Chapter 2 Parallel & Perpendicular Lines

Although the trend in many buildings today is to move away from the standard cube shaped building, most architectural structures begin from a cubic base. What is a cube? Yes, it is three dimensional and has space but as you pull it apart you see that in its simplest form, it is a combination of lines and planes that meet to form the shape. In order for the structure to be sound, architects maximize the benefits of the relationship of parallel and perpendicular lines. In this chapter, you will explore the different relationships formed by parallel and perpendicular lines and planes. Different angle relationships will also be explored and what happens to these angles when lines are parallel. There will also be a review of equations of lines and slopes and how we show algebraically that lines are parallel and perpendicular.

Concepts & Skills

The concepts and skills covered in this chapter include

- Identify pairs of lines and angles.
- Use parallel lines and transversals.
- Identify angles made by transversals.
- Use the Corresponding Angles Postulate.
- Use the Alternate Interior and Exterior Angles Theorem.
- Understand the properties of perpendicular lines to prove theorems about them.
- Explore problems with parallel lines and a perpendicular transversal.
- Solve problems involving complementary adjacent angles.

Section 2.1: Lines and Angles

INSTANT RECALL!

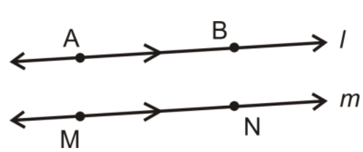
1. What is the equation of a line with slope -2 and passes through the point (0, 3)?
2. What is the equation of the line that passes through (3, 2) and (5, -6).
3. Change 4x-3y=12 into slope-intercept form.

2.1a Defining Parallel, Perpendicular, and Skew

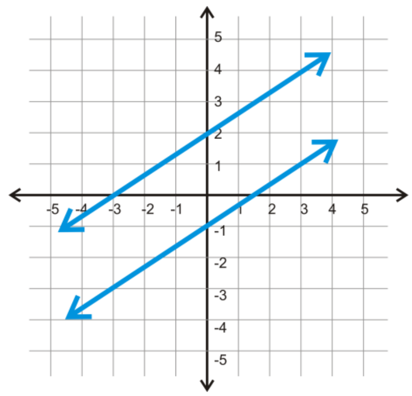
Lines are **parallel** when two or more lines lie in the same plane and never intersect. Parallel lines are also algebraically defined as two or more lines that have the same slope.

Important to note: The symbol for parallel lines is $||$.

To mark lines parallel, draw arrows $(>)$on each parallel line. If there are more than one pair of parallel lines, use two arrows $(>>)$for the second pair. The two lines below can be labeled as $\overleftrightarrow{AB} \,||\, \overleftrightarrow{MN}$ or $l \,||\, m$.



Algebraically, recall from Integrated Math 1 that since parallel lines never cross we determined that they have the same slope. This is seen on the Cartesian Plane below.



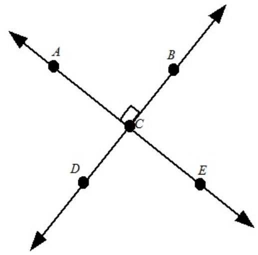
Can you determine the slope of each line? What methods can you use to do this?

YOU TRY!

1. Find the equation of the line that is parallel to $y = -\frac{1}{3}x + 4$ and passes through (9, -5).

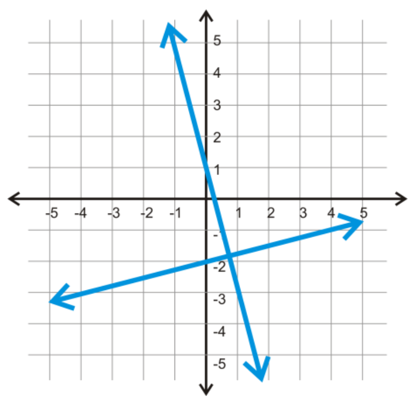
IMPORTANT TO NOTE: The equation of Parallel lines always have the <u>same slope</u> and <u>different $y-$intercepts</u>.

Lines are **perpendicular** when they intersect to form four (4) right angles.



IMPORTANT TO NOTE: The ⊥ (upside down T) symbol, denotes when lines are perpendicular to one-another. In the above diagram $AE \perp DB$

Algebraically, perpendicular lines have slopes that are the opposite reciprocal of one another. For example in the diagram below, the slope a line A is 4, the slope of the line reciprocal to line A has a slope of -1/4. Prove this with an appropriate method.



If given an equation of a line, you can determine the slope of lines that are both parallel and perpendicular to the line.

Example 1: If y= 2x -5 determine the slope of all lines parallel and perpendicular to this line.

The slope of the given line is 2. Therefore, the slope of <u>ALL lines parallel</u> to this line MUST have a slope of 2.

The slope of <u>ALL lines perpendicular</u> to the given line MUST have a slope that is -1/2 because this is the negative reciprocal of 2.

Your try!

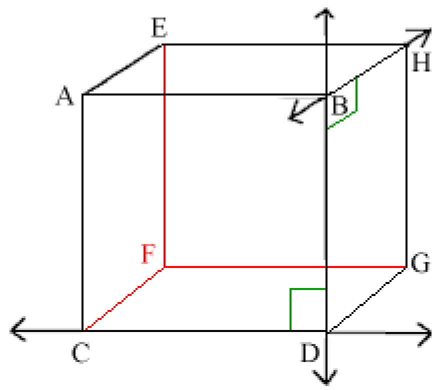
Determine the slope of all lines parallel and perpendicular to each of the following.

a) y= -3/4x – 6
b) m=5
c) A line has a slope of 7 and passes through the point (4,8)
d) y=8 + 11x
e) Find the equation of the line that is perpendicular to $y = -\frac{1}{3}x + 4$and passes through (9, -5).

Planes can also be parallel or perpendicular. The image below shows two parallel planes, with a third blue plane that is perpendicular to both of them.

An example of parallel planes could be the top of a table, the floor, or the composition of a building that we spoke of in the introduction. The legs would be in perpendicular planes to the table top and the floor would be perpendicular to the walls.

Skew lines are lines that are in different planes and never intersect**.**



In the diagram above, lines AB and HG lie in different planes and do not intersect. Can you identify other pairs of skew lines?

IMPORTANT TO NOTE: You will never encounter skew lines when working with polygons or any simple two dimensional shape!

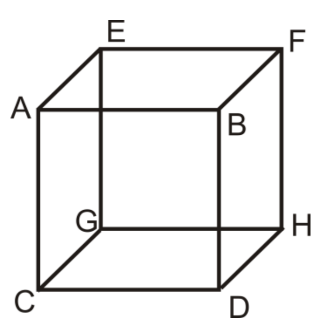
You try!

In the cube below, identify:

a) 3 pairs of parallel planes

b) 2 pairs of perpendicular planes

c) 3 pairs of skew line segments

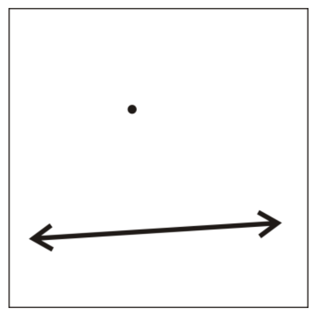


Parallel Line Postulate: For a line and a point not on the line, there is exactly one line parallel to this line through the point. There are infinitely many lines that pass through the point, but only one is parallel to the line.

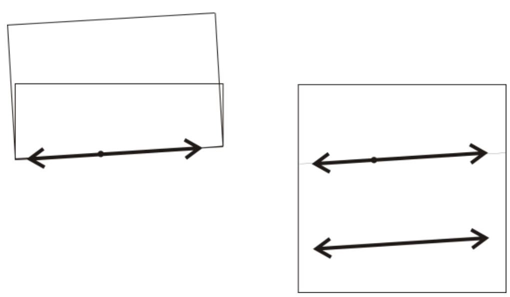
Investigation 2-1: Patty Paper and the Parallel Line Postulate

1. Get a piece of patty paper (a translucent square piece of paper).

Draw a line and a point above the line.



2. Fold up the paper so that the line is over the point. Crease the paper and unfold.



3. Are the lines parallel? Yes, by design, this investigation replicates the line we drew in #1 over the point. Therefore, there is only one line parallel through this point to this line.

Perpendicular Line Postulate: For a line and a point not on the line, there is exactly one line perpendicular to the line that passes through the point. There are infinitely many lines that pass through the point, but only one that is perpendicular to the line.

Investigation 2-2: Perpendicular Line Construction; through a Point NOT on the Line

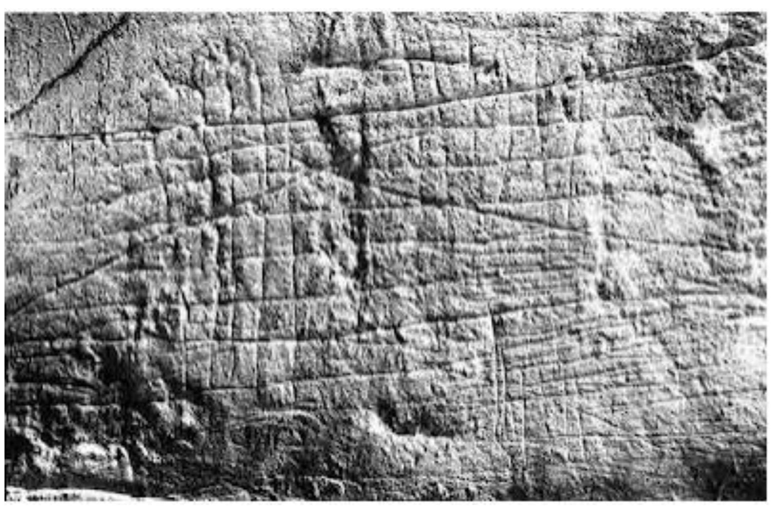
To see how this postulate works with the use of a straightedge and a compass visit the following video link.

http://www.mathsisfun.com/geometry/construct-perpnotline.html

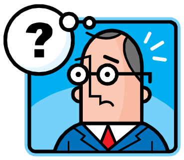
Investigation 2-3: Perpendicular Line Construction; through a Point on the Line

To see how this postulate works with the use of a straightedge and a compass visit the following video link.

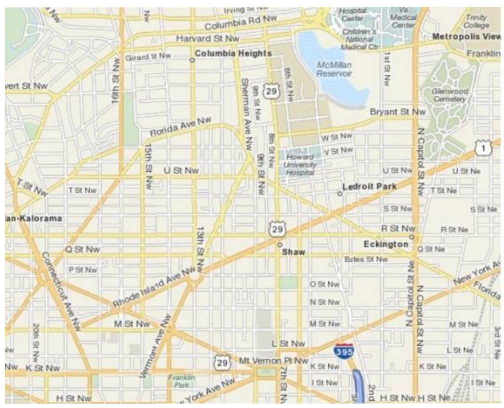
http://www.mathsisfun.com/geometry/construct-perponline.html



Even ancient peoples understood the concepts of parallel and perpendicular lines! The above picture is rock art from the Koonalda Cave located in Nullarbor Plain, South Australia. Located 60 meters underground, these carvings are dated to be 15,000-25,000 years old!



Know What? Below is a partial map of Washington DC. The streets are designed on a grid system, where lettered streets, A through Z run east to west and numbered streets 1st to 30th run north to south. Just to mix things up a little, every state has its own street that runs diagonally through the city. There are, of course other street names, but we will focus on these three groups for this chapter. Can you explain which streets are parallel and perpendicular? Are any skew? How do you know these streets are parallel or perpendicular?



If you are having trouble viewing this map, check out the interactive map here: **http://www.travelguide.tv/washington/map.html**

Internet & Video Links

For more information on the above topics visit the following:

1. **http://www.khanacademy.org/math/geometry/intro-to-euclidean-geo/v/identifying-parallel-and-perpendicular-lines**
2. **http://www.brightstorm.com/math/geometry/geometry-building-blocks/parallel-planes-and-lines/**
3. **http://www.brightstorm.com/math/geometry/geometry-building-blocks/parallel-and-skew-lines/**

2.1b – Properties of Parallel Lines: Transversals & Angles

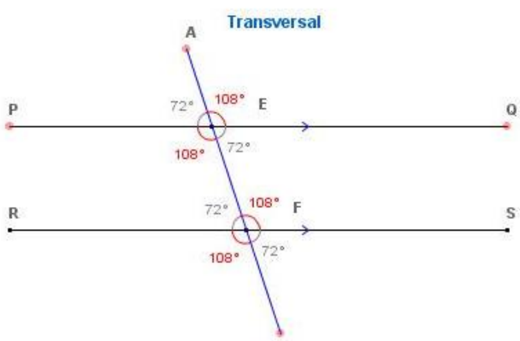
INSTANT RECALL!

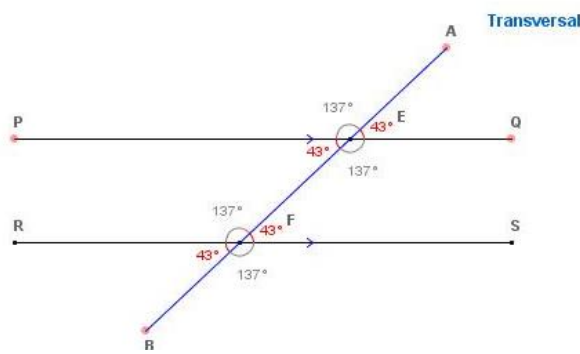
In integrated math 1 you learned about angle relations. Define the following angle relations you learned last year!

Linear Pair Vertical angles Supplementary angles

Complementary angles Congruent angles

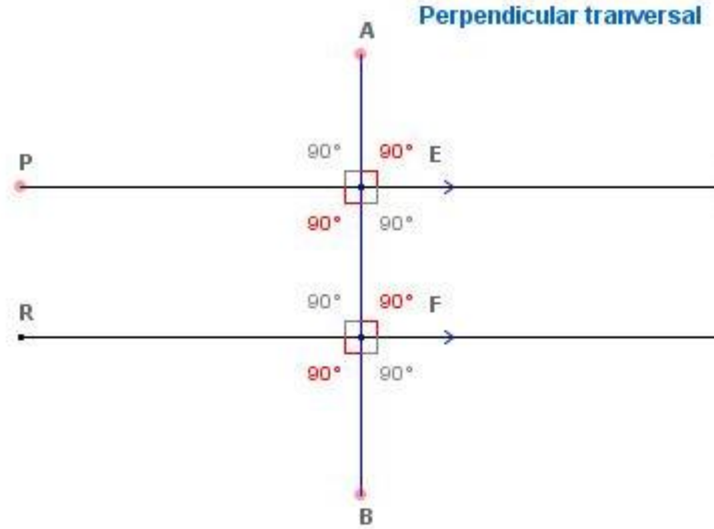
A **transversal** is a line that intersects two or more coplanar lines at different points. These two lines may or may not be parallel.



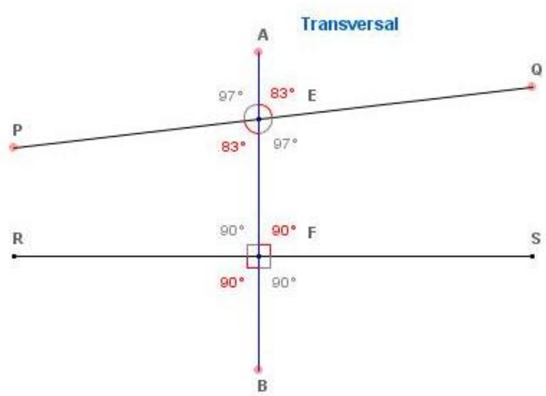


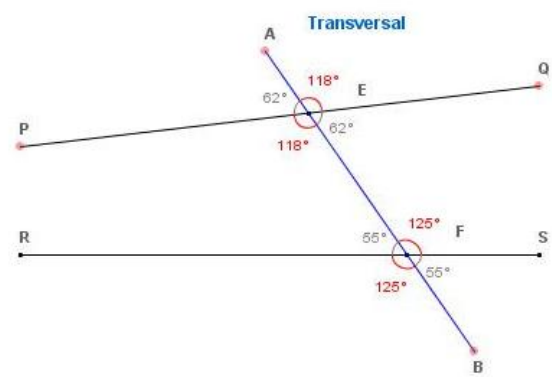
Notice that there are specific angle pair relations created by the transversal and the parallel lines. We will study these relations in depth during this section.

If the transversal cuts parallel lines at a right angle, it is known as a **perpendicular transversal**



If a transversal cuts through non-parallel lines, there may or may not be a limited number of special angle relations between the two angle clusters. This is shown in the diagram below

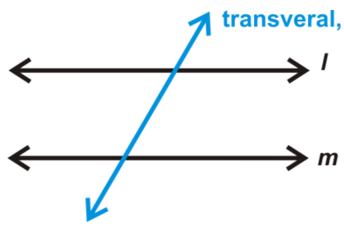




IMPORTANT TO NOTE: The area *between lines PQ and RS* is the called the *interior*. The area *outside lines PQ and RS* is called the *exterior*. These areas form the basis for the terminology used in defining special angle pairs formed by a transversal.

Transversals Create Angles- Transversals with Parallel Lines!

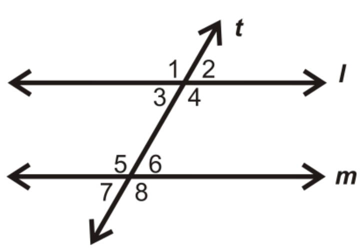
In the simple diagram below we are given lines t, l and m. Line l is parallel to line m and line t is the transversal. In this diagram the transversal forms t 8 angles. As well, several linear pairs vertical angle pairs are formed by the transversal. There are also 4 new angle relationships, defined here:



These four new angle relationships are

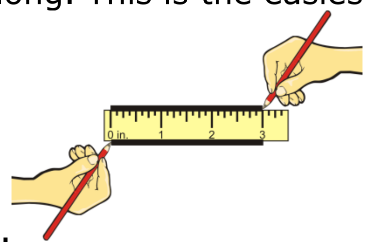
1. Corresponding Angles
2. Alternate Interior Angles
3. Alternate Exterior Angles
4. Same-side Interior Angles

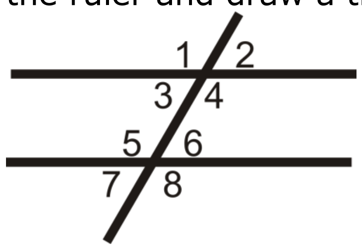
1. **Corresponding Angles** are formed by two angles that are in the "same place" with respect to the transversal, but on different lines. Using the diagram below, imagine sliding the four angles formed with line l and the transversal down to line m. The angles which match up are corresponding. When this happens we see that $\angle 2$ and $\angle 6$ are corresponding angles because they would be on top of one another and they have the same degree of measure. Which other corresponding angles do you see?



Investigation2-4:Corresponding Angles Exploration

You will need: paper, ruler, protractor

1. Place your ruler on the paper. On either side of the ruler, draw lines, 3 inches long. This is the easiest way to ensure that the lines are 

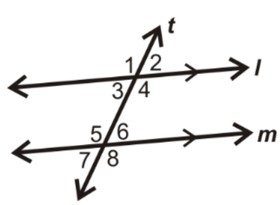
parallel.
2. Remove the ruler and draw a transversal. Label the eight angles as 

shown.
3. Using your protractor, measure all of the angles. What do you notice?

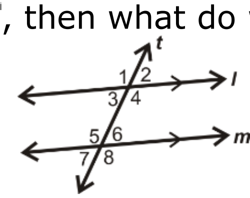
In this investigation, you should see that $m\angle 1 = m\angle 4 = m\angle 5 = m\angle 8$and $m\angle 2 = m\angle 3 = m\angle 6 = m\angle 7$. $\angle 1 \cong \angle 4$, $\angle 5 \cong \angle 8$ by the Vertical Angles Theorem. By the Corresponding Angles Postulate, we can say $\angle 1 \cong \angle 5$and therefore $\angle 1 \cong \angle 8$by the Transitive Property. You can use this reasoning for the other set of congruent angles as well.

You Try!

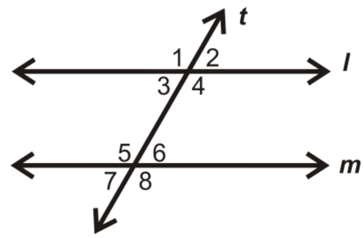
1. In the diagram below, If $m\angle 2 = 76°$, what is $m\angle 6$?



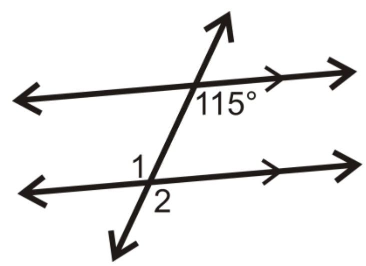
2. Using the measures of $\angle 2$and $\angle 6$ from problem 1, find all the other angle measures.
3. If $m\angle 8 = 110°$and $m\angle 4 = 110°$, then what do we know about lines $l$and m?



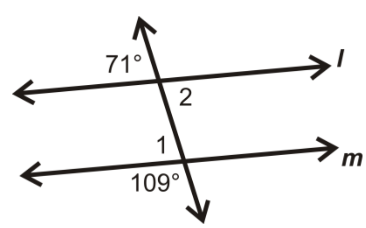
2. **Alternate Interior Angles:** Two angles that are on the <u>interior</u> of lines l and lines m, but on opposite sides of the transversal and have the same degree of measure. In the diagram below $\angle 3$ and $\angle 6$ are alternate interior angles. Which other angle pairs are alternate interior angles?



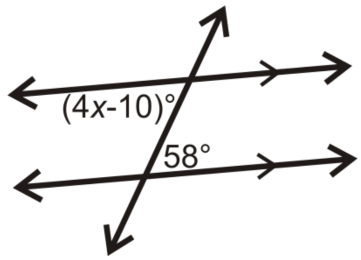
From the information given in the diagram below, find $m\angle 1$. What type of angle is $\angle 2$?



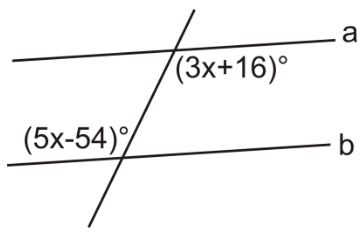
Use alternate interior angles to make the case that *l* || *m*



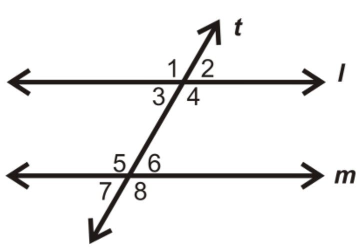
Algebra Connection In the diagram below, solve for the value of x. Hint: Think about the relationship between the $(4x-10)^\circ$ and 58° angles.



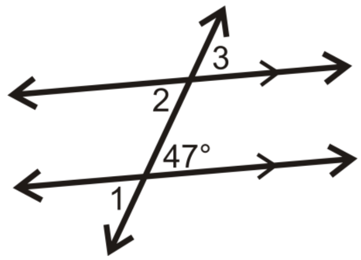
Algebra Connection*:* What does x have to be to make $a \parallel b$? Hint: What assumption must you make about the two angles?



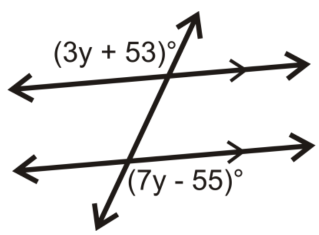
3. **Alternate Exterior Angles:** Two angles that are on the <u>exterior</u> of line l and line m, and on opposite sides of the transversal. In the diagram below $\angle 1$ and $\angle 8$ are alternate exterior angles. What other angle pairs make alternate exterior angles?



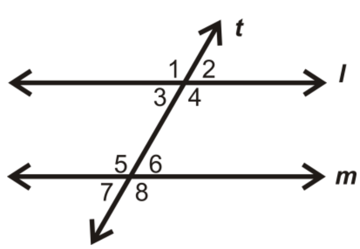
Using the diagram below, find $m\angle 1$and $m\angle 3$. What other information did you need to use to determine these measures?



Algebra Connection Find the value of y to determine the measure of each angle.

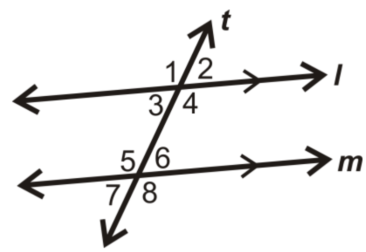


4. **Same Side Interior Angles:** Two angles that are on the same side of the transversal and on the interior of the two lines. In the diagram below $\angle 3$ and $\angle 5$ are same side interior angles. What other angle pairs form same side interior angles?

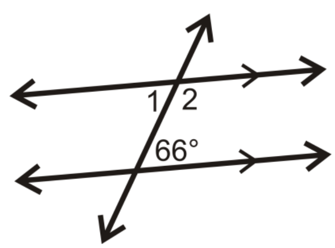


Same Side Angles are a bit different than the previous angles we discussed. Same Side Angles always form supplementary angles when a transversal crosses parallel lines.

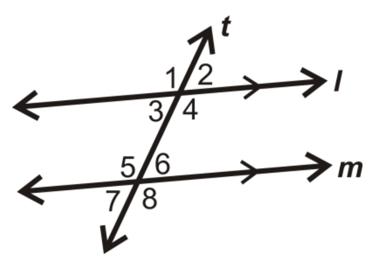
IMPORTANT TO NOTE: If two parallel lines are cut by a transversal, then the same side interior angles are supplementary. In the diagram below, If *l* || *m* and both are cut by *t*, then $m\angle 3 + m\angle 5 = 180°$ and $m\angle 4 + m\angle 6 = 180°$.



Using the diagram below, find $m\angle 2$. How many different ways can you find this angles measure?

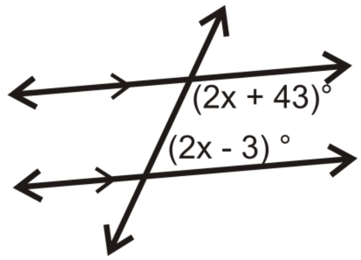


You Try!



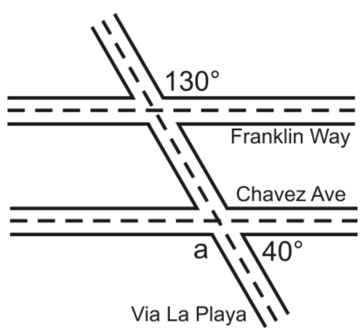
a) If $\angle 2 = 48°$(in the picture above), determine the measure of the remaining angles.
b) What is the relationship between angle 1 and angle 7? What could these angles be called?

Algebra Connection Find the value of x to help you determine the measure of each angle. **Hint:** Remember the relationship of the two angles to one another.

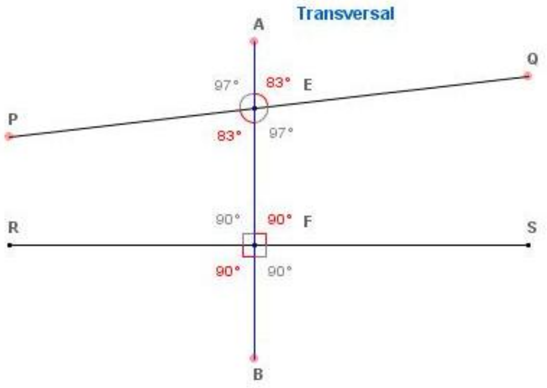


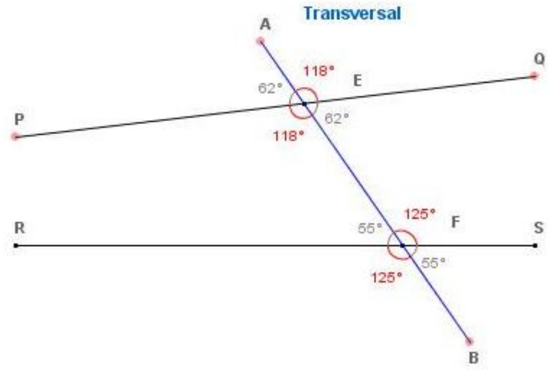
Real-World Situation: The map below shows three roads in Julio's town.

Julio used a surveying tool to measure two angles at the intersections in this picture he drew (NOT to scale). Julio wants to know if Franklin Way is parallel to Chavez Avenue. Using the information we have discussed about angle relations, determine if the two streets are parallel or not. State you reasons why or why not in well-constructed sentences.



IMPORTANT TO NOTE: If the transversal crosses non-parallel lines than the above angles still exist in name but they do not necessarily have the same measure of degree.





From the diagram on the left we know that angle E and angle F do not have an equal measure but because they meet the definition of a corresponding angle, they would still be classified as such.

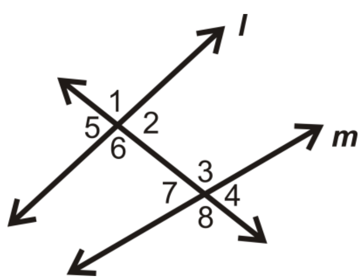
You try!

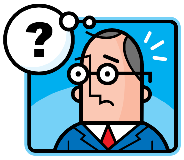
For the diagram below, determine if the following. State if the angle pairs are congruent or not.

a) A corresponding angle to $\angle 3$?

b) An alternate interior angle to $\angle 7$?

c) An alternate exterior angle to $\angle 4$?





Know What? The streets below are in Washington DC. The red street is R St. and the blue street is Q St. These two streets are parallel. The transversals are: Rhode Island Ave. (green) and Florida Ave. (orange).

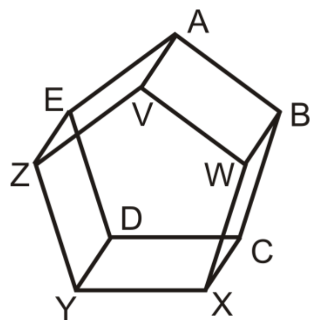
1. If $m\angle FTS = 35^\circ$, determine the other angles that are 35°.
2. If $m\angle SQV = 160^\circ$, determine the other angles that are 160°.

Internet & Video Links

1. **http://www.khanacademy.org/math/geometry/angles/v/angles-formed-between-transversals-and-parallel-lines**
2. **http://www.khanacademy.org/math/geometry/angles/v/angles-of-parallel-lines-2**
3. **http://www.khanacademy.org/math/geometry/angles/v/angles-formed-by-parallel-lines-and-transversals**
4. **http://www.brightstorm.com/math/geometry/reasoning-diagonals-angles-and-parallel-lines/corresponding-angles/**
http://www.brightstorm.com/math/geometry/reasoning-diagonals-angles-and-parallel-lines/alternate-interior-angles/
5. **http://www.brightstorm.com/math/geometry/reasoning-diagonals-angles-and-parallel-lines/alternate-exterior-angles/**
6. **http://www.brightstorm.com/math/geometry/reasoning-diagonals-angles-and-parallel-lines/same-side-interior-and-same-side-exterior-angles/**
7. **http://www.mathsisfun.com/geometry/transversal.html**
8. **http://www.mathsisfun.com/geometry/parallel-lines.html**

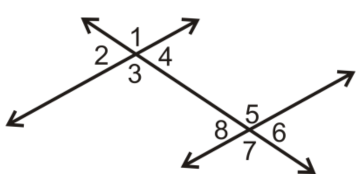
Section 2.1b Problem Set #1

Use the figure below to answer questions 1-5. The two pentagons are parallel and all of the rectangular sides are perpendicular to both of them.



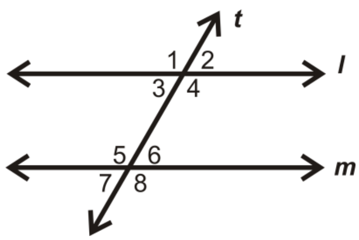
1. Find two pairs of skew lines.
2. List a pair of parallel lines.
3. List a pair of perpendicular lines.
4. For $\overline{AB}$, how many perpendicular lines pass through point V? What line is this?
5. For $\overline{XY}$, how many parallel lines passes through point D? What line is this?

For questions 6-12, use the picture below.



6. What is the corresponding angle to $\angle 4$?
7. What is the alternate interior angle with $\angle 5$?
8. What is the corresponding angle to $\angle 8$?
9. What is the alternate exterior angle with $\angle 7$?
10. What is the alternate interior angle with $\angle 4$?
11. What is the same side interior angle with $\angle 3$?
12. What is the corresponding angle to $\angle 1$?

Use the picture below for questions 13-16.



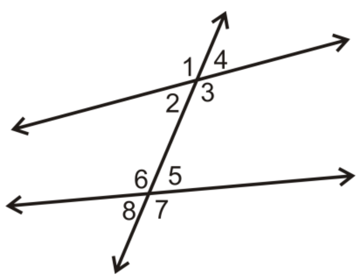
13. If $m\angle 2 = 55°$, what other angles do you know?

14. If $m\angle 5 = 123°$, what other angles do you know?

15. If $t \perp l$, is $t \perp m$? Why or why not?

16. Is $l \parallel m$? Why or why not?

Use the picture below to determine the answers for 17-20:



17. A pair of corresponding angles.

18. A pair of alternate interior angles.

19. A pair of same side interior angles.

20. If $m\angle 4 = 37°$, what other angles do you know?

Geometry is often apparent in nature. Think of examples of each of the following in nature.

21. Parallel Lines or Planes

22. Perpendicular Lines or Planes

23. Skew Lines

Algebra Connection In questions 24-27 explore the concepts of parallel and perpendicular lines in the coordinate plane.

24. Write the equations of two lines parallel to $y = 3$.

25. Write the equations of two lines perpendicular to $y = 5$.

26. Plot the points $A(2, -5),\ B(-3, 1),\ C(0, 4),\ D(-5, 10)$. Draw the lines $\overleftrightarrow{AB}$and $\overleftrightarrow{CD}$. What are the slopes of these lines? What is the geometric relationship between these lines?

27. Plot the points $A(2, 1),\ B(7, -2),\ C(2, -2),\ D(5, 3)$. Draw the lines $\overleftrightarrow{AB}$and $\overleftrightarrow{CD}$. What are the slopes of these lines? What is the geometric relationship between these lines?

Find the equation of the line that is <u>parallel</u> to the given line and passes through (5, -1).

28. $y = 2x - 7$

29. $y = -\frac{3}{5}x + 1$

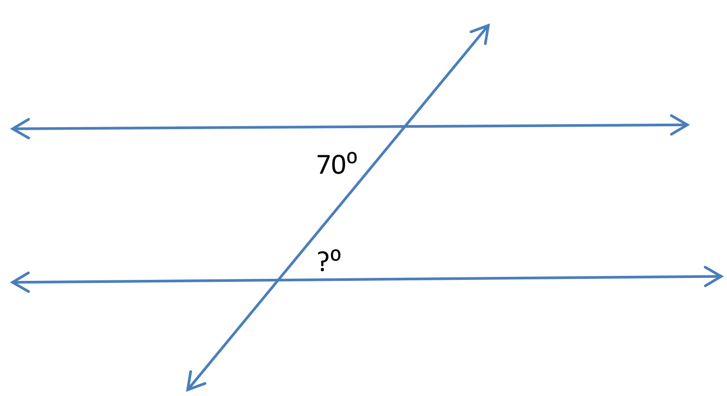
Find the equation of the line that is *<u>perpendicular</u>* to the given line and passes through (2, 3).

30. $y = \frac{2}{3}x - 5$

31. $y = -\frac{1}{4}x + 9$

Critical Thinking

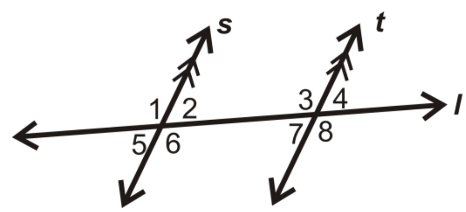
32. Using one or more complete sentences, write a word problem suggested by the question mark. Answers may vary. (You do not have to give the answer to the problem. The two lines are parallel.



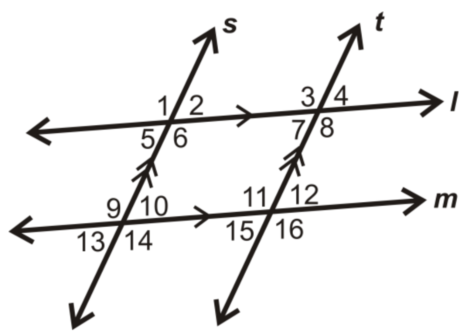
Section 2.1b Problem Set #2

For questions 1-7, determine if each angle pair below is congruent, supplementary or neither.

1. $\angle 1$ and $\angle 7$
2. $\angle 4$ and $\angle 2$
3. $\angle 6$ and $\angle 3$
4. $\angle 5$ and $\angle 8$
5. $\angle 1$ and $\angle 6$
6. $\angle 4$ and $\angle 6$
7. $\angle 2$ and $\angle 3$

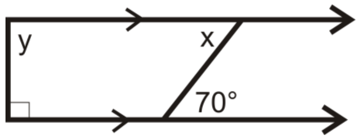


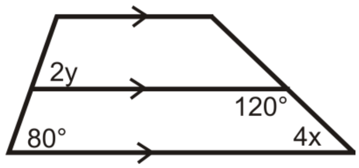
For questions 8-16, determine if the angle pairs below are: Corresponding Angles, Alternate Interior Angles, Alternate Exterior Angles, Same Side Interior Angles, Vertical Angles, Linear Pair or None.

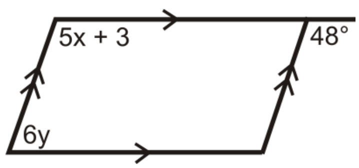


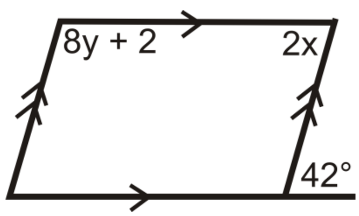
8. $\angle 2$ and $\angle 13$
9. $\angle 7$ and $\angle 12$
10. $\angle 1$ and $\angle 11$
11. $\angle 6$ and $\angle 10$
12. $\angle 14$ and $\angle 9$
13. $\angle 3$ and $\angle 11$
14. $\angle 4$ and $\angle 15$
15. $\angle 5$ and $\angle 16$
16. List all angles congruent to $\angle 8$.

For 17-20, find the values of x and y.

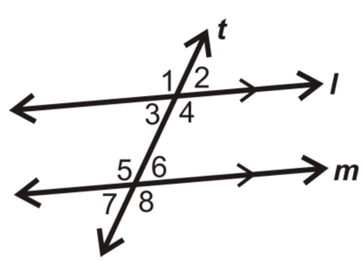
17. 

18. 

19. 

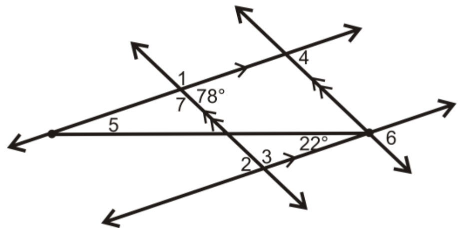
20. 

Algebra Connection For questions 21-25, use the picture to the right. Find the value of $x$and/or y.



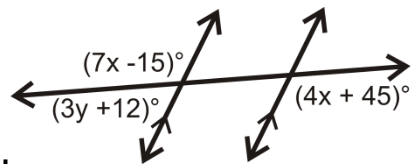
21. $m\angle 1 = (4x + 35)^\circ,\ m\angle 8 = (7x - 40)^\circ$
22. $m\angle 2 = (3y + 14)^\circ,\ m\angle 6 = (8x - 76)^\circ$
23. $m\angle 3 = (3x + 12)^\circ,\ m\angle 5 = (5x + 8)^\circ$
24. $m\angle 4 = (5x - 33)^\circ,\ m\angle 5 = (2x + 60)^\circ$
25. $m\angle 1 = (11y - 15)^\circ,\ m\angle 7 = (5y + 3)^\circ$

26. Find the measures of all the numbered angles in the figure below.

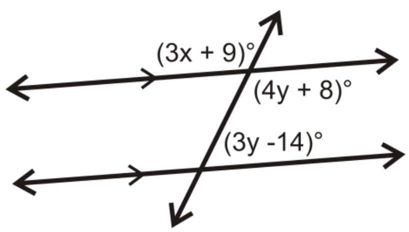


Algebra Connection For 27 and 28, find the values of x and y.

27.

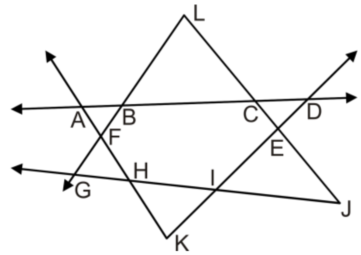


27.



Sections 2.1b Problem Set #3

In 1-6, use the given information to determine which lines are parallel. If there are none, write *none*. Consider each question individually.



1. $\angle LCD \cong \angle CJI$

2. $\angle BCE \text{ and } \angle BAF$ are supplementary

3. $\angle FGH \cong \angle EIJ$

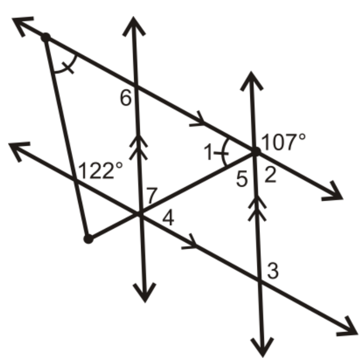
4. $\angle BFH \cong \angle CEI$

5. $\angle LBA \cong \angle IHK$

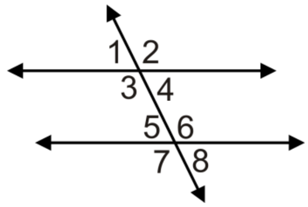
6. $\angle ABG \cong \angle BGH$

In 7-13, find the measure of the lettered angles below.

7. $m\angle 1$
8. $m\angle 2$
9. $m\angle 3$
10. $m\angle 4$
11. $m\angle 5$
12. $m\angle 6$
13. $m\angle 7$



For 14-18, what does the value of x have to be to make the lines parallel?



14. $m\angle 3 = (3x + 25)^{\circ}$ and $m\angle 5 = (4x - 55)^{\circ}$

15. $m\angle 2 = (8x)^{\circ}$ and $m\angle 7 = (11x - 36)^{\circ}$

16. $m\angle 1 = (6x - 5)^{\circ}$ and $m\angle 5 = (5x + 7)^{\circ}$

17. $m\angle 4 = (3x - 7)^{\circ}$ and $m\angle 7 = (5x - 21)^{\circ}$

18. $m\angle 1 = (9x)^{\circ}$ and $m\angle 6 = (37x)^{\circ}$

Section 2.2: Properties of Perpendicular Lines

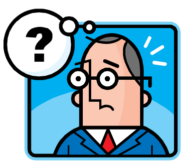
INSTANT RECALL!

1. What conditions must exist for lines to be perpendicular?

Find the slope between the two given points and the state the negative reciprocal of that slope.

2. (-3, 4) and (-3, 1)

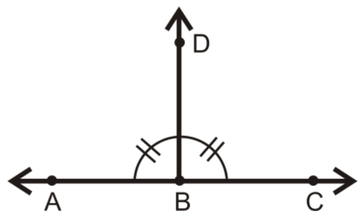
3. (6, 7) and (-5, 7)



Know What? There are several examples of slope in nature. Below are pictures of Half Dome, in Yosemite National Park and the horizon over the Pacific Ocean. These are examples of horizontal and vertical lines in real life. Can you determine the slope of these lines?

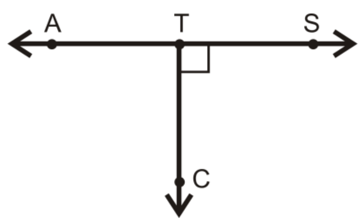
2.2a Congruent Linear Pairs

Recall that a linear pair is a pair of adjacent angles whose outer sides form a straight line. The Linear Pair Postulate says that the angles in a linear pair are supplementary. What happens when the angles in a linear pair are congruent?

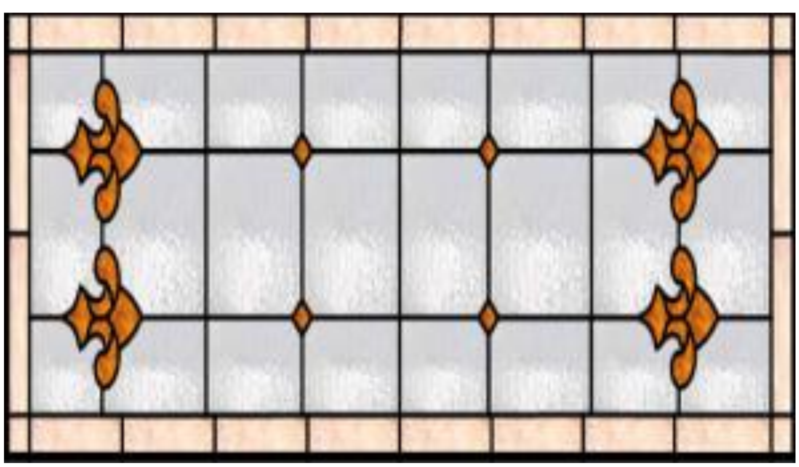


IMPORTANT TO NOTE: Anytime a linear pair is congruent, the angles formed must be 90°

Given the diagram below, what is $m\angle CTA$.

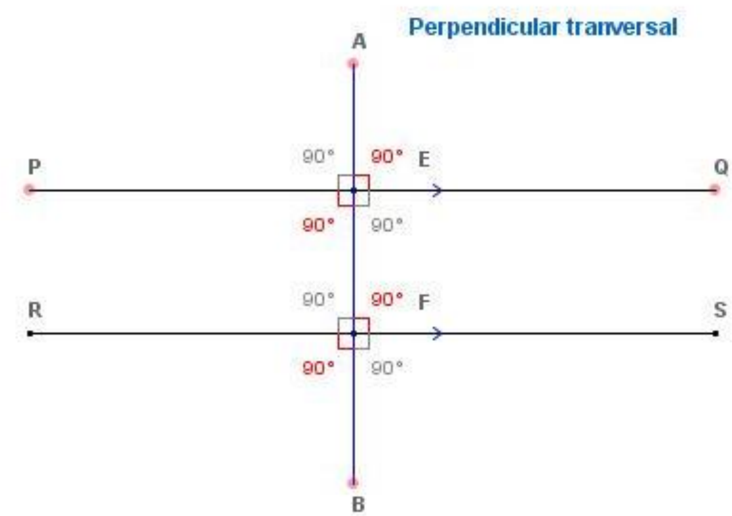


We see congruent linear pairs around us all of the time and just don't realize it! Look at the picture of the stained glass window, how many congruent linear pairs do you see?



2.2b - Perpendicular Transversals

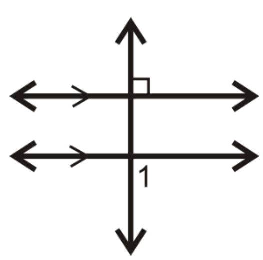
Recall that when two lines intersect, four angles are created. If the two lines are perpendicular, then all four angles are right angles, even though only one needs to be marked with the square. Therefore, all four angles are 90°. When another parallel line is added, a total of eight 90° angles pairs are formed. This is shown in the diagram below.



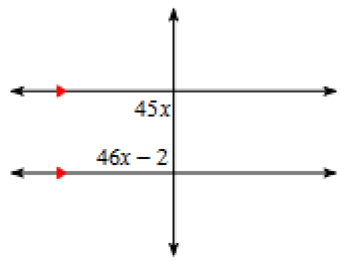
If an additional perpendicular transversal (CD) is added to the diagram above, what would the relationship be between lines AB and lines CD?

YOU TRY!

1. From the diagram below, determine the measure of $\angle 1$.

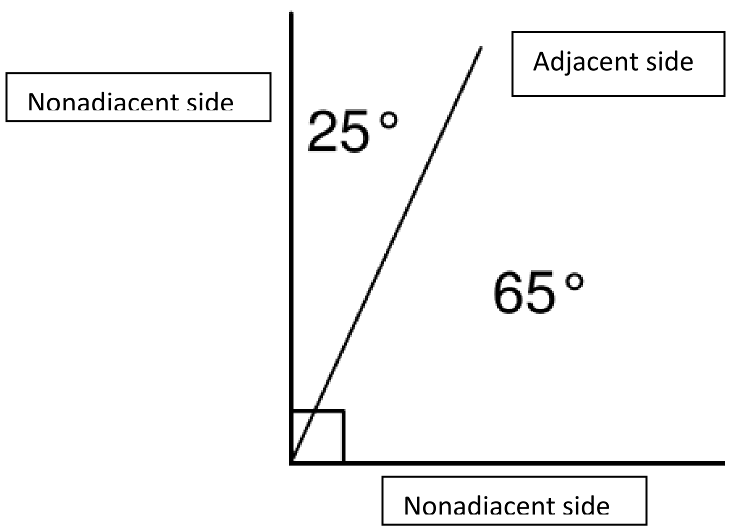


2. Find the value of x in the diagram shown.



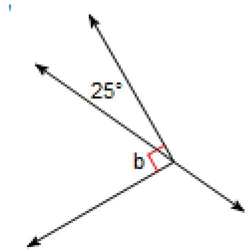
2.2c - Adjacent Complementary Angles

Recall that complementary angles add up to $90°$. If complementary angles are adjacent, their nonadjacent sides are perpendicular rays and therefore create a perpendicular pairing. This can be seen in the diagram below.



Notice that the two nonadjacent sides form a right angle, therefore they are perpendicular to one-another.

Example: Find the measure of angle b in the diagram below

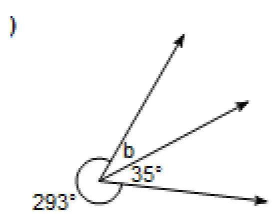


From the diagram we see that the two adjacent angles, b and 25° have nonadjacent sides that form a 90° angle. Therefore we know that the sum of these two angles is 90°. Set up the equation and solve.

Solution: $\angle b + 25° = 90$ (subtract 25° from both sides)

$\angle b = 65°$

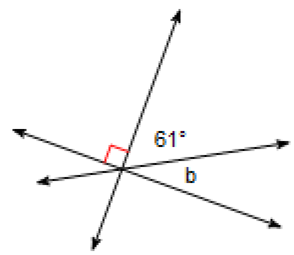
Example: Find the measure of angle b in the diagram below and determine if there is an Adjacent Complementary pair.



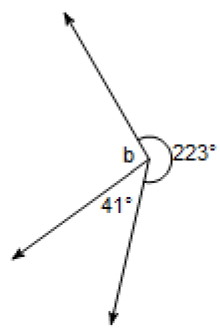
From the diagram there is no indication that angle b and the 35° angle form a right angle therefore it is not likely they are complementary. If you add 35° and 293° you get 328°. Recall that there are 360° in the rotation of an angle so angle b is 360°-328° which 32°. If you add 35° to 32° the adjacent angles add up to 67° which verifies that they are not complementary.

YOU TRY!

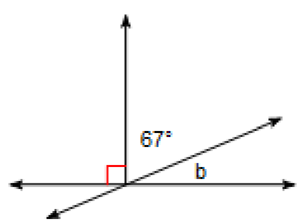
1. Find the measure of angle b and determine if the adjacent angles are complementary.



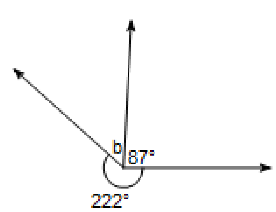
2. Find the measure of angle b and determine if the adjacent angles are complementary.

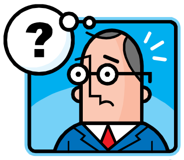


3. Find the measure of angle b and determine if the adjacent angles are complementary.



4. Find the measure of angle b and determine if the adjacent angles are complementary.





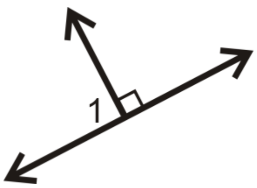
Know What? Revisited

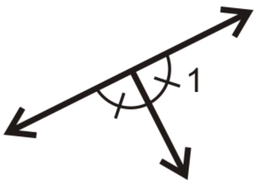
Half Dome is vertical and the slope of any vertical line is undefined. Thousands of people flock to Half Dome to attempt to scale the rock. This front side is very difficult to climb because it is vertical. The only way to scale the front side is to use the provided cables at the base of the rock. http://www.nps.gov/yose/index.htm

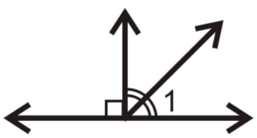
Any horizon over an ocean is horizontal, which has a slope of zero, or no slope. There is no steepness, so no incline or decline. The complete opposite of Half Dome. Actually, if Half Dome was placed on top of an ocean or flat ground, the two would be perpendicular!

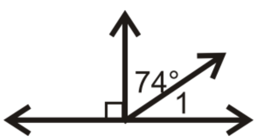
Review Questions

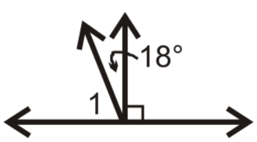
Find the measure of $\angle 1$ for each problem below.

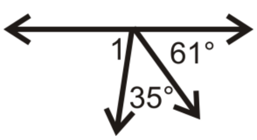
1. 

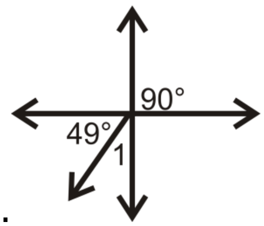
2. 

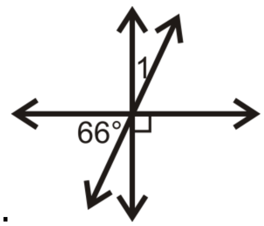
3. 

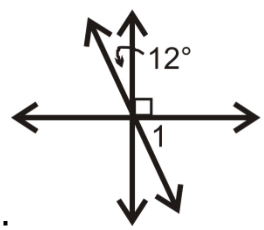
4. 

5. 

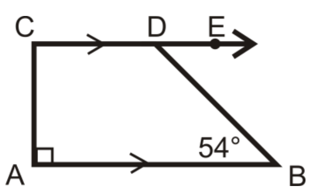
6. 

7. 

8. 

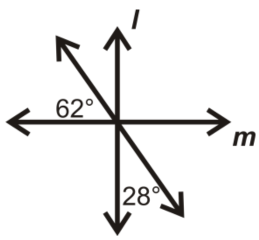
9. 

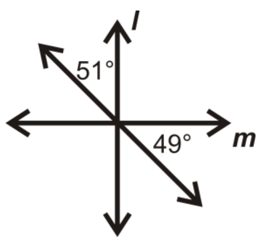
For questions 10-13, use the picture below.

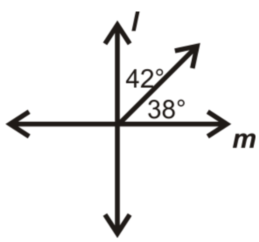


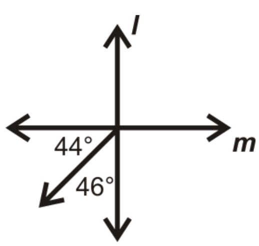
10. Find $m\angle ACD$.
11. Find $m\angle CDB$.
12. Find $m\angle EDB$.
13. Find $m\angle CDE$.

In questions 14-17, determine if $l \perp m$.

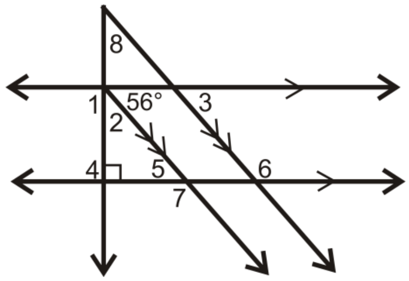
14. 

15. 

16. 

17. 

For questions 18-25, use the picture below.



18. Find $m\angle 1$.

19. Find $m\angle 2$.

20. Find $m\angle 3$.

21. Find $m\angle 4$.

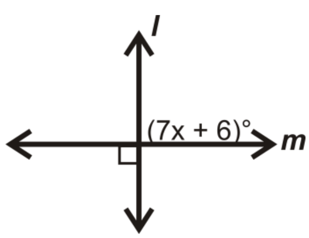
22. Find $m\angle 5$.

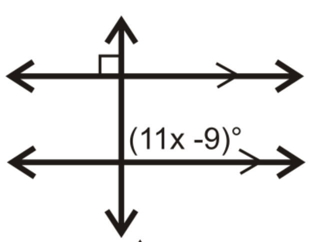
23. Find $m\angle 6$.

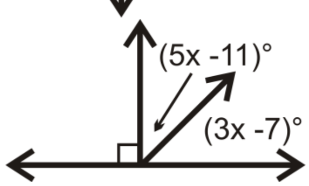
24. Find $m\angle 7$.

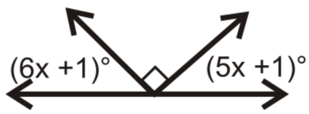
25. Find $m\angle 8$.

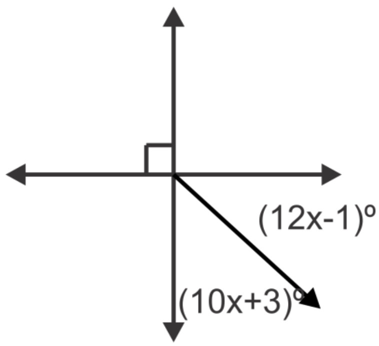
Algebra Connection Find the value of x.

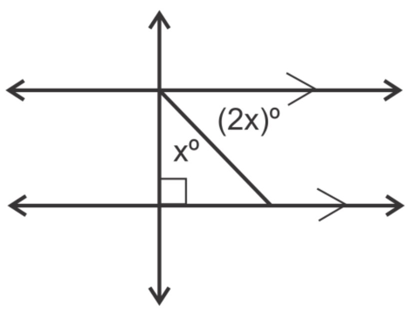
27. 

28. 

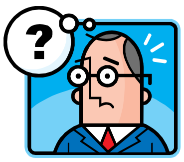
29. 

30. 

31. 

32. 

Section 2.3: Using the Distance Formula with Parallel and Perpendicular lines



Know What? The shortest distance between two points is a straight line. Below is an example of how far apart cities are in the greater Los Angeles area. There are always several ways to get somewhere in Los Angeles. Here, we have the distances between Los Angeles and Orange. Which distance is the shortest? Which is the longest?

2.3a The Distance Formula

INSTANT RECALL!

Remember from Integrated Math 1 The distance between two points (x_1, y_1) and (x_2, y_2) can be defined as $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

IMPORTANT TO REMEMBER: This formula is derived from the Pythagorean Theorem

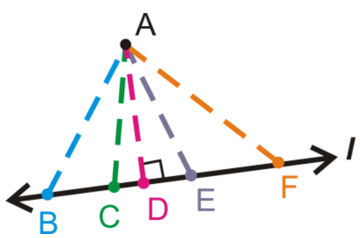
YOU TRY!

Use the distance formula to answer the following

1. Find the distance between (4, -2) and (-10, 3).
2. The distance between two points is 4 units. One point is (1, -6). What is the second point? You may assume that the second point is made up of integers.

2.3b Shortest Distance between a Point and a Line

We know that the shortest distance between two points is a straight line. This distance can be calculated by using the distance formula. Let's extend this concept to the shortest distance between a point and a line.



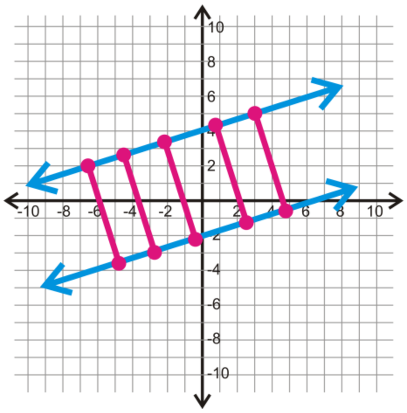
Just by looking at a few line segments from point A to line l, we can tell that the shortest distance between a point and a line is the *perpendicular line* between them. Therefore, AD is the shortest distance between A and line l.

Putting this onto a graph can be a little tougher.

Example: Determine the shortest distance between the point (1, 5) and the line $y = \frac{1}{3}x - 2$.

2.3d Shortest Distance between Two Parallel Lines

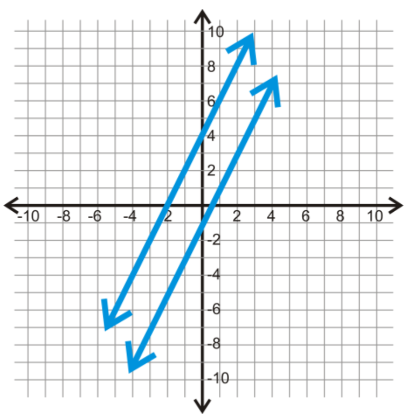
The shortest distance between two parallel lines is the length of the perpendicular segment between them. It doesn't matter which perpendicular line you choose, as long as the two points are on the lines. Recall that there are infinitely many perpendicular lines between two parallel lines.



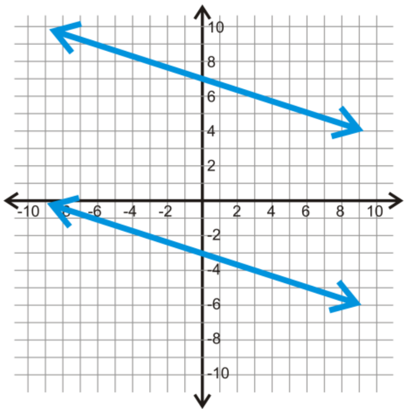
Notice that all of the pink segments are the same length. So, when picking a perpendicular segment, be sure to pick one with endpoints that are integers.

Example 3: Find the distance between $x = 3$ and $x = -5$.

Example 4: What is the shortest distance between $y = 2x + 4$ and $y = 2x - 1$?



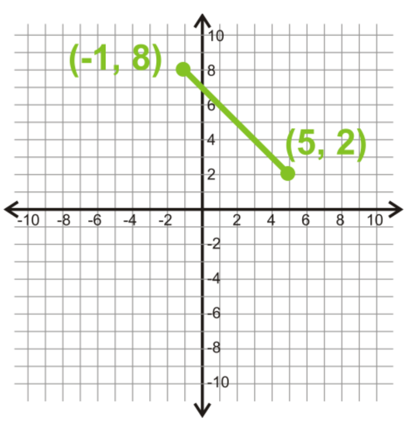
Example 5: Find the distance between the two parallel lines below.



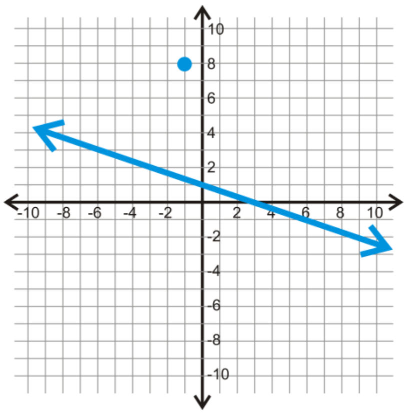
2.3d Perpendicular Bisectors in the Coordinate Plane

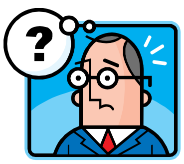
Recall from Integrated Math 1 that the definition of a perpendicular bisector is a perpendicular line that goes through the midpoint of a line segment. Using what we have learned in this chapter and the formula for a midpoint (recall from last year) , we can find the equation of a perpendicular bisector.

Example 6: Find the equation of the perpendicular bisector of the line segment between (-1, 8) and (5, 2).



Example 7: The perpendicular bisector of $\overline{AB}$ has the equation $y = -\frac{1}{3}x + 1$. If A is (-1, 8) what are the coordinates of B?





Know What? Revisited. Draw two intersecting lines. Make sure they are not perpendicular. Label the 26.3 miles along hwy 5. The longest distance is found by adding the distances along the 110 and 405, or 41.8 miles.

Review Questions

Find the distance between each pair of points. Round your answer to the nearest hundredth.

1.(4, 15) and (-2, -1)

2.(-6, 1) and (9, -11)

3.(0, 12) and (-3, 8)

4.(-8, 19) and (3, 5)

5.(3, -25) and (-10, -7)

6.(-1, 2) and (8, -9)

7.(5, -2) and (1, 3)

8.(-30, 6) and (-23, 0)

Determine the shortest distance between the given line and point. Round your answers to the nearest hundredth.

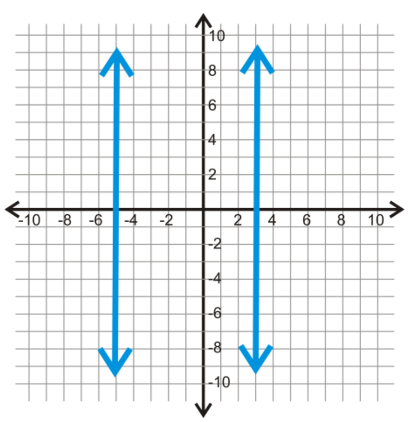
9. $y = \frac{1}{3}x + 4;\ (5,\ -1)$

10. $y = 2x - 4;\ (-7,\ -3)$

11. $y = -4x + 1;\ (4,\ 2)$

12. $y = -\frac{2}{3}x - 8;\ (7,\ 9)$

Use each graph below to determine how far apart each the parallel lines are. Round your answers to the nearest hundredth.

13. 

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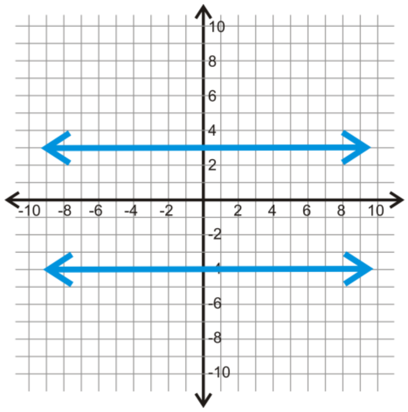
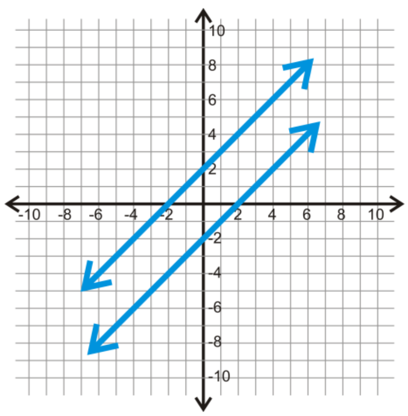

10
8
6
4
2
-10 -8 -6 -4 -2 2 4 6 8 10
-2
-6
-8
-10

15.


10
8
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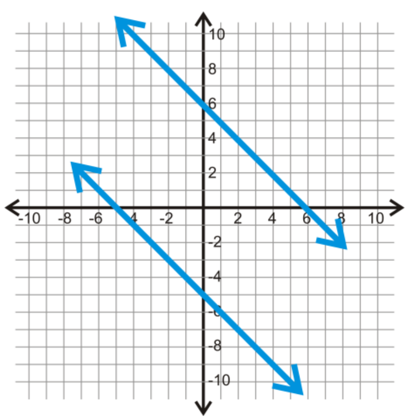
-10 -8 -6 -4 -2 2 4 6 8 10
-2
-4
-6
-8
-10

16.


10
8
6
4
2
-10 -8 -6 -4 -2 2 4 6 8 10
-2
-4
-8
-10

Determine the shortest distance between the each pair of parallel lines. Round your answer to the nearest hundredth.

17. $x = 5, \ x = 1$
18. $y = -6, \ y = 4$
19. $y = x + 5, \ y = x - 3$
20. $y = -\frac{1}{3}x + 2, \ y = -\frac{1}{3}x - 8$
21. $y = 4x + 9, \ y = 4x - 8$
22. $y = \frac{1}{2}x, \ y = \frac{1}{2}x - 5$

Find the equation of the perpendicular bisector for pair of points.

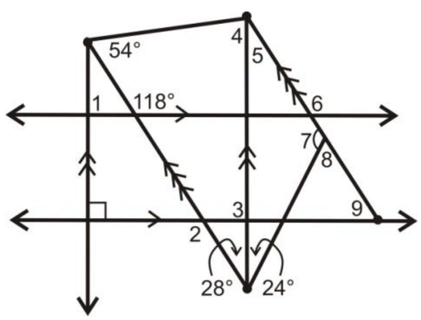
23. (1, 5) and (7, -7)
24. (1, -8) and (7, -6)
25. (9, 2) and (-9, -10)
26. (-7, 11) and (-3, 1)
27. The perpendicular bisector of $\overline{CD}$has the equation $y = 3x - 11$. If $D$is (-3, 0) what are the coordinates of C?
28. The perpendicular bisector of $\overline{LM}$has the equation $y = -x + 5$. If $L$is (6, -3) what are the coordinates of M?
29. *Writing* List the steps you would take to find the distance between two parallel lines, like the two in #24.
30. *Critical Thinking – Complete the following without using any variable names: given two points in coordinate plane, you find the distance between them by* ____.

Chapter 2 Review

Keywords and Theorems

- Parallel
- Skew Lines
- Parallel Postulate
- Perpendicular Line Postulate
- Transversal
- Corresponding Angles
- Alternate Interior Angles
- Alternate Exterior Angles
- Same Side Interior Angles
- Corresponding Angles Postulate
- Alternate Interior Angles Theorem
- Alternate Exterior Angles Theorem
- Same Side Interior Angles Theorem
- Converse of Corresponding Angles Postulate
- Converse of Alternate Interior Angles Theorem
- Converse of the Alternate Exterior Angles Theorem
- Converse of the Same Side Interior Angles Theorem
- Parallel Lines Property
- Theorem 3-1
- Theorem 3-2
- Distance Formula: $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Review -Find the value of each of the numbered angles below.



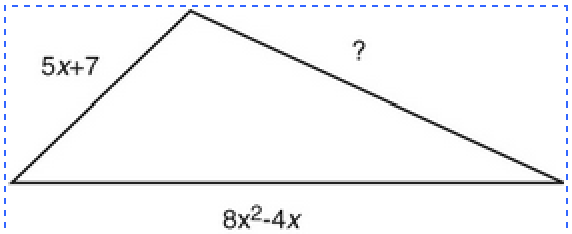
Texas Instruments Resources

In the CK-12 Texas Instruments Geometry FlexBook, there are graphing calculator activities designed to supplement the objectives for some of the lessons in this chapter. See http://www.ck12.org/flexr/chapter/9688.

Pull it All Together! End of Chapter Integration Exercise

1. Simplify: $(r+8)(r-4)$

2. The perimeter of the triangle below is $10x^2 - 6x + 1$.



What is the length of the missing side?

3. What binomial must be added to (-3t+6) so that the sum is (10t-1)?

4. A system of linear equations is shown below.

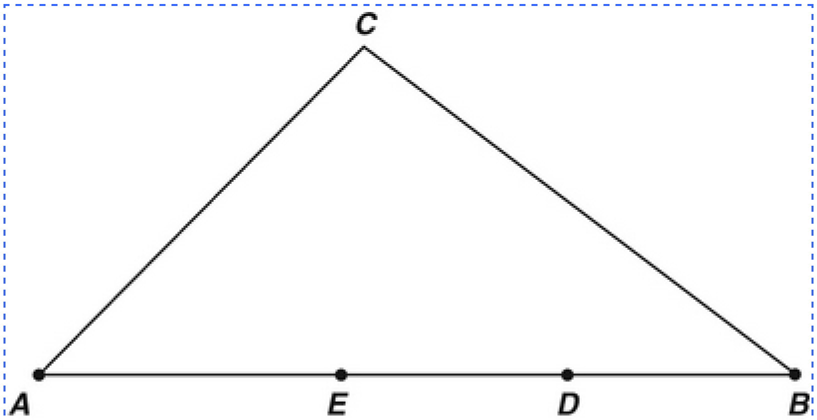
$$\begin{cases} x + y = 3 \\ x + y = 4 \end{cases}$$

Lisa solves the system of equations and finds a result of $0 = -1$. Which statement explains what this result shows about the solution of this system of equations?

5. During the past year, Ms. Abalos invested $9,000. Some of her money was invested in a savings account paying 5% annually and the rest was invested in a different savings account paying 8% annually. She received a total of $600 from these investments for the year. How much did she invest in the savings account paying 5%?

6. Solve by factoring: $2x^2 + x - 15 = 0$

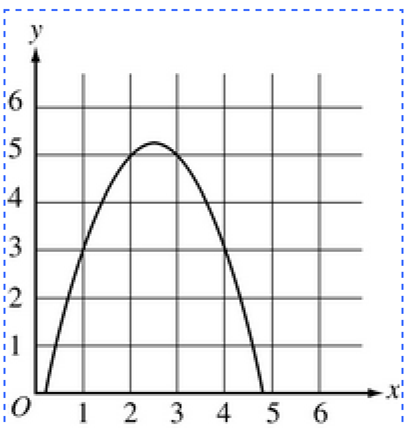
In triangle *ABC*, $\overline{AE}$ is 4 cm, $\overline{BD}$ is 3 cm, and D is the midpoint of $\overline{EB}$.



7. How long is $\overline{AB}$?

8. Find the distance between and midpoint of the segment connecting the points $\left(\frac{3}{2},\frac{1}{2}\right)$ and $\left(-\frac{5}{2},-\frac{3}{2}\right)$

According to the graph of $y = f(x)$ shown below, $f(2) =$



9.

10. Solve the following: $8\sqrt{2x+8}-10=150$

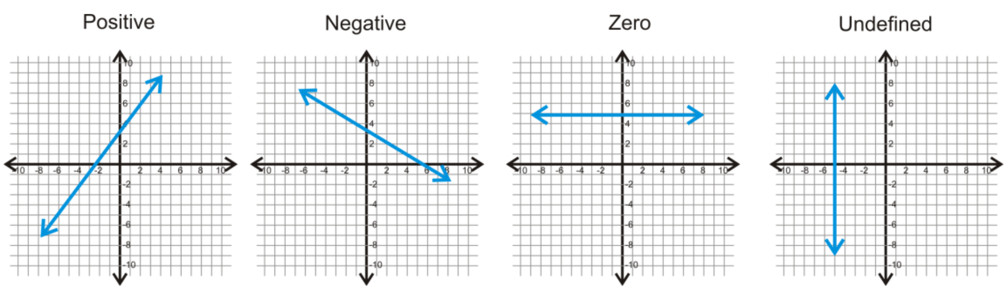
End of Chapter 2

Chapter Appendix: Slope and Linear Functions on the Coordinate Plane

Slope in the Coordinate Plane

Recall from Integrated Math 1, The slope of the line between two points (x_1, y_1) and (x_2, y_2) is $m = \frac{(y_2 - y_1)}{(x_2 - x_1)}$.

Different Types of Slope:



Example 1: What is the slope of the line through (2, 2) and (4, 6)?

Example 2: Find the slope between (-8, 3) and (2, -2).

Example 3: Find the slope between (-5, -1) and (3, -1).

Example 4: What is the slope of the line through (3, 2) and (3, 6)?

Example 6: Find the slope of the perpendicular lines to the lines below.

a) $y = 2x + 3$

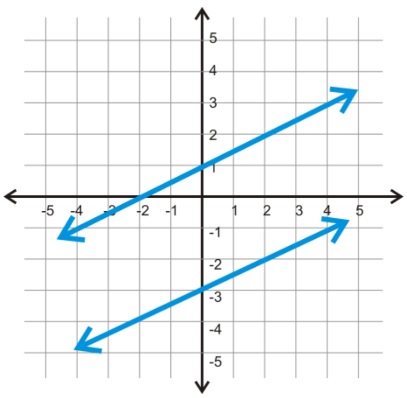
b) $y=-\frac{2}{3}x-5$

c) $y=x+2$

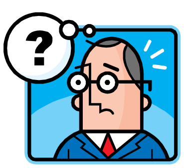
Example 7: Find the equation of the line that is perpendicular to $y=-\frac{1}{3}x+4$ and passes through (9, -5).

Graphing Parallel and Perpendicular Lines

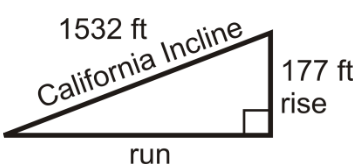
Example 8: Find the equations of the lines below and determine if they are parallel, perpendicular or neither.



Example 9: Graph $3x-4y=8$ and $4x+3y=15$. Determine if they are parallel, perpendicular, or neither.



Know What? Revisited In order to find the slope, we need to first find the horizontal distance in the triangle to the right. This triangle represents the incline and the elevation. To find the horizontal distance, or the run, we need to use the Pythagorean Theorem, $a^2+b^2=c^2$, where c is the hypotenuse.



$$
\begin{aligned}
177^2+run^2&=1532^2\\
31,329+run^2&=2,347,024\\
run^2&=2,315,695\\
run&\approx 1521.75
\end{aligned}
$$

The slope is then $\frac{177}{1521.75}$, which is roughly $\frac{3}{25}$.

Appendix Review Questions

Find the slope between the two given points.

1.(4, -1) and (-2, -3)
2.(-9, 5) and (-6, 2)
3.(7, 2) and (-7, -2)
4.(-6, 0) and (-1, -10)
5.(1, -2) and (3, 6)
6.(-4, 5) and (-4, -3)

Determine if each pair of lines are parallel, perpendicular, or neither. Then, graph each pair on the same set of axes.

7.$y=-2x+3$and $y=\frac{1}{2}x+3$
8.$y=4x-2$and $y=4x+5$
9.$y=-x+5$and $y=x+1$
10. $y=-3x+1$and $y=3x-1$
11. $2x-3y=6$and $3x+2y=6$
12. $5x+2y=-4$and $5x+2y=8$
13. $x-3y=-3$and $x+3y=9$
14. $x+y=6$and $4x+4y=-16$

Determine the equation of the line that is *parallel* to the given line, through the given point.

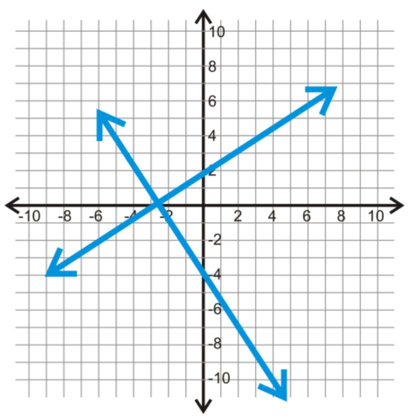
15. $y=-5x+1;\ (-2,\ 3)$
16. $y=\frac{2}{3}x-2;\ (9,1)$
17. $x-4y=12;\ (-16,\ -2)$
18. $3x+2y=10;\ (8,\ -11)$
19. $2x-y=15;\ (3,\ 7)$

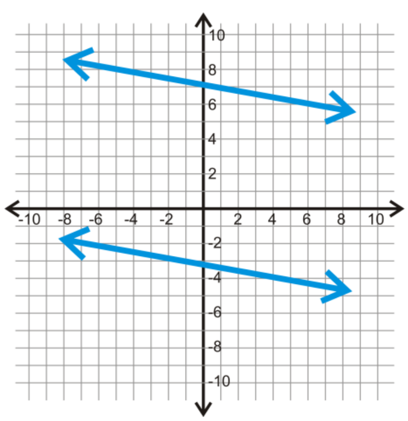
20. $y = x - 5;\ (9,\ -1)$

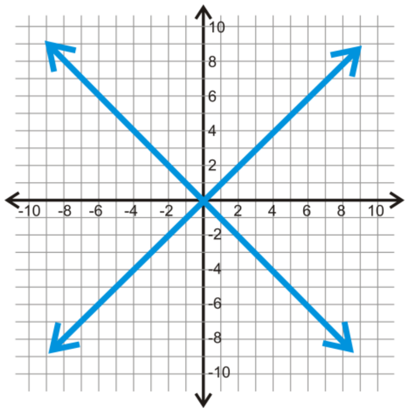
Determine the equation of the line that is *perpendicular* to the given line, through the given point.

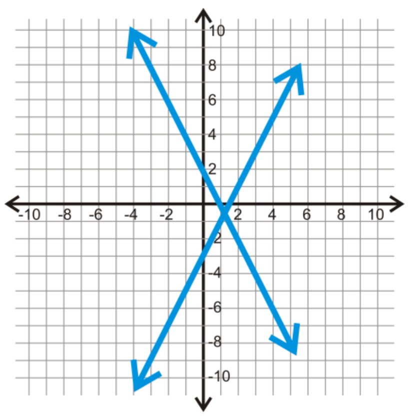
21. $y = x - 1;\ (-6,\ 2)$
22. $y = 3x + 4;\ (9,\ -7)$
23. $5x - 2y = 6;\ (5,\ 5)$
24. $y = 4;\ (-1,\ 3)$
25. $x = -3;\ (1,\ 8)$
26. $x - 3y = 11;\ (0,\ 13)$

Find the equation of the two lines in each graph below. Then, determine if the two lines are parallel, perpendicular or neither.

27. 

28. 

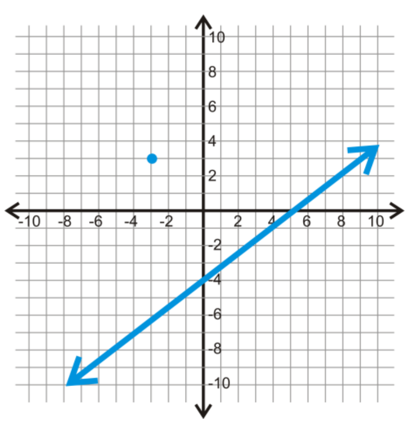
29. 

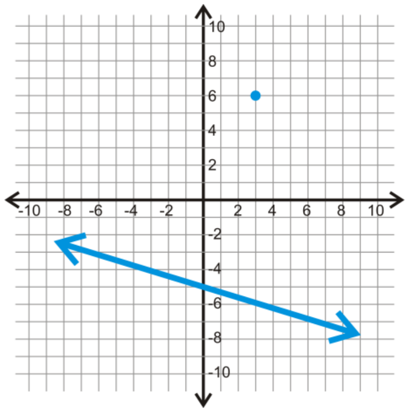
30. 

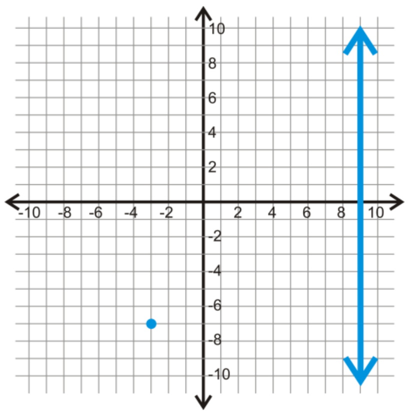
For the line and point below, find:

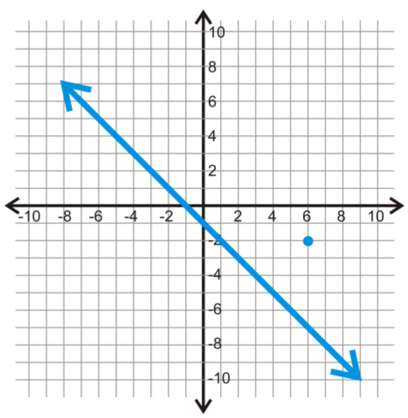
a) A parallel line, through the given point.

b) A perpendicular line, through the given point.

31. 

32. 

33. 

34. 

End of Chapter 2 Appendix