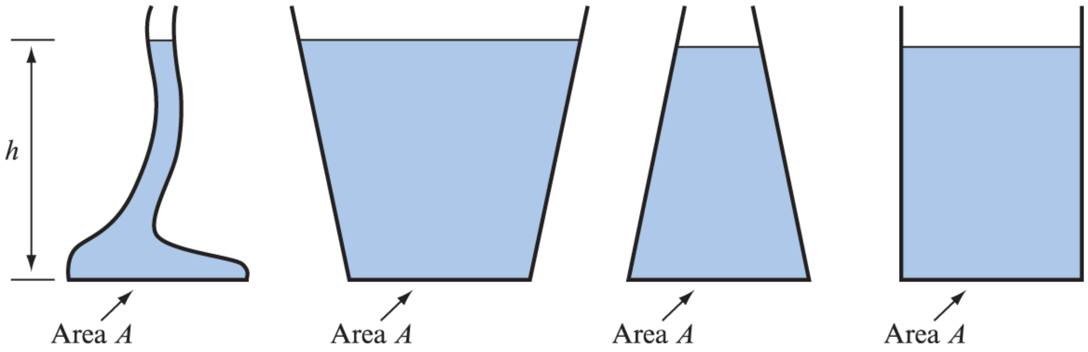
CHAPTER 2
Pressure and Head



Dr. Khalil Mahmoud ALASTAL

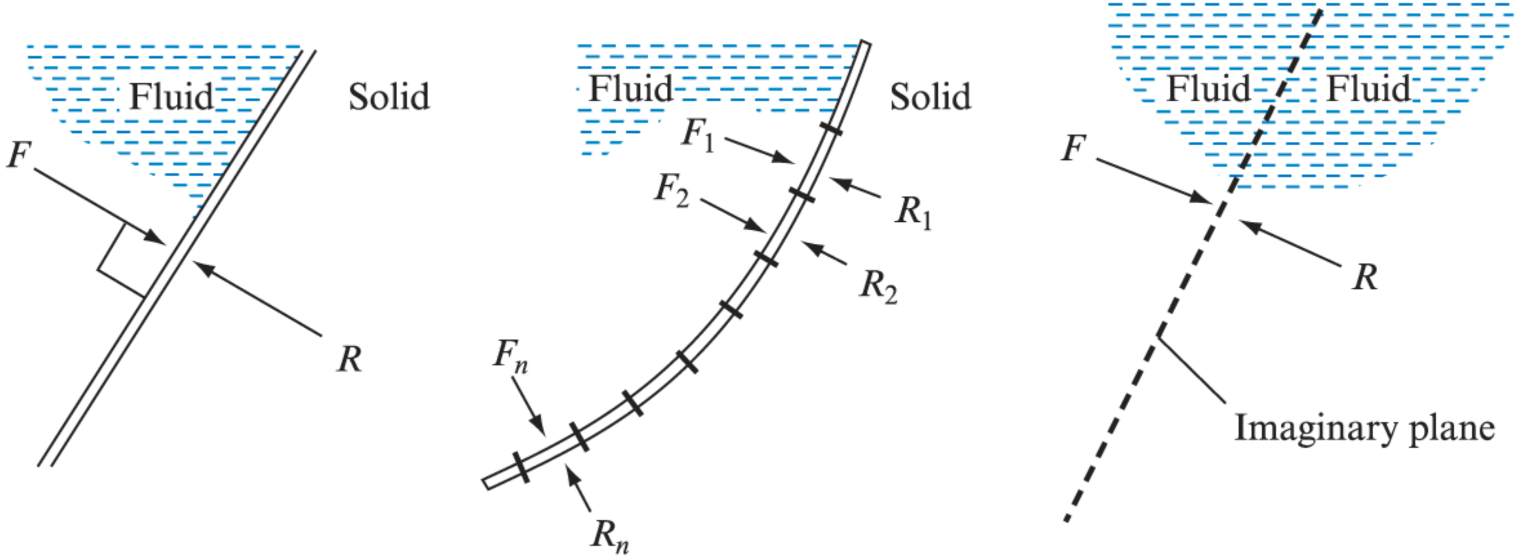
Objectives of this Chapter:

- Introduce the ***concept of pressure***.
- Prove it has a ***unique value*** **at any particular elevation**.
- Show how pressure ***varies with depth*** according to the hydrostatic equation.
- Show how pressure can be expressed in terms of ***head*** of fluid.
- Demonstrate methods of pressure measurement using ***manometer***.

2.1 Statics of Fluid Systems:

From chapter 1:

- A **static fluid** can have **no shearing force** acting on it.
- Any force between the fluid and the boundary must be acting **at right angles to the boundary**.

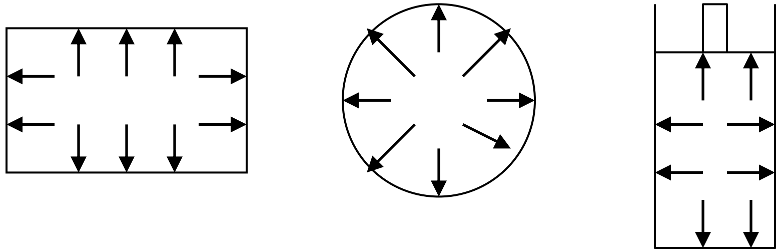


Pressure force normal to the boundary

2.1 Statics of Fluid Systems:

This is also true for:

- **Curved** surfaces (the force acting at any point is normal to the surface at that point).
- Any **imaginary** plane.



- We use this fact in our analysis by considering **elements of fluid bounded by *solid boundaries* or *imaginary planes***.
- And since the fluid at rest: (the element will be in equilibrium)
  - The **sum of the components of forces** in any direction will be zero.
  - The **sum of the moments of forces** on the element about any point must also be zero.

2.2 Pressure:

- As mentioned above a fluid will exert a normal force on any boundary it is in contact with.
- Since these boundaries may be large and the ***force may differ from place to place***

→ it is convenient to work in terms of **pressure** "*p*" which is the **force per unit area**.

2.2 Pressure:

$$\text{Pressure} = \frac{\text{Force}}{\text{Area over which the force is applied}}$$

$$p = \frac{F}{A}$$

Unit:

- **N/m²** or kg m^{-1} s^{-1}
- Also known as a **Pascal**, *Pa*, i.e. 1 *Pa* = 1 *N/m²*
- Also frequently used is **bar**, where 1 bar = 10^5 *Pa*.

Note : Uniform Pressure

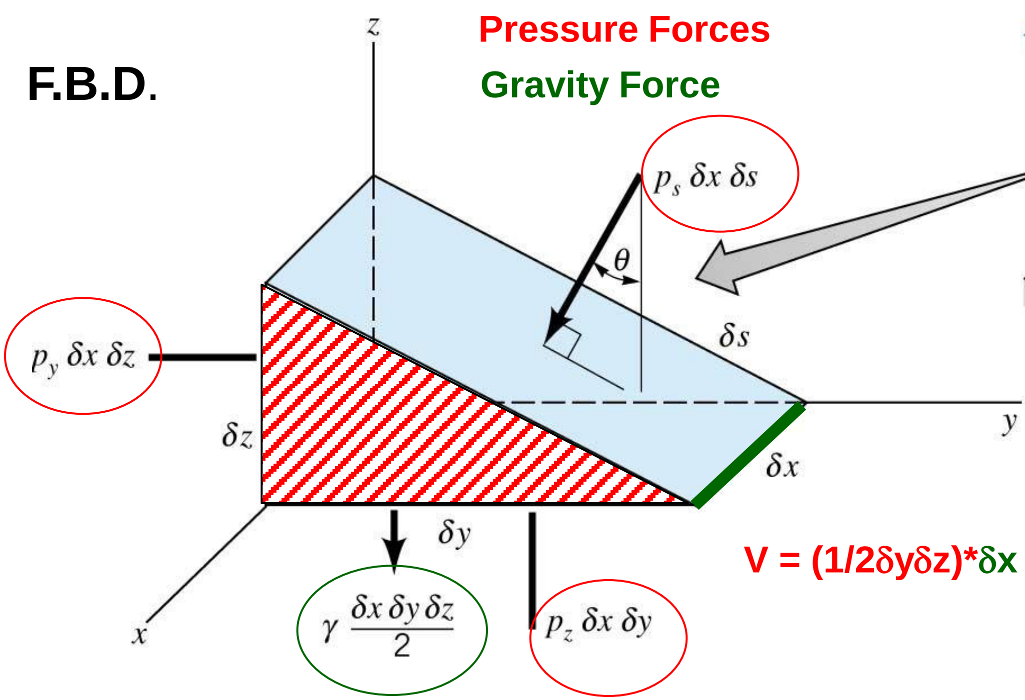
If the **force exerted on each unit area** of a boundary (surface) is **the same**, the pressure is said to be ***uniform***.

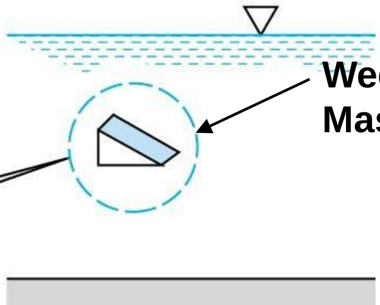
2.3 Pascal's Law for Pressure at a Point:

How does the **pressure at a point *vary* with orientation** of the plane passing through the point?

Blaise Pascal (1623-1662)

F.B.D.





Wedged Shaped Fluid Mass

p is average pressure in the x, y, and z direction.

P_s is the average pressure on the surface

θ is the plane inclination

δ is the length is each coordinate direction, x, y, z

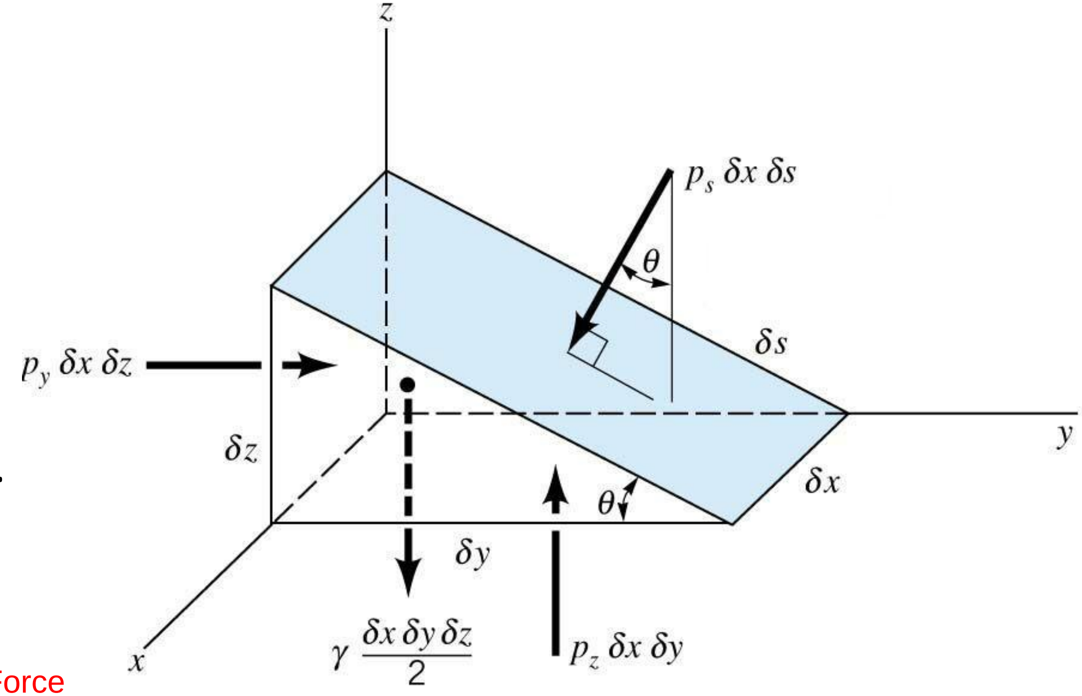
δs is the length of the plane

γ is the specific weight

2.3 Pascal's Law for Pressure at a Point:

Remember:

- No shearing forces
- All forces at right angles to the surfaces



For simplicity, the x-pressure forces cancel and do not need to be shown. Thus to arrive at our solution we **balance only the y and z forces:**

$$\delta y = \delta s \cos\theta \qquad \delta z = \delta s \sin\theta$$

Pressure Force in the y-direction on the y-face

Pressure Force on the plane in the y-direction

$$\sum F_y = p_y\, \delta x\, \delta z - p_s\, \delta x\, \delta s \sin\theta = 0$$

$$p_y = p_s$$

2.3 Pascal's Law for Pressure at a Point:

$$\sum F_z = p_z\,\delta x\,\delta y - p_s\,\delta x\,\delta s\cos\theta - \gamma\frac{\delta x\,\delta y\,\delta z}{2} = \mathbf{0}$$

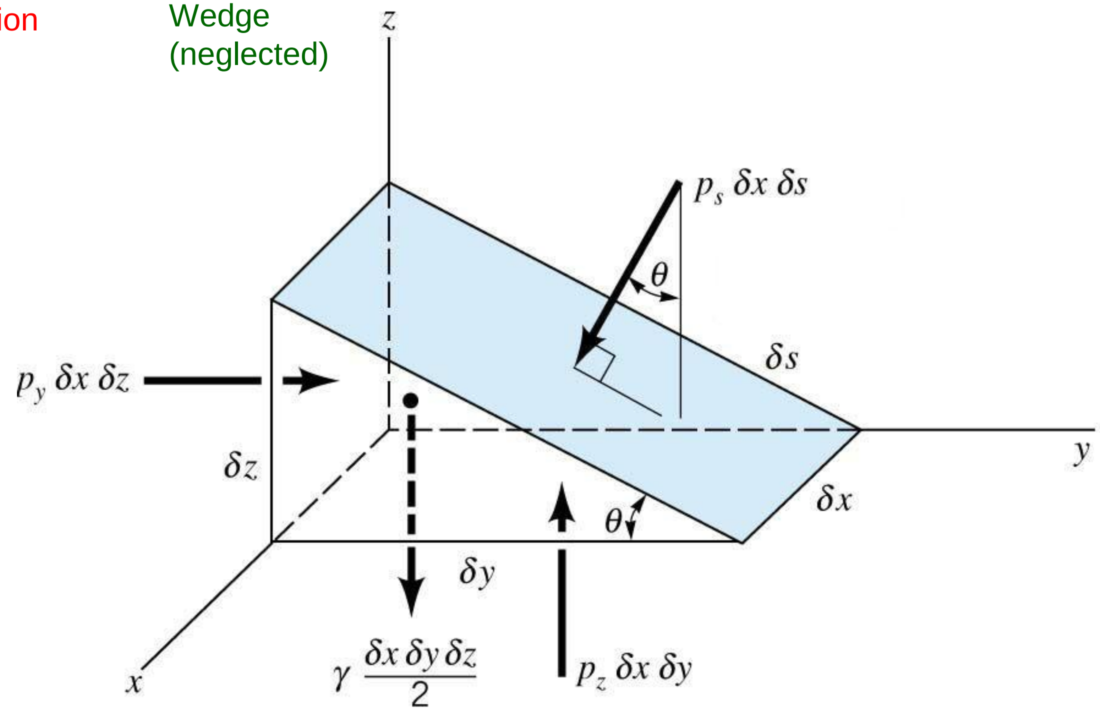
Pressure Force in the z-direction on the z-face

Pressure Force in the plane in the z-direction

Weight of the Wedge (neglected)

The element is small i.e. δx , δy and δz are small, and so **$\delta x\ \delta y\ \delta z$ is very small and considered negligible**

$$\Rightarrow \quad p_z = p_s$$



2.3 Pascal's Law for Pressure at a Point:

Thus:

$$p_x = p_y = p_s$$

Pressure at any point is the *same* in all directions.

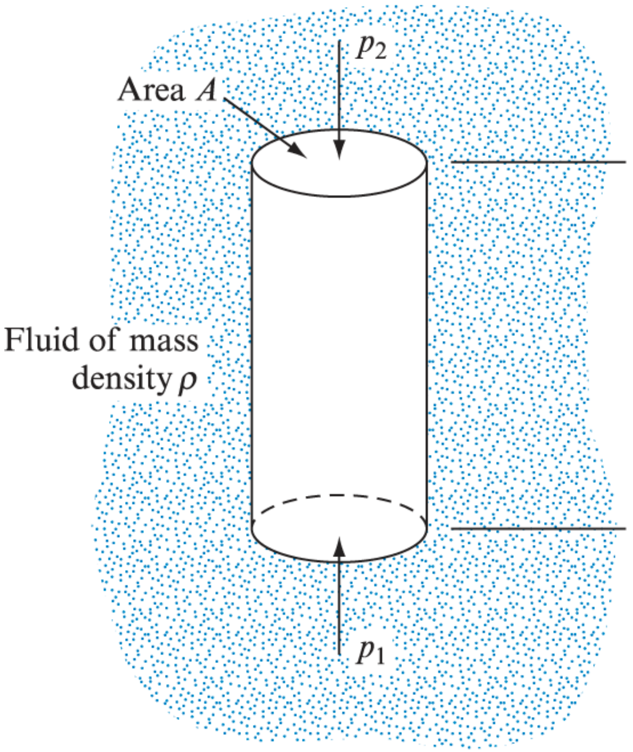
This is known as **Pascal's Law** and applies to fluids at rest.

2.4 Variation of Pressure Vertically in a Fluid under Gravity

The pressure at:

- the bottom of the cylinder is p_1 at level z_1
- the top of the cylinder is p_2 at level z_2

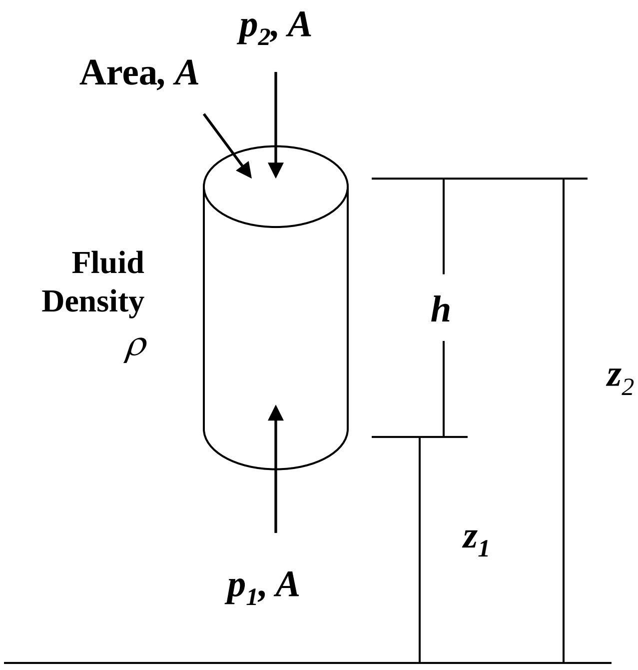
The fluid is at rest and in equilibrium so **all the forces in the vertical direction sum to zero.**



Vertical elemental cylinder of fluid

The forces involved are:

- Force due to p_1 on A (upward) = $\boldsymbol{p_1 A}$
- Force due to p_2 on A (downward) = $\boldsymbol{p_2 A}$
- Force due to weight of element (downward) = $mg = \rho g A(z_2 - z_1)$



$$p_1 A - p_2 A - \rho g A(z_2 - z_1) = 0$$

$$p_2 - p_1 = -\rho g (z_2 - z_1)$$

Thus in a fluid under gravity, pressure decreases linearly with increase in height (h)

Thus, in any fluid under gravity:

an increase in elevation causes a decrease in pressure.

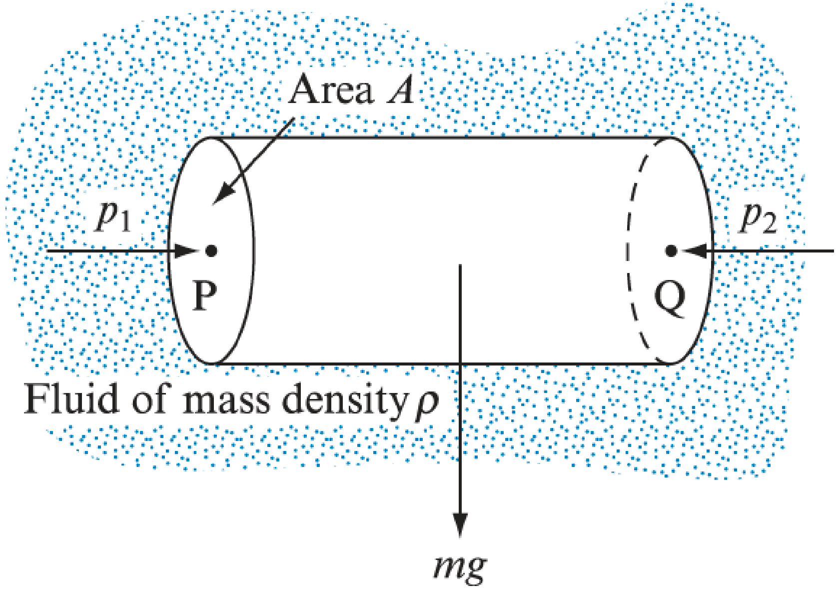
a decrease in elevation causes an increase in pressure.

2.5 Equality of Pressure at the Same Level in a Static Fluid:

For equilibrium
the sum of the forces in the *x* direction is zero.

$$p_l A = p_r A$$

$$p_l = p_r$$



Horizontal cylinder elemental of fluid

Pressure in the horizontal direction is constant

This result is the same for any *continuous* fluid

2.5 Equality of Pressure at the Same Level in a Static Fluid:

- It is still true for two connected tanks which appear not to have any direct connection.

We have shown:

$$p_R = p_S$$

For a vertical pressure change we have:

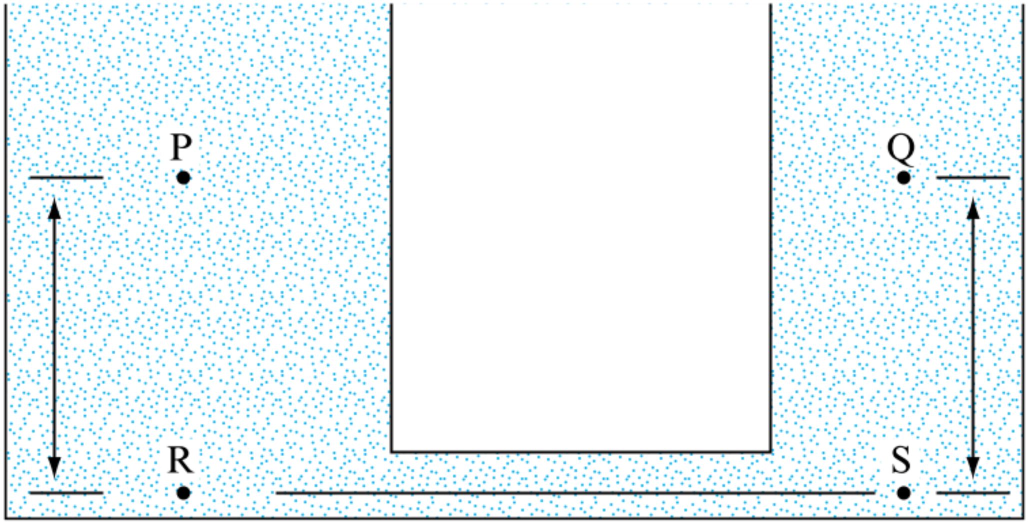
$$p_R = p_P + \rho g z$$

and

$$p_S = p_Q + \rho g z$$

so

$$p_P = p_Q$$



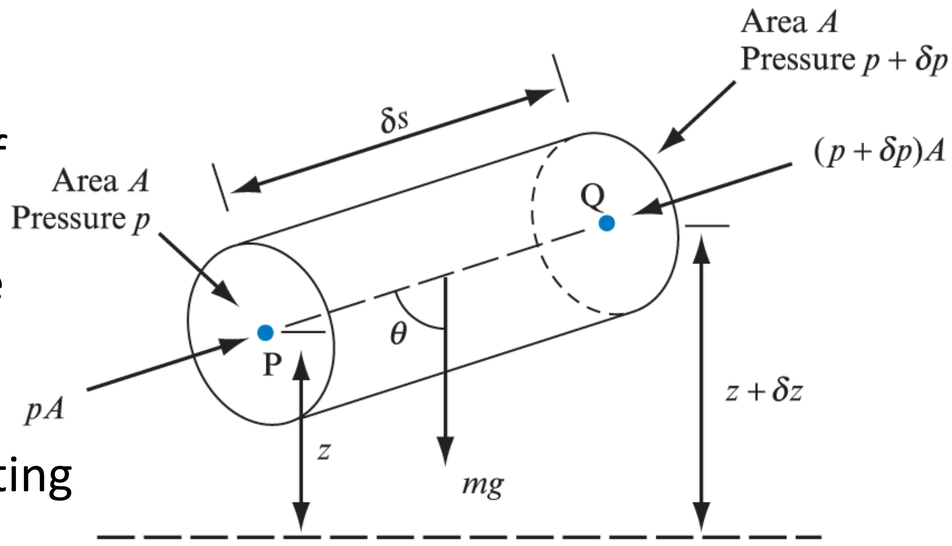
*Equality of pressures in a **continuous** body of fluid*

Pressure at the two equal levels are the same.

2.6 General Equation for Variation of Pressure in a Static Fluid

The forces acting on the element are:

- *pA* acting at right angle to the end of the face at *z*
- $(p + \delta p)\,A$ acting at right angle to the end of the face at $z + \delta z$
- $mg=\rho A\,\delta s\,g$
- forces from the surrounding fluid acting normal to the sides of the element.



Resolving the forces in the direction along the central axis gives:

$$pA - (p + \delta p)A - \rho A\,\delta s\,g\cos\theta = 0$$

$$\delta p = -\rho g\,\delta s\cos\theta$$

In the differential form:

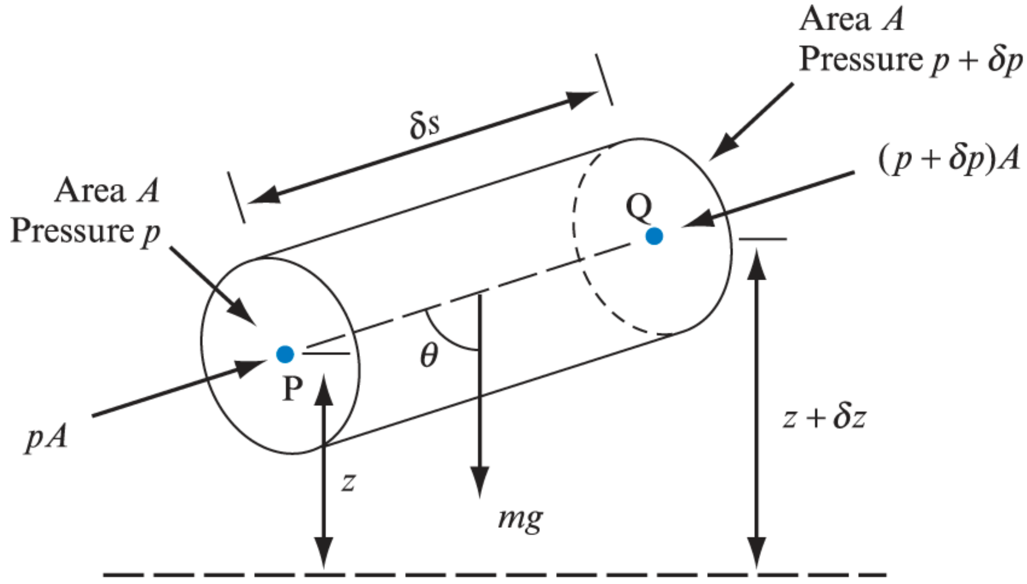
$$\frac{dp}{ds} = -\rho g\cos\theta$$

Horizontal

If $\theta = 90^o$, then *s* is in the *x* or *y* directions, so:

$$\left(\frac{dp}{ds}\right)_{\theta=90^\circ} = \frac{dp}{dx} = \frac{dp}{dy} = 0$$

Confirming that pressure on any horizontal plane is zero.



Vertical

If $\theta = 0^o$ then *s* is in the *z* direction so:

$$\left(\frac{dp}{ds}\right)_{\theta=0^\circ} = \frac{dp}{dz} = -\rho g$$

Confirming the result

$$\frac{p_2 - p_1}{z_2 - z_1} = -\rho g$$

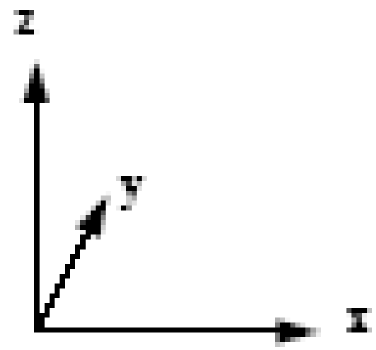
$$p_2 - p_1 = -\rho g(z_2 - z_1)$$

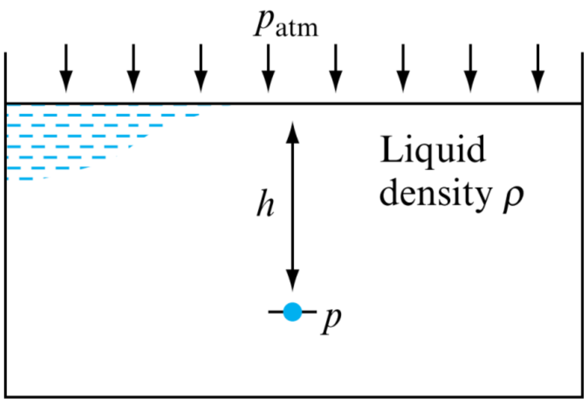
2.13 Pressure And Head

- In a static fluid of constant density we for vertical pressure the relationship:

$$\frac{dp}{dz} = -\rho g$$

- This can be integrated to give $p = -\rho g z + \text{constant}$





- measuring *z* from the free surface so that *z* = -*h*

$$p = \rho g h + \text{constant}$$

- The pressure at the free surface of fluids are normally be atmospheric pressure, $p_{\text{atmospheric}}$. So:

$$p = \rho gh + p_{atm}$$

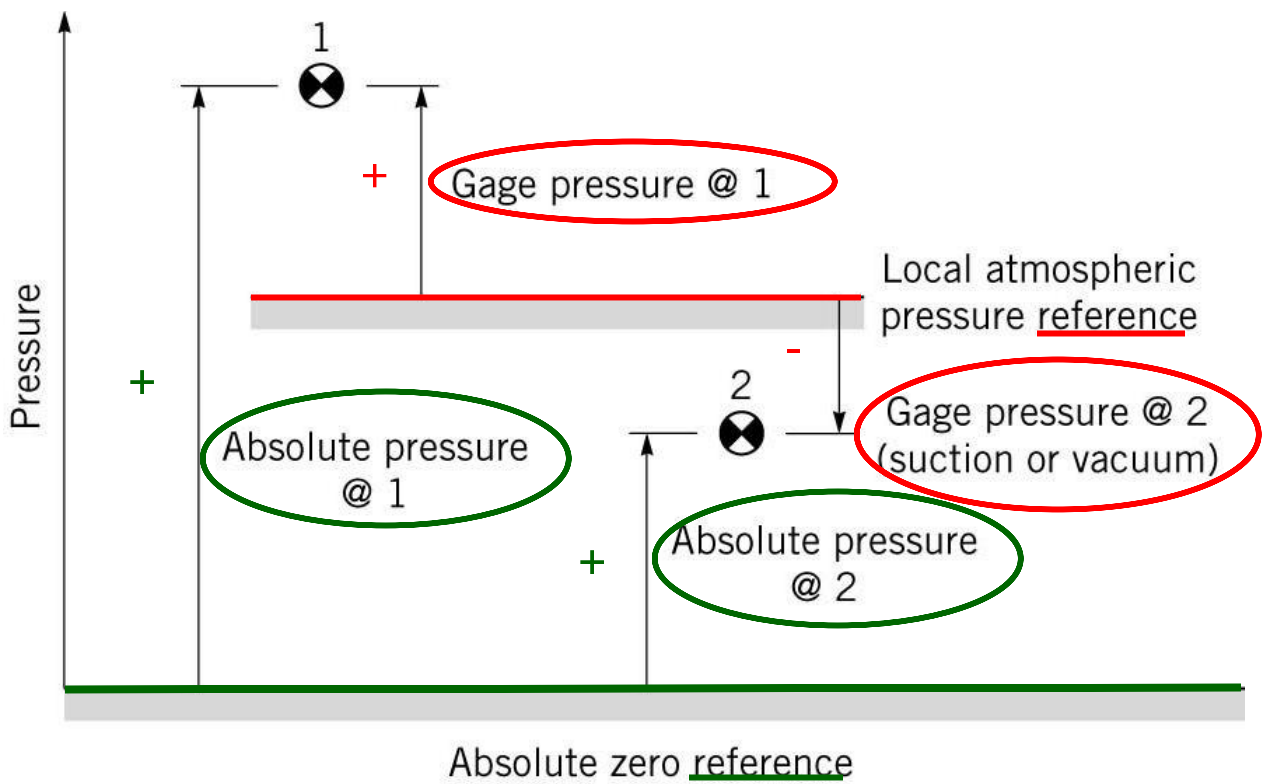
- As we live constantly under the pressure of the atmosphere, and everything else exists under this pressure, **it is convenient to take atmospheric pressure as the datum**. So we quote pressure above or below atmospheric.
- Pressure quoted in this way is known as **gauge pressure** i.e.

$$p_{gauge} = \rho gh$$

- The lower limit of any pressure is the pressure in a perfect vacuum.
- Pressure measured above a perfect vacuum (zero) is known as **absolute pressure.**

$$p_{absolute} = \rho g h + p_{atmospheric}$$

$$p_{absolute} = p_{gauge} + p_{atmospheric}$$



Summary: Absolute and Gauge Pressure

Pressure measurements are generally indicated as being either absolute or gauge pressure.

Gauge pressure

- is the pressure measured above or below the atmospheric pressure (i.e. taking the atmospheric as datum).
- can be positive or negative.
- a negative gauge pressure is also known as vacuum pressure.

Absolute pressure

- uses absolute zero, which is the lowest possible pressure.
- therefore, an absolute pressure will always be positive.
- a simple equation relating the two pressure measuring system can be written as:

$$P_{abs} = P_{gauge} + P_{atm}$$

Atmospheric pressure

- refers to the prevailing pressure in the air around us.
- It varies somewhat with changing weather conditions, and it decreases with increasing altitude.
- At sea level, average atmospheric pressure is 101.3 kPa (abs), 14.7 psi (abs), or 1 atmosphere (1 bar = $1x10^5$ Pa).
- This is commonly referred to as '**standard atmospheric pressure**'.

- Taking $p_{atm} = 0$

$$p = \rho g h$$

Which indicate that:

- As g is (approximately) constant, the gauge pressure can be given by **stating the vertical height of any *fluid*** of density ρ which is equal to this pressure.

- This vertical height is known as **head** of fluid.

Note: If pressure is quoted in *head*, the density of the fluid *must* also be given.

Example:

What is a pressure of 500 kN/m^2 in head of:

- water of density, ρ = 1000 kg/m^3

$$p = \rho g h \quad \Rightarrow \quad h = \frac{p}{\rho g}$$

$$h = \frac{500 \times 10^3}{1000 \times 9.81} = 50.95 \text{ m of water}$$

- mercury of density, ρ = 13600 kg/m^3

$$h = \frac{500 \times 10^3}{13600 \times 9.81} = 3.75 \text{ m of mercury}$$

2.14 The hydrostatic paradox

- The pressure exerted by the fluid depends only on:
  - vertical head of fluid (h).
  - type of fluid (density ρ).
  - **not** on the weight of the fluid present.
- Therefore, all the containers shown would have the same pressure at the bottom – no matter what the size or shape of container and how much fluid they contained.
- This observation is called **Pascal's Paradox**.

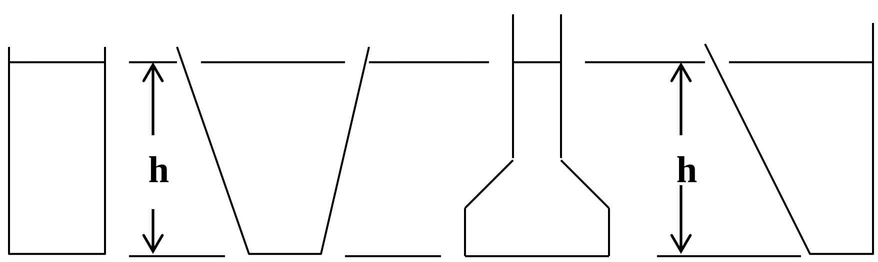
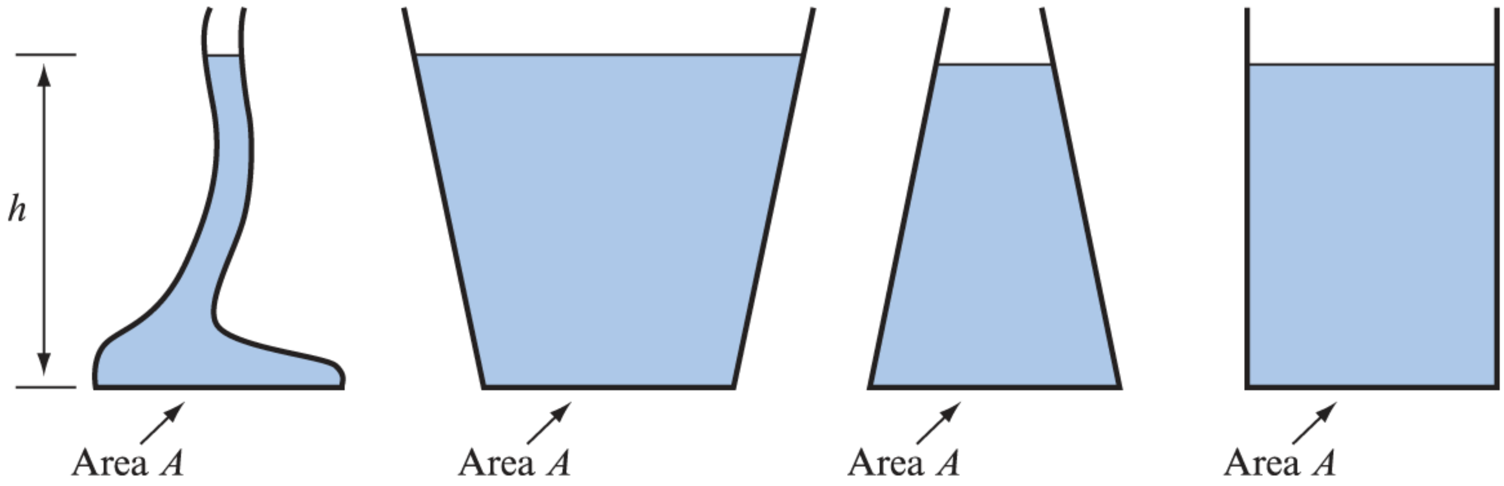


Illustration of Pascal's Paradox

Pressure is the same at the bottom of container: $p=\rho gh$

If the vessels have the **same base area *A***, and filled to the **same height *h*** then the forces are equal



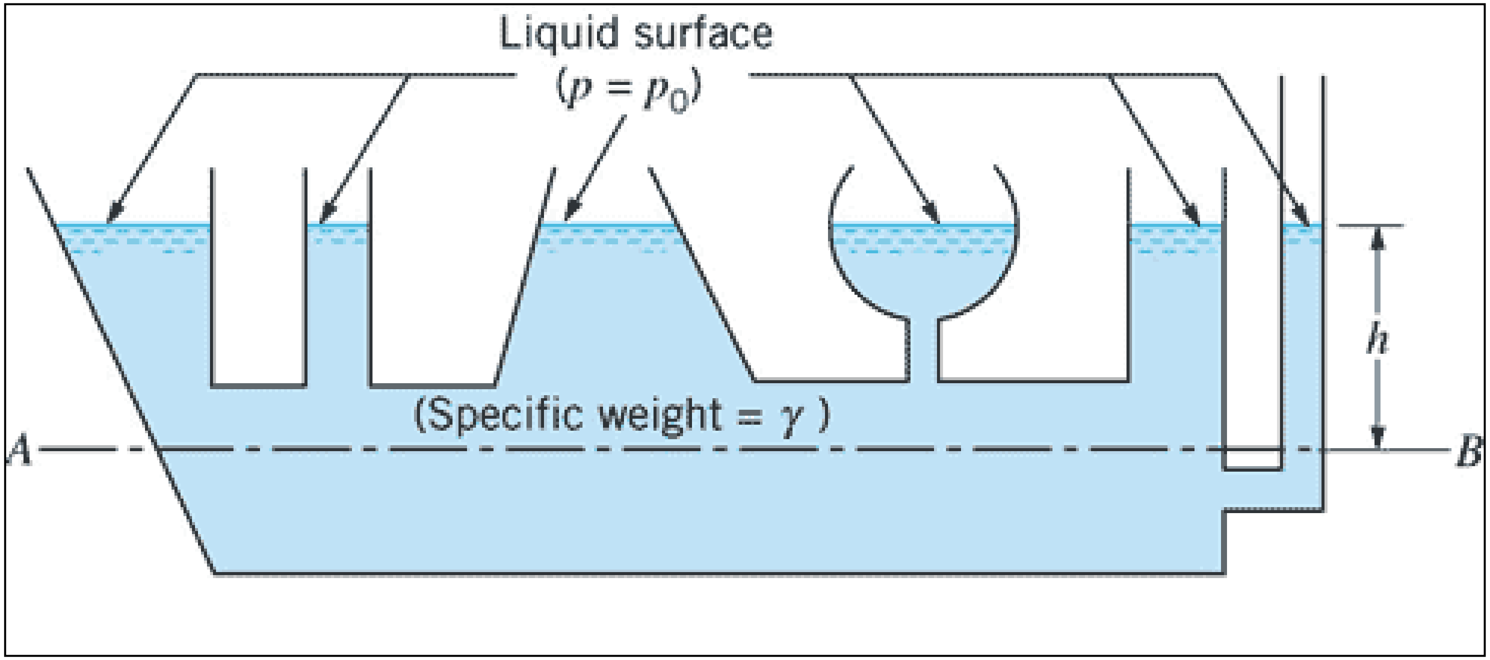
Pressure on the bottom in each case $p = \rho g h$

Force on the bottom = pressure . Area = $\rho g h A$

Although the weight of the fluid varied in each case, the force on the base is the same , depending on *h*, and *A*

Example:

Rank them according to the **pressure** at depth h, greatest first.



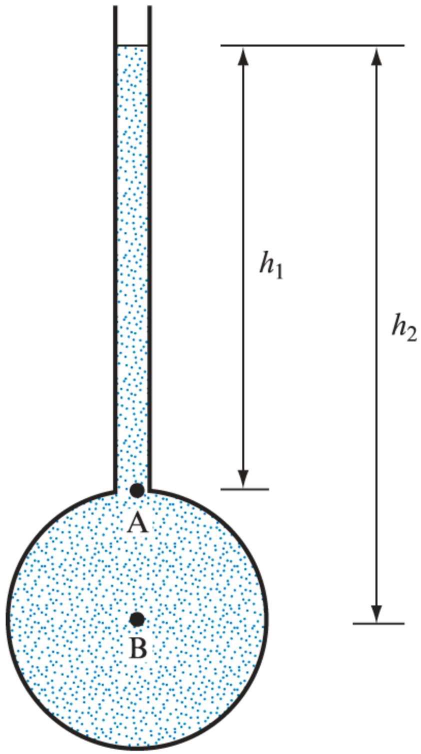
2.15 Pressure Measurement By Manometer:

- The relationship between pressure and head is used to measure pressure with a manometer (also know as a liquid gauge).

- In the following we demonstrate the analysis and use of various types of manometers for pressure measurement.

A. The Piezometer Tube Manometer

- The simplest manometer is a tube, open at the top, which is attached to the top of a vessel or pipe containing liquid at a pressure (higher than atmospheric) to be measured.
- As the tube is open to the atmosphere the pressure measured is relative to atmospheric so is **gauge pressure**.



Pressure at A = pressure due to column of liquid h1 $p_A = \rho g h_1$

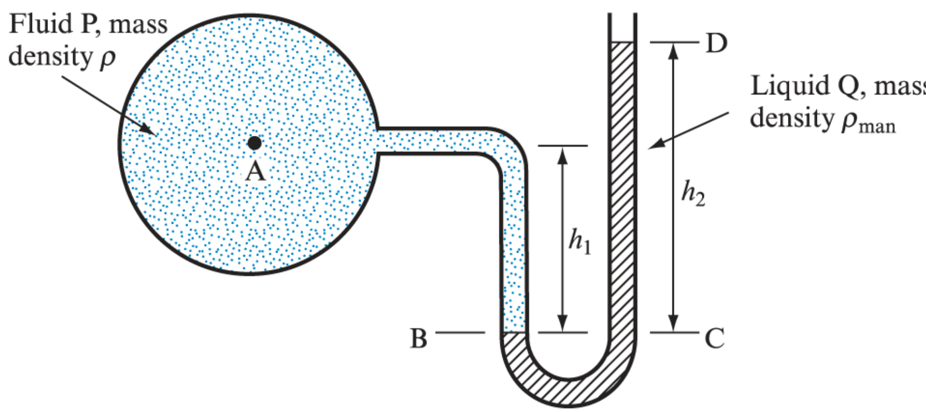
Pressure at B = pressure due to column of liquid h2 $p_B = \rho g h_2$

Problems with the Piezometer:

1. It cannot measure suction pressure which is lower than the atmospheric pressure.
2. The pressure measured is limited by available column height.
3. It can only deal with liquids, not gases.

B. The "U" Tube Manometer

- Using a "U" Tube enables the pressure of both **liquids and gases** to be measured with the same instrument.
- The "U" is connected as shown and filled with a fluid called the ***manometric fluid***.



Important points:

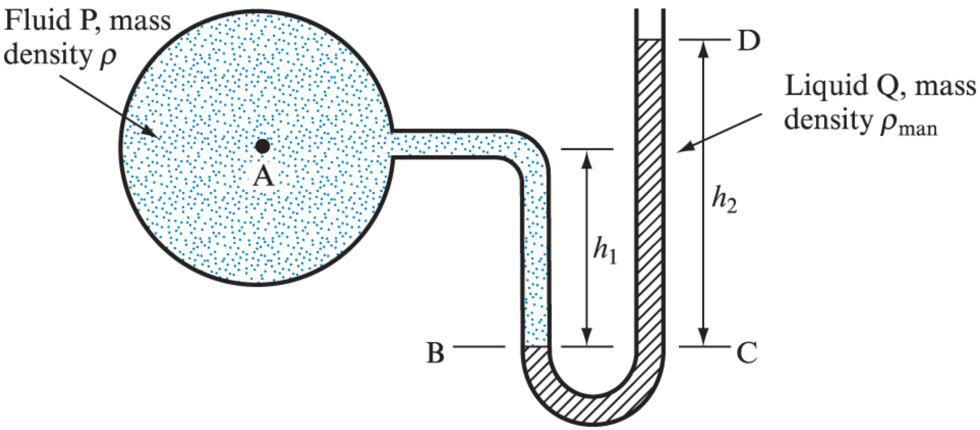
1. The manometric fluid density should be greater than of the fluid measured.

$$\rho_{man} > \rho$$

2. The two fluids should not be able to mix, they must be immiscible.

- Pressure in a continuous static fluid is the same at any horizontal level so:
- pressure at *B* = pressure at *C*

$$p_B = p_C$$



Pressure at *B* = pressure at *A* + pressure due to height h_1 of fluid being measured

$$p_B = p_A + \rho g h_1$$

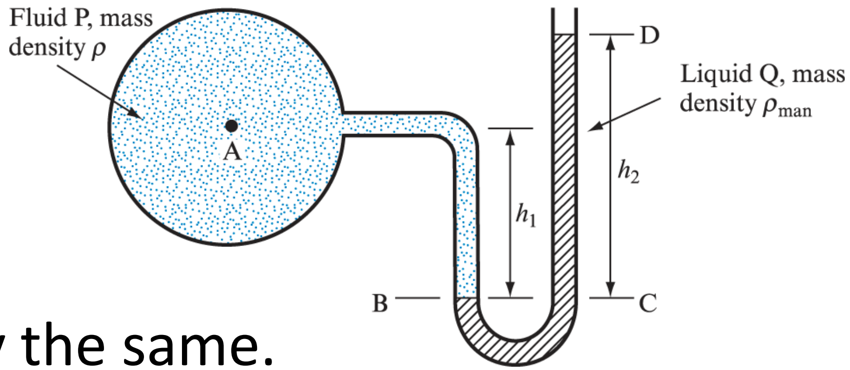
Pressure at *C* = pressure at *D* + pressure due to height h_2 of manometric fluid

$$p_C = p_D + \rho_{man} g h_2$$

But; p_D = Atmospheric pressure = Zero gauge pressure

$$p_C = 0 + \rho_{man} g h_2$$

$$\Rightarrow \quad p_A + \rho g h_1 = \rho_{man} g h_2 \quad \Rightarrow \quad \boxed{p_A = \rho_{man} g h_2 - \rho g h_1}$$



What if the fluid is a gas?

- Nothing changes.
- The manometer work exactly the same.

But:

- the density of gas will probably be very low in comparison to the density of the manometric fluid i.e.

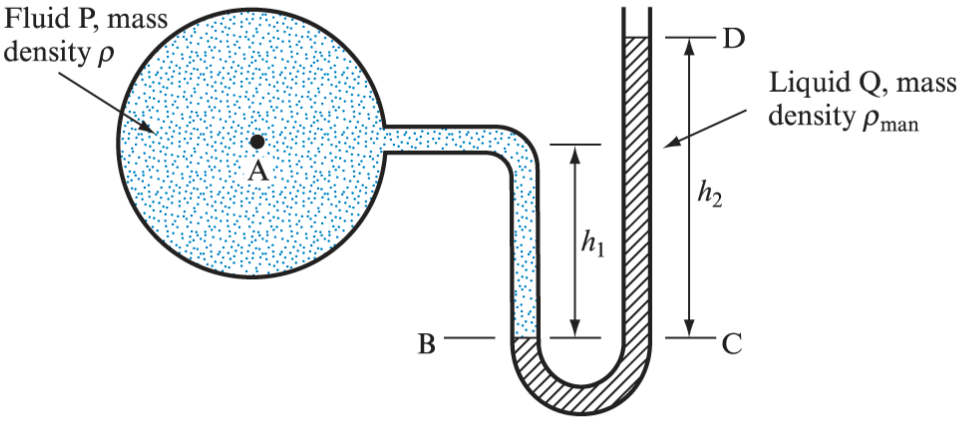
$$\rho_{man} >> \rho$$

- In this case the term $\rho g h_1$ can be neglected, and the gauge pressure is given by:

$$p_A = \rho_{man} g h_2$$

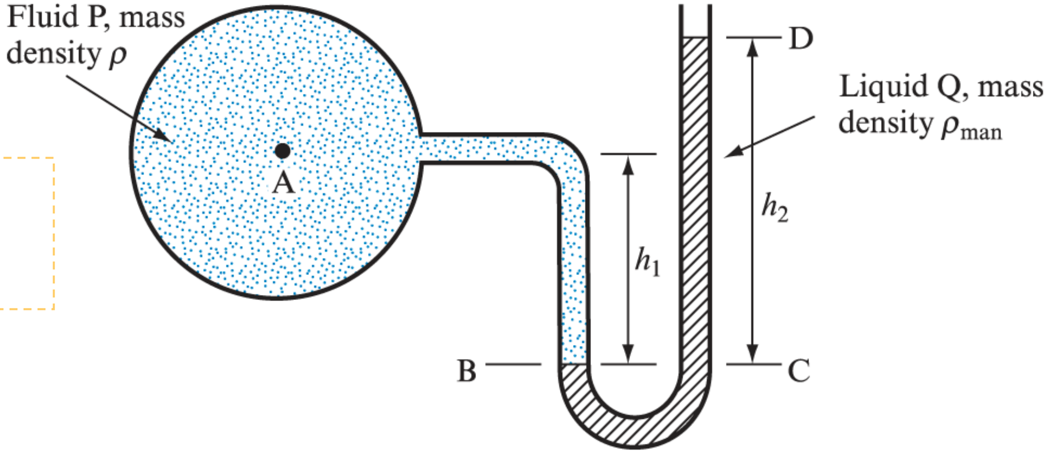
Example:

- Using a U-tube manometer to measure gauge pressure of fluid density $\rho = 700\ kg/m^3$, and the manometric fluid is mercury, with a relative density of 13.6. What is the gauge pressure if:



- $h_1 = 0.4$m and $h_2 = 0.9$m?
- h_1 stayed the same but $h_2 = -0.1$m?

Solution:



h_1 = 0.4m and h_2 = 0.9m?

h_1 stayed the same but h_2 = -0.1m?

$$p_B = p_C$$

$$p_B = p_A + \rho g h_1$$

$$p_B = p_{Atmospheric} + \rho_{man}\, g h_2$$

We are measuring *gauge* pressure so $p_{atmospheric} = 0$

$$p_A = \rho_{man}\, g h_2 - \rho g h_1$$

a) $p_A = 13.6 \times 10^3 \times 9.81 \times 0.9 - 700 \times 9.81 \times 0.4 = 117\,327\ N/m^2\ (1.17\ bar)$

b) $p_A = 13.6 \times 10^3 \times 9.81 \times (-0.1) - 700 \times 9.81 \times 0.4 = -16\,088.4\ N/m^2, (-0.16\ bar)$

The negative sign indicates that the pressure is *below atmospheric*

Measurement of Pressure Difference using a "U" Tube Manometer

The "U"-tube manometer **can be connected at both ends** to measure *pressure difference* between these two points

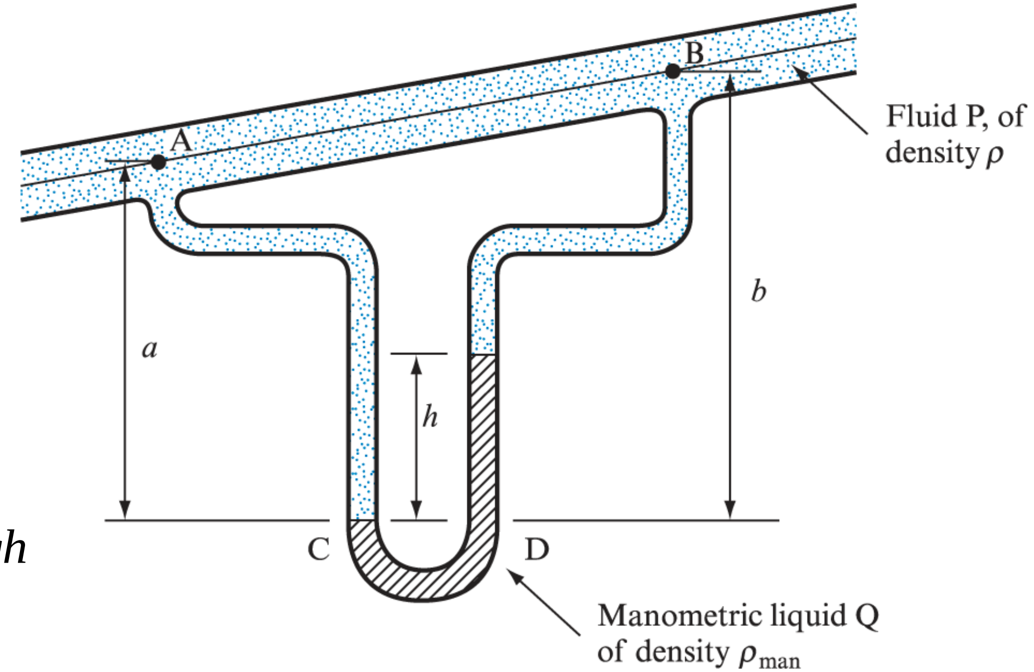
pressure at *C* = pressure at *D*

$$p_C = p_D$$

$$p_C = p_A + \rho g a$$

$$p_D = p_B + \rho g(b-h) + \rho_{man} g h$$

$$p_A + \rho g a = p_B + \rho g(b-h) + \rho_{man} g h$$



$$\Rightarrow \quad p_A - p_B = \rho g(b-a) + (\rho_{man} - \rho) g h$$

- If the fluid whose pressure difference is being measured is a gas and $\rho_{\text{man}} >> \rho$.
- Then the terms involving ρ can be neglected, so:

$$p_A - p_B = \rho_{man} g h$$

return

Example:

- A differential "U"-tube manometer containing mercury of density 13000 kg/m^3 is used to measure the pressure drop along a horizontal pipe.
- If the fluid in the pipe is water and the manometer reading is 0.65*m*, what is the pressure difference between the two tapping points?

pressure at C and D is equal:

$$p_C = p_D$$

$$p_A + \rho_w g\, a = p_B + \rho_w g\,(b - h) + \rho_{Hg} g\, h$$

$$p_A - p_B = \rho_w g\, b - \rho_w g\, h - \rho_w g\, a + \rho_{Hg} g\, h$$

$$= \rho_w g\,(b - a) + hg\,(\rho_{Hg} - \rho_w)$$

As horizontal a = b

$$p_A - p_B = hg\,(\rho_{Hg} - \rho_w)$$

$$= 0.65 \times 9.81 \times (13000 - 1000)$$

$$= 76\,518\ N/m^2$$

$$= 76.5\ kN/m^2$$

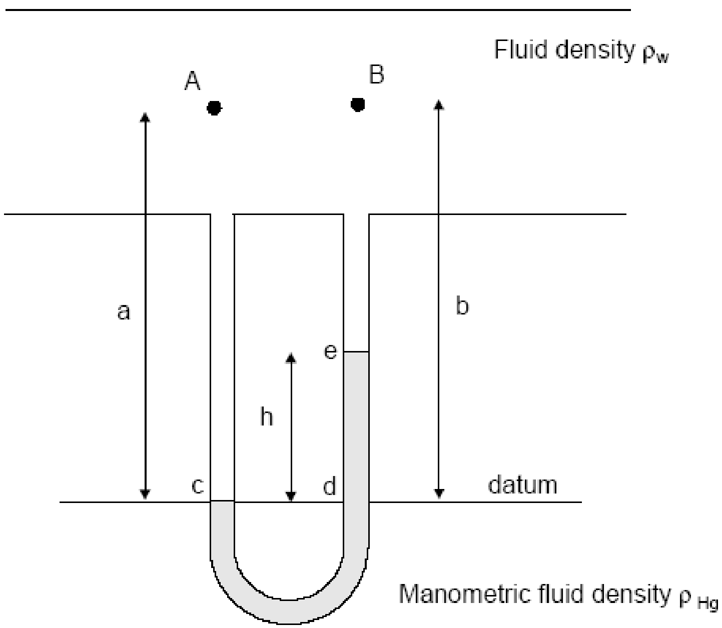
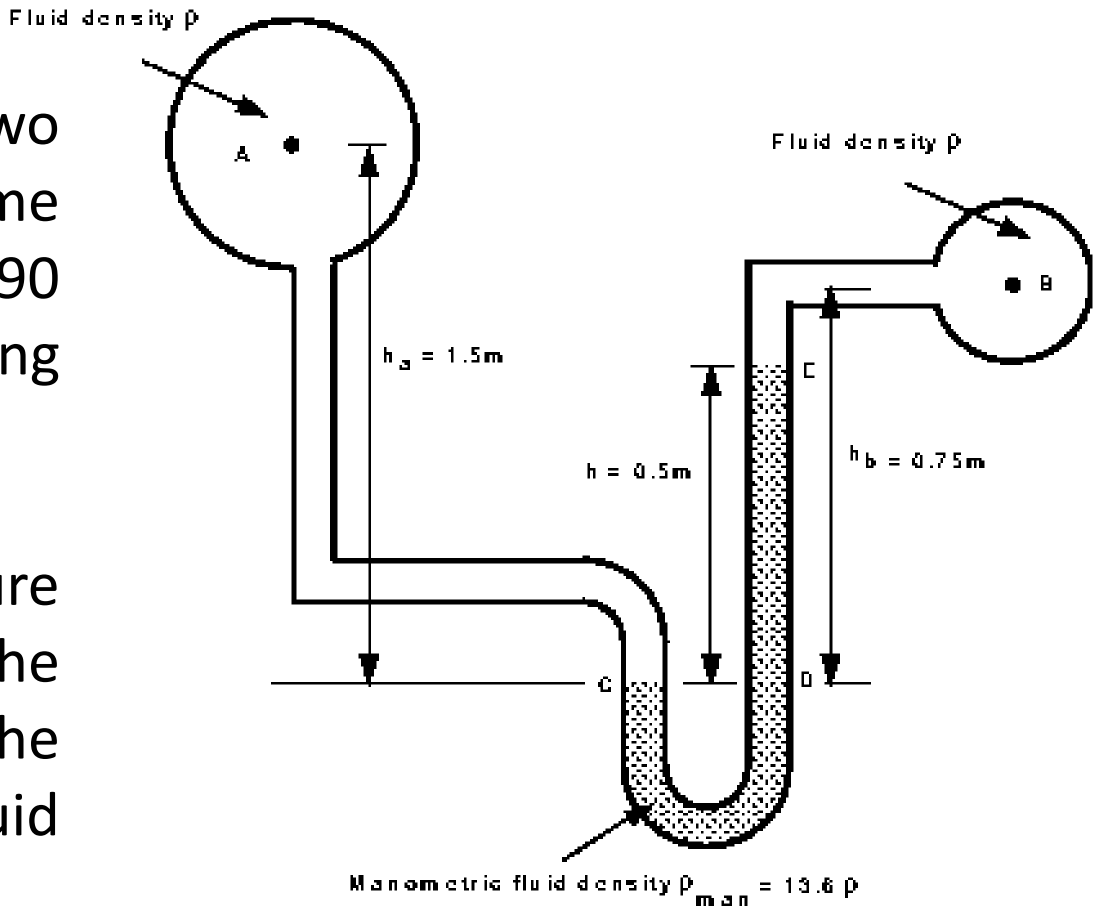


Figure of manometer setup

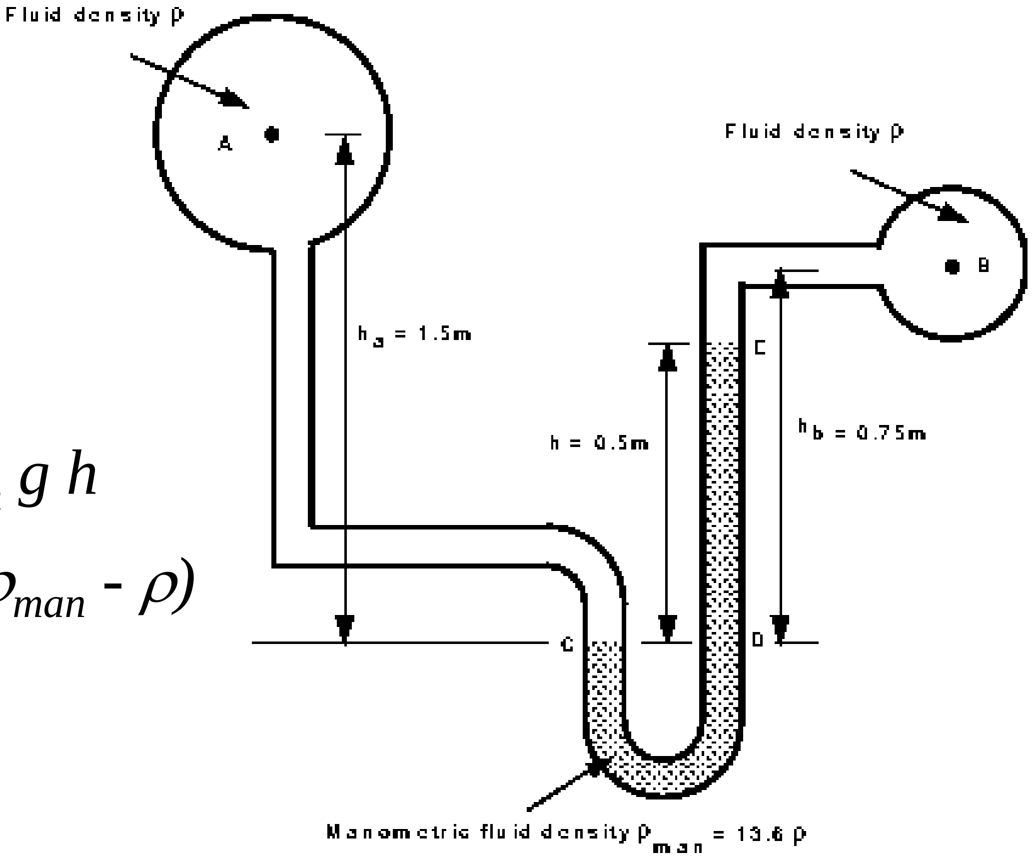
Example:

In the figure below two pipes containing the same fluid of density ρ = 990 kg/m^3 are connected using a u-tube manometer.

What is the pressure difference between the two pipes if the manometer contains fluid of relative density 13.6?



Solution:



$$p_C = p_D$$

$$p_C = p_A + \rho g\, h_A$$

$$p_D = p_B + \rho g\,(h_B - h) + \rho_{man}\, g\, h$$

$$p_A - p_B = \rho g\,(h_B - h_A) + hg(\rho_{man} - \rho)$$

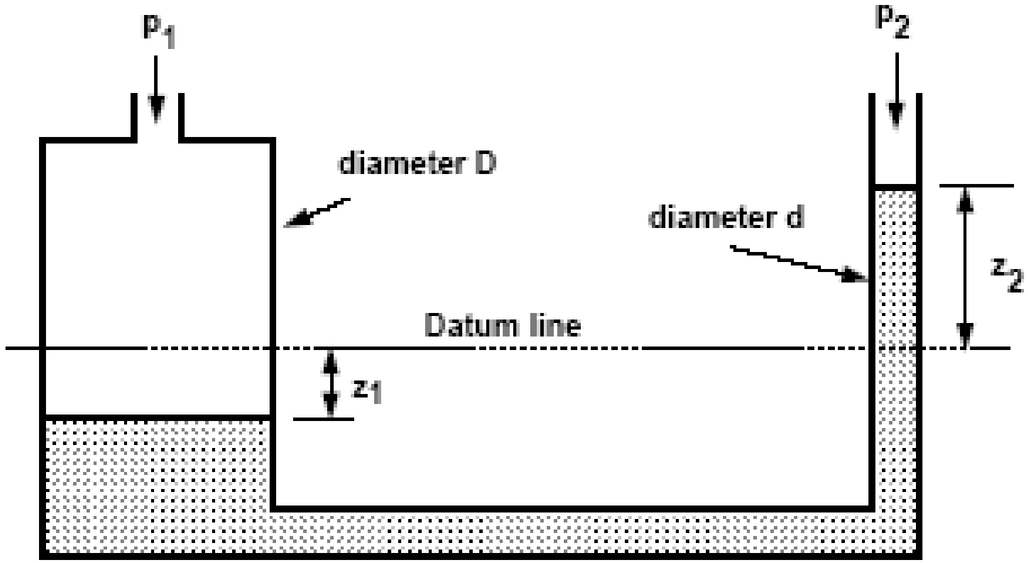
$$= 990 \times 9.81 \times (0.75-1.5) + 0.5 \times 9.81 \times (13.6-0.99) \times 10^3$$

$$= -7284 + 61852$$

$$= 54\,568\ N/m^2 \text{ (or Pa or 0.55 bar)}$$

Advances to “U” Tube Manometer

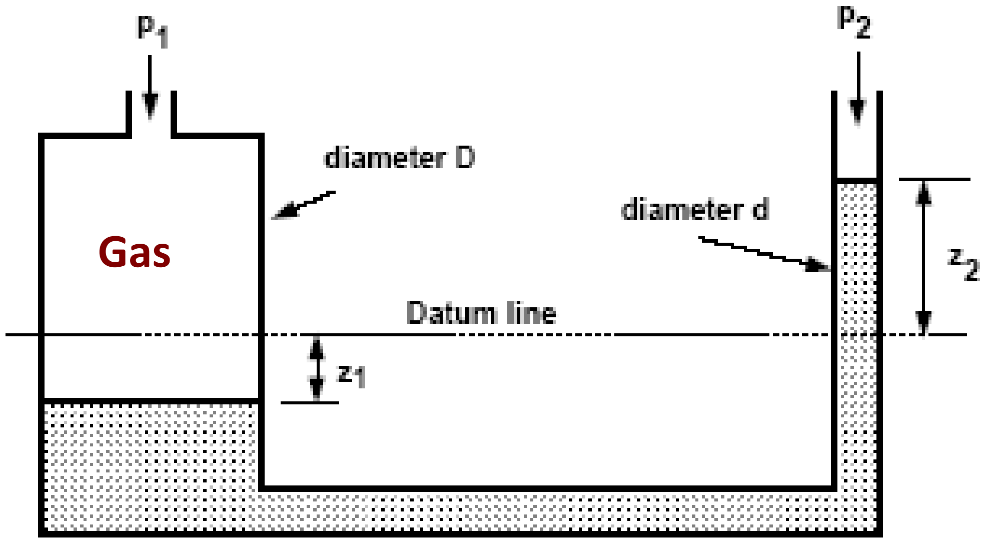
- The “U” tube manometer has the disadvantage that the **change in height of the liquid in both sides must be read**.
- This can be avoided by **making the diameter of one side very large compared to the other**.
- In this case the side with the large area moves very little when the small area side move considerably more.



If the manometer is measuring the pressure difference of a **gas** of $(p_1 - p_2)$ as shown,
we know:

$$p_1 - p_2 = \rho_{man} g h$$

go



Volume of liquid moved from the left side to the right: $= z_2 \times \frac{\pi}{4} d^2$

The fall in level of the left side is:

$$z_1 = \frac{\text{Volume moved}}{\text{Area of left side}} = \frac{z_2 \times \frac{\pi}{4} d^2}{\frac{\pi}{4} D^2} = z_2 \left(\frac{d}{D}\right)^2$$

Putting this in the equation above:

$$p_1 - p_2 = \rho_{man} g \left[z_2 + z_2 \left(\frac{d}{D}\right)^2 \right] = \rho_{man} g z_2 \left[1 + \left(\frac{d}{D}\right)^2 \right]$$

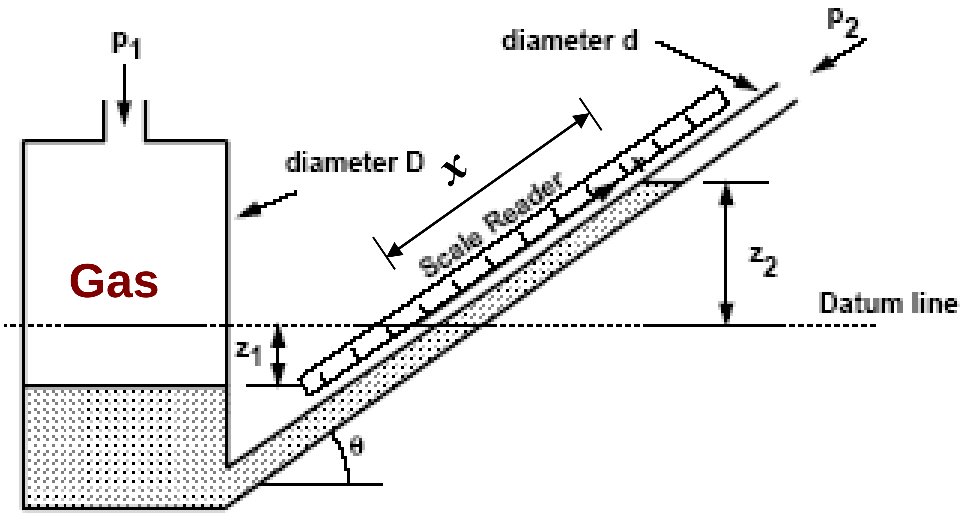
Clearly if D is very much larger than *d* then $(d/D)^2$ is very small so

$$p_1 - p_2 = \rho_{man} g z_2$$

only one reading need be taken to measure the pressure difference.

Inclined manometer

- If the pressure to be measured is **very small.**
- Then **tilting the arm** provides a convenient way of obtaining a larger (more easily read) movement of the manometer.



The pressure difference is still given by the height change of the manometric fluid.

$$p_1 - p_2 = \rho_{man} g z_2$$

- But $z_2 = x \sin\theta$
- Then $p_1 - p_2 = \rho_{man} g \, x \sin\theta$

- The sensitivity to pressure change can be increased further by a greater inclination of the manometer arm.

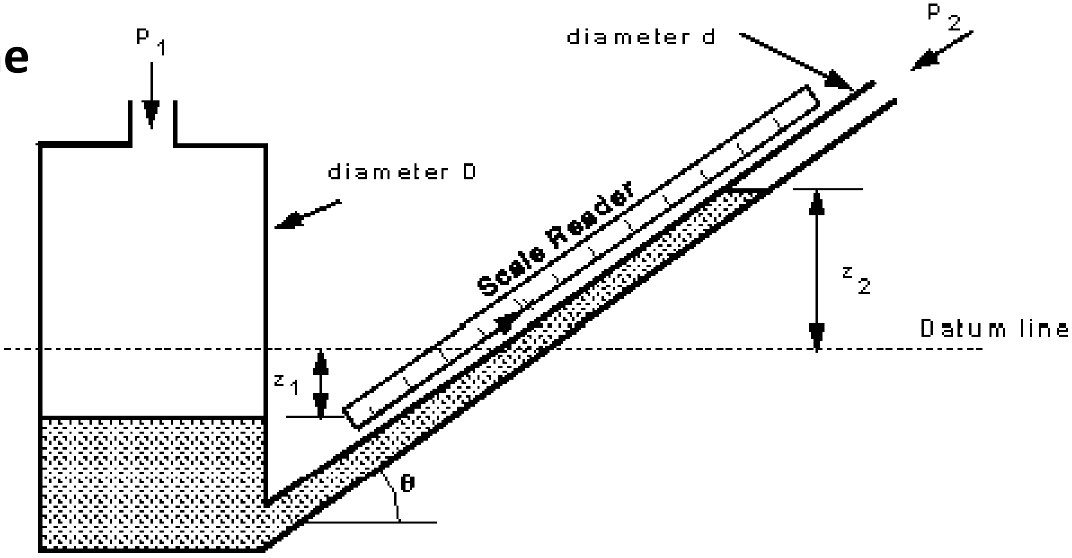
Alternative solution:

- Reduce density of manometric fluid.

Example:

An inclined tube manometer consists of a vertical cylinder 35mm diameter. At the bottom of this is connected a tube 5mm in diameter inclined upward at an angle of 15 to the horizontal, the top of this tube is connected to an air duct. The vertical cylinder is open to the air and the manometric fluid has relative density 0.785.

1. Determine the **pressure in the air duct** if the manometric fluid moved 50mm along the inclined tube.
2. What is the **error** if the movement of the fluid in the vertical cylinder is ignored?



Use the equation derived in the lecture for a manometer where $\rho_{man} >> \rho$.

$$p_1 - p_2 = \rho g h = \rho g(z_1 + z_2)$$

Where: $z_2 = x \sin \theta$, and

$A_1 z_1 = a_2 x$

$z_1 = x (d/D)^2$

where x is the reading on the manometer scale.

p_1 is atmospheric i.e. $p_1 = 0$

$$p_2 = -\rho g x\left(\sin\theta + \left(\frac{d}{D}\right)^2\right)$$

And $x = -50mm = -0.05m$,

$$-p_2 = 0.785\times10^3\times9.81\times(-0.05)\left[\sin 15 + \left(\frac{0.005}{0.035}\right)^2\right]$$

$$p_2 = 107.2\ N$$

If the movement in the large cylinder is ignored the term $(d/D)^2$ will disappear:

$$p_1 - p_2 = \rho g x \sin\theta$$

$$p_2 = 0.785\times10^3\times9.81\times0.05\times\sin 15$$

$$= 99.66\ N$$

So the error induced by this assumption is $error = \frac{107.2 - 99.66}{102.2}100 = 7.3\%$

The Inverted "U" Tube Manometer

Pressure at XX will be the same in both limb

- **For left hand limb:**

$$p_{XX} = p_A - \rho g a - \rho_{man} g h$$

- **For right hand limb:**

$$p_{XX} = p_B - \rho g (b + h)$$

- Thus:

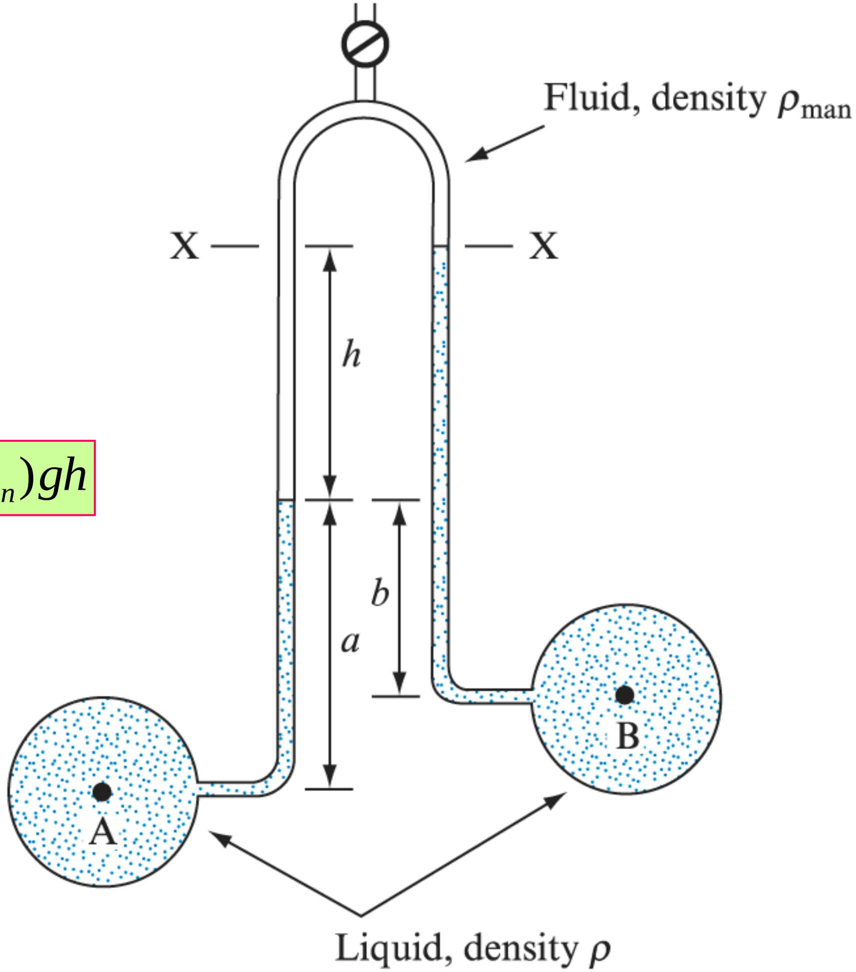
$$p_B - p_A = \rho g (b - a) + (\rho - \rho_{man}) g h$$

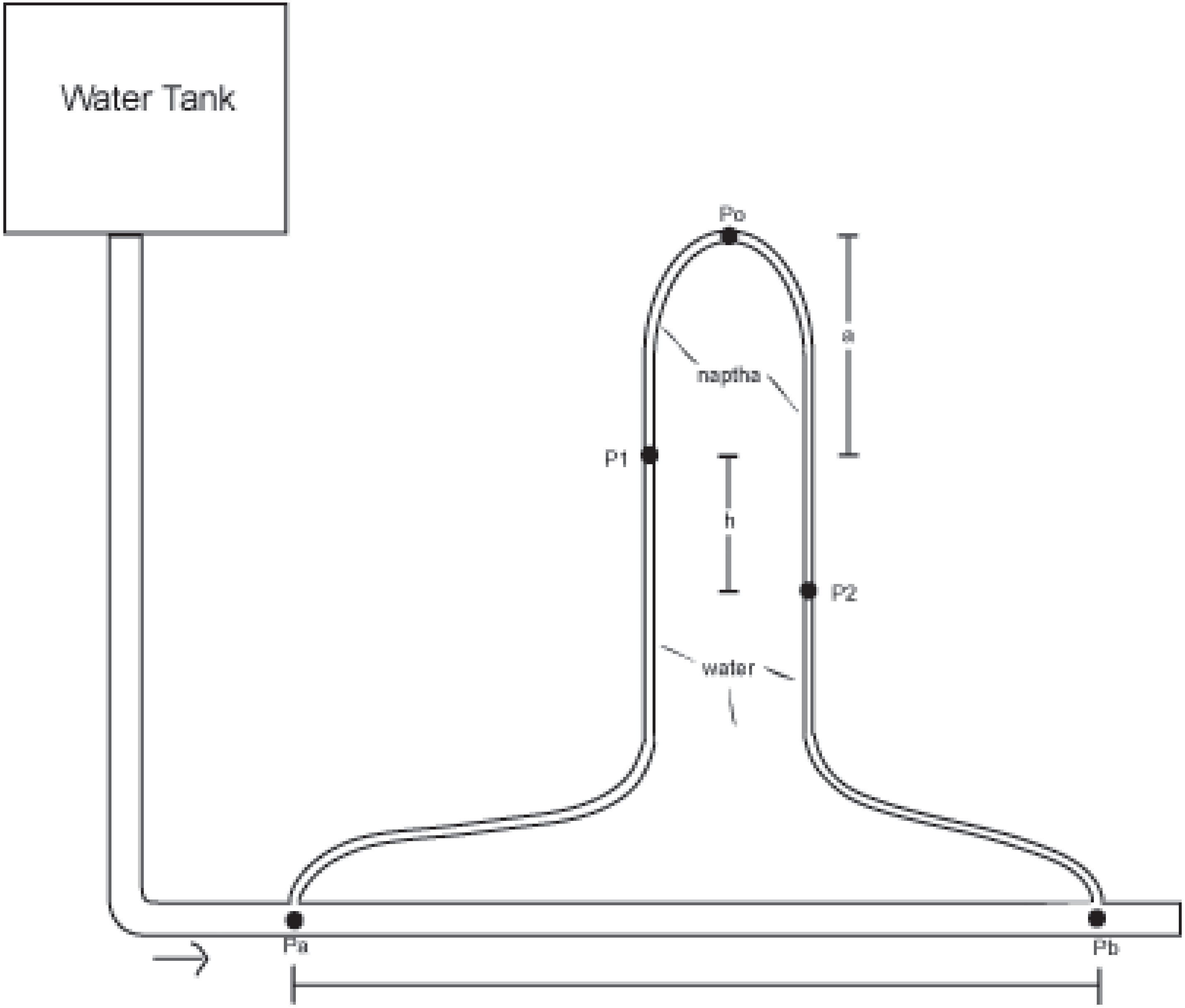
- Or, if *A* and *B* are at the same level:

$$p_B - p_A = (\rho - \rho_{man}) g h$$

- If the top of the tube is filled with air ρ_{man} is negligible compared to ρ and :

$$p_B - p_A = \rho g h$$




Water Tank
Po
a
naptha
P1
h
P2
water
Pa
Pb

Choice of Manometer

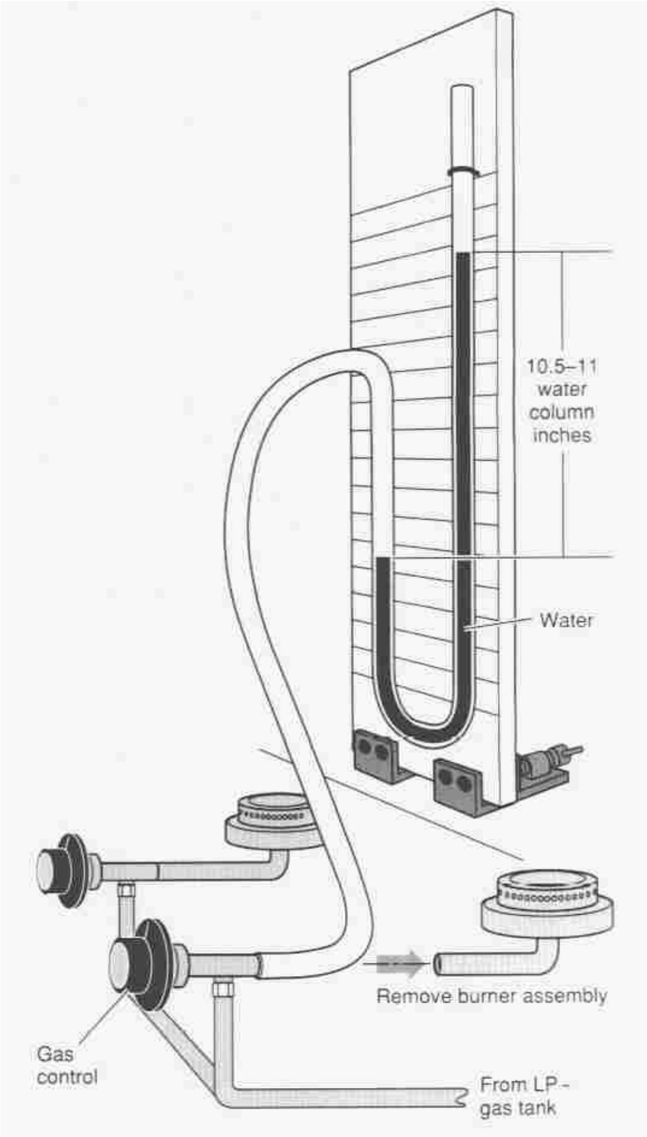
- Take care when fixing the manometer to vessel
- Burrs cause local pressure variations.

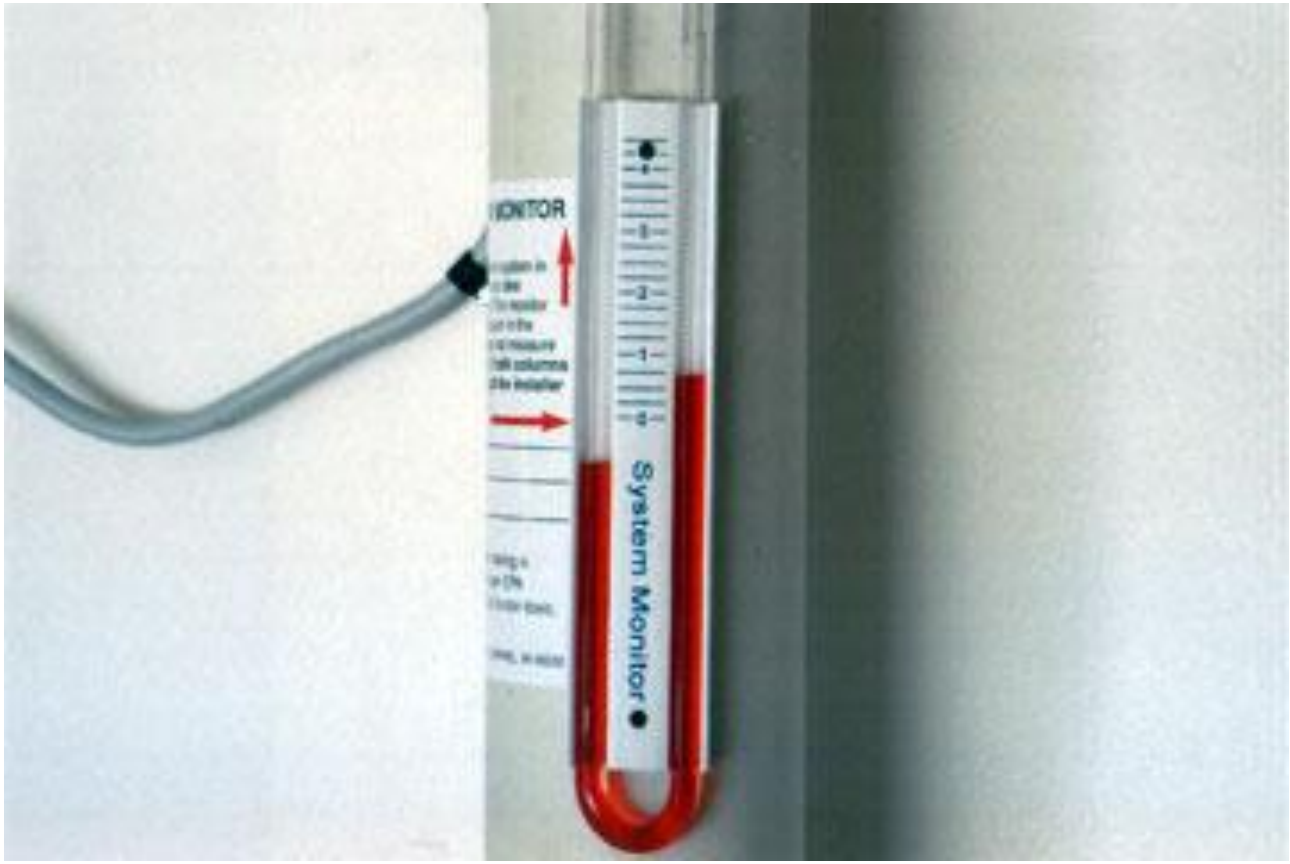
Advantages of manometers:

- They are very simple.
- No calibration is required - the pressure can be calculated from first principles.

Some disadvantages of manometers:

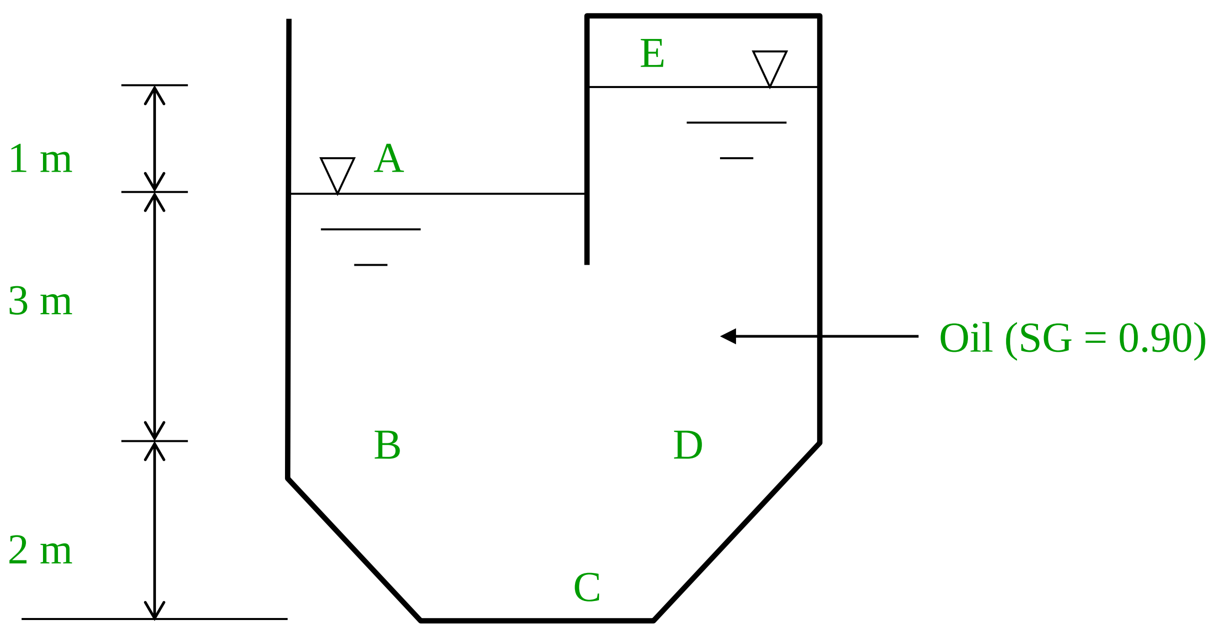
- Slow response - only really useful for very slowly varying pressures - no use at all for fluctuating pressures;
- For the "U" tube manometer two measurements must be taken simultaneously to get the *h* value. (This may be avoided by using a tube with a much larger cross-sectional area on one side of the manometer than the other;)
- It is often difficult to measure small variations in pressure. (a different manometric fluid may be required - alternatively a sloping manometer may be employed).
- It cannot be used for very large pressures unless several manometers are connected in series.
- For very accurate work the temperature and relationship between temperature and ρ must be known;


10.5–11
water
column
inches
Water
Remove burner assembly
Gas
control
From LP -
gas tank


System Monitor

Example:

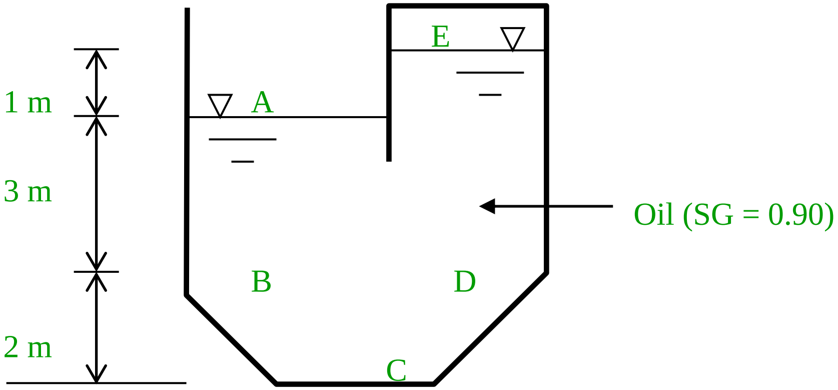
- Figure below shows a tank with one side open to the atmosphere and the other side sealed with air above the oil (SG=0.90). **Calculate the gauge pressure at points A,B,C,D,E.**



Solution:

point A, the oil is exposed to the atmosphere

thus $P_A = P_{atm} = 0$ (gauge)



Point B is 3 m below point A,

Thus $P_B = P_A + \rho_{oil} g h$

$= 0 + 0.9 \times 1000 \times 9.81 \times 3$

$= 26.5$ kPa (gauge)

Point C is 5 m below point A,

Thus $P_C = P_A + \rho_{oil} g h$

$= 0 + 0.9 \times 1000 \times 9.81 \times 5$

$= 44.15$ kPa (gauge)

Point D is at the same level of point B,

thus $P_D = P_B$

$= 26.5$ kPa (gauge)

Point E is higher by 1 m from point A,

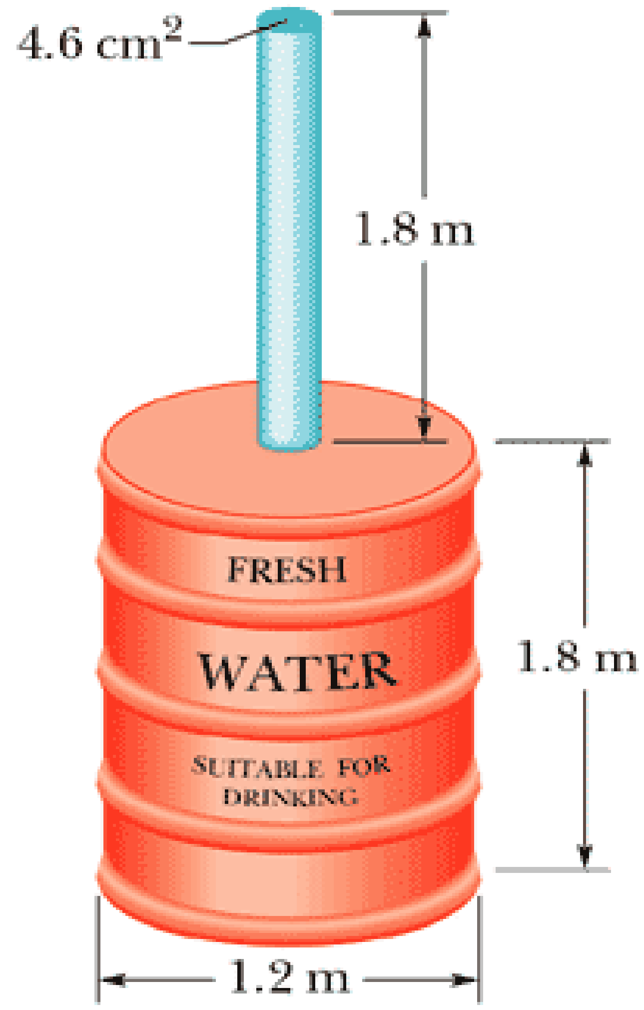
Thus $P_E = P_A - \rho_{oil} g h$

$= 0 - 0.9 \times 1000 \times 9.81 \times 1$

$= -8.83$ kPa (gauge).

Example:

- The figure shows the vessel which is **filled with water** to the top of the tube.
- Calculate the ratio of hydrostatic force at the bottom of the barrel to the gravitational force on the water contained inside the barrel ?
- Why the ratio is not equal to one ?

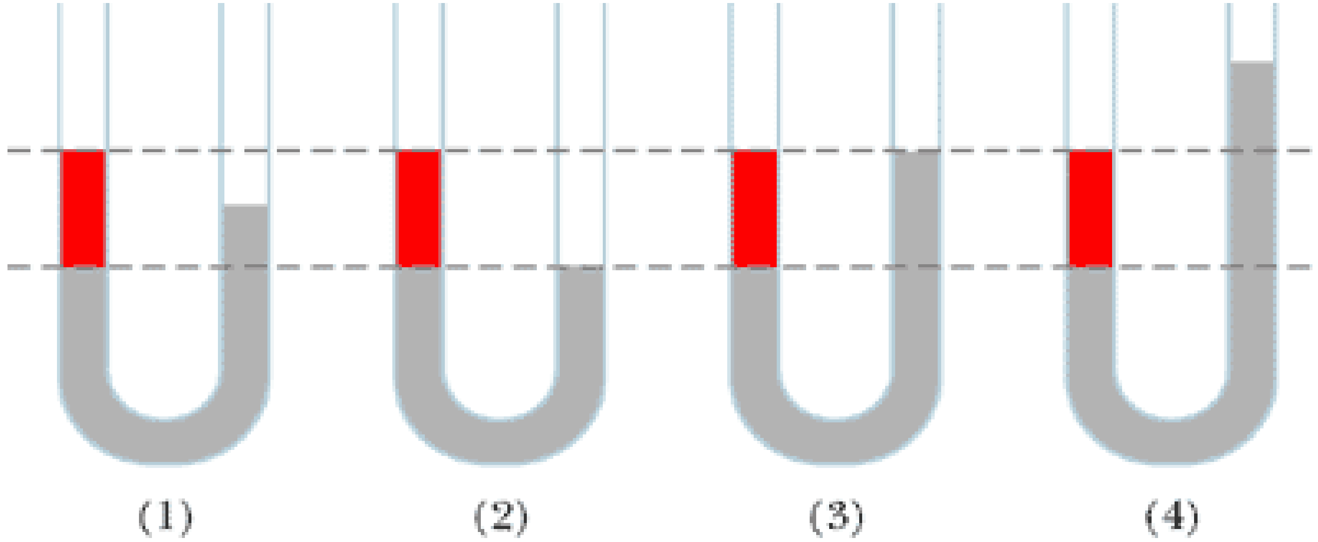


Example:

The figure shows four situations in which **a dark liquids cannot be in static equilibrium**.

(a) which one is that ?

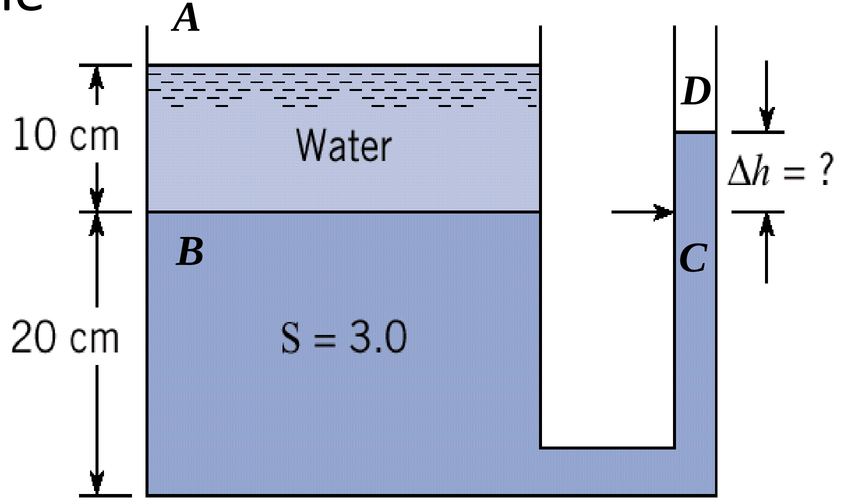
(b) For the others, assume static equilibrium, for each is the density of dark liquid greater than, less than, or equal to the gray one ?



Example:

Find the location of the surface in the manometer

The distance *Δh* is the height of the liquid in the manomoter above the heavier liquid in the tank.



$$p_A + 0.1 * \gamma_w = p_D + \Delta h * \gamma_m$$

$$p_A = P_D = 0$$

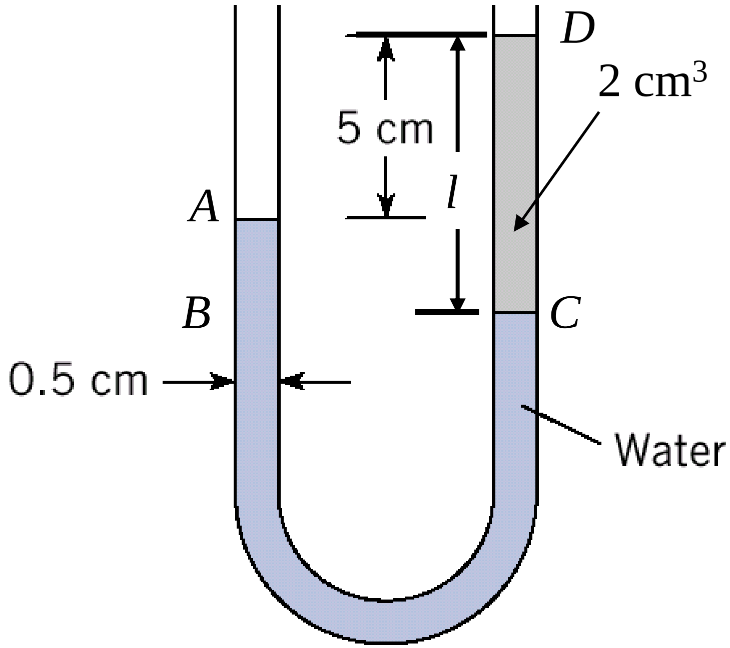
$$\Delta h = 0.1 * \frac{\gamma_w}{\gamma_m}$$

$$\Delta h = 0.1 * \frac{1}{3} = 3.33 cm$$

Example:

Find: Specific weight of fluid

Solution:



$$V = \frac{\pi}{4} d^2 l$$

$$= \frac{\pi}{4} (0.5)^2 l = 2\, cm^3$$

$$l = 10.186\, cm$$

Manometer Equation:

$$p_B = p_C$$

$$p_A + (l - 0.05)\gamma = p_D + l\gamma_{liq}$$

$$\gamma_{liq} = \frac{(l - 0.05)}{l}\gamma$$

$$= \frac{(0.10186 - 0.05)}{0.10186}(9810)$$

$$\gamma_{liq} = 4{,}995\, N/m^3$$

Example: