

# Chapter 2: Interactions of Radiation with Matter

*Diagnostic Radiology Physics:  
A Handbook for Teachers and Students*

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**Special topics I**

**Objective:**

To familiarize students with the physics of events that occur when photons and electrons interact with matter

**IAEA**

International Atomic Energy Agency

## **Chapter 2. TABLE OF CONTENTS**

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## .2.1 INTRODUCTION

This chapter deals with the physics of events that occur when **photons** and **electrons** interact with matter. These are the radiations that are **important for diagnostic radiology**, and only those interactions which result in their **attenuation, absorption and scattering** are dealt with

The energy range used for **diagnostic radiology** is generally on the boundary between **classical and quantum physics** and following the “Complementarity Principle”, the numerical details of the interactions will be treated by classical reasoning where that is appropriate and by quantum mechanical considerations where that gives superior results

## .2.1 INTRODUCTION

The interaction of **photons** and **electrons**, as they traverse matter, are random and obey the laws of chance, although, with very different behaviour:

- **Photons** in general have none, one or a few interactions and are **exponentially attenuated**. **Photon** interactions are expressed in terms of **cross-sections** for individual interactions and **attenuation coefficients** for passage through bulk media
- **Electrons** experience large numbers of interactions and in general gradually lose energy until they are stopped. This is expressed in terms of **electron range** and **material stopping powers**

## .2.2 INTERACTIONS OF PHOTONS WITH MATTER

The **probability** of a single **photon** incident on a slab of material of area  $A$ , that contains one target of cross-sectional area  $\sigma$ , interacting with the target will be the **ratio of the two areas:  $\sigma/A$**

If there are  $\Phi$  photons randomly directed at area  $A$ , that contains  $n$  targets, each with area  $\sigma$ , the expected number of interactions  $\Delta\Phi$  between photons and targets will be  $\Delta\Phi = \Phi n(\sigma/A)$

Another way of saying this is that the **probability** of a projectile making a hit is  $n(\sigma/A)$ , which is just the fraction of the area that is blocked off by the targets

## .2.2 INTERACTIONS OF PHOTONS WITH MATTER

Changing now the targets to: **atoms**, their **cross-section** would be an **atomic cross-section**

This wouldn't be an **actual area** of the atom, but would be an **effective area** – effective for an interaction between the photon and the atom that is being considered

**Cross-sections** are frequently represented by the symbol  $\sigma$  and conventionally expressed in a unit of area called the **barn** (b)  
 $1 \text{ barn} = 10^{-28} \text{ m}^2$

## .2.2 INTERACTIONS OF PHOTONS WITH MATTER

There are **four fundamental X-ray interactions** that we need to consider; each can be associated with a specific cross-section

Interaction	Cross-section
photoelectric effect	$\tau$
coherent scattering	$\sigma_{coh}$
incoherent scattering	$\sigma_{incoh}$
pair and triplet production	$\kappa$

The first three of these interactions are important in the diagnostic energy range up to 150 keV

Pair and triplet production are only important at much higher energies and are only treated here for completeness

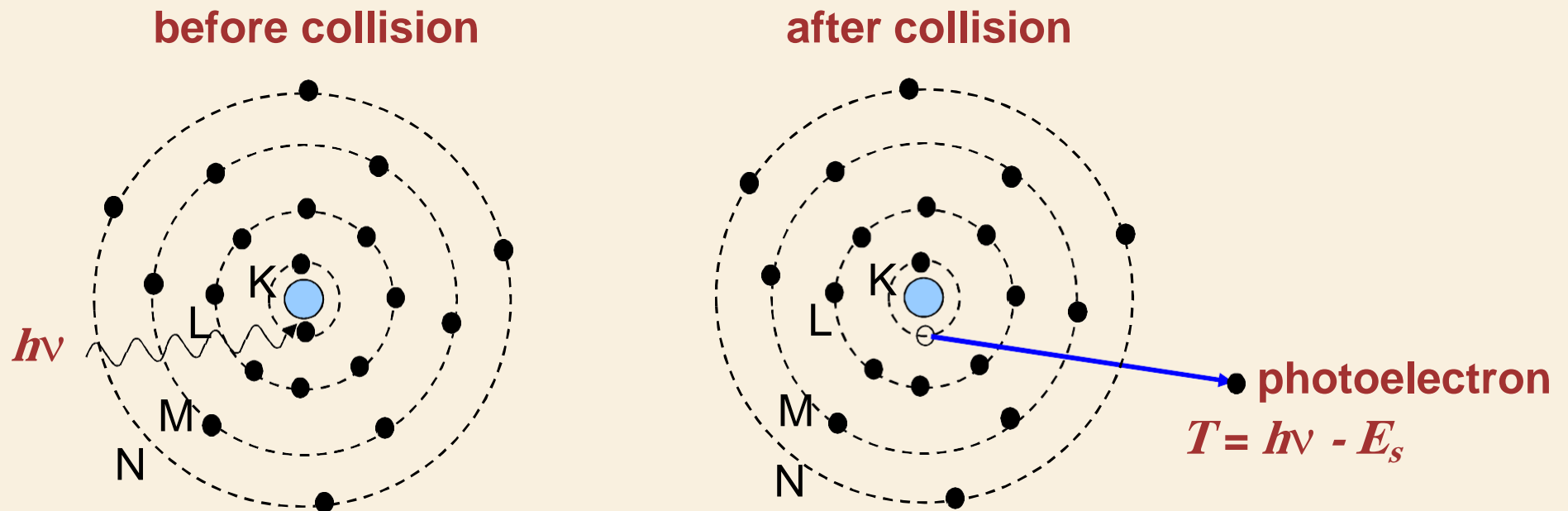
## .2.2 INTERACTIONS OF PHOTONS WITH MATTER

### .2.2.1 Photoelectric Effect

In the **photoelectric effect** the incident photon interacts with an atom and disappears. A **photoelectron** is ejected

The **photoelectric effect** can only take place if the photon energy  $h\nu > E_s$  (binding energy)

The most probable electron shell to lose an electron is the one that satisfies this constraint and has the highest binding energy





## 2. INTERACTIONS OF PHOTONS WITH MATTER

### 1. *Photoelectric Effect*

Calculation of the probability of the interaction is very complicated and quantum mechanics is required

In the diagnostic energy range up to 150 keV, the photoelectric effect cross-section per atom,  $\tau$ , is approximately given by:

$$\tau(h\nu, Z) = k \frac{Z^n}{(h\nu)^m}$$

$k$  is a constant

$Z$  is the atomic number

$n$  is an exponent in the range 3.6-5.3, being largest for low atomic numbers

$m$  is an exponent in the range 2.5-3.5, again being largest for low atomic numbers

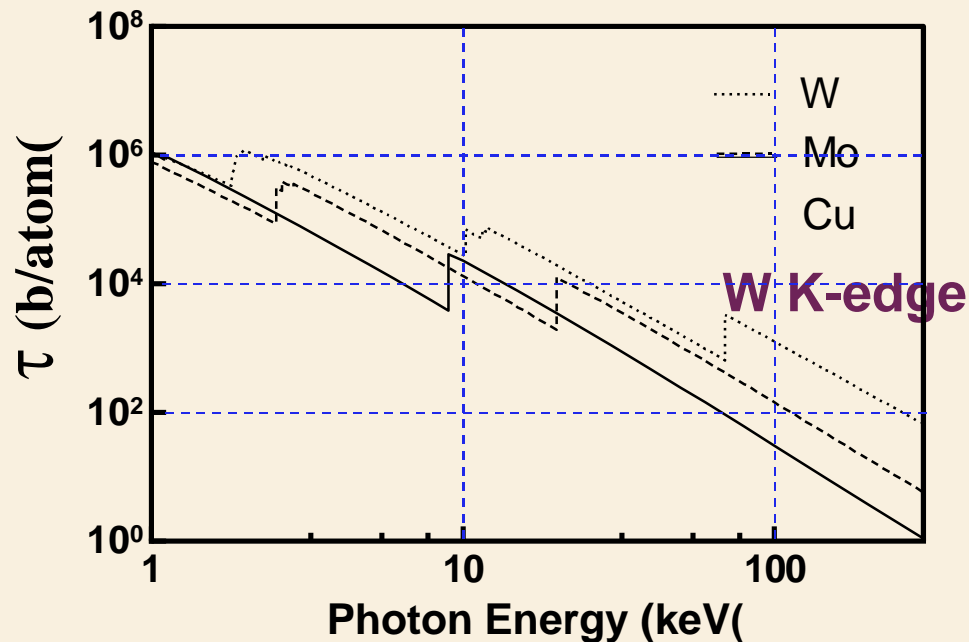
A typical dependence of  $\tau$  in the diagnostic photon energy range is:

$$\tau \propto \frac{Z^4}{(h\nu)^3}$$

## .2.2 INTERACTIONS OF PHOTONS WITH MATTER

### *.2.2.1 Photoelectric Effect*

The **sharp discontinuities** correspond to the positions of the **absorption edges** for the different materials, which increase in energy with increasing atomic number and shell binding energy



Atomic photoelectric cross sections for copper, molybdenum and tungsten

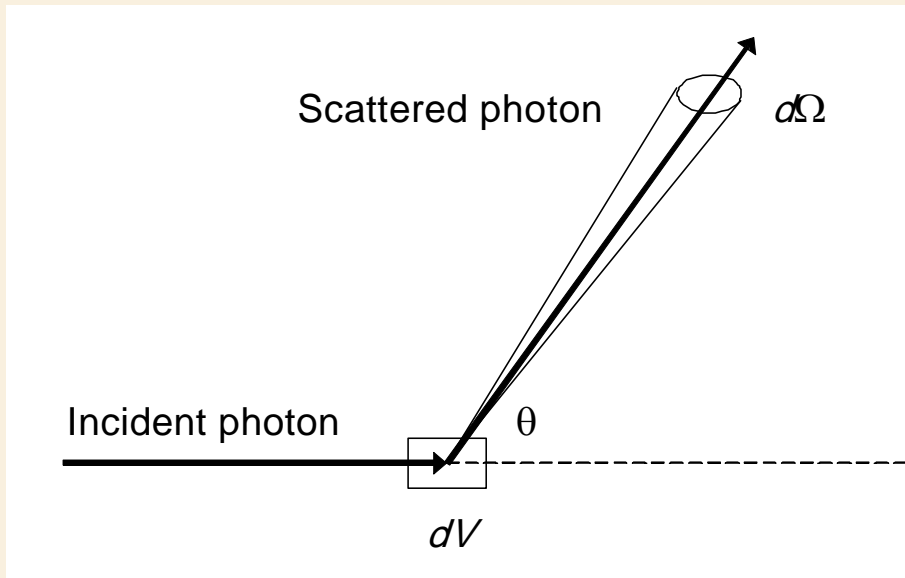
For **tungsten**, the discontinuity seen at **69.5keV** represents the effect of the K shell. At an energy just less than this, the cross section is  $6.4 \times 10^2$  b/atom, while just above this energy, the coefficient is  $3.3 \times 10^3$  b/atom. This represents a sudden **increase in cross-section** of about a **factor of five** when the photon energy increases above that of the K shell (**K-edge**)

Thus the **major contribution** to the **cross-section** above the **K-edge** comes from interactions with the two **K-shell electrons**

## .2.2 INTERACTIONS OF PHOTONS WITH MATTER

### .2.2.2 Thomson Scattering

**J.J. Thomson** gave the first treatment of the scattering of photons by electrons in the very early years of the 20<sup>th</sup> century



The probability that the photon will interact and be scattered into solid angle  $d\Omega$  is proportional to:

$$\frac{d\sigma}{d\Omega} d\Omega$$

$\frac{d\sigma}{d\Omega}$  is the differential cross section

A photon incident on a small volume element  $dV$  is scattered through angle  $\theta$  into the solid angle element  $d\Omega$

The total cross section is obtained by integrating over all solid angles:

$$\sigma = \int \frac{d\sigma}{d\Omega} d\Omega$$

## .2.2 INTERACTIONS OF PHOTONS WITH MATTER

### *.2.2.2 Thomson Scattering*

Thomson's derivation of the scattering of a photon by a single free electron at rest is based on classical physics and results in a description that is only meaningful at the low energy limit of this interaction

Thomson showed that the differential cross section, at scattering angle  $\theta$ , is given by:

$$\frac{d\sigma_{Th}}{d\Omega} = \frac{r_0^2}{2} (1 + \cos^2 \theta)$$

$$r_0 = \frac{k e^2}{m_0 c^2} = 2.81794 \times 10^{-15} \text{ m}$$

$r_0$  is “classical radius of the electron”

$k$  is a proportionality constant from Coulomb's law

$e$  is the charge on the electron

$m_0$  is the rest mass of the electron

$c$  is the speed of light

## .2.2 INTERACTIONS OF PHOTONS WITH MATTER

### *.2.2.2 Thomson Scattering*

Differential cross-section

$$\frac{d\sigma_{Th}}{d\Omega} = \frac{r_0^2}{2} (1 + \cos^2 \theta)$$

This equation predicts that the same amount of energy will be scattered forward as backward, and also that the energy scattered at right angles will be half this amount. Except at low energies, this result does not agree with observation or with the predictions made by quantum mechanics

Total cross-section

$$\sigma_{Th} = \frac{r_0^2}{2} \int_0^\pi 2\pi (1 + \cos^2 \theta) \sin \theta d\theta = \frac{8\pi r_0^2}{3} = 66.52 \times 10^{-30} \text{ m}^2$$

$\sigma_{Th}$  is constant, predicting that the classical scattering probability is independent of electromagnetic radiation energy

This, of course, is not correct

## **.2.2 INTERACTIONS OF PHOTONS WITH MATTER**

### ***.2.2.3 Coherent (Rayleigh) Scattering***

**In deriving the expression for Thomson scattering it was assumed that the electron was free, alone and at rest**

**In reality the photon is scattered collectively by the atomic electrons which are not free, and their proximity to one another is not very different from the wavelength of the radiation**

**In coherent scattering essentially no energy is lost by the photon as it transfers momentum to the atom and is scattered through angle  $\theta$**

**The scattering by the different electrons is in phase and the resultant angular distribution is determined by an interference pattern which is characteristic of the atom**

## .2.2 INTERACTIONS OF PHOTONS WITH MATTER

### .2.2.3 Coherent (Rayleigh) Scattering

The differential cross section is then given by:  $\frac{d\sigma_{coh}}{d\Omega} = \frac{d\sigma_{Th}}{d\Omega} F^2(x, Z)$

$$\frac{d\sigma_{Th}}{d\Omega}$$

is the Thomson differential scattering coefficient and  $F$  is known as the coherent form factor

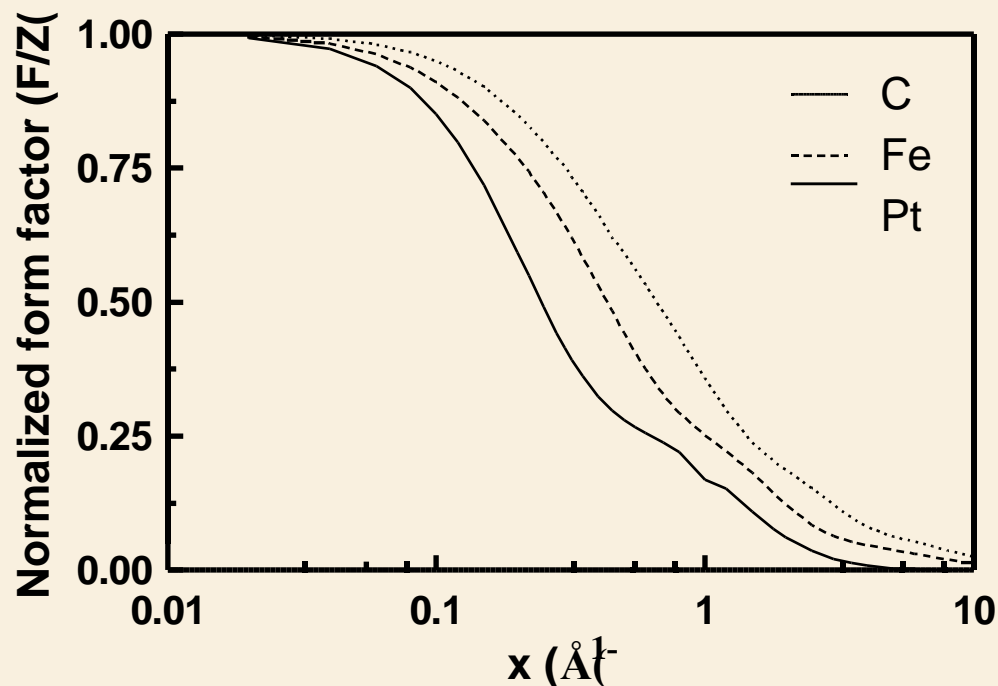
It may be calculated using quantum mechanical models

$$x = \frac{\sin(\theta/2)}{\lambda}$$

$\lambda$  is the wavelength of the incident photon. The parameter  $x$  is proportional to the transfer of momentum between the initial and scattered photon directions

## .2.2 INTERACTIONS OF PHOTONS WITH MATTER

### .2.2.3 Coherent (Rayleigh) Scattering



Variation of the normalized form factor  $F/Z$  for coherent scattering with the momentum transfer parameter  $x$

Data from Hubbell and Øverbø, J Phys Chem Ref Data 8, 69-106 ((1979

For scattering in the forward direction all the atomic electrons act together, and  $F$  is equal to the atomic number and the differential cross section depends upon  $Z^2$

As the scattering angle increases,  $F$  decreases because it becomes increasingly difficult for all the electrons to scatter in phase without any energy transfer

However, for a given value of the scattering angle, the normalized coherent form factor  $F/Z$ , increases with increasing atomic number



## **.2.2 INTERACTIONS OF PHOTONS WITH MATTER**

### ***.2.2.4 Compton scattering by free electrons***

**Compton Scattering, like Thomson scattering, is the interaction between electromagnetic radiation and a free electron, but in this case there is an energy transfer to the electron**

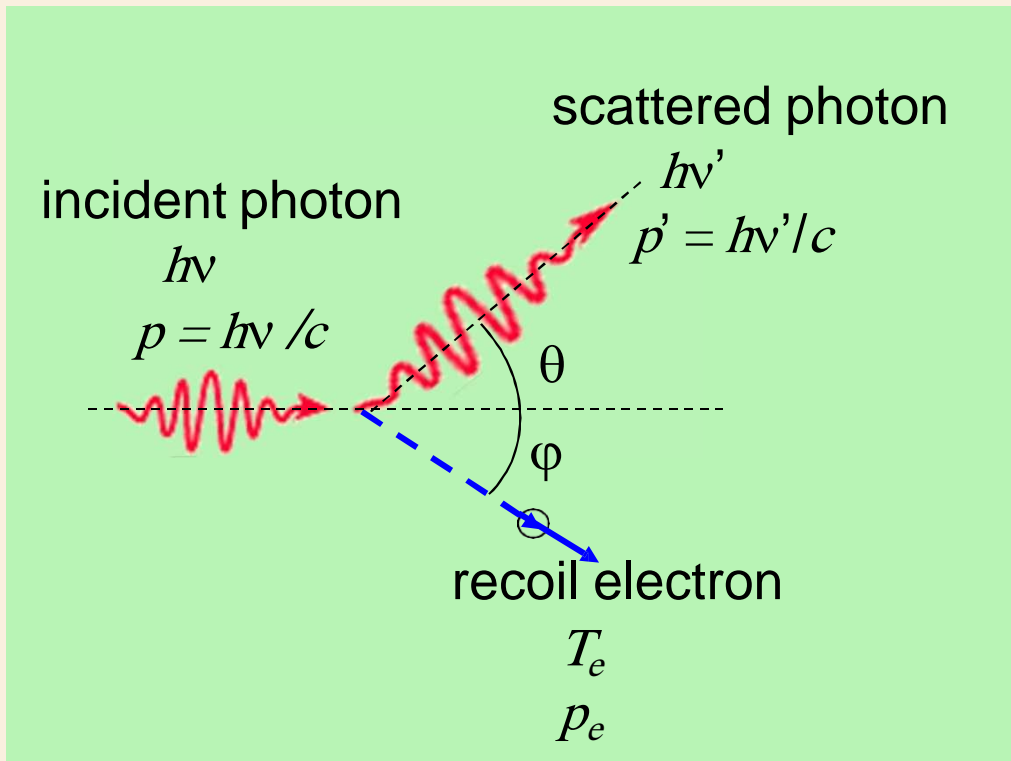
**The energy range is such that relativity and quantum mechanics must be used to derive expressions for the cross section**

**Both the photon and the electron must be considered as particles**

## .2.2 INTERACTIONS OF PHOTONS WITH MATTER

### .2.2.4 Compton scattering by free electrons

Using conservation of energy and momentum  
Compton relations can be derived:



$$\frac{h\nu'}{h\nu} = \frac{1}{1 + \alpha (1 - \cos\theta)}$$

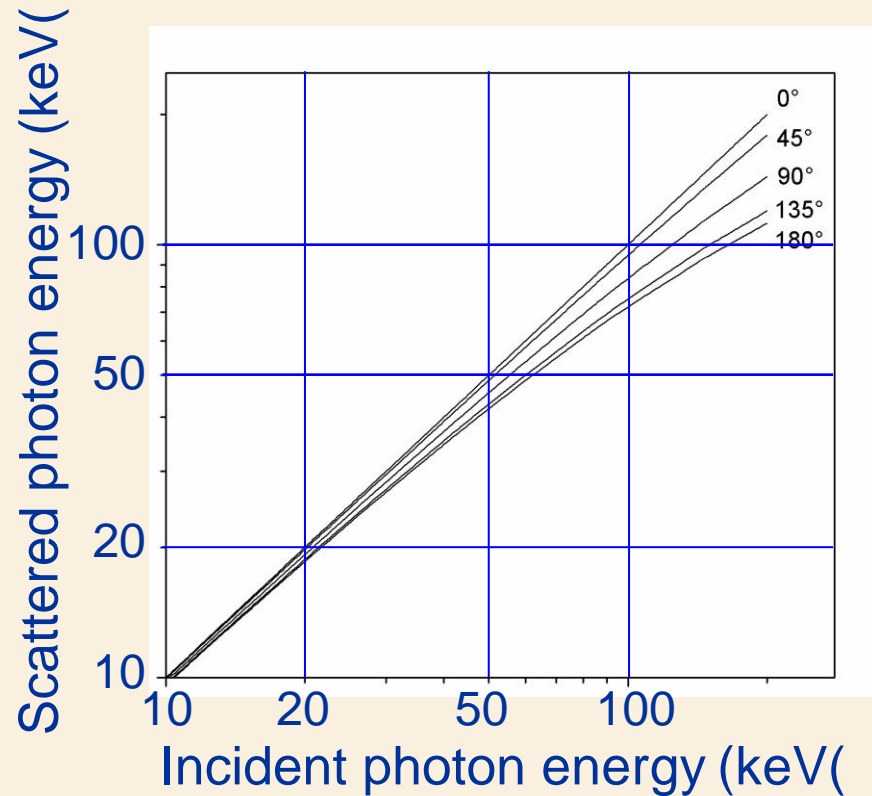
$$\alpha = \frac{h\nu}{m_0 c^2}$$

$$\cot\phi = (1 + \alpha) \tan\left(\frac{\theta}{2}\right)$$

$$T_e = h\nu - h\nu' = h\nu \frac{\alpha (1 - \cos\theta)}{1 + \alpha (1 - \cos\theta)}$$

## .2.2 INTERACTIONS OF PHOTONS WITH MATTER

### *.2.2.4 Compton scattering by free electrons*



$$\frac{h\nu'}{h\nu} = \frac{1}{1 + \alpha(1 - \cos\theta)}$$

$$T_e = h\nu - h\nu' = h\nu \frac{\alpha(1 - \cos\theta)}{1 + \alpha(1 - \cos\theta)}$$

In the diagnostic energy range

$$\alpha = \frac{h\nu}{m_0c^2}$$

is small, and as a consequence the energy transfer to the recoil electron is also small, being zero in the forward direction and taking its largest value when the photon is backscattered

For 20 keV, 50 keV and 100 keV incident photons, the maximum energy transfers to the recoil electron are:

1.5 keV, 8.2 keV and 28.1 keV respectively

## .2.2 INTERACTIONS OF PHOTONS WITH MATTER

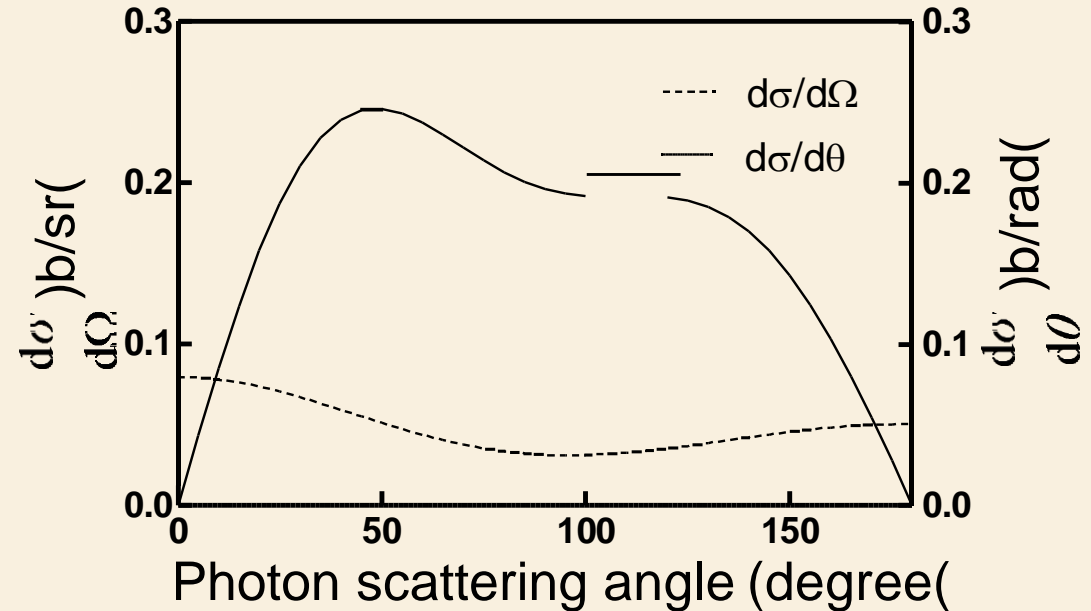
### .2.2.4 Compton scattering by free electrons

The cross-section for the scattering of a photon, with energy  $h\nu$  through a given angle  $\theta$ , was first derived in 1928 by Klein and Nishina using the Dirac theory of the electron

The expression for the differential cross-section for scattering of photons by a single free electron is:

$$\frac{d\sigma_{KN}}{d\Omega} = \frac{r_0^2}{2} (1 + \cos^2 \theta) f_{KN}$$

$$f_{KN} = \left\{ \frac{1}{1 + \alpha(1 - \cos\theta)} \right\}^2 \left\{ 1 + \frac{\alpha^2(1 - \cos\theta)^2}{1 + \alpha(1 - \cos\theta)[1 + \cos^2\theta]} \right\}$$



Compton differential cross-sections for scattering of 70 keV photons

This cross-section reduces to the Thomson cross-section when  $\alpha \rightarrow 0$  (that is  $h\nu \ll h\nu_0 \rightarrow 1$ )

## .2.2 INTERACTIONS OF PHOTONS WITH MATTER

### *.2.2.4 Compton scattering by free electrons*

**The total Compton cross-section (probability of interaction per electron) for a photon of energy  $h\nu$ , is obtained by integrating the differential cross section**

$$\frac{d\sigma_{KN}}{d\Omega} = \frac{r_0^2}{2} (1 + \cos^2\theta) f_{KN}$$

**over solid angle. The result is:**

$$\sigma_{KN}(h\nu) = 2\pi r_0^2 \left\{ \left( \frac{1+\alpha}{\alpha^2} \right) \left( \frac{2(1+\alpha)}{1+2\alpha} - \frac{\ln(1+2\alpha)}{\alpha} \right) + \frac{\ln(1+2\alpha)}{2\alpha} - \frac{1+3\alpha}{(1+2\alpha)^2} \right\}$$

## .2.2 INTERACTIONS OF PHOTONS WITH MATTER

### *.2.2.4 Compton scattering by free electrons*

$\sigma_s$  : Scattering coefficient

$\sigma_{tr}$  : Energy transfer coefficient

$\sigma_{KN}$  : Total cross section for Compton scattering

In the incoherent free electron scattering process the initial photon energy is divided between the scattered photon and the recoiling electron. The differential energy transfer coefficient is given by:

$$\frac{d\sigma_{tr}}{\Omega} = \frac{r_0^2}{2} (1 + \cos^2\theta) f_{KN} \left( \frac{\alpha(1 - \cos\theta)}{1 + \alpha(1 - \cos\theta)} \right)$$

$\sigma_{tr}$  can be obtained integrating over all angles with  $d\Omega = 2\pi \sin\theta d\theta$

The scattering coefficient is, then, by definition:

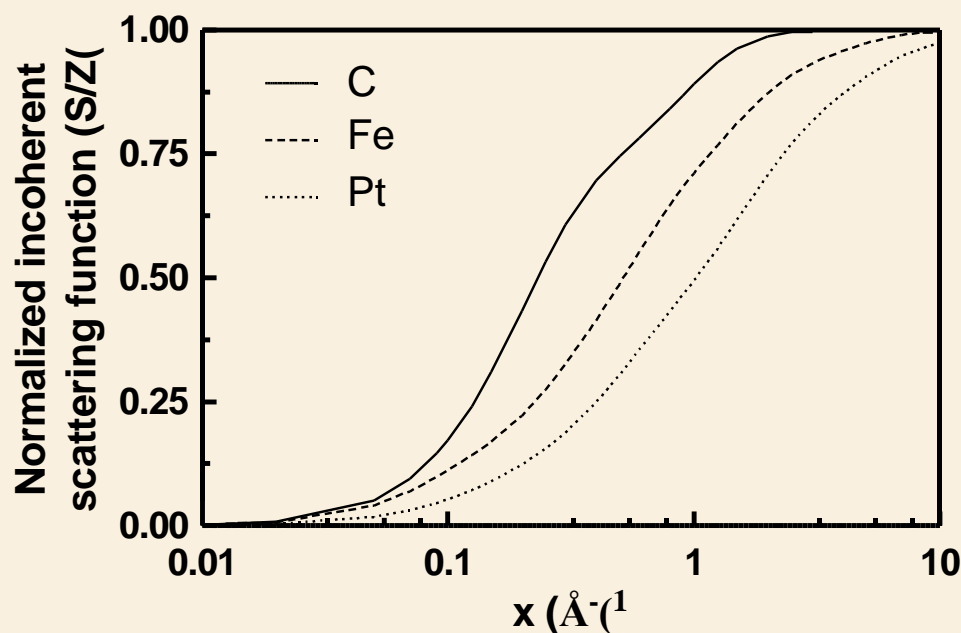
$$\sigma_s = \sigma_{KN} - \sigma_{tr}$$

## .2.2 INTERACTIONS OF PHOTONS WITH MATTER

### .2.2.5 Incoherent scattering

For incoherent scattering by bound atomic electrons, the contributions from individual electrons are added and the differential cross section takes the form:

$$\frac{d\sigma_{incoh}}{d\Omega} = \frac{r_0^2}{2} (1 + \cos^2 \theta) f_{KN} S(x, Z)$$



Data from Hubbell, Veigele et al., J Phys Chem Ref Data 4(3) ((1975

$S$  is known as the incoherent scattering function, and is a universal function of the momentum transfer quantity  $x$  and the atomic number.  $S$  is zero in the forward direction and increases with increasing momentum transfer reaching the value of  $Z$ , the number of electrons per atom. This increase becomes slower as the atomic number increases

## .2.2 INTERACTIONS OF PHOTONS WITH MATTER

### *.2.2.5 Incoherent scattering*

The total cross section for incoherent scattering is obtained by numerical integration of equation:

$$\frac{d\sigma_{incoh}}{d\Omega} = \frac{r_0^2}{2} (1 + \cos^2 \theta) f_{KN} \mathcal{S}(X, Z)$$

In many situations it is very nearly equal to the single electron cross section multiplied by the number of electrons in the atom

$$\sigma_{incoh} \approx Z \sigma_{KN}$$



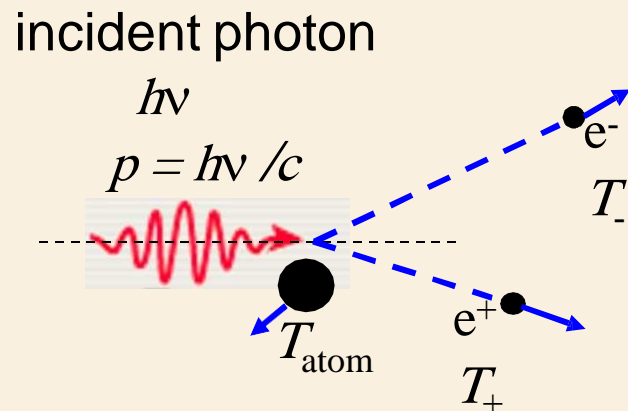
## .2.2 INTERACTIONS OF PHOTONS WITH MATTER

### .2.2.6 Pair and Triplet production

#### *Pair production*

When a high energy photon passes near to an atomic nucleus, the photon may interact with the nuclear Coulomb field by a process called pair production. The photon is converted into an electron-positron pair, each with its own kinetic energy. The energy balance is:

$$h\nu = T_+ + T_- + 2m_0c^2$$



Pair production cannot take place for photons with energies less than the energy threshold  $2m_0c^2 = 1022\text{ keV}$

As pair production occurs in the field of the nucleus, the cross-section for this interaction varies approximately as  $Z^2$  where  $Z$  is the nuclear charge

## .2.2 INTERACTIONS OF PHOTONS WITH MATTER

### *.2.2.6 Pair and Triplet production*

#### *Triplet production*

When a high energy photon passes near to an orbital electron, the photon may interact with the Coulomb field of this electron by a process called triplet production

The target electron is itself ejected with considerable energy. Two electrons and one positron are thus set into motion

The energy threshold for Triplet production is  $4 m_0 c^2$

Thresholds for pair and triplet production are much higher than the photon energies relevant to diagnostic radiology

## .2.3 PHOTON ATTENUATION COEFFICIENTS

**Photons** may undergo more than one interaction as they pass through bulk material:

An initial scatter interaction might be followed by a second scattering process which in turn might be followed by a third scatter, photoelectric absorption or no further interactions with the photon leaving the bulk material, for example

To consider this macroscopic behaviour of primary photons traversing matter

- linear attenuation coefficients and
- mass attenuation coefficients

are used, which are simply related to the total cross section

The exit beam from the bulk material will also comprise both primary and scattered photons. Such effects are best estimated using Monte Carlo techniques

## .2.3 PHOTON ATTENUATION COEFFICIENTS

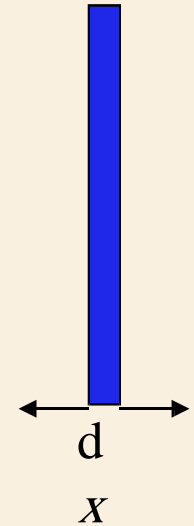
### *.2.3.1 Linear attenuation coefficient*

Consider a **thin uniform slab** of material of thickness  $d$ , which is irradiated with a **beam of photons incident normally** on the slab

Individual photons may pass through the slab without interacting, or they may be absorbed or scattered

The probability that an individual photon will interact in this thin section is given by:

$$N_a \sigma dx$$



**Linear attenuation coefficient:**  $N_a \sigma = \mu \text{ m}^{-1} \left( = \frac{1000 N_A \rho}{A_r} \sigma \right) \text{ S.I.}$

$N_a$  is the number of interaction centres (atoms) per unit volume

$\sigma$  is the total interaction cross section per atom

For scattering by atoms,  $N_a$  may be calculated from the Avogadro constant  $N_A$ , the atomic weight  $A_r$  and the density  $\rho$

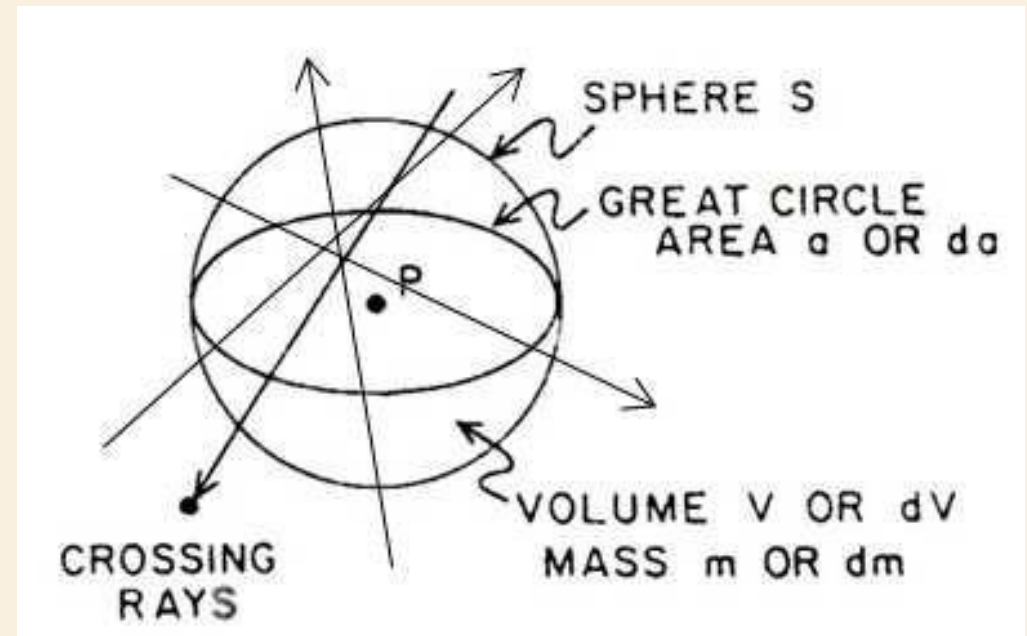
## .2.3PHOTON ATTENUATION COEFFICIENTS

### *.2.3.2Exponential attenuation*

#### Definition of fluence $\Phi$

A **radiation field** at a point P can be quantified by the physical non-stochastic quantity **fluence**, which is usually expressed in units of  $m^{-2}$  or  $cm^{-2}$ , and is given by:

$$\Phi = \frac{dN}{da}$$



where  $dN$  is the differential of the expectation value of the number of particles (photons, or massive particles) striking an infinitesimal sphere with a great-circle area  $da$  surrounding point P

## .2.3 PHOTON ATTENUATION COEFFICIENTS

### *.2.3.2 Exponential attenuation*

Now consider a **thick slab** of material.  $\Phi(x)$  represents the **fluence of photons which have not interacted in the slab after passage through thickness  $x$** . The **expected change  $d\Phi$  in this fluence after passage through a further thickness  $dx$  is given by:**

$$d\Phi = -\Phi \mu dx$$

Integrating: 
$$\Phi = \Phi_0 e^{-\mu x}$$

$\Phi_0$  is the initial fluence

This equation, known as **Beer's law**, describes the **exponential attenuation of a photon beam**

## .2.3 PHOTON ATTENUATION COEFFICIENTS

### *.2.3.2 Exponential attenuation*

$$\Phi = \Phi_0 e^{-\mu x}$$

**Beer's law, describes the exponential attenuation of a photon beam**

It should be noted that it describes the number of photons which have not interacted, also known as **primary photons**

At **diagnostic energies** other photons may be present at depth resulting from photon scattering interactions or the emission of fluorescent photons following a photoelectric interaction

## .2.3 PHOTON ATTENUATION COEFFICIENTS

### *.2.3.3 Mass attenuation coefficient*

The linear attenuation coefficient  $\mu$  ( $\text{m}^{-1}$ ) is dependent on density which in turn is dependent on the physical state of the material

As a consequence  $\mu$  is not a suitable quantity for data compilations

The quantity **mass attenuation coefficient**  $\frac{\mu}{\rho}$   $\left(\frac{\text{m}^2}{\text{kg}}\right)$  is independent of density

Physical state	Energy of photon (keV)	$\mu$ ( $\text{m}^{-1}$ )	$\mu/\rho$ ( $\text{m}^2/\text{kg}$ )
water	50	21.4	0.0226
ice	50	19.6	0.0226



## .2.3 PHOTON ATTENUATION COEFFICIENTS

### .2.3.4 Mass energy transfer coefficients and mass energy absorption coefficients

For dosimetric purposes it is necessary to know the energy transferred to secondary electrons as a result of the initial interaction

$$d(h\nu)_{tr} = \Phi h\nu \mu_{tr} dx$$

energy  $d(h\nu)_{tr}$  transferred by interactions to kinetic energy of electrons when photons of energy  $h\nu$  traverse a distance  $dx$  in a material

$$\mu_{tr} = \mu \frac{\langle T \rangle}{h\nu}$$

$\langle T \rangle$  is the expectation value of the energy converted to secondary electrons

$h\nu$  is the energy of photon

$\mu$	linear attenuation coefficient
$\mu_{tr}$	linear transfer coefficient
$\mu_{tr}/\rho$	mass energy transfer coefficient
$\mu_{en}/\rho$	mass energy absorption coefficient
$K = d$	Kerma

$$(h\nu)_{tr}/dm$$

## .2.3 PHOTON ATTENUATION COEFFICIENTS

### *.2.3.4 Mass energy transfer coefficients and mass energy absorption coefficients*

$$K = \frac{d(h\nu)_{tr}}{dm} = \frac{\Phi h\nu \mu_{tr} dx}{dm} = \Phi h\nu \left( \frac{\mu_{tr}}{\rho} \right)$$

**$K$  (Kerma): energy transferred from uncharged particles to matter**

$$\frac{dm}{dx} = \rho$$

**Some of the energy transferred to the secondary charged particles is lost to radiative processes in the material, mainly Bremsstrahlung**

**This is taken account by the mass energy absorption coefficient:**

$$\left( \frac{\mu_{en}}{\rho} \right) = \left( \frac{\mu_{tr}}{\rho} \right) (1-g)$$

**$g$  gives the energy fraction lost to radiative processes. For the energies used in diagnostic radiology,  $g$  may be taken as zero**

## .2.3 PHOTON ATTENUATION COEFFICIENTS

### *.2.3.5 Contribution of individual interactions to the total mass attenuation coefficient*

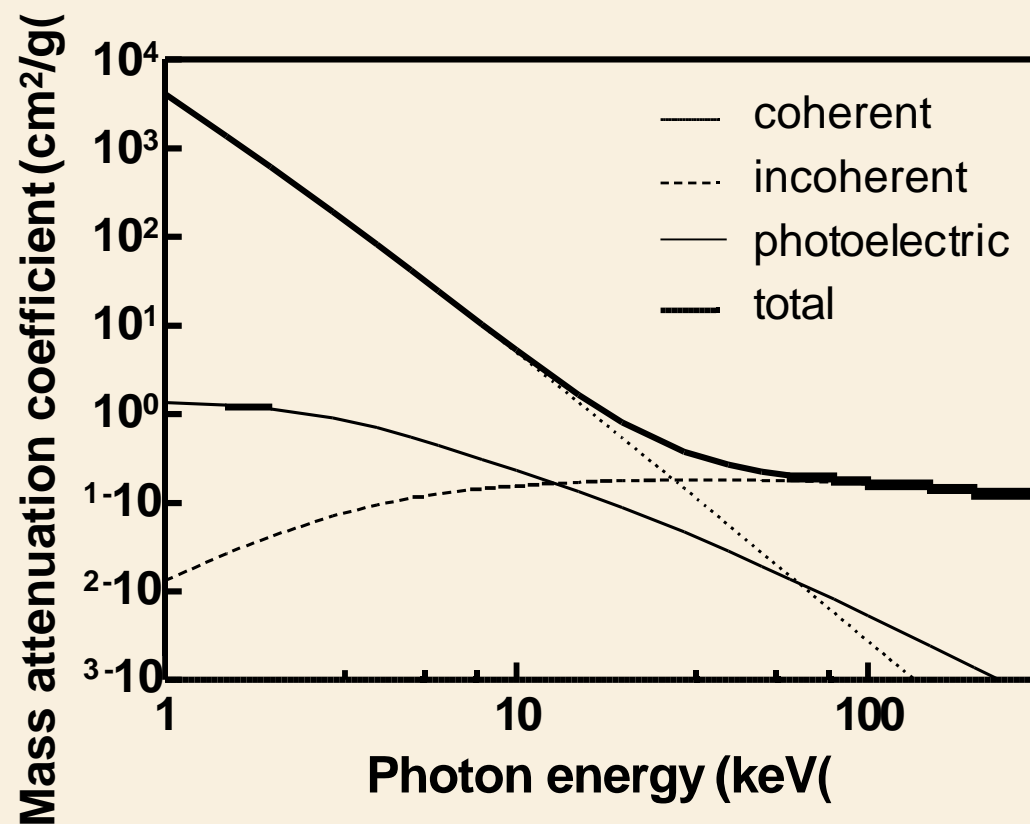
The four distinctly different mechanisms by which photons may interact with matter compete, according to their individual probabilities. The total mass attenuation coefficient is therefore the sum of all the individual mass attenuation coefficients:

$$\left(\frac{\mu}{\rho}\right) = \left(\frac{\tau}{\rho}\right) + \left(\frac{\mu_{coh}}{\rho}\right) + \left(\frac{\mu_{inc}}{\rho}\right) + \left(\frac{\kappa}{\rho}\right) = (\tau + \sigma_{coh} + \sigma_{inc} + \kappa) \frac{N_A}{A_r} 1000$$

The size of each attenuation coefficient will depend on the photon energy and the atomic number of the material

## .2.3 PHOTON ATTENUATION COEFFICIENTS

### *.2.3.5 Contribution of individual interactions to the total mass attenuation coefficient*

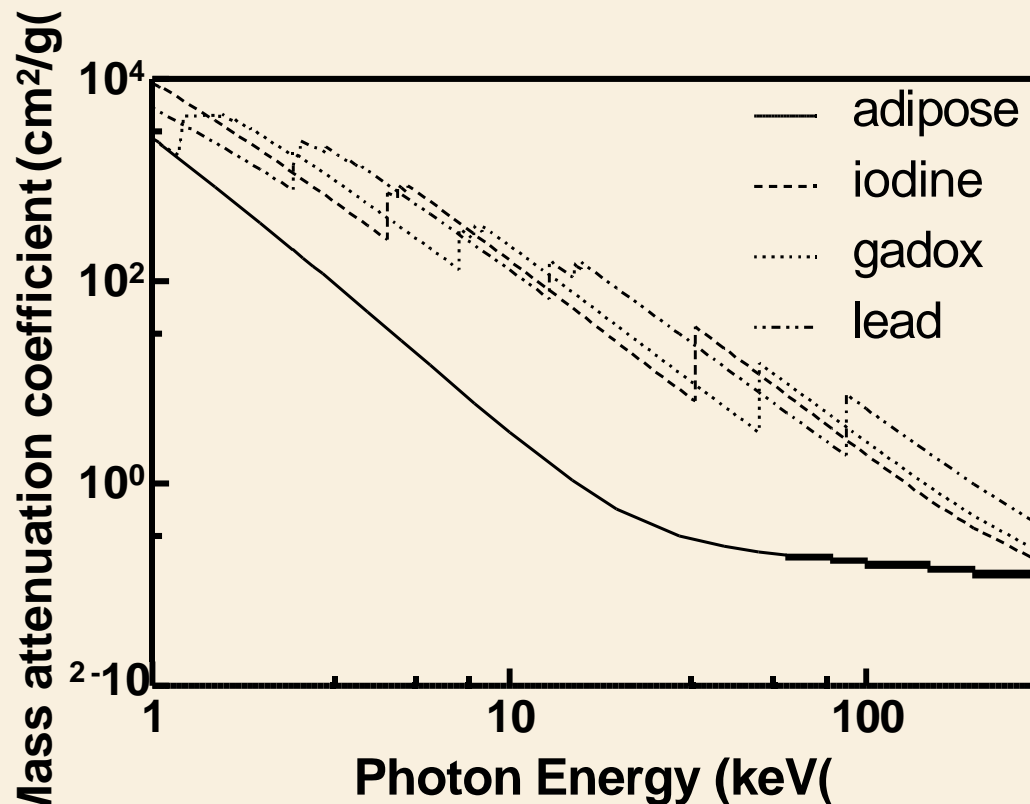


Mass attenuation coefficients for water for photon energies from 1 keV to 300 keV

The **photoelectric interaction** makes the dominant contribution to the **total interaction cross section at the lowest energies**. The steep decrease at the lower photon energies is characteristic for the photoelectric effect and ends when **incoherent (Compton) scattering** becomes dominant and remains so for the rest of the diagnostic energy range. The cross-over position for these two interactions depends on the atomic number **For water is about 30 keV**

## .2.3 PHOTON ATTENUATION COEFFICIENTS

### *.2.3.5 Contribution of individual interactions to the total mass attenuation coefficient*



**Total mass interaction coefficients** for materials relevant to diagnostic radiology (adipose tissue, iodine, gadolinium oxysulphide (gadox) and lead

In the energy range up to 100 keV and for the high  $Z$  materials, the discontinuities arising from the differences in **photoelectric absorption** at K, L and M edges are evident

The **differences in absorption** that this creates are important for the design of filters used to shape **X-ray spectra** (particularly in mammography and for imaging using iodinated contrast agents)

## .2.3 PHOTON ATTENUATION COEFFICIENTS

### *.2.3.6 Coefficients for compounds and mixtures*

**Mass attenuation coefficients and mass energy-transfer coefficients for compounds and intimate mixtures can be obtained by a weighted summation of the coefficients of the constituents:**

$$\left(\frac{\mu}{\rho}\right) = \sum_i \left(\frac{\mu}{\rho}\right)_i w_i$$

$$\left(\frac{\mu_{tr}}{\rho}\right) = \sum_i \left(\frac{\mu_{tr}}{\rho}\right)_i w_i$$

$$\left(\frac{\mu_{en}}{\rho}\right) = \sum_i \left(\frac{\mu_{en}}{\rho}\right)_i w_i$$

$w_i$  are the normalized weight fractions of the elements  $i$  (or mixture components  $i$ ) present in the absorber

The **mass energy-absorption coefficient** for an element accounts for the radiative losses of the secondary electrons

## .2.4 INTERACTIONS OF ELECTRONS WITH MATTER

### Energy loss by electrons



#### Ionizational or collisional losses

Principal process:  
collisions with other electrons

Energy losses and the changes  
in direction can be quite large

The electron which leaves the  
collision with the most energy is  
assumed to be the original  
incident electron. This means the  
maximum energy exchange is  
half of the original energy

#### Radiative losses or Bremsstrahlung

Interaction with the electric field  
of nuclei and be decelerated so  
rapidly that some of its energy  
may be radiated away

## .2.4 INTERACTIONS OF ELECTRONS WITH MATTER

Energy lost by charged particles in passing through matter is generally described using a quantity called **stopping power  $S$**

$dT$  is the loss in kinetic energy of the particle as it travels a distance  $dx$ .  $\rho$  is the density of the material

**Stopping power**

$$\underline{S} = \frac{dT}{dx}$$

**Mass stopping power**

$$\frac{S}{\rho} = \frac{1}{\rho} \frac{dT}{dx}$$



## **.2.4 INTERACTIONS OF ELECTRONS WITH MATTER**

### ***.2.4.1 Ionizational (Collisional) Interactions and Ionizational Stopping Power***

**This process involves collisions between electrons travelling through matter and electrons from atoms that are part of the material, leaving the atoms ionized**

**It is rather difficult to measure the rate of energy lost by these interactions but is relatively easy to calculate it. Relativity and Quantum Mechanics must be used**

## .2.4 INTERACTIONS OF ELECTRONS WITH MATTER

### *.2.4.1 Ionizational (Collisional) Interactions and Ionizational Stopping Power*

The problem was first solved by Bethe in the early part of the 20<sup>th</sup> century. The **Ionizational Mass Stopping Power ( $S_{ion}/\rho$ ) (MeV·cm<sup>2</sup>·g<sup>-1</sup>)** was derived by **Bethe-Bloch** and extended by **Sternheimer**:

$$\frac{S_{ion}}{\rho} = 2\pi r_0^2 N_e \frac{\mu_0}{\beta^2} \left[ \ln \frac{T^2 (T + 2\mu_0)}{2\mu_0 I^2} + \frac{T^2 / 8 - (2T + \mu_0)\mu_0 \ln 2}{(T + \mu_0)^2} + 1 - \beta^2 - \delta \right]$$

$r_0$  is the “classical radius of the electron”

$N_e = N_A(Z/A_r)$ ,  $N_A$  is the Avogadro constant

$\mu_0 = m_0 c^2$  is the rest mass of the electron multiplied by the speed of light squared

$T$  is the kinetic energy

$\beta$  is the ratio of the speed of the electron to that of light

The density correction term  $\delta$ , was added later by Sternheimer. Its effect is to reduce energy losses, but only at high energies. At 100 MeV it can be as great as 20%

$I$  is a semi-empirical quantity called the “Mean Excitation Energy” which is a property of the material and increases as the atomic number of the material increases. Values of  $I$  for a large number of materials are given by NIST

## .2.4 INTERACTIONS OF ELECTRONS WITH MATTER

### *.2.4.1 Ionizational (Collisional) Interactions and Ionizational Stopping Power*

$$\frac{S_{ion}}{\rho} = 2\pi r_0^2 N_e \frac{\mu_0}{\beta^2} \left[ \ln \frac{T^2 (T + 2\mu_0)}{2\mu_0 I^2} + \frac{T^2 / 8 - (2T + \mu_0)\mu_0 \ln 2}{(T + \mu_0)^2} + 1 - \beta^2 - \delta \right]$$

**Below  $100 \text{ keV}$  or so, the term  $2\pi r_0^2 N_e \mu_0 / \beta^2$  is the most important**

**The factor  $1/\beta^2 = c^2/v^2$  makes the stopping power nearly inversely proportional to the kinetic energy**

**For energies **above  $100 \text{ keV}$**   $\beta$  is essentially 1 and the term in front becomes nearly constant**

**The terms inside the square bracket increase slowly with energy and the stopping power passes through a minimum in the neighborhood of  $1 \text{ MeV}$**

## .2.4 INTERACTIONS OF ELECTRONS WITH MATTER

### *.2.4.1 Ionizational (Collisional) Interactions and Ionizational Stopping Power*

$$\frac{S_{ion}}{\rho} = 2\pi r_0^2 N_e \frac{\mu_0}{\beta^2} \left[ \ln \frac{T^2 (T + 2\mu_0)}{2\mu_0 I^2} + \frac{T^2 / 8 - (2T + \mu_0)\mu_0 \ln 2}{(T + \mu_0)^2} + 1 - \beta^2 - \delta \right]$$

**Dependence on atomic number is not strong**

The factor in front of the square brackets contains the number of electrons per unit mass, given by  $N_A(Z/A_r)$ , and remembering  $Z/A_r$  is 0.5 or slightly less, for all materials except hydrogen, the mass stopping power decreases only slightly as atomic number increases

The mean excitation energy,  $I$ , increases as atomic number increases, which also serves to make  $S_{ion}$  smaller for high atomic number materials

## .2.4 INTERACTIONS OF ELECTRONS WITH MATTER

### .2.4.2 Radiative Interactions and Radiative Stopping Power

When an **electron** passes close to a **nucleus** it will experience a substantial **Coulomb force** and will be **decelerated**, radiating energy in the form of electromagnetic radiation (**Bremsstrahlung**)

The quantum mechanical solution is complicated and approximations must be made, and the most appropriate form for the result depends on the energy range

The approximate Radiative Mass Stopping Power ( $S_{rad}/\rho$ ) (MeV·cm<sup>2</sup>·g<sup>-1</sup>) for **diagnostic energies** is:

$$\frac{S_{rad}}{\rho} = \sigma_0 \frac{N_A}{A} Z^2 (T + \mu_0) \bar{B}$$

$$\sigma_0 = \frac{1}{137} \left( \frac{e}{\mu_0} \right)^2 = 0.580 \text{ barn/nucleus}$$

The function  $B = B(h\nu/T)$  is a slowly varying function of  $T$  and  $Z$  with an average for non-relativistic energies,  $T \ll m_0 c^2$ , of  $\bar{B} = 16/3$

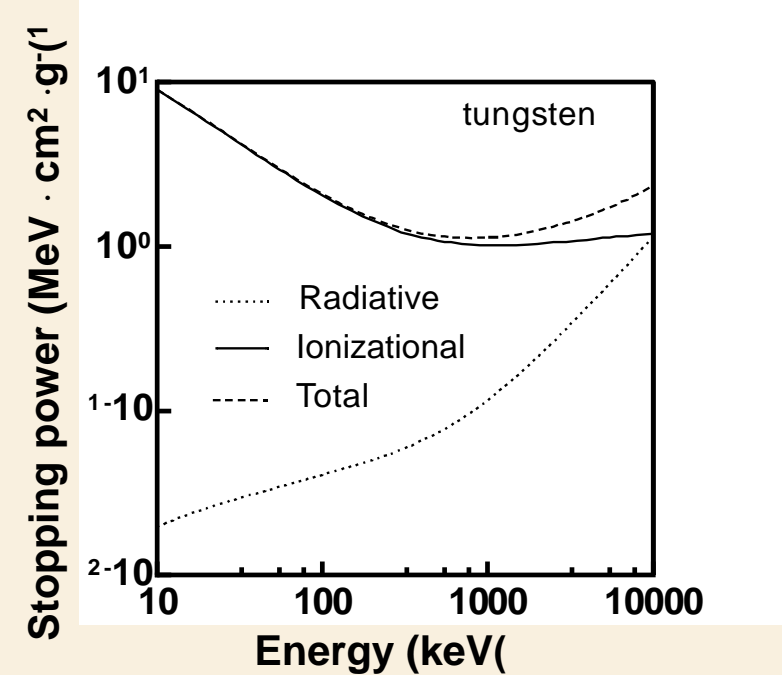
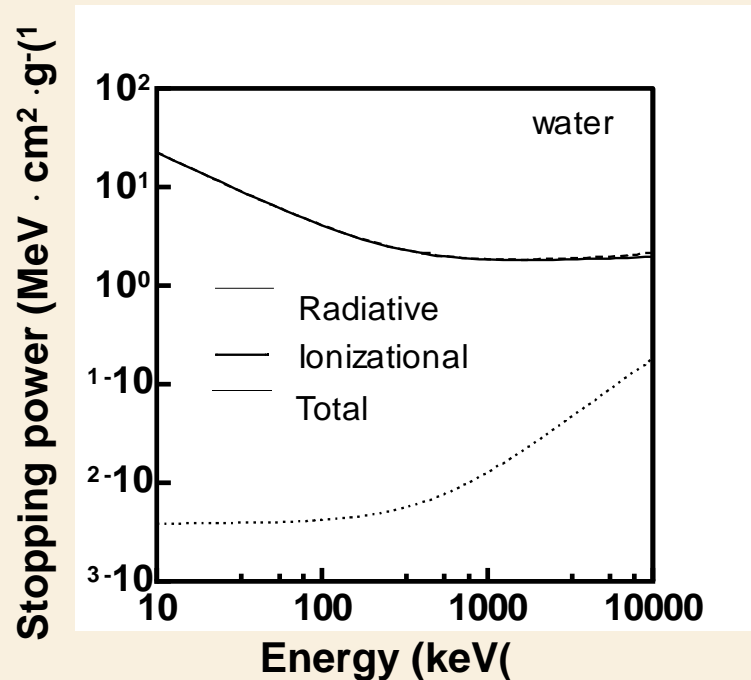
The **energy loss** due to this process is quite strongly dependent on the atomic number, as can be seen by the  **$Z^2$**  term

## .2.4 INTERACTIONS OF ELECTRONS WITH MATTER

### .2.4.3 Total Stopping Power

The total stopping power is the sum of the ionizational and radiative stopping powers:

$$S_{tot} = S_{ion} + S_{rad}$$



Ionizational, radiative and total stopping powers for water and tungsten for electron energies from 10 keV to 10 MeV

## .2.4 INTERACTIONS OF ELECTRONS WITH MATTER

### *.2.4.4 Stopping power in Compounds and Mixtures*

The mass stopping power can be approximated for compounds or intimate mixtures by a weighed addition of the mass stopping power of the elemental constituents assuming independent contribution to stopping power:

$$\left( \frac{S}{\rho} \right) = \sum_i \left( \frac{S}{\rho} \right)_i w_i$$

$w_i$  are the normalized weight fractions of the elements  $i$  present in the material. The influence of chemical binding is neglected here

## .2.4 INTERACTIONS OF ELECTRONS WITH MATTER

### .2.4.5 Linear Energy Transfer

The mass collision stopping power characterizes the energy loss of the electron due to all collisions. The secondary electrons resulting from hard collisions ( $\delta$ -rays) carry part of the energy some distance away from the track and may escape the volume of interest

In radiobiology, the use of the collision stopping power can result in an overestimation of dose. In such situations, the restricted stopping power may be used which relates to the energy lost in the absorber by secondary particles not exceeding an energy limit  $\Delta$  (typically 10 keV), thus limiting the volume of interest to the range of electrons with energy  $\Delta$

The restricted stopping power may be expressed as mass restricted stopping power  $(S/\rho)_{Res}$  or, for a description of the energy passed to the medium per unit track length, it is common to use the linear energy transfer  $L: \Delta$

Linear energy transfer  $L_{\Delta}$  (keV/ $\mu\text{m}$ ):

$$L_{\Delta} = \frac{\rho}{10} \left( \frac{1}{\rho} \frac{dT}{dx} \right)_{\Delta}$$



## **.2.5 DATA SOURCES**

**Data for photon attenuation coefficients and stopping powers can be found in many textbooks but are conveniently obtained from web based data sources provided by:**

- **NIST (Berger, Hubbell et al., 2005; Berger, Coursey, et al, (2005**

**or from the tabulations in:**

- **ICRU ((1992**

**The composition of body tissues and phantom materials are given in:**

- **ICRU (1989) and**
- **ICRP ((2003**

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