

## Chapter 2

Describing Motion: Kinematics in One Dimension


## Opening Question?

Two small heavy balls have the same diameter but one weighs twice as much as the other. The balls are dropped from a second story balcony at the exact same time. The time to reach the ground below will be:
(a) twice as long for the lighter ball than the heavier one.
(b) longer for the lighter ball but not twice as long.
(c) twice as long for the heavier ball than the lighter one.
(d) longer for the heavier ball but not twice as long.
(e) $\sqrt{ }$ nearly the same for both balls.

## Units of Chapter 2

- Reference Frames and Displacement
- Average Velocity
- Instantaneous Velocity
- Acceleration
- Motion at Constant Acceleration
- Solving Problems
- Freely Falling Objects

Note: Optional sections 2-8 \& 2-9 are not required.

## 2-1 Reference Frames and Displacement

Any measurement of position, distance, or speed must be made with respect to a reference frame.

For example, if you are sitting on a train and someone walks down the aisle, the person's speed with respect to the train is a few $\mathrm{km} / \mathrm{hr}$, at most. The person's speed with respect to the ground is much higher.
A person walks toward the front of a train at $5 \mathrm{~km} / \mathrm{h}$. The train is moving $80 \mathrm{~km} / \mathrm{h}$ with respect to the ground, so the walking person's speed, relative to the ground, is $85 \mathrm{~km} / \mathrm{h}$.


## 2-1 Reference Frames and Displacement

We make a distinction between distance and displacement.

Displacement (blue line) is how far the object is from its starting point, regardless of how it got there.

Distance traveled (dashed line) is measured along the actual path.



A person walks 70 m east, then 30 m west. The total distance traveled is 100 m (path is shown dashed in black); but the displacement, shown as a solid blue arrow, is $\mathbf{4 0} \mathbf{~ m}$ to the east.

Note that we often use the $x$ axis for horizontal motion and the $y$ axis for vertical motion but this is arbitrary.

## 2-1 Reference Frames and Displacement

The displacement is written: $\Delta x=x_{2}-x_{1}$.

Left:
Displacement is positive.


The arrow represents the displacement $x_{2}-y_{1}$.
Distances are in $m$.
For the displacement $\Delta x=x_{2}-x_{1}=10.0 m-30.0 \mathrm{~m}$, the displacement vector points to the left. ( $\Delta x=-20.0 \mathrm{~m}$ )

## 2-2 Average Speed and Velocity

Speed is how far an object travels in a given time interval:

$$
\text { average speed }=\frac{\text { distance traveled }}{\text { time elapsed }} .
$$

Velocity includes directional information:

$$
\text { average velocity }=\frac{\text { displacement }}{\text { time elapsed }} .
$$

## 2-2 Average Velocity

Example 2-1: Runner's average velocity.
The position of a runner as a function of time is plotted as moving along the $x$ axis of a coordinate system. During a 3.00 s time interval, the runner's position changes from $x_{1}=50.0 \mathrm{~m}$ to $x_{2}=30.5 \mathrm{~m}$, as shown. What was the runner's average velocity?


Divide the displacement by the elapsed time.
$\bar{v}=\frac{\Delta x}{\Delta t}=\frac{30.5-50.0}{3.00}=\frac{-19.5}{3.00}=-6.50 \mathrm{~m} / \mathrm{s}$
Answer could also be written as 6.5 m to left.

A bar over a quantity is used to indicate average.

## 2-2 Average Velocity

Example 2-2: Distance a cyclist travels.
How far can a cyclist travel in 2.5 h along a straight road if her average velocity is $18 \mathrm{~km} / \mathrm{h}$ ?

$$
\bar{v}=\frac{\Delta x}{\Delta t} \rightarrow \Delta x=\bar{v} \Delta t=18 \times 2.5=45 \mathrm{~km}
$$

## 2-3 Instantaneous Velocity

The instantaneous velocity is the average velocity in the limit as the time interval becomes infinitesimally short.

$$
v=\lim _{\Delta t \rightarrow 0} \frac{\Delta x}{\Delta t}=\frac{d x}{d t} .
$$



Ideally, a speedometer would measure instantaneous velocity; in fact, it measures average velocity, but over a very short time interval.

## 2-3 Instantaneous Velocity

The instantaneous speed always equals the magnitude of the instantaneous velocity; it only equals the average velocity if the velocity is constant.



## 2-3 Instantaneous Velocity

On a graph of a particle's position vs. time, the instantaneous velocity is the tangent to the curve at any point.


The slope of the straight line $P_{1} P_{2}$ represents the average velocity of the particle during the time interval $\Delta t=t_{2}-t_{1}$.

## 2-3 Instantaneous Velocity <br> 

The average velocity over the time interval $t_{i}-t_{1}$ (which is the slope of $P_{1} P_{i}$ ) is less than the average velocity over the time interval $t_{2}-t_{1}$. The slope of the thin line tangent to the curve at point $P_{1}$ equals the instantaneous velocity at time



This is the same $x$ vs. $t$ curve as in previous slide showing the slope at four different points.
At $P_{3}$ the slope is zero (instantaneous velocity is zero).
At $\mathrm{P}_{4}$ the slope is negative so $v<0$.

## 2-3 Instantaneous Velocity

Example 2-3: Given $x$ as a function of $t$.
A jet engine moves along an experimental track (which we call the $x$ axis) as shown. We will treat the engine as if it were a particle. Its position as a function of time is given by the equation $x=A t^{2}+B$, where $A=$ $2.10 \mathrm{~m} / \mathrm{s}^{2}$ and $B=2.80 \mathrm{~m}$. (a) Determine the displacement of the engine during the time interval from $t_{1}=3.00 \mathrm{~s}$ to $t_{2}=$ 5.00 s. (b) Determine the average velocity during this time interval. (c) Determine the magnitude of the instantaneous velocity at $t=5.00 \mathrm{~s}$.


Tangent at $P_{2}$ whose

(a) Determine the displacement of the engine during the time interval from $t_{1}=3.00 \mathrm{~s}$ to $t_{2}=5.00 \mathrm{~s}$.

$$
\begin{aligned}
& x=A t^{2}+B=2.10 t^{2}+2.80 \\
& \text { at } t_{1}=3 \mathrm{~s} \\
& x_{1}=2.10 \times 3^{2}+2.80=21.7 \mathrm{~m} \\
& \text { at } t_{2}=5 \mathrm{~s}
\end{aligned} x_{2}=2.10 \times 5^{2}+2.80=55.3 \mathrm{~m} . ~ l
$$

displacement : $\Delta x=x_{2}-x_{1}=55.3-21.7=33.6 m$
(b) Determine the average velocity during this time interval.

$$
\bar{v}=\frac{\Delta x}{\Delta t}=\frac{x_{2}-x_{1}}{t_{2}-t_{1}}=\frac{55.3-21.7}{5-3}=\frac{33.6}{2}=16.8 \mathrm{~m} / \mathrm{s}
$$

(c) Determine the magnitude of the instantaneous velocity at $t=5.00 \mathrm{~s}$.

Instantaneous velocity:

$$
\begin{aligned}
x & =A t^{2}+B \quad\left[A=2.10 \mathrm{~m} / \mathrm{s}^{2} \quad B=2.80 \mathrm{~m} \quad t=5 \mathrm{~s}\right] \\
v & =\frac{d x}{d t}=\frac{d}{d t}\left(A t^{2}+B\right)=2 A t \\
& =2 \times 2.10 \times 5=21.0 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

## 2-4 Acceleration

Acceleration is the rate of change of velocity.

$$
\text { average acceleration }=\frac{\text { change of velocity }}{\text { time elapsed }} .
$$

Example 2-4: Average acceleration.


[^0]

## 2-4 Acceleration

Conceptual Example 2-5: Velocity and acceleration.
(a) If the velocity of an object is zero, does it mean that the acceleration is zero?
a. No; if this were true nothing could ever change from a velocity of zero!
(b) If the acceleration is zero, does it mean that the velocity is zero? Think of some examples.
b. No, but it does mean the velocity is constant.

## 2-4 Acceleration

Example 2-6: Car slowing down.
An automobile is moving to the right along a straight highway, which we choose to be the positive $x$ axis. Then the driver puts on the brakes. If the initial velocity (when the driver hits the brakes) is $v_{1}=15.0 \mathrm{~m} / \mathrm{s}$, and it takes 5.0 s to slow down to $v_{2}=5.0 \mathrm{~m} / \mathrm{s}$, what was the car's average acceleration?
$\bar{a}=\frac{\Delta v}{\Delta t}=\frac{v_{2}-v_{1}}{t_{2}-t_{1}}=\frac{5.0-15.0}{5.0-0}$


## 2-4 Acceleration

There is a difference between negative acceleration and deceleration:

Negative acceleration is acceleration in the negative direction as defined by the coordinate system.

Deceleration occurs when the acceleration is opposite in direction to the velocity.


The car of the previous example, now moving to the left and decelerating. The acceleration is $+2.0 \mathrm{~m} / \mathrm{s}$

## 2-4 Acceleration

The instantaneous acceleration is the average acceleration in the limit as the time interval becomes infinitesimally short.

$$
a=\lim _{\Delta t \rightarrow 0} \frac{\Delta v}{\Delta t}=\frac{d v}{d t} .
$$

Slope is average acceleration during $\Delta t=t_{2}-t_{1}$


## 2-4 Acceleration

Example 2-7: Acceleration given $x(t)$.
A particle is moving in a straight line so that its position is given by the relation
$x=\left(2.10 \mathrm{~m} / \mathrm{s}^{2}\right) t^{2}+(2.80 \mathrm{~m})$. Calculate (a) its average acceleration during the time interval from $t_{1}=3.00 \mathrm{~s}$ to $t_{2}=5.00 \mathrm{~s}$, and (b) its instantaneous acceleration as a function of time.



(a) For the motion $x=\mathrm{A} t^{2}+\mathrm{B}\left[\mathrm{A}=2.10 \mathrm{~m} / \mathrm{s}^{2}, \mathrm{~B}=2.80 \mathrm{~m}\right]$
$v=\frac{d x}{d t}=\frac{d}{d t}\left(\mathrm{~A} t^{2}+\mathrm{B}\right)=2 \mathrm{~A} t=2 \times 2.10 t=4.2 t \mathrm{~m} / \mathrm{s}$
at $t_{1}=3.00 \mathrm{~s} \rightarrow v_{1}=4.2 \times 3.00=12.6 \mathrm{~m} / \mathrm{s}$
at $t_{2}=5.00 \mathrm{~s} \rightarrow v_{2}=4.2 \times 5.00=21.0 \mathrm{~m} / \mathrm{s}$
$\bar{a}=\frac{\Delta v}{\Delta t}=\frac{v_{2}-v_{1}}{t_{2}-t_{1}}=\frac{21.0-12.6}{5.00-3.00}=4.2 \mathrm{~m} / \mathrm{s}^{2}$
(b) For the motion $x=\mathrm{A} t^{2}+\mathrm{B} \quad\left[\mathrm{A}=2.10 \mathrm{~m} / \mathrm{s}^{2}, \mathrm{~B}=2.80 \mathrm{~m}\right]$ from (a) $v=2 \mathrm{~A} t$
$a=\frac{d v}{d t}=2 \mathrm{~A}=2 \times 2.10=4.2 \mathrm{~m} / \mathrm{s}^{2}$




Note that $a$ is constant which accounts for instantaneous and average acceleration being equal.

## 2-4 Acceleration

Conceptual Example 2-8: Analyzing with graphs.
This figure shows the velocity as a function of time for two cars accelerating from 0 to $100 \mathrm{~km} / \mathrm{h}$ in a time of 10.0 s . Compare (a) the average acceleration; (b) instantaneous acceleration; and (c) total distance traveled for the two cars.


a. Average acceleration is the same; both have the same change in speed over the same time.
b. Car A accelerates faster than B at the beginning but then slower than B towards the end (look at the slope of the lines).
c. Car A is always going faster than car B, so it will travel farther.

## 2-5 Motion at Constant Acceleration

The average velocity of an object during a time interval $t$ is

$$
\bar{v}=\frac{\Delta x}{\Delta t}=\frac{x-x_{0}}{t-t_{0}}=\frac{x-x_{0}}{t} .
$$

Note that $x_{0}$ and $\mathrm{t}_{0}$ are initial positions and time.
It is normal to take $\mathrm{t}_{0}=0$ and omit it from the equations.

The acceleration, assumed constant, is

$$
a=\frac{v-v_{0}}{t} . \longleftrightarrow v=v_{0}+a t
$$

## 2-5 Motion at Constant Acceleration

Since $\bar{v}=\frac{\left(x-x_{0}\right)}{t} \rightarrow \bar{v} t=x-x_{0} \quad \rightarrow \quad x=x_{0}+\bar{v} t$ and for constant acceleration $\bar{v}=\frac{v_{0}+v}{2}$
Combining these equations and $v=v_{0}+a t$ gives:

$$
x=x_{0}+v_{0} t+\frac{1}{2} a t^{2}
$$

## 2-5 Motion at Constant Acceleration

We can also combine these equations so as to eliminate $t$ :

$$
v^{2}=v_{0}^{2}+2 a\left(x-x_{0}\right) .
$$

We now have all the equations we need to solve constant-acceleration problems.

$$
\begin{aligned}
v & =v_{0}+a t \\
x & =x_{0}+v_{0} t+\frac{1}{2} a t^{2} \\
v^{2} & =v_{0}^{2}+2 a\left(x-x_{0}\right) \\
\bar{v} & =\frac{v+v_{0}}{2} .
\end{aligned}
$$

## 2-6 Solving Problems

1. Read the whole problem and make sure you understand it. Then read it again.
2. Decide on the objects under study and what the time interval is.
3. Draw a diagram and choose coordinate axes.
4. Write down the known (given) quantities, and then the unknown ones that you need to find.
5. What physics applies here? Plan an approach to a solution.

## 2-6 Solving Problems

6. Which equations relate the known and unknown quantities? Are they valid in this situation? Solve algebraically for the unknown quantities, and check that your result is sensible (correct dimensions).
7. Calculate the solution and round it to the appropriate number of significant figures.
8. Look at the result-is it reasonable? Does it agree with a rough estimate?
9. Check the units again.

## 2-6 Solving Problems

Example 2-10: Acceleration of a car.
How long does it take a car to cross a 30.0 m wide intersection after the light turns green, if the car accelerates from rest at a constant $2.00 \mathrm{~m} / \mathrm{s}^{2}$ ?



Use: $x=x_{0}+v_{0} t+\frac{1}{2} a t^{2}=\frac{1}{2} a t^{2}$

$$
x_{0}=v_{0}=0
$$

$$
x=30.0 \mathrm{~m}
$$

$$
\therefore t=\sqrt{\frac{2 x}{a}}=\sqrt{\frac{2 \times 30.0}{2.00}}=5.48 \mathrm{~s} \quad \begin{aligned}
& x=30.0 \mathrm{~m} \\
& a=2.00 \mathrm{~m} / \mathrm{s}^{2} \\
& t=?
\end{aligned}
$$

A small plane can accelerate at $2.00 \mathrm{~m} / \mathrm{s}^{2}$. If its takeoff speed is $100 \mathrm{~km} / \mathrm{hr}$, what is the minimum length of runway required?

$$
\begin{aligned}
& v_{0}=0 \\
& v=\frac{100 \mathrm{~km}}{\mathrm{hr}}=\frac{100 \times 1000 \mathrm{~m}}{60 \times 60 \mathrm{~s}}=27.8 \mathrm{~m} / \mathrm{s} \\
& a=2 \mathrm{~m} / \mathrm{s}^{2} \\
& x_{0}=0 \\
& x=? \\
& \text { Use : } v^{2}=v_{0}^{2}+2 a\left(x-x_{0}\right) \\
& (27.8)^{2}=0+2 \times 2(x-0) \\
& \left.\quad x=\frac{(27.8)^{2}}{4}=193 \mathrm{~m} \text { (approx. } 200 \mathrm{~m}\right)
\end{aligned}
$$

## 2-6 Solving Problems

## Example 2-11: Air bags.

Suppose you want to design an air bag system that can protect the driver at a speed of 100 $\mathrm{km} / \mathrm{h}(60 \mathrm{mph})$ if the car hits a brick wall. Estimate how fast the air bag must inflate to effectively protect the driver. How does the use of a seat belt help the driver?


Example 2-11. An air bag deploying on impact.
Solution: Assume the acceleration is constant; the car goes from $100 \mathrm{~km} / \mathrm{h}(27.8 \mathrm{~m} / \mathrm{s})$ to zero in a distance of about 1 m (the crumple zone). This takes a time of about $t=0.07 \mathrm{~s}$, so the air bag has to inflate faster than this.
The seat belt keeps the driver in position, and also assures that the driver decelerates with the car, rather than by hitting the dashboard


$$
\begin{array}{ll}
v_{0}=\frac{100 \mathrm{~km}}{\mathrm{hr}}=\frac{100 \times 1000 \mathrm{~m}}{60 \times 60 \mathrm{~s}}=27.8 \mathrm{~m} / \mathrm{s} \\
v=0 & \\
x_{0}=0 & \mathrm{a}=? \\
x=1 \mathrm{~m} & \mathrm{t}=?
\end{array}
$$

$1: v^{2}=v_{0}^{2}+2 a\left(x-x_{0}\right)$
$0=(27.8)^{2}+2 a(1-0)$
$a=\frac{-(27.8)^{2}}{2}=-386 \mathrm{~m} / \mathrm{s}^{2}$
$2: v=v_{0}+a t$
$t=\frac{v-v_{0}}{a}=\frac{0-27.8}{-386}=0.07 \mathrm{~s}$

## Example 2-12: Braking distances.

Estimate the minimum stopping distance for a car. The problem is best dealt with in two parts, two separate time intervals. (1) The first time interval begins when the driver decides to hit the brakes, and ends when the foot touches the brake pedal. This is the "reaction time," about 0.50 s , during which the speed is constant, so $a=0$.


1. $v=x / t \rightarrow x=v t=14 \times 0.5=7 \mathrm{~m}$
(2) The second time interval is the actual braking period when the vehicle slows down $(a \neq 0)$ and comes to a stop. The stopping distance depends on the reaction time of the driver, the initial speed of the car (the final speed is zero), and the acceleration of the car. Calculate the total stopping distance for an initial velocity of $50 \mathrm{~km} / \mathrm{h}(=13.9 \mathrm{~m} / \mathrm{s})$ and assume the acceleration of the car is $-6.0 \mathrm{~m} / \mathrm{s}^{2}$ (the minus sign appears because the velocity is taken to be in the positive $x$ direction and its magnitude is decreasing).

$$
\begin{aligned}
& v^{2}=v_{0}^{2}+2 a\left(x-x_{0}\right) \\
& 0=(13.9)^{2}+2(-6.0)(x-0) \\
& \therefore \quad x=\frac{(13.9)^{2}}{2 \times 6.0}=16 \mathrm{~m}
\end{aligned}
$$


$\therefore$ Total Distance $=7+16=23 \mathrm{~m}$

## 2-6 Solving Problems

Example 2-13: Two moving objects: Police and speeder.
A car speeding at $150 \mathrm{~km} / \mathrm{h}$ passes a still police car which immediately takes off in hot pursuit. Using simple assumptions, such as that the speeder continues at constant speed, estimate how long it takes the police car to overtake the speeder. Then estimate the police car's speed at that moment and decide if the assumptions were reasonable.


We can always attempt to solve a complex problem by making assumptions and approximations. The question is are these assumptions and approximations good and justified?

Look at the implications of the assumptions made in this question:

Speeding Car: $\quad v=150 \mathrm{~km} / \mathrm{hr}=41.7 \mathrm{~m} / \mathrm{s}$

$$
x=v t=41.7 t
$$

Police Car : Can go from 0 to $100 \mathrm{~km} / \mathrm{hr}=27.8 \mathrm{~m} / \mathrm{s}$ in 5 s
Constant acceleration : $a=\frac{\Delta v}{\Delta t}=\frac{27.8}{5.0}=5.56 \mathrm{~m} / \mathrm{s}^{2}$
Using $x=x_{0}+v_{0} t+\frac{1}{2} a t^{2} \quad\left(x_{0}=0, v_{0}=0\right)$
gives $x=2.78 t^{2}$
Combining equations: $41.7 t=2.78 t^{2} \rightarrow t=\frac{41.7}{2.78}=15.0 \mathrm{~s}$

For Police Car : $v=v_{0}+a t=0+5.56 \times 15=83.4 \mathrm{~m} / \mathrm{s}=300 \mathrm{~km} / \mathrm{hr}$
This is clearly not possible. The acceleration of the police car would not be constant. The motor would not be able to maintain a constant torque and air resistance would increase with speed (roughly as the square of the speed). Also the speeding car may have slowed down during the pursuit. The graph on the right may be a better representation of the chase but we do not have sufficient information do determine this.


## 2-7 Freely Falling Objects

Near the surface of the Earth, all objects experience approximately the same acceleration due to gravity.


This is one of the most common examples of motion with constant acceleration.

Multiflash photograph of a falling apple, at equal time intervals. The apple falls farther during each successive interval, which means it is accelerating.

## 2-7 Freely Falling Objects


(a) A ball and a light piece of paper are dropped at the same time.

(b) Repeated, with the paper wadded up.

In the absence of air resistance, all objects fall with the same acceleration, although this may be tricky to tell by testing in an environment where there is air resistance.


## 2-7 Freely Falling Objects

Suppose a ball is thrown downward with an initial velocity of $3.00 \mathrm{~m} / \mathrm{s}$, instead of being dropped. (a) What then would be its position after 1.00 s and 2.00 s? (b) What would its speed be after 1.00 s and 2.00 s? Compare with the speeds of a dropped ball.

Solution: This is the same as previous example, except that the initial speed is not zero. Show that:
(a) At $t=1.00 \mathrm{~s}, \boldsymbol{y}=7.90 \mathrm{~m}$. At $\boldsymbol{t}=2.00 \mathrm{~s}, \boldsymbol{y}=25.6 \mathrm{~m}$.
(b) At $t=1.00 \mathrm{~s}, v=12.8 \mathrm{~m} / \mathrm{s}$. At $\boldsymbol{t}=2.00 \mathrm{~s}, v=22.6 \mathrm{~m} / \mathrm{s}$.

This is the case where initial speed is ZERO
'The speed is always $3.00 \mathrm{~m} / \mathrm{s}$ faster than a dropped ball.
Hint: If you choose downwards as positive you avoid negative signs in equation.

You should attempt the following four examples from the text before looking at the worked solutions as an exercise before the next lecture. Answers are given here but not worked solutions.

Be careful in your choice of reference frame and the origin.
It is best to take the point where the object is thrown as zero ( $y=y_{0}=0$ at this point).

All upwards displacements (positions) and velocities will be positive. A downwards velocity (or a displacement below the origin will be negative.

The acceleration (acceleration due to gravity ( $\mathrm{g}=9.8 \mathrm{~m} / \mathrm{s}^{2}$ ) is always negative as it is directed downwards.

## 2-7 Freely Falling Objects



A person throws a ball upward into the air with an initial velocity of 15.0 m/s. Calculate (a) how high it goes, and (b) how long the ball is in the air before it comes back to the hand. Ignore air resistance.
An object thrown into the air leaves the thrower's hand at A, reaches its maximum height at $B$, and returns to the original position at C.
Solution: a. At the highest position, the speed is zero, so we know the acceleration, the initial and final speeds, and are asked for the distance. Substituting gives $y=11.5 \mathrm{~m}$. b. Now we want the time; $\mathrm{t}=3.06 \mathrm{~s}$.

## 2-7 Freely Falling Objects

Conceptual Example: Two possible misconceptions.
Give examples to show the error in these two common misconceptions: (1) that acceleration and velocity are always in the same direction, and (2) that an object thrown upward has zero acceleration at the highest point.
1.If acceleration and velocity were always in the same direction, nothing could ever slow down!
2. At its highest point, the speed of thrown object is zero. If its acceleration were also zero, it would just stay at that point.

## 2-7 Freely Falling Objects



Let us consider again a ball thrown upward, and make more calculations. Calculate (a) how much time it takes for the ball to reach the maximum height, and (b) the velocity of the ball when it returns to the thrower's hand (point C).
a.The time is 1.53 s , half the time for a round trip (since we are ignoring air resistance).
b. $v=-15.0 \mathrm{~m} / \mathrm{s}$

## 2-7 Freely Falling Objects

Ball thrown upward, III; the quadratic formula.
For a ball thrown upward at an initial speed of $15.0 \mathrm{~m} / \mathrm{s}$, calculate at what time $t$ the ball passes a point 8.00 m above the person's hand.



Note that this is a quadratic equation; there are two solutions: $\mathbf{t}=0.69 \mathrm{~s}$ and $t=2.37 \mathrm{~s}$. The first is the ball going up and the second is the ball coming back down. In many problems only one of the solutions is valid. In this case both are valid.

## 2-7 Freely Falling Objects

## Example 2-20: Ball thrown upward at edge of

 cliff.

Suppose that a ball is thrown upward at a speed of $15.0 \mathrm{~m} / \mathrm{s}$ by a person standing on the edge of a cliff, so that the ball can fall to the base of the cliff 50.0 m below. (a) How long does it take the ball to reach the base of the cliff? (b) What is the total distance traveled by the ball? Ignore air resistance (likely to be significant, so our result is an approximation).

$$
\begin{aligned}
& v_{0}=15 \mathrm{~m} / \mathrm{s} \\
& v=0 \\
& y_{0}=0 \\
& y=-50 \mathrm{~m} \\
& a=g=-9.8 \mathrm{~m} / \mathrm{s}^{2} \\
& \text { (a) } y=y_{0}+v_{0} t+\frac{1}{2} a t^{2} \\
& -50=0+15 t-0.5 \times 9.8 t^{2} \\
& 4.9 t^{2}-15 t-50=0 \\
& t=\frac{-\mathrm{b} \pm \sqrt{\mathrm{b}^{2}-4 \mathrm{ac}}}{2 \mathrm{a}}
\end{aligned}
$$

$$
\begin{aligned}
t= & \frac{15 \pm \sqrt{15^{2}+4 \times 4.9 \times 50}}{2 \times 4.9} \\
& =1.53 \pm 3.54 \mathrm{~s} \\
& =5.07 \mathrm{~s} \text { or }-2.01 \mathrm{~s}
\end{aligned}
$$

5.07 s is the answer we want.
-2.01 s is a time before the ball was thrown.

## Answer:

Time for the ball to reach the base of the cliff $=\mathbf{5 . 0 7} \mathrm{s}$.
(b) What is the total distance traveled by the ball? Ignore air resistance (likely to be significant, so our result is an approximation).

Firstly calculate the maximum height.

$$
\begin{aligned}
v^{2} & =v_{0}^{2}+2 a\left(y-y_{0}\right) \\
0 & =15^{2}-2 \times 9.8(y-0) \\
y & =\frac{15^{2}}{2 \times 9.8}=11.5 \mathrm{~m}
\end{aligned}
$$

Total Distance $:=2 \times 11.5+50=73 \mathrm{~m}$

## Summary of Chapter 2

- Kinematics is the description of how objects move with respect to a defined reference frame.
- Displacement is the change in position of an object.
- Average speed is the distance traveled divided by the time it took; average velocity is the displacement divided by the time.
- Instantaneous velocity is the average velocity in the limit as the time becomes infinitesimally short.


## Summary of Chapter 2

- Average acceleration is the change in velocity divided by the time.
- Instantaneous acceleration is the average acceleration in the limit as the time interval becomes infinitesimally small.
- The equations of motion for constant acceleration are given in the text; there are four, each one of which requires a different set of quantities.
- Objects falling (or having been projected) near the surface of the Earth experience a gravitational acceleration of $9.80 \mathrm{~m} / \mathrm{s}^{2}$.


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