Chapter 2

Describing Motion: Kinematics in One Dimension

Units of Chapter 2

- Reference Frames and Displacement
- Average Velocity
- Instantaneous Velocity
- Acceleration
- Motion at Constant Acceleration
- Solving Problems
- Freely Falling Objects

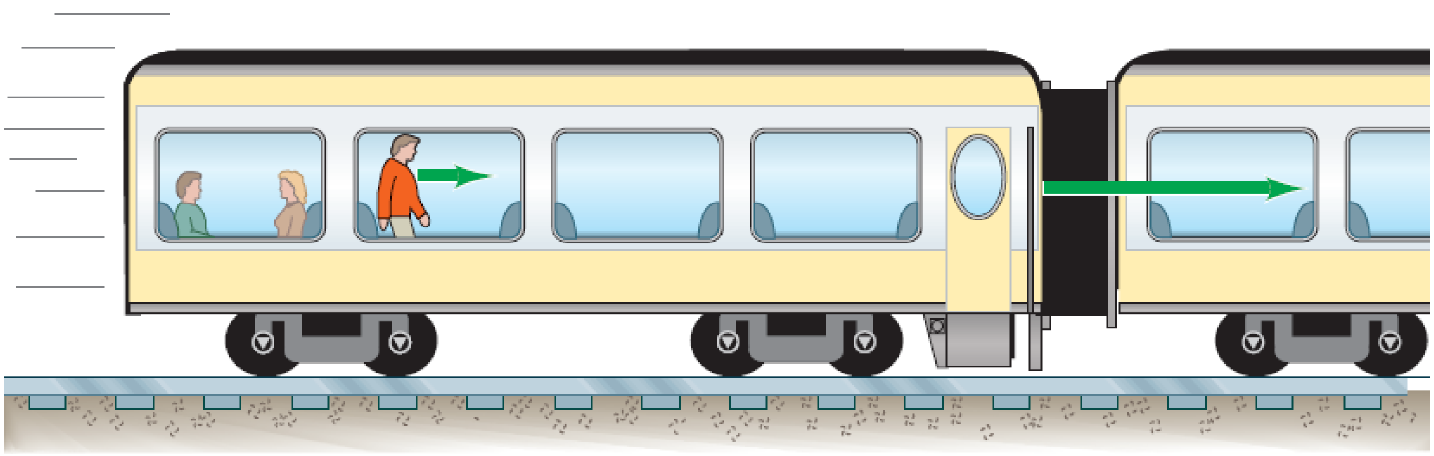
Units of Chapter 2

- **Variable Acceleration; Integral Calculus**
- **Graphical Analysis and Numerical Integration**

2-1 Reference Frames and Displacement

Any measurement of position, distance, or speed must be made with respect to a reference frame.

For example, if you are sitting on a train and someone walks down the aisle, the person's speed with respect to the train is a few miles per hour, at most. The person's speed with respect to the ground is much higher.

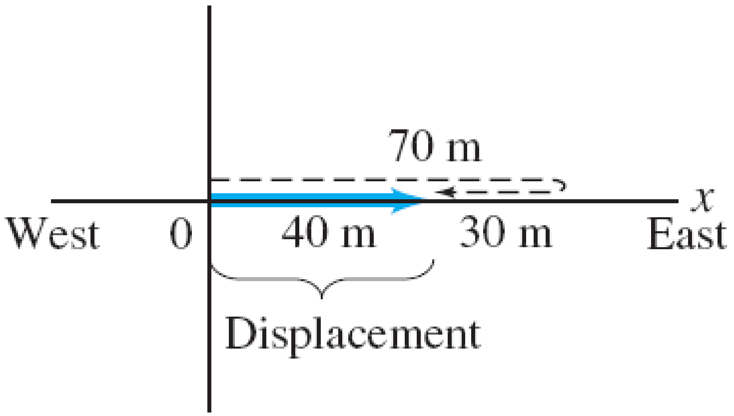


2-1 Reference Frames and Displacement

We make a distinction between distance and displacement.

Displacement (blue line) is the change in position, which represents how far the object is from its starting point, regardless of how it got there.

Distance traveled (dashed line) is measured along the actual path.

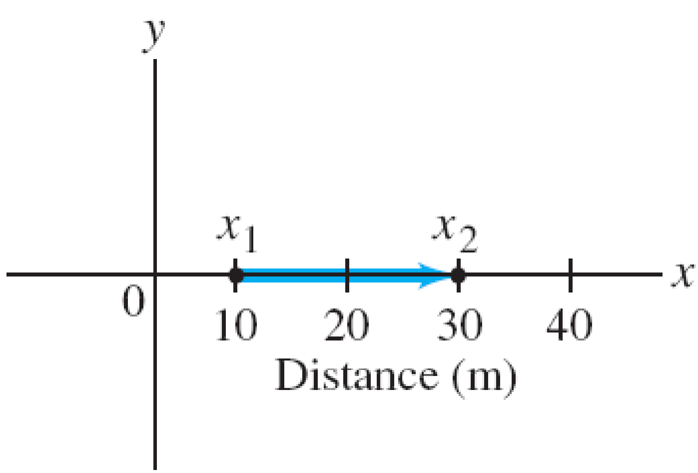


2-1 Reference Frames and Displacement

The displacement is written: $\Delta x = x_2 - x_1.$

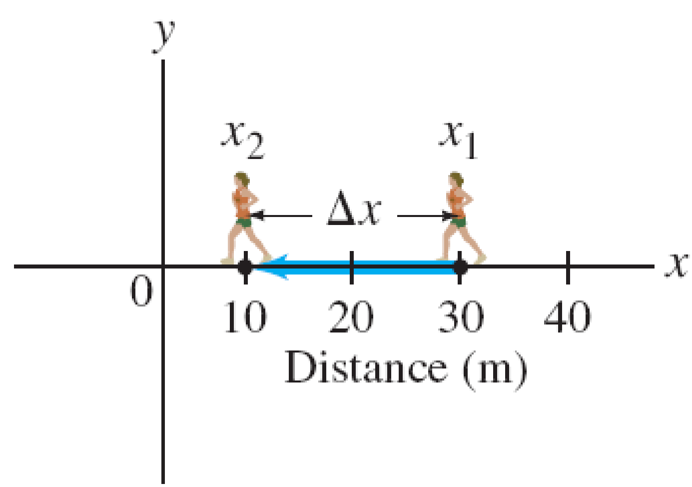
Left:

Displacement is positive.



Right:

Displacement is negative.



2-2 Average Velocity

Speed is how far an object travels in a given time interval:

$$\text{average speed} = \frac{\text{distance traveled}}{\text{time elapsed}}.$$

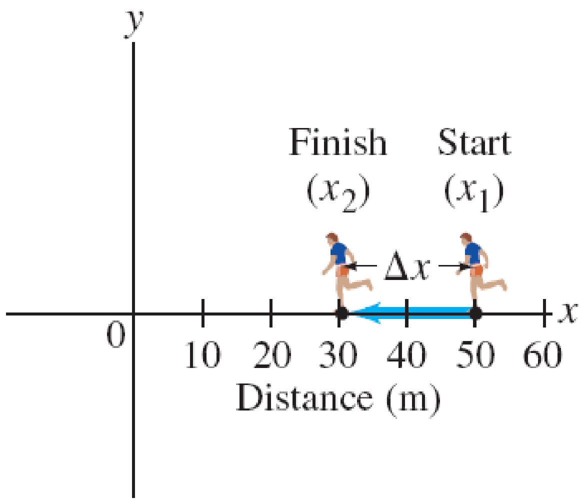
Velocity includes directional information:

$$\text{average velocity} = \frac{\text{displacement}}{\text{time elapsed}}.$$

2-2 Average Velocity

Example 2-1: Runner's average velocity.

The position of a runner as a function of time is plotted as moving along the x axis of a coordinate system. During a 3.00-s time interval, the runner's position changes from x_1 = 50.0 m to x_2 = 30.5 m, as shown. What was the runner's average velocity?



2-2 Average Velocity

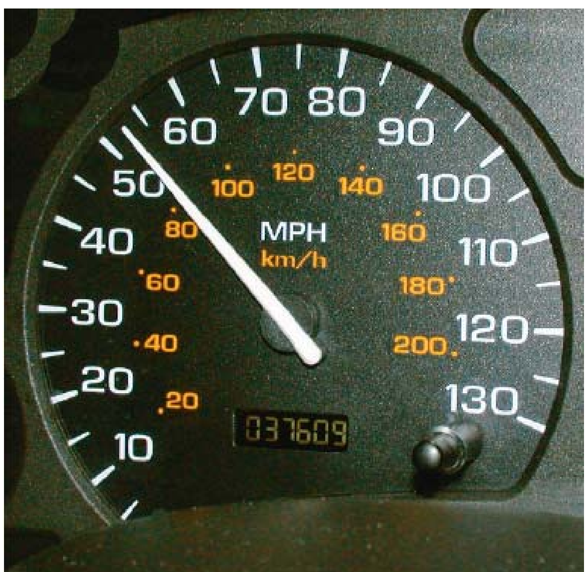
Example 2-2: Distance a cyclist travels.

How far can a cyclist travel in 2.5 h along a straight road if her average velocity is 18 km/h?

2-3 Instantaneous Velocity

The instantaneous velocity is the average velocity in the limit as the time interval becomes infinitesimally short.

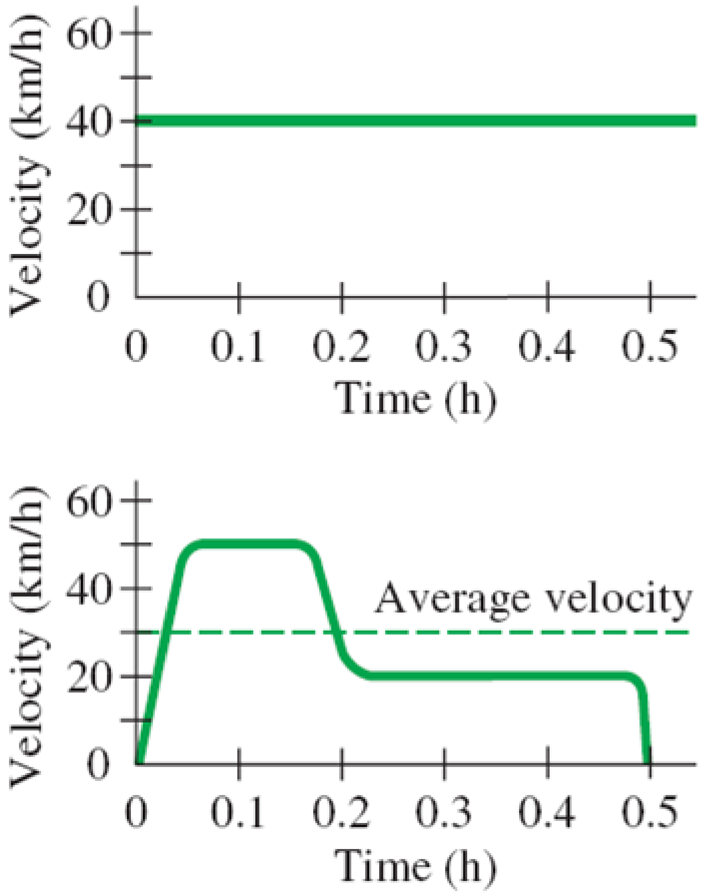
$$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}.$$



Ideally, a speedometer would measure instantaneous velocity; in fact, it measures average velocity, but over a very short time interval.

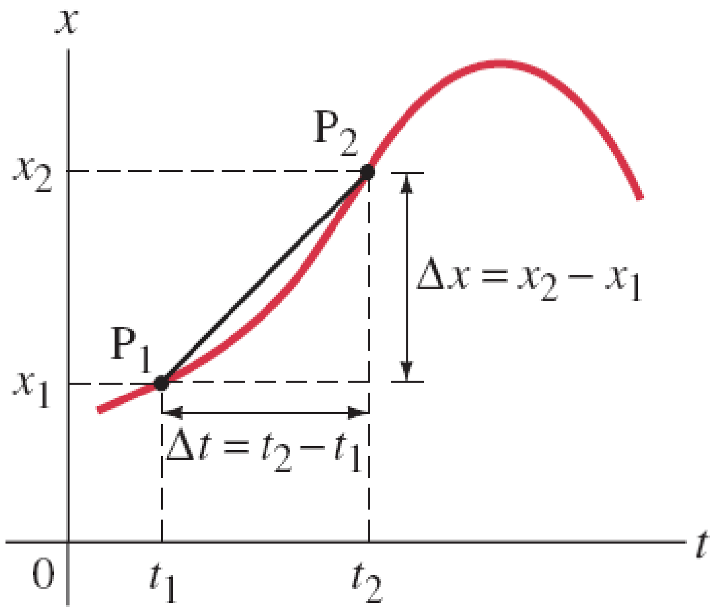
2-3 Instantaneous Velocity

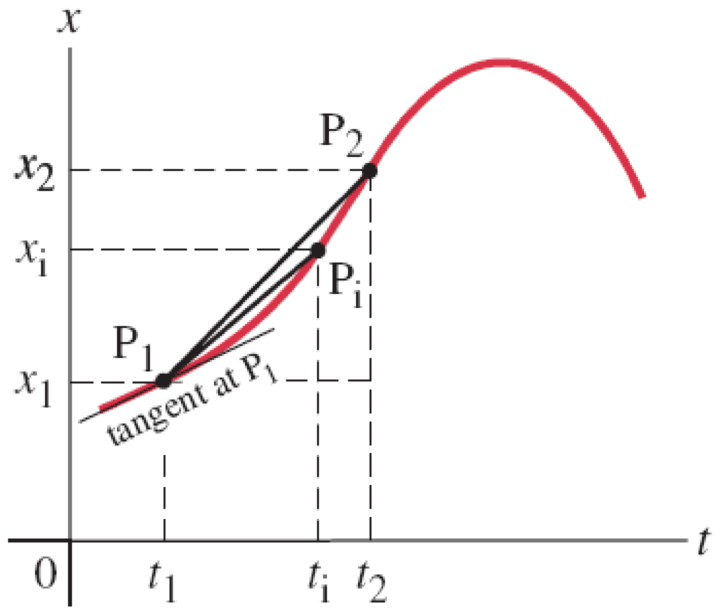
The instantaneous speed always equals the magnitude of the instantaneous velocity; it only equals the average velocity if the velocity is constant.



2-3 Instantaneous Velocity

On a graph of a particle's position vs. time, the instantaneous velocity is the tangent to the curve at any point.

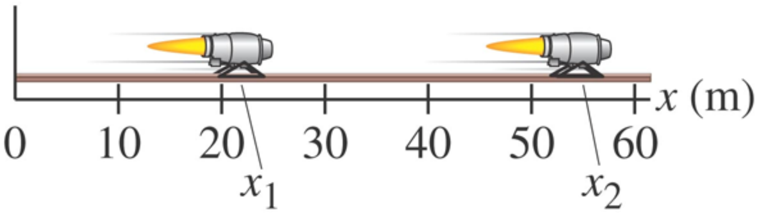


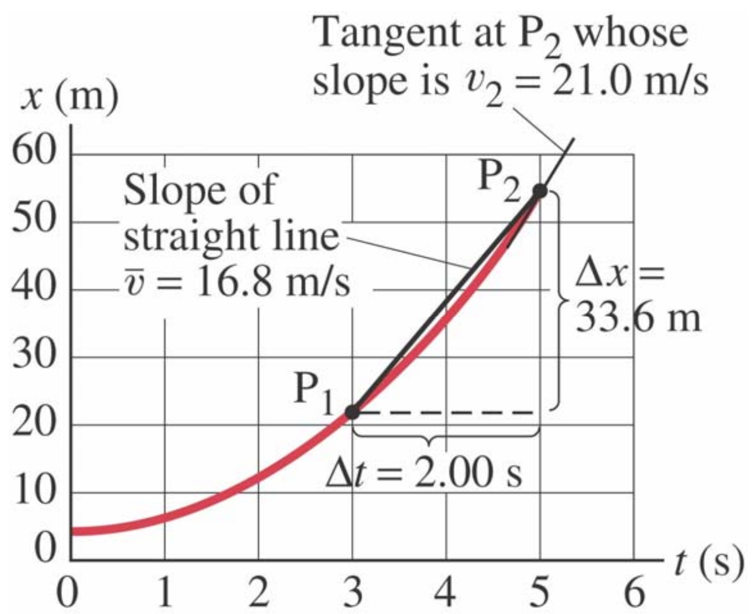


2-3 Instantaneous Velocity

Example 2-3: Given x as a function of t.

A jet engine moves along an experimental track (which we call the x axis) as shown. We will treat the engine as if it were a particle. Its position as a function of time is given by the equation $x = At^2 + B$, where A = 2.10 m/s^2 and B = 2.80 m. (a) Determine the displacement of the engine during the time interval from t_1 = 3.00 s to t_2 = 5.00 s. (b) Determine the average velocity during this time interval. (c) Determine the magnitude of the instantaneous velocity at t = 500 s.





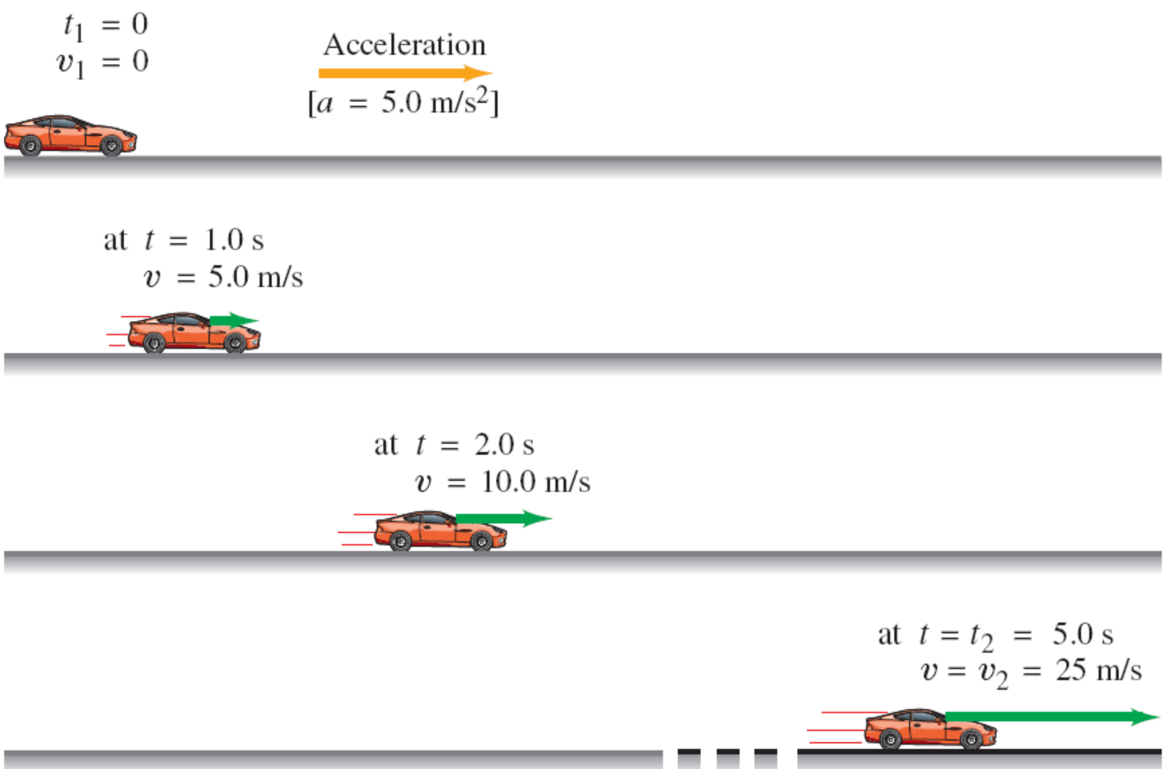
2-4 Acceleration

Acceleration is the rate of change of velocity.

$$\text{average acceleration} = \frac{\text{change of velocity}}{\text{time elapsed}}.$$

Example 2-4: Average acceleration.

A car accelerates along a straight road from rest to 90 km/h in 5.0 s. What is the magnitude of its average acceleration?



2-4 Acceleration

Conceptual Example 2-5: Velocity and acceleration.

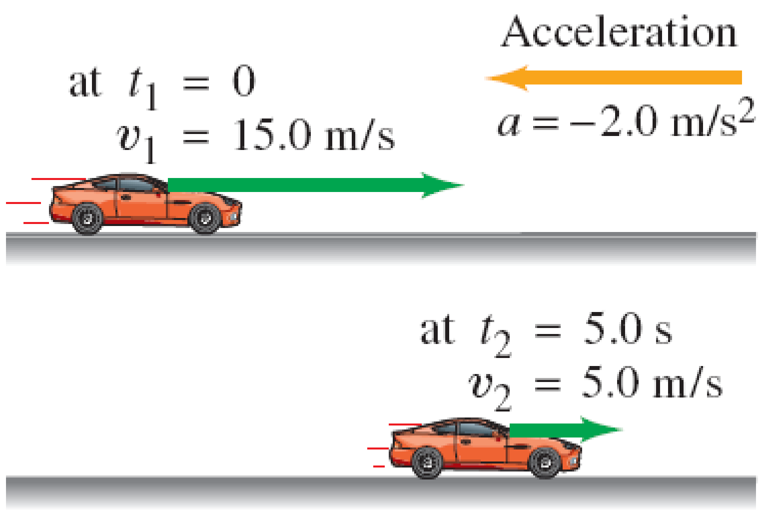
(a) If the velocity of an object is zero, does it mean that the acceleration is zero?

(b) If the acceleration is zero, does it mean that the velocity is zero? Think of some examples.

2-4 Acceleration

Example 2-6: Car slowing down.

An automobile is moving to the right along a straight highway, which we choose to be the positive x axis. Then the driver puts on the brakes. If the initial velocity (when the driver hits the brakes) is v_1 = 15.0 m/s, and it takes 5.0 s to slow down to v_2 = 5.0 m/s, what was the car's average acceleration?

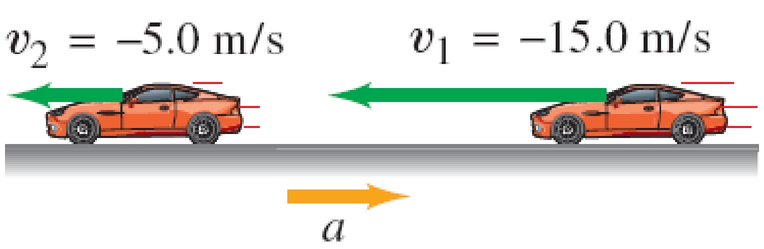


2-4 Acceleration

There is a difference between negative acceleration and deceleration:

Negative acceleration is acceleration in the negative direction as defined by the coordinate system.

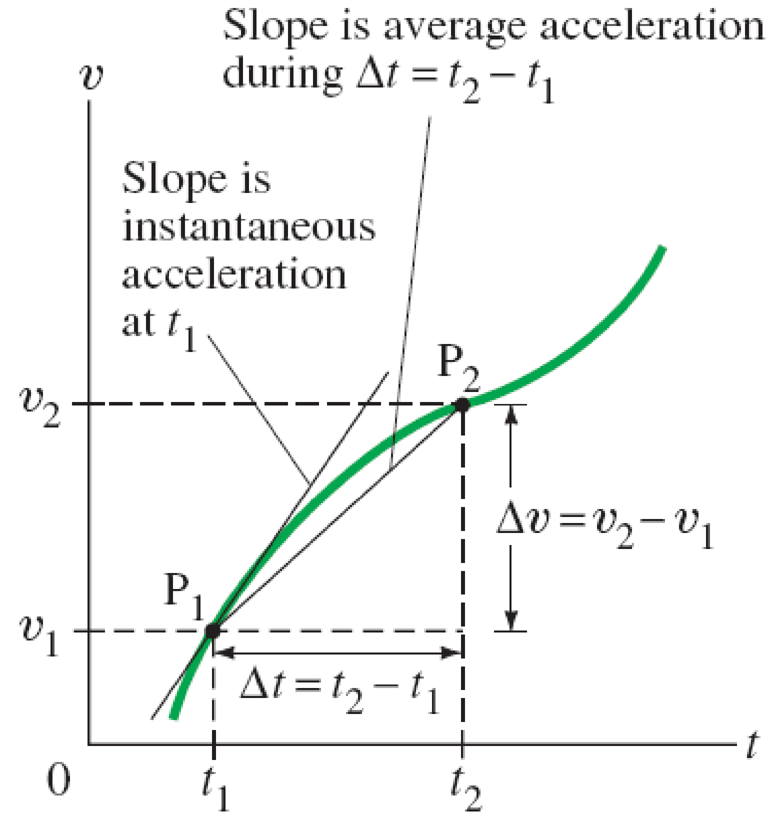
Deceleration occurs when the acceleration is opposite in direction to the velocity.



2-4 Acceleration

The instantaneous acceleration is the average acceleration in the limit as the time interval becomes infinitesimally short.

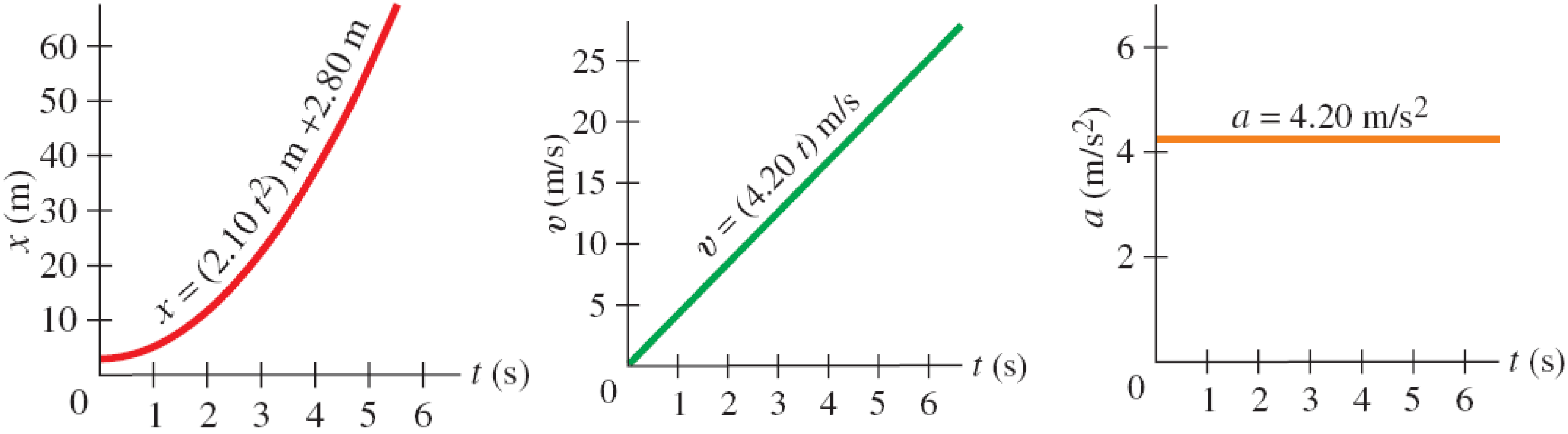
$$a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt}.$$



2-4 Acceleration

Example 2-7: Acceleration given $x(t)$.

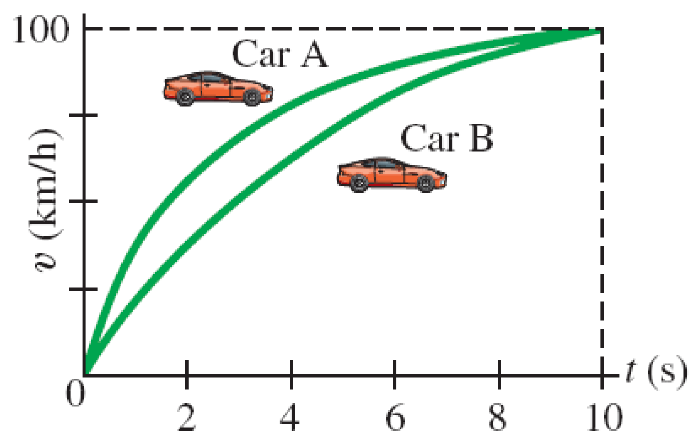
A particle is moving in a straight line so that its position is given by the relation $x = (2.10\ \text{m/s}^2)t^2 + (2.80\ \text{m})$. Calculate (a) its average acceleration during the time interval from $t_1 = 3.00$ s to $t_2 = 5.00$ s, and (b) its instantaneous acceleration as a function of time.



2-4 Acceleration

Conceptual Example 2-8: Analyzing with graphs.

This figure shows the velocity as a function of time for two cars accelerating from 0 to 100 km/h in a time of 10.0 s. Compare (a) the average acceleration; (b) instantaneous acceleration; and (c) total distance traveled for the two cars.



2-5 Motion at Constant Acceleration

The average velocity of an object during a time interval *t* is

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{x - x_0}{t - t_0} = \frac{x - x_0}{t}.$$

The acceleration, assumed constant, is

$$a = \frac{v - v_0}{t}.$$

2-5 Motion at Constant Acceleration

In addition, as the velocity is increasing at a constant rate, we know that

$$\bar{v} = \frac{v_0 + v}{2}.$$

Combining these last three equations, we find:

$$x = x_0 + v_0 t + \tfrac{1}{2} a t^2.$$

2-5 Motion at Constant Acceleration

We can also combine these equations so as to eliminate *t*:

$$v^2 = v_0^2 + 2a(x - x_0).$$

We now have all the equations we need to solve constant-acceleration problems.

$$v = v_0 + at$$

$$x = x_0 + v_0 t + \tfrac{1}{2}at^2$$

$$v^2 = v_0^2 + 2a(x - x_0)$$

$$\bar{v} = \frac{v + v_0}{2}.$$

2-6 Solving Problems

1. Read the whole problem and make sure you understand it. Then read it again.
2. Decide on the objects under study and what the time interval is.
3. Draw a diagram and choose coordinate axes.
4. Write down the known (given) quantities, and then the unknown ones that you need to find.
5. What physics applies here? Plan an approach to a solution.

2-6 Solving Problems

6. Which equations relate the known and unknown quantities? Are they valid in this situation? Solve algebraically for the unknown quantities, and check that your result is sensible (correct dimensions).

7. Calculate the solution and round it to the appropriate number of significant figures.

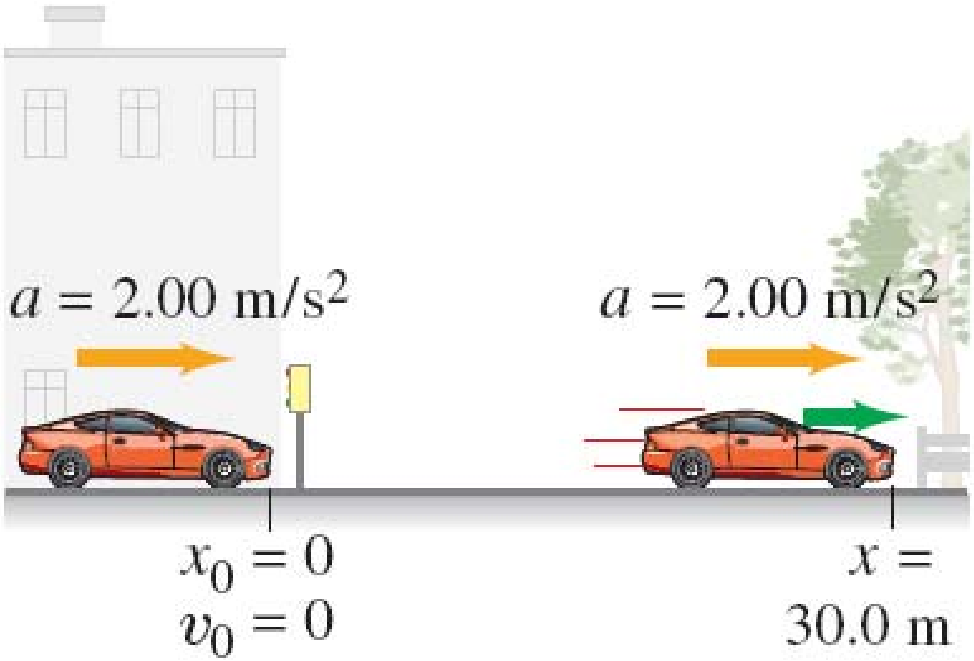
8. Look at the result—is it reasonable? Does it agree with a rough estimate?

9. Check the units again.

2-6 Solving Problems

Example 2-10: Acceleration of a car.

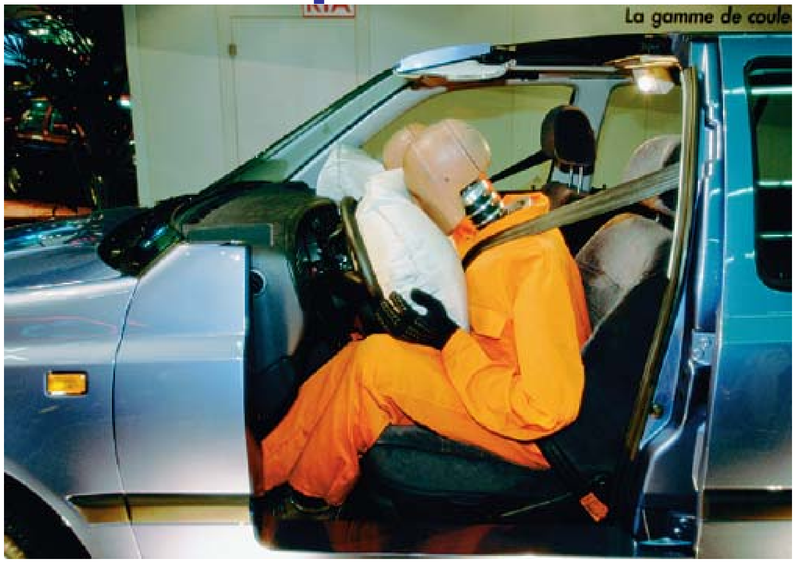
How long does it take a car to cross a 30.0-m-wide intersection after the light turns green, if the car accelerates from rest at a constant 2.00 m/s^2?



2-6 Solving Problems

Example 2-11: Air bags.

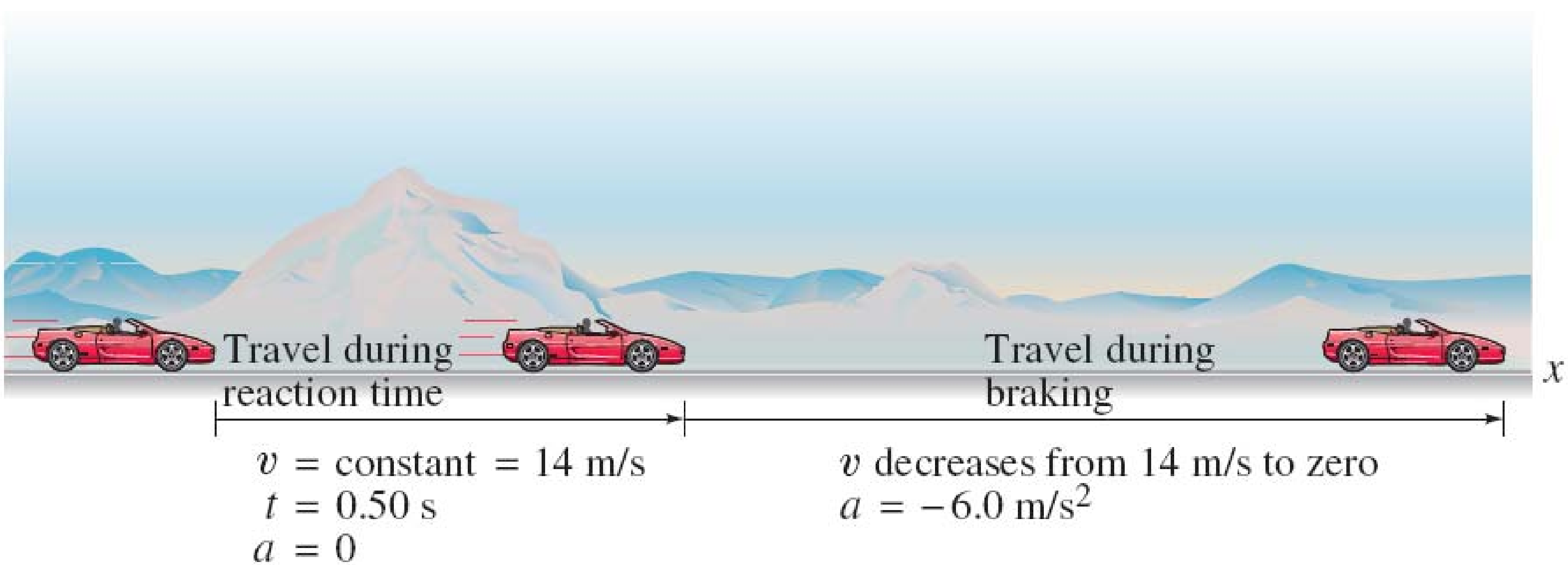
Suppose you want to design an air bag system that can protect the driver at a speed of 100 km/h (60 mph) if the car hits a brick wall. Estimate how fast the air bag must inflate to effectively protect the driver. How does the use of a seat belt help the driver?



2-6 Solving Problems

Example 2-12: Braking distances.

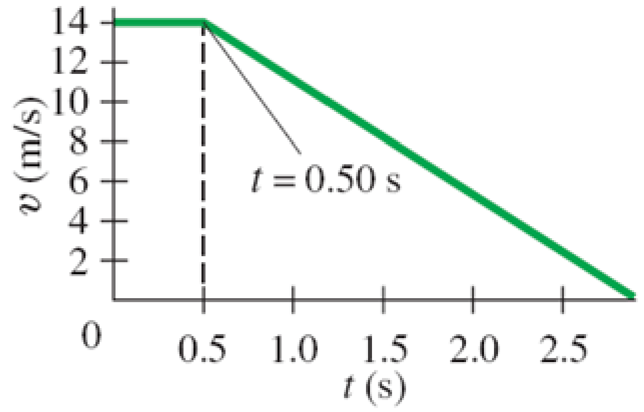
Estimate the minimum stopping distance for a car. The problem is best dealt with in two parts, two separate time intervals. (1) The first time interval begins when the driver decides to hit the brakes, and ends when the foot touches the brake pedal. This is the "reaction time," about 0.50 s, during which the speed is constant, so a = 0.

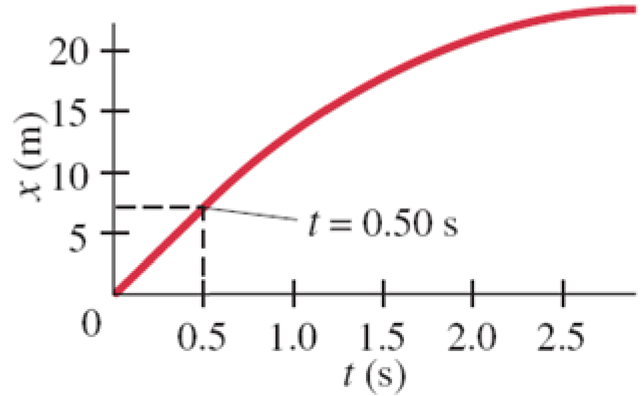


2-6 Solving Problems

Example 2-12: Braking distances.

(2) The second time interval is the actual braking period when the vehicle slows down ($a \neq 0$) and comes to a stop. The stopping distance depends on the reaction time of the driver, the initial speed of the car (the final speed is zero), and the acceleration of the car. Calculate the total stopping distance for an initial velocity of 50 km/h (= 14 m/s ≈ 31 mi/h) and assume the acceleration of the car is -6.0 m/s^2 (the minus sign appears because the velocity is taken to be in the positive x direction and its magnitude is decreasing).

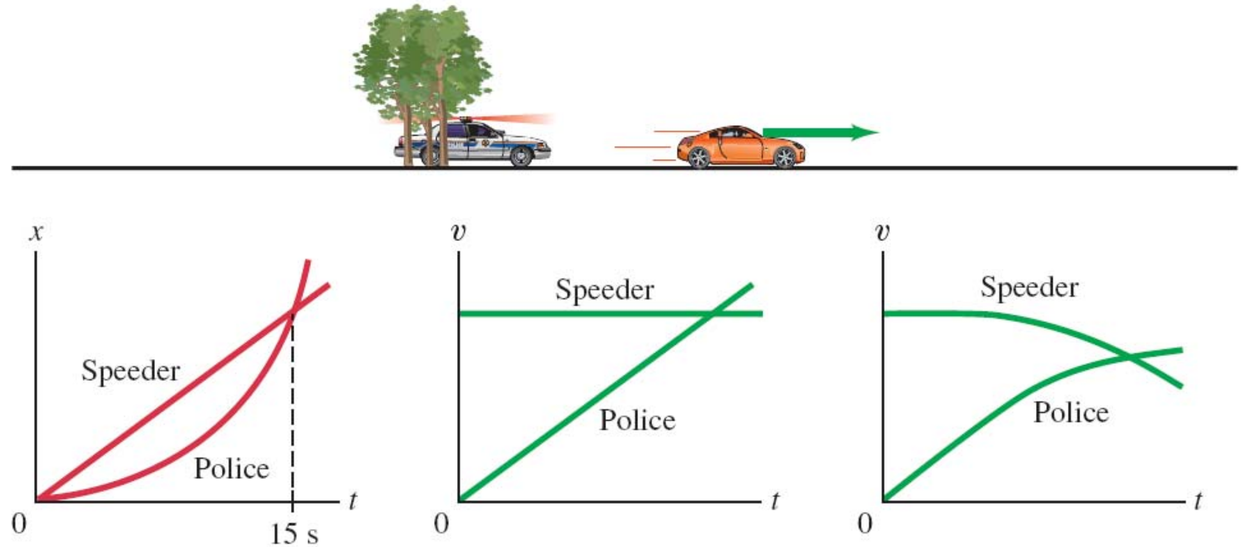




2-6 Solving Problems

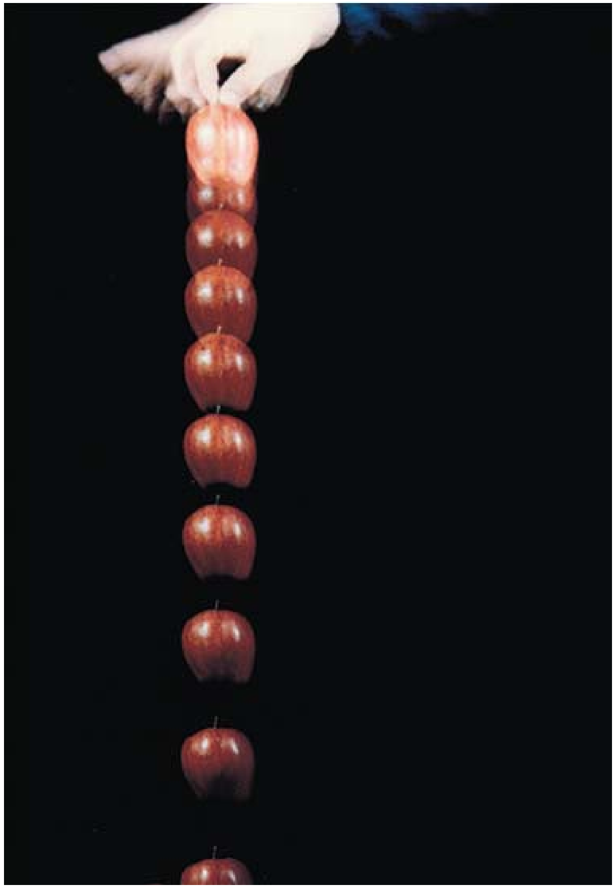
Example 2-13: Two moving objects: Police and speeder.

A car speeding at 150 km/h passes a still police car which immediately takes off in hot pursuit. Using simple assumptions, such as that the speeder continues at constant speed, estimate how long it takes the police car to overtake the speeder. Then estimate the police car's speed at that moment and decide if the assumptions were reasonable.



2-7 Freely Falling Objects

Near the surface of the Earth, all objects experience approximately the same acceleration due to gravity.

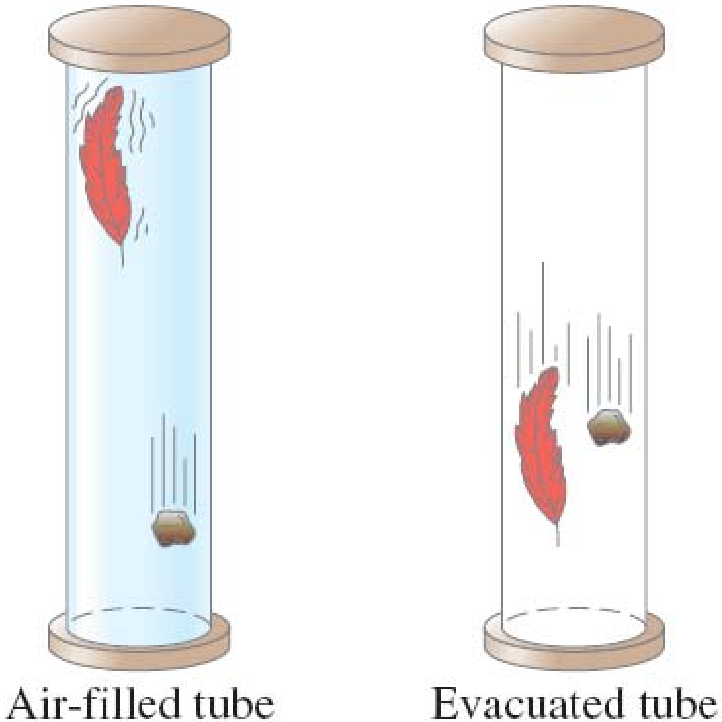


This is one of the most common examples of motion with constant acceleration.

2-7 Freely Falling Objects

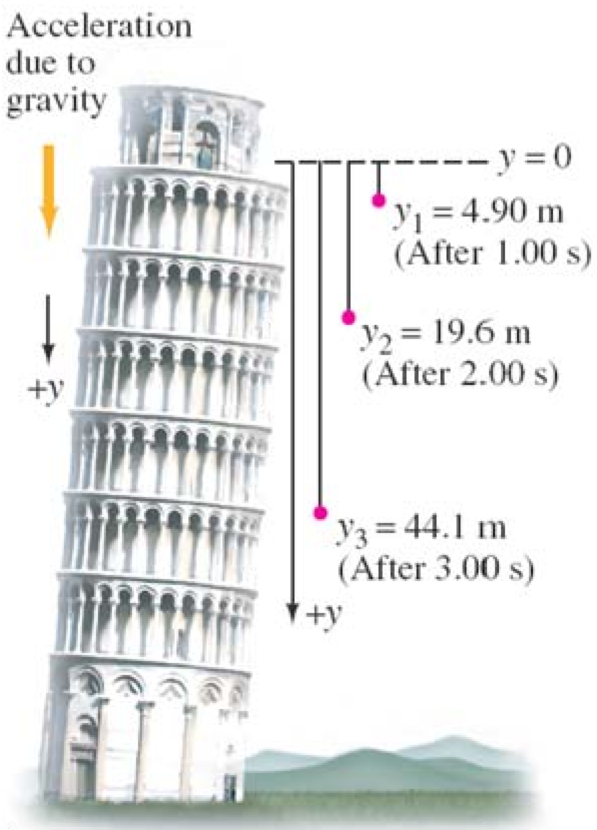
In the absence of air resistance, all objects fall with the same acceleration, although this may be tricky to tell by testing in an environment where there is air resistance.

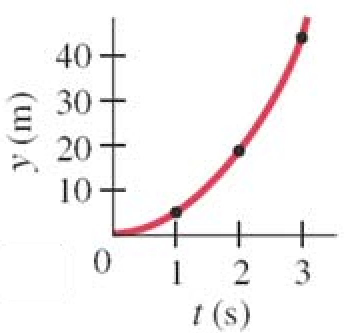
2-7 Freely Falling Objects



The acceleration due to gravity at the Earth's surface is approximately 9.80 m/s^2. At a given location on the Earth and in the absence of air resistance, all objects fall with the same constant acceleration.

2-7 Freely Falling Objects





Example 2-14: Falling from a tower.

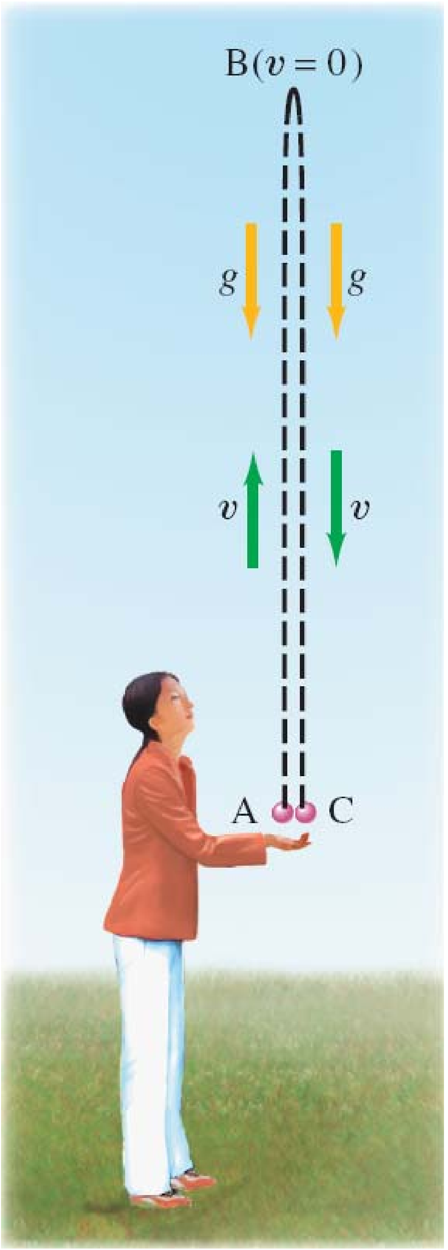
Suppose that a ball is dropped (v_0 = 0) from a tower 70.0 m high. How far will it have fallen after a time t_1 = 1.00 s, t_2 = 2.00 s, and t_3 = 3.00 s? Ignore air resistance.

2-7 Freely Falling Objects

Example 2-15: Thrown down from a tower.

Suppose a ball is thrown downward with an initial velocity of 3.00 m/s, instead of being dropped. (a) What then would be its position after 1.00 s and 2.00 s? (b) What would its speed be after 1.00 s and 2.00 s? Compare with the speeds of a dropped ball.

2-7 Freely Falling Objects



Example 2-16: Ball thrown upward, I.

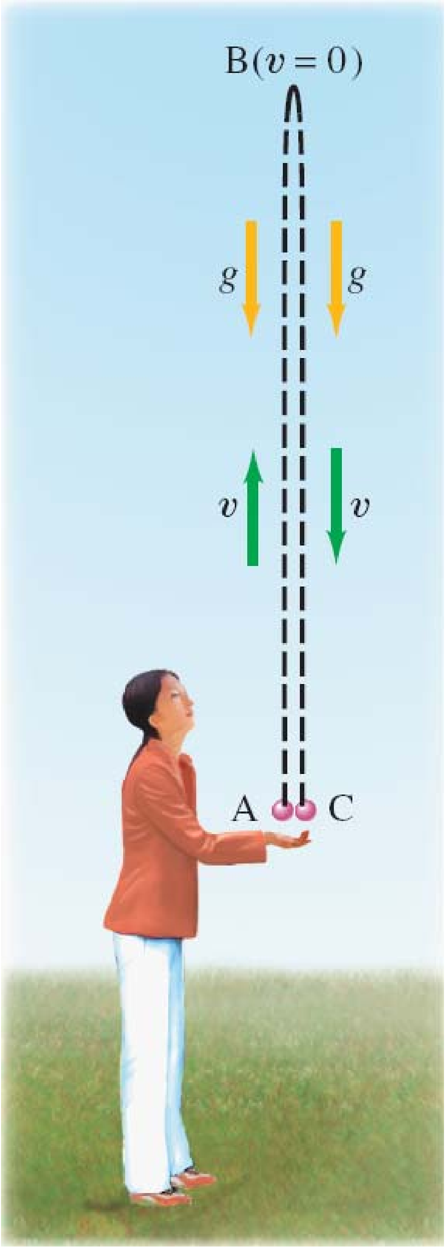
A person throws a ball upward into the air with an initial velocity of 15.0 m/s. Calculate (a) how high it goes, and (b) how long the ball is in the air before it comes back to the hand. Ignore air resistance.

2-7 Freely Falling Objects

Conceptual Example 2-17: Two possible misconceptions.

Give examples to show the error in these two common misconceptions: (1) that acceleration and velocity are always in the same direction, and (2) that an object thrown upward has zero acceleration at the highest point.

2-7 Freely Falling Objects



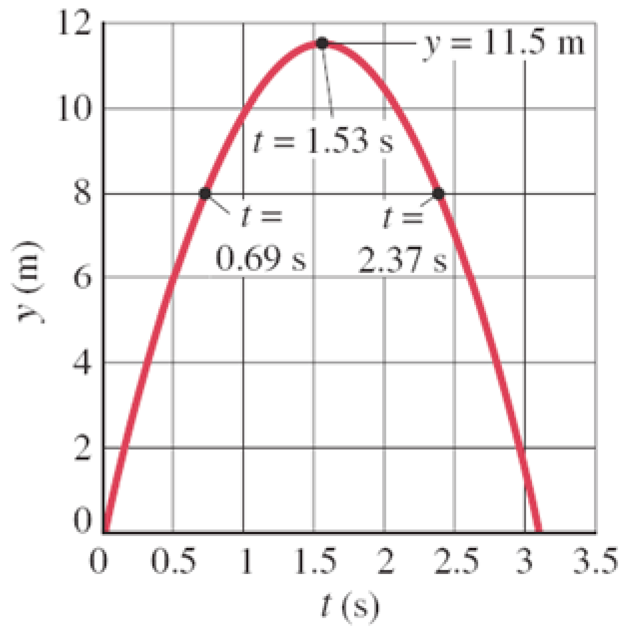
Example 2-18: Ball thrown upward, II.

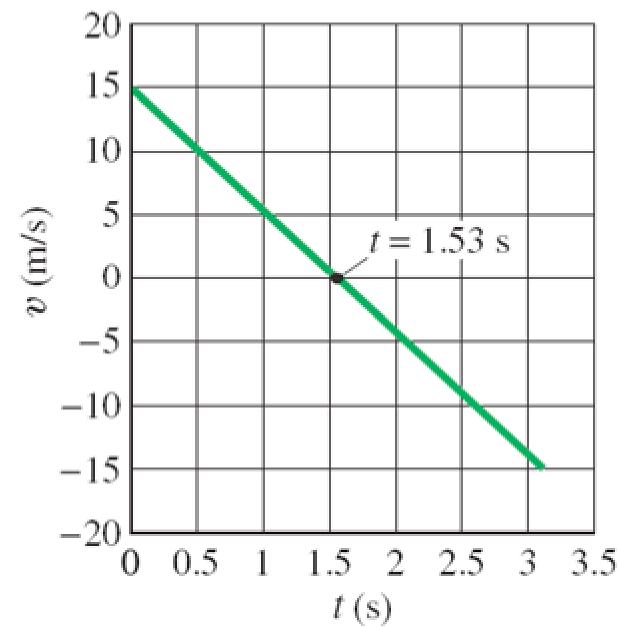
Let us consider again a ball thrown upward, and make more calculations. Calculate (a) how much time it takes for the ball to reach the maximum height, and (b) the velocity of the ball when it returns to the thrower's hand (point C).

2-7 Freely Falling Objects

Example 2-19: Ball thrown upward, III; the quadratic formula.

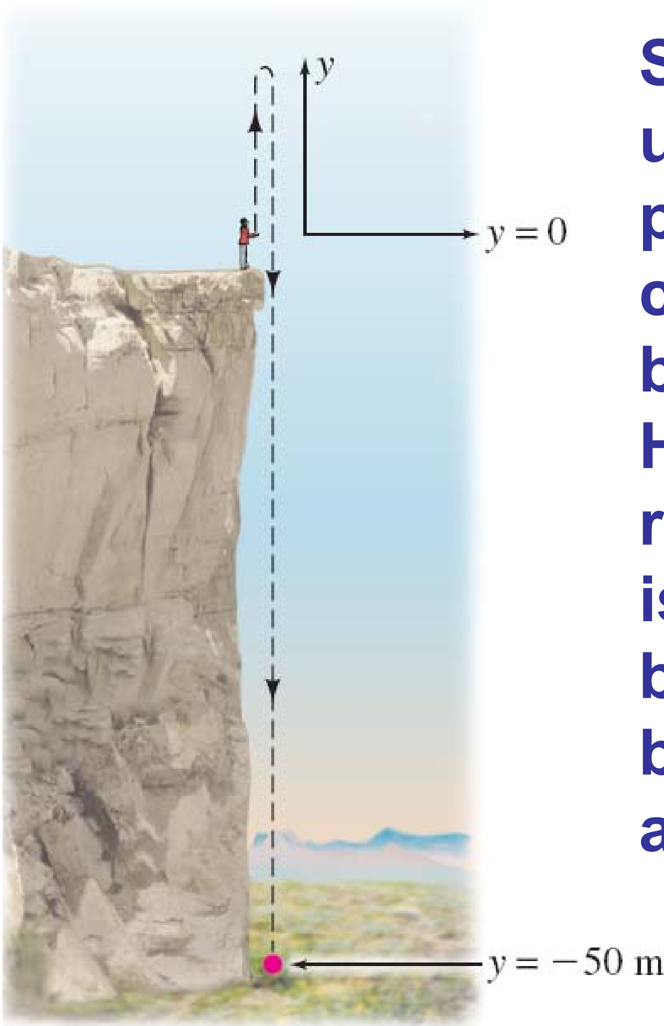
For a ball thrown upward at an initial speed of 15.0 m/s, calculate at what time *t* the ball passes a point 8.00 m above the person's hand.





2-7 Freely Falling Objects

Example 2-20: Ball thrown upward at edge of cliff.



Suppose that a ball is thrown upward at a speed of 15.0 m/s by a person standing on the edge of a cliff, so that the ball can fall to the base of the cliff 50.0 m below. (a) How long does it take the ball to reach the base of the cliff? (b) What is the total distance traveled by the ball? Ignore air resistance (likely to be significant, so our result is an approximation).

2-8 Variable Acceleration; Integral Calculus

Deriving the kinematic equations through integration:

$$dv = a\,dt$$

$$\int_{v=v_0}^{v} dv = \int_{t=0}^{t} a\,dt.$$

For constant acceleration,

$$v - v_0 = at.$$

2-8 Variable Acceleration; Integral Calculus

Then:

$$\begin{aligned} dx &= v\,dt \\ &= (v_0 + at)\,dt \end{aligned}$$

$$\int_{x=x_0}^{x} dx = \int_{t=0}^{t} (v_0 + at)\,dt.$$

For constant acceleration,

$$x - x_0 = v_0 t + \tfrac{1}{2}at^2.$$

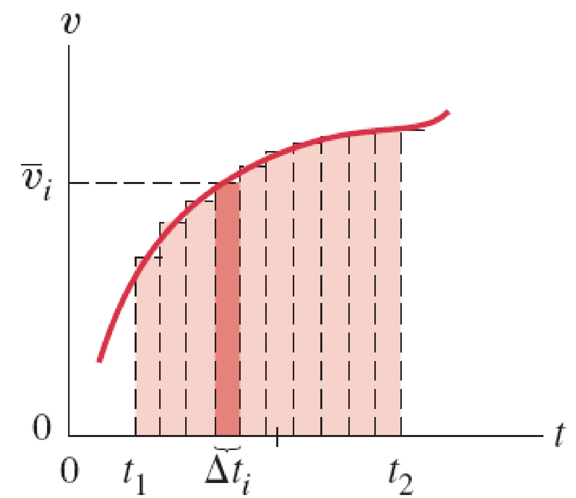
2-8 Variable Acceleration; Integral Calculus

Example 2-21: Integrating a time-varying acceleration.

An experimental vehicle starts from rest (v_0 = 0) at t = 0 and accelerates at a rate given by a = (7.00 m/s^3)t. What is (a) its velocity and (b) its displacement 2.00 s later?

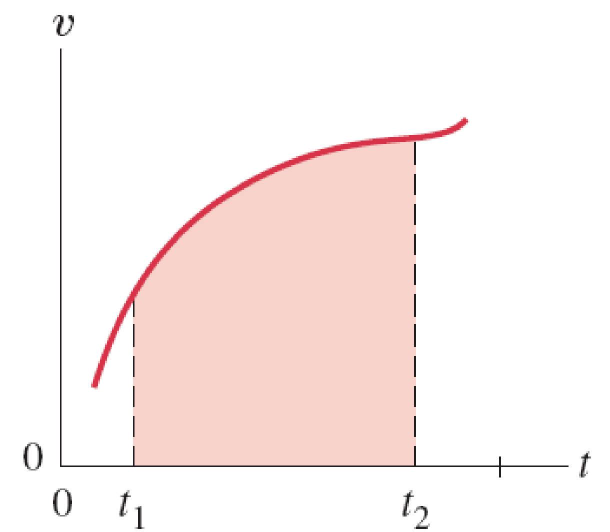
2-9 Graphical Analysis and Numerical Integration

The total displacement of an object can be described as the area under the *v-t* curve:



$$x_2 - x_1 = \lim_{\Delta t \to 0} \sum_{t_1}^{t_2} \overline{v}_i \, \Delta t_i$$

$$= \int_{t_1}^{t_2} v(t)\, dt.$$



2-9 Graphical Analysis and Numerical Integration

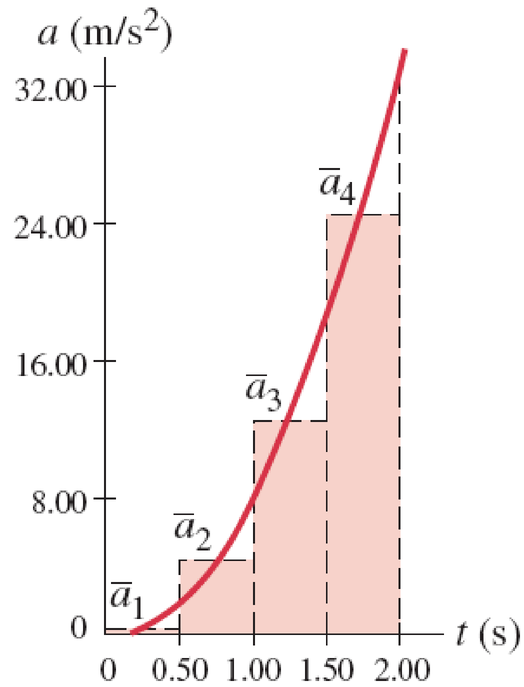
Similarly, the velocity may be written as the area under the *a-t* curve.

However, if the velocity or acceleration is not integrable, or is known only graphically, numerical integration may be used instead.

2-9 Graphical Analysis and Numerical Integration

Example 2-22: Numerical integration.

An object starts from rest at $t = 0$ and accelerates at a rate $a(t) = (8.00\ \mathrm{m/s^4})t^2$. Determine its velocity after 2.00 s using numerical methods.



Summary of Chapter 2

- Kinematics is the description of how objects move with respect to a defined reference frame.

- Displacement is the change in position of an object.

- Average speed is the distance traveled divided by the time it took; average velocity is the displacement divided by the time.

- Instantaneous velocity is the average velocity in the limit as the time becomes infinitesimally short.

Summary of Chapter 2

- Average acceleration is the change in velocity divided by the time.
- Instantaneous acceleration is the average acceleration in the limit as the time interval becomes infinitesimally small.
- The equations of motion for constant acceleration are given in the text; there are four, each one of which requires a different set of quantities.
- Objects falling (or having been projected) near the surface of the Earth experience a gravitational acceleration of 9.80 m/s^2.