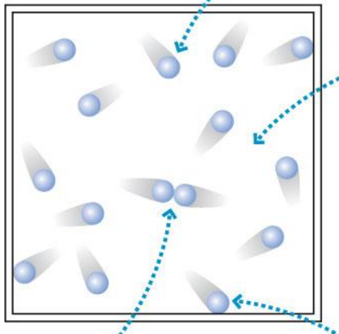
CHAPTER 15
Fluids and Elasticity

In this chapter, we will aim to understand macroscopic systems that flow or deform. We will study the connection between density, pressure, and buoyancy (i.e. the "tendency to sink or float"). At the end of the chapter, we investigate "non-rigid" bodies under conditions of stress.

15.1 Fluids



In everyday speech, we tend to use the terms "fluid" and "liquid" interchangeably. In fact, **a fluid is a substance that flows**, and therefore both gases and liquids are fluids (as are **plasmas**, but that's beyond the scope of PHYS 212).

In a **gas**, each molecule* moves freely in space, occasionally colliding with other molecules or with the walls of its container. The latter collisions, taken on average, are said to **exert pressure** on the walls.

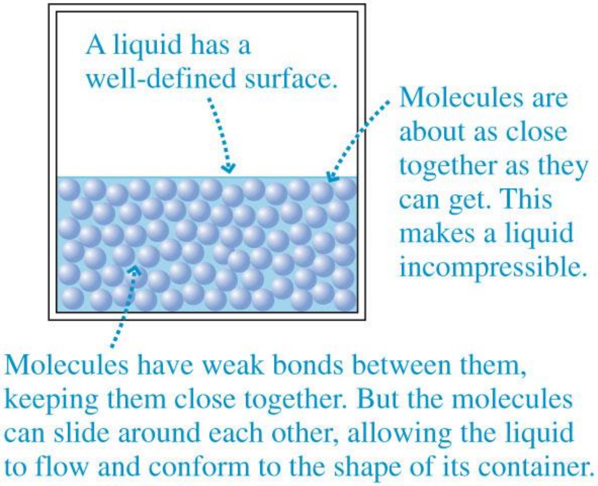
Gases are also **compressible**. Since most of the volume (V) of a container of gas consists of empty space, this volume can easily be increased or decreased.

*or atom, depending on the type of gas.

15.1 Fluids

On the other hand, liquids are effectively **incompressible** (their volumes can only be changed by a tiny fraction). The molecules of a liquid are very close together; they interact weakly with each other – weak enough that they can flow past each other, but strong enough that the interactions can affect the physical properties of the flow.

Liquids will deform to fit the shape of their container, and in the absence of any external factors, they will have a well-defined surface.



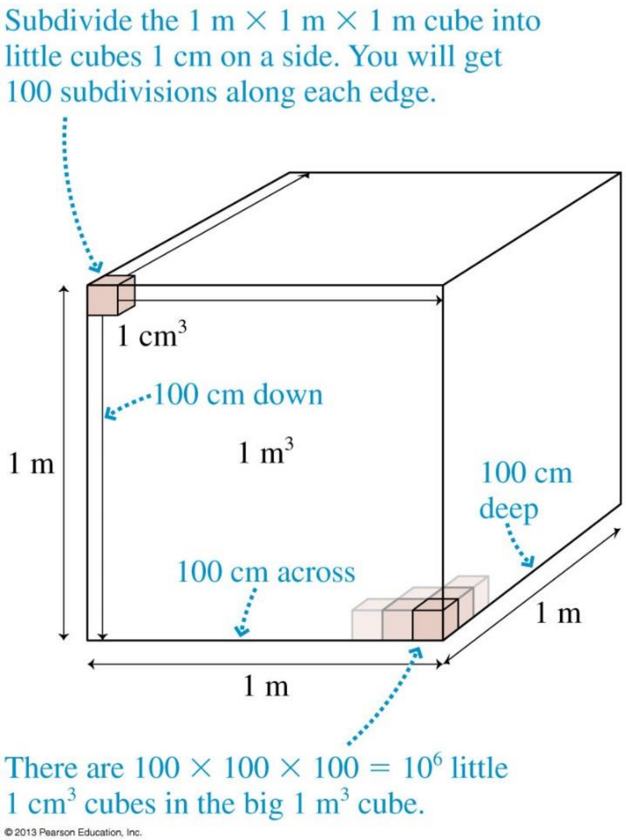
15.1 Fluids

Volume and Density

In studying fluids and thermodynamics, we frequently require the parameter of a system's **volume**. The SI unit for volume is the **cubic meter** (m^3). However, this is a very large quantity for most areas of research (although not for marine science!)

The most common metric unit for volume is the **liter** (L). There are 1000 liters in 1 m^3. In many fields (particularly medicine), the **cubic centimeter** (cm^3) is used. There are 1000 cm^3 in 1 L, and 10^6 cm^3 in 1 m^3. To see this, note that 1 m = 100 cm, so

$$1\ m^3 = (1\ m)^3 = (100\ cm)^3 = (100)^3\ cm^3 = 10^6\ cm^3.$$



15.1 Fluids

TABLE 15.1 Densities of fluids at standard temperature (0°C) and pressure (1 atm)

Substance	ρ (kg/m^3)
Air	1.28
Ethyl alcohol	790
Gasoline	680
Glycerin	1260
Helium gas	0.18
Mercury	13,600
Oil (typical)	900
Seawater	1030
Water	1000

Volume and Density *cont'*

Every object or system of objects has both a mass and a volume. The ratio of these parameters is the **mass density**, ρ

$$\rho = \frac{m}{V}$$

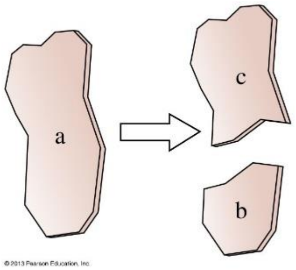
(Often, we just use the term "density". However, there are many other types of density used in different areas of physics, so it's a good idea to be specific).

As the ratio of a mass to a volume, mass density has SI units of kg/m^3. Often, it is more appropriate to use g/cm^3 (1 g/cm^3 = 1000 kg/m^3). The table shown here illustrates a few mass densities. Note that all physical data such as this will be provided for you on exams.

Problem #1: Broken Glass

RDK STT. 15.1

	A piece of glass is broken into two pieces of different size. Rank pieces *a*, *b*, and *c* in order of mass density (from largest to smallest).
A	$a > c > b$
B	$b > c > a$
C	All pieces have the same density



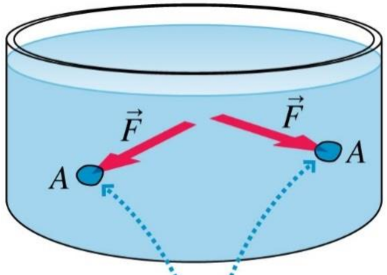
Problem #2: Swimming Pool

RDK Ex. 15.3

A rectangular swimming pool is 6.0 m wide by 12.0 m long. Its depth slopes linearly from a 1.0 m depth at one end to a 3.0 m depth in the other. **What is the mass of the water in the pool?**

Solution: in class

15.2 Pressure



Consider a container that is filled with a fluid, and a small portion of the container's inner surface, which has area A. As we learned in the last section, the fluid's molecules are constantly colliding with this surface. Averaged over time, this represents a force, $\vec{F}$, applied to the surface.

The **pressure** at this portion of the surface is defined as the ratio of the force's magnitude to the area on which the force is exerted:

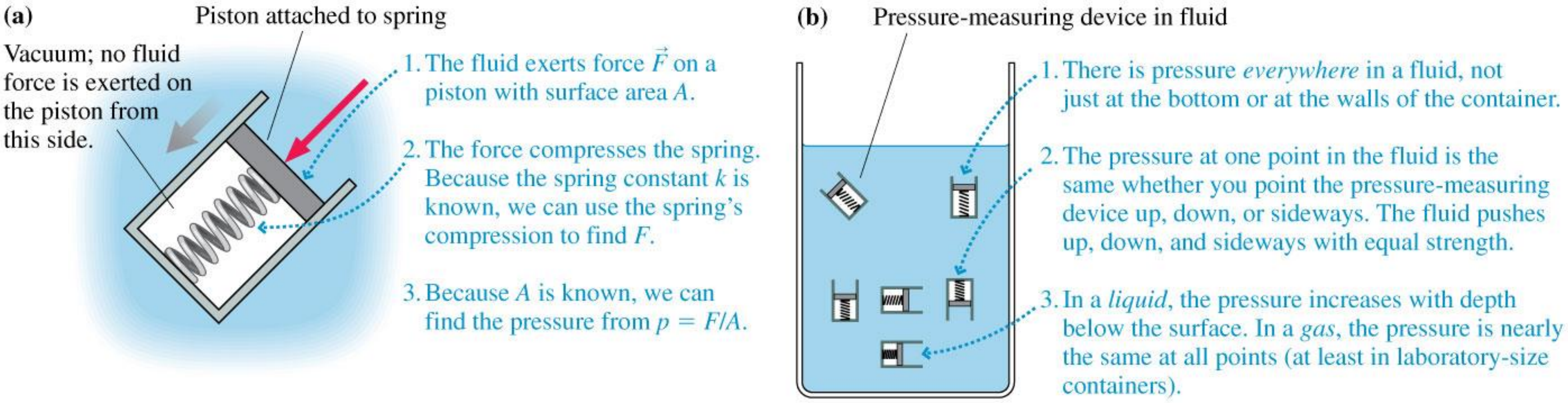
$$p = \frac{F}{A}$$

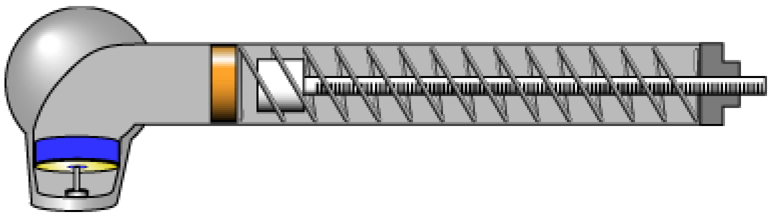
(try not to confuse pressure with linear momentum, which also uses the symbol p).

As the ratio of a force to an area, pressure has SI units of N/m^2. A common SI term is the **Pascal** (1 Pa = 1 N/m^2).

15.2 Pressure

A simple device for measuring pressure is shown below. We'll discuss it in class. This device is the basis for a tire pressure gauge.





15.2 Pressure

Causes of Pressure

There are two primary contributions to pressure. The first is *gravitational* – the fluid is pulled down toward the bottom of its container, causing pressure on the bottom surface (what type of force exists at the surface? Think back to PHYS 211). Since it can flow, the sides of the container beneath the fluid's surface experience pressure as well.

The second contribution is *thermal*. As mentioned previously, collisions between the molecules and the container walls exert tiny forces on the walls. The net force due to all of the collisions results in a pressure. This force depends on the number of collisions per unit time, which depends on the average speed of the molecules, which depends on their temperature (more about that in a future chapter).

15.2 Pressure

Pressure in Gases

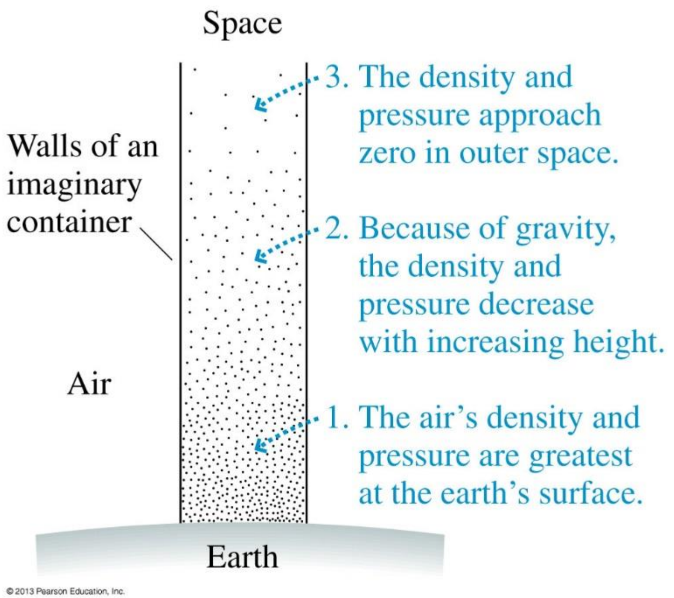
In a container of gas (of the size you might find in the lab), the gravitational contribution to pressure is negligible, since the gravitational force on an individual molecule is tiny (its gravitational acceleration does little to affect its speed, which can be 100s or 1000s of m/s). We can safely assume that **the pressure is constant everywhere in the container**.

If we reduce the number of molecules in the container, the pressure decreases accordingly, simply because there are fewer collisions per unit time. Removing all of the molecules results in a pressure of 0 Pa. However, it is not possible to remove all of the molecules in a container. A "vacuum" really just refers to a situation where the pressure is at least a few orders of magnitude lower than atmospheric pressure.

15.2 Pressure

Atmospheric Pressure

The earth's atmosphere can be thought of as an extraordinarily tall container (with "imaginary" walls). With a height of 10's of km, the gravitational contribution to density and pressure is noticeable – the density slowly decreases with increasing height.



The average pressure of the atmosphere at sea level is 101.3 kPa. This is defined as one **atmosphere:**

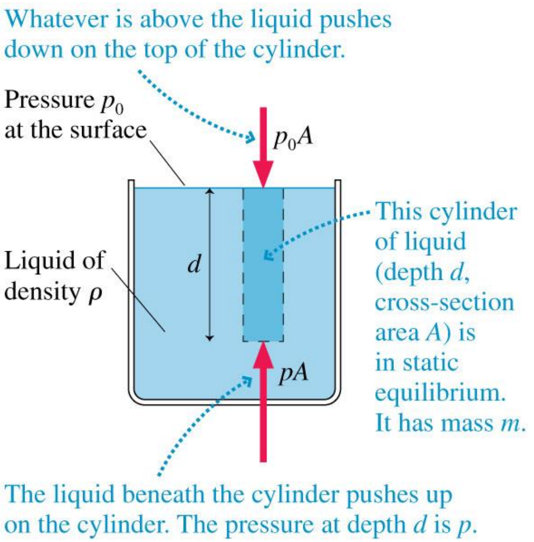
$$1\ \mathrm{atm} = 101.3\ \mathrm{kPa} = 1.0135 \times 10^5\ \mathrm{Pa}$$

Note that the atm is not an SI unit.

15.2 Pressure

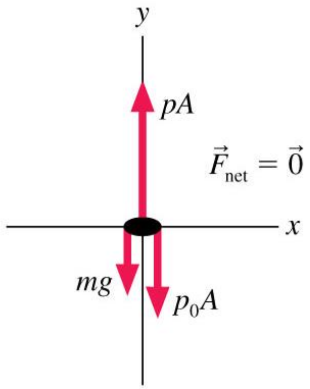
Pressure in Liquids

Due to gravity, a liquid will settle to the bottom of its container. The pressure at the surface is p_0 (usually 1 atm). As we will see, the pressure p within the liquid depends on the depth d below the surface. Assume for now that the liquid has a mass density ρ, and is at rest (it's not flowing) – therefore, Newton's 2nd law tells us that it must be in equilibrium, with $\vec{F}_{\text{net}} = 0$.



We will examine a cylinder of liquid of cross-section area A and height d, extending downward from the surface. There are 3 forces acting on the cylinder: gravitational (mg), a force p_0A pushing downward on the top of the cylinder, and a force pA pushing upward on the bottom of the cylinder.

15.2 Pressure



Free-body diagram of the column of liquid

Pressure in Liquids *cont'*

Since the upward and downward forces must balance, we find that

$$pA = p_0A + mg$$

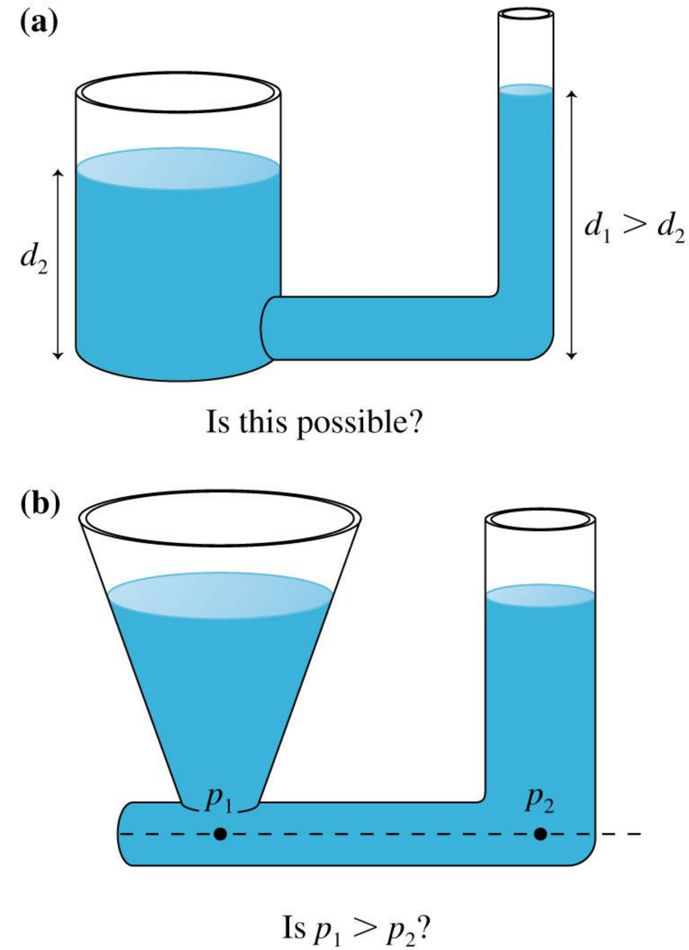
Furthermore, we know that the volume of the cylinder is $V = Ad$, and that its mass is $m = \rho V = \rho Ad$. Combining these equations, we find that the **hydrostatic pressure at a depth d below the surface of a liquid is**

$$p = p_0 + \rho g d$$

(the term "hydrostatic" indicates that the fluid is not moving). Note that this derivation assumes that the density of the liquid is constant with depth; i.e. that the liquid is **incompressible.**

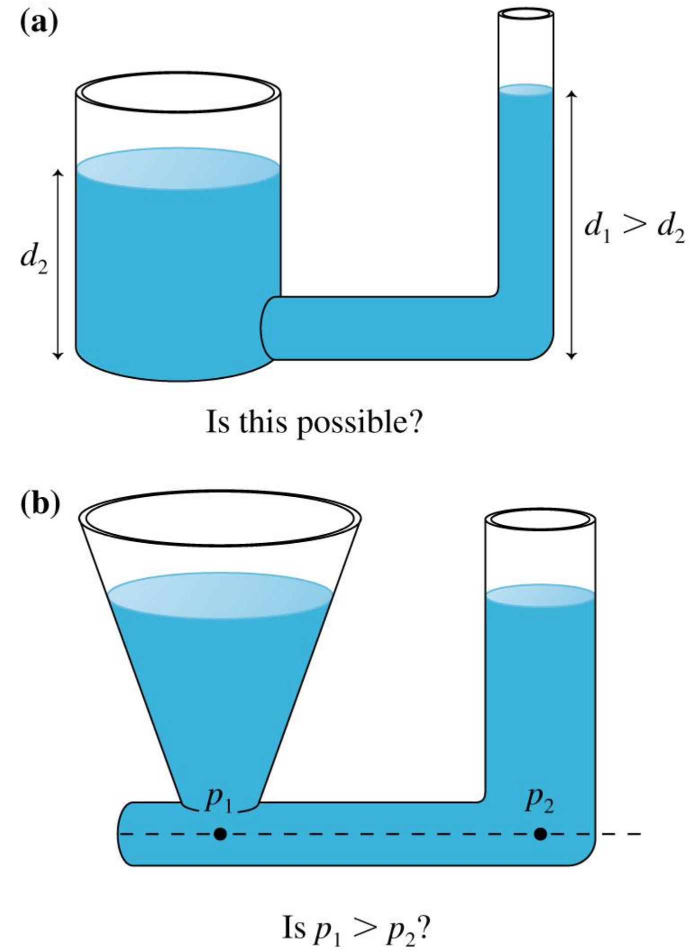
15.2 Pressure

Pressure in Liquids – *Resulting Properties*
Based on the preceding equation, we can see that the situation shown in the top figure is not possible (the two cylinders are open to the atmosphere). If $d_1 > d_2$, then the pressure at the bottom of the narrow cylinder would be greater than that at the bottom of the wide cylinder, and the liquid would flow until the levels equalized. Altogether, we can say that **a connected liquid in hydrostatic equilibrium rises to the same height in all open regions of the container**. If one or both of the cylinders has a closed top, the situation is different, as shown in Example 15.4



15.2 Pressure

Pressure in Liquids – *Resulting Properties*
As a result, **the pressure is the same at all points on a horizontal line through a connected liquid in hydrostatic equilibrium.** This is shown in the bottom figure.
Finally, changing the surface pressure from p_0 to p_1 results in a change in the pressure at depth d from $p = p_0 + \rho g d$ to $p' = p_1 + \rho g d$. The change in pressure is $\Delta p = p_1 - p_0$, and it is the same at all points in the fluid. This is know as **Pascal's principle**



Problem #3: Submarine

RDK Ex. 15.9

A research submarine has a 20-cm-diameter window 8.0 cm thick. The manufacturer says that the window can withstand forces up to 1.0 X 10^6 N. If the pressure inside the submarine is maintained at 1.0 atm, **what is the maximum safe depth of the submarine?**

Solution: in class

Problem #4: Snowshoes

Snow is a powdery white stuff that exists in other parts of the world. People who live there sometimes use **snowshoes** to walk on the surface of deep snow. Let's discuss how these work.

Problem #5: Large Room

RDK Ex. 15.31

A gymnasium is 16 m high (from floor to ceiling). **By what percentage is the air pressure at the floor greater than the air pressure at the ceiling?**

Solution: in class

15.3 Measuring and Using Pressure

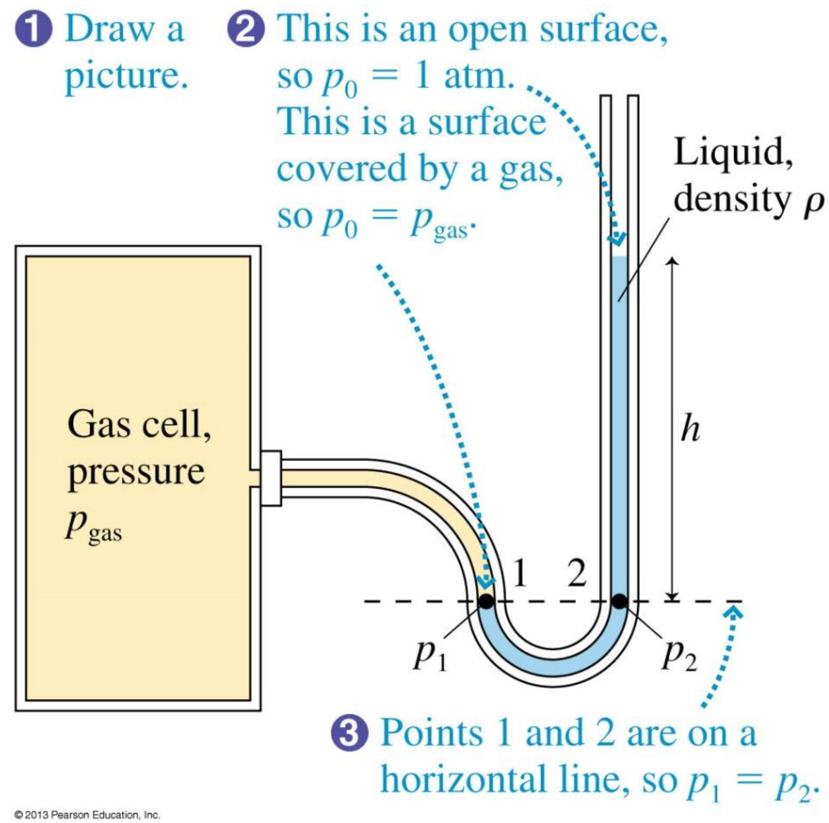
The pressure gauge was described on slide 9. It is important to realize that when the gauge isn't "in use", it's still measuring the ambient pressure (1 atm or 101.3 kPa). Since we usually want to measure pressures *relative to the ambient pressure*, the gauge is calibrated to give a reading of zero when it's not "in use". Overall, we can say that the **gauge pressure** (p_g, the value read off of the gauge) is related to the absolute pressure (p) as

$$p_g = p - 1\text{ atm}$$

Pressure gauges can be found in many labs on campus, particularly on gas cylinders. You may also have seen them on the propane tank of a grill, where the pressure reading gives a good indication of the amount of propane remaining in the tank.

Manometers

A manometer is a device used to measure gas pressure. It is simply a U-shaped tube, connected to the gas container at one end and open to the atmosphere at the other end. It is filled with a liquid of density *ρ.*



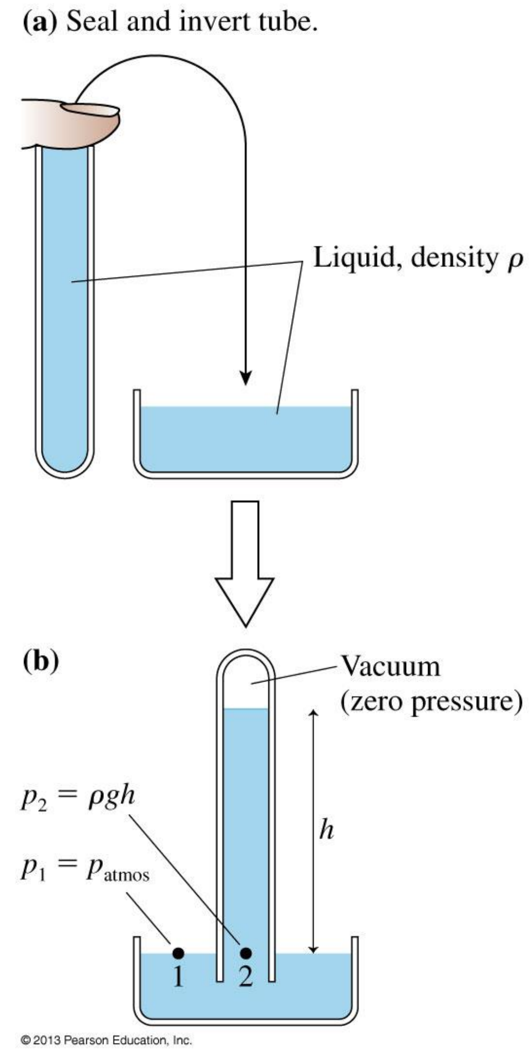
From section 15.2, we know that the pressure at the gas/liquid interface (point 1) is equal to p_{gas}, and that the pressure at the top of the liquid column is p_0 = 1 atm. Furthermore, we know from slide 16 that $p_1 = p_2$. Therefore, we can easily calculate that

$$p_{gas} = 1\text{ atm} + \rho g h$$

15.3 Measuring and Using Pressure

Barometers

A barometer is used to measure the atmospheric pressure (which can differ from 101.3 kPa). We will discuss it in class.



15.3 Measuring and Using Pressure

Other Units of Pressure

Several common (but non-SI) units of pressure are listed in the table below.

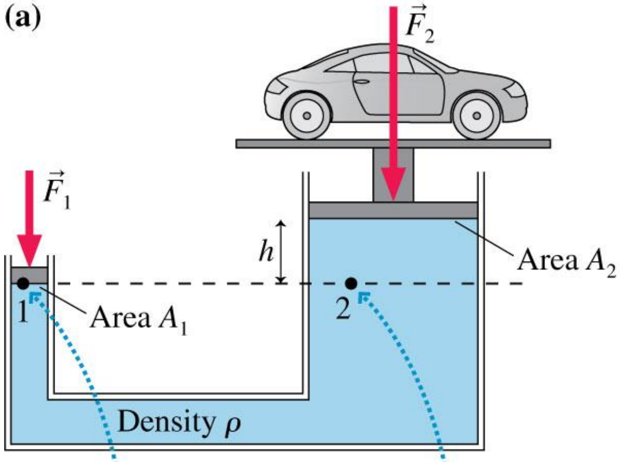
TABLE 15.2 Pressure units

Unit	Abbreviation	Conversion to 1 atm	Uses
pascal	Pa	101.3 kPa	SI unit: 1 Pa = 1 N/m^2
atmosphere	atm	1 atm	general
millimeters of mercury	mm of Hg	760 mm of Hg	gases and barometric pressure
inches of mercury	in	29.92 in	barometric pressure in U.S. weather forecasting
pounds per square inch	psi	14.7 psi	engineering and industry

15.3 Measuring and Using Pressure

Hydraulic Lift

Pascal's principle tells us that connected columns of fluid can be used to transmit pressure from one point to another.



In the figure, a force $\vec{F}_1$ is applied to a piston of area A_1, while a force $\vec{F}_2$ is applied to a piston of area A_2. Both pistons are also subjected to the atmospheric pressure p_0.

Since the pressure at points 1 and 2 must be equal, we can write

$$p_0 + \frac{F_1}{A_1} = p_0 + \frac{F_2}{A_2} + \rho g h$$

Or,

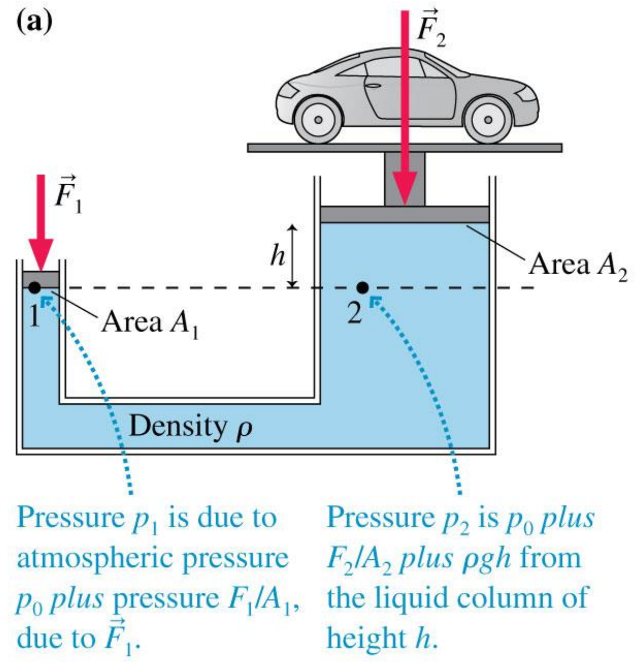
$$F_2 = \frac{A_2}{A_1} F_1 - \rho g h A_2$$

15.3 Measuring and Using Pressure

Hydraulic Lift *cont'*

In the case that h is small, the last term in the previous equation can be neglected, resulting in

$$F_2 = \frac{A_2}{A_1} F_1$$

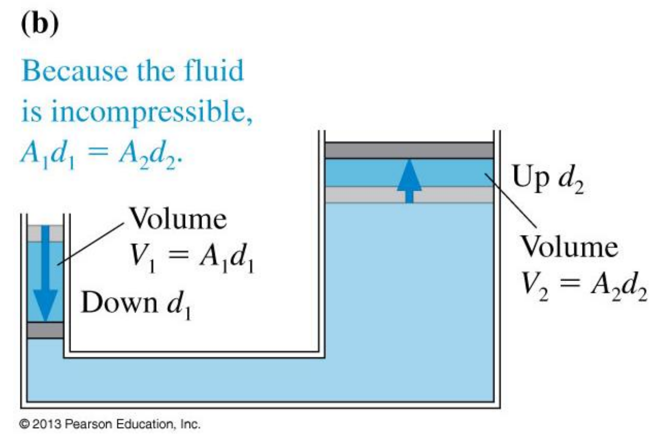


If $A_2 \gg A_1$, a very large mass (resulting in a very large $F_2 = mg$) can be supported by a relatively small force F_1. This is the principle of the hydraulic lift.

15.3 Measuring and Using Pressure

Hydraulic Lift *cont'*

If we wish to actually *raise* the mass – rather than simply support it – we need only to push piston down. Since the total volume of the fluid is constant (it's incompressible), pushing the piston down by a distance d_1 will raise the opposite piston (the one that supports the mass) by a distance



$$d_2 = \frac{A_1}{A_2} d_1$$

Thus, there is a trade-off in designing the cylinder areas of a hydraulic lift: if the ratio A_2/A_1 is large, a small force can support a large mass, but the piston must be moved a large distance in order to raise the heavy mass by a small amount.

Problem #6: Cheerleader

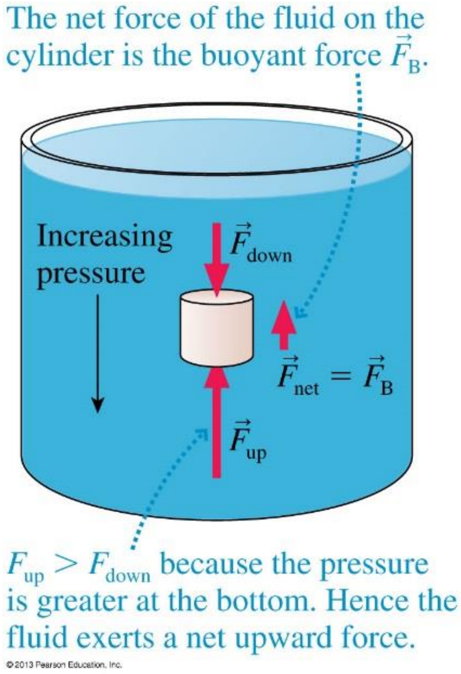
RDK Ex. 15.41

A 55 kg cheerleader uses an oil-filled hydraulic lift to hold four 110 kg football players at a height of 1.0 m. If her piston is 16 cm in diameter, **what is the diameter of the football players' piston?**

Solution: in class

15.4 Buoyancy

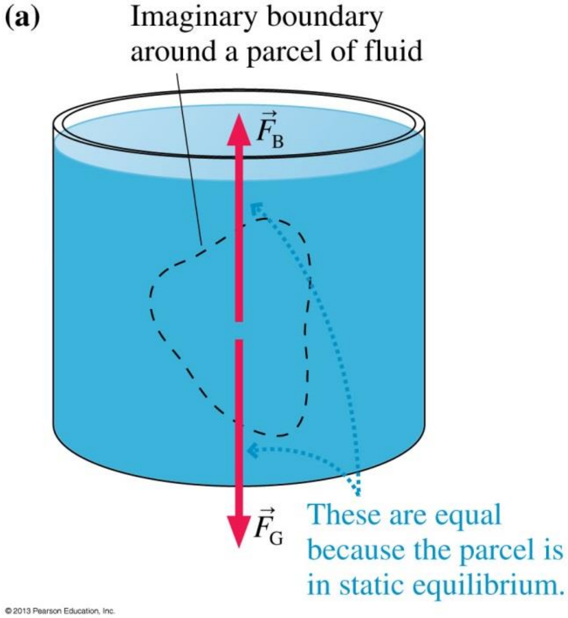
Buoyancy refers to the ability (or lack thereof) of a particular object to float in a particular fluid. As you may expect, the ability to float depends on properties of both the object and the fluid. It is not strictly a property of the object's mass – the textbook points out that a penny will sink in the ocean while an aircraft carrier will float.



The ability of an object to float depends upon (among other things), the **buoyant force**, $\vec{F}_B$. This is the upward force that you feel when you try to hold a beach ball under water, for instance. It arises due to the pressure gradient with depth; the greater pressure at the bottom of the object means that the upward force exceeds the downward force, resulting in a net upward $\vec{F}_B$.

15.4 Buoyancy

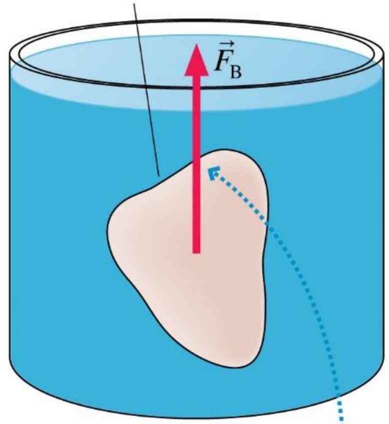
To visualize the buoyant force, first picture a container of fluid in hydrostatic equilibrium. Now, imagine that we place an imaginary boundary around an arbitrary portion of the fluid. As this portion is in equilibrium, the net vertical force acting on it is zero – thus, the buoyant force is equal in magnitude to the weight of the portion of fluid: $F_B = mg$.



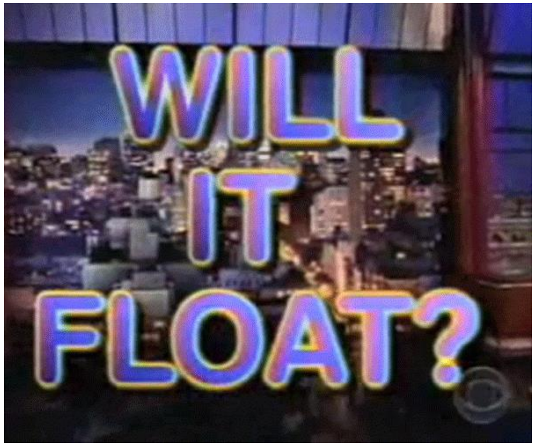
15.4 Buoyancy

Now, we're going to imagine removing the fluid within the boundary and replacing it with an object of the same shape and size. Since the fluid surrounding this object hasn't changed, it must be supplying the same $\vec{F}_B$ as before – *regardless of the weight of the new object*. In other words, $\vec{F}_B$ is determined not by the weight of the object, but by the weight of the fluid that it displaces. This is **Archimedes' Principle**.

For example, if a fluid has density ρ_f and the object displaces a volume V_f of fluid, the mass of the displaced fluid is $m_f = \rho_f V_f$, and its weight is $m_f g = \rho_f V_f g$. Archimedes' principle tells us that the magnitude of the buoyant force is $F_B = \rho_f V_f g$.



15.4 Buoyancy



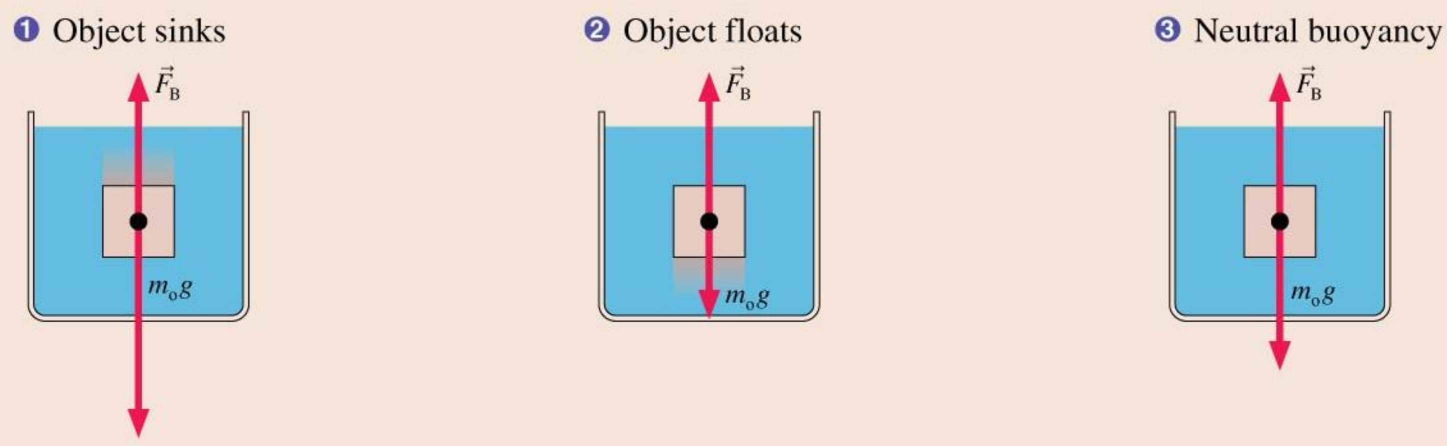
The most common buoyancy problem is simply this: *will a given object sink or float in a particular fluid, and if it floats, what fraction of it lies above and below the fluid surface?*

To answer this question, we need to draw a **free-body diagram** to analyze the net vertical force acting on the object (the net *horizontal* force is zero). The forces to consider are $\vec{F}_B$ (directed upward) and the object's weight, mg, directed downward.

Since we're comparing the magnitudes $\rho_f V_f g$ and mg, we can cancel a factor of g and simply compare **the mass of the displaced fluid** $(\boldsymbol{\rho_f V_f})$ and **the mass of the object** $(\boldsymbol{m})$.

15.4 Buoyancy

The three possibilities are shown below. We'll discuss them in class.



An object sinks if it weighs more than the fluid it displaces—that is, if its average density is greater than the density of the fluid:

$$\rho_{avg} > \rho_f$$

An object floats on the surface if it weighs less than the fluid it displaces—that is, if its average density is less than the density of the fluid:

$$\rho_{avg} < \rho_f$$

An object hangs motionless if it weighs exactly the same as the fluid it displaces—that is, if its average density equals the density of the fluid:

$$\rho_{avg} = \rho_f$$

15.4 Buoyancy

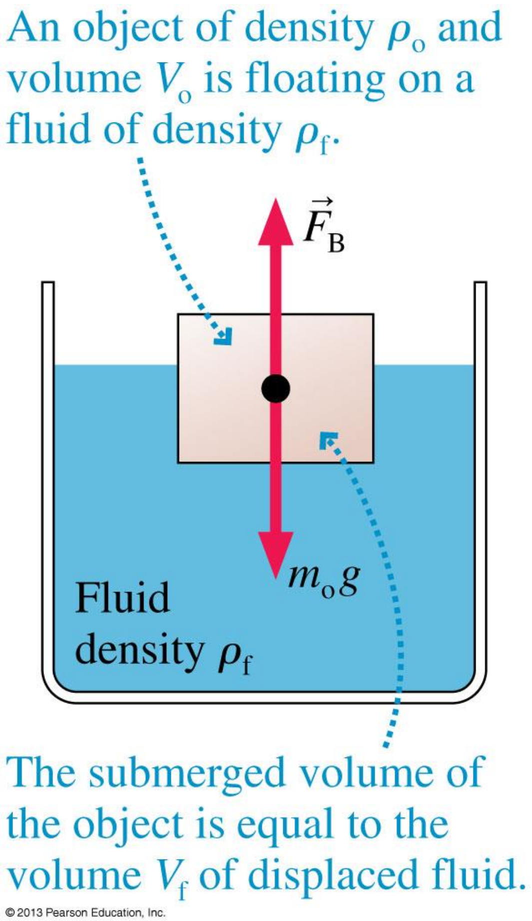
Properties of a Floating Object

A floating object is in **static equilibrium**, and therefore

$$F_B = \rho_f V_f g = m_0 g = \rho_0 V_0 g$$

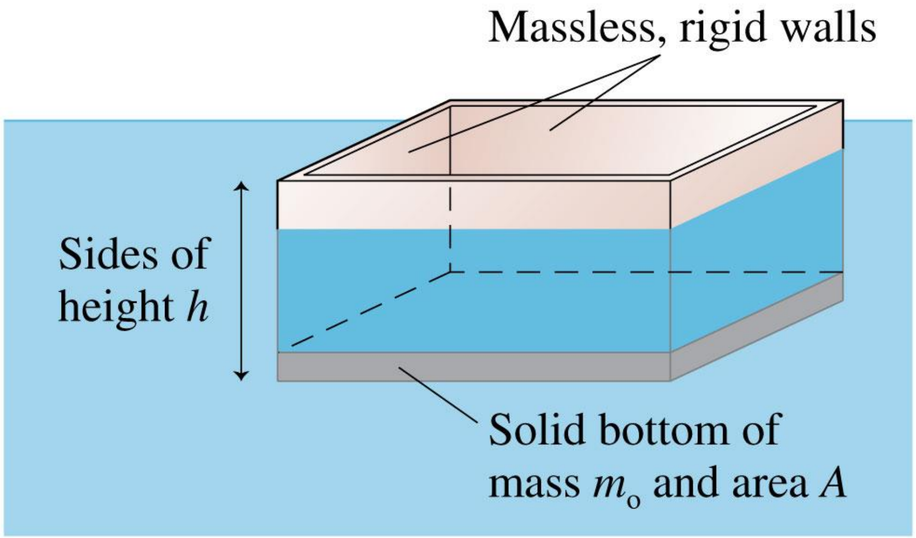
If we aren't in a situation of neutral buoyancy, then the volume of the object and the volume of displaced fluid are not the same, since a portion of the object lies above the fluid's surface. Rearranging the above equation, we find that the volume of displaced fluid is

$$V_f = \frac{\rho_0}{\rho_f} V_0$$



Since $\rho_0 < \rho_f$ (a necessary condition to float), we know that **a portion of the object must exist above the surface** (see: icebergs).

15.4 Boats

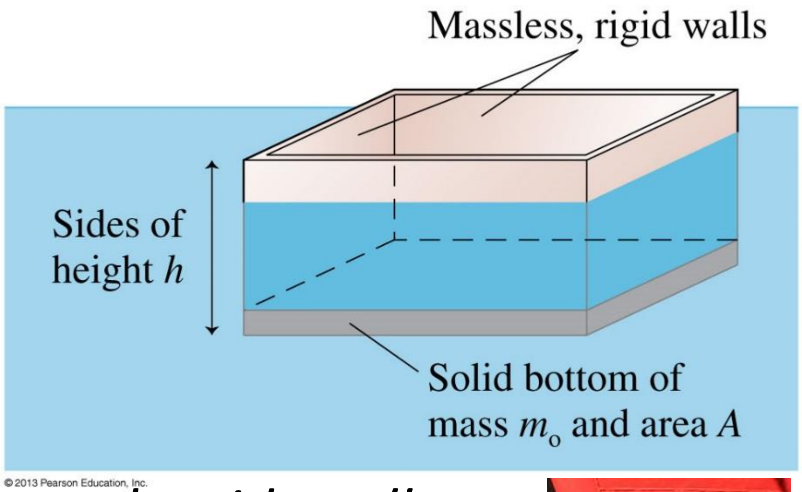


Boats

Let's examine a highly simplified version of a boat. It consists of a solid steel bottom of mass m_0 and area A, with four massless but rigid sidewalls of height h.

Since steel is denser than water, the bottom plate on its own will sink. However, the presence of the sidewalls allows the boat to displace a volume of water that is much greater than the volume of the steel alone. Most of this volume is air, with a very small ρ. The boat will float if $\rho_{\text{avg}} < \rho_f$, where ρ_{avg} is the boat's average density. In this case, the boat's volume is $V = Ah$, so its average density is $\rho_{\text{avg}} = m_0/Ah$ (only the steel bottom contributes to the mass).

15.4 Boats



Boats cont'

Therefore, the boat will float if

$$\frac{m_0}{Ah} < \rho_f$$

A better question would be *how high must the sidewalls be in order for the boat to float?* Rearranging the previous equation gives us our answer:

$$h_{\min} = \frac{m_0}{\rho_f A}$$

Many of you may know that boats have a "draft" specification. This is a measurement of the vertical distance from the waterline to the keel (the bottom of the hull). This indicates the minimum depth of water in which the boat can safely navigate.

Problem #7: Floating Stick

RDK Ex. 15.15

A 2.0 cm x 2.0 cm x 6.0 cm stick floats in water with its long axis vertical. The length of the block above water is 2.0 cm. **What is the block's mass density?**

Solution: in class

Problem #8: Beach Ball

RDK Ex. 15.20

You and your friends are playing in the swimming pool with a beach ball of 60-cm diameter. **How much force would be needed to push the ball completely under water?**

Solution: in class

Problem #9: Sink or Float?

A ball floats in water with ¾ of its volume submerged. Then, we take the ball and place it in a container of oil, which has a mass density that is half as great as that of water. **What happens to the ball?**	
A	It floats, with ½ of its volume submerged
B	It floats, with ¾ of its volume submerged
C	It experiences neutral buoyancy
D	It sinks

15.5 Fluid Dynamics

Sadly, not every fluid problem is a static one. We are often required to examine fluids that flow, an area of study known as **fluid dynamics.** In truth, fluid dynamics is an incredibly complex field, due to the nature of the interactions of a fluid with itself and its container. However, we can still draw some simple conclusions at the level of PHYS 212 by using an **ideal-fluid model**. This model makes three assumptions:

1. The fluid is **incompressible** (already, not so accurate for gasses!)
2. The fluid is **nonviscous**. Viscosity represents a resistance to flow (pancake syrup is more viscous than water, for example). Physically, viscosity is related to kinetic friction (PHYS 211!).
3. The flow is **steady**. That is, at each point in space, the fluid velocity is constant; it does not fluctuate in time. This is also called **laminar flow**, in contrast to **turbulent flow.**

15.5 Fluid Dynamics

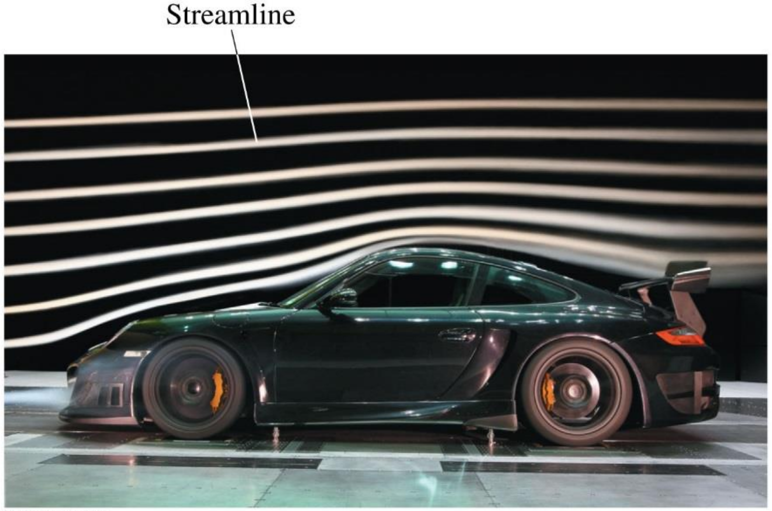
The Equation of Continuity

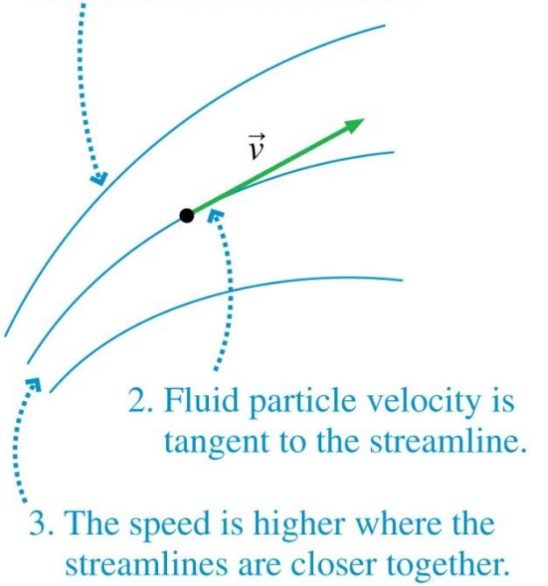
Imagine that we could track a single molecule of an ideal fluid as it flowed. The trajectory that it follows is called a **streamline**.

In an ideal fluid, streamlines exhibit three important properties:

- they never cross
- at any point, the molecule's velocity is tangent to the streamline
- the speed is highest in regions where the streamlines are closest together

Where in the car photo do you think the air is traveling the fastest?

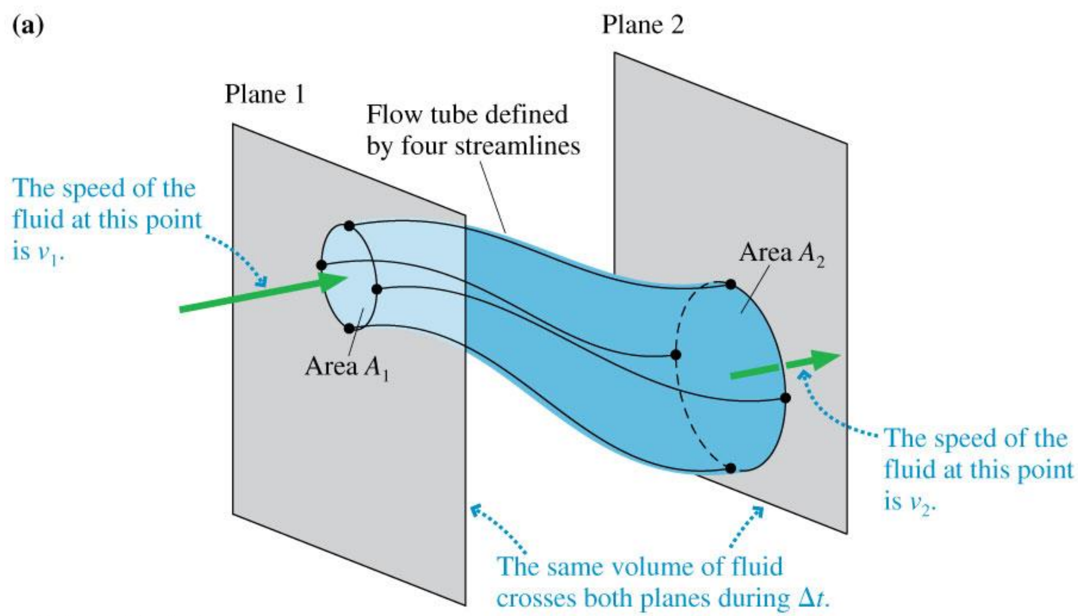




15.5 Fluid Dynamics

The Equation of Continuity *cont'*

A bundle of neighboring streamlines is called a **flow tube**, as shown in the figures below. Since streamlines never cross, all streamlines that cross plane 1 within area A_1 will eventually cross plane 2 within area A_2 (the only way to violate this would involve crossing of streamlines). Thus, we can think of the flow tube as acting just like a real tube…the portions of fluid inside and outside of the flow tube can never mix.



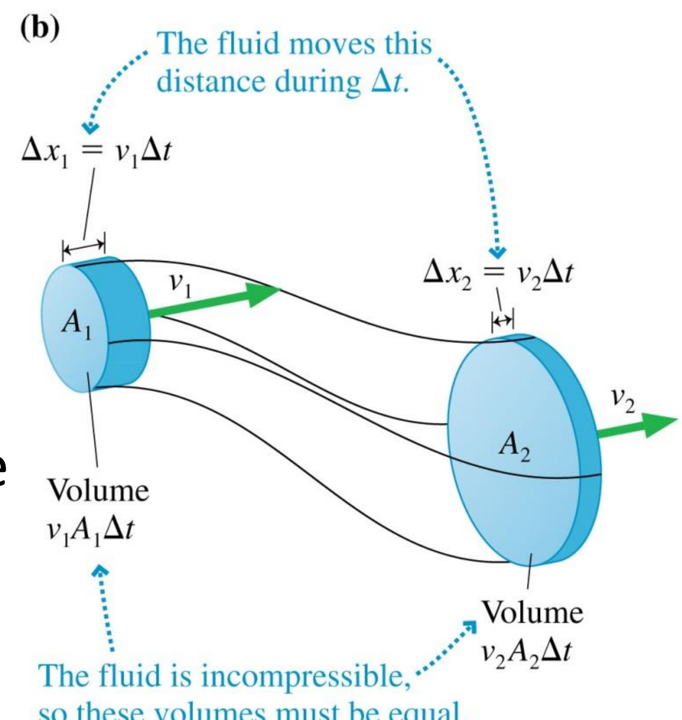
15.5 Fluid Dynamics

The Equation of Continuity *cont'*

Now let's track a particular volume of fluid as it traverses the flow tube. We keep in mind that the fluid **can not be created, destroyed, or stored** – it can only move from one point to another.

As a result, if a volume V enters the flow tube through area A_1 in a time Δt, then an equal volume V must leave the flow tube through area A_2 during the same time interval.

To find this volume, we recall that velocity is the change in position divided by the change in time. If the velocity while crossing A_1 is v_1, then the fluid moves a distance $\Delta x_1 = v_1 \Delta t$, and likewise for v_2 crossing A_2.



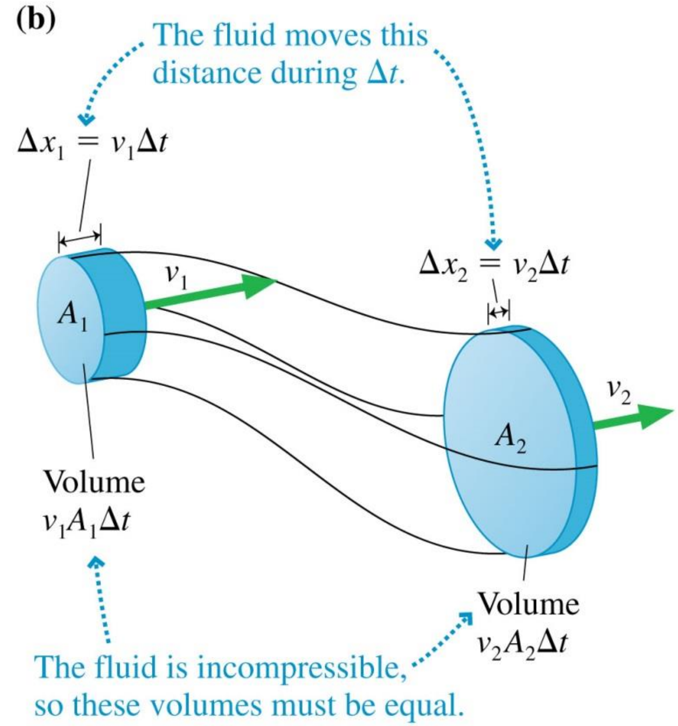
15.5 Fluid Dynamics

The Equation of Continuity *cont'*

Finally, since each volume is area multiplied by Δx, we can conclude that

$$v_1 A_1 = v_2 A_2$$

This is the **equation of continuity**. In plain english, it simply indicates that **for an incompressible fluid, the volume entering one part of a flow tube must be matched by an equal volume leaving the flow tube at some point downstream**.



15.5 Fluid Dynamics

The Equation of Continuity *cont'*

This concept is easily* demonstrated by observing the downward flow of water from a faucet. Due to gravitational acceleration, the water speeds up as it falls. This must be accompanied by a corresponding narrowing of the water column, in order to satisfy the equation of continuity.

The **volume flow rate** is defined as $Q = vA$; it has SI units of m^3/s. The equation of continuity tells us that **the volume flow rate is constant at all points in a flow tube.**

* well, it's easier said than done. Water isn't a perfectly ideal fluid, and the internal workings of the faucet conspire to produce a water output that doesn't really resemble a cylindrical tube. Still, the narrowing is evident in this photo.

15.5 Fluid Dynamics

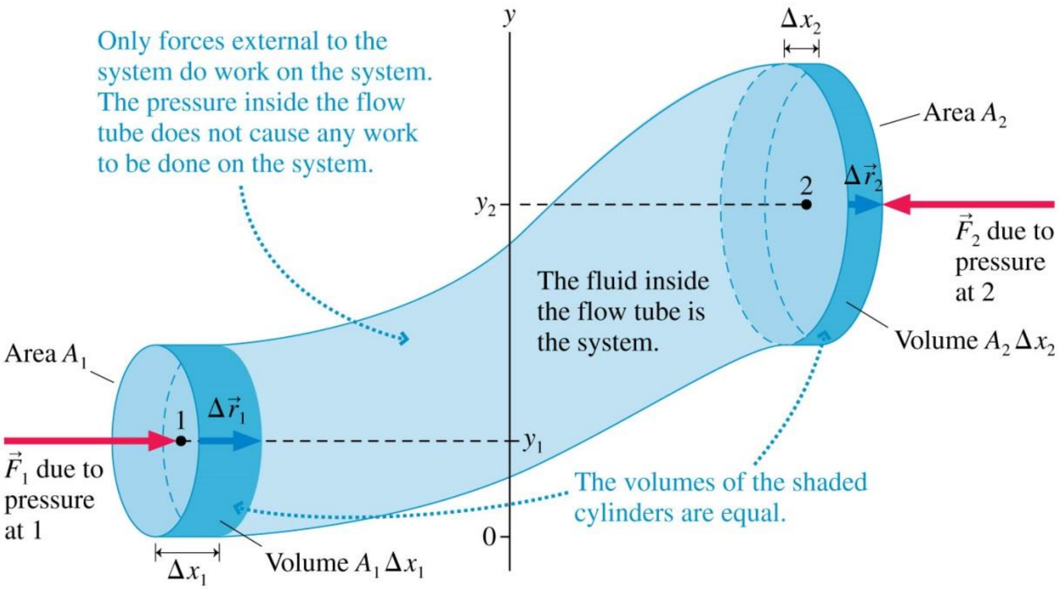
Bernoulli's Equation

Our next task is to examine the flow from a viewpoint of energy conservation. Looking back to PHYS 211, you will hopefully recall that this is expressed as

$$\Delta K + \Delta U = W_{\text{ext}}$$

That is, the sum of kinetic and potential energy changes by an amount equal to the work done on the system by any *external* forces.

The upcoming derivation will require frequent reference to this figure →



15.5 Fluid Dynamics

Bernoulli's Equation *cont'*

In this case, the system in question is the volume of fluid within the flow tube. External work is done on this system by the pressure forces of the surrounding fluid (why can we ignore pressure forces from the fluid within the flow tube)?

At point 1, a force $\vec{F}_1$ is exerted on the system due to the pressure of the fluid external to the flow tube. This pushes the fluid at this point through a parallel displacement $\Delta\vec{r}_1$. The work done on the fluid at this point is therefore

$$W_1 = \vec{F}_1 \cdot \Delta\vec{r}_1 = \vec{F}_1 \Delta\vec{r}_1 = p_1 A_1 \Delta x_1 = p_1 V$$

(this equation will be clarified in class)

The situation is the same at point 2, except than now the force is directed opposite to the displacement. Here,

$$W_2 = \vec{F}_2 \cdot \Delta\vec{r}_2 = -F_2 \Delta r_2 = -p_2 A_2 \Delta x_2 = -p_2 V$$

15.5 Fluid Dynamics

Bernoulli's Equation *cont'*

Essentially, the pressure at point 1 tries to increase the speed of the fluid (it does positive work), while the pressure at point 2 tries to decrease its speed (negative work). The net external work is

$$W_{\text{ext}} = W_1 + W_2 = p_1V - p_2V$$

Next, we will see how the potential energy of the system changes between point 1 and point 2. This change is due to a difference in height between the two points. From PHYS 211, you should recall that

$$\Delta U = mgy_2 - mgy_1 = \rho Vgy_2 - \rho Vgy_1$$

where we have once again used the relation $m = \rho V$.

15.5 Fluid Dynamics

Bernoulli's Equation *cont'*

Now we must examine the change in kinetic energy. This is simply

$$\Delta K = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2 = \frac{1}{2}\rho V v_2^2 - \frac{1}{2}\rho V v_1^2$$

We can now combine the work, kinetic, and potential energy terms as shown on Slide 45, to produce

$$\frac{1}{2}\rho V v_2^2 - \frac{1}{2}\rho V v_1^2 + \rho V g y_2 - \rho V g y_1 = p_1 V - p_2 V$$

Finally, we can cancel out the volume V from each term (this *must* be the case, since we were working with an arbitrary V):

$$p_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = p_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2$$

15.5 Fluid Dynamics

Bernoulli's Equation *cont'*

This is known as **Bernoulli's equation**, named for Daniel Bernoulli (not to be confused with Jacob I, Jacob II, Johann I, Johann II, Johann III, Nicolaus I, or Nicolaus II Bernoulli – all fairly prominent scientists and mathematicians in their day).

In practice, Bernoulli's equation is usually used as a conservation law. It can be written

$$p + \frac{1}{2}\rho v^2 + \rho g y = \text{constant}$$

Examples 15.11 and 15.12 in the text provide some opportunity to practice using this equation.

Problem #10: Chimney Smoke

How is the smoke drawn up a chimney affected when there is a wind blowing outside?	
A	Smoke rises more rapidly in the chimney
B	Smoke is unaffected by the wind blowing
C	Smoke rises more slowly up the chimney
D	Smoke is forced back down the chimney

Problem #11: Tree Sap Flow

RDK Ex. 15.59

A tree loses water due by the process of *transpiration* at a rate of 110 g/h. This water is replaced by the upward flow of sap through vessels in the trunk. If the trunk contains 2000 vessels, each 100 μm in diameter, **what is the upward speed of the sap in each vessel?** The density of tree sap is 1040 kg/m^3.

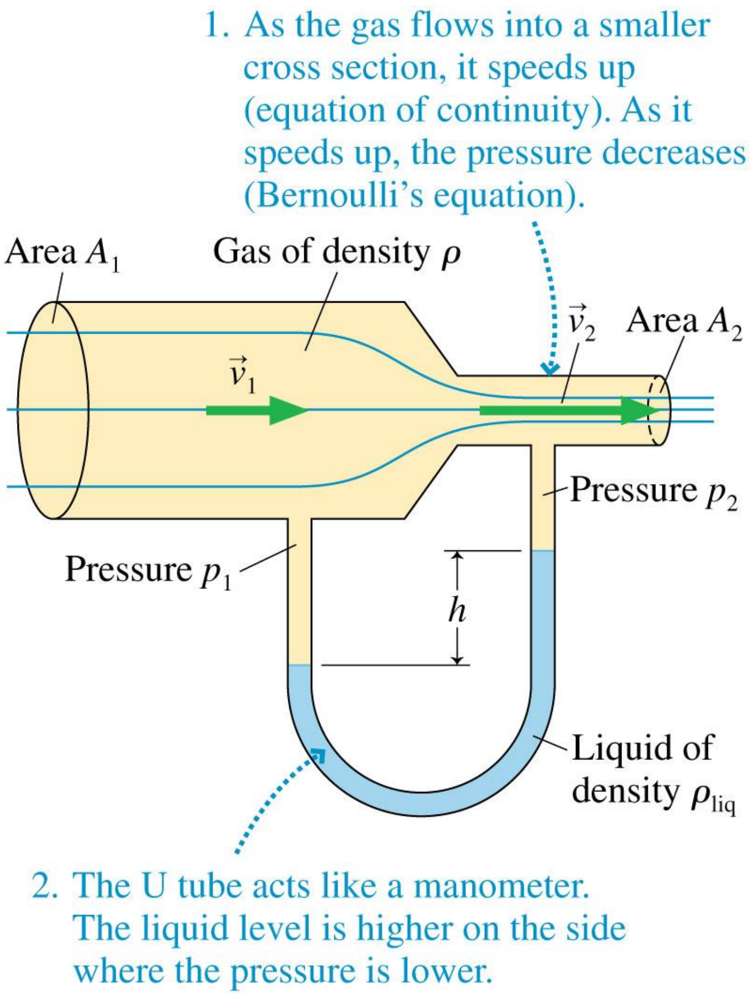
Solution: in class

15.5 Fluid Dynamics

Venturi Tube

One relatively simple application of Bernoulli's equation (coupled with the continuity equation) is the **Venturi tube**, which is used to measure the speed of a flowing gas.

We will discuss its operation in class.

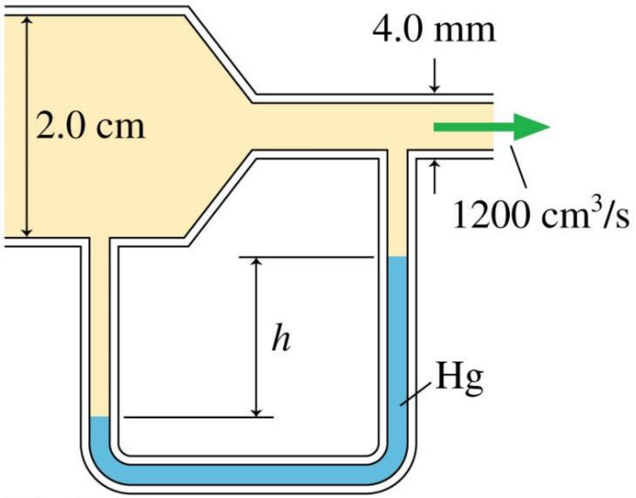


Problem #12: Venturi Tube

RDK Ex. 15.63

Air flows through the tube shown in the figure at a rate of 1200 cm^3/s. Assume that air is an ideal fluid. **What is the height h of mercury in the right side of the tube?**

Solution: in class

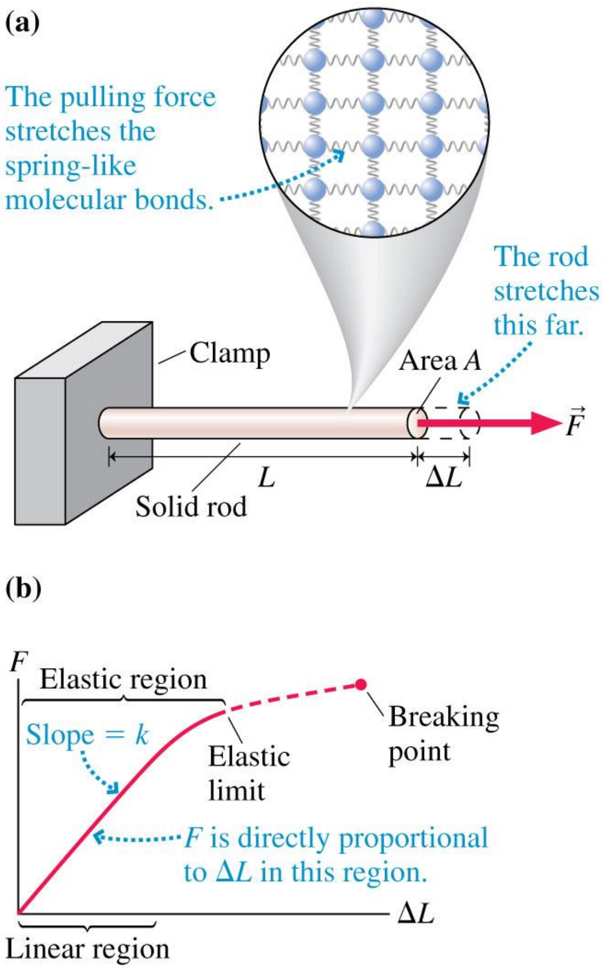


15.6 Elasticity

The subject of elasticity applies primarily to solids, although it is similar in spirit to many of the aspects of fluids that we have seen so far in this chapter.

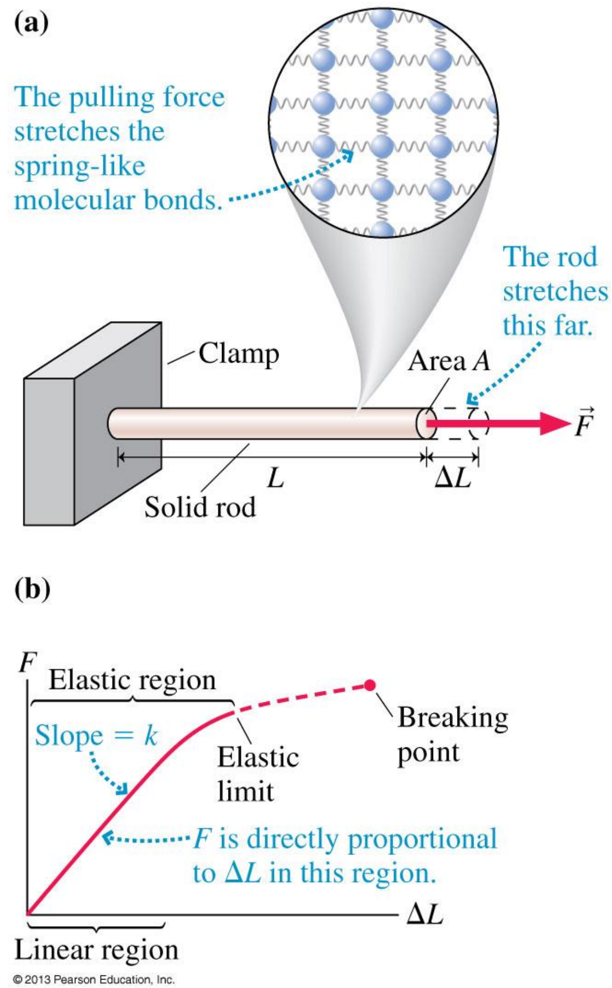
Suppose that we have a solid rod of a particular material, of length L. One end is clamped in place, while the other end can be stretched along the rod's axis by an applied force $\vec{F}$.

In PHYS 211, we only dealt with rigid (i.e. non-deformable) bodies. However, a microscopic view of a real solid material suggests that it can be stretched, even if only very slightly.



15.6 Elasticity

The bottom graph illustrates the relationship between the magnitude of the applied force and the increase in the rod's length, ΔL. For small forces, this relationship is linear. Furthermore, if the force is removed, the rod will return to its original length. This is referred to as the **elastic region**. Stretching the rod past the **elastic limit** means that it is permanently deformed; removing the force no longer results in a return to the original length (ever loan a shirt to someone who's a few sizes too large? That's what happened). Finally, there will be an ultimate point at which the rod breaks.



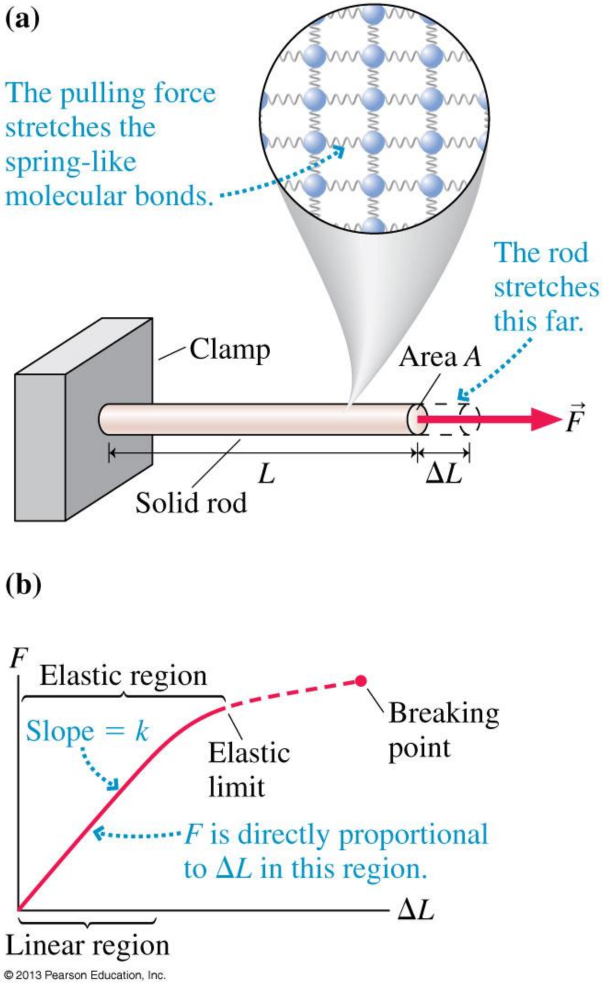
15.6 Elasticity

In PHYS 212, we are only concerned with the linear region. This linear relationship can be expressed as

$$F = k\Delta L$$

where k is the slope of the F vs. ΔL plot. This equation is familiar…if we replaced the rod with a spring, this would simply be Hooke's law.

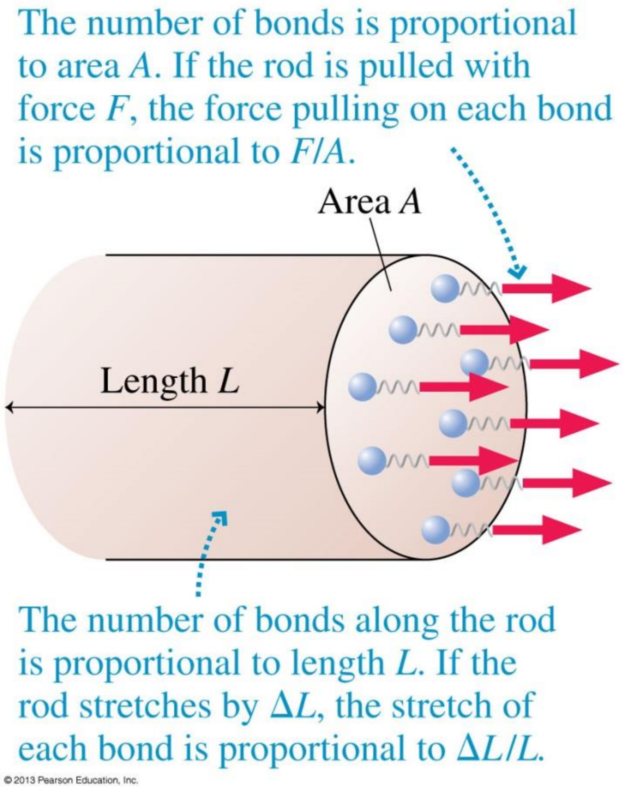
Unlike the spring, here k depends not only on the rod's material, but also on its shape (length and cross-sectional area). A long, narrow rod will elongate more easily than a short, stumpy rod.



15.6 Elasticity

It would be beneficial to find a way to characterize the elastic properties of a particular material, irrespective of its shape. Referring to the figure, the elasticity is related to the spring constant of the individual molecular bonds in the material.

If a force is applied to the end of the rod, it is distributed across the area of the end face – thus, the force pulling on *each bond* is proportional to $F\ /\ A$. The result of the force is that each bond is stretched by an amount proportional to $\Delta L/L$ (since the entire rod stretches by ΔL and the number of bonds along the length of the rod is proportional to L).



15.6 Elasticity

There's two unknown proportionality constants in the preceding description, but that's OK. Hooke's law tells us that the force pulling on a bond is proportional to the distance by which the bond stretches. In other words,

$$\frac{F}{A} = Y\frac{\Delta L}{L}$$

The proportionality constant Y is called **Young's modulus**, and it has SI units of N/m^2. To reiterate, it depends purely on a material's composition, and not on its geometry.

In the preceding equation, $F\ /\ A$ is termed the **tensile stress** applied to the rod (units of N/m^2), and $\Delta L/L$ is called **strain** (dimensionless). Thus, we see that Y is the ratio of tensile stress to strain.

We note, finally, that the preceding derivation is equally valid in the case of compressive – rather than tensile – stress.

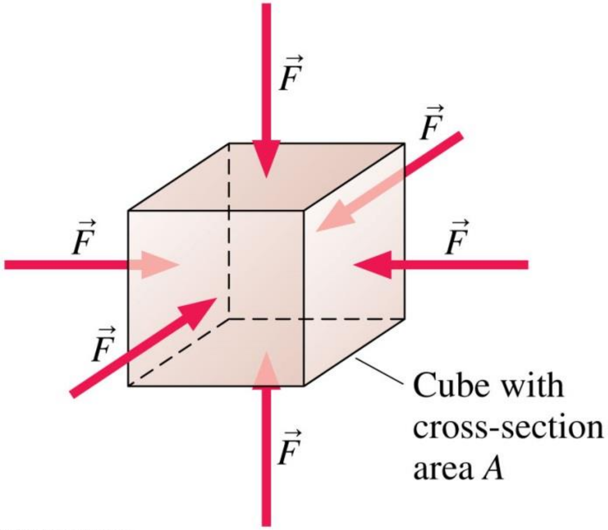
15.6 Elasticity

Volume Stress and the Bulk Modulus

The concept of Young's modulus only applies to an object that is being stretched or compressed along one axis. As we have seen, an object that is submerged in a liquid is "squeezed" from all directions. In this case, the term $F\ /\ A$ is termed the **volume stress**, and in fact is identical to our definition of pressure.

In response to volume stress, an object will experience **volume strain**, a fractional change in volume $(\Delta V/V)$.

Volume strain must be a negative number, since volume stress decreases the volume.



15.6 Elasticity

Volume Stress and the Bulk Modulus *cont'*

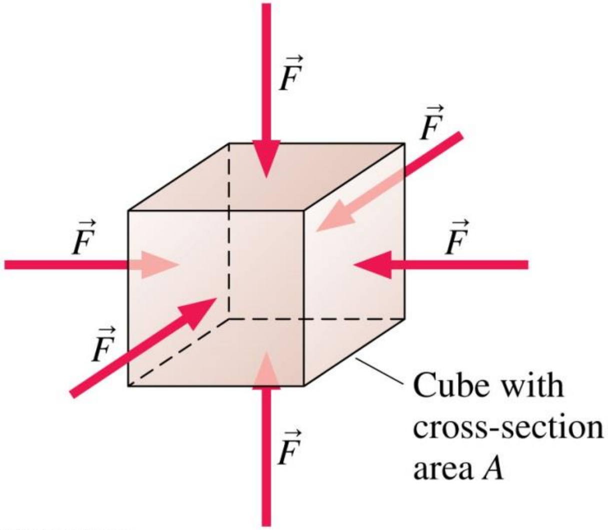
As was the case 2 slides ago, volume stress and volume strain are proportional to each other:

$$\frac{F}{A} = p = -B\frac{\Delta V}{V}$$

where B is called the **bulk** modulus of the material. The negative sign ensures that both p and B are positive quantities.

TABLE 15.3 Elastic properties of various materials

Substance	Young's modulus (N/m^2)	Bulk modulus (N/m^2)
Aluminum	7×10^{10}	7×10^{10}
Concrete	3×10^{10}	–
Copper	11×10^{10}	14×10^{10}
Mercury	–	3×10^{10}
Plastic (polystyrene)	0.3×10^{10}	–
Steel	20×10^{10}	16×10^{10}
Water	–	0.2×10^{10}
Wood (Douglas fir)	1×10^{10}	–



Problem #13: Spinal Disks

RDK Ex. 15.68

There is a disk of cartilage between each pair of vertebrae in your spine. Young's modulus for cartilage is 1.0×10^6 N/m^2. Supposed that a relaxed disk is 4.0 cm in diameter and 5.0 mm thick. If a disk in the lower spine supports half the weight of a 66 kg person, **by how many mm does the disk compress?**

Solution: in class