

# Chapter 2 Reasoning and Proof

## Prerequisite Skills for the lesson "Reasoning and Proof"

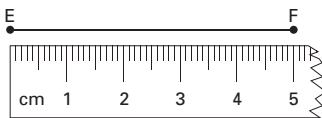
1. Sample answer:  $\angle CGA$
2. Sample answer:  $\angle BGA$  and  $\angle DGE$
3. Sample answer:  $\angle CGB$  and  $\angle CGE$
4. Sample answer:  $\angle DGE$  and  $\angle EGF$
5. Line segment with endpoints  $A$  and  $B$



6. Line containing points  $C$  and  $D$



7. The measure of the length of the segment from point  $E$  to  $F$ .



8. Ray with endpoint  $G$  containing point  $H$ .



9.  $3x + 5 = 20$   
 $3x = 15$   
 $x = 5$
10.  $4(x - 7) = -12$   
 $4x - 28 = -12$   
 $4x = 16$   
 $x = 4$

11.  $5(x + 8) = 4x$   
 $5x + 40 = 4x$   
 $x + 40 = 0$   
 $x = -40$

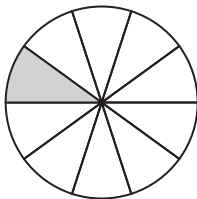
12. Check drawings.  $\angle 1$  and  $\angle 2$  should be vertical angles.

13. Check drawings.  $\angle ABD$  and  $\angle DBC$  should be complementary angles.  $\angle ABC$  should be a right angle.

## Lesson 2.1 Use Inductive Reasoning

### Guided Practice for the lesson "Use Inductive Reasoning"

1.



2. Each number in the pattern is 0.02 more than the previous number: 5.09, 6.01, 6.03
3. Continue the pattern from Example 3. You can connect 6 collinear points  $10 + 5$ , or 15 different ways and then the 7 collinear points  $15 + 6$ , or 21 different ways.

4. Find a pattern:

$$(-1)(-2)(-3) = -6$$

$$(-4)(-5)(-6) = -120$$

$$(-2)(-5)(-7) = -70$$

$$(-4)(-7)(-8) = -224$$

Conjecture: The sign of the product of any three negative integers is negative.

5. Find a value of  $x$  that is greater than or equal to  $x^2$ .

$$\text{When } x = \frac{1}{2}, x^2 = \frac{1}{4}$$

$$\frac{1}{4} \not\geq \frac{1}{2}$$

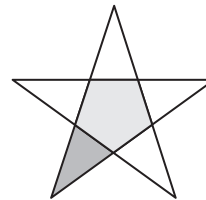
Because a counterexample exists, the conjecture is false.

6. Sample answer: The number of girls playing soccer will increase; the number of girls playing soccer has increased every year for more than 10 years.

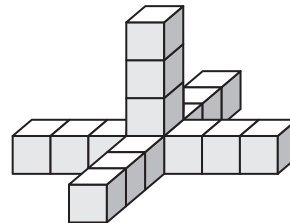
## Exercises for the lesson "Use Inductive Reasoning"

### Skill Practice

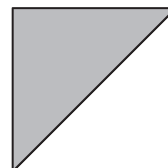
1. Sample answer: A conjecture is a statement about an observation that can be true or false.
2. The word counter means contrary or opposite. A counterexample is an example that is contrary to the given conjecture.
3. The shaded portion of the figure is decreasing by one section in each figure. Sketch the fourth figure by shading only the center and the lower left section of the figure.



4. Each figure is created by adding one block to the outer-most edges of the previous figure. Sketch the fourth figure by adding one block to the outermost edges of the third figure.



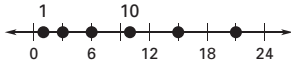
5. C; Each figure is the previous figure rotated  $90^\circ$  counterclockwise. The fourth figure is the third figure rotated  $90^\circ$  counterclockwise.





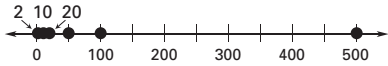
26.  $1, \underbrace{3}_{+2}, \underbrace{6}_{+3}, \underbrace{10}_{+4}, \underbrace{15}_{+5}, \dots$

You add 2 to get the second number, then you add 3 to get the third number, then you add 4 to get the fourth number, then add 5 to get the fifth number. To find the sixth number, add the next consecutive integer, which is 6. The next number is 21.



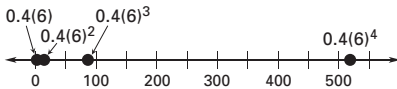
27.  $2, \underbrace{20}_{\times 10}, \underbrace{10}_{\div 2}, \underbrace{100}_{\times 10}, \underbrace{50}_{\div 2}, \dots$

You multiply by 10 to get the second number, then you divide by 2 to get the third number, then multiply by 10 to get the fourth number, then divide by 2 to get the fifth number. To find the sixth number, continue the pattern and multiply by 10. The next number is 500.



28.  $0.4(6), 0.4(6)^2, 0.4(6)^3, \dots$

The exponent of the 6 in each number in the pattern is one more than the exponent of the 6 in the previous number. The next number is  $0.4(6)^4$ .



29. For all values of  $r$  where  $r > 1$ , the values of the numbers in the pattern are increasing. For all values of  $r$  where  $0 < r < 1$ , the values of the numbers in the pattern are decreasing. Raising numbers greater than 1 by successive natural number powers increases the results while raising numbers between 0 and 1 by successive natural number powers decreases the result.

30.  $1, \underbrace{2}_{+1}, \underbrace{4}_{+2}, \dots$

Yes; you can add 1 to get the second number, then add 2 to get the third number. To find the fourth number, add the next consecutive integer, which is 3. The next number is 7.

31. a.  $1, \underbrace{1\frac{1}{2}}_{+\frac{1}{2}}, \underbrace{1\frac{3}{4}}_{+\frac{1}{4}}, \underbrace{1\frac{7}{8}}_{+\frac{1}{8}}, \dots$

You add  $\frac{1}{2}$  to get the second number, then add  $\frac{1}{4}$  to get the third number, then add  $\frac{1}{8}$  to get the fourth number. To find the fifth number, add  $\frac{1}{2}$  times the amount added to the previous number, or  $\frac{1}{2}(\frac{1}{8}) = \frac{1}{16}$ . To find the sixth number, add  $\frac{1}{2}$  times the amount added to the previous number, or  $\frac{1}{2}(\frac{1}{16}) = \frac{1}{32}$ .

To find the seventh number, add  $\frac{1}{2}$  times the amount added to the previous number, or  $\frac{1}{2}(\frac{1}{32}) = \frac{1}{64}$ . The

next three numbers are  $1\frac{15}{16}$ ,  $1\frac{31}{32}$ , and  $1\frac{63}{64}$ .

- b. The values of the numbers are increasing.
- c. As the pattern continues, the values of the numbers get closer and closer to 2. This is a reasonable conjecture because the fraction part of the mixed number gets closer and closer to 1, so the value of the number gets closer and closer to 2.

**Problem Solving**

32.  $F, C, F, F, C, C, F, F, F$

The pitcher throws one of each pitch first, then throws two of each pitch, then continues to increase the number of throws of each pitch by 1. The next five pitches will be  $C, C, C, F, F$ .

33. Conjecture: More person-to-person e-mail messages will be sent in 2004 than in 2003. This is a reasonable conjecture because the graph shows an increase each year from 1996–2003.

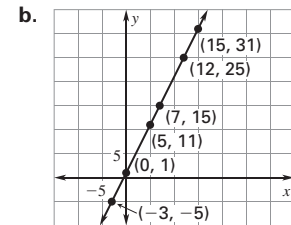
34. a.

Figure	1	2	3	4	5
Distance	4	8	12	16	20

- b. The distance around each figure is four times the figure number.
- c. The distance around the 20th figure is  $4(20) = 80$  units.

35. a.

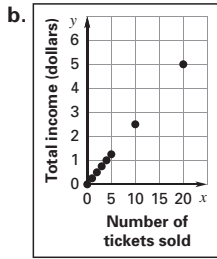
x	y
-3	-5
0	1
5	11
7	15
12	25
15	31



- c. The value of  $y$  is one more than twice the value of  $x$ . So, an equation relating  $x$  and  $y$  is  $y = 2x + 1$ .

36. a.

Number of tickets sold	0	1	2	3	4	5	10	20
Class income	0	0.25	0.50	0.75	1.00	1.25	2.5	5.00



There is a linear relationship between the number of tickets sold and the income. The income is 0.25 times the number of tickets sold.

c. Let  $y$  be your income and  $x$  be the number of tickets sold. An equation for your income is  $y = 0.25x$ .

d. When  $y = 14$ :  
 $14 < 0.25x$   
 $56 < x$

Your class must sell more than 56 tickets to make a profit.

e. To make a profit of \$50, your class must sell  $14 + 50 = \$64$  worth of tickets. Find the value of  $x$  when  $y = 64$ .

$$64 = 0.25x$$

$$256 = x$$

Your class must sell 256 tickets to make a profit of \$50.

37. a. After the first two numbers, each number is the *sum* of the *two* previous numbers.

b. 144, 233, 377

c. *Sample answer:* Spiral patterns on the head of a sunflower

38. a. *Sample answer:* A counterexample is 15, which is a multiple of 5 but not a multiple of 8. So 15 is a member of set A, but not a member of set B. The conjecture is false because a counterexample exists.

b. *Sample answer:* A counterexample is 99, which is less than 100 but not a member of set A or set B. The conjecture is false because a counterexample exists.

c. A counterexample is 40, which is in both set A and set B. The conjecture is false because a counterexample exists.

## Lesson 2.2 Analyze Conditional Statements

### Guided Practice for the lesson "Analyze Conditional Statements"

- If an angle is a  $90^\circ$  angle, then it is a right angle.
- If  $x = -3$ , then  $2x + 7 = 1$ .
- If  $n = 9$ , then  $n^2 = 81$ .
- If a tourist is at the Alamo, then the tourist is in Texas.
- Converse:* If a dog is large, then it is a Great Dane.  
False, not all large dogs are Great Danes.  
*Inverse:* If a dog is not a Great Dane, then it is not large.  
False, a dog could be large but not a Great Dane.  
*Contrapositive:* If a dog is not large, then it is not a Great Dane.  
True, a dog that is not large cannot be a Great Dane.

6. *Converse:* If a polygon is regular, then the polygon is equilateral.

True, all regular polygons are equilateral.

*Inverse:* If a polygon is not equilateral, then it is not regular.

True, a polygon that is not equilateral cannot be regular.

*Contrapositive:* If a polygon is not regular, then the polygon is not equilateral.

False, a polygon that is not regular can still be equilateral.

7. True.  $\angle JMF$  and  $\angle FMG$  form a linear pair so they are supplementary.

8. False. It is not known that  $M$  bisects  $\overline{FH}$ . So, you cannot state that  $M$  is the midpoint for  $\overline{FH}$ .

9. True.  $\angle JMF$  and  $\angle HMG$  are vertical angles because their sides form two pairs of opposite rays.

10. False. It is not shown that  $\overline{FH}$  and  $\overline{JG}$  intersect to form right angles. So, you cannot state that  $\overline{FH} \perp \overline{JG}$ .

11. An angle is a right angle if and only if it measures  $90^\circ$ .

12. Mary will be in the fall play if and only if she is in theater class.

### Exercises for the lesson "Analyze Conditional Statements"

#### Skill Practice

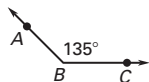
- The *converse* of a conditional statement is found by switching the hypothesis and the conclusion.
- Collinear points are points that lie on the same line. Points are collinear if and only if they lie on the same line.
- If  $x = 6$ , then  $x^2 = 36$ .
- If an angle is a straight angle, then it measures  $180^\circ$ .
- If a person is allowed to vote, then that person is registered to vote.
- The error is in identifying the correct hypothesis and conclusion when writing the if-then form of the statement. The hypothesis is "a student is in high school" and the conclusion is "the student takes four English courses."  
*If-then statement:* If a student is in high school, then the student takes four English courses.
- If-then:* If two angles are complementary, then they add to  $90^\circ$ .  
*Converse:* If two angles add to  $90^\circ$ , then they are complementary.  
*Inverse:* If two angles are not complementary, then they do not add to  $90^\circ$ .  
*Contrapositive:* If two angles do not add to  $90^\circ$ , then they are not complementary.
- If-then:* If an animal is an ant, then it is an insect.  
*Converse:* If an animal is an insect, then it is an ant.  
*Inverse:* If an animal is not an ant, then it is not an insect.  
*Contrapositive:* If an animal is not an insect, then it is not an ant.

9. *If-then:* If  $x = 2$ , then  $3x + 10 = 16$ .  
*Converse:* If  $3x + 10 = 16$ , then  $x = 2$ .  
*Inverse:* If  $x \neq 2$ , then  $3x + 10 \neq 16$ .  
*Contrapositive:* If  $3x + 10 \neq 16$ , then  $x \neq 2$ .
10. *If-then:* If a point is a midpoint, then it bisects a segment.  
*Converse:* If a point bisects a segment, then it is a midpoint.  
*Inverse:* If a point is not a midpoint, then it does not bisect a segment.  
*Contrapositive:* If a point does not bisect a segment, then it is not a midpoint.
11. False; a polygon can have 5 sides without being a regular pentagon.

Counterexample:



12. True.
13. False; two angles can be supplementary without being a linear pair. Counterexample:



14. True.
15. False; Counterexample: The number 5 is real, but not irrational.
16. True;  $\angle ABC$  is a right angle, so  $m\angle ABC = 90^\circ$ .
17. False; It is not known that  $\angle 1$  is a right angle, so you cannot conclude that  $\overrightarrow{PQ} \perp \overrightarrow{ST}$ .
18. True;  $\angle 2$  and  $\angle 3$  are adjacent angles whose noncommon sides form opposite rays, so  $\angle 2$  and  $\angle 3$  are a linear pair. Angles in a linear pair are supplementary, so  $m\angle 2 + m\angle 3 = 180^\circ$ .
19. An angle is obtuse if and only if its measure is between  $90^\circ$  and  $180^\circ$ .
20. Two angles are a linear pair if and only if they are adjacent angles whose noncommon sides are opposite rays.
21. Points are coplanar if and only if they lie in the same plane.
22. This is not a valid definition. The converse of the statement is not true. Rays can have a common endpoint without being opposite rays.
23. The statement is a valid definition.
24. The statement is not a valid definition. The converse of the statement is false. If the measure of an angle is greater than that of an acute angle, the angle is not necessarily a right angle.
25. A; If you do your homework, then you can go to the movie afterwards. This is the if-then form of the given statement.
26. If  $x > 0$ , then  $x > 4$ . A counterexample is  $x = 2$ . Note that  $2 > 0$ , but  $2 \not> 4$ . Because a counterexample exists, the converse is false.

27. If  $-x > -6$ , then  $x < 6$ . The converse is true.
28. If  $x \leq 0$ , then  $x \leq -x$ . The converse is true.
29. *Sample answer:* If  $x = 2$ , then  $x^2 > 0$ .
30. If  $\angle 1$  and  $\angle 2$  are linear pairs, then  $m\angle 2$  is  $90^\circ$ ; if  $\angle 1$  and  $\angle 4$  are linear pairs, then  $m\angle 4$  is  $90^\circ$ ; if  $\angle 4$  and  $\angle 3$  are linear pairs, then  $m\angle 3$  is  $90^\circ$ .

### Problem Solving

31. *Statement:* If a fragment has a diameter greater than 64 millimeters, then it is called a block or bomb.  
*Converse:* If a fragment is called a block or bomb, then it has a diameter greater than 64 millimeters.  
 Both the statement and its converse are true. So, the biconditional statement is true.
32. *Counterexample:* a fragment with a diameter of 1 millimeter  
 The diameter is less than 64 millimeters, but the fragment is not called a lapilli. Because a counterexample exists, the biconditional statement is false.
33. You can show that the statement is false by finding a counterexample. Some sports do not require helmets, such as swimming or track.
34. a. The statement is true. The mean is the average value of the data, so it will lie between the least and greatest values in the data set.  
 b. If the mean of your data set is between  $x$  and  $y$ , then  $x$  and  $y$  are the least and greatest values in your data set.  
 The converse is false. The mean is between any two numbers in a data set where one of the numbers is less than the mean and the other is greater than the mean. The numbers do not have to be the least and greatest values in the data set.  
 c. If a data set has a mean, median, and mode, then the *mode* of the data set will always be one of the measurements.  
 The mode is the data value that occurs most frequently in a data set. So, if the mode exists, then it will always be one of the data values. The median is one of the data values only when there is an odd number of values in the data set. The mean does not have to be a data value.
35. *Sample answer:* If a student is in the jazz band, then the student is in the band.
36. a. If a rock is formed from the cooling of molten rock, then it is igneous rock.  
 If a rock is formed from pieces of other rocks, then it is sedimentary rock.  
 If a rock is formed by changing temperature, pressure, or chemistry, then it is metamorphic rock.  
 b. If a rock is igneous rock, then it is formed from the cooling of molten rock.  
 If a rock is sedimentary, then it is formed from pieces of other rocks.  
 If a rock is metamorphic, then it is formed by changing temperature, pressure, or chemistry.

The converse of each statement is true.

If a rock is classified in one of these ways, it must be formed in the manner described.

- c. *Sample answer:* If it is a rock, then it can be formed in different ways. The converse of the statement is false. If something can be formed in different ways, it doesn't necessarily mean it has to be a rock. It could be soil for example.
37. The statement cannot be written as a true biconditional. The biconditional is false because  $x = -3$  also makes the statement true. A counterexample exists, so the biconditional statement is false.
38. For a statement to be a true biconditional, both the original statement and the converse must be true. If the contrapositive of a statement is true, then you know that the original statement is true. However, you do not know if the converse is true. So, you don't know if it can be written as a true biconditional.
39. It is Tuesday. Because it is Tuesday, I have art class. Because I have art class, I do not have study hall. Because I do not have study hall, I must have music class.

### Lesson 2.3 Apply Deductive Reasoning

#### Investigating Geometry Activity for the lesson "Apply Deductive Reasoning"

	n-dimensional geometry	Differential calculus	Math for theory of relativity	Perspective drawing	Pythagorean Theorem	Did not eat beans	Studied moonlight	Wrote a math book at 17	Fluent in Latin	Played piano
Maria Agnesi	X	0	X	X	X	X	X	0	X	X
Anaxagoras	X	X	0	X	X	0	X	0	X	X
Emmy Noether	X	X	0	X	X	X	X	X	0	0
Julio Rey Pastor	0	X	X	X	X	X	X	0	X	X
Pythagoras	X	X	X	0	0	0	X	X	X	X
Did not eat beans	X	X	X	0	0					
Studied moonlight	X	X	0	X	0					
Wrote a math book at 17	0	X	X	X	X					
Fluent in Latin	X	0	X	X	X					
Played piano	X	X	0	X	X					

1. *If-then form:* If the mathematician is Julio Rey Pastor, then the mathematician wrote a book at age 17.  
*Contrapositive:* If the mathematician did not write a book at age 17, then the mathematician is not Julio Rey Pastor.  
 The contrapositive is a helpful clue because it allows you to eliminate anyone who did not write a book at age 17 as possible choices for Julio Rey Pastor.
2. After clue 6, you know that the person who played the piano was the person who is either responsible for the math for the theory of relativity or used perspective drawing. You know that the person who played the piano was either Maria Agnesi or Emmy Noether. You also know that the person fluent in Latin was either Maria Agnesi or Emmy Noether. The person who is fluent

in Latin contributed to differential calculus, so Emmy Noether could not have been fluent in Latin. Emmy Noether had to play the piano.

3. Before Clue 7, you knew that the person who used perspective drawing was either Maria Agnesi, Anaxagoras, or Julio Rey Pastor. Clue 7 stated that the person who used perspective drawing was not Maria Agnesi or Julio Rey Pastor. So, Anaxagoras had to be the one who first used perspective drawing.

#### Guided Practice for the lesson "Apply Deductive Reasoning"

- Because  $m\angle R = 155^\circ$  satisfies the hypothesis of a true conditional statement, the conclusion is true. So,  $\angle R$  is obtuse.
- The conclusion of the first statement is the hypothesis of the second statement, so you can write the following new statement.  
 If Jenelle gets a job, then she will drive to school.
- The Law of Syllogism is illustrated. The conclusion of the first statement is the hypothesis of the second statement, so the new statement is written using the Law of Syllogism.
- Because  $x = 4$  satisfies the hypothesis of a true conditional statement, the Law of Detachment states that the conclusion is also true. So,  $x + 9 > 20$ .
- Look for a pattern:  
 $1 + 1 = 2$ ;  $2 + 2 = 4$ ;  $3 + 3 = 6$ ;  
 $8 + 8 = 16$ ;  $10 + 10 = 20$ ;  $15 + 15 = 30$   
*Conjecture:* The sum of a number and itself is twice the number.  
 Let  $n$  be any number. Then  $n + n = 2n$ .  
 So, the sum of a number and itself is 2 times the number.
- Sample answer:* If it is a weekday, a newspaper should be delivered to my house. No newspaper was delivered to my house today. So, today is not a weekday.

#### Exercises for the lesson "Apply Deductive Reasoning"

##### Skill Practice

- If the hypothesis of a true if-then statement is true, then the conclusion is also true by the Law of *Detachment*.
- The man is standing in front of a mirrored ball. You can see the reflections of people standing near him in the mirror.
- There is a light source to the window side of the pears. You can see shadows cast by the pears opposite the window side.
- Because  $m\angle A = 90^\circ$  satisfies the hypothesis, the conclusion is also true. So,  $\angle A$  is a right angle.
- Because  $x = 15$  satisfies the hypothesis, the conclusion is also true. So,  $-15 < -12$ .
- Because reading a biography satisfies the hypothesis, the conclusion is also true. So, the book you are reading is nonfiction.

7. If a rectangle has four equal side lengths, then it is a regular polygon.
8. If  $y > 0$ , then  $2y - 5 \neq -5$ .
9. If you play the clarinet, then you are a musician.
10. If  $\frac{1}{2}a = 1\frac{1}{2}$ , then  $5a = 15$ .
11.  $2 + 4 = 6$ ;  $6 + 10 = 16$ ;  $4 + 14 = 18$ ;  
 $8 + 12 = 20$ ;  $10 + 12 = 22$ ;  $12 + 16 = 28$   
*Conjecture:* even integer + even integer = even integer  
 Let  $n$  and  $m$  be any two integers.  
 $2n$  and  $2m$  are even integers because any integer multiplied by 2 is even.  
 $2n + 2m = 2(n + m)$   
 $2(n + m)$  is the product of 2 and an integer  $n + m$ .  
 So,  $2(n + m)$  is an even integer. The sum of an even integer and an even integer is an even integer.
12. B; Because  $\angle 1$  and  $\angle 2$  are vertical angles satisfies the hypothesis, the conclusion is also true.  
 So,  $m\angle 1 = m\angle 2$ .
13. In the second statement, the hypothesis and conclusion have been switched, which does not make a true statement.  
 If two angles are a linear pair, then they are supplementary. Angles  $C$  and  $D$  are a linear pair, so they are supplementary.
14. a.  $AB = \sqrt{(3 - 1)^2 + (6 - 3)^2} = \sqrt{4 + 9} = \sqrt{13}$   
 $CD = \sqrt{(6 - 4)^2 + (7 - 4)^2} = \sqrt{4 + 9} = \sqrt{13}$   
 $EF = \sqrt{(9 - 7)^2 + (4 - 1)^2} = \sqrt{4 + 9} = \sqrt{13}$
- b. *Sample answer:* Conjecture: If one endpoint is 2 units to the right and 3 units above the other end point, then the segment is congruent to the given segments.  
 Let  $M(2, 1)$  and  $N(4, 4)$  be the endpoints of  $\overline{MN}$ .  
 $MN = \sqrt{(4 - 2)^2 + (4 - 1)^2} = \sqrt{2^2 + 3^2} = \sqrt{13}$   
 Because  $MN = \sqrt{13}$ ,  $\overline{MN} \cong \overline{AB} \cong \overline{CD} \cong \overline{EF}$ .
- c. Let  $S(x, y)$  be one endpoint of the segment. Then  $T(x + 2, y + 3)$  is the other endpoint.  
 $ST = \sqrt{[(x + 2) - x]^2 + [(y + 3) - y]^2}$   
 $= \sqrt{2^2 + 3^2} = \sqrt{13}$   
 The length of  $\overline{ST}$  will always be  $\sqrt{13}$ , so it will be congruent to  $\overline{AB}$ ,  $\overline{CD}$ , and  $\overline{EF}$ .
- d.  $MN = \sqrt{(5 - 3)^2 + (2 - 5)^2} = \sqrt{2^2 + (-3)^2} = \sqrt{13}$   
 $PQ = \sqrt{(4 - 1)^2 + (-3 - (-1))^2}$   
 $= \sqrt{3^2 + (-2)^2}$   
 $= \sqrt{13}$   
 $RS = \sqrt{(1 - (-2))^2 + (4 - 2)^2} = \sqrt{3^2 + 2^2} = \sqrt{13}$   
 The student is correct. Each segment is congruent to the given segments because each segment has a length of  $\sqrt{13}$ .

15. The Law of Syllogism works when used with the contrapositives of a pair of statements. The contrapositive of a true statement is true. So, you can use the Law of Syllogism with the contrapositive of each true statement to write a new true statement.

If a creature is not a marsupial, then it is not a wombat.

If a creature does not have a pouch, then it is not a marsupial.

The conclusion of the second true statement is the hypothesis of the first true statement, so you use the Law of Syllogism to write the following new statement.

If a creature does not have a pouch, then it is not a wombat.

### Problem Solving

16. Because you saved \$1200 does not satisfy the hypothesis, the conclusion is not true. So, you cannot buy a car.
17. Write each statement in if-then form.  
 If the revenue is greater than the costs, the bakery makes a profit.  
 If the bakery makes a profit, then you will get a raise.  
 So, if the revenue is greater than the costs, then you will get a raise.
18. So, Simone may have visited Mesa Verde National Park.
19. So, Billy is with a park ranger.
20. a. *Sample answer:* If calcite is scratched on gypsum, then a scratch mark is left on the gypsum.  
 If fluorite is scratched on calcite, then a scratch mark is left on the calcite.  
 If calcite is scratched on talc, then a scratch mark is left on the talc.
- b. You can conclude that Mineral C is talc because it is the only mineral that can be scratched by all 3 other minerals.  
 Mineral A cannot be fluorite because fluorite cannot be scratched by any of the other minerals. So, Mineral A must be gypsum or calcite.  
 Mineral B cannot be gypsum because gypsum can only scratch talc, which is Mineral C. So, Mineral B must be calcite or fluorite.
- c. Test to see if Mineral D can scratch Mineral B. If Mineral D can scratch Mineral B, then Mineral D is fluorite because it is the only mineral that cannot be scratched. If Mineral D is fluorite, then Mineral B is calcite and Mineral A is gypsum. If Mineral D cannot scratch Mineral B, then Mineral B is fluorite; If Mineral B is fluorite, take Mineral A and Mineral D and do one scratch test to identify them.
21. Deductive reasoning; The conclusion is reached by using laws of logic and the facts about your school rules and what you did that day.
22. Inductive reasoning; The conclusion is reached by using a pattern of past activities to make a conclusion on a future activity.

23. *Conjecture:* The sum of an even integer and an odd integer is an odd integer.  
*Reasoning:* Let  $2m$  be an even integer and  $2n + 1$  be an odd integer.  
 $2m + (2n + 1) = (2m + 2n) + 1 = 2(m + n) + 1$   
 $2(m + n)$  is the product of 2 and an integer  $(m + n)$ , so  $2(m + n)$  is an even integer.  
 $2(m + n) + 1$  is one more than an even integer, so  $2(m + n) + 1$  is an odd integer.
24. Use the Law of Syllogism to write a conditional statement for the first two statements.  
 For want of a nail the horse is lost.  
 Use the Law of Syllogism to write a new conditional statement for the statements in the poem.  
 For want of a nail the rider is lost.
25. The conclusion is true. The game is not sold out, so Arlo went to the game and he bought a hot dog.
26. The conclusion is true. The game is not sold out, so Arlo and Mia went to the game.
27. The conclusion is false. The statements never mention Mia buying a hot dog. So you cannot make that conclusion.
28. The conclusion is false. The statements never mention Arlo eating popcorn. So, you cannot make that conclusion.
29. a. If Adam is telling the truth, then Bob is lying. So, Bob's statement about Charlie lying is itself a lie. Charlie must be telling the truth in his statement.  
 b. Assume Adam is telling the truth. Then Bob is lying and Charlie is telling the truth. Charlie's statement says that Adam and Bob are both lying, which is a contradiction of the original assumption.  
 c. From part (b) you know that Adam is lying. Then from Adam's statement, Bob is telling the truth. Charlie says that Adam and Bob are both lying, which cannot be true because Bob is telling the truth. So, Bob is telling the truth, and Adam and Charlie are both lying.

**Quiz for the lessons "Use Inductive Reasoning", "Analyze Conditional Statements" and "Apply Deductive Reasoning"**

1. To find a counterexample, you need to find a product of two positive numbers that is positive.  
*Sample answer:*  $6(4) = 24$   
 Both numbers are positive and the product is positive. Because a counterexample exists, the conjecture is false.
2. To find a counterexample, you need to find a sum that is less than the greater number.  
*Sample answer:*  $-2 + 6 = 4$   
 $4 \not> 6$   
 Because a counterexample exists, the conjecture is false.
3. *If-then form:* If points lie on the same line, then they are called collinear points.  
*Contrapositive:* If points are not collinear, then they do not lie on the same line.

4. *If-then form:* If  $x = 5$ , then  $2x - 8 = 2$ .  
*Contrapositive:* If  $2x - 8 \neq 2$ , then  $x \neq 5$ .
5. Because  $98^\circ\text{F}$  satisfies the hypothesis, the conclusion must be true. So, I will wear shorts.
6. A multiple of 3 is, by definition, divisible by 3. If a number is divisible by a multiple of 3, then the number is a multiple of 3. So, if a number is divisible by a multiple of 3, then it is divisible by 3.

**Extension for the lesson "Apply Deductive Reasoning"**

1. Use the symbol for negation ( $\sim$ ) with the conclusion of the conditional statement, then use the arrow ( $\rightarrow$ ) to connect the conclusion and the hypothesis of the new statement, then use the symbol for negation again with the hypothesis of the conditional statement. So,  $\sim q \rightarrow \sim p$ .
2. If polygon  $ABCDE$  is equiangular and equilateral, then it is a regular polygon.
3. Polygon  $ABCDE$  is not equiangular and equilateral.
4. If polygon  $ABCDE$  is not a regular polygon, then it is not equiangular and equilateral.
5. Polygon  $ABCDE$  is equiangular and equilateral if and only if it is a regular polygon.

6.

$p$	$q$	$\sim q$	$p \rightarrow \sim q$
T	T	F	F
T	F	T	T
F	T	F	T
F	F	T	T

7.

$p$	$q$	$\sim p$	$\sim p \rightarrow q$
T	T	F	T
T	F	F	T
F	T	T	T
F	F	T	F

8.

$p$	$q$	$p \rightarrow q$	$\sim(p \rightarrow q)$
T	T	T	F
T	F	F	T
F	T	T	F
F	F	T	F

9.

$p$	$q$	$q \rightarrow p$	$\sim(q \rightarrow p)$
T	T	T	F
T	F	T	F
F	T	F	T
F	F	T	F



10. a. The last column for “ $\sim(p \text{ or } q)$ ” is F, F, F, T.  
 b. The last column for “ $(\sim p \text{ and } \sim q)$ ” is F, F, F, T.  
 c. The truth tables are the same.

## Lesson 2.4 Use Postulates and Diagrams

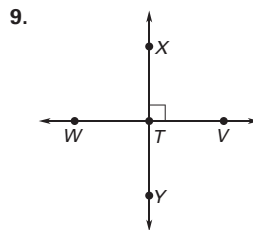
### Guided Practice for the lesson “Use Postulates and Diagrams”

1. Postulate 11: If two planes intersect, then their intersection is a line.
2. Postulate 5: Through points  $A$  and  $B$ , there exists a line  $n$ .  
 Postulate 6: Line  $n$  contains points  $A$  and  $B$ .  
 Postulate 7: Line  $m$  and line  $n$  intersect at point  $A$ .
3. Mark  $\overline{PW}$  and  $\overline{QW}$  congruent using two tick marks for each segment to make them different from  $\overline{TW}$  and  $\overline{WV}$ .
4. *Sample answer:*  $\angle TWP$  and  $\angle PWV$  are supplementary because they form a linear pair.
5. Yes, you can assume plane  $S$  intersects plane  $T$  at  $\overline{BC}$  because it is shown in the diagram.
6. Because the diagram shows that  $\overrightarrow{AB}$  is perpendicular to plane  $S$ , then  $\overrightarrow{AB}$  is perpendicular to every line in plane  $S$  that intersects  $\overrightarrow{AB}$  at point  $B$  by the definition of a line perpendicular to a plane.  $\overline{BC}$  is in plane  $S$  and intersects  $\overrightarrow{AB}$  at point  $B$ , so  $\overrightarrow{AB} \perp \overrightarrow{BC}$ .

### Exercises for the lesson “Use Postulates and Diagrams”

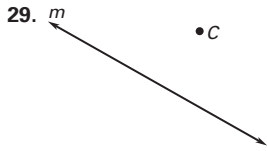
#### Skill Practice

1. A line perpendicular to a plane is a line that intersects the plane in a point and is perpendicular to every line in the plane that intersects it.
2. You cannot assume  $\angle BMA \cong \angle CJA$  because you do not know their measures.
3. Postulate 5: Through any two points there exists exactly one line.
4. Postulate 9: A plane contains at least 3 noncollinear points.
5. a. *If-then form:* If three points are noncollinear, then there exists exactly one plane that contains all three points.  
 b. *Converse:* If there exists exactly one plane that contains three points, then the three points are noncollinear.  
*Inverse:* If three points are collinear, then there does not exist exactly one plane that contains all three points.  
*Contrapositive:* If there does not exist exactly one plane that contains three points, then the three points are collinear.
- c. The converse, inverse, and contrapositive are true.
6. Line  $q$  contains the three points  $J$ ,  $H$ , and  $K$ .
7. Line  $p$  intersects line  $q$  at point  $H$ .
8. Through points  $G$ ,  $K$ , and  $L$ , there exists exactly one plane,  $M$ .



$\overline{WT}$  does not have to be congruent to  $\overline{TV}$  because  $T$  is not given as the midpoint of  $\overline{WV}$ .

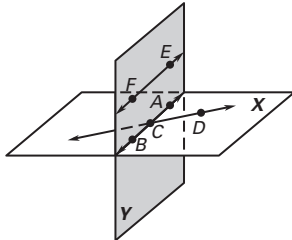
10. B; No drawn line connects  $F$ ,  $B$ , and  $G$ , so you cannot assume they are collinear.
11. The statement is false. Through any two points there exists exactly one line not through any three points. For example, consider two cars parked on one side of a street and one car parked on the other side of the street.
12. It is true that a point can be in more than one plane.
13. The statement is false. Two planes do not have to intersect each other. For example, consider the planes that contain opposite faces of a rectangular prism, which do not intersect.
14. True. The diagram shows that planes  $W$  and  $X$  intersect at  $\overline{KL}$ .
15. False. No drawn line connects  $Q$ ,  $J$ , and  $M$ , so you cannot say they are collinear.
16. False. The diagram shows that points  $K$ ,  $L$ ,  $M$ , and  $R$  are not coplanar. Points  $M$  and  $R$  lie in different planes.
17. False. The diagram does not show that  $\overline{MN}$  and  $\overline{RP}$  intersect.
18. False.  $\overline{RP}$  lies in plane  $W$ , so they cannot be perpendicular.
19. True. The diagram shows that  $\overline{JK}$  lies in plane  $X$ .
20. False. With no right angle marked, you cannot assume that  $\angle PLK$  is a right angle.
21. True. The sides of  $\angle NKL$  and  $\angle JKM$  form two pairs of opposite rays, so they are vertical angles.
22. True.  $\angle NKJ$  and  $\angle JKM$  are a linear pair, so they are supplementary angles.
23. False. The diagram does not show that  $\angle JKM$  is congruent to  $\angle KLP$ , so you cannot assume  $\angle JKM \cong \angle KLP$ .
24. C; The diagram shows  $\overline{LN}$ ,  $\overline{AB}$ , and  $\overline{DC}$  intersecting at point  $M$ ,  $\overline{AB}$  bisecting  $\overline{LN}$ , and  $\overline{DC} \perp \overline{LN}$ .
25. Answers will vary.
26. *Sample answer:* A line contains at least two points. Three points are sometimes contained in a line.
27. You know that a plane contains at least three noncollinear points by Postulate 9. You also know that there exists exactly one line through any two points by Postulate 5. So, there is at least one line in every plane.
28. You know that plane  $M$  contains at least three noncollinear points, one of which is point  $X$ , by Postulate 9. Let  $A$  and  $B$  be the other two points. You know that lines  $\overline{XA}$  and  $\overline{XB}$  exist in plane  $M$  by Postulate 6.



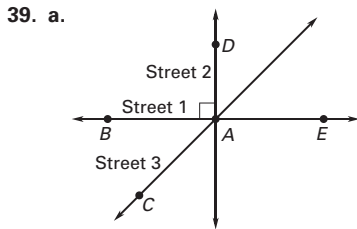
You know that line  $m$  contains at least two points by Postulate 6. You also know that through two points on line  $m$  and point  $C$  not on line  $m$  there exists exactly one plane by Postulate 8. So, there is one plane that can be drawn so that line  $m$  and point  $C$  lie in the plane.

**Problem Solving**

- 30. Postulate 5 is suggested by the photo. Through one point, the person's hand, and the other point, the dog, there exists exactly one line, the leash.
- 31. Postulate 7 is suggested by the photo. If two lines, the two swords, intersect, then their intersection is exactly one point.
- 32. Postulate 11 is suggested by the photo. If two planes, two sides of the poster board, intersect, then their intersection is a line.
- 33. Answers will vary.
- 34. *Sample answer:*

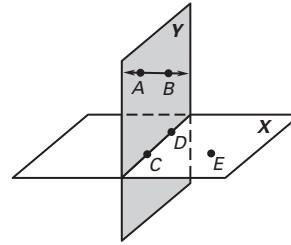


- 35. *Sample answer:* Through points  $Z$  and  $U$  there exists  $\overleftrightarrow{ZU}$ .
- 36. *Sample answer:*  $\overleftrightarrow{SZ}$  intersects  $\overleftrightarrow{ZU}$  at point  $Z$ .
- 37. *Sample answer:* The plane of the pyramid contains the points  $S$ ,  $T$ , and  $Y$ .
- 38. *Sample answer:* The points  $S$  and  $Z$  lie in the plane of the pyramid, so  $\overleftrightarrow{SZ}$  lies in the plane of the pyramid.

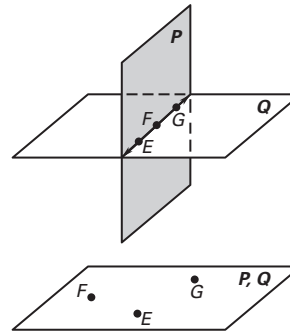


- b. Streets 1 and 2 intersect at Building A.
  - c. Building B is directly west of Building A and Building D is directly north of Building A, so the angle formed by streets 1 and 2 is a right angle.
  - d. Building E is not between Buildings A and B because if it were, then  $\angle CAE$  would be an acute angle.
  - e. Building E is on Street 1.
40. a. See diagram.  
 b. See diagram.  
 c. Through points  $A$  and  $B$ , there exists line  $\overleftrightarrow{AB}$ .

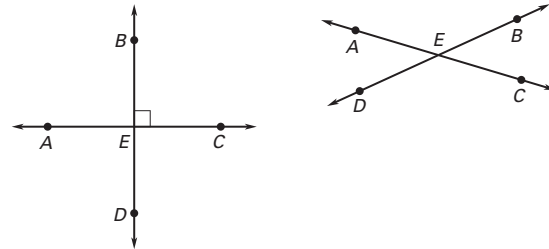
- d. Point  $C$  would lie on the intersection of plane  $X$  and plane  $Y$ .
- e. Plane  $X$  contains points  $C$ ,  $D$ , and  $E$ .



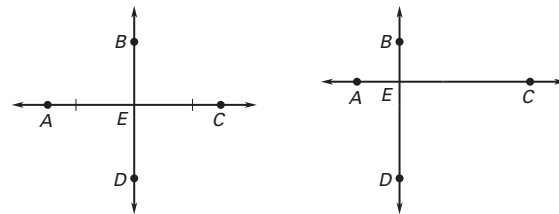
41. If  $P$  and  $Q$  are different planes, then points  $E$ ,  $F$ , and  $G$  must be collinear and lie on the line of intersection between planes  $P$  and  $Q$ . If  $P$  and  $Q$  are the same plane, the points  $E$ ,  $F$ , and  $G$  must be noncollinear.



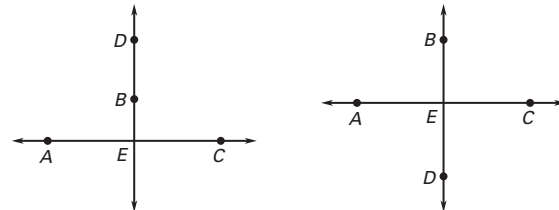
42.



43.



44.



45. You know that through any three noncollinear points there exists exactly one plane. When the lengths of all the legs are different there are four different combinations of three of the four leg ends. So, there are four planes determined by the lower ends of the legs. When exactly three of the legs of the second chair have the same length there are two different combinations of leg ends. So, there are two different planes determined by the lower ends of the legs of the second chair.

**Mixed Review of Problem Solving for the lessons "Use Inductive Reasoning", "Analyze Conditional Statements", "Apply Deductive Reasoning", and "Use Postulates and Diagrams"**

- The time of sunrise gets earlier each day from Jan. 1 through June 1, then gets later on July 1 and Aug. 1.
  - Sample answer:* 6:00 A.M.
- The statement is true.
  - The statement is false.  
A counterexample is a hurricane with a wind speed of 100 miles per hour, which is less than 130 miles per hour, but which represents a category 2 hurricane.
- $$1, \underbrace{2}_{+1}, \underbrace{5}_{+3}, \underbrace{10}_{+5}, \underbrace{17}_{+7}, \underbrace{26}_{+9}, \dots$$

You add 1 to get the second number, then add 3 to get the third number, then add 5 to get the fourth number, then add 7 to get the fifth number, then add 9 to get the sixth number. To find the seventh number, add the next consecutive odd integer, which is 11. So, the next number is 37.
- The statement is the result of inductive reasoning because the conclusion is based on an observation in the pattern of the data.
  - The statement is the result of deductive reasoning because the conclusion compares the values given in the graph.
  - The statement is the result of inductive reasoning because the conclusion is based on an observation.
- You *must have* a library card. Without a library card, you could not have checked out the book.
  - Bob *may have visited* volcanoes. He has never been to the volcanoes in Hawaii, but he may have visited other volcanoes.
- 

$\angle PNR$  and  $\angle QNS$  are acute angles. You know that  $\angle PNR$  and  $\angle PNS$  are supplementary angles because they are a linear pair. Because  $\angle PNS$  is an obtuse angle,  $\angle PNR$  must be an acute angle. Similarly  $\angle QNS$  and  $\angle QNR$  are supplementary angles, so  $\angle QNS$  is also an acute angle.

**Lesson 2.5 Reason Using Properties from Algebra**

**Investigating Geometry Activity for the lesson "Reason Using Properties from Algebra"**

**Explore: Step 2**

*Sample answer:*

- |                   |                   |                   |
|-------------------|-------------------|-------------------|
| a. 16             | a. 42             | a. 63             |
| b. $16 \cdot 2$   | b. $42 \cdot 2$   | b. $63 \cdot 2$   |
| c. $32 + 4$       | c. $84 + 4$       | c. $126 + 4$      |
| d. $36 \cdot 5$   | d. $88 \cdot 5$   | d. $130 \cdot 5$  |
| e. $180 + 12$     | e. $440 + 12$     | e. $650 + 12$     |
| f. $192 \cdot 10$ | f. $452 \cdot 10$ | f. $662 \cdot 10$ |
| g. $1920 - 320$   | g. $4520 - 320$   | g. $6620 - 320$   |
| h. 1600           | h. 4200           | h. 6300           |

Your answer is always the number you picked in part (a).

**Draw Conclusions**

- $x$
  - $2x$
  - $2x + 4$
  - $(2x + 4) \cdot 5$
  - $(10x + 20) + 12$
  - $(10x + 32) \cdot 10$
  - $(100x + 320) - 320$
  - $100x \div 100$
- $x$  is chosen,  $2x$  doubles  $x$ ,  $2x + 4$  is four more than  $2x$ ,  $5(2x + 4)$  is five times the previous number,  $5(2x + 4) + 12$  is 12 more than the previous number,  $10[5(2x + 4) + 12]$  multiplies the previous number by 10,  $10[5(2x + 4) + 12] - 320$  reduces the previous number by 320, crossing out the zeros (dividing by 100) leaves  $x$ .
- Sample answer:*

  - 5
  - $5 \cdot 2$
  - $10 + 18$
  - $28 \div 2$
  - $14 - 5$

Your answer is 9. Your answer does not depend on the number you choose. Your answer will always be 15 if you change Step 3 to "Add 30 to your answer." You have to add twice the value you want as an answer. Then when you divide your answer in Step 3 by 2 and subtract your original answer, you will always get the value you want as an answer.
- Answers will vary.

**Guided Practice for the lesson "Reason Using Properties from Algebra"**

- |                              |                               |
|------------------------------|-------------------------------|
| $4x + 9 = -3x + 2$           | Given                         |
| $4x + 9 + 3x = -3x + 2 + 3x$ | Addition Property of Equality |
| $7x + 9 = 2$                 | Simplify.                     |

- $7x + 9 - 9 = 2 - 9$       Subtraction Property of Equality  
 $7x = -7$       Simplify.  
 $x = -1$       Division Property of Equality
2.  $14x + 3(7 - x) = -1$       Given  
 $14x + 21 - 3x = -1$       Distributive Property  
 $11x + 21 = -1$       Simplify.  
 $11x + 21 - 21 = -1 - 21$       Subtraction Property of Equality  
 $11x = -22$       Simplify.  
 $x = -2$       Division Property of Equality
3.  $A = \frac{1}{2}bh$   
 $2A = bh$   
 $\frac{2A}{h} = b$
4. Symmetric Property of Equality  
 5. Transitive Property of Equality  
 6. Reflexive Property of Equality

**Exercises for the lesson "Reason Using Properties from Algebra"**

**Skill Practice**

1. Reflexive Property of Equality for angle measure  
 2. To check your answer, substitute  $a = \frac{r - 154}{-0.70}$  into the original equation. Simply to see if the result is a true statement.  
 3.  $3x - 12 = 7x + 8$       Given  
 $-4x - 12 = 8$       Subtraction Property of Equality  
 $-4x = 20$       Addition Property of Equality  
 $x = -5$       Division Property of Equality  
 4.  $5(x - 1) = 4x + 13$       Given  
 $5x - 5 = 4x + 13$       Distributive Property  
 $x - 5 = 13$       Subtraction Property of Equality  
 $x = 18$       Addition Property of Equality  
 5. D; The statement illustrates the Transitive Property of Equality for segment length.  
 6.  $5x - 10 = -40$       Given  
 $5x = -30$       Addition Property of Equality  
 $x = -6$       Division Property of Equality  
 7.  $4x + 9 = 16 - 3x$       Given  
 $7x + 9 = 16$       Addition Property of Equality  
 $7x = 7$       Subtraction Property of Equality  
 $x = 1$       Division Property of Equality

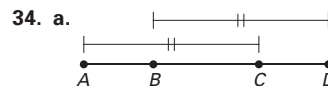
8.  $5(3x - 20) = -10$       Given  
 $15x - 100 = -10$       Distributive Property  
 $15x = 90$       Addition Property of Equality  
 $x = 6$       Division Property of Equality  
 9.  $3(2x + 11) = 9$       Given  
 $6x + 33 = 9$       Distributive Property  
 $6x = -24$       Subtraction Property of Equality  
 $x = -4$       Division Property of Equality  
 10.  $2(-x - 5) = 12$       Given  
 $-2x - 10 = 12$       Distributive Property  
 $-2x = 22$       Addition Property of Equality  
 $x = -11$       Division Property of Equality  
 11.  $44 - 2(3x + 4) = -18x$       Given  
 $44 - 6x - 8 = -18x$       Distributive Property  
 $-6x + 36 = -18x$       Simplify.  
 $36 = -12x$       Addition Property of Equality  
 $-3 = x$       Division Property of Equality  
 12.  $4(5x - 9) = -2(x + 7)$       Given  
 $20x - 36 = -2x - 14$       Distributive Property  
 $22x - 36 = -14$       Addition Property of Equality  
 $22x = 22$       Addition Property of Equality  
 $x = 1$       Division Property of Equality  
 13.  $2x - 15 - x = 21 + 10x$       Given  
 $x - 15 = 21 + 10x$       Simplify.  
 $-9x - 15 = 21$       Subtraction Property of Equality  
 $-9x = 36$       Addition Property of Equality  
 $x = -4$       Division Property of Equality  
 14.  $3(7x - 9) - 19x = -15$       Given  
 $21x - 27 - 19x = -15$       Distributive Property  
 $2x - 27 = -15$       Simplify.  
 $2x = 12$       Addition Property of Equality  
 $x = 6$       Division Property of Equality  
 15.  $5x + y = 18$       Given  
 $y = -5x + 18$       Subtraction Property of Equality  
 16.  $-4x + 2y = 8$       Given  
 $2y = 4x + 8$       Addition Property of Equality  
 $y = 2x + 4$       Division Property of Equality  
 17.  $12 - 3y = 30x$       Given  
 $-3y = 30x - 12$       Subtraction Property of Equality  
 $y = -10x + 4$       Division Property of Equality  
 18.  $3x + 9y = -7$       Given  
 $9y = -3x - 7$       Subtraction Property of Equality  
 $y = -\frac{1}{3}x - \frac{7}{9}$       Division Property of Equality

19.  $2y + 0.5x = 16$  Given  
 $2y = -0.5x + 16$  Subtraction Property of Equality  
 $y = -0.25x + 8$  Division Property of Equality
20.  $\frac{1}{2}x - \frac{3}{4}y = -2$  Given  
 $-\frac{3}{4}y = -\frac{1}{2}x - 2$  Subtraction Property of Equality  
 $y = \frac{2}{3}x + \frac{8}{3}$  Multiplication Property of Equality
21. If  $AB = 20$ , then  $AB + CD = 20 + CD$ .
22. If  $m\angle 1 = m\angle 2$ , then  $m\angle 2 = m\angle 1$ .
23. If  $AB = CD$ , then  $AB + EF = CD + EF$ .
24. If  $5(x + 8) = 2$ , then  $5x + 40 = 2$ .
25. If  $m\angle 1 = m\angle 2$  and  $m\angle 2 = m\angle 3$ , then  $m\angle 1 = m\angle 3$ .
26. In step 2, the Subtraction Property of Equality should have been used instead of the Addition Property of Equality.  
 $7x = x + 24$  Given  
 $6x = 24$  Subtraction Property of Equality  
 $x = 4$  Division Property of Equality
27. Answers will vary.
28. Equation (Reason)  
 $AD = AB, DC = BC$  (Given)  
 $AC = AC$  (Reflexive Property of Equality)  
 $AD + DC = AB + DC$  (Addition Property of Equality)  
 $AD + DC = AB + BC$  (Substitution)  
 $AD + DC + AC = AB + BC + AC$  (Addition Property of Equality)
29. Equation (Reason)  
 $AD = CB, DC = BA$  (Given)  
 $AC = AC$  (Reflexive Property of Equality)  
 $AD + DC = CB + DC$  (Addition Property of Equality)  
 $AD + DC = CB + BA$  (Substitution)  
 $AD + DC + AC = CB + BA + AC$  (Addition Property of Equality)
30.  $ZY + YX = ZX \rightarrow ZY = ZX - YX$   
 $YX + XW = YW \rightarrow XW = YW - YX$   
 $ZY = (5x + 17) - 3 = 5x + 14$   
 $XW = (10 - 2x) - 3 = 7 - 2x$   
 Because  $\overline{ZY} \cong \overline{XW}$ ,  $ZY = XW$ .  
 $5x + 14 = 7 - 2x$   
 $7x + 14 = 7$   
 $7x = -7$   
 $x = -1$   
 So,  $ZY = 5(-1) + 14 = 9$  and  $XW = 7 - 2(-1) = 9$ .

### Problem Solving

31.  $P = 2\ell + 2w$  Given  
 $P - 2w = 2\ell$  Subtraction Property of Equality  
 $\frac{P}{2} - w = \ell$  Division Property of Equality  
 When  $P = 55$  and  $w = 11$ :  
 $\ell = \frac{55}{2} - 11 = 16.5$   
 The length is 16.5 meters.
32.  $A = \frac{1}{2}bh$  Given  
 $2A = bh$  Multiplication Property of Equality  
 $\frac{2A}{b} = h$  Division Property of Equality  
 When  $A = 1768$  and  $b = 52$ :  
 $h = \frac{2(1768)}{52} = 68$   
 The height is 68 inches.

Equation	Explanation	Reason
$m\angle 1 = m\angle 4$ , $m\angle EHF = 90^\circ$ , $m\angle GHF = 90^\circ$	Marked in diagram.	Given
$m\angle EHF = \angle GHF$	Substitute $m\angle GHF$ for $90^\circ$ .	Substitution Property of Equality
$m\angle EHF = m\angle 1 + m\angle 2$ $m\angle GHF = m\angle 3 + m\angle 4$	Add measures of adjacent angles.	Angle Addition Postulate
$m\angle 1 + m\angle 2 = m\angle 3 + m\angle 4$	Write expressions equal to the angle measures.	Substitution Property of Equality
$m\angle 1 + m\angle 2 = m\angle 3 + m\angle 1$	Substitute $m\angle 1$ for $m\angle 4$ .	Substitution Property of Equality
$m\angle 2 = m\angle 3$	Subtract $m\angle 1$ from each side.	Subtraction Property of Equality



- b.  $AB + BC = AC$   
 $BC + CD = BD$   
 Because  $AC = BD$ ,  $AB + BC = BC + CD$ ,  
 so,  $AB$  must equal  $CD$ .

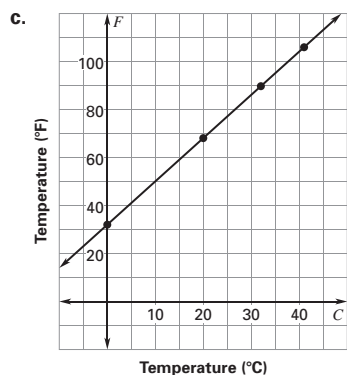
- c.  $AC = BD$  Given  
 $BC = BC$  Reflexive Prop. of Equality  
 $AC = AB + BC$  Segment Add. Postulate  
 $BD = BC + CD$  Segment Add. Postulate  
 $AB + BC = BC + CD$  Substitution Prop. of Equality  
 $AB = CD$  Subtraction Prop. of Equality

35.  $m\angle 1 + m\angle 2 + m\angle 3 = 180^\circ$   
 $m\angle 1 + m\angle 2 + m\angle 1 = 180^\circ$   
 $m\angle 2 = 180^\circ - 2m\angle 1$   
 $m\angle 1 + m\angle 2 = 148^\circ$   
 $m\angle 2 = 148^\circ - m\angle 1$   
 $180^\circ - 2m\angle 1 = 148^\circ - m\angle 1$   
 $32^\circ = m\angle 1$   
 So,  $m\angle 2 = 180^\circ - 2(32^\circ) = 116^\circ$ .

36. a.  $C = \frac{5}{9}(F - 32)$  Given  
 $C = \frac{5}{9}F - \frac{160}{9}$  Distributive Property  
 $C + \frac{160}{9} = \frac{5}{9}F$  Addition Property of Equality  
 $\frac{9}{5}\left(C + \frac{160}{9}\right) = F$  Mult. Property of Equality  
 $\frac{9}{5}C + 32 = F$  Distributive Property

b.

Temperature (°C)	0	20	32	41
Temperature (°F)	32	68	89.6	105.8



This is a linear function.

37. The relationship is symmetric. “Yen worked the same hours as Jim” is the same as “Jim worked the same hours as Yen.”
38. The relationship is transitive. One example of an explanation is that  $-4$  is less than  $-2$  and  $-2$  is less than  $-1$ , so  $-4$  is less than  $-1$ .

### Quiz for the lessons “Use Postulates and Diagrams” and “Reason Using Properties from Algebra”

- True
- False
- True
- $x + 20 = 35$  Given  
 $x = 15$  Subtraction Property of Equality
- $5x - 14 = 16 + 3x$  Given  
 $2x - 14 = 16$  Subtraction Property of Equality  
 $2x = 30$  Addition Property of Equality  
 $x = 15$  Division Property of Equality
- If  $AB = CD$ , then  $AB - EF = CD - EF$ .
- If  $a = b$  and  $b = c$ , then  $a = c$ .

### Lesson 2.6 Prove Statements about Segments and Angles

#### Guided Practice for the lesson “Prove Statements about Segments and Angles”

Statements	Reasons
1. $AC = AB + AB$	1. Given
2. $AB + BC = AC$	2. Segment Addition Postulate
3. $AB + AB = AB + BC$	3. Substitution Property
4. $AB = BC$	4. Subtraction Property of Equality

- Reflexive Property of Segment Congruence
- Symmetric Property of Angle Congruence
- Step 5 would be  $MB + MB = AB$ .  
 Step 6 would be  $2MB = AB$ .  
 Step 7 would be  $MB = \frac{1}{2}AB$ .
- It does not matter what the actual distances are in order to prove the relationship between  $AB$  and  $CD$ . What matters are the positions of the stores relative to each other.
- 

From Example 4, you know that  $AB = CD$ . You also know that  $\overline{BE} \cong \overline{EC}$  by the definition of a midpoint. By the definition of congruent segments,  $BE = EC$ . So,  $AB + BE = EC + CD$ , or  $AE = ED$ . The food court and the bookstore are also the same distance from the clothing store.

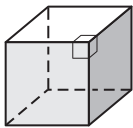
**Exercises for the lesson "Prove Statements about Segments and Angles"**

**Skill Practice**

1. A theorem is a statement that can be proven. A postulate is a rule that is accepted without proof.
2. *Sample answer:* You can use postulates such as the Angle Addition Postulate. You can use properties such as the Reflexive Property of Equality. You can also use definitions such as the definition of a right angle.

Statements	Reasons
1. $AB = 5, BC = 6$	1. Given
2. $AC = AB + BC$	2. Segment Addition Postulate
3. $AC = 5 + 6$	3. Substitution Property of Equality
4. $AC = 11$	4. Simplify.

4. A; Transitive Property of Equality
5.  $\overline{SE} \cong \overline{SE}$
6. If  $\angle JKL \cong \angle RST$ , then  $\angle RST \cong \angle JKL$ .
7. If  $\angle F \cong \angle J$  and  $\angle J \cong \angle L$ , then  $\angle F \cong \angle L$ .
8. Symmetric Property of Congruence
9. Reflexive Property of Congruence
10. Transitive Property of Congruence
11. Reflexive Property of Equality
12. C; Symmetric Property of Congruence
13. The reason that  $\overline{MN} \cong \overline{PN}$  should be the Transitive Property of Segment Congruence, not the Reflexive Property of Segment Congruence  
Because  $\overline{MN} \cong \overline{LQ}$  and  $\overline{LQ} \cong \overline{PN}$ , then  $\overline{MN} \cong \overline{PN}$  by the Transitive Property of Congruence.
14. Six planes that intersect at right angles:



15. Shops along the boardwalk:



Statements	Reasons
1. $RT = 5, RS = 5,$ $\overline{RT} \cong \overline{TS}$	1. Given
2. $RS = RT$	2. Transitive Property of Equality
3. $RT = TS$	3. Definition of congruent segments
4. $RS = TS$	4. Transitive Property of Equality
5. $\overline{RS} \cong \overline{TS}$	5. Definition of congruent segments

17. If  $\overline{QR} \cong \overline{PQ}$  and  $\overline{RS} \cong \overline{PQ}$ , then  $QR = PQ$  and  $RS = PQ$  by the definition of congruent segments.  $PQ = RS$  by the Symmetric Property of Equality. If  $QR = PQ$  and  $PQ = RS$ , then  $QR = RS$  by the Transitive Property of Equality.

$$QR = RS$$

$$2x + 5 = 10 - 3x \quad \text{Given}$$

$$5x + 5 = 10 \quad \text{Addition Property of Equality}$$

$$5x = 5 \quad \text{Subtraction Property of Equality}$$

$$x = 1 \quad \text{Division Property of Equality}$$

18. You know that  $6x^\circ + (3x - 9)^\circ = m\angle ABC$  by the Angle Addition Postulate. You are given that  $m\angle ABC = 90^\circ$ . So,  $6x + (3x - 9) = 90$ .

$$6x + (3x - 9) = 90$$

$$9x - 9 = 90 \quad \text{Simplify}$$

$$9x = 99 \quad \text{Addition Property of Equality}$$

$$x = 11 \quad \text{Division Property of Equality}$$

19. Writing a proof is an example of deductive reasoning because you are using facts, definitions, accepted properties, and laws of logic to reach a desired conclusion.
20. a. You are given that  $P$  is the midpoint of  $\overline{MN}$ ,  $\overline{AB} \cong \overline{MP}$ , and  $PN = x$ . You know that  $\overline{MP} \cong \overline{PN}$  by the definition of midpoint. If  $\overline{AB} \cong \overline{MP}$  and  $\overline{MP} \cong \overline{PN}$ , then  $\overline{AB} \cong \overline{PN}$  by the Transitive Property of Segment Congruence.  $AB = PN$  by the definition of congruent segments. So,  $AB = x$ .  
b. You are given that  $P$  is the midpoint of  $\overline{MN}$  and  $PN = x$ . You know that  $MN = MP + PN$  by the Segment Addition Postulate and  $\overline{MP} \cong \overline{PN}$  by the definition of midpoint.  $MP = PN$  by the definition of congruent segments.  $MN = PN + PN$  by the Substitution Property of Equality. So,  $MN = x + x = 2x$ .  
c. You are given that  $P$  is the midpoint of  $\overline{MN}$ ,  $Q$  is the midpoint of  $\overline{MP}$ , and  $PN = x$ . You know that  $\overline{MP} \cong \overline{PN}$  and  $\overline{MQ} \cong \overline{QP}$  by the definition of midpoint.  $MP = PN$  and  $MQ = QP$  by the definition of congruent segments.  $MP = MQ + QP$  by the Segment Addition Postulate.  $MP = MQ + MQ$ , and so  $PN = MQ + MQ$ , or  $x = 2MQ$ , by the Substitution Property of Equality. So,  $MQ = \frac{1}{2}x$  by the Division Property of Equality.

- d. You know that  $MN = MQ + NQ$  by the Segment Addition Postulate. From part (b) you know that  $MN = 2x$  and from part (c) you know that  $MQ = \frac{1}{2}x$ . Using substitution,  $2x = \frac{1}{2}x + NQ$ . So,  $NQ = \frac{3}{2}x$ .

### Problem Solving

21.

Statements	Reasons
1. $\overrightarrow{TV}$ bisects $\angle UTW$ .	1. Given
2. $\angle 1 \cong \angle 2$	2. Definition of angle bisector
3. $\angle 2 \cong \angle 3$	3. Given
4. $\angle 1 \cong \angle 3$	4. Transitive Property of Angle Congruence

22.

Statements	Reasons
1. $\overrightarrow{QS}$ is an angle bisector of $\angle PQR$ .	D. Given
2. $\angle PQS \cong \angle SQR$	A. Definition of angle bisector
3. $m\angle PQS = m\angle SQR$	F. Definition of congruent angles
4. $m\angle PQS + m\angle SQR = m\angle PQR$	C. Angle addition Postulate
5. $m\angle PQS + m\angle PQS = m\angle PQR$	G. Substitution Property of Equality
6. $2 \cdot m\angle PQS = m\angle PQR$	B. Distributive Property
7. $m\angle PQS = \frac{1}{2}m\angle PQR$	E. Division Property of Equality

23.

Statements	Reasons
1. $2AB = AC$	1. Given
2. $AB + AB = AC$	2. Distributive Property
3. $AB + BC = AC$	3. Segment Addition Postulate
4. $AB + AB = AB + BC$	4. Transitive Property of Equality
5. $AB = BC$	5. Subtraction Property of Equality

24.

Statements	Reasons
1. $m\angle 1 + m\angle 2 = 180^\circ$ and $m\angle 1 = 62^\circ$	1. Given
2. $m\angle 2 = 180^\circ - m\angle 1$	2. Subtraction Property of Equality
3. $m\angle 2 = 180^\circ - 62^\circ$	3. Substitution Property of Equality
4. $m\angle 2 = 118^\circ$	4. Simplify.

25.

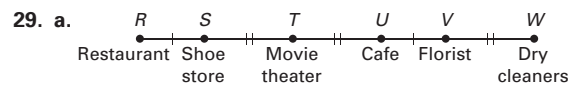
Statements	Reasons
1. $A$ is an angle.	1. Given
2. $m\angle A = m\angle A$	2. Reflective Property of Equality
3. $\angle A \cong \angle A$	3. Definition of congruent angles

26.

Statements	Reasons
1. $\overline{WX} \cong \overline{XY}$ and $\overline{XY} \cong \overline{YZ}$	1. Given
2. $WX = XY$ and $XY = YZ$	2. Definition of congruent segments
3. $WX = YZ$	3. Transitive Property of Equality
4. $\overline{WX} \cong \overline{YZ}$	4. Definition of congruent segments

27. You know that  $\angle 1 \cong \angle 3$  by the Transitive Property of Congruent Angles, so all the angles are the same. The sculpture is an equiangular triangle.

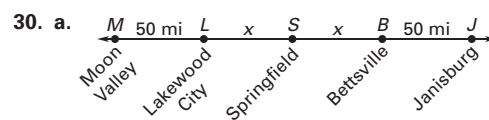
28. *Sample answer:* All of the line segments are congruent because when you copy and paste a line segment, the length of each line segment is the same. So, all of the segments are congruent by the definition of congruent segments.



b. Given:  $RS = UV$   
 $ST = TU = VW$   
 Prove:  $RT = UW$

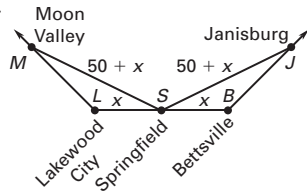
c.

Statements	Reasons
1. $RS = UV$ and $ST = TU = VW$	1. Given
2. $RT = RS + ST$	2. Segment Addition Postulate
3. $UW = UV + VW$	3. Segment Addition Postulate
4. $RT = UV + VW$	4. Substitution Property of Equality
5. $RT = UW$	5. Transitive Property of Equality





b. Sample answer:



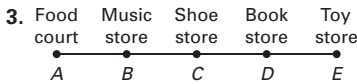
c. In the first diagram, all of the cities lie in a straight line, so you know the distance between Moon Valley and Lakewood City, and between Bettsville and Janisburg, is 50 miles. You do not know this distance from the second diagram.

**Problem Solving Workshop for the lesson “Prove Statements about Segments and Angles”**

- a. The proofs both use the definition of midpoint and properties of equality to prove the same conclusion. The proof on page 120 uses the Reflexive Property of Equality and the Substitution Property of Equality to prove  $FM = SB$ . The proof on page 115 uses the Transitive Property of Congruence to prove  $AB = CD$ . The letters chosen to represent the locations are different but the conclusion is the same.

b. Answers will vary.

2. Substitution Property of Equality, because you could substitute  $m\angle C$  for  $m\angle B$ .



Given:  $B$  is halfway between  $A$  and  $C$

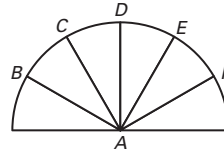
$C$  is halfway between  $B$  and  $D$

$D$  is halfway between  $C$  and  $E$

Prove:  $AB = DE$

Statements	Reasons
1. $B$ is halfway between $A$ and $C$ .	1. Given
2. $C$ is halfway between $B$ and $D$ .	2. Given
3. $D$ is halfway between $C$ and $E$ .	3. Given
4. $B$ is the midpoint of $\overline{AC}$ .	4. Definition of midpoint
5. $C$ is the midpoint of $\overline{BD}$ .	5. Definition of midpoint
6. $D$ is the midpoint of $\overline{CE}$ .	6. Definition of midpoint
7. $AB = BC$ , $BC = CD$ , and $CD = DE$	7. Definition of midpoint
8. $AB = CD$	8. Transitive Property of Equality
9. $BC = DE$	9. Transitive Property of Equality
10. $AB = DE$	10. Substitution Property of Equality

4.



Given:  $\angle BAC \cong \angle CAD \cong \angle DAE \cong \angle EAF$

Prove:  $m\angle CAE = \frac{1}{2}m\angle BAF$

Statements	Reasons
1. $\angle BAC \cong \angle CAD \cong \angle DAE \cong \angle EAF$	1. Given
2. $m\angle BAC = m\angle CAD = m\angle DAE = m\angle EAF$	2. Definition of congruent angles
3. $m\angle CAE = m\angle CAD + m\angle DAE$	3. Angle Addition Postulate
4. $m\angle BAF = m\angle BAC + m\angle CAD + m\angle DAE + m\angle EAF$	4. Angle Addition Postulate
5. $m\angle BAF = m\angle CAD + m\angle DAE + m\angle CAD + m\angle DAE$	5. Substitution Property of Equality

Statements	Reasons
6. $m\angle BAF = m\angle CAE + m\angle CAE$	6. Substitution Property of Equality
7. $m\angle BAF = 2 \cdot m\angle CAE$	7. Distributive Property
8. $2m\angle CAE = m\angle BAF$	8. Symmetric Property of Equality
9. $m\angle CAE = \frac{1}{2}m\angle BAF$	9. Division Property of Equality

- a. Both proofs use the same reasoning to prove the Symmetric Property of Angle Congruence and the Symmetric Property of Segment Congruence. The difference is that one proof deals with angle congruence and the other proof deals with segment congruence.

b. If  $\overline{FG} \cong \overline{DE}$  is the second statement, then the reason would have to be the Symmetric Property of Segment Congruence. This is not a valid reason in this proof because the Symmetric Property of Segment Congruence is what is trying to be proven, so it is an unproven theorem.

**Lesson 2.7 Prove Angle Pair Relationships**

**Investigating Geometry Activity for the lesson “Prove Angle Pair Relationships”**

1.  $\angle AEC$  and  $\angle AED$  are linear, so they are supplementary.
2.  $\angle AED$  and  $\angle DEB$  are linear, so they are supplementary.
3.  $m\angle AEC$  is equal to  $m\angle DEB$ .

4. When you move  $C$  to a different position it changes the measure of the angles, but it does not change the angle relationships.
- Two angles supplementary to the same angle are congruent.
5. Yes, let  $\angle A$  and  $\angle B$  be two angles supplementary to  $\angle C$ . Then  $m\angle A + m\angle C = 180^\circ$ ,  $m\angle B + m\angle C = 180^\circ \rightarrow m\angle A + m\angle C = m\angle B + m\angle C \rightarrow m\angle A = m\angle B$ , so  $\angle A \cong \angle B$ .
6. Yes, the angle measures change, but the angle relationship stays the same.
7. If the non-adjacent sides of  $\angle CEG$  and  $\angle GEB$  are perpendicular, then  $\angle CEG$  and  $\angle GEB$  are complementary angles.
8. If two angles are vertical angles formed by intersecting lines, then the two angles are congruent.
9. The vertical angles are:  
 $\angle AEC$  and  $\angle BED$ ,  $\angle CEG$  and  $\angle FED$ ,  $\angle GEB$  and  $\angle AEF$ ,  $\angle CEB$  and  $\angle DEA$ ,  $\angle GED$  and  $\angle CEF$ ,  $\angle AEG$  and  $\angle BEF$ . The vertical angles in each pair are congruent.

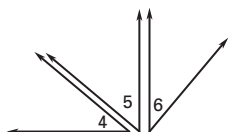
### Guided Practice for the lesson "Prove Angle Pair Relationships"

1. You save two steps using the Right Angles Congruence Theorem. The following is the proof without using the Right Angle Congruence Theorem.

Statements	Reasons
1. $\overline{AB} \perp \overline{BC}, \overline{DC} \perp \overline{BC}$	1. Given
2. $\angle B$ and $\angle C$ are right angles.	2. Definition of perpendicular lines
3. $m\angle B = 90^\circ$ , $m\angle C = 90^\circ$	3. Definition of right angle
4. $m\angle B = m\angle C$	4. Transitive Property of Equality
5. $\angle B \cong \angle C$	5. Definition of congruent angles

2. *Case 1:* Two angles complementary to the same angle are congruent.

Given:  $\angle 4$  and  $\angle 5$  are complements.  
 $\angle 5$  and  $\angle 6$  are complements.  
 Prove:  $\angle 4 \cong \angle 6$



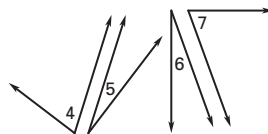
*Case 2:* Two angles complementary to congruent angles are congruent.

Given:  $\angle 4$  and  $\angle 5$  are complements.

$\angle 6$  and  $\angle 7$  are complements.

$\angle 5 \cong \angle 6$

Prove:  $\angle 4 \cong \angle 7$



3. Given:  $m\angle 1 = 112^\circ$   
 $m\angle 3 = m\angle 1 = 112^\circ$   
 $m\angle 1 + m\angle 2 = 180^\circ$   
 $112^\circ + m\angle 2 = 180^\circ$   
 $m\angle 2 = 68^\circ$   
 $m\angle 4 = m\angle 2 = 68^\circ$
4. Given:  $m\angle 2 = 67^\circ$   
 $m\angle 4 = m\angle 2 = 67^\circ$   
 $m\angle 1 + m\angle 2 = 180^\circ$   
 $m\angle 1 + 67^\circ = 180^\circ$   
 $m\angle 1 = 113^\circ$   
 $m\angle 3 = m\angle 1 = 113^\circ$
5. Given:  $m\angle 4 = 71^\circ$   
 $m\angle 2 = m\angle 4 = 71^\circ$   
 $m\angle 3 + m\angle 4 = 180^\circ$   
 $m\angle 3 + 71^\circ = 180^\circ$   
 $m\angle 3 = 109^\circ$   
 $m\angle 1 = m\angle 3 = 109^\circ$
6. Congruent Supplements Theorem
7.  $32 + (3x + 1) = 180$   
 $33 + 3x = 180$   
 $3x = 147$   
 $x = 49$
8.  $m\angle TPS = m\angle QPR$   
 $m\angle QPR = 3(49) + 1 = 148^\circ$   
 $m\angle TPS = 148^\circ$

### Exercises for the lesson "Prove Angle Pair Relationships"

#### Skill Practice

- If two lines intersect at a point, then the *vertical* angles formed by the intersecting lines are congruent.
- The sum of the measures of complementary angles is  $90^\circ$ . The sum of the measures of supplementary angles is  $180^\circ$ . The measures of vertical angles are equal. The sum of the angle measures of a linear pair is  $180^\circ$ .
- $\angle PSM$  and  $\angle PSR$  are both right angles. So,  $\angle PSM \cong \angle PSR$  by the Right Angles Congruence Theorem.  $m\angle MSN = 50^\circ$  and  $m\angle PSQ = 50^\circ$ , so  $\angle MSN \cong \angle PSQ$  by the definition of congruent angles.  $\angle PSN$  is the complement of  $\angle MSN$  and  $\angle RSQ$  is the complement of  $\angle PSQ$ . So,  $\angle PSN \cong \angle RSQ$  by the Congruent Complements Theorem.
- $\angle ABC \cong \angle DEF$  and  $\angle CBE \cong \angle FEB$  by the Congruent Supplements Theorem.
- $\angle FGH \cong \angle WXZ$ ;  $\angle WXZ$  is a right angle because  $58^\circ + 32^\circ = 90^\circ$ , so  $\angle FGH \cong \angle WXZ$  by the Right Angles Congruence Theorem.

6.  $\angle KMJ$  and  $\angle KMG$  are both right angles. So,  $\angle KMJ \cong \angle KMG$  by the Right Angles Congruence Theorem.  $\angle GML$  and  $\angle HMJ$  and  $\angle GMH$  and  $\angle LMJ$  are two pairs of vertical angles. So,  $\angle GML \cong \angle HMJ$  and  $\angle GMH \cong \angle LMJ$  by the Vertical Angles Congruence Theorem.
7. The four angles are congruent right angles. They are all right angles by the definition of perpendicular lines. All right angles are congruent by the Right Angles Congruence Theorem.
8. Given:  $m\angle 1 = 155^\circ$   
 $m\angle 3 = m\angle 1 = 155^\circ$   
 $m\angle 1 + m\angle 2 = 180^\circ$   
 $155^\circ + m\angle 2 = 180^\circ$   
 $m\angle 2 = 25^\circ$   
 $m\angle 4 = m\angle 2 = 25^\circ$
9. Given:  $m\angle 3 = 168^\circ$   
 $m\angle 1 = m\angle 3 = 168^\circ$   
 $m\angle 3 + m\angle 4 = 180^\circ$   
 $168^\circ + m\angle 4 = 180^\circ$   
 $m\angle 4 = 12^\circ$   
 $m\angle 2 = m\angle 4 = 12^\circ$
10. Given:  $m\angle 4 = 27^\circ$   
 $m\angle 2 = m\angle 4 = 27^\circ$   
 $m\angle 4 + m\angle 3 = 180^\circ$   
 $27^\circ + m\angle 3 = 180^\circ$   
 $m\angle 3 = 153^\circ$   
 $m\angle 1 = m\angle 3 = 153^\circ$
11. Given:  $m\angle 2 = 32^\circ$   
 $m\angle 4 = m\angle 2 = 32^\circ$   
 $m\angle 1 + m\angle 2 = 180^\circ$   
 $m\angle 1 + 32^\circ = 180^\circ$   
 $m\angle 1 = 148^\circ$   
 $m\angle 3 = m\angle 1 = 148^\circ$
12. Using the Vertical Angles Congruence Theorem:  
 $8x + 7 = 9x - 4$   
 $11 = x$   
 $5y = 7y - 34$   
 $-2y = -34$   
 $y = 17$
13. Using the Vertical Angles Congruence Theorem:  
 $4x = 6x - 26$   
 $-2x = -26$   
 $x = 13$   
 $6y + 8 = 7y - 12$   
 $20 = y$
14. Using the Vertical Angles Congruence Theorem:  
 $10x - 4 = 6(x + 2)$   
 $10x - 4 = 6x + 12$   
 $4x = 16$   
 $x = 4$   
 $18y - 18 = 16y$   
 $-18 = -2y$   
 $9 = y$
15. The error is assuming that  $\angle 1$  and  $\angle 4$  and  $\angle 2$  and  $\angle 3$  are vertical angle pairs. They are not formed by the intersection of two lines. So,  $\angle 1 \neq \angle 4$  and  $\angle 2 \neq \angle 3$ .
16. D;  $m\angle A + m\angle D = 90^\circ$   
 $4x^\circ + m\angle D = 90^\circ$   
 $m\angle D = 90 - 4x^\circ$
17.  $30^\circ$ ; If  $m\angle 3 = 30^\circ$ , then  $m\angle 6 = 30^\circ$  by the Vertical Angles Congruence Theorem.
18.  $25^\circ$ ; If  $m\angle BHF = 115^\circ$ , then  $m\angle 2 = 65^\circ$  by the Linear Pair Postulate. Because  $m\angle BHG = 90^\circ$ ,  $m\angle BHD = 90^\circ$  by the Linear Pair Postulate.  $\angle 3$  is the complement of  $\angle 2$  because  $m\angle BHD = 90^\circ$ . So,  $m\angle 3 = 25^\circ$ .
19.  $27^\circ$ ; If  $m\angle 6 = 27^\circ$ , then  $m\angle 1 = 27^\circ$  by the Congruent Complements Theorem.
20.  $133^\circ$ ; If  $m\angle DHF = 133^\circ$ , then  $m\angle CHG = 133^\circ$  by the Vertical Angles Congruence Theorem.
21.  $58^\circ$ ; If  $m\angle BHG = 90^\circ$ , then  $m\angle BHD = 90^\circ$  by the Linear Pair Postulate.  $\angle 2$  is the complement of  $\angle 3$  because  $m\angle BHD = 90^\circ$ . So,  $m\angle 2 = 58^\circ$ .
22. The statement is false.  $\angle 1$  and  $\angle 2$  are a linear pair and you know the intersecting lines are not perpendicular, so  $\angle 1 \neq \angle 2$ .
23. The statement is true.  $\angle 1$  and  $\angle 3$  are vertical angles.
24. The statement is false.  $\angle 1$  and  $\angle 4$  are a linear pair and you know that the intersecting lines are not perpendicular, so  $\angle 1 \neq \angle 4$ .
25. The statement is false.  $\angle 2$  and  $\angle 3$  are a linear pair and you know that the intersecting lines are not perpendicular, so  $\angle 3 \neq \angle 2$ .
26. The statement is true.  $\angle 2$  and  $\angle 4$  are vertical angles.
27. The statement is true.  $\angle 3$  and  $\angle 4$  are a linear pair, so they are supplementary.
28. Using the Linear Pair Postulate:  
 $10y + 3y + 11 = 180$   
 $13y = 169$   
 $y = 13$   
 $7x + 4 + 4x - 22 = 180$   
 $11x - 18 = 180$   
 $11x = 198$   
 $x = 18$   
 The measure of each angle is:  
 $3(13) + 11 = 50^\circ$   
 $10(13) = 130^\circ$   
 $4(18) - 22 = 50^\circ$   
 $7(18) + 4 = 130^\circ$
29. Using the Vertical Angle Congruence Theorem:  
 $2(5x - 5) = 6x + 50$   
 $10x - 10 = 6x + 50$   
 $4x = 60$   
 $x = 15$   
 $5y + 5 = 7y - 9$   
 $14 = 2y$   
 $7 = y$

The measure of each angle is:

$$5(7) + 5 = 40^\circ$$

$$2(5 \cdot 15 - 5) = 140^\circ$$

$$7(7) - 9 = 40^\circ$$

$$6(15) + 50 = 140^\circ$$

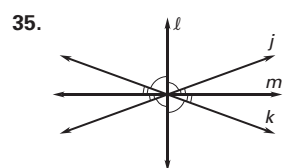
30. *Sample answer:*  $m\angle ABY = 80^\circ$  because  $\overrightarrow{XY}$  bisects  $\angle ABC$ .  $m\angle CBX = 100^\circ$  because  $\angle CBY$  and  $\angle CBX$  are supplementary.

31.  $\angle EGH \cong \angle FGH$  by the definition of angle bisector.

32.  $\angle 1 \cong \angle 9$  by the Congruent Supplements Theorem.

33. *Sample answer:*  $\angle AED \cong \angle BEC$  by the definition of perpendicular lines and the Vertical Angles Congruence Theorem.

34.  $\angle 5 \cong \angle 1$  by the Congruent Complements Theorem.



Lines  $l$  and  $m$  bisect supplementary angles. The sum of supplementary angles is  $180^\circ$ ; so half the sum of each angle pair is  $90^\circ$ . Line  $l$  is perpendicular to line  $m$  by the definition of perpendicular lines.

### Problem Solving

36.	Statements	Reasons
	1. $\angle 1$ and $\angle 2$ are supplements. $\angle 3$ and $\angle 4$ are supplements. $\angle 1 \cong \angle 4$	1. Given
	2. $m\angle 1 + m\angle 2 = 180^\circ$ $m\angle 3 + m\angle 4 = 180^\circ$	2. Definition of supplementary angles
	3. $m\angle 1 + m\angle 2 = m\angle 3 + m\angle 4$	3. Transitive Property of Equality
	4. $m\angle 1 = m\angle 4$	4. Definition of congruent angles
	5. $m\angle 1 + m\angle 2 = m\angle 3 + m\angle 1$	5. Substitution Property of Equality
	6. $m\angle 2 = m\angle 3$	6. Subtraction Property of Equality
	7. $\angle 2 \cong \angle 3$	7. Definition of congruent angles

37.	Statements	Reasons
	1. $\angle 1$ and $\angle 2$ are complements. $\angle 1$ and $\angle 3$ are complements.	1. Given
	2. $m\angle 1 + m\angle 2 = 90^\circ$ $m\angle 1 + m\angle 3 = 90^\circ$	2. Definition of complementary angles
	3. $m\angle 1 + m\angle 2 = m\angle 1 + m\angle 3$	3. Transitive Property of Equality
	4. $m\angle 2 = m\angle 3$	4. Subtraction Property of Equality
	5. $\angle 2 \cong \angle 3$	5. Definition of congruent angles

38.	Statements	Reasons
	1. $\angle ABD$ is a right angle. $\angle CBE$ is a right angle.	1. Given
	2. $m\angle ABD = 90^\circ$ ; $m\angle CBE = 90^\circ$	2. Definition of right angle
	3. $m\angle ABC + m\angle CBD = m\angle ABD$ $m\angle CBD + m\angle DBE = m\angle CBE$	3. Angle Addition Postulate
	4. $\angle ABC$ and $\angle CBD$ are complements. $\angle CBD$ and $\angle DBE$ are complements.	4. Definition of complementary angles
	6. $\angle ABC \cong \angle DBE$	5. Congruent Supplements Theorem

39.	Statements	Reasons
	1. $\overline{JK} \perp \overline{JM}, \overline{KL} \perp \overline{ML}$ $\angle J \cong \angle M,$ $\angle K \cong \angle L$	1. Given
	2. $\angle J$ is a right angle; $\angle L$ is a right angle.	2. Definition of perpendicular lines
	3. $m\angle J = 90^\circ$ $m\angle L = 90^\circ$	3. Definition of a right angle
	4. $m\angle J = m\angle L$ $m\angle L = m\angle K$	4. Definition of congruent angles
	5. $m\angle M = 90^\circ$ $m\angle K = 90^\circ$	5. Transitive Property of Equality
	6. $\angle M$ and $\angle K$ are right angles.	6. Definition of a right angle
	7. $\overline{JM} \perp \overline{ML}, \overline{JK} \perp \overline{KL}$	7. Definition of perpendicular lines

40. a. Given:  $m\angle 1 = x^\circ$   
 $m\angle 2 = (180 - x)^\circ$  because  $\angle 1$  and  $\angle 2$  are supplements.  
 $m\angle 3 = x^\circ$  because  $\angle 1$  and  $\angle 3$  are vertical angles.  
 $m\angle 4 = (180 - x)^\circ$  because  $\angle 3$  and  $\angle 4$  are supplements.
- b. *Sample answer:*  $x = 120$   
 $m\angle 1 = m\angle 3 = 120^\circ$   
 $m\angle 2 = m\angle 4 = 180 - 120 = 60^\circ$
- c. As  $\angle 4$  gets smaller,  $\angle 2$  gets smaller and  $\angle 1$  and  $\angle 3$  get larger.  $\angle 1$  and  $\angle 4$  are supplementary and  $\angle 2$  and  $\angle 3$  are supplementary. As one angle measure gets smaller, the other must get larger to keep the sum of  $180^\circ$ .

41. Given:  $\angle 4$  and  $\angle 5$  are complementary.  
 $\angle 6$  and  $\angle 7$  are complementary.  
 $\angle 5 \cong \angle 7$   
 Prove:  $\angle 4 \cong \angle 6$

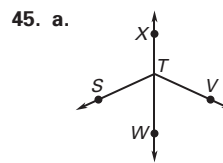


Statements	Reasons
1. $\angle 4$ and $\angle 5$ are complementary. $\angle 6$ and $\angle 7$ are complementary. $\angle 5 \cong \angle 7$	1. Given
2. $m\angle 4 + m\angle 5 = 90^\circ$ ; $m\angle 6 + m\angle 7 = 90^\circ$	2. Definition of complementary angles
3. $m\angle 4 + m\angle 5 = m\angle 6 + m\angle 7$	3. Transitive Property of Equality
4. $m\angle 5 = m\angle 7$	4. Definition of congruent angles
5. $m\angle 4 + m\angle 5 = m\angle 6 + m\angle 5$	5. Substitution Property of Equality
6. $m\angle 4 = m\angle 6$	6. Subtraction Property of Equality
7. $\angle 4 \cong \angle 6$	7. Definition of congruent angles

Statements	Reasons
1. $\angle 1 \cong \angle 3$	1. Given
2. $\angle 1 \cong \angle 2$	2. Vertical Angles Congruence Theorem
3. $\angle 3 \cong \angle 4$	3. Vertical Angles Congruence Theorem
4. $\angle 1 \cong \angle 4$	4. Transitive Property
5. $\angle 2 \cong \angle 4$	5. Transitive Property

Statements	Reasons
1. $\angle QRS$ and $\angle PSR$ are supplementary.	1. Given
2. $\angle QRS$ and $\angle QRL$ are a linear pair.	2. Definition of linear pair
3. $\angle QRS$ and $\angle QRL$ are supplementary.	3. Linear Pair Postulate
4. $\angle QRL \cong \angle PSR$	4. Congruent Supplements Theorem

Statements	Reasons
1. $\angle 1$ is complementary to $\angle 3$ . $\angle 2$ is complementary to $\angle 4$ .	1. Given
2. $\angle 2 \cong \angle 3$	2. Vertical Angles Congruence Theorem
3. $\angle 1 \cong \angle 4$	3. Congruent Complements Theorem



- b. Given:  $\overline{TW}$  bisects  $\angle STV$  and  $\overline{TX}$  and  $\overline{TW}$  are opposite rays.  
 Prove:  $\angle STX \cong \angle VTX$

c.	Statements	Reasons
	1. $\overrightarrow{TW}$ bisects $\angle STV$ . $\overrightarrow{TX}$ and $\overrightarrow{TW}$ are opposite rays.	1. Given
	2. $\angle STW \cong \angle VTW$	2. Definition of angle bisector
	3. $\angle STW$ and $\angle STX$ are a linear pair. $\angle VTW$ and $\angle VTX$ are a linear pair.	3. Definition of a linear pair
	4. $\angle STW$ and $\angle STX$ are supplements. $\angle VTW$ and $\angle VTX$ are supplements.	4. Linear Pair Postulate
	5. $\angle STX \cong \angle VTX$	5. Congruent Supplements Theorem

46. a.  $m\angle 3 = m\angle 7$ ; They are both right angles.  
 b.  $m\angle 4 = m\angle 6$ ; They are vertical angles.  
 c.  $m\angle 8 + m\angle 6 < 150^\circ$ ; The sum must be equal to  $90^\circ$  because  $m\angle 8 + m\angle 7 + m\angle 6 = 180^\circ$ .  
 d. If  $m\angle 4 = 30^\circ$ ; then  $m\angle 5 > m\angle 4$ ;  $\angle 4$  and  $\angle 5$  are a linear pair.

47.	Statements	Reasons
	1. $m\angle WYZ = m\angle TWZ = 45^\circ$	1. Given
	2. $\angle WYZ \cong \angle TWZ$	2. Definition of congruent angles
	3. $\angle WYZ$ and $\angle XYW$ are a linear pair. $\angle TWZ$ and $\angle SWZ$ are a linear pair.	3. Definition of linear pair
	4. $\angle WYZ$ and $\angle XYW$ are supplements. $\angle TWZ$ and $\angle SWZ$ are supplements.	4. Linear Pair Postulate
	5. $\angle SWZ \cong \angle XYW$	5. Congruent Supplements Theorem

48.	Statements	Reasons
	1. The hexagon is regular.	1. Given
	2. The interior angles are congruent.	2. Definition of regular polygon
	3. The measures of the interior angles are equal.	3. Definition of congruent angles
	4. $\angle 2$ and its adjacent interior angle are a linear pair.	4. Definition of linear pair
	5. $\angle 2$ and its adjacent interior angle are supplements.	5. Linear Pair Postulate
	6. The sum of $m\angle 2$ and the measure of its adjacent interior angle is $180^\circ$	6. Definition of supplementary angles
	7. The sum of $m\angle 2$ and the measure of any interior angle is $180^\circ$ .	7. Substitution Property of Equality
	8. $\angle 1$ and the interior angle whose sides form two pairs of opposite rays are vertical angles.	8. Definition of vertical angles
	9. $\angle 1$ and the interior angle whose sides form two pairs of opposite rays are congruent.	9. Vertical Angles Congruence Theorem
	10. $m\angle 1$ and the measure of any interior angle are equal.	10. Definition of congruent angles
	11. $m\angle 2 + m\angle 1 = 180^\circ$	11. Substitution Property of Equality

**Quiz for the lessons "Prove Statements about Segments and Angles" and "Prove Angle Pair Relationships"**

1. B; Symmetric Property of Congruence  
 2. C; Transitive Property of Congruence  
 3. A; Reflexive Property of Congruence

4.	Statements	Reasons
	1. $\angle XWY$ is a straight angle. $\angle ZWV$ is a straight angle.	1. Given
	2. $\angle XWV$ and $\angle ZWY$ are vertical angles.	2. Definition of vertical angles
	3. $\angle XWV \cong \angle ZWY$	3. Vertical Angles Congruence Theorem

**Mixed Review of Problem Solving for the lessons “Reason Using Properties from Algebra”, “Prove Statements about Segments and Angles”, and “Prove Angle Pair Relationships”**

1. a.	Statements	Reasons
	1. $\overrightarrow{BD}$ bisects $\angle ABC$ and $\overrightarrow{BC}$ bisects $\angle DBE$ .	1. Given
	2. $\angle ABD \cong \angle DBC$ and $\angle DBC \cong \angle CBE$	2. Definition of Angle Bisector
	3. $\angle ABD \cong \angle CBE$	3. Transitive Property of Congruence
	4. $m\angle ABD = m\angle CBE$	4. Definition of congruent angles

b.  $m\angle ABE = m\angle ABD + m\angle DBC + m\angle CBE$

$m\angle ABD = m\angle DBC = m\angle CBE$

Let  $x = m\angle ABD$ .

$99 = x + x + x$

$99 = 3x$

$33 = x$

So,  $m\angle DBC = 33^\circ$ .

2. All of the strips will have the same width. They are cut into congruent pieces each time, so all of the strips will have the same width.

3.  $m\angle 1 + m\angle 2 + m\angle 3 + m\angle 4 = 360^\circ$

$m\angle 1 + m\angle 1 + 80^\circ + 80^\circ = 360^\circ$

$2 \cdot m\angle 1 = 200^\circ$

$m\angle 1 = 100^\circ$

4. *Sample answer:*  $\angle 1$  is a right angle, so  $m\angle 1 = 90^\circ$  by the definition of right angle.  $\angle 1$  and  $\angle 2$  are supplementary, so  $m\angle 1 + m\angle 2 = 180^\circ$  by the definition of supplementary angles. Use properties of equality to show that  $m\angle 2 = 180^\circ - m\angle 1 = 180^\circ - 90^\circ = 90^\circ$ . Because  $m\angle 2 = 90^\circ$ ,  $\angle 2$  is a right angle by the definition of right angle.

5. a.  $T = c(1 + s)$  Given  
 $T = c + sc$  Distributive Property  
 $T - c = sc$  Subtraction Property of Equality  
 $\frac{T - c}{c} = s$  Division Property of Equality

b. When  $T = \$26.75$  and  $c = \$25$ :

$s = \frac{26.75 - 25}{25} = 0.07$

So, the sales tax rate is 7%.

c. *Sample answer:* You could have first divided each side by  $c$ , by the Division Property of Equality. Then subtract 1 from each side by the Subtraction Property of Equality.

6. *Sample answer:* Either  $m\angle BAC$  or  $m\angle CAD$ ; because  $\angle GAD$  is a straight angle, if two of the three angles are known, the third angle can be found.
7. You know that  $\angle 1$  and  $\angle 2$  are a pair of vertical angles because  $m\angle 1 = m\angle 2$ , while  $m\angle 3 = 3m\angle 1$ .  $m\angle 3 = m\angle 4$  because  $\angle 3$  and  $\angle 4$  are the other pair of vertical angles.  $\angle 1$  and  $\angle 3$  are a linear pair, so  $m\angle 1 + m\angle 3 = 180^\circ$ . Let  $m\angle 1 = x$ , then  $m\angle 3 = 3x$  and  $x + 3x = 180$ , or  $x = 45$ . So,  $m\angle 1 = 45^\circ$ ,  $m\angle 2 = 45^\circ$ ,  $m\angle 3 = 3(45) = 135^\circ$ , and  $m\angle 4 = 135^\circ$ .
8.  $\angle BAC$  and  $\angle CAF$  are a linear pair because  $\overrightarrow{AB}$  and  $\overrightarrow{AF}$  are opposite rays.  $m\angle BAC + m\angle CAF = 180^\circ$  by the Linear Pair Postulate.  $m\angle CAF = m\angle CAE + m\angle EAF$  and  $m\angle CAF = m\angle CAD + m\angle DAE$  by the Angle Addition Postulate. You know that  $m\angle CAD + m\angle DAE = 90^\circ$  because  $\angle CAD$  and  $\angle DAE$  are complements.  $m\angle BAC + (m\angle CAE + m\angle EAF) = 180^\circ$  and  $m\angle CAE = 90^\circ$  by the Substitution Property of Equality. So,  $m\angle BAC + 90^\circ + m\angle EAF = 180^\circ$ , or  $m\angle BAC + m\angle EAF = 90^\circ$ , so  $\angle BAC$  and  $\angle EAF$  are complements.

**Chapter Review for the chapter “Reasoning and Proof”**

1. A statement that can be proven is called a *theorem*.
2. The inverse negates the hypothesis and conclusion of a conditional statement. The converse exchanges the hypothesis and conclusion of a conditional statement.
3. When  $m\angle A = m\angle B$  and  $m\angle B = m\angle C$ , then  $m\angle A = m\angle B$ .
4.  $-20,480, -5120, -1280, -320, \dots$   
 $\underbrace{\hspace{1.5cm}}_{\div 4} \quad \underbrace{\hspace{1.5cm}}_{\div 4} \quad \underbrace{\hspace{1.5cm}}_{\div 4} \quad \underbrace{\hspace{1.5cm}}_{\div 4}$   
 Each number in the pattern is the previous number divided by 4. The next three numbers are  $-80, -20, -5$ .
5. *Counterexample:*  $\frac{-24}{-8} = 3$   
 Because a counterexample exists, the conjecture is false.
6. *If-then:* If an angle measures  $34^\circ$ , then it is an acute angle.  
*Converse:* If an angle is an acute angle then it measures  $34^\circ$ .  
*Inverse:* If an angle does not measure  $34^\circ$ , then it is not an acute angle.  
*Contrapositive:* If an angle is not an acute angle, then it does not measure  $34^\circ$ .
7. This is a valid definition because it can be written as a true biconditional statement.
8. All the interior angles of a polygon are congruent if and only if the polygon is an equiangular polygon.
9. Because  $\angle B$  is a right angle it satisfies the hypothesis, so the conclusion is also true. So,  $\angle B$  measures  $90^\circ$ .

10. The conclusion of the second statement is the hypothesis of the first statement, so you can write the following new statement if  $4x = 12$ , then  $2x = 6$ .

11. Look for a pattern:

$$1 + 3 = 4, 5 + 7 = 12, 9 + 3 = 12$$

Conjecture: Odd integer + odd integer = even integer

Let  $2n + 1$  and  $2m + 1$  be any two odd integers

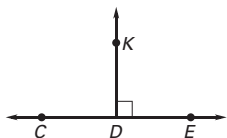
$$(2n + 1) + (2m + 1) = 2n + 2m + 2 = 2(n + m + 1)$$

$2(n + m + 1)$  is the product of 2 and an integer

$(n + m + 1)$ . So,  $2(m + n + 1)$  is an even integer.

The sum of any two odd integers is an even integer.

12.



13.  $\overline{BD}$ ; With no right angle marked, you cannot assume  $\overline{CD} \perp$  plane  $P$ .

$-9x - 21 = -20x - 87$	Given
$11x - 21 = -87$	Addition Property of Equality
$11x = -66$	Addition Property of Equality
$x = -6$	Division Property of Equality

$15x + 22 = 7x + 62$	Given
$8x + 22 = 62$	Subtraction Property of Equality
$8x = 40$	Subtraction Property of Equality
$x = 5$	Division Property of Equality

$3(2x + 9) = 30$	Given
$6x + 27 = 30$	Distributive Property
$6x = 3$	Subtraction Property of Equality
$x = \frac{1}{2}$	Division Property of Equality

$5x + 2(2x - 23) = -154$	Given
$5x + 4x - 46 = -154$	Distributive Property
$9x - 46 = -154$	Simplify.
$9x = -108$	Addition Property of Equality
$x = -12$	Division Property of Equality

18. Symmetric Property of Congruence

19. Reflexive Property of Congruence

20. Transitive Property of Equality

21. Given:  $\angle 1 \cong \angle 2$  and  $\angle 2 \cong \angle 3$

Prove:  $\angle 1 \cong \angle 3$

Statements	Reasons
1. $\angle 1 \cong \angle 2$ $\angle 2 \cong \angle 3$	1. Given
2. $m\angle 1 = m\angle 2$ $m\angle 2 = m\angle 3$	2. Definition of congruent angles
3. $m\angle 1 = m\angle 3$	3. Transitive Property of Equality
4. $\angle 1 \cong \angle 3$	4. Definition of congruent angles

22. Given: $m\angle 1 = 114^\circ$	23. Given: $m\angle 4 = 57^\circ$
$m\angle 3 = m\angle 1 = 114^\circ$	$m\angle 2 = m\angle 4 = 57^\circ$
$m\angle 1 + m\angle 2 = 180^\circ$	$m\angle 4 + m\angle 3 = 180^\circ$
$114^\circ + m\angle 2 = 180^\circ$	$57^\circ + m\angle 3 = 180^\circ$
$m\angle 2 = 66^\circ$	$m\angle 3 = 123^\circ$
$m\angle 4 = m\angle 2 = 66^\circ$	$m\angle 1 = m\angle 3 = 123^\circ$

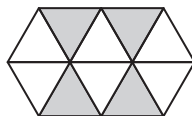
Statements	Reasons
1. $\angle 3$ and $\angle 2$ are complementary. $m\angle 1 + m\angle 2 = 90^\circ$	1. Given
2. $\angle 1$ and $\angle 2$ are complementary.	2. Definition of complementary angles
3. $\angle 3 \cong \angle 1$	3. Congruent Complements Theorem

### Chapter Test for the chapter "Reasoning and Proof"

1. The figure is rotated counterclockwise. The next figure is:



2. Two pieces are added to the figure alternating between unshaded and shaded pattern. The next figure is:



3.  $-6, -1, 4, 9, \dots$

$\underbrace{\quad}_{+5} \quad \underbrace{\quad}_{+5} \quad \underbrace{\quad}_{+5} \quad \underbrace{\quad}_{+5}$

Each number in the pattern is five more than the previous number. The next number is 14.

4.  $100, -50, 25, -12.5, \dots$

$\times \underbrace{-\frac{1}{2}} \times \underbrace{-\frac{1}{2}} \times \underbrace{-\frac{1}{2}} \times \underbrace{-\frac{1}{2}}$

Each number in the pattern is  $-\frac{1}{2}$  times the previous number. The next number is 6.25.



5. *If-then form:* If two angles are right angles, then they are congruent.  
*Converse:* If two angles are congruent, then they are right angles.  
*Inverse:* If two angles are not right angles, then they are not congruent.  
*Contrapositive:* If two angles are not congruent, then they are not right angles.
6. *If-then form:* If an animal is a frog, then it is an amphibian.  
*Converse:* If an animal is an amphibian, then it is a frog.  
*Inverse:* If an animal is not a frog, then it is not an amphibian.  
*Contrapositive:* If an animal is not an amphibian, then it is not a frog.
7. *If-then form:* If  $x = -2$ , then  $5x + 4 = -6$ .  
*Converse:* If  $5x + 4 = -6$ , then  $x = -2$ .  
*Inverse:* If  $x \neq -2$ , then  $5x + 4 \neq -6$ .  
*Contrapositive:* If  $5x + 4 \neq -6$ , then  $x \neq -2$ .
9. Because you are going to the football game satisfies the hypothesis, the conclusion is true. So, you will miss band practice.
10. The conclusion of the first statement is the hypothesis of the second statement, so you write the following new statement.  
 If Margot goes to college, then she will need to buy a lab manual.
11. *Sample answers:* Line  $\overleftrightarrow{TQ}$  contains points  $N$ ,  $Q$ , and  $T$ .
12. *Sample answer:* Plane  $Y$  contains points  $Q$ ,  $R$ , and  $S$ .
13. Planes  $X$  and  $Y$  intersect at  $\overleftrightarrow{TQ}$ .
14.  $9x + 31 = -23$       Given  
 $9x = -54$       Subtraction Property of Equality  
 $x = -6$       Division Property of Equality
15.  $-7(-x + 2) = 42$       Given  
 $7x - 14 = 42$       Distributive Property  
 $7x = 56$       Addition Property of Equality  
 $x = 8$       Division Property of Equality
16.  $26 + 2(3x + 11) = -18x$       Given  
 $26 + 6x + 22 = -18x$       Distributive Property  
 $6x + 48 = -18x$       Simplify.  
 $48 = -24x$       Subtraction Property of Equality  
 $-2 = x$       Division Property of Equality
17. B; Symmetric Property of Congruence
18. A; Reflexive Property of Congruence
19. C; Transitive Property of Congruence

$$20. \begin{aligned} 7y &= 5y + 36 \\ 2y &= 36 \\ y &= 18 \\ 3x - 21 &= 2x + 4 \\ x &= 25 \end{aligned}$$

The measures of the angles are:

$$\begin{aligned} 7(18) &= 126^\circ \\ 2(25) + 4 &= 54^\circ \\ 5(18) + 36 &= 126^\circ \\ 3(25) - 21 &= 54^\circ \end{aligned}$$

21.	Statements	Reasons
	1. $\overline{AX} \cong \overline{DX}$ , $\overline{XB} \cong \overline{XC}$	1. Given
	2. $AX = DX$ , $XB = XC$	2. Definition of congruent segments
	3. $AX + XC = AC$ $DX + XB = BD$	3. Segment Addition Postulate
	4. $DX + XC = AC$ $DX + XC = BD$	4. Substitution Property of Equality
	5. $AC = BD$	5. Transitive Property of Equality
	6. $\overline{AC} \cong \overline{BD}$	6. Definition of congruent segments

### Algebra Review for the chapter "Reasoning and Proof"

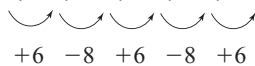
- $\frac{5x^4}{20x^2} = \frac{\cancel{5} \cdot \cancel{x} \cdot \cancel{x} \cdot \cancel{x} \cdot x}{\cancel{5} \cdot 4 \cdot \cancel{x} \cdot \cancel{x}} = \frac{x^2}{4}$
- $\frac{-12ab^3}{9a^2b} = \frac{-4 \cdot \cancel{3} \cdot \cancel{a} \cdot \cancel{b} \cdot b \cdot b}{3 \cdot \cancel{3} \cdot \cancel{a} \cdot a \cdot \cancel{b}} = \frac{-4b^2}{3a}$
- $\frac{5m + 35}{5} = \frac{\cancel{5}(m + 7)}{\cancel{5}} = m + 7$
- $\frac{36m - 48m}{6m} = \frac{-12m}{6m} = -2$
- $\frac{k + 3}{-2k + 3}$ ; cannot be simplified
- $\frac{m + 4}{m^2 + 4m} = \frac{\cancel{m} + 4}{m(\cancel{m} + 4)} = \frac{1}{m}$
- $\frac{12x + 16}{8 + 6x} = \frac{2(\cancel{6}x + 8)}{\cancel{8} + \cancel{6}x} = 2$
- $\frac{3x^3}{5x + 8x^2} = \frac{3 \cdot \cancel{x} \cdot x \cdot x}{\cancel{x}(5 + 8x)} = \frac{3x^2}{5 + 8x}$
- $\frac{3x^2 - 6x}{6x^2 - 3x} = \frac{\cancel{3}\cancel{x}(x - 2)}{\cancel{3}\cancel{x}(2x - 1)} = \frac{x - 2}{2x - 1}$
- $\sqrt{75} = \sqrt{25} \cdot \sqrt{3} = 5\sqrt{3}$
- $-\sqrt{180} = -\sqrt{36} \cdot \sqrt{5} = -6\sqrt{5}$
- $\pm\sqrt{128} = \pm\sqrt{64} \cdot \sqrt{2} = \pm 8\sqrt{2}$
- $\sqrt{2} - \sqrt{18} + \sqrt{6} = \sqrt{2} - 3\sqrt{2} + \sqrt{6} = \sqrt{6} - 2\sqrt{2}$

14.  $\sqrt{28} - \sqrt{63} - \sqrt{35} = 2\sqrt{7} - 3\sqrt{7} - \sqrt{35}$   
 $= -\sqrt{7} - \sqrt{35}$
15.  $4\sqrt{8} + 3\sqrt{32} = 8\sqrt{2} + 12\sqrt{2} = 20\sqrt{2}$
16.  $(6\sqrt{5})(2\sqrt{2}) = 12\sqrt{10}$
17.  $(-4\sqrt{10})(-5\sqrt{5}) = 20\sqrt{50} = 20 \cdot 5\sqrt{2} = 100\sqrt{2}$
18.  $(2\sqrt{6})^2 = (2)^2 \cdot (\sqrt{6})^2 = 4 \cdot 6 = 24$
19.  $\sqrt{(25)^2} = 25$
20.  $\sqrt{x^2} = x$
21.  $\sqrt{(-a)^2} = \sqrt{a^2} = a$
22.  $\sqrt{(3y)^2} = \sqrt{9y^2} = 3y$
23.  $\sqrt{3^2 + 2^2} = \sqrt{9 + 4} = \sqrt{13}$
24.  $\sqrt{h^2 + k^2}$ ; cannot be simplified

### Extra Practice

#### For the chapter "Reasoning and Proof"

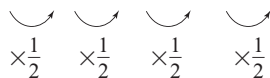
1. 17, 23, 15, 21, 13, 19, ...



The pattern alternates between adding 6 to the previous number and subtracting 8 from the previous number.

The next number in the pattern is  $19 - 8 = 11$ .

2. 1, 0.5, 0.25, 0.125, 0.0625, ...



Each number is  $\frac{1}{2}$  the previous number.

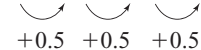
The next number in the pattern is  $\frac{1}{2}(0.625) = 0.3125$ .

3. 2, 3, 5, 7, 11, 13, ...

The numbers are consecutive prime numbers.

The next prime number in the pattern is 17.

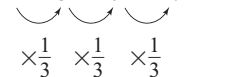
4. 7.0, 7.5, 8.0, 8.5, ...



Each number is 0.5 more than the previous number.

The next number in the pattern is  $8.5 + 0.5 = 9.0$

5. 1,  $\frac{1}{3}$ ,  $\frac{1}{9}$ ,  $\frac{1}{27}$ , ...

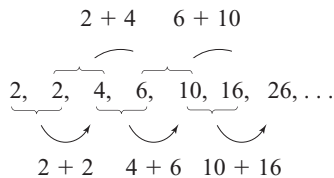


Each number is  $\frac{1}{3}$  the previous number.

The next number in the pattern is  $\frac{1}{3}\left(\frac{1}{27}\right) = \frac{1}{81}$ .

6. Starting with the third number, each number is the sum of the previous two numbers.

The next number in the pattern is  $16 + 26 = 42$ .



7. *Sample answer:*

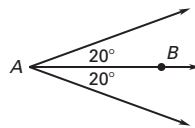
The difference of 10 and 12 is  $10 - 12 = -2$  and  $-2$  is not between 10 and 12. Because a counterexample exists, the conjecture is false.

8. *Sample answer:*

If  $x = 0$ , then  $2x = 0$ , so  $2x \not> x$ . Because a counterexample exists, the conjecture is false.

9. *Sample answer:*

Suppose  $m\angle A = 40^\circ$ .  $\overrightarrow{AB}$  can be drawn so that it divides  $\angle A$  into two  $20^\circ$  angles. So  $\angle A$  can be bisected, but  $\angle A$  is not obtuse. Because a counterexample exists, the conjecture is false.



10. *If-then form:* If two lines intersect, then they form two pairs of vertical angles.

*Converse:* If two lines form two pairs of vertical angles, then the two lines intersect.

*Inverse:* If two lines do not intersect, then they do not form two pairs of congruent angles.

*Contrapositive:* If two lines do not form two pairs of vertical angles, then the two lines do not intersect.

11. *If-then form:* If a figure is a square, then it is a four-sided regular polygon.

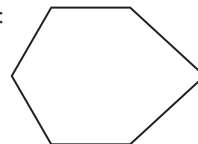
*Converse:* If a figure is a four-sided regular polygon, then it is a square.

*Inverse:* If a figure is not a square, then it is not a four-sided regular polygon.

*Contrapositive:* If a figure is not a four-sided regular polygon, then it is not a square.

12. False; Not all hexagons are regular.

Counterexample:



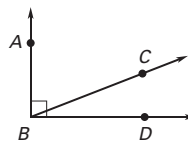
13. True; By definition, two angles are complementary angles if the sum of their measures is  $90^\circ$ .

14. By the Law of Syllogism, the given conditional statements lead to the following conditional statement:

"If a triangle is equilateral, then it is regular."

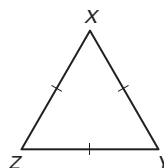
15. By the Law of Syllogism, the given conditional statements lead to the following conditional statement: "If two coplanar lines are not parallel, then they form congruent vertical angles."
16. John only does his math homework when he is in study hall. John is doing his math homework. So, John is in study hall.
17. May sometimes buys pretzels when she goes to the supermarket. May is at the supermarket. So, she might buy pretzels.
18. Cannot be determined; The marking shows  $\overleftrightarrow{SV} \perp \overleftrightarrow{TW}$ , but that does not guarantee that  $\overleftrightarrow{SV}$  is perpendicular to every line in plane  $Z$  that it intersects.
19. True; The diagram shows that point  $Y$  lies in plane  $Z$  and  $\overleftrightarrow{XU}$  passes through point  $Z$ .
20. True; The diagram shows that  $\overleftrightarrow{TW}$  lies in plane  $Z$ .
21. False;  $\angle SYT$  and  $\angle WYS$  are adjacent because they share side  $YS$ .
22. False;  $\angle SYT$  and  $\angle TYV$  are both right angles, so they are supplementary.
23. True; The noncommon sides  $\overleftrightarrow{YT}$  and  $\overleftrightarrow{YW}$  of  $\angle TYU$  and  $\angle UYW$  are opposite rays.
24. Cannot be determined; The diagram does not give enough information.
25.  $4x + 15 = 39$  Write original equation.  
 $4x = 24$  Subtraction Property of Equality  
 $x = 6$  Division Property of Equality
26.  $6x + 47 = 10x - 9$  Write original equation.  
 $47 = 4x - 9$  Subtraction Property of Equality  
 $56 = 4x$  Addition Property of Equality  
 $14 = x$  Division Property of Equality
27.  $2(-7x + 3) = -50$  Write original equation.  
 $-14x + 6 = -50$  Distributive Property  
 $-14x = -56$  Subtraction Property of Equality  
 $x = 4$  Division Property of Equality
28.  $54 + 9x = 3(7x + 6)$  Write original equation.  
 $54 + 9x = 21x + 18$  Distributive Property  
 $54 = 12x + 18$  Subtraction Property of Equality  
 $36 = 12x$  Subtraction Property of Equality  
 $3 = x$  Division Property of Equality
29.  $13(2x - 3) - 20x = 3$  Write original equation.  
 $26x - 39 - 20x = 3$  Distributive Property  
 $6x - 39 = 3$  Simplify.  
 $6x = 42$  Addition Property of Equality  
 $x = 7$  Division Property of Equality
30.  $31 + 25x = 7x - 14 + 3x$  Write original equation.  
 $31 + 25x = 10x - 14$  Simplify.  
 $31 + 15x = -14$  Subtraction Property of Equality  
 $15x = -45$  Subtraction Property of Equality  
 $x = -3$  Division Property of Equality

31. If  $m\angle JKL = m\angle GHI$  and  $m\angle GHI = m\angle ABC$ , then  $m\angle JKL = m\angle ABC$ .  
 Transitive Property of Equality
32. If  $m\angle MNO = m\angle PQR$ , then  $m\angle PQR = m\angle MNO$ .  
 Symmetric Property of Equality
33.  $m\angle XYZ = m\angle XYZ$  Reflexive Property of Equality
34. Given: Point  $C$  is in the interior of  $\angle ABD$ .  
 $\angle ABD$  is a right angle.  
 Prove:  $\angle ABC$  and  $\angle CBD$  are complementary.



Statements	Reasons
1. $\angle ABD$ is a right angle.	1. Given
2. $m\angle ABD = 90^\circ$	2. Definition of right angle
3. Point $C$ is in the interior of $\angle ABD$ .	3. Given
4. $m\angle ABD = m\angle ABC + m\angle CBD$	4. Angle Addition Postulate
5. $90^\circ = m\angle ABC + m\angle CBD$	5. Substitution Property of Equality
6. $\angle ABC$ and $m\angle CBD$ are complementary.	6. Definition of complementary angles

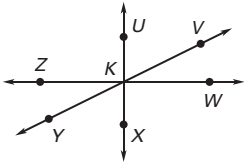
35. Given:  $\overline{XY} \cong \overline{YZ} \cong \overline{ZX}$   
 Prove: The perimeter of  $\triangle XYZ$  is  $3 \cdot XY$ .



Statements	Reasons
1. $\overline{XY} \cong \overline{YZ} \cong \overline{ZX}$	1. Given
2. $XY = YZ = ZX$	2. Definition of congruent segments
3. Perimeter of $\triangle XYZ = XY + YZ + ZX$	3. Perimeter formula
4. Perimeter of $\triangle XYZ = XY + XY + XY$	4. Substitution
5. Perimeter of $\triangle XYZ = 3 \cdot XY$	5. Simplify.

36. If  $m\angle CGF = 158^\circ$ , the  $m\angle EGD = 158^\circ$ .  $\angle CGF$  and  $\angle EGD$  are vertical angles, so they are congruent by the Vertical Angles Congruence Theorem.

37. If  $m\angle EGA = 67^\circ$ , then  $m\angle FGD = 23^\circ$ .  
 Because  $m\angle AGD = 90^\circ$ ,  $m\angle AGC = 180^\circ - 90^\circ = 90^\circ$  by the Linear Pair Postulate. Then,  $m\angle EGC = m\angle AGC - m\angle EGA = 90^\circ - 67^\circ = 23^\circ$ .  
 Because  $\angle EGC$  and  $\angle FGD$  are vertical angles, they are congruent. So,  $m\angle EGC = m\angle FGD = 23^\circ$ .
38. If  $m\angle FGC = 149^\circ$ , then  $m\angle EGA = 59^\circ$ .  
 By the Vertical Angles Congruence Theorem,  $m\angle BGC = 90^\circ$ . So,  $m\angle FGB = m\angle FGC - m\angle BGC = 149^\circ - 90^\circ = 59^\circ$ . Because  $\angle FGB$  and  $\angle EGA$  are vertical angles, they are congruent. So,  $m\angle FGB = m\angle EGA = 59^\circ$ .
39.  $m\angle DGB = 90^\circ$   
 Because  $m\angle AGD = 90^\circ$ ,  $m\angle DGB = 180^\circ - 90^\circ = 90^\circ$  by the Linear Pair Postulate.
40.  $m\angle FGH = 90^\circ$   
 $\angle AGD$  and  $\angle BGD$  form a linear pair, and  $m\angle AGD = 90^\circ$ . So,  $m\angle BGD = 90^\circ$ .  $m\angle BGD = m\angle FGB + m\angle FGD$ . It is given that  $\angle FGD \cong \angle BGH$ , so  $m\angle FGD = m\angle BGH$ . Using substitution,  $m\angle FGB + m\angle BGH = 90^\circ$ . So,  $m\angle FGH = 90^\circ$ .
41. Given:  $\angle UKV$  and  $\angle VKW$  are complements.  
 Prove:  $\angle YKZ$  and  $\angle XKY$  are complements.



Statements	Reasons
1. $\angle UKV$ and $\angle VKW$ are complements.	1. Given
2. $m\angle UKV + m\angle VKW = 90^\circ$	2. Definition of complementary angles
3. $\angle UKV \cong \angle XKY$ , $\angle VKW \cong \angle YKZ$	3. Vertical angles are congruent.
4. $m\angle UKV = m\angle XKY$ , $m\angle VKW = m\angle YKZ$	4. Definition of congruent angles
5. $m\angle YKZ + m\angle XKY = 90^\circ$	5. Substitution
6. $\angle YKZ$ and $\angle XKY$ are complements.	6. Definition of complementary angles