Chapter 17: Image Post Processing and Analysis

Slide set of 176 slides based on the chapter authored by P.A. Yushkevich of the IAEA publication (ISBN 978-92-0-131010-1):

Diagnostic Radiology Physics: A Handbook for Teachers and Students

Objective:

To familiarize the student with the most common problems in image post processing and analysis, and the algorithms to address them.

IAEA
International Atomic Energy Agency

Slide set prepared by E. Berry (Leeds, UK and The Open University in London)

TABLE OF CONTENTS

17.1 INTRODUCTION

17.1

Introduction (1 of 2)

- For decades, scientists have used computers to enhance and analyze medical images
- Initially simple computer algorithms were used to enhance the appearance of interesting features in images, helping humans read and interpret them better
- Later, more advanced algorithms were developed, where the computer would not only enhance images, but also participate in understanding their content

Introduction (2 of 2)

- **Segmentation** algorithms were developed to detect and extract specific anatomical objects in images, such as malignant lesions in mammograms
- **Registration** algorithms were developed to align images of different modalities and to find corresponding anatomical locations in images from different subjects
- These algorithms have made computer-aided detection and diagnosis, computer-guided surgery, and other highly complex medical technologies possible
- Today, the field of image processing and analysis is a complex branch of science that lies at the intersection of applied mathematics, computer science, physics, statistics, and biomedical sciences

Overview

- This chapter is divided into two main sections
  - classical image processing algorithms
    - image filtering, noise reduction, and edge/feature extraction from images.
  - more modern image analysis approaches
    - including segmentation and registration

Image processing vs. Image analysis

- The main feature that distinguishes image analysis from image processing is the use of external knowledge about the objects appearing in the image
- This external knowledge can be based on
    - heuristic knowledge
    - physical models
    - data obtained from previous analysis of similar images
- Image analysis algorithms use this external knowledge to fill in the information that is otherwise missing or ambiguous in the images

Example of image analysis

- A biomechanical model of the heart may be used by an image analysis algorithm to help find the boundaries of the heart in a CT or MR image
- This model can help the algorithm tell true heart boundaries from various other anatomical boundaries that have similar appearance in the image

The most important limitation of image processing

- Image processing cannot increase the amount of information available in the input image
- Applying mathematical operations to images can only remove information present in an image
  - sometimes, removing information that is not relevant can make it easier for humans to understand images
- Image processing is always limited by the quality of the input data
  - if an imaging system provides data of unacceptable quality, it is better to try to improve the imaging system, rather than hope that the “magic” of image processing will compensate for poor imaging

17.1 INTRODUCTION
17.1

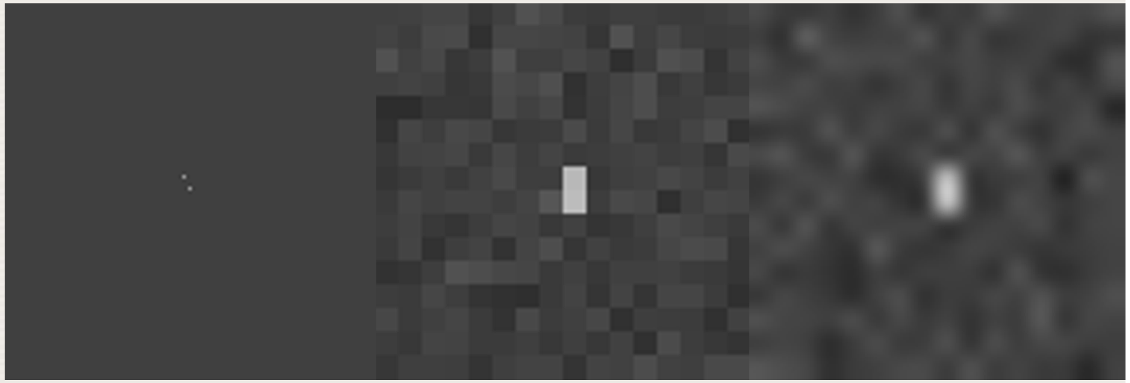
Example of image denoising

- Image noise cannot be eliminated without degrading contrast between small details in the image
- Note that although noise removal gets rid of the noise, it also degrades anatomical features

- From left to right
  - a chest CT slice
  - same slice with added noise
  - same slice processed with an edge-preserving noise removal algorithm

Image from the Lung Cancer Alliance *Give a Scan* database (giveascan.org)

Changing resolution of an image



- The fundamental resolution of the input image (i.e. the ability to separate a pair of nearby structures) is limited by the imaging system and cannot be improved by image processing
  - in centre image, the system's resolution is less than the distance between the impulses - we cannot tell from the image that there were two impulses in the data.
  - in the processed image at right we still cannot tell that there were two impulses in the input data
- From left to right
  - the input to an imaging system, it consists of two nearby point impulses
  - a 16x16 image produced by the imaging system
  - image resampled to 128x128 resolution using cubic interpolation

17.2 DETERMINISTIC IMAGE PROCESSING AND FEATURE ENHANCEMENT

17.2

17.2 DETERMINISTIC IMAGE PROCESSING AND FEATURE ENHANCEMENT

17.2.1 SPATIAL FILTERING AND NOISE REMOVAL

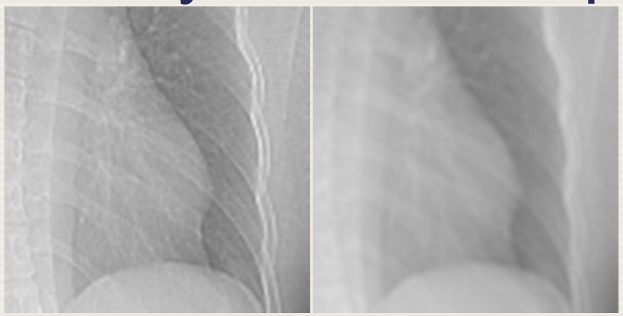
Filtering

- Filtering is an operation that changes the observable quality of an image, in terms of
  - resolution
  - contrast
  - noise
- Typically, filtering involves applying the same or similar mathematical operation at every pixel in an image
  - for example, spatial filtering modifies the intensity of each pixel in an image using some function of the neighbouring pixels
- Filtering is one of the most elementary image processing operations

Mean filtering in the image domain

- A very simple example of a spatial filter is the mean filter
- Replaces each pixel in an image with the mean of the N x N neighbourhood around the pixel
- The output of the filter is an image that appears more “smooth” and less “noisy” than the input image

Input X ray image



Input image convolved with a 7x7 mean filter

- Averaging over the small neighbourhood reduces the magnitude of the intensity discontinuities in the image

Mean filtering

- Mathematically, the mean filter is defined as a convolution between the image and a constant-valued N x N matrix

$$I_{\text{filtered}} = I \circ K; \qquad K = \frac{1}{N^2}\begin{bmatrix} 1 & 1 & \cdots & 1 \\ 1 & 1 & \cdots & 1 \\ \vdots & \vdots & \ddots & \vdots \\ 1 & 1 & \cdots & 1 \end{bmatrix}$$

- The N x N mean filter is a low-pass filter
- A low-pass filter reduces high-frequency components in the Fourier transform (FT) of the image

Convolution and the Fourier transform

- The relationship between Fourier transform (FT) and convolution is

$$F\{A \circ B\} = F\{A\}F\{B\}.$$

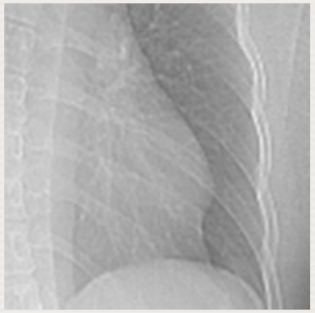
- Convolution of a digital image with a matrix of constant values is the discrete equivalent of the convolution of a continuous image function with the *rect* (boxcar) function
- The FT of the rect function is the *sinc* function
- So, mean filtering is equivalent to multiplying the FT of the image by the *sinc* function
  - this mostly preserves the low-frequency components of the image and diminishes the high-frequency components

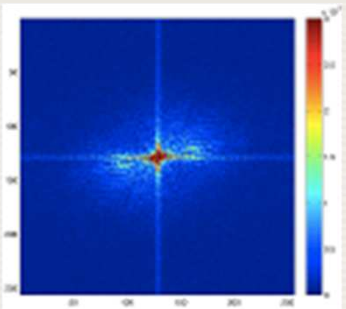
17.2 DETERMINISTIC IMAGE PROCESSING AND FEATURE ENHANCEMENT

17.2.1 Spatial Filtering and Noise Removal

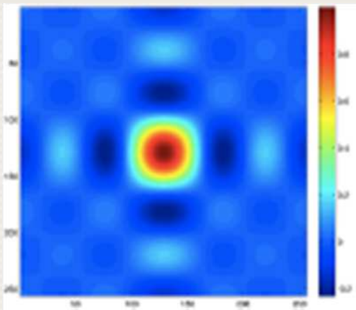
Mean filtering in the Fourier domain

Input X ray image

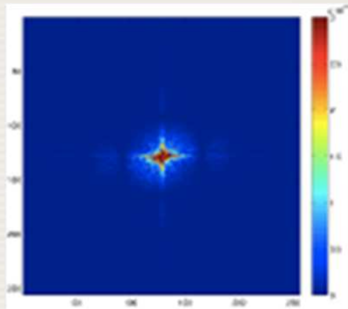




Fourier transform of the input image (magnitude)



Fourier transform of the 7x7 mean filter, i.e., a product of *sinc* functions in x and y



Fourier transform of the filtered image

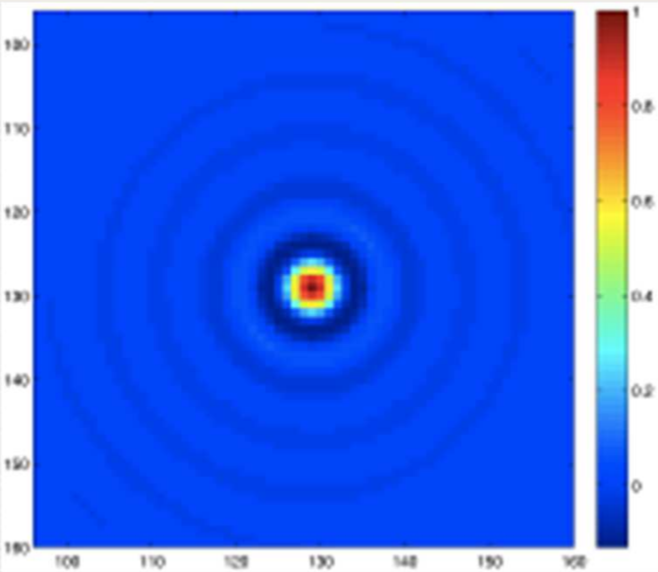
Image smoothing

- Mean filtering is an example of an image smoothing operation
- Smoothing and removal of high-frequency noise can help human observers understand medical images
- Smoothing is also an important intermediate step for advanced image analysis algorithms
- Modern image analysis algorithms involve numerical optimization and require computation of derivatives of functions derived from image data
    - smoothing helps make derivative computation numerically stable

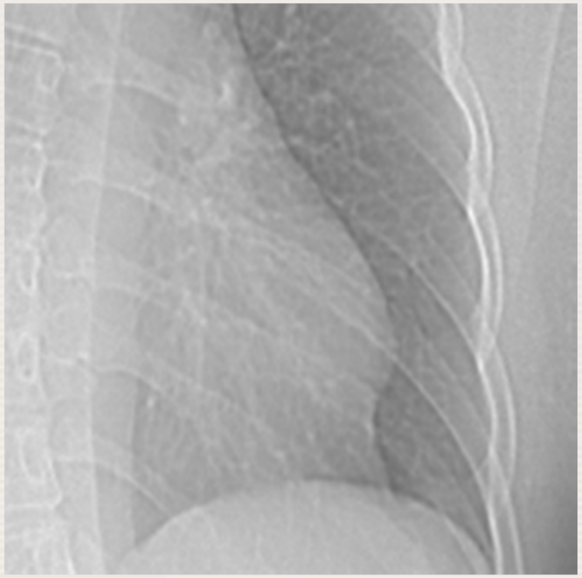
Ideal Low-Pass Filter

- The so-called ideal low-pass filter cuts off all frequencies above a certain threshold in the FT of the image
  - in the Fourier domain, this is achieved by multiplying the FT of the image by a cylinder-shaped filter generated by rotating a one-dimensional *rect* function around the origin
  - theoretically, the same effect is accomplished in the image domain by convolution with a one-dimensional *sinc* function rotated around the origin
- Assumes that images are periodic functions on an infinite domain
  - in practice, most images are not periodic
  - convolution with the rotated *sinc* function results in an artefact called ringing
- Another drawback of the ideal low-pass filter is the computational cost, which is very high in comparison to mean filtering

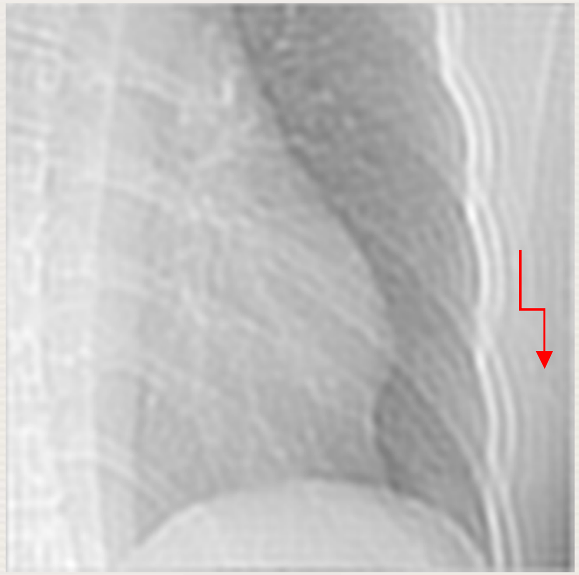
Ideal low-pass filter and ringing artefact



The ideal low-pass filter, i.e., a *sinc* function rotated around the centre of the image



The original image



The image after convolution with the low-pass filter. Notice how the bright intensity of the rib bones on the right of the image is replicated in the soft tissue to the right

Gaussian Filtering

- The Gaussian filter is a low-pass filter that is not affected by the ringing artefact
- In the continuous domain, the Gaussian filter is defined as the normal probability density function with standard deviation σ, which has been rotated about the origin in x,y space
- Formally, the Gaussian filter is defined as

$$G_\sigma(x,y) = \frac{1}{2\pi\sigma^2} e^{-\frac{x^2+y^2}{2\sigma^2}}$$

where the value σ is called the width of the Gaussian filter

FT of Gaussian filter

- The FT of the Gaussian filter is also a Gaussian filter with reciprocal width $1/\sigma$

$$F\left(G_{\sigma}(x,y)\right)=G_{1/\sigma}(\eta,\nu)$$

where η,υ are spatial frequencies

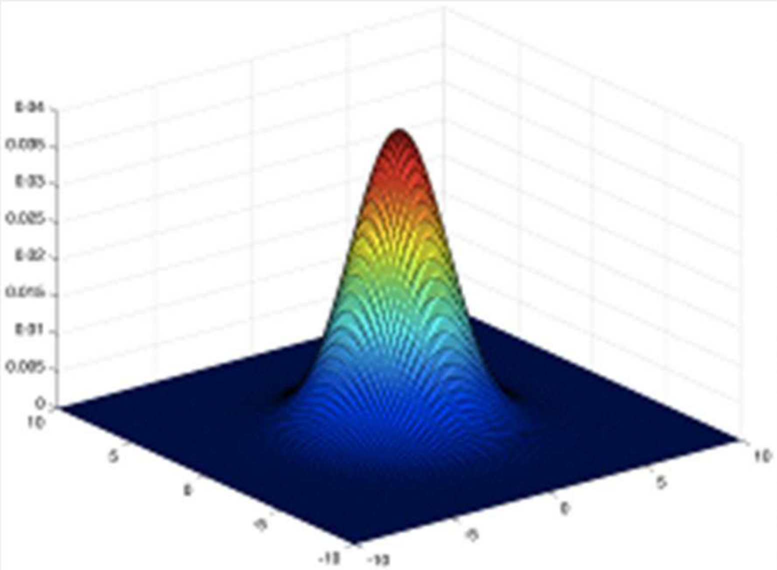
Discrete Gaussian filter

- The discrete Gaussian filter is a $(2N+1)\times(2N+1)$ matrix
- Its elements, G_{ij}, are given by

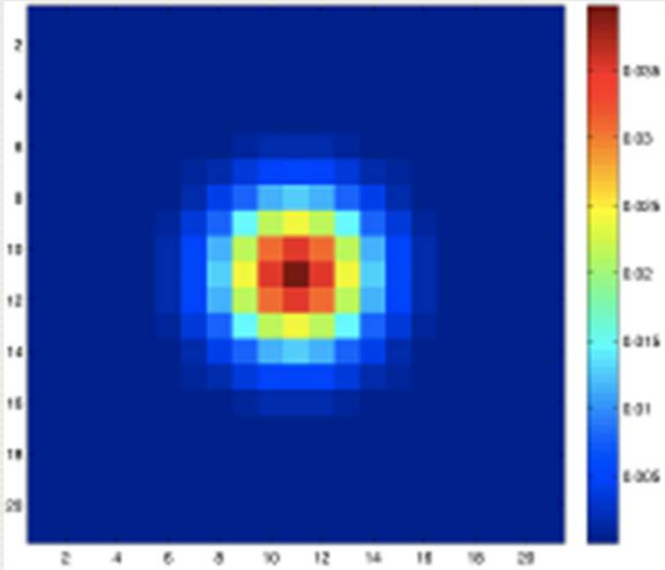
$$G_{ij} = G_{\sigma}(i-N-1, j-N-1)$$

- The size of the matrix, $2N+1$, determines how accurately the discrete Gaussian approximates the continuous Gaussian
- A common choice is $N >= 3\sigma$

Examples of Gaussian filters



A continuous 2D Gaussian with σ = 2

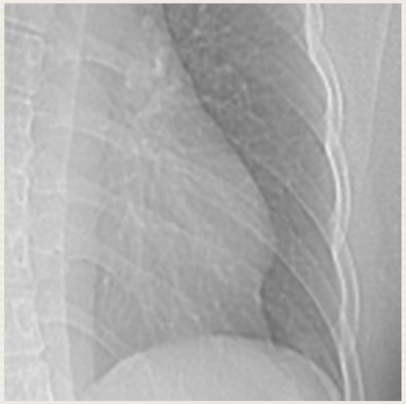


A discrete 21x21 Gaussian filter with σ = 2

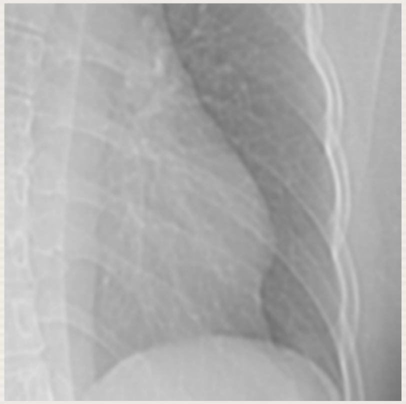
Application of the Gaussian filter

- To apply low-pass filtering to a digital image, we
  - perform convolution between the image and the Gaussian filter
  - this is equivalent to multiplying the FT of the image by a Gaussian filter with width $1/\sigma$
- The Gaussian function decreases very quickly as we move away from the peak
  - at the distance 4σ from the peak, the value of the Gaussian is only 0.0003 of the value at the peak
- Convolution with the Gaussian filter
  - removes high frequencies in the image
  - low frequencies are mostly retained
  - the larger the standard deviation of the Gaussian filter, the smoother the result of the filtering

An image convolved with Gaussian filters with different widths

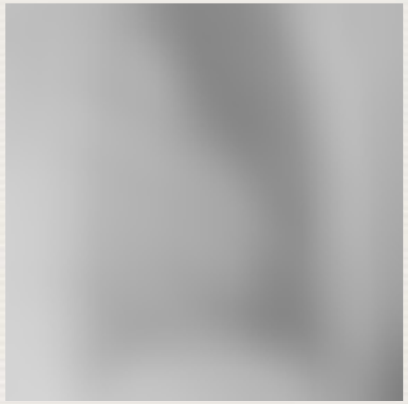


Original image



σ=1

σ=4



σ=16

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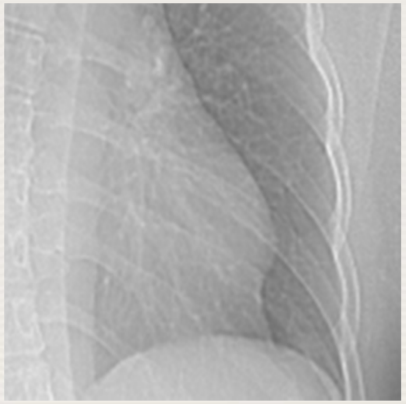
Median Filtering

- The median filter replaces each pixel in the image with the median of the pixel values in an N x N neighbourhood
- Taking the median of a set of numbers is a non-linear operation
  - therefore, median filtering cannon be represented as convolution
- The median filter is useful for removing impulse noise, a type of noise where some isolated pixels in the image have very high or very low intensity values
- The disadvantage of median filtering is that it can remove important features, such as thin edges

Example of Median Filtering



Original image

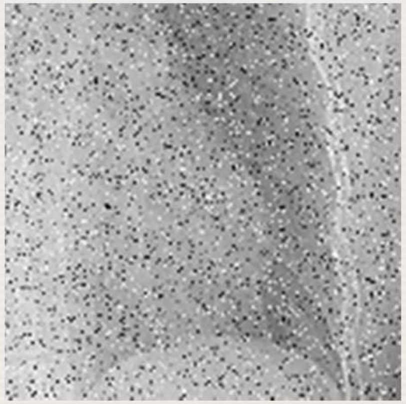
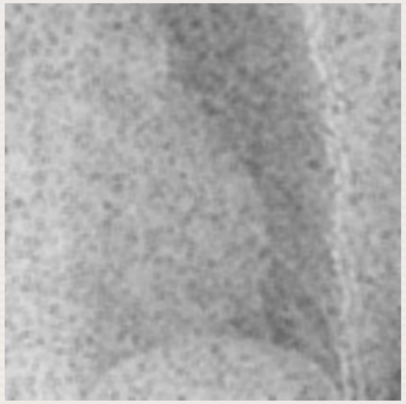
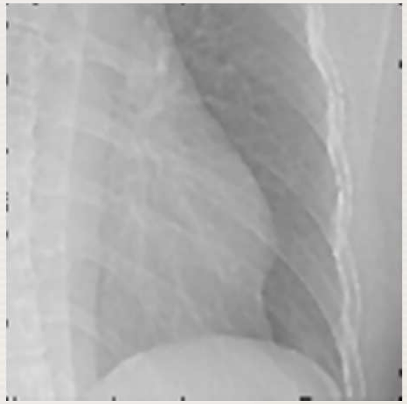


Image degraded by adding "salt and pepper" noise. The intensity of a tenth of the pixels has been replaced by 0 or 255



The result of filtering the degraded image with a 5x5 mean filter



The result of filtering with a 5x5 median filter. Much of the salt and pepper noise has been removed – but some of the fine lines in the image have also been removed by the filtering

Edge-preserving smoothing and de-noising

- When we smooth an image, we remove high-frequency components
- This helps reduce noise in the image, but it also can remove important high-frequency features such as edges
  - an edge in image processing is a discontinuity in the intensity function
  - for example, in an X ray image, the intensity is discontinuous along the boundaries between bone and soft tissue
- Some advanced filtering algorithms try to remove noise in images without smoothing edges
  - e.g. the anisotropic diffusion algorithm (Perona and Malik)

Anisotropic Diffusion algorithm

- Mathematically, smoothing an image with a Gaussian filter is analogous to simulating heat diffusion in a homogeneous body
- In anisotropic diffusion, the image is treated as an inhomogeneous body, with different heat conductance at different places in the image
  - near edges, the conductance is lower, so heat diffuses more slowly, preventing the edge from being smoothed away
  - away from edges, the conductance is faster
- The result is that less smoothing is applied near image edges
- The approach is only as good as our ability to detect image edges

17.2 DETERMINISTIC IMAGE PROCESSING AND FEATURE ENHANCEMENT

17.2.2 EDGE, RIDGE AND SIMPLE SHAPE DETECTION

Edges

- One of the main applications of image processing and image analysis is to detect structures of interest in images
- In many situations, the structure of interest and the surrounding structures have different image intensities
- By searching for discontinuities in the image intensity function, we can find the boundaries of structures of interest
  - these discontinuities are called edges
  - for example, in an X ray image, there is an edge at the boundary between bone and soft tissue

Edge detection

- Edge detection algorithms search for edges in images automatically
- Because medical images are complex, they have very many discontinuities in the image intensity
  - most of these are not related to the structure of interest
  - may be discontinuities due to noise, imaging artefacts, or other structures
- Good edge detection algorithms identify edges that are more likely to be of interest
- However, no matter how good an edge detection algorithm is, it will frequently find irrelevant edges
  - edge detection algorithms are not powerful enough to completely automatically identify structures of interest in most medical images
  - instead, they are a helpful tool for more complex segmentation algorithms, as well as a useful visualization tool

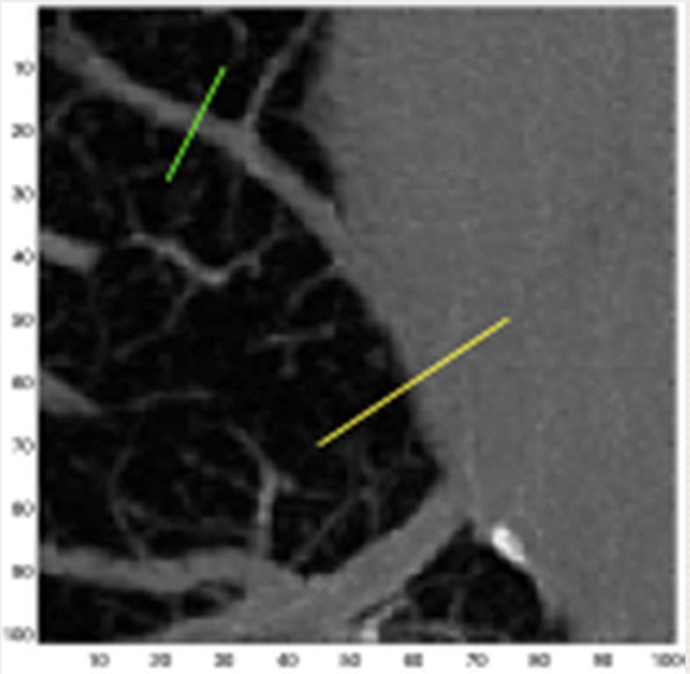
Tube detection

- Some structures in medical images have very characteristic shapes
- For example, blood vessels are tube-like structures with
  - gradually varying width
  - two edges that are roughly parallel to each other
- This property can be exploited by special tube-detection algorithms

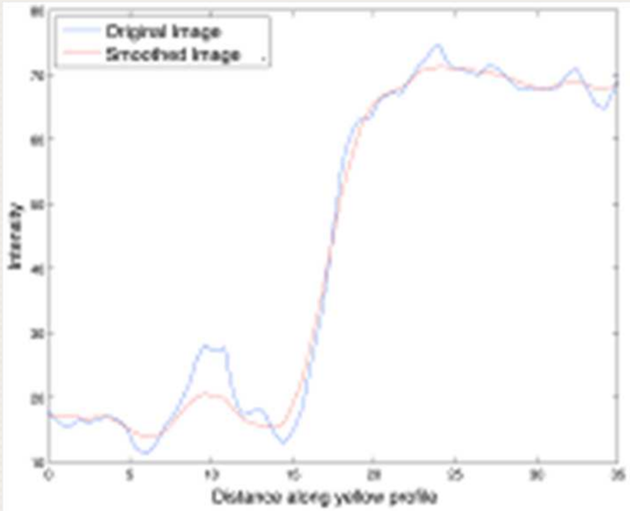
17.2 DETERMINISTIC IMAGE PROCESSING AND FEATURE ENHANCEMENT

17.2.2 Edge, Ridge and Simple Shape Detection

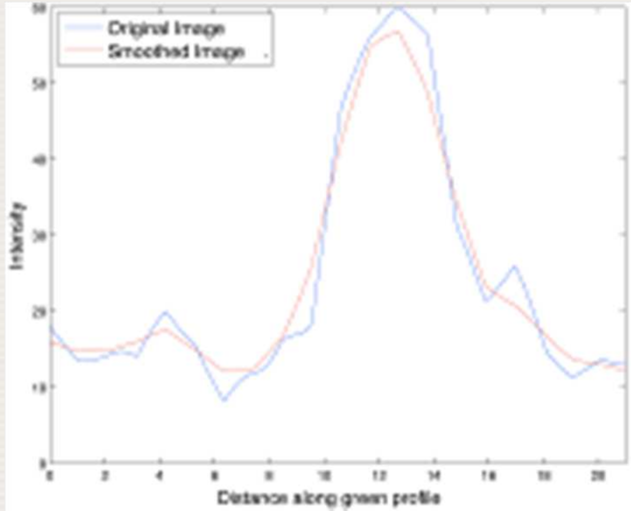
Illustration of edges and tubes in an image



Detail from a chest CT image – The yellow profile crosses an edge, and the green profile crosses a tube-like structure



Plot (blue) of image intensity along the yellow profile and a plot (red) of image intensity after smoothing the input image with a Gaussian filter with $\sigma = 1$



Plot of image intensity along the green profile. Edge and tube detectors use properties of image derivative to detect edges and tube

How image derivatives are computed

- An edge is a discontinuity in the image intensity
- Therefore, the directional derivative of the image intensity in the direction orthogonal to the edge must be large, as seen in the preceding figure
- Edge detection algorithms exploit this property
- In order to compute derivatives, we require a continuous function, but an image is just an array of numbers
- One solution is to use the finite difference approximation of the derivative

Finite difference approximation in 1D

- From the Taylor series expansion, it is easy to derive the following approximation of the derivative

$$f'(x) = \frac{f(x+\delta) - f(x-\delta)}{2\delta} + O(\delta^2),$$

where

- δ is a real number
- $O(\delta^2)$ is the error term, involving δ to the power of two and greater
- when $\delta \ll 1$ these error terms are very small and can be ignored for the purpose of approximation

Finite difference approximation in 2D (1 of 2)

- Likewise, the partial derivatives of a function of two variables can be approximated as

$$\frac{\partial f(x,y)}{\partial x} = \frac{f(x+\delta_x, y) - f(x-\delta_x, y)}{2\delta_x} + O(\delta_x^2),$$

$$\frac{\partial f(x,y)}{\partial y} = \frac{f(x, y+\delta_y) - f(x, y-\delta_y)}{2\delta_y} + O(\delta_y^2).$$

Finite difference approximation in 2D (2 of 2)

- If we
  - treat a digital image as a set of samples from a continuous image function
  - set δ_x and δ_y to be equal to the pixel spacing
- We can compute approximate image derivatives using these formulae
- However, the error term is relatively high, of the order of 1 pixel width
- In practice, derivatives computed using finite difference formulae are dominated by noise

Computing image derivatives by filtering (1 of 3)

- There is another, often more effective, approach to computing image derivatives
- We can reconstruct a continuous signal from an image by convolution with a smooth kernel (such as a Gaussian), which allows us to take the derivative of the continuous signal

Computing image derivatives by filtering (2 of 3)

$$f(x,y) = (I \circ G)(x,y);$$
$$D_{\mathbf{v}}(f)(x,y) = D_{\mathbf{v}}(I \circ G)(x,y)$$

- In the above, $D_{\mathbf{v}}$ denotes the directional derivative of a function in the direction $\mathbf{v}$
- One of the most elegant ways to compute image derivatives arises from the fact that differentiation and convolution are commutable operations
  - both are linear operations, and the order in which they are applied does not matter

Computing image derivatives by filtering (3 of 3)

- Therefore, we can achieve the same effect by computing the convolution of the image with the derivative of the smooth kernel

$$D_{\mathbf{v}} f(x, y) = I \circ (D_{\mathbf{v}} G)(x, y)$$

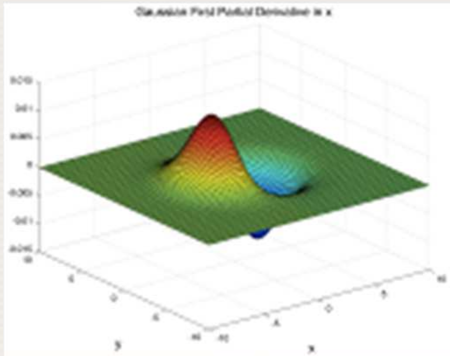
- This leads to a very practical and efficient way of computing derivatives
  - create a filter, which is just a matrix that approximates $D_{\mathbf{v}} G$
  - compute numerical convolution between this filter and the image
  - this is just another example of filtering described earlier

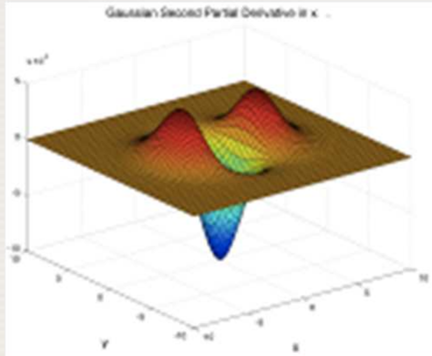
Computing image derivatives by Gaussian filtering

- Most frequently G is a Gaussian filter
- The Gaussian is infinitely differentiable, so it is possibly to take an image derivative of any order using this approach
- The width of the Gaussian is chosen empirically
  - the width determines how smooth the interpolation of the digital image is
  - the more smoothing is applied, the less sensitive will the derivative function be to small local changes in image intensity
  - this can help selection between more prominent and less prominent edges

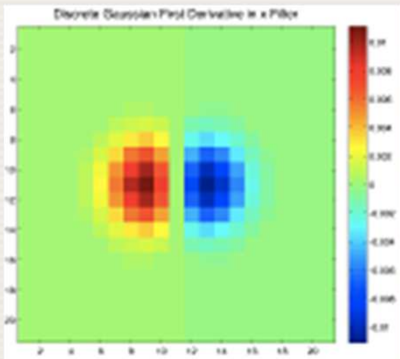
Examples of Gaussian derivative filters

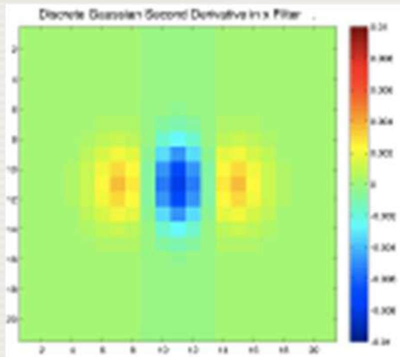
First and second partial derivatives in x of the Gaussian with σ =2





Corresponding 21 × 21 discrete Gaussian derivative filters





Edge Detectors Based on First Derivative

- A popular and simple edge detector is the Sobel operator
- To apply this operator, the image is convolved with a pair of filters

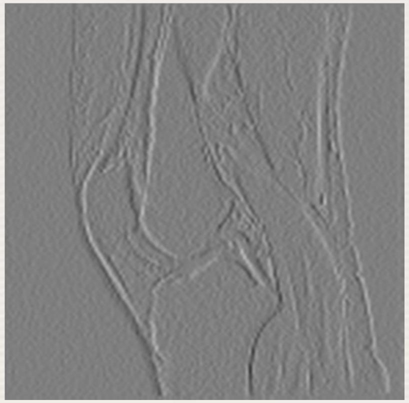
$$S_x = \begin{pmatrix} 1 & 0 & -1 \\ 2 & 0 & -2 \\ 1 & 0 & -1 \end{pmatrix}; S_y = \begin{pmatrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ -1 & -2 & -1 \end{pmatrix}$$

Sobel operator

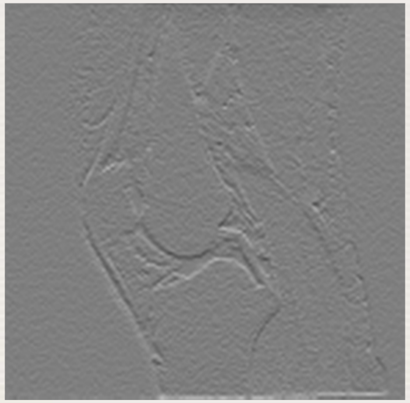
- It can be shown that this convolution is quite similar to the finite difference approximation of the partial derivatives of the image
- In fact, the Sobel operator
  - approximates the derivative at the given pixel and the two neighbouring pixels
  - computes a weighted average of these three values with weights (1,2,-1)
- This averaging makes the output of the Sobel operator slightly less sensitive to noise than simple finite differences

Illustration of the Sobel operator

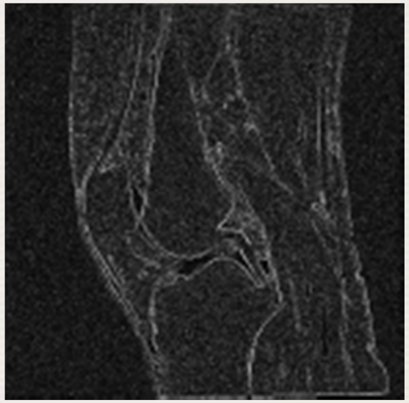
MR image of the knee



Convolution of the image with the Sobel x derivative filter S_x



Convolution of the image with the Sobel y derivative filter S_y



Gradient magnitude image

- The gradient magnitude is high at image edges, but also at isolated pixels where image intensity varies due to noise

Image from the U.S. National Biomedical Imaging Archive Osteoarthritis Initiative (https://imaging.nci.nih.gov/ncia)

Gradient magnitude image

- The last image is the so-called gradient magnitude image, given by

$$\sqrt{(S_x^2 + S_y^2)}$$

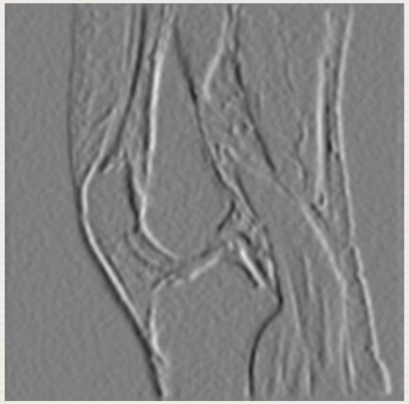
  - large values of the gradient magnitude correspond to edges
  - low values are regions where intensity is nearly constant
- However, there is no absolute value of the gradient magnitude that distinguishes an edge from non-edge
  - for each image, one has to empirically come up with a threshold to apply to the gradient magnitude image in order to separate the edges of interest from spurious edges caused by noise and image artefact
- This is one of the greatest limitations of edge detection based on first derivatives

Convolution with Gaussian derivative filters

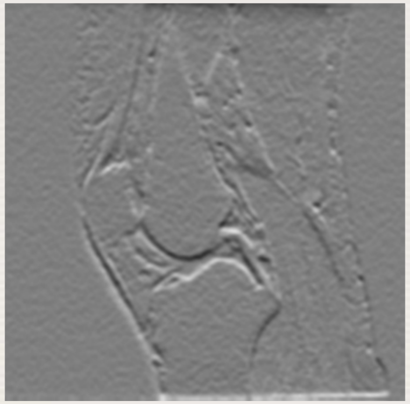
- Often, the small amount of smoothing performed by the Sobel operator is not enough to eliminate the edges associated with image noise
- If we are only interested in very strong edges in the image, we may want to perform additional smoothing
- A common alternative to the Sobel filter is to compute the partial derivatives of the image intensity using convolution of the image with Gaussian derivative operators
- $D_x G$ and $D_y G$

Illustration of Gaussian derivative filters

MR image of the knee



Convolution of the image with the Gaussian x derivative filter, with σ=2



Convolution of the image with the Gaussian y derivative filter, with σ=2

Gradient magnitude image

- The gradient magnitude is higher at the image edges, but less than for the Sobel operator at isolated pixels where image intensity varies due to noise

Image from the U.S. National Biomedical Imaging Archive Osteoarthritis Initiative (https://imaging.nci.nih.gov/ncia)

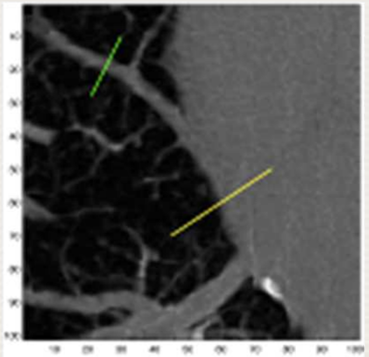
First derivative filters and noise

- Of course, too much smoothing can remove important edges too
- Finding the right amount of smoothing is a difficult and often ill-posed problem

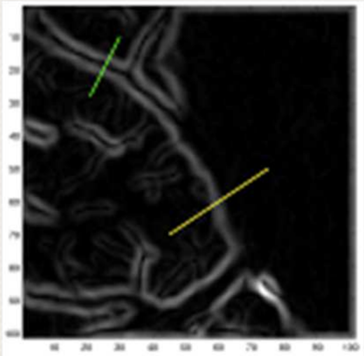
Edge Detectors Based on First Derivative

- Imagine a particle crossing an edge in a continuous smooth image F, moving in the direction orthogonal to the edge (i.e. in the direction of the image gradient)
- If we plot the gradient magnitude of the image along the path of the particle, we see that at the edge, there is a local maximum of the gradient magnitude

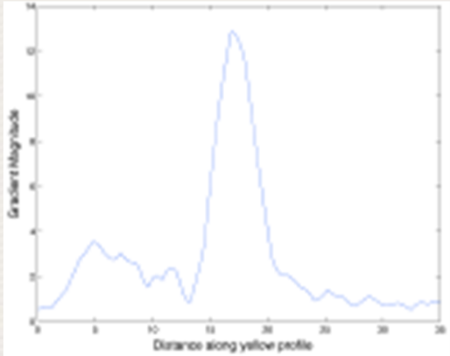
Gradient magnitude at image edges



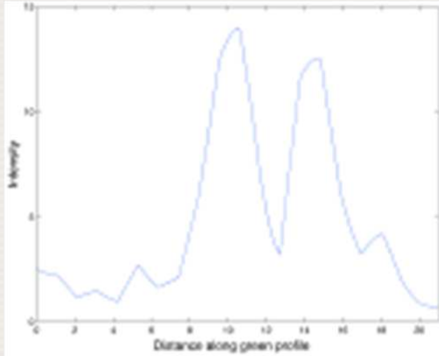
Detail from chest CT image



Corresponding gradient magnitude image



A plot of gradient magnitude image across the edge (yellow profile)



A plot of gradient magnitude image across the tube-like structure (green profile)

- The gradient magnitude reaches its maximum at the points where the profiles cross the image edge

Local maximum of gradient magnitude

- Let us denote the unit vector in the particle's direction as $\mathbf{v}$, and the point where the particle crosses the edge as $\mathbf{x}$
- The gradient magnitude of the image F at $\mathbf{x}$ is simply

$$\nabla F(\mathbf{x}) = D_{\mathbf{v}} F \mid \mathbf{x}$$

- The gradient magnitude reaches a local maximum at x in the direction $\mathbf{v}$ if and only if

$$D_{\mathbf{vv}} F = 0 \quad \text{and} \quad D_{\mathbf{vvv}} F \leq 0$$

- Several edge detectors leverage this property

Marr-Hildreth edge detector (1 of 2)

- The earliest of these operators is the Marr-Hildreth edge detector
- It is based on the fact that the necessary (but not sufficient) condition for $D_{vv}F = 0$ and $D_{vvv}F \leq 0$ is

$$D_{xx}F + D_{yy}F = 0$$

- The operator $D_{xx} + D_{yy}$ is the Laplacian operator
- By finding the set of all points in the image where the Laplacian of the image is zero, we find the superset of all the points that satisfy $D_{vv}F = 0$ and $D_{vvv}F \leq 0$

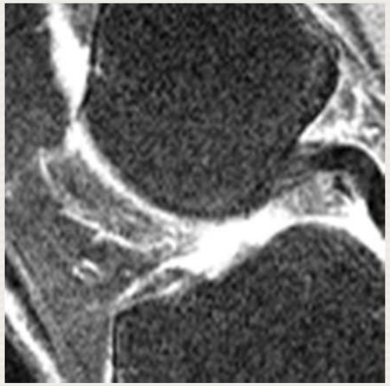
Marr-Hildreth edge detector (2 of 2)

- When dealing with discrete images, we must use convolution with a smooth filter (such as the Gaussian) when computing the second derivatives and the Laplacian
- The Marr-Hildreth edge detector convolves the discrete image I with the Laplacian of Gaussian (LoG) filter:

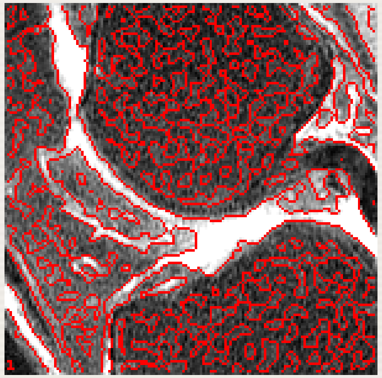
$$J = I \circ (D_{xx} G + D_{yy} G) = D_{xx}(I \circ G) + D_{yy}(I \circ G)$$

- Next, the Marr-Hildreth edge detector finds contours in the image where $J=0$
    - these contours are closed and form the superset of edges in the image
- The last step is to eliminate the parts of the contour where the gradient magnitude of the input image is below a user-specified threshold

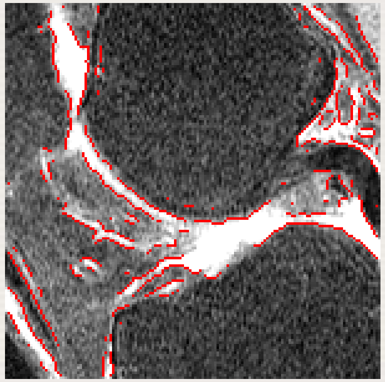
Illustration of Marr-Hildreth edge detector



Input image



Zero crossings of the convolution of the image with the LoG operator



Edges produced by the Marr-Hildreth detector, i.e., a subset of the zero crossings that have gradient magnitude above a threshold

Image from the U.S. National Biomedical Imaging Archive Osteoarthritis Initiative (https://imaging.nci.nih.gov/ncia)

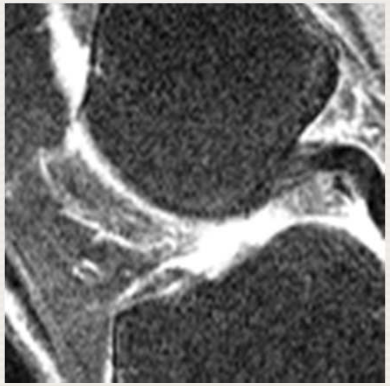
Canny edge detector

- The Canny edge detector is also rooted in the fact that the second derivative of the image in the edge direction is zero
  - applies Gaussian smoothing to the image
  - finds the pixels in the image with high gradient magnitude using the Sobel operator and thresholding
  - eliminates pixels that do not satisfy the maximum condition

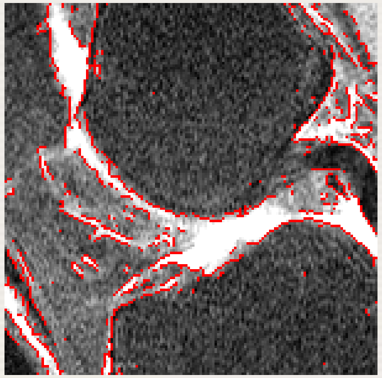
$$D_{vv}F = 0 \text{ and } D_{vvv}F \leq 0$$

  - uses a procedure called hysteresis to eliminate very short edges that are most likely the product of noise in the image
- The Canny edge detector has very good performance characteristics compared to other edge detectors
  - and is very popular in practice

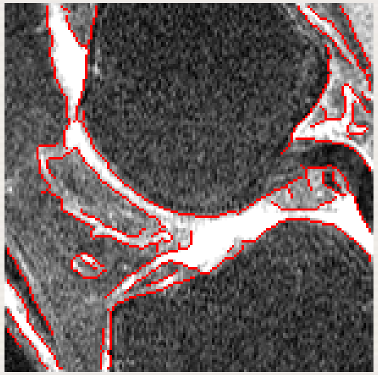
Illustration of Canny edge detector



Input image



Edges produced by the Sobel detector



Edges produced by the Canny detector

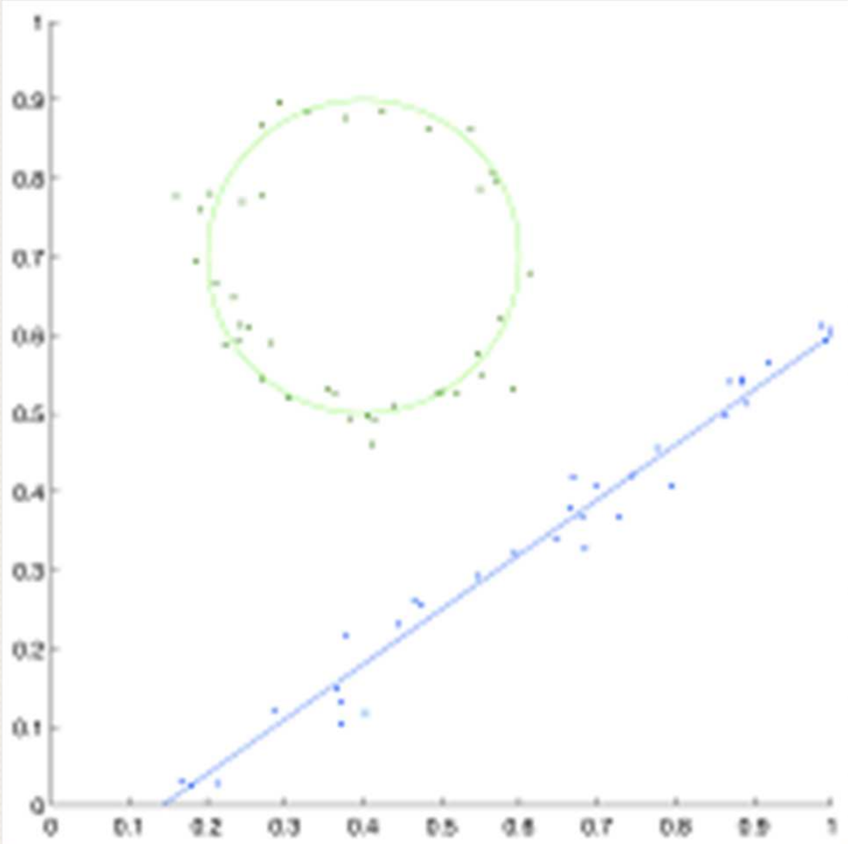
Image from the U.S. National Biomedical Imaging Archive Osteoarthritis Initiative (https://imaging.nci.nih.gov/ncia)

Hough Transform

- So far, we have discussed image processing techniques that search for edges
- Sometimes, the objects that we are interested in detecting have a very characteristic shape: circles, tubes, lines
- In these cases, we are better off using detectors that search for these shapes directly, rather than looking at edges
- The Hough transform is one such detector

Hough Transform – simplified problem (1 of 2)

- Given a set of points in the plane
- Find lines, circles or ellipses approximately formed by these points



Hough Transform – simplified problem (2 of 2)

- Simple shapes, like lines, circles and ellipses, can be described by a small number of parameters
  - circles are parameterized by the centre (2 parameters) and radius (1 parameter)
  - ellipses are parameterized by four parameters
  - lines are naturally parameterized by the slope and intercept (2 parameters)
    - however, this parameterization is asymptotic for vertical lines
    - an alternative parameterization by Duda and Hart (1972) uses the distance from the line to the origin and the slope of the normal to the line as the two parameters describing a line

Hough Transform – parameter space

- Each line, circle, or ellipse corresponds to a single point in the corresponding 2, 3 or 4 dimensional parameter space
- The set of all lines, circles, or ellipses passing through a certain point (x,y) in the image space corresponds to an infinite set of points in the parameter space
- These points in the parameter space form a manifold
- For example
    - all lines passing through (x,y) form a sinusoid in the Duda and Hart 2-dimensional line parameter space
    - all circles passing through (x,y) form a cone in the 3-dimensional circle parameter space

Hough Transform – image domain to parameter domain

- This is the meaning of the Hough transform
- It transforms points in the image domain into curves, surfaces or hypersurfaces in the parameter domain
- If several points in the image domain belong to a single line, circle or ellipse, then their corresponding manifolds in the parameter space intersect at a single point $(p_1, \ldots, p_k)$
- This gives rise to the shape detection algorithm

Hough Transform – shape detection algorithm

- The 2, 3 or 4-dimensional parameter space is divided into a finite set of bins and every bin j is assigned a variable q_j that is initialized to zero
- For every point (x_I, y_I) in the image domain
  - compute the corresponding curve, surface, or hypersurface in the parameter space
  - find all the bins in the parameter space though which the manifold passes
- Every time that the curve, surface or hyper-surface passes through the bin j
  - increment the corresponding variable q_j by 1
- Once this procedure is completed for all N points
  - look for the bins where q_j is large
  - these bins correspond to a set of q_j points that approximately form a line, circle or ellipse

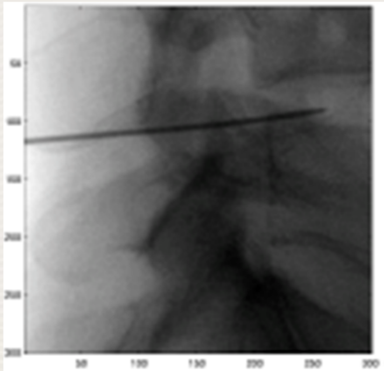
Hough Transform for shape detection

- The Hough transform, combined with edge detection, can be used to search for simple shapes in digital images
  - the edge detector is used to find candidate boundary points

$$(x_1, y_1)\ldots(x_N, y_N)$$

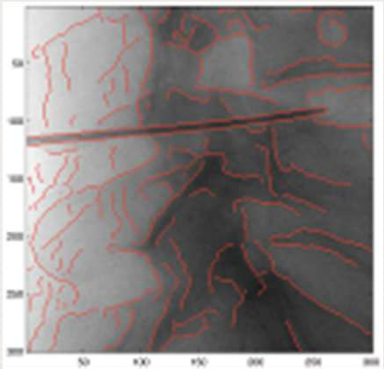
  - then the Hough transform is used to find simple shapes
- The Hough transform is an elegant and efficient approach, but it scales poorly to more complex objects
  - objects more complex than lines, circles, and ellipses require a large number of parameters to describe them
  - the higher the dimensionality of the parameter space, the more memory- and computationally-intensive the Hough transform

Illustration of Hough transform

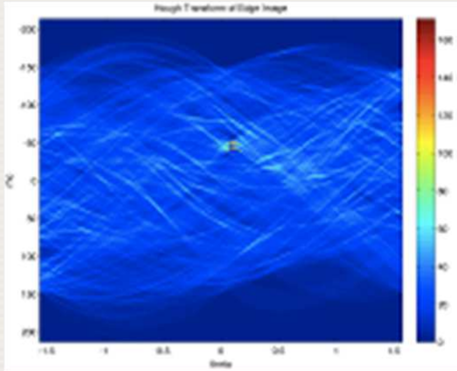


a.

An input fluoroscopy image of a surgical catheter. The catheter is almost straight, making it a good candidate for detection with the Hough transform

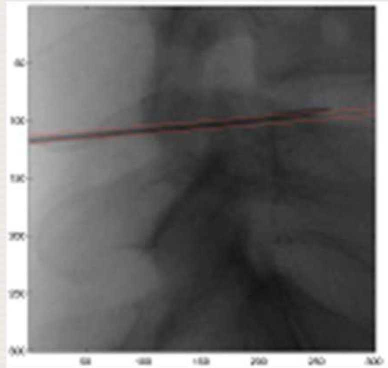


Edge map produced by the Canny edge detector



d.

Superimposed Hough transforms of the edge points. The Hough transform of a point in image space is a sinusoid in Hough transform space. The plot shows the number of sinusoids that pass through every bin in the Hough transform space



The lines shown correspond with the two bins through which many sinusoids pass

17.3 IMAGE SEGMENTATION

17.3

17.3 IMAGE SEGMENTATION

17.3

- 17.3.1 Object representation
- 17.3.2 Thresholding
- 17.3.3 Automatic tissue classification
- 17.3.4 Active contour segmentation methods
- 17.3.5 Atlas-based segmentation

Image segmentation

- The problem of finding objects in images, known as segmentation, is the central problem in the field of image analysis
- It is also a highly complex problem, and there are many types of segmentation problems
  - finding and outlining a specific anatomical structure in a medical image
  - finding pathology in medical images
- These problems are very different depending on the anatomy and imaging modality

Challenges of image segmentation

- Heart segmentation in CT is very different from heart segmentation in MRI, which is very different from brain segmentation in MRI
- Some structures move during imaging, while other structures are almost still
- Some structures have a simple shape that varies little from subject to subject, while others have complex, unpredictable shapes
- Some structures have good contrast with surrounding tissues, and others do not
- More often than not, a given combination of anatomical structure and imaging modality requires a custom segmentation algorithm

17.3 IMAGE SEGMENTATION

17.3.1 OBJECT REPRESENTATION

IAEA

Methods of representing objects in images

- Binary image or label image
- Geometric boundary representations
- Level sets of real-valued images

- Several other representations are available, but they are not discussed here

Binary image or label image

- These are very simple ways to represent an object or a collection of objects in an image
- Given an image I that contains some object O, we can construct another image S of the same dimensions as I, whose pixels have values 0 and 1 according to

$$S(\mathbf{x}) = \begin{cases} 1 & \text{if } \mathbf{x} \in \text{Object} \\ 0 & \text{otherwise} \end{cases}$$

- Such an image is called the binary image of O

Label image

- When I contains multiple objects of interest, we can represent them
  - as separate binary images (although this would not be very memory-efficient)
  - or as a single label image L

$$L(\mathbf{x}) = \begin{cases} 1 & \text{if } \mathbf{x} \in \text{Object 1} \\ 2 & \text{if } \mathbf{x} \in \text{Object 2} \\ & \vdots \\ 0 & \text{otherwise} \end{cases}$$

Limitations of binary and label images

- Their accuracy is limited by the resolution of the image I
- They represent the boundaries of objects as very non-smooth (piecewise linear) curves or surfaces
  - whereas the actual anatomical objects typically have smooth boundaries

17.3 IMAGE SEGMENTATION
17.3.1 Object Representation

Methods of representing objects in images

- Binary image or label image
- Geometric boundary representations
- Level sets of real-valued images

Geometric boundary representations

- Objects can be described by their boundaries
  - more compact than the binary image representation
  - allows sub-pixel accuracy
  - smoothness can be ensured
- The simplest geometric boundary representation is defined by
  - a set of points on the boundary of an object, called vertices
  - a set of line segments called edges (or in 3D, a set of polygons, called faces) connecting the vertices
  - or connect points using smooth cubic or higher order curves and surfaces
- Such geometric constructs are called meshes
- The object representation is defined by the coordinates of the points and the connectivity between the points

17.3 IMAGE SEGMENTATION

17.3.1 Object Representation

Methods of representing objects in images

- Binary image or label image
- Geometric boundary representations
- Level sets of real-valued images

Level sets of real-valued images (1 of 2)

- This representation combines attractive properties of the two preceding representations
  - like binary images, this representation uses an image F of the same dimensions as I to represent an object O in the image I
  - unlike the binary representation, the level set representation can achieve sub-pixel accuracy and smooth object boundaries
- Every pixel (or voxel) in F
  - has intensity values in the range $[-M,M]$
  - where M is some real number

Level sets of real-valued images (2 of 2)

- The boundary of O is given by the zero level set of the function F

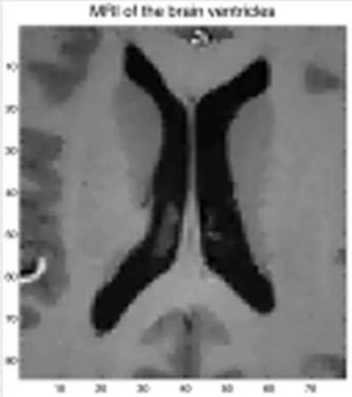
$$B(O) = \left\{ \mathbf{x} \in \mathbb{R}^N : F(\mathbf{x}) = 0 \right\}$$

- F is a discrete image and this definition requires a continuous function
  - in practice, linear interpolation is applied to the image F

Conversion to geometric boundary representations

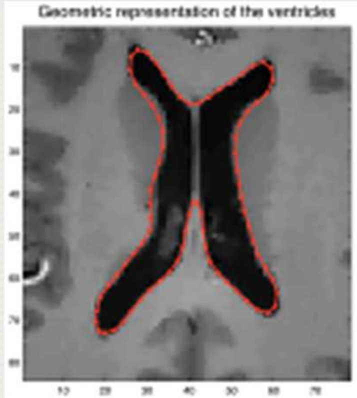
- Binary and level set representations can be converted to geometric boundary representations using contour extraction algorithms
  - such as the marching cubes algorithm
- A binary or geometric boundary representation can be converted to a level set representation using the distance map algorithm

Examples of object representation

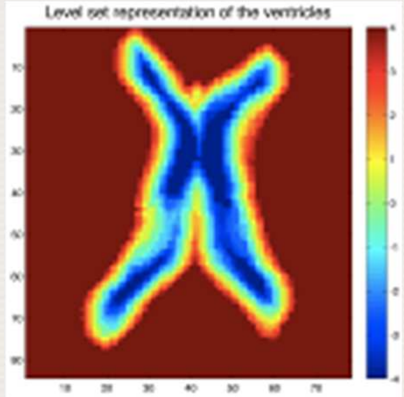


Original image, axial slice from a brain MRI

Binary representation of the lateral ventricles in the image



Geometric representation of the lateral ventricles in the image



Level sets representation of the lateral ventricles in the image

17.3 IMAGE SEGMENTATION

17.3.2 THRESHOLDING

Thresholding (1 of 2)

- Thresholding is the simplest segmentation technique possible
- It is applicable in situations where the structure of interest has excellent contrast with all other structures in the image
- For example, in CT images, thresholding can be used to identify bone, muscle, water, fat and air because these tissue classes have different attenuation levels

Thresholding (2 of 2)

- Thresholding produces a binary image using the following simple rule

$$S(\mathbf{x}) = \begin{cases} 1 & \mathrm{T_{lower}} \le \mathrm{I}(\mathbf{x}) < T_{\mathrm{upper}} \\ 0 & \text{otherwise} \end{cases}$$

- Here, T_{lower} is a value called the lower threshold and T_{upper} is the upper threshold
  - for example, for bone in CT, $T_{lower} = 400$ and $T_{upper} = \infty$
- The segmentation is simply the set of pixels that have intensity between the upper and lower thresholds

Example of thresholding

Original CT image

Thresholded image

Thresholded image, using a higher T_{lower}

Disadvantages of thresholding

- In most medical image segmentation problems, thresholding does not produce satisfactory results
  - in noisy images, there are likely to be pixels inside the structure of interest that are incorrectly labelled because their intensity is below or above the threshold
  - in MRI images, intensity is usually inhomogeneous across the image, so that a pair of thresholds that works in one region of the image is not going to work in a different region
  - the structure of interest may be adjacent to other structures with very similar intensity
- In all of these situations, more advanced techniques are required

Choice of threshold value

- In some images, the results of thresholding are satisfactory, but the value of the upper and lower threshold is not known *a priori*
- For example, in brain MRI images, it is possible to apply intensity inhomogeneity correction to the image, to reduce the effects of inhomogeneity
  - but to segment the grey matter or white matter in these images would typically need a different pair of thresholds for every scan
- In these situations, automatic threshold detection is required

17.3 IMAGE SEGMENTATION

17.3.3 AUTOMATIC TISSUE CLASSIFICATION

Automatic Tissue Classification

- We have to partition an image into regions corresponding to a fixed number of tissue classes, k
- In the brain, for example, there are three important tissue classes
  - white matter, gray matter, and cerebrospinal fluid (CSF)
- In T1-weighted MRI, these tissue classes produce different image intensities: bright white matter and dark CSF
  - unlike CT, the range of intensity values produced by each tissue class in MRI is not known *a priori*
  - because of MRI inhomogeneity artefacts, noise, and partial volume effects, there is much variability in the intensity of each tissue class
- Automatic tissue classification is a term used to describe various computational algorithms that partition an image into tissue classes based on statistical inference

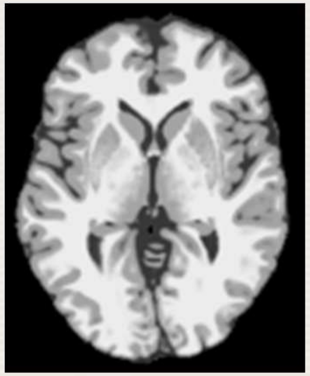
Automatic Tissue Classification by thresholding

- The simplest automatic tissue classification algorithms are closely related to thresholding
- Assume that the variance in the intensity of each tissue class is not too large
- In this case, we can expect the histogram of the image to have peaks corresponding to the *k* tissue classes
- Tissue classification simply involves finding thresholds that separate these peaks
- In a real MR image, the peaks in the histogram are not as well separated and it is not obvious from just looking at the histogram what the correct threshold values ought to be

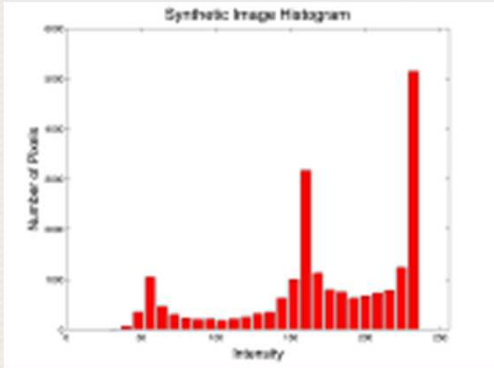
17.3 IMAGE SEGMENTATION

17.3.3 Automatic Tissue Classification

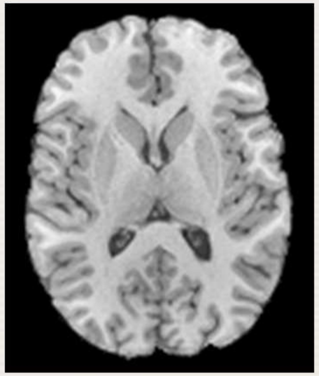
Examples of tissue classification by thresholding



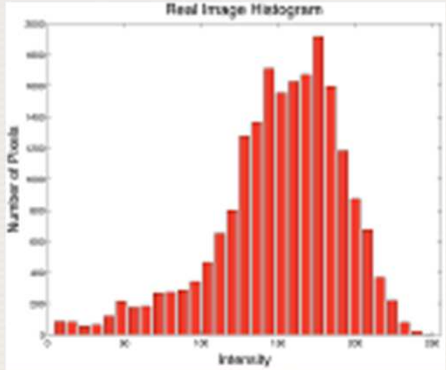
A slice from the digital brain MRI phantom from BrainWeb (Collins et al 1998). This synthetic image has very little noise and intensity inhomogeneity, so the intensity values of all pixels in each tissue class are very similar



The histogram of the synthetic image, with clearly visible peaks corresponding to CSF, gray matter and white matter. A threshold at intensity value 100 would separate the CSF class from the gray matter class, and a threshold of 200 would separate the gray matter from the white matter



A slice from a real brain MRI, with the skull removed



The histogram of the real MRI image. Peaks in the histogram are much less obvious

k-means clustering (1 of 3)

- There are several automatic tissue classification methods that examine the image histogram and determine thresholds that are optimal, according to a certain criterion
- The simplest of these is *k-means clustering*
- This approach groups intensity values in the image histogram into clusters
- The algorithm seeks to minimize the variability of the intensity within each cluster
- Formally, k-means clustering is defined an energy minimization problem

k-means clustering (2 of 3)

$$\alpha_1^* \ldots \alpha_N^* = \arg\min_{\{\alpha_1 \ldots \alpha_N \in [1 \ldots k]\}} \sum_{j=1}^{k} \sum_{\{q:\alpha_q = j\}} (I_q - \mu_j)^2,$$

where

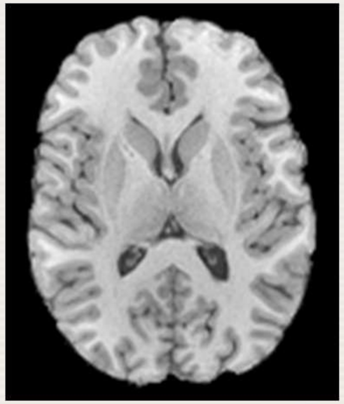
- α_i is the cluster to which the pixel is assigned
- N is the number of pixels in the image
- I_q is the intensity of the pixel q
- μ_j is the mean of the cluster j , i.e., the average intensity of all pixels assigned the label j

❑ $\arg\min\limits_{x \in \Omega} f(x)$ is read as “the point in the domain Ω where the function $f(x)$ attains its minimum”

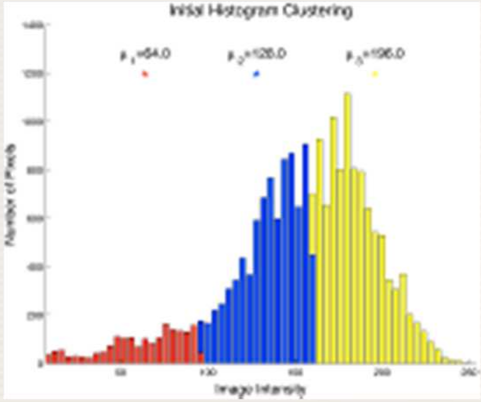
k-means clustering (3 of 3)

- Theoretically, the optimization problem is intractable
  - but a simple iterative approach yields good approximations of the global minimum in practice
- This iterative approach requires the initial means of the clusters to be specified
- One of the drawbacks of k-means clustering is that it can be sensitive to initialization

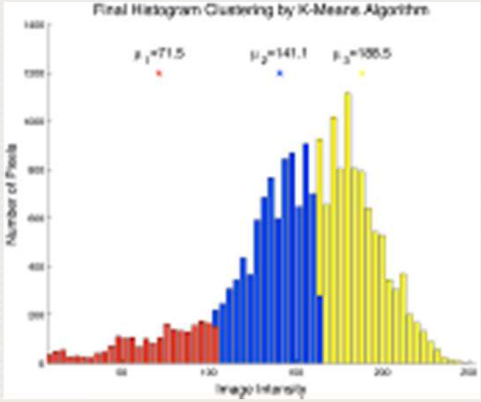
Example of segmentation by k-means clustering



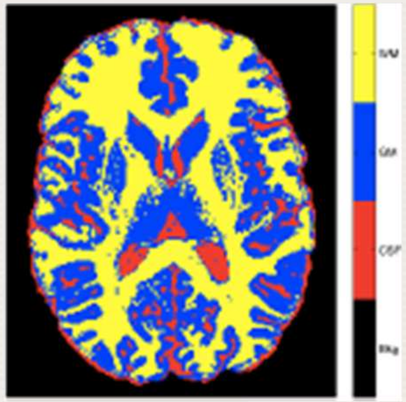
A slice from a brain MRI



Partitioning of the image histogram into clusters based on initial cluster means



Partitioning of the histogram into clusters after 10 iterations



Segmentation of the image into gray matter (GM), white matter (WM) and cerebrospinal fluid (CSF)

Fuzzy c-means clustering

- In fuzzy c-means clustering, cluster membership is not absolute
- Instead, fuzzy set theory is used to describe partial cluster membership
- This results in segmentations where uncertainty can be adequately represented

Gaussian mixture modelling (1 of 3)

- Gaussian mixture modelling assumes that the pixel intensities $x_i, \ldots, x_N$ in an image are samples from a random variable X with a probability density function $f(x)$ that is a weighted sum of n Gaussian probability densities, with respective weights $\alpha_1, \ldots, \alpha_n$

$$f(x; \{\alpha_m, \mu_m, \sigma_m\}) = \sum_{m=1}^{k} \alpha_m z(\frac{x - \mu_m}{\sigma_m})$$

where

- z is the standard normal distribution
- $\alpha_m, \mu_m, \sigma_m$ are unknown

Gaussian mixture modelling (2 of 3)

- The expectation-minimization algorithm (EM) is used to find the maximum likelihood estimate of $\alpha_m, \mu_m, \sigma_m$

$$\{\alpha_m^*, \mu_m^*, \sigma_m^*\} = \arg\max_{\{\alpha_m, \mu_m, \sigma_m\}} P(x_1, \ldots, x_N; \{\alpha_m, \mu_m, \sigma_m\}) = \arg\max_{\{\alpha_m, \mu_m, \sigma_m\}} \prod_{i=1}^{n} f(x_i; \{\alpha_m, \mu_m, \sigma_m\})$$

- Intuitively, Gaussian mixture modelling fits the image histogram with a weighted sum of Gaussian densities
- Once the optimal parameters have been found, the probability that pixel belongs to a tissue class is found as

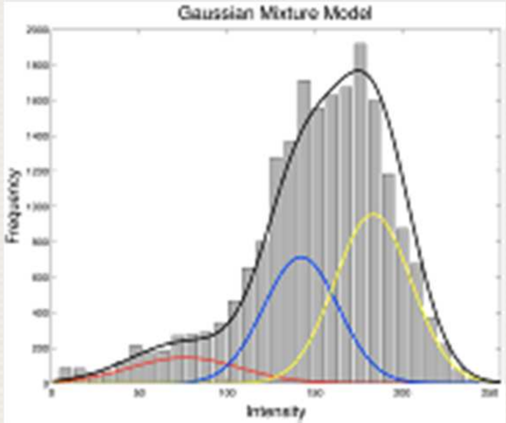
$$P(L_j = m \mid x_j) = z\left(\frac{x_j - \mu_m}{\sigma_m}\right).$$

Gaussian mixture modelling (3 of 3)

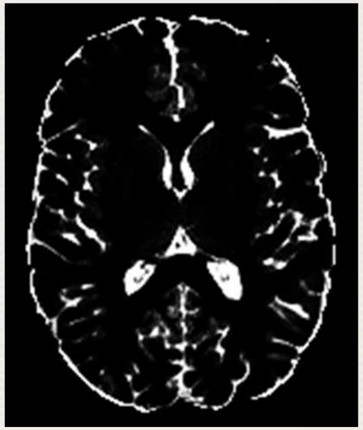
- Like fuzzy c-means, Gaussian mixture modelling can describe uncertainty
- For each tissue class, a probability image is generated
  - estimating the probability that a given pixel belongs to a given tissue class
- It is possible to model partial volume effects
  - for example, a pixel may be assigned 0.5 probability of being white matter, 0.4 probability of being gray matter and 0.1 probability of being CSF
  - which can be interpreted as a partial volume effect, i.e., both white matter and gray matter tissues present in the pixel

17.3 IMAGE SEGMENTATION
17.3.3 Automatic Tissue Classification

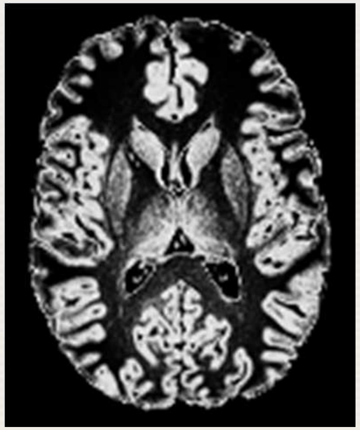
Example of segmentation using Gaussian mixture models



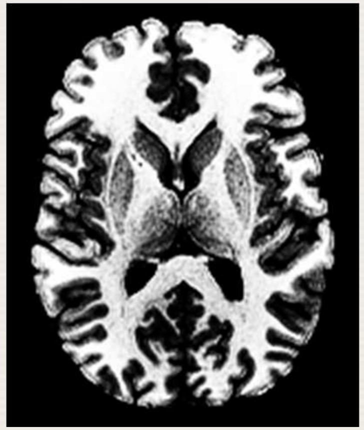
A mixture model fitted to the histogram of brain MRI image



CSF probability map generated by the method



Grey matter probability map generated by the method



White matter probability map generated by the method

- The mixture model (black curve) is a weighted sum of three Gaussian probability densities, one for each tissue type (red, blue and yellow curves)

Improving the performance of Gaussian mixture modelling

- The performance of Gaussian mixture modelling can be further improved by introducing constraints on consistency of the segmentation between neighbouring pixels
- Methods that combine Gaussian mixture modelling with such spatial regularization constraints are among the most widely used in brain tissue segmentation from MRI

17.3 IMAGE SEGMENTATION

17.3.4 ACTIVE CONTOUR SEGMENTATION METHODS

Active contours

- The term active contours is used to describe a family of image segmentation algorithms
  - emerged from the seminal work of Kass and Witkin (1988) on active snakes
- Before active snakes, the mainstream approach to object segmentation involved edge detection, followed by linking edges to form object boundaries
- Such a deterministic approach is limited to simple segmentation problems
- Active snakes were a radical shift from the deterministic paradigm
  - an early example of knowledge-based image analysis, where prior knowledge about the shape and smoothness of object boundaries is used to guide segmentation

Object segmentation

- Unlike automatic tissue classification, active contour methods address the problem of object segmentation
- The goal is to identify a specific anatomical structure, or small set of structures, in a biomedical image
- The structure is represented by a contour (a closed curve in 2D, or a closed surface in 3D)
- The goal is to find a contour C that minimizes an energy function $E(C)$

Minimising the energy function

- The energy function typically comprises of two terms
  - a term that measures how well the contour coincides with the boundaries of objects in the image
  - a term that measures how simple the contour C is
- As an example, consider the 2D contour energy function proposed by Caselles et al. (1997)

$$E(C) = \int_0^1 g_I(C(t))\left|C'(t)\right| \mathrm{d}t \quad \text{where} \quad g_I(x,y) = \frac{1}{1 + |\nabla(G_\sigma \circ I)(x,y)|}$$

2D contour energy function

$$E(C) = \int_0^1 g_I(C(t))\left|C'(t)\right| \mathrm{d}t \quad \text{where} \quad g_I(x,y) = \frac{1}{1+|\nabla(G_\sigma \circ I)(x,y)|}$$

- The contour C is parameterized by the variable $t, 0 \leq t \leq 1$
- The function g_I is called the speed function
  - it is a monotonically decreasing function of the image gradient magnitude
  - it has very small values along the edges of I, and is close to 1 away from the edges of I
- It is easy to verify that the energy $E(C)$ is decreased by
  - making C fall on edges in I, where g_I is reduced
  - making C shorter, which reduces $\left|C'(t)\right|$

Evolution equation

- Active contour methods are usually described not in terms of the energy function $E(C)$, but instead, in terms of an evolution equation

$$\frac{\partial C}{\partial T} = F\,\vec{N}$$

where

- F is a scalar function of the image and the contour
- $\vec{N}$ is the unit normal vector to C

- This equation describes how to evolve the contour over time T such that the energy $E(C)$ decreases

Deriving the evolution equation

- The evolution equation and the function F can be derived from the energy function using the calculus of variations
- Different active contour methods use different functions F, which correspond to different energy functions
    - for example, the evolution equation for the 2D Caselles energy function has the form

$$\frac{\partial C}{\partial T} = \left(g_I \kappa - \nabla g_I \cdot \vec{N} \right) \vec{N}$$

where κ is the curvature of the contour C

- The same equation is used to describe contour evolution in 3D, except that is used to describe the mean curvature of the surface

Boundary representation for active contours (1 of 3)

- In early active contour methods, the segmentation was represented using a geometric boundary representation
  - i.e. a piecewise cubic curve for 2D segmentation
  - a surface for 3D segmentation
- In modern active contour methods, the level set representation is used instead, because of its numerical stability and simplicity
- The evolution equation can be adapted to the level set representation

Boundary representation for active contours (2 of 3)

- If ϕ is a function on $\mathbb{R}^n$, such that $C = \{\mathbf{x} \in \mathbb{R}^n : \phi(\mathbf{x}) = 0\}$
  - i.e. C is the zero level set of ϕ
- Then the evolution equation can be rewritten in terms of ϕ as

$$\partial\phi / \partial T = F\left|\nabla\phi\right|$$

- For instance, for the Caselles energy, the level set evolution equation has the form

$$\frac{\partial\phi}{\partial T} = \left[g_I \operatorname{div}\left(\frac{\nabla\phi}{\left|\nabla\phi\right|} \right) + \nabla g_I \frac{\nabla\phi}{\left|\nabla\phi\right|} \right] \left|\nabla\phi\right|$$

Boundary representation for active contours (3 of 3)

- The level set representation of the active contour has a number of advantages
  - level set methods are numerically robust and simple to implement
  - with the level set representation, the topology of the segmentation can change: multiple contours can merge into a single contour
  - because the active contour is represented as a level set, the contour is always a closed manifold

Active contours and tissue classification

- Active contour segmentation can be used in conjunction with automatic tissue classification
  - using a method developed by Zhu and Yuille (1996)
- This method uses the following definition for F

$$F(\mathbf{x}) = \alpha\left[\log P(\mathbf{x} \in \mathrm{Object}) - \log P(\mathbf{x} \in \mathrm{Background})\right] - \beta\,\kappa$$

where

- $P(\mathbf{x} \in \mathrm{Object})$ and $P(\mathbf{x} \in \mathrm{Background})$ are the probabilities that a pixel at position x belongs to the object of interest or to the background, respectively
  - these probabilities can be estimated from the image I using automatic tissue classification or manual thresholding
- the constants α and β are user-specified weights that provide a trade-off between the terms in the speed function

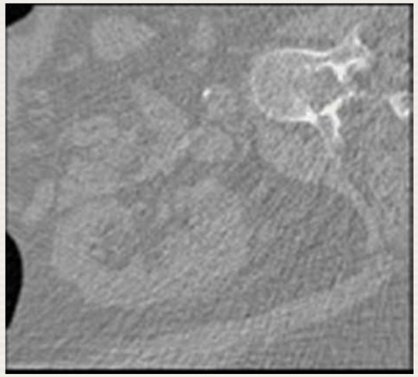
Interpretation of evolution for tissue classification

- The evolution has a very intuitive interpretation
- The component of the force weighted by α
  - pushes the contour outwards if it lies inside of the object
    - i.e. $P(\mathbf{x} \in \text{Object}) > P(\mathbf{x} \in \text{Background})$
  - pushes the contour inwards if it lies outside of the object
- The component $-\beta\kappa$
  - pushes the contour inward at points with large negative curvature
  - pushes it outwards at points with large positive curvature
- The effect is to smooth out the sharp corners in the contour, keeping the shape of the contour simple

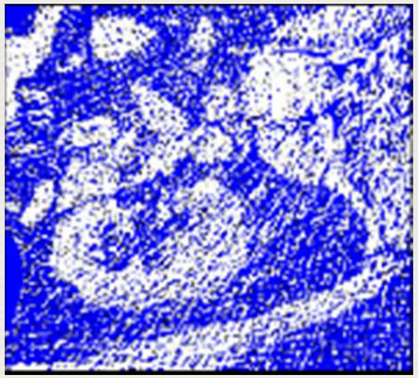
Active contours and segmentation

- Regardless of the flavour of the active contour method, the segmentation proceeds as follows
  - the user provides an initial segmentation: for example, a circle or sphere placed inside the object of interest
  - contour evolution is then simulated by repeatedly applying the evolution equation
  - evolution is repeated
    - until convergence
    - or until the user interrupts it

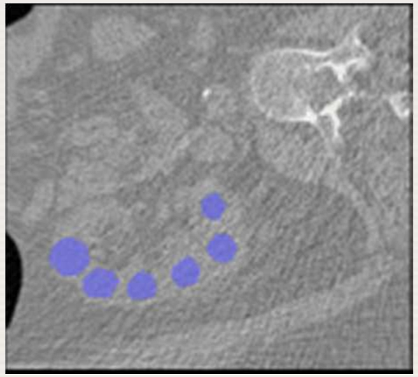
Example of active contour segmentation



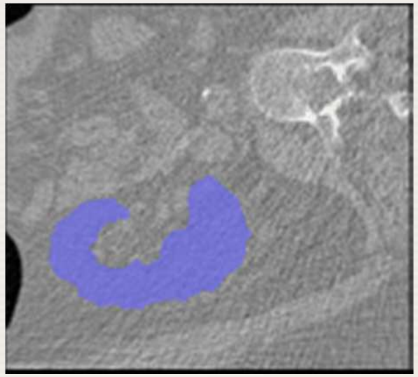
An axial slice from a low-contrast CT volume



A map of $P(\mathbf{x} \in \text{Object}) - P(\mathbf{x} \in \text{Background})$ computed using tissue classification



Initialization of the active contour method using five spherical seeds



Segmentation after 1000 iterations of evolution. Note that the contours have merged to form a single surface

- In the probability map white pixels have high object probability; blue points have high background probability
- Simple thresholding of the probability map will lead to a very noisy segmentation

Extensions to active contour segmentation

- Active contour segmentation is an area of active research
- Numerous extensions to the methods have been proposed in the recent years, including
  - more general shape priors
  - constraints on the topology of the segmentation
  - various application-specific image-based criteria

17.3 IMAGE SEGMENTATION

17.3.5 ATLAS-BASED SEGMENTATION

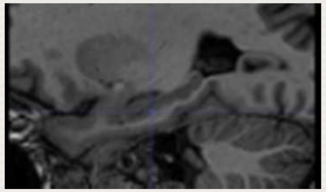
Atlas-based segmentation (1 of 2)

- Deformable image registration is a technique that
  - automatically finds correspondences between pairs of images
  - in recent years, has also become a popular tool for automatic image segmentation
- Perform registration between
  - one image, called the atlas, in which the structure of interest has been segmented, say manually
  - another image I, in which we want to segment the structure of interest

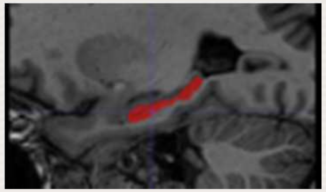
Atlas-based segmentation (2 of 2)

- By performing registration between the image and the atlas
  - we obtain a mapping $\phi(x)$ that maps every point in the image into a corresponding point in the atlas
  - this mapping can be used to transform the segmentation from the atlas into image I
- The quality of the segmentation is limited only by the quality of the registration

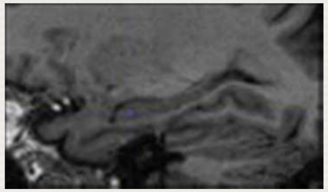
Example of atlas-based segmentation



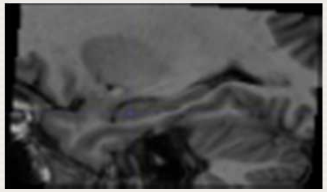
A brain MR image used as the atlas



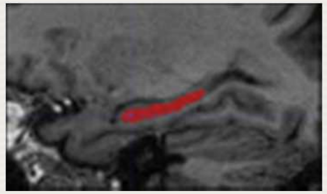
Manual segmentation of the hippocampus in the atlas



The target image, in which we want to segment the hippocampus



Atlas warped to the target image using deformable registration



The atlas-based segmentation of the target image, overlaid on the target image

Multiple atlas-based segmentation

- Several authors have extended this simple idea to using multiple atlases
- Each atlas is registered to the image I, and segmentation from each atlas is mapped into I
- Because of registration errors, these warped segmentations $S_1 \ldots S_K$ do not overlap perfectly
- A voting scheme is used to derive a consensus segmentation from $S_1 \ldots S_K$

Advantages of atlas-based segmentation

- The appeal of atlas-based segmentation is that it is very easy to implement
- Several image registration software applications are available in the public domain
- All that the user needs to perform atlas-based segmentation is an image, or several images, where the object of interest has been manually segmented
- Atlas-based segmentation can be applied in various imaging modalities
  - but its quality may not be as high as methods that use shape priors

17.4 IMAGE REGISTRATION

17.4

Image registration

- Often in medical image analysis, we have to process information from multiple images
  - images with different modalities (CT, PET, MRI) from the same subject
  - images acquired at different time points from a single subject
  - images of the same anatomical regions from multiple subjects
- In all these, and many other situations, we need a way to find and align corresponding locations in multiple images
- Image registration is a field that studies optimal ways to align and normalise images

17.4 IMAGE REGISTRATION
17.4

Image registration and transformations

- Image registration is the problem of finding transformations between images
- Given an image $I : \Omega \in \mathbb{R}^n \rightarrow \mathbb{R}$ and an image $J : \Omega \in \mathbb{R}^n \rightarrow \mathbb{R}$
- Seek a transformation $\phi : \Omega \rightarrow \Omega$
  - such that $I(x)$ and $J(\phi(x))$ are "similar" for all x in Ω
- The meaning of "similar" depends on the application
  - in the context of medical image analysis, "similar" usually means "describing the same anatomical location"
  - however, in practice such anatomical similarity cannot be quantified, and "similar" means "having similar image intensity features"

Characterisation of image registration problems

- There are many different types of image registration problems
- They can be characterized by two main components
  - the transformation model
  - the similarity metric

Image Registration

17.4 IMAGE REGISTRATION

17.4.1 TRANSFORMATION MODELS

Image Registration

Linear vs. non-linear transformations

- The transformation can take many forms
- The transformation is called linear when it has the form

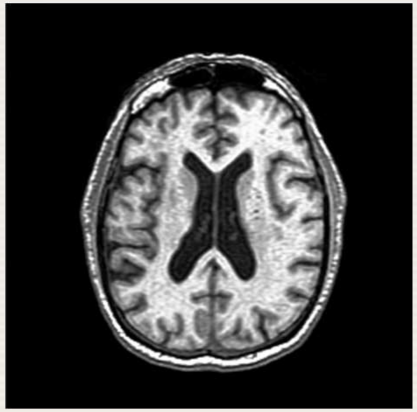
$$\phi(\mathbf{x}) = A\mathbf{x} + b$$

where
  - A is an n × n matrix
  - b is an n × 1 vector
- Otherwise, the transformation is non-linear

Rigid vs. non-rigid transformations

- A special case of linear transformations are rigid transformations
- The matrix in rigid transformations is a rotation matrix
- Rigid transformations describe rigid motions
- They are used in applications when the object being imaged moves without being deformed
- Non-rigid linear transformations, as well as non-linear transformations, are called deformable transformations

Examples of spatial transformations



Original image

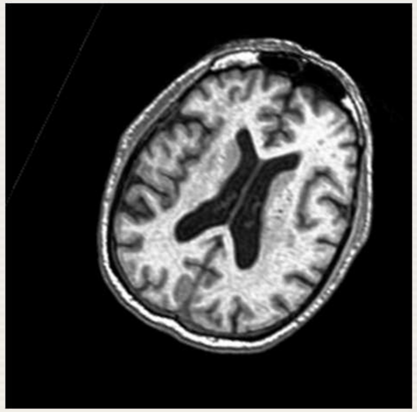


Image transformed by rigid transformation (rotation and translation)

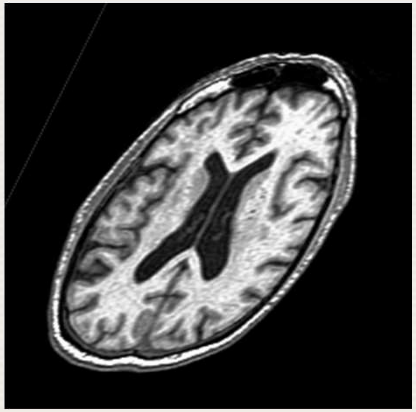


Image transformed by linear affine transformation

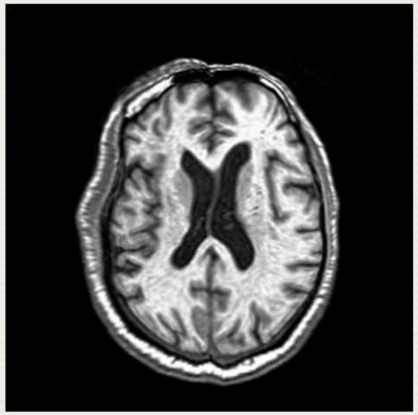


Image transformed by non-linear deformable transformation

Parametric transformations

- Non-linear transformations can be parametric or non-parametric
- Parametric transformations have the form

$$\phi(\mathbf{x}) = \mathbf{x} + \sum_{i=1}^{n} W_{i_1} f_i(\mathbf{x}) e_1 + W_{i_2} f_i(\mathbf{x}) e_2 + \ldots$$

where

- $\{f_i(\mathrm{x}) : \Omega \to \mathbb{R}$ is a basis, such as the Fourier basis or the B-spline basis
- e_1, e_2 are unit vectors in the cardinal coordinate directions
- W_{i_1}, W_{i_2} are the coefficients of the basis functions

Parametric vs. non-parametric transformations

- Usually, a relatively small number of low-frequency basis functions is used to represent a parametric transformation
- The resulting transformations vary smoothly across Ω
- Such transformations are called low-dimensional non-linear transformations
- Non-parametric transformations do not have such a parametric form
- Instead, at every point in Ω, a vector ***v(x)*** is defined, and the transformation is simply given by

$$\phi(\mathbf{x}) = \mathbf{x} + \mathbf{v}(\mathbf{x})$$

Diffeomorphic transformations

- Diffeomorphic transformations are a special class of non-parametric deformable transformations
  - they are differentiable on Ω and have a differentiable inverse
  - e.g. in one dimension ($n=1$), diffeomorphic transformations are monotonically increasing (or monotonically decreasing) functions
- Very useful for medical image registration because they describe realistic transformations of anatomy, without singularities such as tearing or folding
- Registration algorithms that restrict deformations to be diffeomorphic exploit the property that the composition of two diffeomorphic transformations is also diffeomorphic
  - the deformation between two images is constructed by composing many infinitesimal deformations, each of which is itself diffeomorphic

17.4 IMAGE REGISTRATION

17.4.2 REGISTRATION SIMILARITY METRICS

Image Registration

Similarity metrics

- Image registration tries to match places in images that are similar
- Since true anatomical similarity is not known, surrogate measures based on image intensity are used
- Many metrics have been proposed
- We will only review three such metrics
  - Mean squared intensity difference
  - Mutual information
  - Cross-correlation

Mean squared intensity difference

- The similarity is measured as difference in image intensity
- The similarity of images I and J is given by

$$\mathrm{Sim}(I, J \mid \phi) = \frac{1}{|\Omega|} \int_{\Omega} \left[I(\mathbf{x}) - J(\phi(\mathbf{x})) \right]^2 \mathrm{d}\mathbf{x}$$

- Simple to compute
- Appropriate when anatomically similar places can reasonably be expected to have similar image intensity values
- Not appropriate for
  - registration of images with different modalities
  - MRI registration, because MRI intensity values are not consistent across scans

Mutual information (1 of 3)

- Very useful for multimodality image registration
- A pair of images of the body are acquired with different modalities
  - in modality 1, bone may have intensity range 100-200 and soft tissue may have range 10-20
  - in modality 2, bone may have intensity between 3000 and 5000, and soft tissue may have intensity between 10000 and 20000
- The mean square intensity difference metric would return very large values if these two images are aligned properly
- Another metric is needed that does not directly compare the intensity values

Mutual information (2 of 3)

- The mutual information metric is derived from information theory
- To compute mutual information between images I and J, we treat the pairs of intensity values (I_k, J_k) as samples from a pair of random variables X, Y
- One such sample exists at each pixel
- Mutual information is a measure of how dependent random variables X and Y are on each other

Mutual information (3 of 3)

Mutual information is given by

$$\iint p(x,y)\log\left(\frac{p(x,y)}{p(x)p(y)}\right)dxdy$$

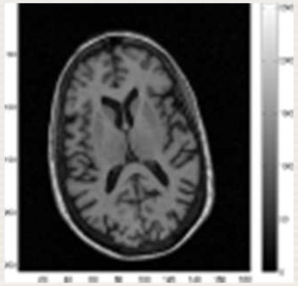
where

- $p(x,y)$ is the joint density of X and Y
- $p(x)$ is the marginal density of X, $p(y)$ is the marginal density of Y

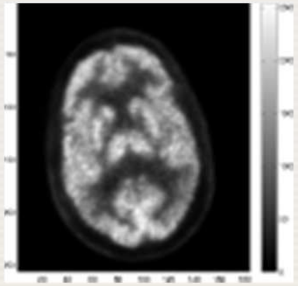
- The marginal densities are estimated by the histograms of the images I and J
- The joint density is estimated by the two-dimensional joint histogram of the images I and J

17.4.2 Registration Similarity Metrics

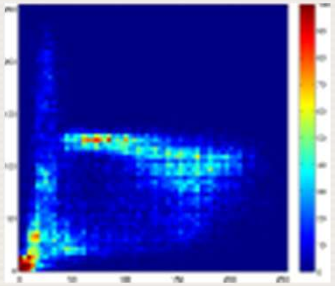
Illustration of the joint histogram used in the computation of the mutual information metric



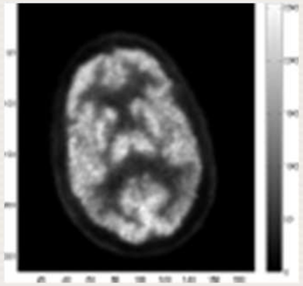
Axial slice from an MR image



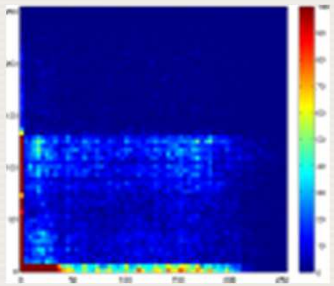
Axial slice from a PET image aligned with the MRI



Joint histogram of the MR and PET images



PET slice rotated out of alignment with the MRI



Joint histogram of the MRI and misaligned PET slice

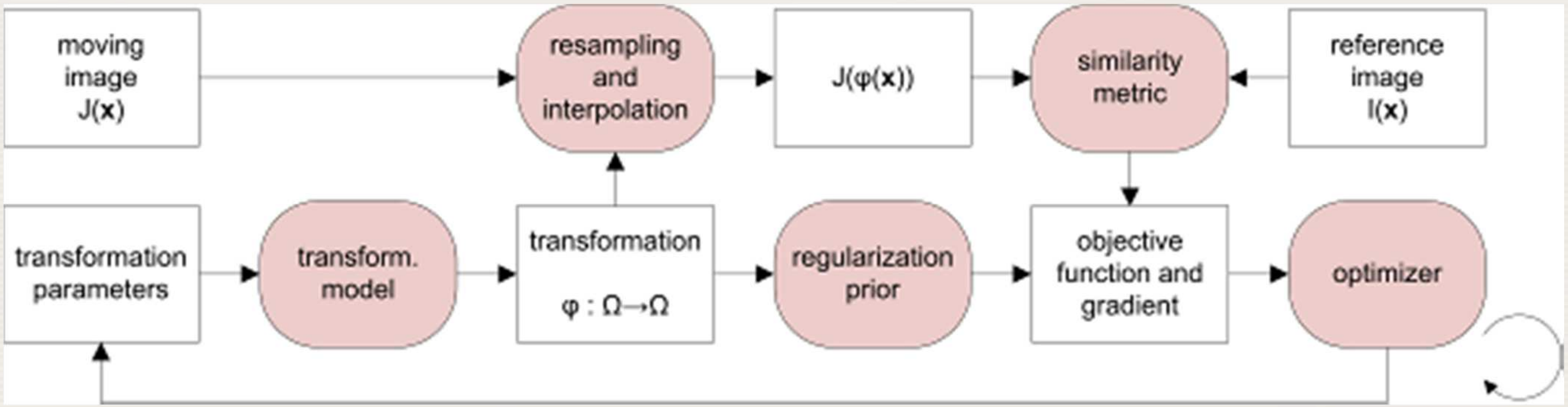
Cross-correlation

- The cross-correlation metric is computed as follows
    - at each pixel index k, we compute the correlation coefficient between the values of image I in a small neighbourhood of pixels surrounding k, and the values of image J over the same neighbourhood
    - the correlation coefficients are summed up over the whole image
- The cross-correlation metric is robust to noise because it considers neighbourhoods rather than individual pixels
- However, it is expensive in computation time

17.4 IMAGE REGISTRATION

17.4.3 THE GENERAL FRAMEWORK FOR IMAGE REGISTRATION

General algorithmic framework for image registration



General algorithmic framework – transformation ϕ

- Usually, one of the images is designated as a reference image and the other image is the moving image
- Transformations are applied to the moving image, while the reference image remains unchanged
- The transformation ϕ is defined by some set of parameters
  - small set for linear registration
  - bigger for parametric non-linear registration
  - and very large for non-parametric non-linear registration

General algorithmic framework – similarity metric

- Some initial parameters are supplied
  - usually these initial parameters correspond to the identity transformation
- The transformation is applied to the moving image
  - this involves resampling and interpolation because the values of $\phi(\boldsymbol{x})$ fall between voxel centres
- The resampled image $J(\phi(\boldsymbol{x}))$ is compared to the reference image $I(\boldsymbol{x})$ using the similarity metric
  - this results in a dissimilarity value
  - registration seeks to minimize this dissimilarity value

General algorithmic framework – regularization prior

- In many registration problems, an additional term, called the regularization prior, is minimized
- This term measures the complexity of the transformation ϕ
  - favours smooth, regular transformations over irregular transformations
  - can be though of as an Occam's razor prior for transformations

General algorithmic framework – objective function value

- Together, the dissimilarity value and the regularization prior value are combined into an objective function value
- The gradient of the objective function with respect to the transformation parameters is also computed
- Numerical optimization updates the values of the transformation parameters so as to minimize the objective function

17.4 IMAGE REGISTRATION

17.4.4 APPLICATIONS OF IMAGE REGISTRATION

Applications of Image Registration

- There are many image analysis problems that require image registration
- Different problems require different transformation models, and different similarity metrics
- We can group medical image registration problems into two general categories
  - registration that accounts for differences in image acquisition
  - registration that accounts for anatomical variability (image normalisation)

Registration that accounts for differences in image acquisition

- In many biomedical applications, multiple images of the same subject are acquired
  - images may have completely different modalities (MRI vs. CT, CT vs. PET, etc.)
  - images may be acquired on the same piece of equipment using different imaging parameters
  - even when parameters are identical, the position of the subject in the scanner may change between images
- To co-analyse multiple images of the same subject, it is necessary to match corresponding locations in these images
- This is accomplished using image registration
- Within this category, there are several distinct subproblems that require different methodology

17.4 IMAGE REGISTRATION

17.4.4 Applications of Image Registration

Accounting for Subject's Motion

- When multiple images of a subject are acquired in a short span of time, the subject may move
  - for example, in fMRI studies, hundreds of scans are acquired during an imaging session
- To analyse the scans, they must first be aligned, so that the differences due to subject motion are factored out
- Motion correction typically uses image registration with rigid transformation models
- Simple image similarity metrics suffice

Alignment of Multi-Modality 3D Images

- Often information from different imaging modalities must be combined for purposes of visualisation, diagnosis, and analysis
  - for example, CT and PET images are often co-analysed, with CT providing high-resolution anatomical detail, and PET capturing physiological measures, such as metabolism
- The images have very different intensity patterns
  - so registration requires specialised image similarity metrics, such as mutual information
- Often rigid transformations suffice
  - however, some modalities introduce geometric distortions to images and low-dimensional parametric transformations may be necessary to align images

Alignment of 3D and 2D Imaging Modalities

- Sometimes registration is needed to align a 2D image of the subject to a 3D image
  - this problem arises in surgical and radiotherapy treatment contexts
  - a 3D scan is acquired and used to plan the intervention
  - during the intervention, X ray or angiographic images are acquired and used to ensure that the intervention is being performed according to the plan
  - corrections to the intervention are made based on the imaging
- For this to work, image registration must accurately align images of different dimensions and different modality
- This is a challenging problem that typically requires the image registration algorithm to simulate 2D images via data from the 3D image

Registration that Accounts for Anatomical Variability (Image Normalisation)

- The other major application of image registration is to match corresponding anatomical locations
  - in images of different subjects
  - in images where the anatomy of a single subject has changed over time
- The term commonly used for this is image normalisation
- Again, there are several different applications

Cross-sectional morphometry (1 of 2)

- Often we are interested in measuring how the anatomy of one group of subjects differs from another
  - in a clinical trial, we may want to compare the anatomy of a cohort receiving a trial drug to the cohort receiving a placebo
- We may do so by matching every image to a common template image using image registration with non-linear transformations
- We may then compare the transformations from the template to the images in one cohort to the transformations to the images in the other cohort

Cross-sectional morphometry (2 of 2)

- Specifically, we may examine the Jacobian of each transformation
- The Jacobian of the transformation describes the local change in volume caused by the transformation
- If
  - an infinitesimal region in the template has volume δV_0,
  - the transformation ϕ maps this region into a region of volume δV_1
- Then the ratio $\delta V_1 / \delta V_0$ equals the determinant of the Jacobian of the transformation

Longitudinal morphometry

- We may acquire multiple images of a subject at different time points when studying the effect on human anatomy of
  - disease
  - intervention
  - aging
- To measure the differences over time, we can employ parametric or non-parametric deformable registration
- Because the overall anatomy does not change extensively between images, the regularisation priors and other parameters of registration may need to be different than for cross-sectional morphometry

17.5 OPEN-SOURCE TOOLS FOR IMAGE ANALYSIS

17.5

Open-source tools for image analysis

- This section briefly reviews several mature image processing and analysis tools that were available freely on the Internet at the time of writing
- The reader can experiment with the techniques described in this chapter by downloading and running these tools
- Most tools run on Apple and PC computers (with Linux and Windows operating systems)
- These are just of few of many excellent tools available to the reader
- The Neuroimaging Informatics Tools and Resources Clearinghouse (NITRC, http://www.nitrc.org) is an excellent portal for finding free image analysis software

Open-source tools for image analysis: URLs

- ImageJ — http://rsbweb.nih.gov/ij
- ITK-SNAP* — http://itksnap.org
- FSL — http://www.fmrib.ox.ac.uk/fsl
- OsiriX — http://www.osirix-viewer.com
- 3D Slicer — http://slicer.org

* Disclaimer: the book chapter author is involved in development of ITK-SNAP

17.5 OPEN-SOURCE TOOLS FOR IMAGE ANALYSIS

ImageJ

- ImageJ provides a wide array of image processing operations that can be applied to 2D and 3D images
- In addition to basic image processing (filtering, edge detection, resampling), ImageJ provides some higher-level image analysis algorithms
- ImageJ is written in Java
- ImageJ can open many common 2D image files, as well as DICOM format medical imaging data

ITK-SNAP

- ITK-SNAP is a tool for navigation and segmentation of 3D medical imaging data
- ITK-SNAP implements the active contour automatic segmentation algorithms by Caselles et al. (1997) and Zhu and Yuille (1996)
- It also provides a dynamic interface for navigation in 3D images
- Several tools for manual delineation are also provided. ITK-SNAP can open many 3D image file formats, including DICOM, NIfTI and Analyze

FSL

- FSL is a software library that offers many analysis tools for MRI brain imaging data
- It includes tools for linear image registration (FLIRT), non-linear image registration (FNIRT), automated tissue classification (FAST) and many others
- FSL supports NIfTI and Analyze file formats, among others

OsiriX

- OsiriX is a comprehensive PACS workstation and DICOM image viewer
- It offers a range of visualization capabilities and a built-in segmentation tool
- Surface and volume rendering capabilities are especially well-suited for CT data
- OsiriX requires an Apple computer with MacOS X

3D Slicer

- Slicer is an extensive software platform for image display and analysis
- It offers a wide range of plug-in modules that provide automatic segmentation, registration and statistical analysis functionality
- Slicer also includes tools for image-guided surgery
- Many file formats are supported

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