
11. 12.6 sq units
12. 25,221.0 sq units
13. 122.0 sq units
14. 36,615.3 sq units
15. $400\sqrt{3}$ m²
16. $36\sqrt{2}$ cm²
17. 480 ft²

pg. 567

**Developing Skills**

3. $3\sqrt{6}$
4. 48
5. $4\sqrt{3}$
6. $4\sqrt{6}$
7. 12.5
8. $\frac{64}{3}$
9. 23.5
10. 31.4
11. 44.5
12. 18.3
13. 97.7
14. 16.9
15. 6.93
16. **a.** 8.85 cm **b.** 32.2 cm
17. **a.** 31.1 in. **b.** 83 in.
18. $\frac{a}{\sin A} = \frac{c}{\sin 90°}$
    $\frac{a}{\sin A} = \frac{c}{1}$
    $\sin A = \frac{a}{c}$

**Applying Skills**

19. **a.** 3.18 ft **b.** 12.3 ft
20. 138.0 ft, 250.2 ft
21. **a.** 14.0 ft **b.** 18.6 ft
22. 3.1 mi
23. $5,909

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## Day 4 - Ambiguous Case/ Law of Sines : pg. 574

**Developing Skills**

3. **a.** 2 **b.** $\{20°, 155°, 5°\}, \{20°, 25°, 135°\}$
4. **a.** 1 **b.** $\{30°, 60°, 90°\}$
5. **a.** 1 **b.** $\{39°, 49°, 92°\}$
6. **a.** 0
7. **a.** 1 **b.** $\{29°, 31°, 120°\}$
8. **a.** 1 **b.** $\{3°, 27°, 150°\}$
9. **a.** 0
10. **a.** 2 **b.** $\{15°, 20°, 145°\}, \{5°, 15°, 160°\}$
11. **a.** 0
12. **a.** 1 **b.** $\{135°, 30°, 15°\}$
13. **a.** 1 **b.** $\{30°, 60°, 90°\}$
14. **a.** 2 **b.** $\{45°, 62°, 73°\}, \{45°, 17°, 118°\}$

**Applying Skills**

15. **a.** 60.07°
    **b.** No, the triangle formed by the ladder, wall, and ground is a right triangle.
16. Yes, there can be only one garden.
    Angles: $\{37°, 68°, 75°\}$
    Sides: $\{5 \text{ ft}, 7.7 \text{ ft}, 8 \text{ ft}\}$

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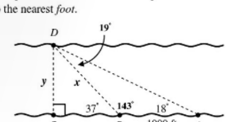
## Day 5 – Applications of Law of Sines/Cosines

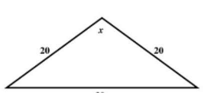
1. Measuring Across Another River – A surveyor would like to measure the distance across a river, similar to *Exercise* #1. The surveyor spots a point $D$ across the river at an angle of 18° from point $A$ as marked on the diagram. The surveyor then moves 1000 feet downstream to point $B$ and measures the angle to $D$ as 37°. Determine the distance across the river from point $C$ to $D$ to the nearest *foot*.

$$\frac{x}{\sin 18°} = \frac{1000}{\sin 19°}$$
$$x = \frac{1000 \sin 18°}{\sin 19°} = 949.162...$$
$$\frac{y}{\sin 37°} = \frac{x}{\sin 90°}$$
$$y = \frac{x \sin 37°}{\sin 90°} = 571.220... \approx 571 \text{ ft}$$

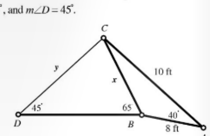



2. A portion of a barn, in the shape of an isosceles triangle, must be painted. The base of the triangle measures 30 feet long and the legs measure 20 feet each. A can of weatherproofing paint will cover 50 square feet of area. What is the minimum number of cans needed to cover this triangular portion? Justify your answer.

$$30^2 = 20^2 + 20^2 - 2(20)(20)\cos x$$
$$900 = 800 - 800\cos x$$
$$100 = -800\cos x \Rightarrow \cos x = \frac{100}{-800}$$
$$x = \cos^{-1}\left(-\frac{100}{800}\right) = 97.180...$$
$$\text{Area} = \frac{1}{2}(20)(20)\sin x = 198.431... \text{ ft}^2$$
$$\# \text{ of Cans} = \frac{198.431... \cancel{\text{ft}^2}}{50 \cancel{\text{ft}^2}/\text{can}} = 3.968... = 4 \text{ cans}$$




3. A crane is being created by four steel members (bold) and a cable, as shown in the diagram below. It is known that $AC = 10$ ft, $AB = 8$ ft, $m\angle A = 40°$, $m\angle CBD = 65°$, and $m\angle D = 45°$.




(a) Determine the length of support member $\overline{BC}$ to the nearest *hundredth* of a foot.

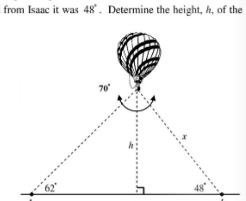
$$x^2 = 8^2 + 10^2 - 2(8)(10)\cos 40°$$
$$x^2 = 41.432...$$
$$x = \sqrt{41.432...} = 6.436... \approx 6.44 \text{ ft}$$

(b) Determine the length of the cable $\overline{CD}$ to the nearest *hundredth* of a foot.

$$\frac{y}{\sin 65°} = \frac{6.456...}{\sin 45°}$$
$$y = 8.2501... \approx 8.25 \text{ ft}$$

4. Isaac and Albert spot a hot air balloon flying at a low altitude. They decide to calculate its height by standing 400 feet apart on a level surface and measuring the angle of elevation to the balloon. From Albert, the balloon was at an angle of elevation of 62° and from Isaac it was 48°. Determine the height, $h$, of the balloon above the ground to the nearest foot.

$$\frac{x}{\sin 62°} = \frac{400}{\sin 70°}$$
$$x = \frac{400 \sin 62°}{\sin 70°} = 375.845...$$
$$\frac{h}{\sin 48°} = \frac{x}{\sin 90°}$$
$$h = \frac{x \sin 48°}{\sin 90°} = 279.307... \approx 279 \text{ ft}$$




## Day 6 - Forces

1. 56.4
   79
2. 65.27
3. 116
4. *a* 50.8; *b* 28.3
5. *a* 26.1; *b* 29
6. 96.44
7. *a* 72.7; *b* 37.7° *or* 37°40'
8. *a* 45.6; *b* 40
9. *a* 32.1; *b* 49
10. 69