

Chapter 2: Segments and Angles

2.1 Segment Bisectors

goal: Bisect a segment. Find the coordinates of segment

Vocabulary

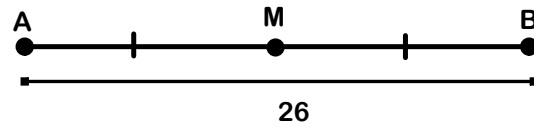
Midpoint: Is the point on the segment that divides it into two congruent segments

Segment bisector: Is a segment, ray, line, or plane that intersects a segment at its midpoint

Bisect: To divide the segment into two congruent segments

Example 1: Find Segment Lengths

M is the midpoint of \overline{AB} . Find AM and MB.

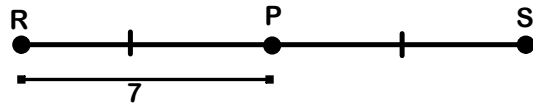


Solution

Example 2: Find Segment Lengths

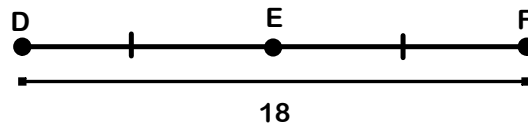
P is the midpoint of \overline{RS} . Find PS and RS.

Solution

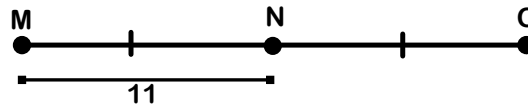


Checkpoint: Complete the following exercises

1. Find DE and EF



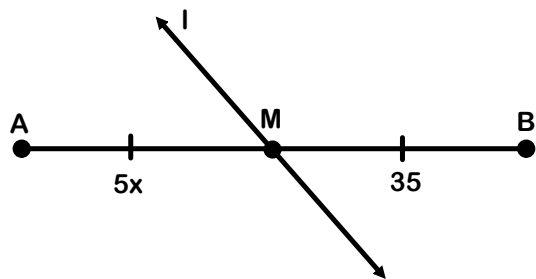
2. Find NO and MO



Example 3: Use Algebra with Segment Lengths

Line l is a segment bisector of \overline{AB} . Find the value of x

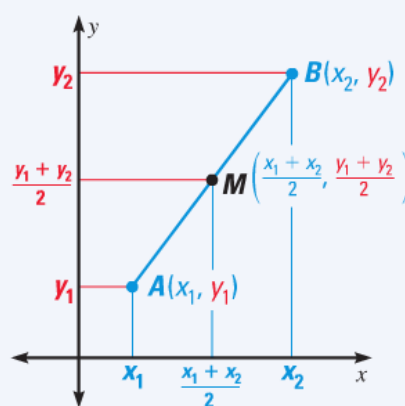
Solution



THE MIDPOINT FORMULA

Words The coordinates of the midpoint of a segment are the averages of the x -coordinates and the y -coordinates of the endpoints.

Symbols The midpoint of the segment joining $A(x_1, y_1)$ and $B(x_2, y_2)$ is $M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$.

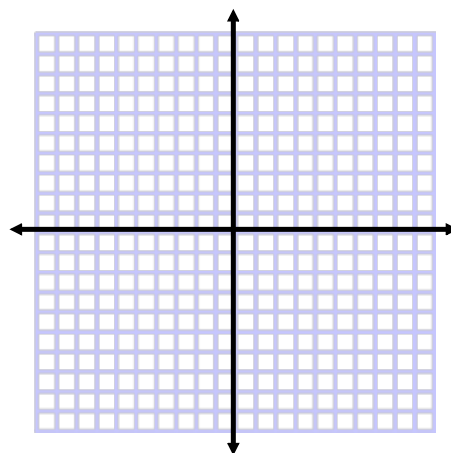


Example 4: Use the Midpoint Formula

Find the coordinates of the midpoint of \overline{AB} for
A(1,2) and B(7,4)

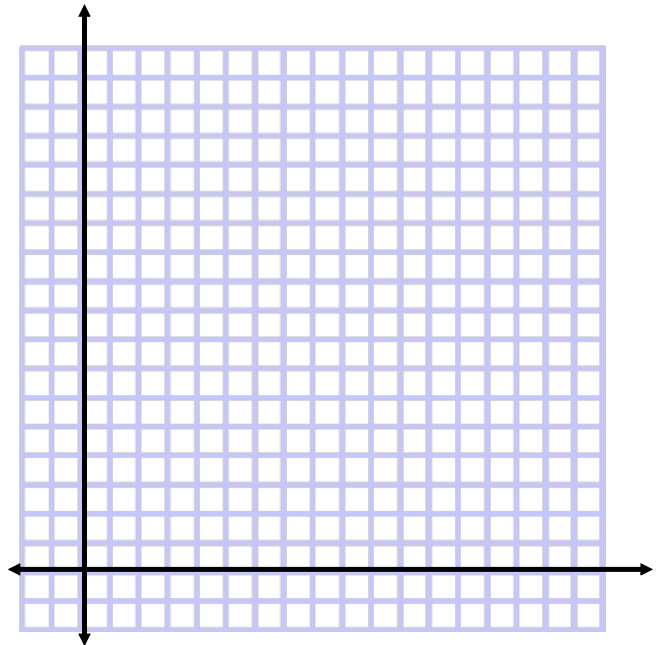
Solution:

$$M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$



Checkpoint: Sketch \overline{PQ} . Then Find the coordinates of its midpoint.

$P(2,5)$ $Q(4,3)$

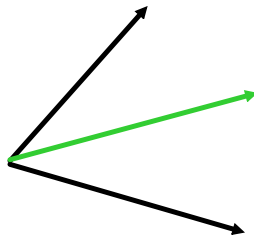


2.2 Angle Bisectors

goal: Bisect an angle.

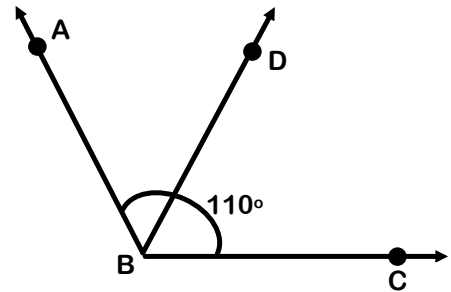
Vocabulary

Angle bisector: is a ray that divides an angle into two angles that are congruent.

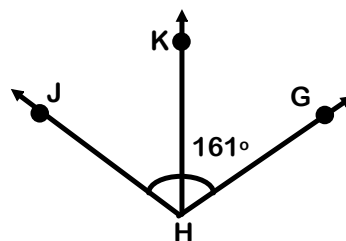
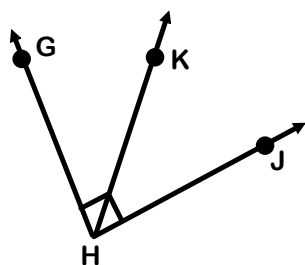
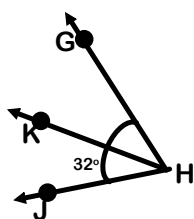


Example 1: Find Angle Measures

\overrightarrow{BD} bisects $\angle ABC$, and $\angle ABC = 110^\circ$ Find $m\angle ABD$ and $m\angle DBC$

Solution

Checkpoint: \overrightarrow{HK} bisects $\angle GHJ$ find $m\angle GHK$ and $m\angle KHJ$

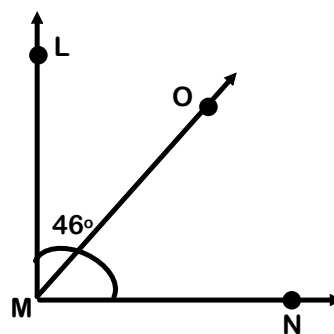


Example 2: Angle Measures and Classification

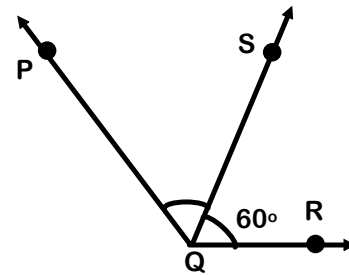
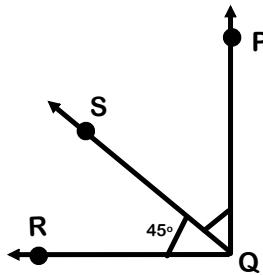
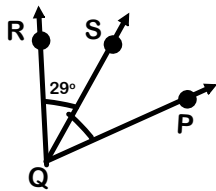
\overrightarrow{MO} bisects $\angle LMN$, and $m\angle LMO = 46^\circ$

1. Find $m\angle OMN$ and $m\angle LMN$
2. Determine whether $\angle LMN$ is acute, right, obtuse, or straight

Solution

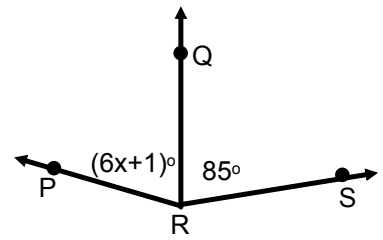


Checkpoint: \overrightarrow{QS} bisects $\angle PQR$ find $m\angle SQP$ and $m\angle PQR$
 Then tell whether $\angle PQR$ is acute, right, obtuse, or straight.

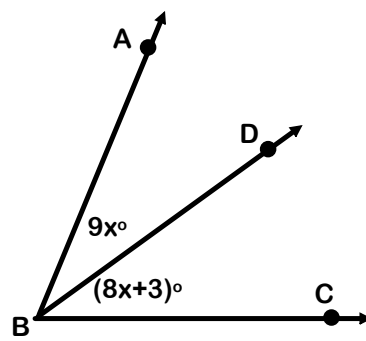
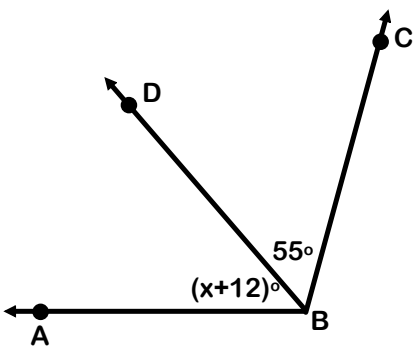


Example 3: Use Algebra with Angle Measures

\overrightarrow{RQ} bisects $\angle PRS$ Find the value of x .



Checkpoint: \overrightarrow{BD} bisects $\angle ABC$ find the value of x .



2.3 Complementary and Supplement

goal: Find measures of complementary and supplement

Vocabulary

Complementary angles: Two angles whose measures have a sum of 90°

Complement: The sum of the measures of an angle and its complement is 90°

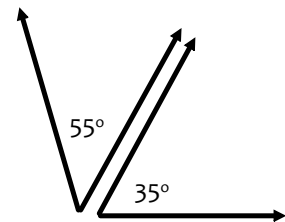
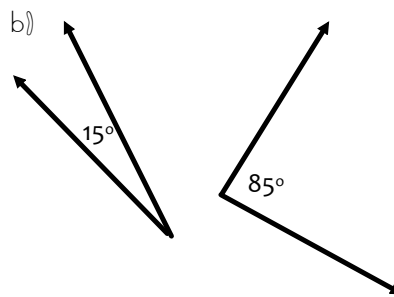
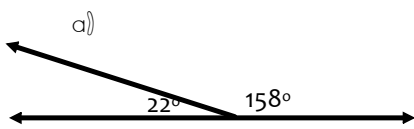
Supplementary angles: Two angles whose measures have a sum of 180°

Supplement: The sum of the measures of an angle and its supplement is 180°

Adjacent angles: Two angles with a common vertex and side but no common interior points

Theorem: A true statement that follows from other true statements.

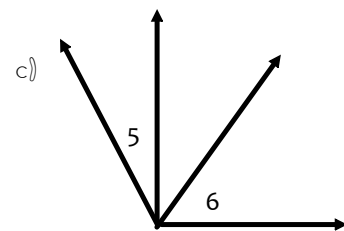
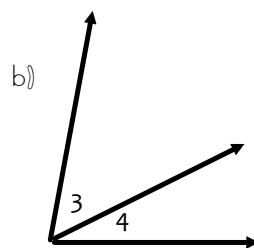
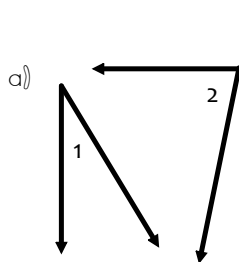
Example 1: Identify angles



Solution

Example 2: Identify Adjacent Angles

State whether the numbered angles are adjacent or nonadjacent.



Solution

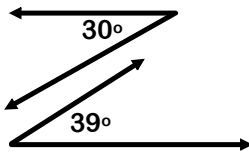
Example 3: Complements and Supplements

- a) $\angle A$ is a complement of $\angle C$ and $m\angle A = 47^\circ$ find $m\angle C$
- b) $\angle P$ is a supplement of $\angle R$ and $m\angle R = 36^\circ$ find $m\angle P$

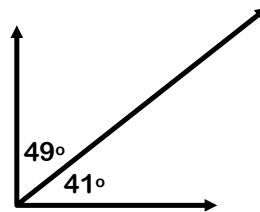
Solution

Checkpoint: State whether the angles are complementary, supplementary or neither.

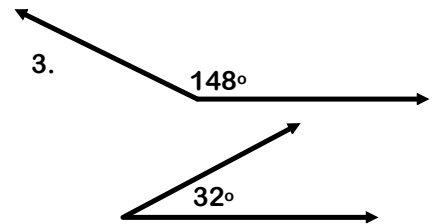
1.



2.



3.



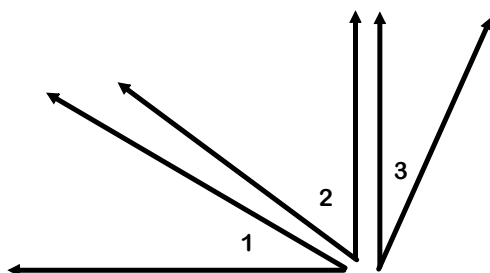
4. $\angle B$ is a complement of $\angle D$, and $m\angle D = 79^\circ$ Find $m\angle B$

5. $\angle G$ is a supplement of $\angle H$, and $m\angle G = 115^\circ$ Find $m\angle H$

Theorem 2.1: Congruent Complements Theorem

Words: If two angles are complementary to the same angle, then they are _____.

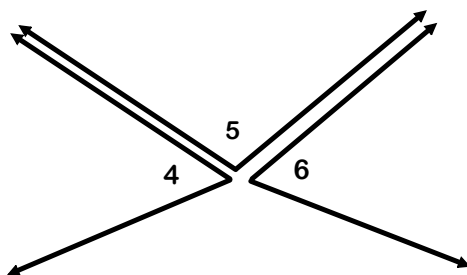
Symbols: If $m\angle 1 + m\angle 2 = 90^\circ$ and $m\angle 2 + m\angle 3 = 90^\circ$ then $\angle \underline{\quad} \cong \angle \underline{\quad}$



Theorem 2.2: Congruent Supplements Theorem

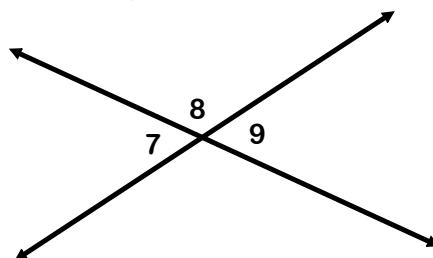
Words: If two angles are supplementary to the same angle, then they are _____.

Symbols: If $m\angle 4 + m\angle 5 = 180^\circ$ and $m\angle 5 + m\angle 6 = 180^\circ$ then $\angle _ \cong \angle _$



Example 4: Use a Theorem

$\angle 7$ and $\angle 8$ are supplementary, and $\angle 8$ and $\angle 9$ are supplementary. Name a pair of congruent angles. Explain your reasoning

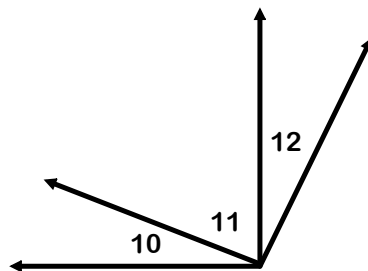
**Solution**

Checkpoint: Complete the following exercise

In the diagram, $m\angle 10 + m\angle 11 = 90^\circ$

and $m\angle 11 + m\angle 12 = 90^\circ$

Name a pair of congruent angles, explain your reasoning

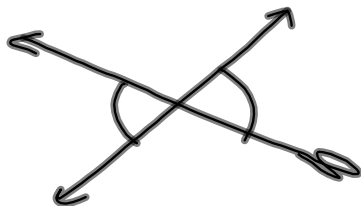


2.4 Vertical Angles

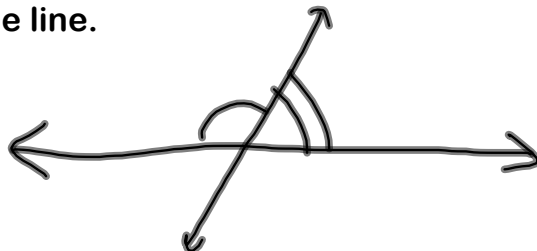
goal: Find the measures of angles formed by in

Vocabulary

Vertical angles: Two angles are vertical angles if they are not adjacent and their sides are formed by two intersecting lines.

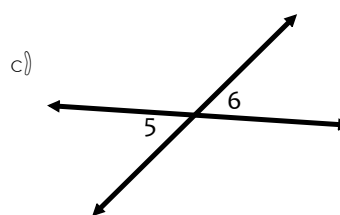
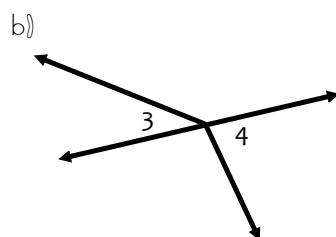
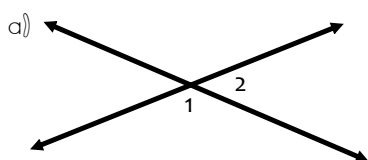


Linear pair: Two adjacent angles are a linear pair if their noncommon sides are on the same line.



Example 1: Vertical Angles and Linear Pairs

Determine whether the labeled angles are vertical angles, a linear pair, or neither.

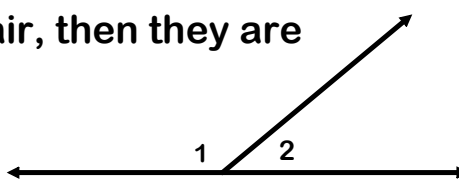


Solution

Postulate 7: Linear Pair Postulate

Words: If two angles form a linear pair, then they are supplementary

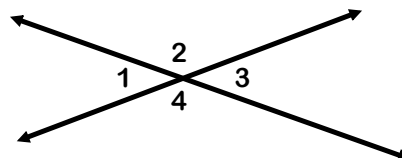
Symbols: $m\angle 1 + m\angle 2 = \underline{\hspace{2cm}}$



Theorem 2.3: Vertical Angles Theorem

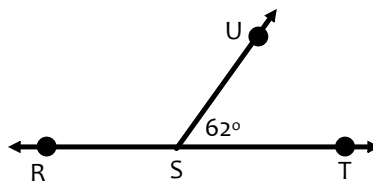
Words: Vertical angles are _____.

Symbols: $\angle 1 \cong \angle 3$ and $\angle \underline{\hspace{1cm}} \cong \angle \underline{\hspace{1cm}}$

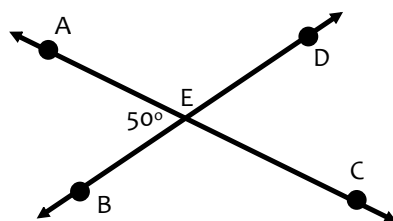


Example 2: Use the Linear Pair Postulate

Find the measure of $\angle RSU$

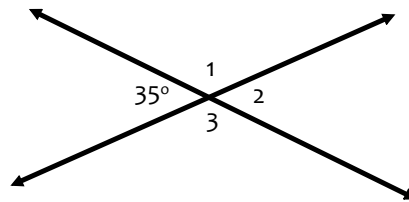


Solution

Example 3: Use the Vertical Angles TheoremFind the measure of $\angle CED$ **Solution**

Example 4: Find Angle Measures

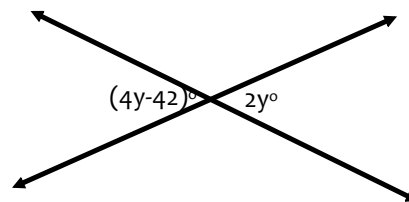
Find $m\angle 1$, $m\angle 2$, and $m\angle 3$

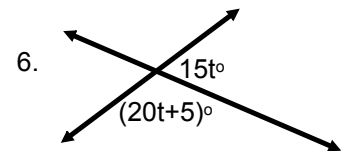
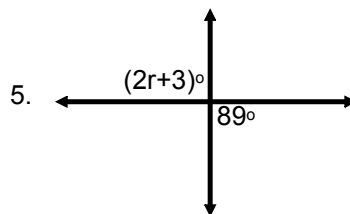
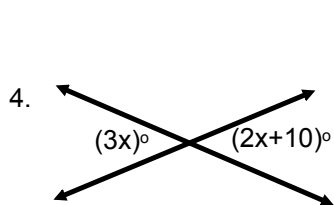
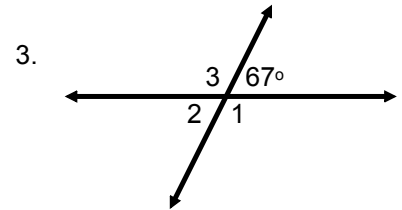
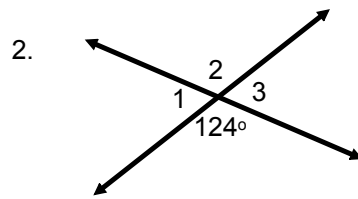
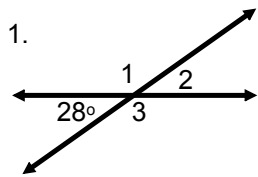


Solution

Example 5: Use Algebra with Vertical Angles

Find the value of y .

**Solution**



2.5 If-Then Statements and Dec

goal: Use if-then statements. Apply laws of log

Vocabulary

if-then statement: an if-then statement has two parts. The "if" part contains the hypothesis. The "then" part contains the conclusion

Hypothesis: The "if" part of an if-then statement.

Conclusion: The 'then' part of an if-then statement.

Deductive reasoning: Using facts, definitions, accepted properties, and laws of logic to make a logical argument

LAW OF DETACHMENT

If the hypothesis of a true if-then statement is true, then the conclusion is _____.

LAW OF SYLLOGISM

If the following two statements are true, then the third statement is _____.

If statement p, then statement q.

If statement q, then statement r.

If statement _____, then statement _____.

Example 1: Identify the Hypothesis and Conclusion

Identify the hypothesis and conclusion of the if-then statement.

"If I pass the driving test, then I will get me driver's license."

Solution

Hypothesis:

Conclusion:

Example 2: Write If-Then Statements

Rewrite the statement as an if-then statement.

- a. Every game on my computer is fun to play.
- b. I will buy the CD if it costs less than \$15.

Solution

a. If _____, then _____.

b. If _____, then _____.

Checkpoint: Rewrite the statement as an if-then statement. Then identify the hypothesis and the conclusion.

1. An angle is obtuse if its measure is 170°
-

Hypothesis:

Conclusion:

Example 3: Use the Law of Detachment

What can you conclude from the following true statement?

If you wash the cotton T-shirt in hot water, then it will shrink.
You wash the cotton T-shirt i hot water.

Solution

The hypothesis of a true if-then statement (the first statement) is true because of the second statement. By the law of detachment the conclusion is also _____.

Answer

You can conclude that _____.

Checkpoint: What can you conclude from the given true statements?

2. If x has a value of 7, then $2x - 3$ has a value of 11. The value of x is 7.

3. If you study at least 2 hours for the test, then you will pass the test. You study 3 hours for the test.

Example 4: Judge the Correctness of an Argument

Which argument about the statement below is correct?

If two angles are vertical angles, then they are congruent.

Argument 1:

$\angle 1$ and $\angle 2$ are congruent
So, $\angle 1$ and $\angle 2$ are vertical
angles.

Argument 2:

$\angle 1$ and $\angle 2$ are vertical angles.
So, $\angle 1$ and $\angle 2$ are congruent

Solution

Argument _____ is correct. In this argument, the hypothesis (two angles are _____) is true, which implies that the conclusion (they are _____) is also true

Example 5: Use the Law of Syllogism

Write the statement that follows from the true statements.

If the daily high temperature is 32°F or less, then the water in the pipe is frozen.

If the water in the pipe is frozen, then the pipe will break.

Solution

Use the Law of Syllogism

If the daily high temperature is 32°F or less, then

Checkpoint: Write the statement that follows from the true statements.

4. If the ball is thrown at the window, it will hit the window.
If the ball hits the window, then the window will break.

2.6 Properties of Equality and C

goal: Use properties of equality and congruenc

Reflexive Property

Equality

$$AB = AB$$
$$m\angle A = \underline{\hspace{2cm}}$$

Congruence

$$\overline{AB} \cong \overline{AB}$$
$$\angle A \cong \underline{\hspace{2cm}}$$

JUST LIKE A MIRROR.....REFLECTION!!!

Symmetric Property

Equality

if $AB = CD$, then $CD = AB$
if $m\angle A = m\angle B$ then

Congruence

$\overline{AB} \cong \overline{CD}$, and $\overline{CD} \cong \overline{AB}$.

if $\angle A \cong \angle B$, then _____

Transitive Property

Equality

if $AB = CD$, then $CD = EF$,
then $AB = EF$.
if $m\angle A = m\angle B$ and
 $m\angle B = m\angle C$, then

Congruence

$\overline{AB} \cong \overline{CD}$, and $\overline{CD} \cong \overline{EF}$,
then $\overline{AB} \cong \overline{EF}$.
if $\angle A \cong \angle B$ and $\angle B \cong \angle C$,
then _____.

Example 1: Properties of Equality and Congruence

Name the property that the statement illustrates

a) $DE = DE$

b) if $\angle P \cong \angle Q$ and $\angle Q \cong \angle R$, then $\angle P \cong \angle R$.

Solution

Checkpoint: Name the property that the statement illustrates

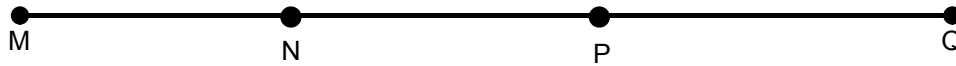
1. if $DF = FG$ and $FG = GH$, then $DF = GH$

2. $\angle P \cong \angle P$

3. if $m\angle S = m\angle T$, then $m\angle T = m\angle S$

Example 2: Use properties of equality

In the diagram, N is the midpoint of \overline{MP} , and P is the midpoint of \overline{NQ} . Show them that $MN = PQ$



Solution

$$MN = \underline{\hspace{2cm}}$$

$$\underline{\hspace{2cm}} = PQ$$

$$MN = PQ$$

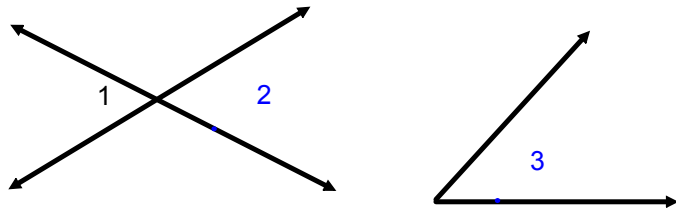
Definition of a midpoint

Definition of a midpoint

$\underline{\hspace{2cm}}$ property of $\underline{\hspace{2cm}}$

Checkpoint: Complete the following exercise

4. $\angle 1$ and $\angle 2$ are vertical angles and $\angle 2 \cong \angle 3$. Show that $\angle 1 \cong \angle 3$



$\angle 1 \cong \angle 2$ _____ Theorem
 $\angle 2 \cong \angle 3$ Given
 $\angle 1 \cong \angle 3$ _____ Property of congruence

Addition Property of Equality

Adding the same number to each side of a true equation produces a true equation

$$x - 3 = 7$$

Subtraction Property of Equality

Subtracting the same number to each side of a true equation produces a true equation

$$y + 5 = 11$$

Multiplication Property of Equality

Multiplying each side of a true equation by the same nonzero number produces a true equation

$$\frac{1}{4}z = 6$$

Division Property of Equality

Dividing each side of a true equation by the same nonzero number produces a true equation

$$8x = 16$$

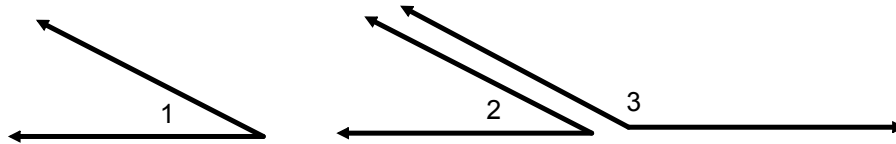
Substitution Property of Equality

Substituting a number for a variable in a true equation produces a true equation.

$$x = 7 \quad 2x + 4$$

Example 3: Justify a Theorem

$\angle 1$ and $\angle 2$ are both supplementary to $\angle 3$. Show that $\angle 1 \cong \angle 2$.



Solution

$m \angle 1 + m \angle 3 = \underline{\hspace{2cm}}$

$m \angle 2 + m \angle 3 = \underline{\hspace{2cm}}$

$m \angle 1 + m \angle 3 = m \angle 2 + m \angle 3$

$\underline{\hspace{2cm}} = \underline{\hspace{2cm}}$

$\angle 1 \cong \angle 2$

Definition of supplementary angles

Definition of supplementary angles

Subtraction property of Equality

Checkpoint: Complete the following exercise

5. In the diagram M is the midpoint of \overline{AB} . Show that $AB = 2 \cdot AM$.

$AM = MB$	Definition of _____
$AM + AM = AB$	_____ Postulate
$AM + AM = AB$	_____ Property of equality
$2 \cdot AM = AB$	Distributive property