Chapter 2
Linear Waveshaping: High-pass Circuits

1. A ramp shown in Fig.2p.1 is applied to a high-pass *RC* circuit. Draw to scale the output waveform for the cases: (i) $T = RC$, (ii) $T = 0.2RC$, (iii) $T = 5RC$.

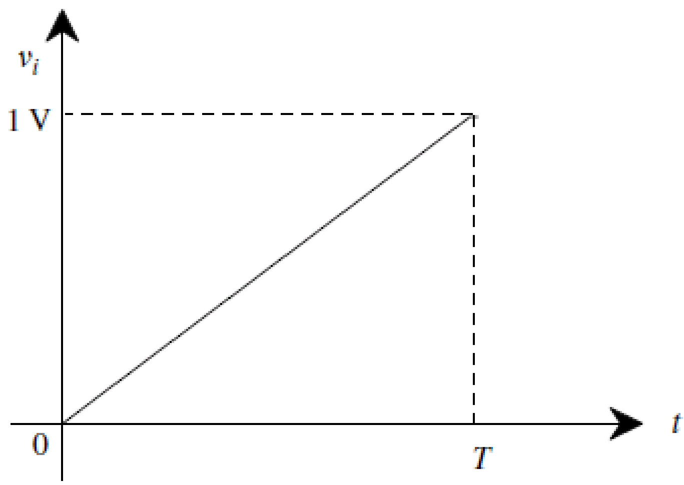

Fig.2p.1 A ramp as input

Solution:
From Eq. (2.64):

$$v_o = \alpha\tau\left(1 - e^{-t/\tau}\right)$$

$$\alpha = \frac{V}{T}$$

$$v_o = V\left(\frac{\tau}{T}\right)\left(1 - e^{-t/\tau}\right)$$

The peak of the output will occur at $t = T$. We know:

$$v_{o(peak)} = V\left(\frac{\tau}{T}\right)\left(1 - e^{-T/\tau}\right)$$

(i) When $T = \tau$:

$\left(\frac{\tau}{T}\right) = 1$

$v_{o\,(peak)} = (1)\left(1 - e^{-1}\right) = 0.632$ V.

(ii) When $T = 0.2\,\tau$:

$\left(\frac{T}{\tau}\right) = 0.2, \quad \left(\frac{\tau}{T}\right) = 5$

$v_{o\,(peak)} = (5)\left(1 - e^{-0.2}\right) = 0.907$ V

(iii) When $T = 5\,\tau$:

$$\left(\frac{T}{\tau}\right) = 5 \qquad \left(\frac{\tau}{T}\right) = 0.2$$

$v_{o\,(peak)} = (0.2)\ \left(1 - e^{-5}\right) = 0.198$ V.

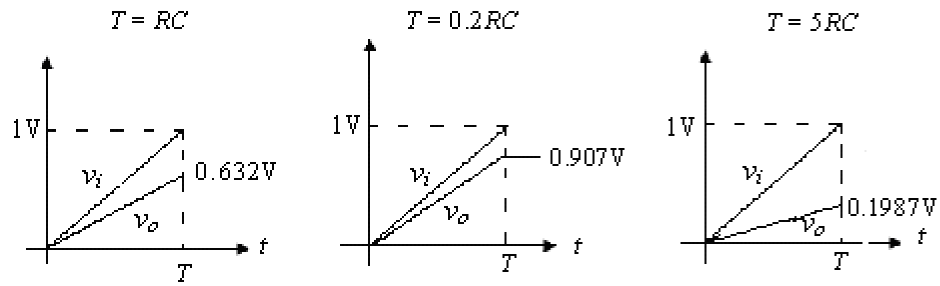

Fig.1 Response of the high-pass circuit for ramp input

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2. A waveform shown in Fig.2p.2 is applied as input to an *RC* high-pass circuit whose time constant is 250 ps. If the maximum output voltage across the resistor is 50 V, what is the peak value of the input waveform?

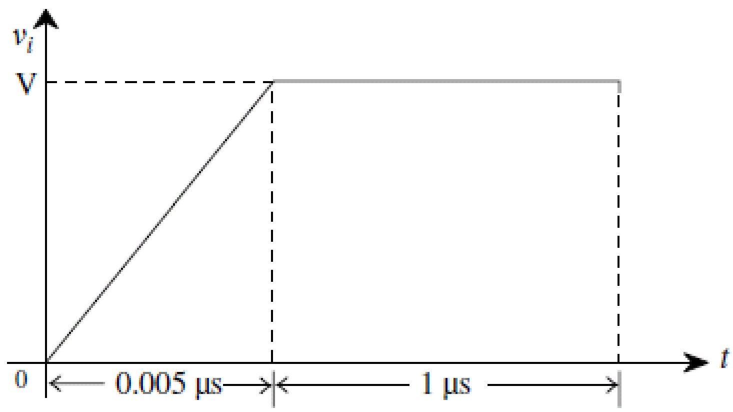

Fig.2p.2 Input to the high-pass *RC* circuit

Solution:

For a ramp input

$$v_o(t) = \alpha RC(1 - e^{-t/RC})$$

At $t = t_1 = 0.005 \times 10^{-6} s$

$$50 = \frac{v_i}{0.005 \times 10^{-6}} \times 250 \times 10^{-12} \left(1 - e^{\frac{-0.005 \times 10^{-6}}{250 \times 10^{-12}}}\right)$$

$$v_i = \frac{1}{50 \times 0.005 \times 10^{-6}} \times 250 \times 10^{-12} \left(1 - e^{\frac{-0.005 \times 10^{-6}}{250 \times 10^{-12}}}\right)$$

$$v_i(\max) \cong 1000\,\text{V}$$

For $t > t_1$

$$v_o = 50 \times e^{-t/250\times10^{-12}}$$

$$v_o = 50 \times e^{\frac{-1\times10^{-6}}{250\times10^{-12}}}$$

$$v_o = 0 \text{ V}$$

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3. A limited ramp shown in Fig.2p.3 is applied to an *RC* high-pass circuit of Fig.2.2 (a). The time constant of the *RC* circuit is 2 ms. Calculate the maximum value of output voltage and the output at the end of the input waveform.

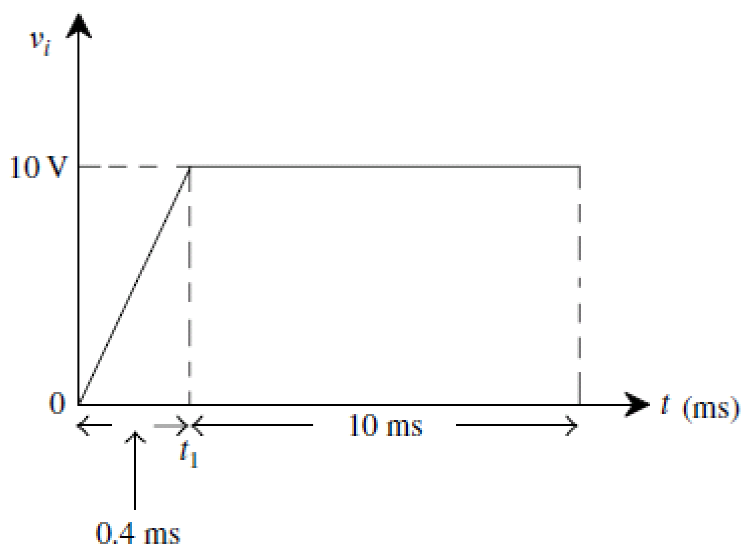

Fig.2p.3 Input to the high-pass circuit

Solution:

For a ramp input

$$v_o(t) = \alpha RC(1 - e^{-t/RC})$$

At $t = t_1 = 0.4\times10^{-3}$ s

$$v(t) = \frac{10}{0.4\times10^{-3}} \times 2\times10^{-3}(1 - e^{\frac{-0.4\times10^{-3}}{2\times10^{-3}}})$$

The peak value occurs only at $t = t_1 = 0.4\times10^{-3}$ s

$$\therefore v_o(\text{max}) = 9.063 \text{ V}$$

For $t > t_1$

$$v_o = 9.063 \times e^{-t/2\times10^{-3}}$$

The voltage at t = 10.4 ms is:

$$v_o = 9.063 \times e^{-10\times10^{-3}/2\times10^{-3}}$$

$$\therefore\ v_o = 0.061\,\text{V}$$

The voltage at t = 10.4 ms is 0.061 V. The output waveform is shown in Fig.3.

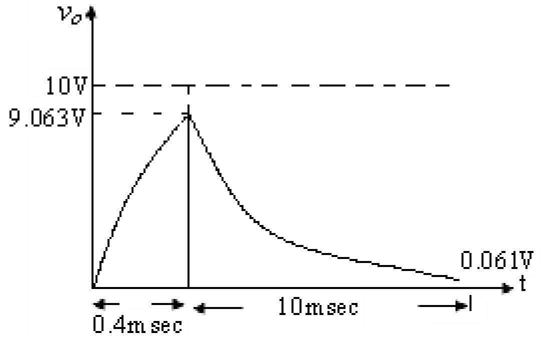

Fig. 3 Output of the high-pass circuit for the given input

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4. The periodic waveform shown in Fig.2p.4 is applied to an *RC* differentiating circuit whose time constant is 10 μs. Sketch the output and calculate the maximum and minimum values of the output voltage with respect to the ground.

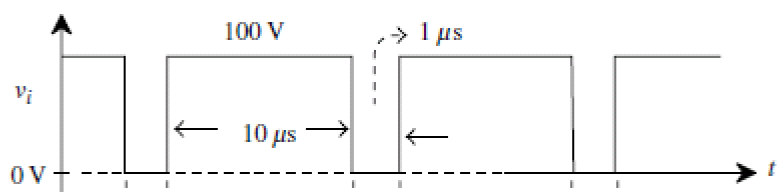

Fig.2p.4 Periodic square wave as an input to the high-pass circuit

Solution:

Given T_1 =100 μs , T_2=1 μs , τ = 10 μs

The steady-state output waveform is drawn by calculating $V_1, V_1^{'}, V_2$ and $V_2^{'}$.

At $t = 0^-, v_o = V_2^{'}$ and at $t = 0^+, v_o = V_1$

For $0 < t < T_1, \quad v_o = V_1 e^{\frac{-t}{\tau}}$

At $t = T_1$

For

At

$$\text{At} \quad t = T_1, \quad v_o = V_1' = V_1 e^{\frac{-T_1}{\tau}} = V_1 e^{\frac{-100}{10}} = 0$$

$$\text{For} \quad T_1 < t < (T_1 + T_2), \quad v_o = V_2 e^{\frac{-T_2}{\tau}}$$

$$\text{At} \quad t = T_2, \quad v_o = V_2' = V_2 e^{\frac{-T_2}{\tau}} = V_2 e^{\frac{-1}{10}} = 0.904\ V_2$$

Peak-to-peak input is 100 V.

$V_1' - V_2 = 100$

$0 - V_2 = 100$

$V_2 = -100$ V

$V_1 - V_2' = 100$

$V_1 + 0.904 \times 100 = 100$

$V_1 = 100 - 0.904 \times 100 = 9.6$ V

$V_1' = V_1 e^{\frac{-T_1}{\tau}} = 9.6 e^{\frac{-100}{10}} = 0$

$V_2' = V_2 e^{\frac{-T_2}{\tau}} = V_2 e^{\frac{-1}{10}} = 0.904\ V_2 = 0.904 \times -100 = -90.4$ V

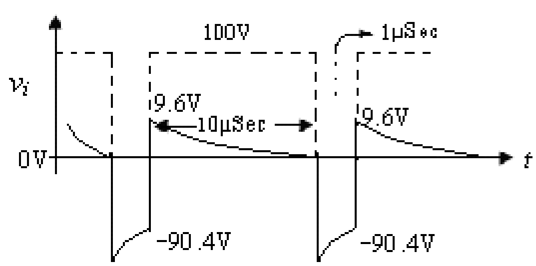

Fig. 4 Output of the high-pass circuit for the specified input

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5. The periodic ramp voltage as shown in Fig.2p.5 is applied to a high-pass *RC* circuit. Find equations from which to determine the steady-state output waveform when $T_1 = T_2 = RC$.

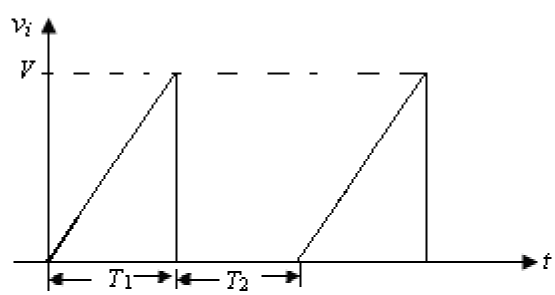

Fig.5 A periodic ramp as input

Solution:

$$v_o = \alpha \tau \left(1 - e^{-t/\tau}\right) \qquad \textbf{(1)}$$

If there is an initial voltage of V_1 on C, Eq.(1) gets modified as follows:

$$v_o = \alpha\tau\left(1-e^{-t/\tau}\right)+V_1e^{-t/\tau}$$

For the ramp input, the slope $\alpha = \dfrac{V}{T_1}$.

The capacitor charges from V_1 to V_2 in time T_1. At $t = T_1+$, the capacitor does not respond for sudden changes. Hence, the output changes to $(V_2 - V)$. During T_2, the capacitor blocks the dc. So the capacitor discharges from V_3 to V_1.

Given $T_1 = T_2 = \tau$

At $t = T_1$ $v_o(t) = V_2$

Using (1)

$$V_2 = \frac{V}{T_1}\times T_1\ (1-e^{-1})+V_1e^{-1} = 0.632\,V + 0.367\,V_1 \qquad \textbf{(2)}$$

$v_o(T_1+) = V_2 - V$

$$V_1 = (V - V)e^{-T_2/r} = (0.632V + 0.367V_1 - 1)e^{-1}$$

$$V_1 = (1-0.134) = -0.135$$

$$V_1 = \frac{-0.135}{0.864} = -0.156V$$

$$V_2 = = 0.632V + 0.367V_1 = 0.632V + 0.367\times -0.156V = 0.575V$$

$v_o(T_1+) = V_2 - V = 0.575V - V = -0.425V$

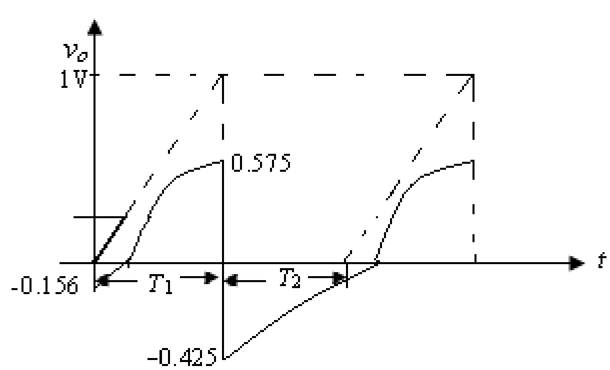

Fig. 5 The changes in voltage with time

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6. A square wave of pulse width 2 ms and peak amplitude of 12 V as shown in Fig.2p.6 is applied to high-pass *RC* circuit with time constant 4 ms. Plot the first four cycles of the output waveform.

$T/2$ = 2 ms

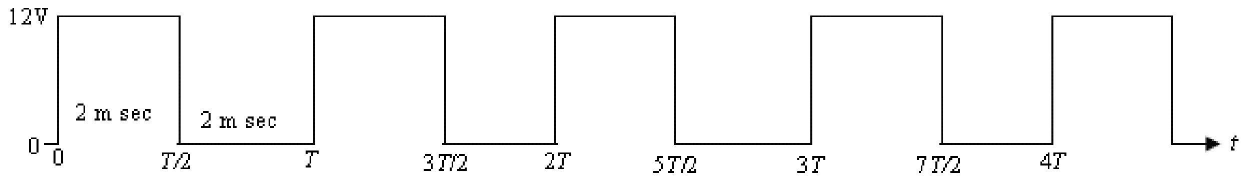

Fig.2p.6 Symmetric square wave as an input

Solution:

Given $T_1 = T_2 = 0.2$ ms, $\tau = 4$ ms

(i) For $t < 0$, $v_i = 0$, and hence $v_o = 0$

At $t = 0$, v_i jumps to 12 V

As the voltage across capacitor cannot change instantaneously, v_o is also equal to 12 V.

At $t = 0$, $v_o = V_a = 12$ V.

(ii) During the period $0 < t < 2.0$ ms, as the input is constant the output decays.

$$v_o = V_a e^{\frac{-t}{\tau}}$$

At $t = 2.0$ ms, $v_o = V_b = V_a e^{\frac{-t}{\tau}} = 12 e^{\frac{-2}{4}} = 7.27$ V

At $t = 2$ ms, the input falls by 12 V. The output also falls by 12 V.

$V_c = V_b - 12 = 7.27 - 12 = -4.73$ V

(iii) For $2.0 < t < 4.0$ $v_o = V_c\, e^{\frac{-(t-T/2)}{\tau}}$

$\therefore$ At $t = T$=4 ms, $v_o = V_d = V_c e^{\frac{-2}{4}} = -4.73e^{-0.5} = -2.86$ V

At $t = 4$ ms, the input rises by 12 V. The output also rises by 12 V.

$V_e = V_d + 100 = -2.86 + 12 = 9.14$ V

(iv) During the period $T < t < 3T/2$, that is between 4 to 6 ms, the output decays.

At $t = 6$ ms $v_o = V_f = V_e e^{\frac{-2}{4}} = 9.14(0.606) = 5.53$ V

At 6 ms, the input falls by 12 V.

Hence $V_g = V_f - 12 = -6.47$ V

(v) During $3T/2 < t < 2T$, that is, during 6 to 8 ms, the output decays.

At $t = 2T$=8 ms, $v_o = V_h = V_g e^{\frac{-2}{4}} = -6.47\, e^{-0.5} = -3.92$ V.

$V_j = V_h + 100$ V $= -3.92 + 12 = 8.08$ V.

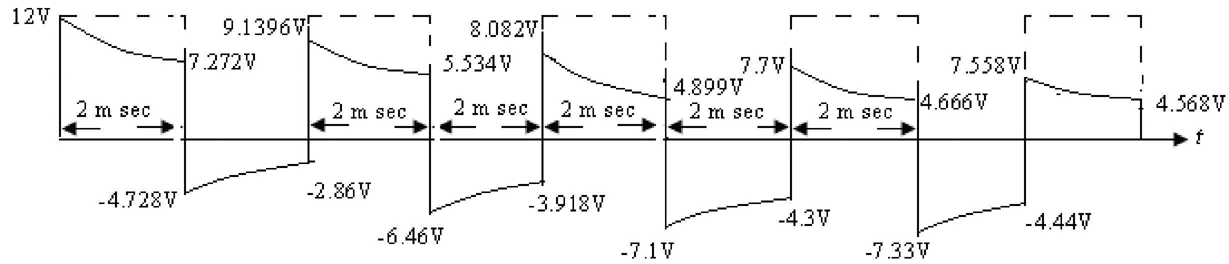

Fig.6 The output waveform

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7. A 20-Hz symmetric square wave referenced to 0 volts and, with a peak-to-peak amplitude of 10 V, is fed to an amplifier through the coupling network shown in Fig. 2p.7. Calculate and plot the output waveform when the lower 3-dB frequency is: (i) 0.6 Hz, (ii) 6 Hz and (iii) 60 Hz.

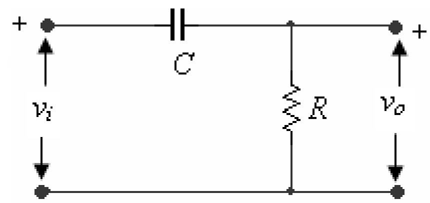

Fig.2p.7 The given coupling network

Solution:

Given $V = 10$ V

(i) $f_1 = 0.6$ Hz

$$\tau = RC = \frac{1}{2\pi f_1} = \frac{1}{2\pi(0.6)} = 0.265 \text{ s}$$

$$T = \frac{1}{f} = \frac{1}{20} = 0.05 \text{ s.}$$

$$\therefore \frac{T}{2} = 0.025 \text{ s.}$$

$$V_1 = \frac{V}{1+e^{-T/2\tau}} = \frac{10}{1+e^{-0.025/0.265}} = 5.25 \text{ V.}$$

$$V_1' = V_1 e^{\frac{-T}{2\tau}} = 5.25e^{\frac{-0.025}{0.265}} = 5.25(0.91) = 4.8 \text{ V.}$$

$\therefore \quad V_1 = -V_2$ and $V_1^{'} = -V_2^{'}$

$V_1 = |V_2| = 5.25$ V

$V_1^{'} = |V_2^{'}| = 4.8$ V

The output in this case is plotted in Fig.7.1.

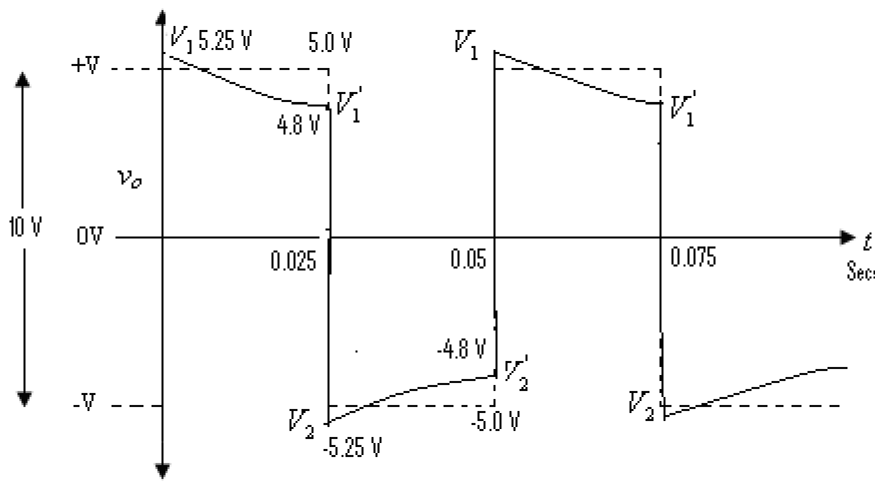

Fig.7.1Output when f_1 = 0.6 Hz

(ii) f_1 = 6 Hz

$$\tau = RC = \frac{1}{2\pi f_1} = \frac{1}{2\pi(6)} = 0.0265 \text{ s}$$

$$V_1 = \frac{V}{1+e^{-T/2\tau}} = \frac{10}{1+e^{-0.025/0.0265}} = 7.20 \text{ V}$$

$$V_1^{'} = V_1 e^{\frac{-T}{2\tau}} = 7.20 e^{\frac{-0.025}{0.0265}} = 7.20(0.389) = 2.8 \text{ V.}$$

$\therefore \quad V_1 = -V_2$ and $V_1^{'} = -V_2^{'}$

$V_1 = |V_2|$ =7.20 V

$V_1^{'} = |V_2^{'}| = 2.8$ V

The output for this condition is plotted in Fig.7.2.

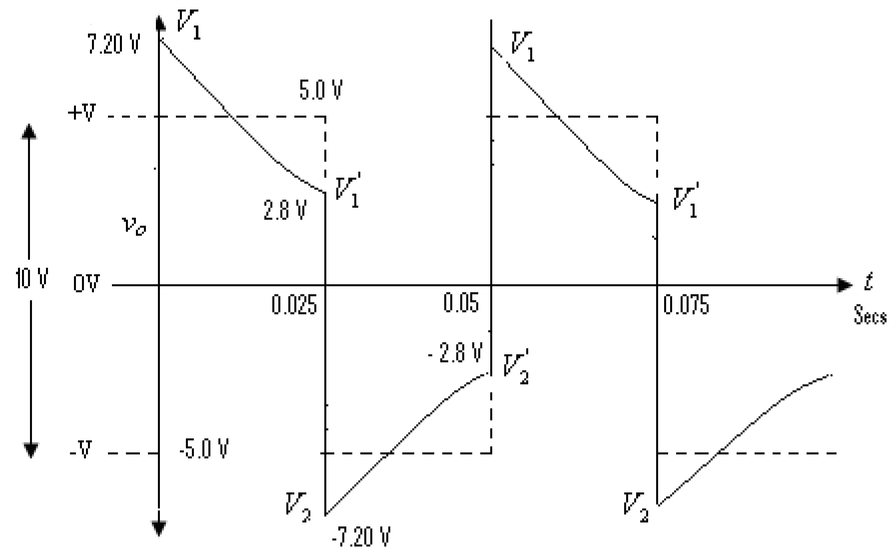

Fig.7.2 Output when f_1 = 6 Hz

(iii) f_1 = 60 Hz

$$\tau = RC = \frac{1}{2\pi f_1} = \frac{1}{2\pi(60)} = 0.00265 \text{ s}$$

$$V_1 = \frac{V}{1+e^{-T/2\tau}} = \frac{2}{1+e^{-0.025/0.00265}} = 10.0 \text{ V}$$

$$V_1' = V_1 e^{\frac{-T}{2\tau}} = 10(0.00008) = 0.0008 \text{ V.}$$

$$\therefore \quad V_1 = -V_2 \text{ and } V_1' = -V_2'$$

$V_1 = |V_2| = 10.00$ V

$V_1' = |V_2'| = 0.0008$ V

The output for this case is plotted in Fig.7.3..

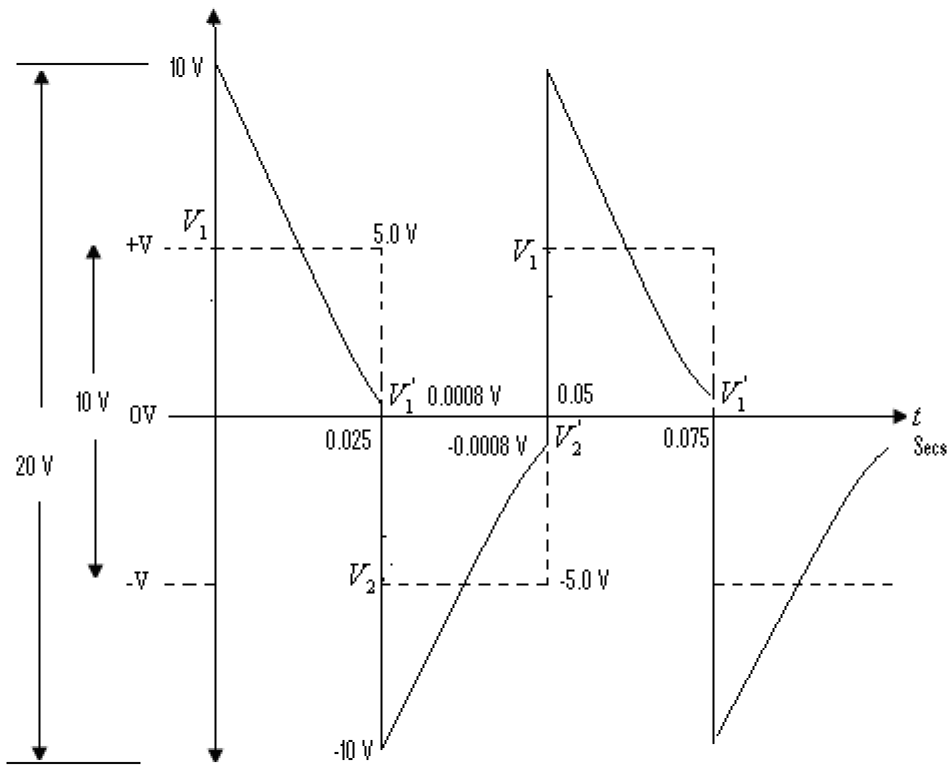

Fig.7.3 Output when f_1 = 60 Hz

8. A square wave is applied as input to an amplifier through a coupling condenser of $10\ \mu\text{F}$. The amplifier has input resistance of $10\,\text{k}\Omega$. Determine the lowest frequency if the tilt is not to exceed 10 per cent.

Solution:

We have $P = 0.1$, $R = 10\ \text{k}\Omega$ and $C = 10\ \mu\text{F}$

Per cent tilt, $P = \dfrac{T}{2\tau} \times 100$ per cent

$$f = \frac{1}{2\tau P} = \frac{1}{2\times 10\times 10^{3}\times 10\times 10^{-6}\times 0.1} = 50\ \text{Hz}$$

$$P = \frac{\pi f_1}{f}\times 100 \text{ per cent}$$

$$f_1 = \frac{Pf}{\pi} = \frac{0.1\times 50}{\pi} = 1.59\ \text{Hz}$$

9. A pulse of 10 V amplitude and duration 1 ms is applied to a high-pass RC circuit with $R = 20\,\text{k}\Omega$ and $C = 0.5\ \mu\text{F}$. Plot the output waveform to scale and calculate the per cent tilt in the output.

Solution:

$\tau = RC = 10$ ms

For $0 < t < t_p$

$v_i = 10$ V

$v_o = 10\ e^{-t/10\times10^{-3}}$

At $t = t_{p-}$, $v_o = V_1^{'} = 10e^{-1\times10^{-3}/10\times10^{-3}} = 9.05$ V

At $t = t_{p+}$, $v_o = V_2 = V_1^{'} - V = 9.05 - 10 = -0.95$ V

For $t > t_p$, $v_o = -0.95\ e^{-(t-1\times10^{-3})/10\times10^{-3}}$

$$\therefore \text{per cent tilt} = \frac{V - V_1^{'}}{V}\times100 = \frac{10-9.05}{10}\times100 = 9.5 \text{ per cent}$$

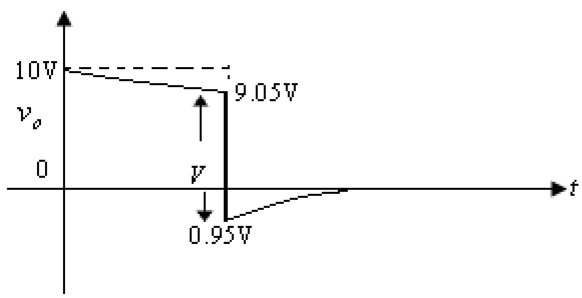

Fig. 9 The output waveform

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10. The input to the high-pass circuit in Fig. 2p.10 is the waveform shown in Fig. 2p.10. Calculate and plot the output waveform to scale, given that $RC = \tau = 0.1$ ms .

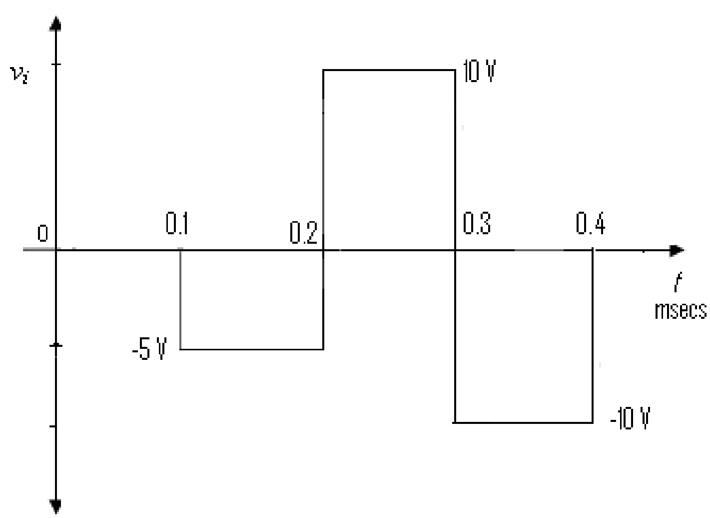

Fig.2p.10 Input to the high-pass circuit

Solution:

For $t < 0.1$ ms $v_i = 0$, $v_o = 0$

(i) At $t = 0.1$ ms, the input suddenly falls to –5 V, and the output also changes by the same amount as the capacitor acts as a short circuit.
For $0.1 < t < 0.2$, v_i remains constant at –5 V. Therefore, v_o decays exponentially with the time constant 0.1 ms.

(ii) At $t = 0.2$ ms,

$$v_o = -5e^{\frac{-0.1\times10^{-3}}{0.1\times10^{-3}}} = -1.839\text{ V}$$

At $t = 0.2$ ms, the input suddenly rises by 15 V, v_o also rises by the same amount.

v_o ($t = 0.2$ ms) = –1.839 + 15 = 13.16 V

For 0.2 ms < t < 0.3 ms, v_i remains at 10 V. Hence v_o decays exponentially with the time constant 0.1 ms

(iii) At $t = 0.3$ ms

$$v_o = 13.16e^{-\frac{0.1\times10^{-3}}{0.1\times10^{-3}}} = 4.84\text{ V}$$

At $t = 0.3$ ms, input suddenly falls by 20 V. The output also changes by the same amount.

v_o ($t = 0.3$ ms) = 4.84 – 20 = –15.16 V

For 0.3 ms < t < 0.4 ms, v_i remains constant at –10 V. Hence, v_o will decay exponentially with the time constant 0.1 ms.

(iv) At $t = 0.4$ ms,

$$v_o = -15.16e^{-\frac{0.1\times10^{-3}}{0.1\times10^{-3}}} = -5.58\text{ V V}$$

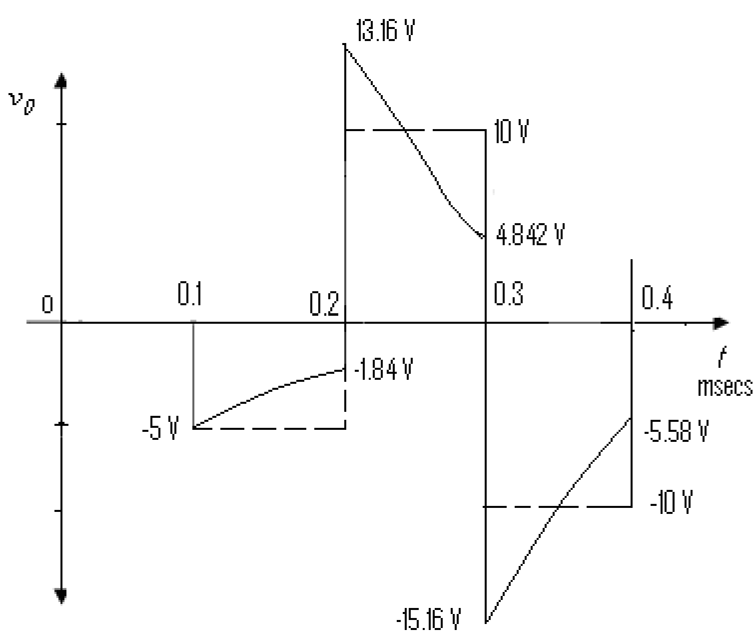

Fig.10 The output waveform

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11. A pulse of 10-V amplitude with a pulse width of 0.5 ms, as shown in Fig.2p.9, is applied to a high-pass *RC* circuit of Fig. 2.1(a), having time constant 10 ms. Sketch the output waveform and determine the per cent tilt in the output.

Solution:

$\tau = 10\ ms$

For, 0< $t < t_p$

v_i =10 V

v_o=10 $e^{-t/10\times10^{-3}}$

At $t = t_{p-}$, $v_o = V_1' = 10e^{-0.5\times10^{-3}/10\times10^{-3}} = 9.512$ V

At $t = t_{p+}$, $v_o = V_2 = V_1' - V = 9.512 - 10 = -0.488V$

For $t > t_p$, $v_o = -0.488\ e^{-(t-0.5\times10^{-3})/10\times10^{-3}}$

$\therefore$ per cent tilt $= \frac{V - V_1'}{V}\times100 = \frac{10-9.512}{10}\times100 = 4.88$ per cent

The output is also shown in Fig. 2p.9.

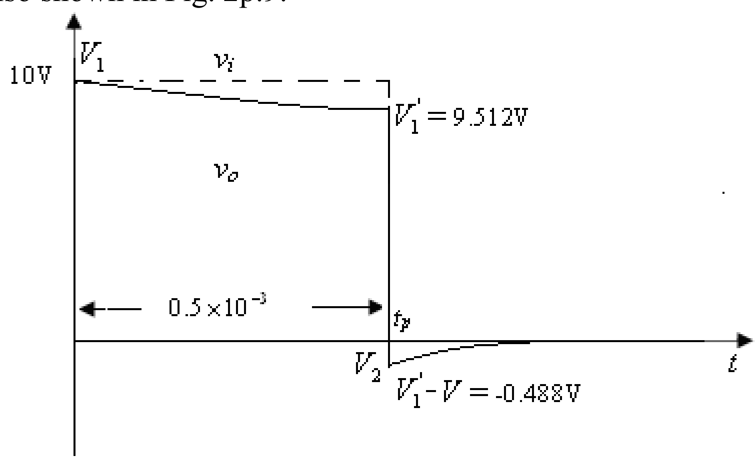

Fig.2p.9 Input and output of the high-pass circuit

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12. A high-pass *RC* circuit is desired to pass a 3-ms sweep (ramp input) with less than 0.4 per cent transmission error. Calculate the highest possible value of the lower 3-dB frequency.

Solution:

Consider the circuit in Fig. 2.1(a).

$T = 3\times10^{-3}$ s

per cent $e_{t(\max)}$ =0.4 per cent or $e_{t(\max)} = 0.004$

$$e_t = \frac{T}{2\tau} = \pi f_1 T$$

$$0.004 = \pi f_1 \times 3 \times 10^{-3}$$

$$\therefore f_1 = \frac{0.004}{\pi \times 3 \times 10^{-3}} = 0.4244 \text{ Hz}$$

13. A symmetric square wave with f = 500 kHz shown in Fig.2p.13 is fed to an RC high-pass network of Fig.2.1(a). Calculate and plot the transient and the steady-state response if: (i) $\tau = 5T$ and (ii) $\tau = T/20$.

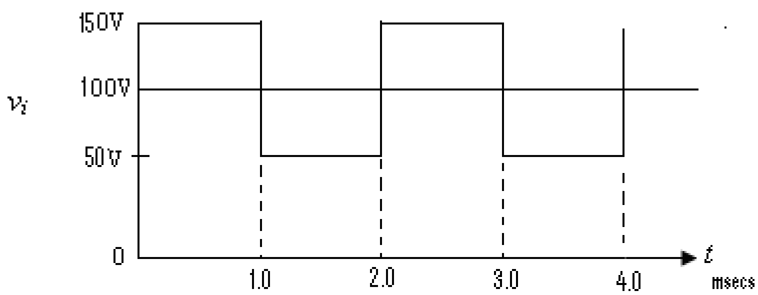

Fig.2p.13 Input to the coupling network

Solution:
Given f = 500 Hz, hence T = 2.0 ms.
Case 1:
Given**,** $\tau = 5T$ =10 ms.
When τ is large, the capacitor charges and discharges very slowly. The output has a small tilt. The voltages are calculated to plot the transient response.
i. For $t < 0$, v_i = 0, and hence v_o = 0
At t = 0, v_i jumps to 150 V.
As the voltage across capacitor cannot change instantaneously, v_o is also equal to 150 V.
At t = 0 $v_o = V_a$ = 150 V.
ii**.** During the period 0 < t < 1.0 ms, as the input is constant the output decays.

$$v_o = V_a e^{\frac{-t}{\tau}}$$

At t = 1.0 ms, $v_o = V_b = V_a e^{\frac{-t}{\tau}} = 150 e^{\frac{-1}{10}} = 135.72$ V.
At t = 1.0 ms, the input falls by 100 V. The output also falls by 100 V.

$$V_c = V_b - 100 = 135.72 - 100 = 35.72 \text{ V.}$$

iii. For 1.0 < t < 2.0, $v_o = V_c\, e^{\frac{-(t-T/2)}{\tau}}$

$\therefore$ At $t = T$ = 2 ms, $v_o = V_d = V_c e^{\frac{-1.0}{10}} = 35.72 e^{-0.1} = 32.32$ V.
At t = 2 ms, the input rises by 100 V. The output also rises by 100 V.

$$V_e = V_d + 100 = 32.32 + 100 = 132.32 \text{ V.}$$

iv. During the period $T < t < 3T/2,$ that is, between 2 to 3 ms, the output decays.

At $t = 3$ ms $v_o = V_f = V_e e^{\frac{-1.0}{10}} = 132.32\ (0.9048) = 119.73$ V.

At 3 ms, the input falls by 100 V. Hence

$V_g = V_f - 100 = 19.73$ V

v. During $3T/2 < t < 2T$, that is, during 3 to 4 ms, the output decays.

At $t = 2T = 4$ ms, $v_o = V_h = V_g e^{\frac{-1.0}{10}} = 19.73\, e^{-0.1} = 17.85$ V.

$V_j = V_h + 100$ V = 17.85+100=117.85 V.

In a few cycles, the output reaches the steady state.

Steady-state response:

Under steady state, the output is symmetrical with respect to zero volts, since the capacitor blocks dc. Therefore, the dc component in the output is zero.

Let V_1 be the voltage at $t = 0$

For $0 < t < T/2$, $v_o = V_1 e^{\frac{-t}{\tau}}$

At $t = T/2 = 1$ ms, $v_o = V_1' = V_1 e^{-0.1} = 0.905\ V_1$

$V_1' = 0.905\ V_1$ **(3)**

As the input abruptly falls, output also falls by the same amount to V_2.

For $T/2 < t < T$ $v_o = V_2 e^{\frac{-(t-T/2)}{\tau}}$

At $t = T,$ $v_o = V_2' = V_2 e^{-0.1} = 0.905\ V_2$

$V_2' = 0.905\ V_2$ **(4)**

For symmetrical wave

$V_1' = -V_2'$ and $V_1 = -V_2$ **(5)**

$V_1' - V_2 = 100$ V and $V_1 - V_2' = 100$ V **(6)**

From (6), we have $V_1' - V_2 = 100$ V **(7)**

And from (3), we have $V_1 = -V_2$ **(8)**

Substituting (8) in (7), we have $V_1' + V_1 = 100$ V **(9)**

From (3), we have $V_1' = 0.905\ V_1$

Substituting in (9)

$0.905 V_1 + V_1 = 100$ V

$1.905 V_1 = 100$ V.

$V_1 = 52.49$ V and $V_1' = 0.905 V_1 = (0.905)(52.49) = 47.50$ V

From (5) as $V_1' = -V_2'$ and $V_1 = -V_2$

$V_2 = -52.49$ V $\quad$ $V_2' = -47.50$ V

We can now plot the steady-state response as we know

$\therefore\ V_1 = 52.49$ V $\quad$ $V_1' = 47.50$ V

$V_2 = -52.49$ V $\qquad V_2' = -47.50$ V

The transient and steady-state responses are plotted in Figs.13.1 and 13.2.

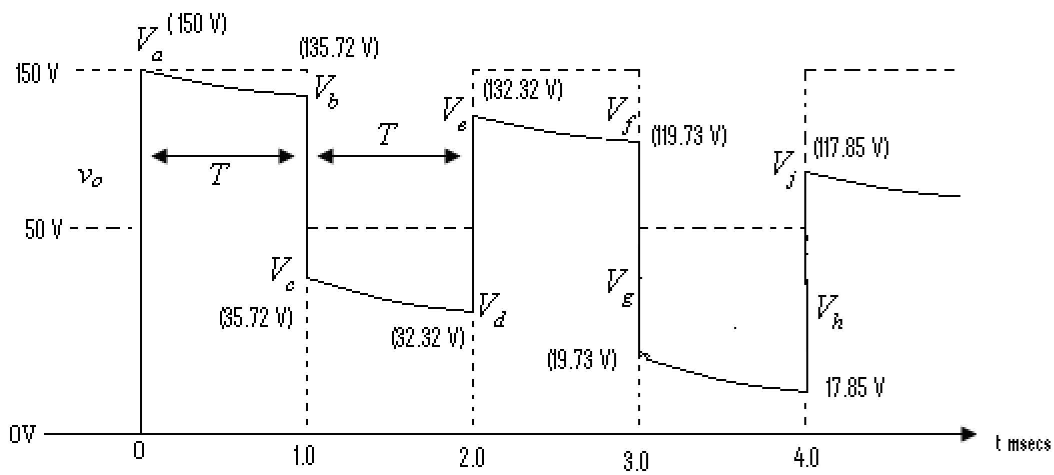

Fig.13.1 Transient response

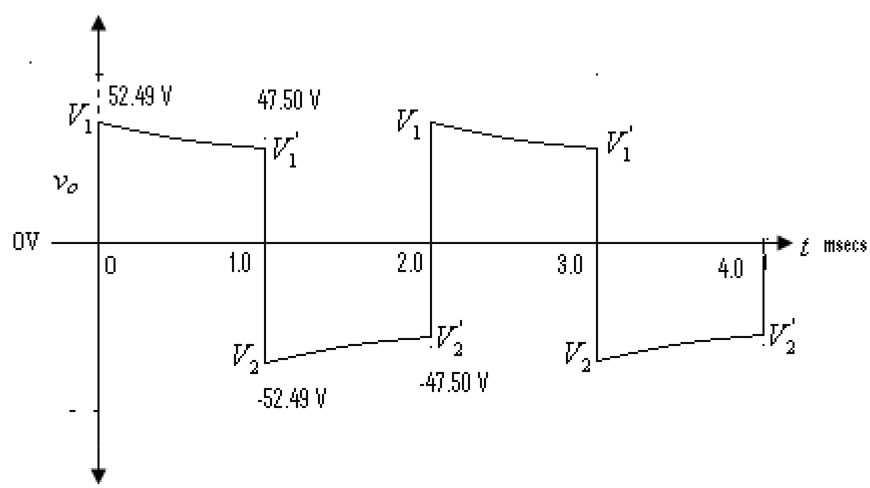

Fig.13.2 Steady-state response

Case 2:

For very low time constant, i.e. when $\tau = T/20 = 0.1$ ms.

Since the time constant is very small, the capacitor charges and discharges very fast.

$\therefore$ The input and output are shown in Fig.13.3.

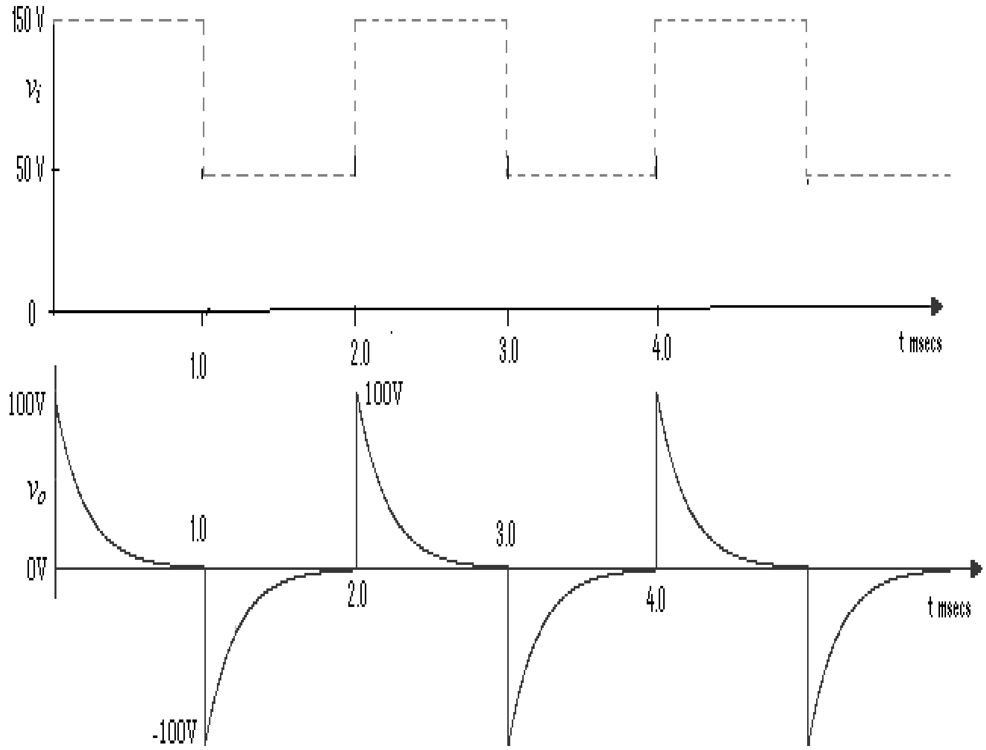

Fig.13.3 Output for the given input when time constant is very small

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14. A current pulse of amplitude 5 A in Fig.2p.11 is applied to a parallel *RC* combination shown in Fig.2p.12. Plot to scale the waveforms of the current flowing through capacitor for the cases: (i) t_p =0.1*RC*, (ii) t_p = *RC*, (iii) t_p = 5*RC*

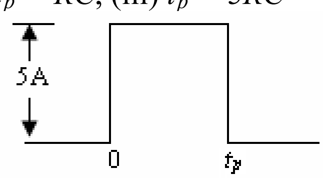

Fig.2p.11The given input to the circuit

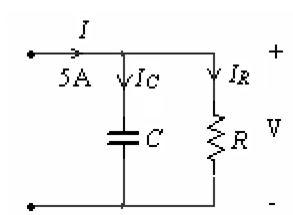

Fig. 2p.12 The given circuit

Solution:

Till $t = t_p$, using Laplace transforms, the circuit can be drawn as in Fig.14.1.

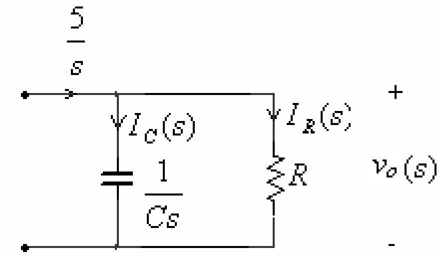

Fig.14.1 Circuit in terms of Laplace transforms

Applying KCL, we have

$$I_C(s) = R\frac{5/s}{R+\frac{1}{Cs}} = \frac{5\text{RCs}}{(\text{RCs}+1)s} = \frac{5}{\left(s+\frac{1}{RC}\right)}$$

Taking Laplace inverse, the charging current is

$$i_C(t) = 5e^{-t/RC}$$

At $t = t_p$, the current suddenly falls from 5 A to 0. The voltage across the capacitor at $t = t_p$ is $[5 - i_C(t_p)]R$

Therefore for $t \geq t_p$,

$$I_C(s) = -\frac{[5-i_C(t_p)]R}{s(R+\frac{1}{Cs})} = -\frac{[5-i_C(t_p)]}{(s+\frac{1}{RC})}$$

Taking Laplace inverse $i_C(t) = -[5 - i_C(t_p)]e^{\frac{-(t-t_p)}{RC}}$

The circuit that represents the discharge of the condenser is presented in Fig.14.2.

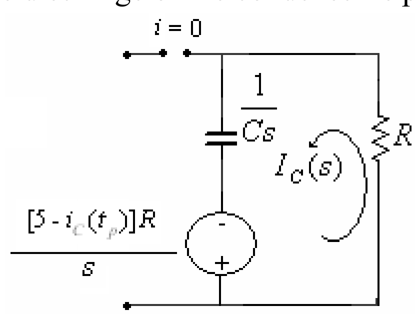

Fig.14.2 Circuit that indicates the discharge of the condenser

Case 1:

For $0 < t < t_p$

$$i_C(t) = 5e^{-t/RC}$$

i_C decays exponentially,

$$\text{at } t = t_p,\ i_C(t_p) = 5 \times e^{\frac{-0.1RC}{RC}} = 4.524 \text{ A}$$

For $t > t_p$, i_C rises exponentially as

$$i_C = -[5 - i_C(t_p)]\ e^{\frac{-(t-t_p)}{RC}} = -0.4758\ e^{-\frac{(t-t_p)}{RC}}$$

Case 2:

For $0 < t < t_p$

$$i_C(t) = 5e^{-t/RC}$$

i_C decays exponentially,

at $t = t_p$, $I_c(t_p) = 5e^{\frac{-RC}{RC}} = 5e^{-1} = 1.839$ A

For $t > t_p$ i_C decays exponentially as

$$-[5 - I_C(t_p)]\ e^{-\frac{(t-t_p)}{RC}} = -3.16\ e^{-\frac{(t-t_p)}{RC}}$$

Case 3:

For $0 < t < t_p$

i_C decays exponentially, $i_C(t) = 5e^{-t/RC}$

at $t = t_p$, $I_c(t_p) = 5 \times e^{\frac{-5RC}{RC}} = 5e^{-5} = 0.0336$ A

For $t > t_p$ i_C rises exponentially as

$$-[5 - i_C(t_p)]e^{-\frac{(t-t_p)}{RC}} = -4.966\ e^{-\frac{(t-t_p)}{RC}}$$

The input and outputs are plotted in Fig.14.3.

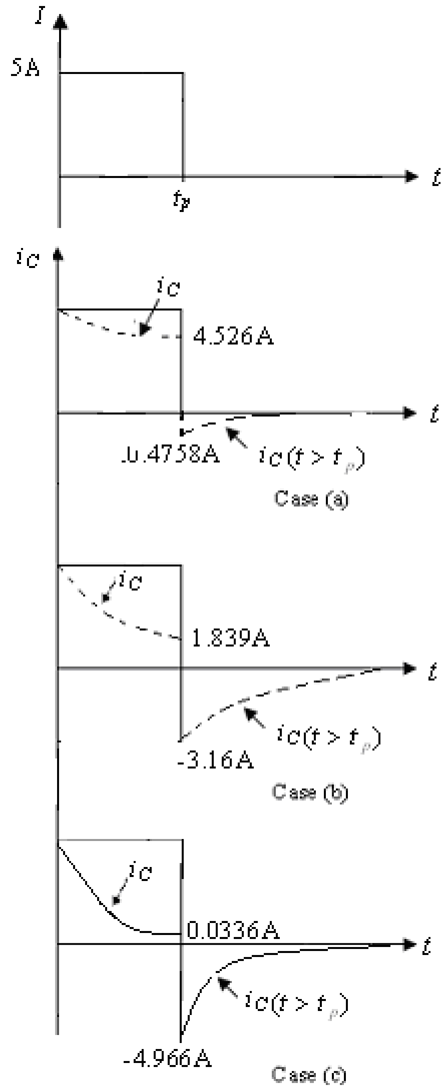

Fig.14.3 Input and outputs for the given circuit

15. Draw the output waveform if the waveform shown in Fig.2p.15(a) is applied at the input of the *RC* circuit shown in Fig.2p.15(b).

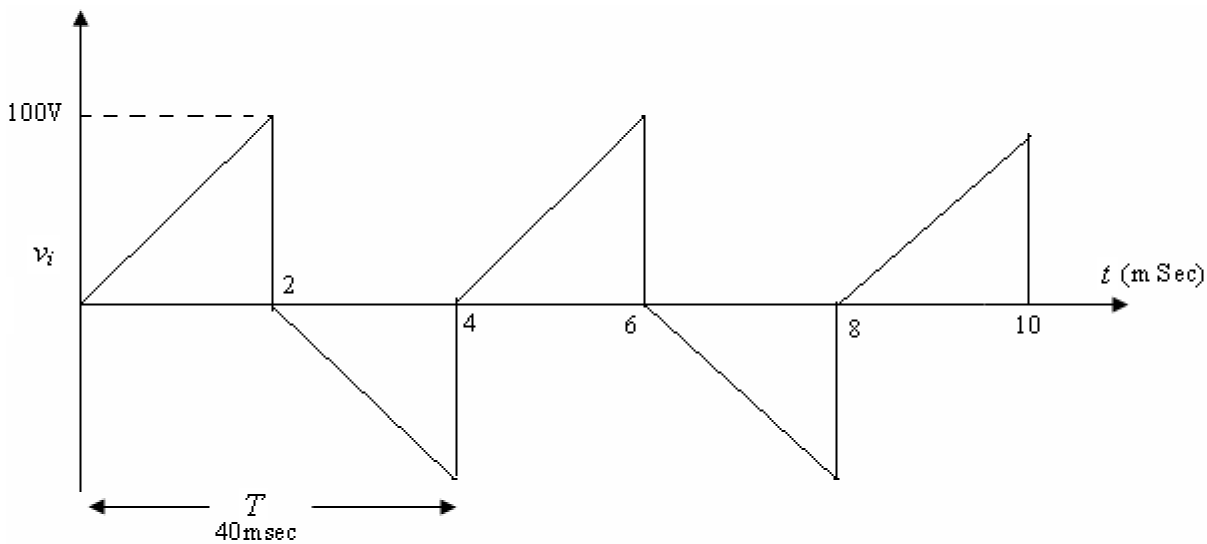

Fig.2p.15(a) The input to the high-pass circuit in Fig.2p.15(a)

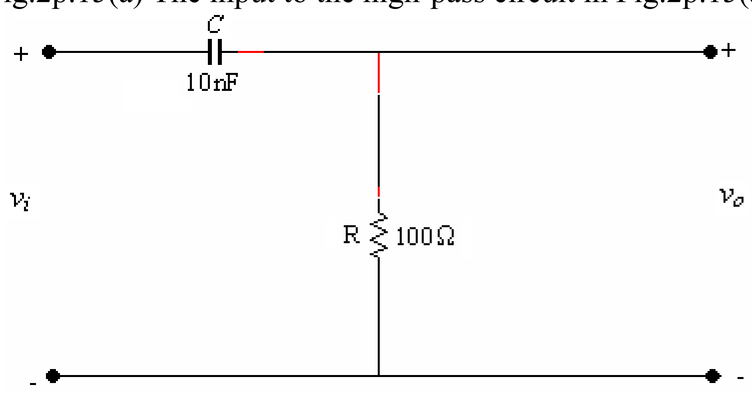

Fig.2p.15(b) The given high-pass circuit

Solution:

$$\text{Time constatnt} = RC = 100 \times 10 \times 10^{-9}$$

$$= 1000 \times 10^{-9} s$$

Time period of input waveform is

T = 4 ms

Since $RC << T$, the RC circuit acts as a good differentiator.

$\therefore$ The expression for output is $v_o = RC\dfrac{dv_i}{dt} = 1000 \times 10^{-9} \times \dfrac{dv_i}{dt}$

$$\text{For } 0 < t < 2 \text{ ms}, v_i = \frac{100}{2 \times 10^{-3}} t,$$

$$v_o = 1000 \times 10^{-9} \times \frac{d}{dt}\frac{100}{2 \times 10^{-3}} t = 1000 \times 10^{-9} \times \frac{100}{2 \times 10^{-3}} = 50 \text{ mV}$$

v_o remains at 50 mV. At t =2 ms, v_i falls by 100 V. Since capacitor acts as a short circuit, v_o also falls by the same amount.

$v_o(t = 2 \text{ ms}) = 0.05 - 100 = -99.95$ V

For $2 < t < 4$ ms, $v_i = \frac{100}{2\times10^{-3}}(t-2\times10^{-3})$

$$\therefore v_o = 1000\times10^{-9}\frac{d}{dt}\left(\frac{-100}{2\times10^{-3}}\right)(t-2\times10^{-3})$$

$$= 1000\times10^{-9}\times\frac{(-100)}{2\times10^{-3}} = -50 \text{ mV}$$

The output waveform is shown in Fig. 15..

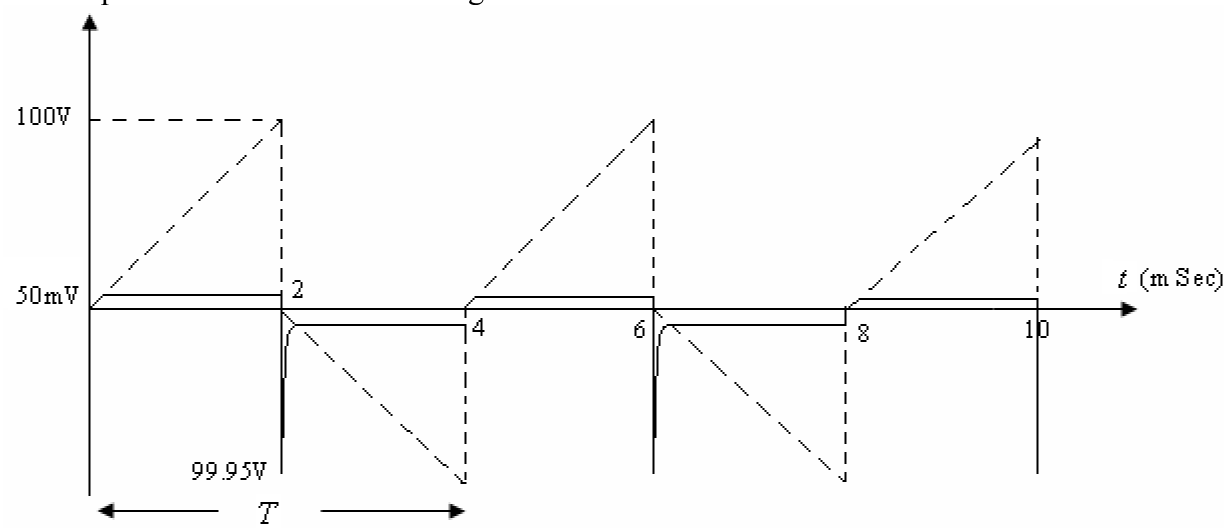

Fig.15 Output of the high-pass circuit
