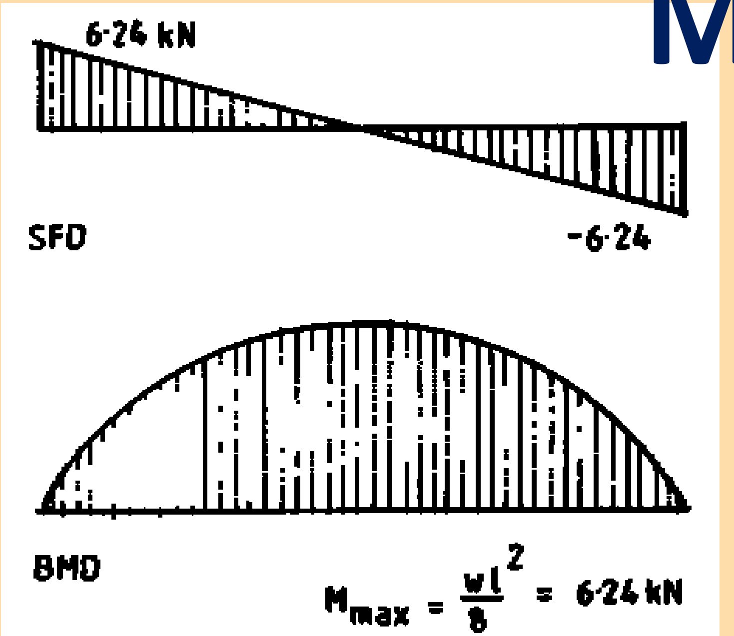
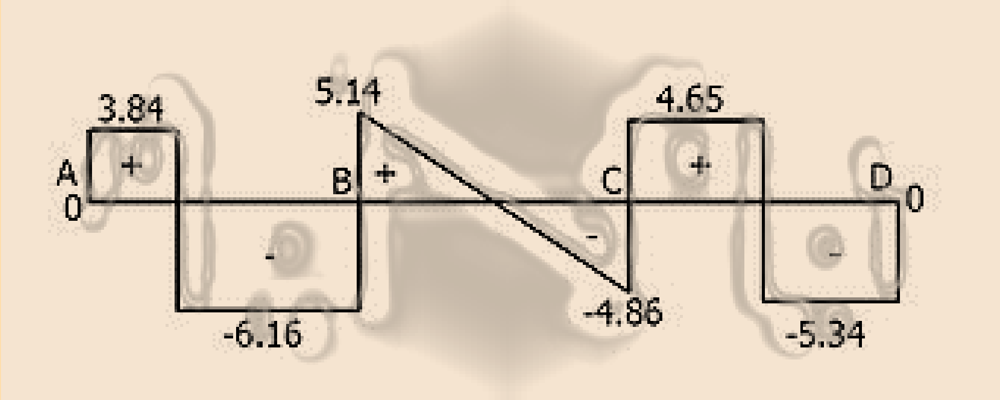
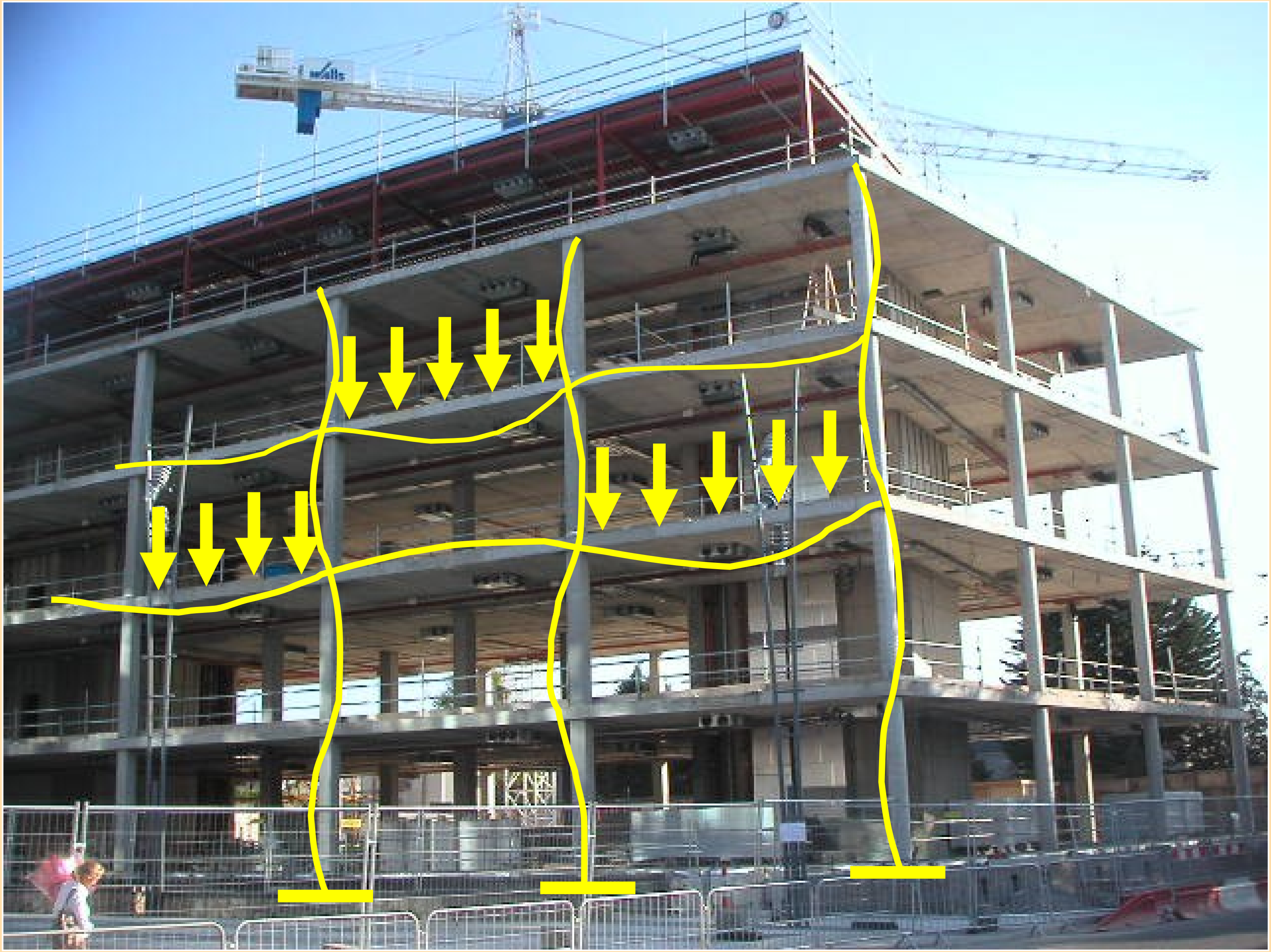
UTM
UNIVERSITI TEKNOLOGI MALAYSIA
RESEARCH UNIVERSITY

CHAPTER 2
Shear Force And Bending Moment







INTRODUCTION

	Effects	Action
Loading	Shear Force	Design Shear reinforcement
Loading	Bending Moment	Design flexure reinforcement

SKAA 2223

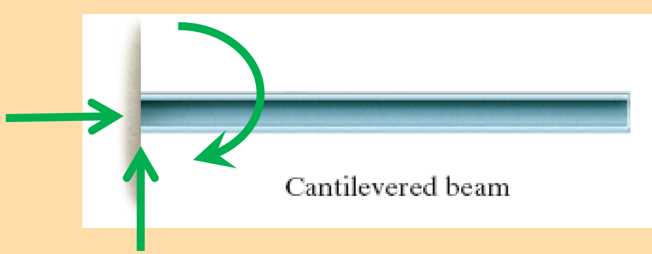
SKAA 3353

SHEAR FORCE & BENDING MOMENT

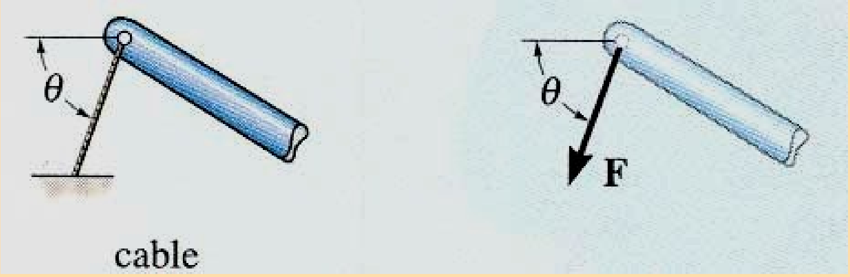
- Introduction
  - Types of beams
  - Effects of loading on beams
  - The force that cause shearing is known as shear force
  - The force that results in bending is known as bending moment
  - Draw the shear force and bending moment diagrams

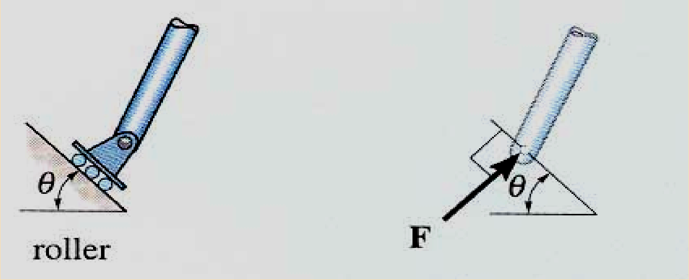
SHEAR FORCE & BENDING MOMENT

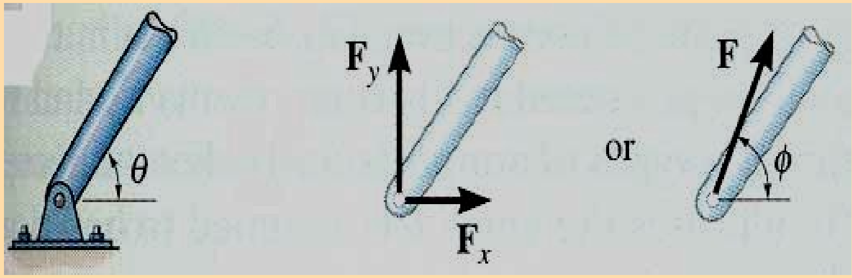
- Members with support loadings applied perpendicular to their longitudinal axis are called ***beams***.
- Beams classified according to the way they are supported.

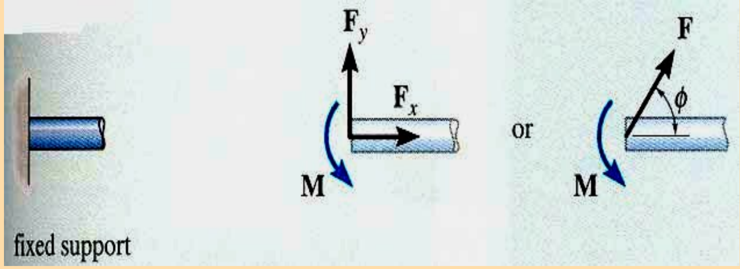


TYPES OF SUPPORT









As a general rule, if a ***support prevents translation*** of a body in a given direction, then ***a force is developed*** on the body in the opposite direction. Similarly, if ***rotation is prevented, a couple moment*** is exerted on the body.

SHEAR FORCE & BENDING MOMENT

- Types of beam

a) Determinate Beam

The force and moment of reactions at supports can be determined by using the 3 equilibrium equations of statics i.e.

$\Sigma F_x = 0$, $\Sigma F_y = 0$ and $\Sigma M = 0$

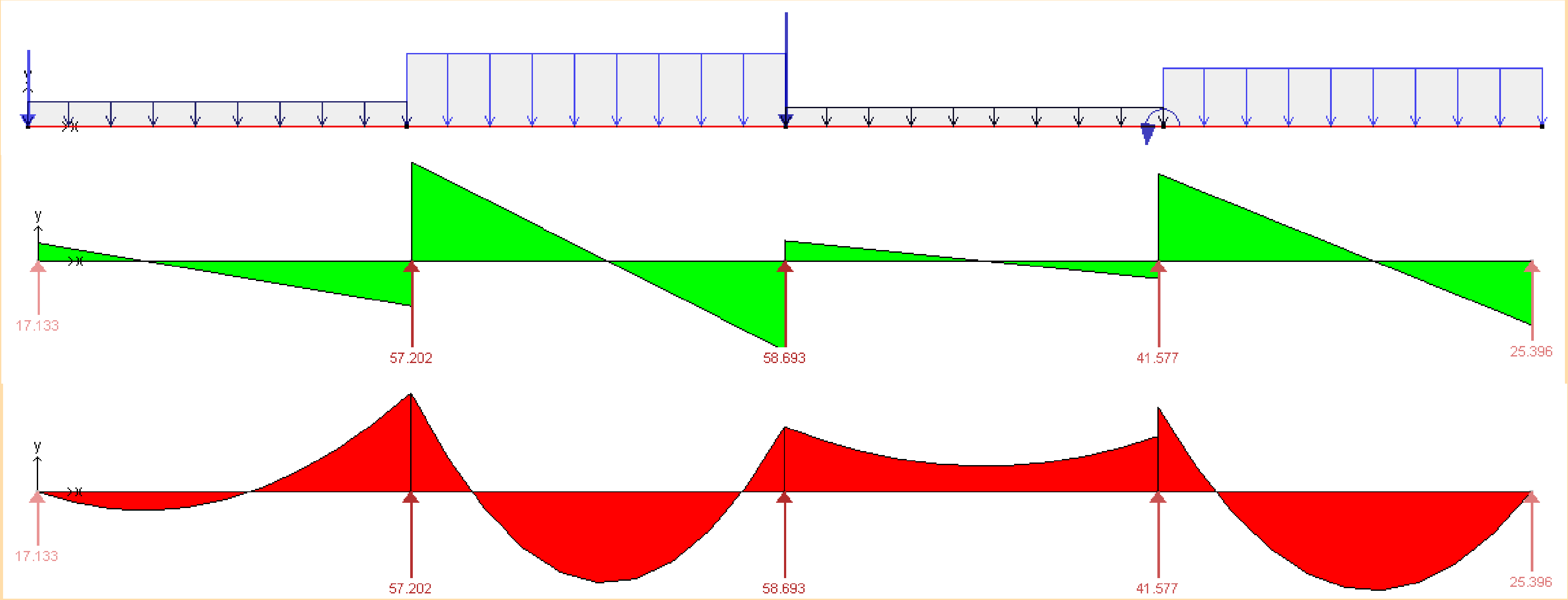
b) Indeterminate Beam

The force and moment of reactions at supports are more than the number of equilibrium equations of statics.

(The extra reactions are called redundant and represent the amount of degrees of indeterminacy).

SHEAR FORCE & BENDING MOMENT

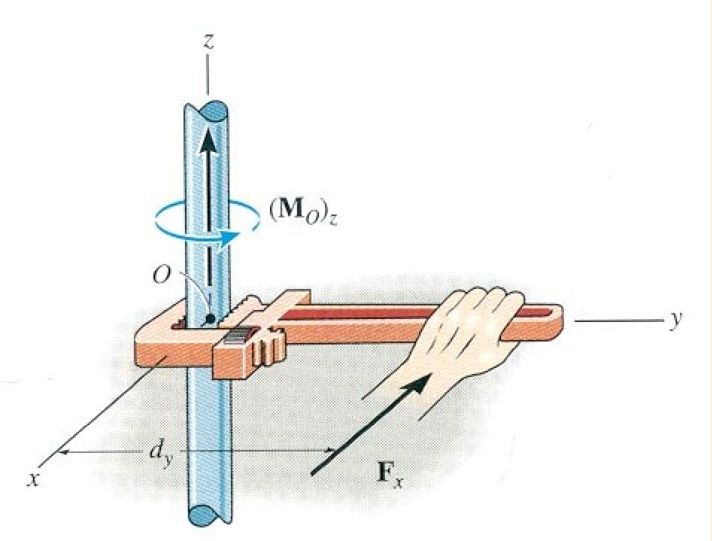
- In order to properly design a beam, it is important to know the ***variation*** of the shear and moment along its axis in order to find the points where these values are a maximum.

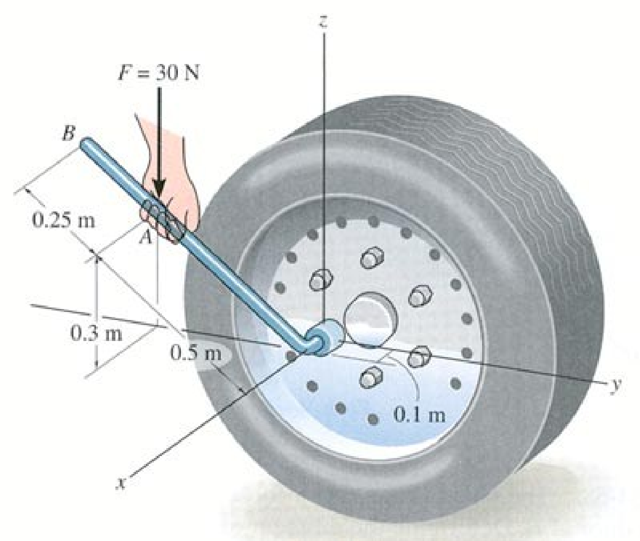


PRINCIPLE OF MOMENTS

- The moment of a force indicates the tendency of a body to turn about an axis passing through a specific point O.
- The principle of moments, which is sometimes referred to as ***Varignon's Theorem*** (Varignon, 1654 – 1722) states that ***the moment of a force about a point is equal to the sum of the moments of the force's components about the point***.

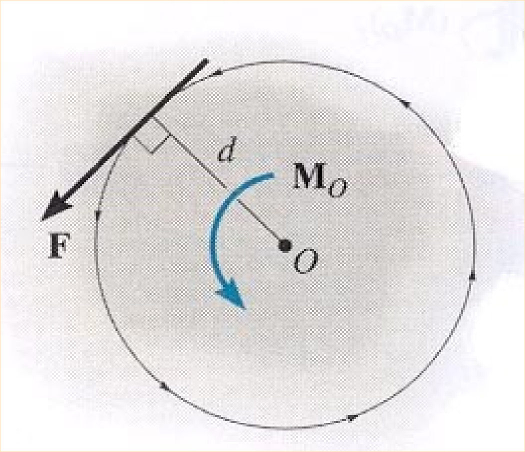
PRINCIPLE OF MOMENTS





In the 2-D case, the magnitude of the moment is:

M_o = Force x distance



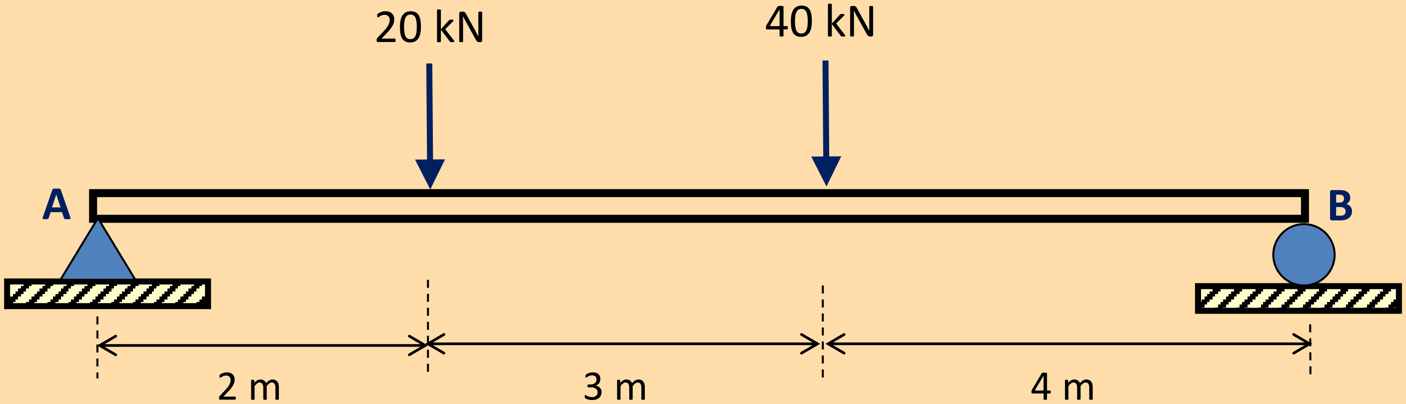
BEAM'S REACTION

- If a support ***prevents translation*** of a body in a particular direction, then the support exerts a ***force*** on the body in that direction.

- Determined using $\Sigma F_x = 0$, $\Sigma F_y = 0$ and $\Sigma M = 0$

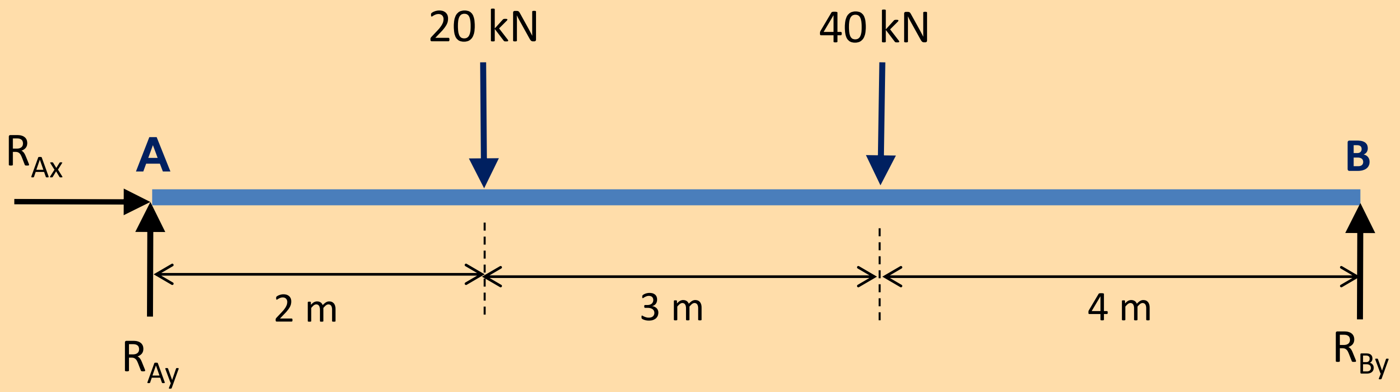
EXAMPLE 1

The beam shown below is supported by a pin at A and roller at B. Calculate the reactions at both supports due to the loading.



EXAMPLE 1 – Solution

Draw the free body diagram:



By taking the moment at B,

$\Sigma M_B = 0$

$R_{Ay} \times 9 - 20 \times 7 - 40 \times 4 = 0$

$9R_{Ay} = 140 + 160$

$R_{Ay} = 33.3$ kN

$\Sigma F_y = 0$

$R_{Ay} + R_{By} - 20 - 40 = 0$

$R_{By} = 20 + 40 - 33.3$

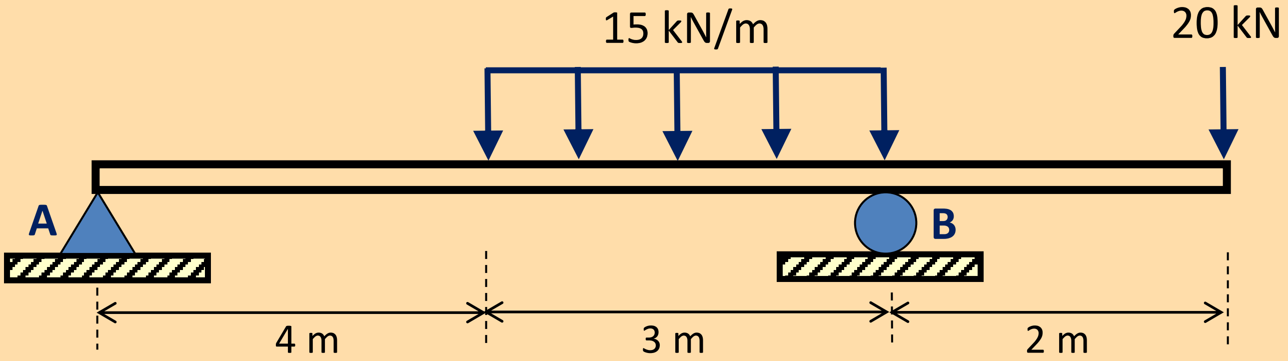
$R_{By} = 26.7$ kN

$\Sigma F_x = 0$

$R_{Ax} = 0$

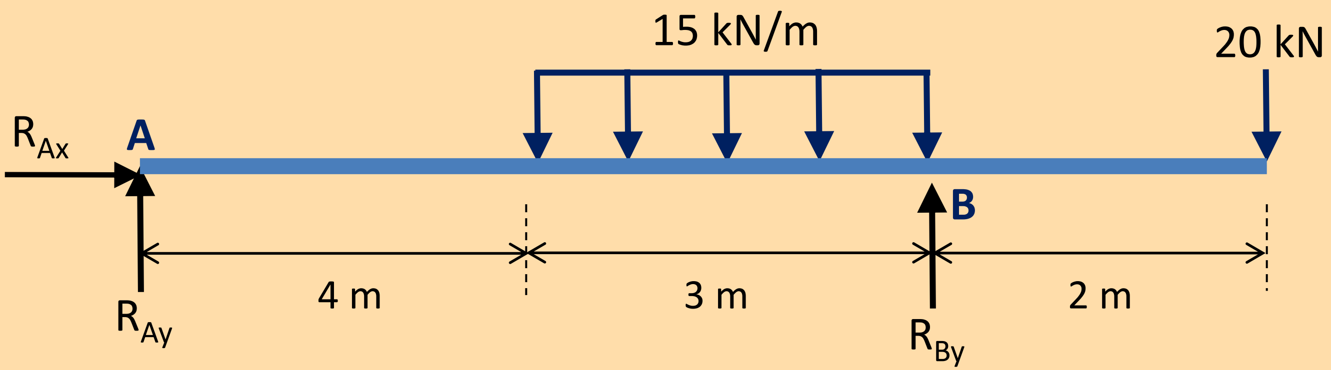
EXAMPLE 2

Determine the reactions at support A and B for the overhanging beam subjected to the loading as shown.



EXAMPLE 2 – Solution

Draw the free body diagram:



By taking the moment at A:

$\Sigma M_A = 0$

$-R_{By} \times 7 + 20 \times 9 - (15 \times 3) \times 5.5 = 0$

$7R_{By} = 247.5 + 180$

$R_{By} = 61.07$ kN

$\Sigma F_y = 0$

$R_{Ay} + R_{By} - 20 - 45 = 0$

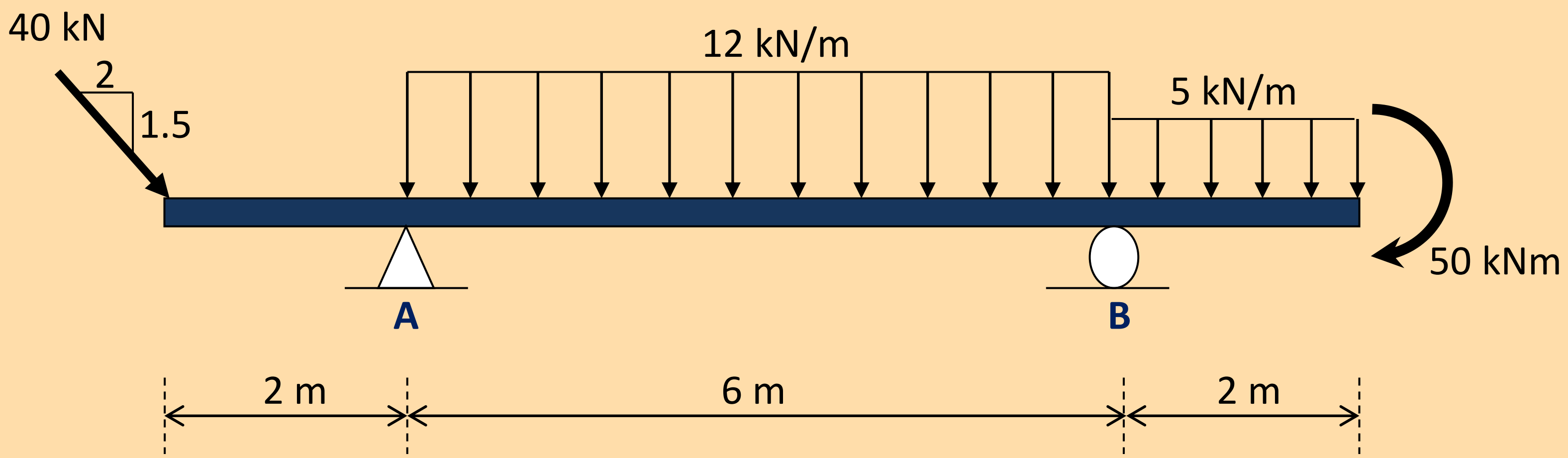
$R_{Ay} = 20 + 45 - 61.07$

$R_{Ay} = 3.93$ kN

$\Sigma F_x = 0$

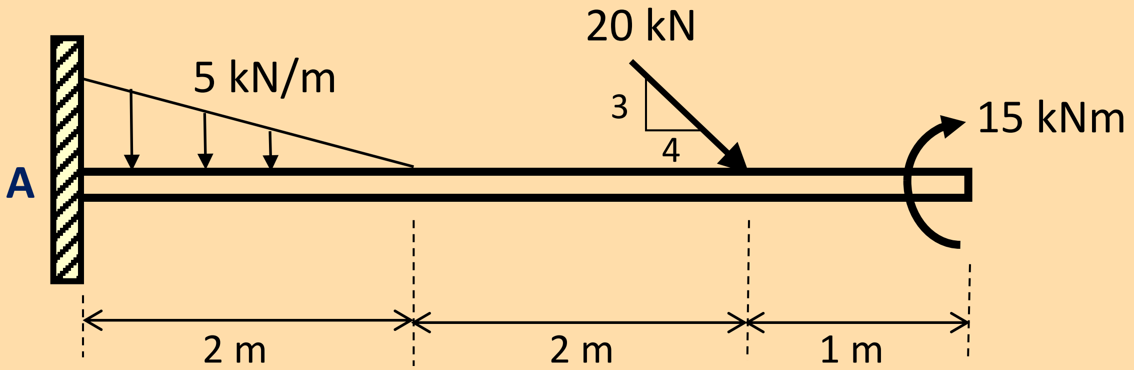
$R_{Ax} = 0$

CLASS EXERCISE – 5 mins?



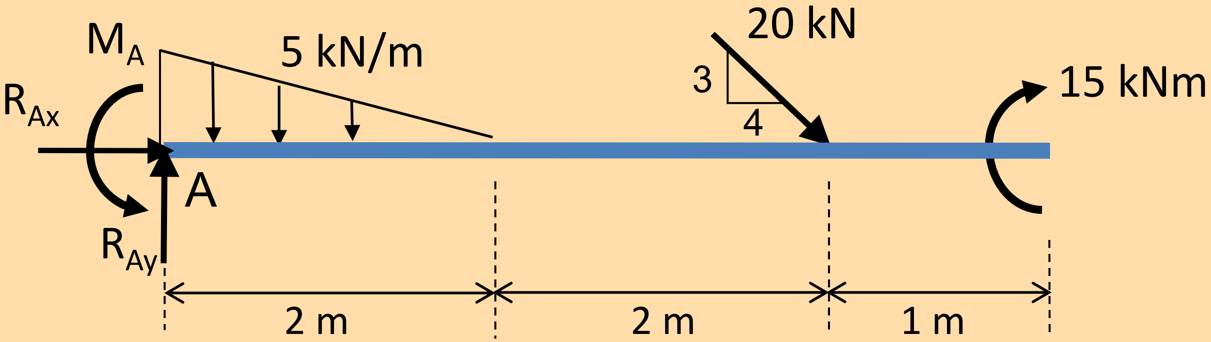
EXAMPLE 3

A cantilever beam is loaded as shown. Determine all reactions at support A.



EXAMPLE 3 – Solution

Draw the free body diagram:



$\Sigma F_x = 0$

$-R_{Ax} + 20\,(4/5) = 0$

$-R_{Ax} = 16$ kN

$\Sigma F_y = 0$

$R_{Ay} - 0.5\,(5)(2) - 20(3/5) = 0$

$R_{Ay} - 5 - 12 = 0$

$R_{Ay} = 17$ kN

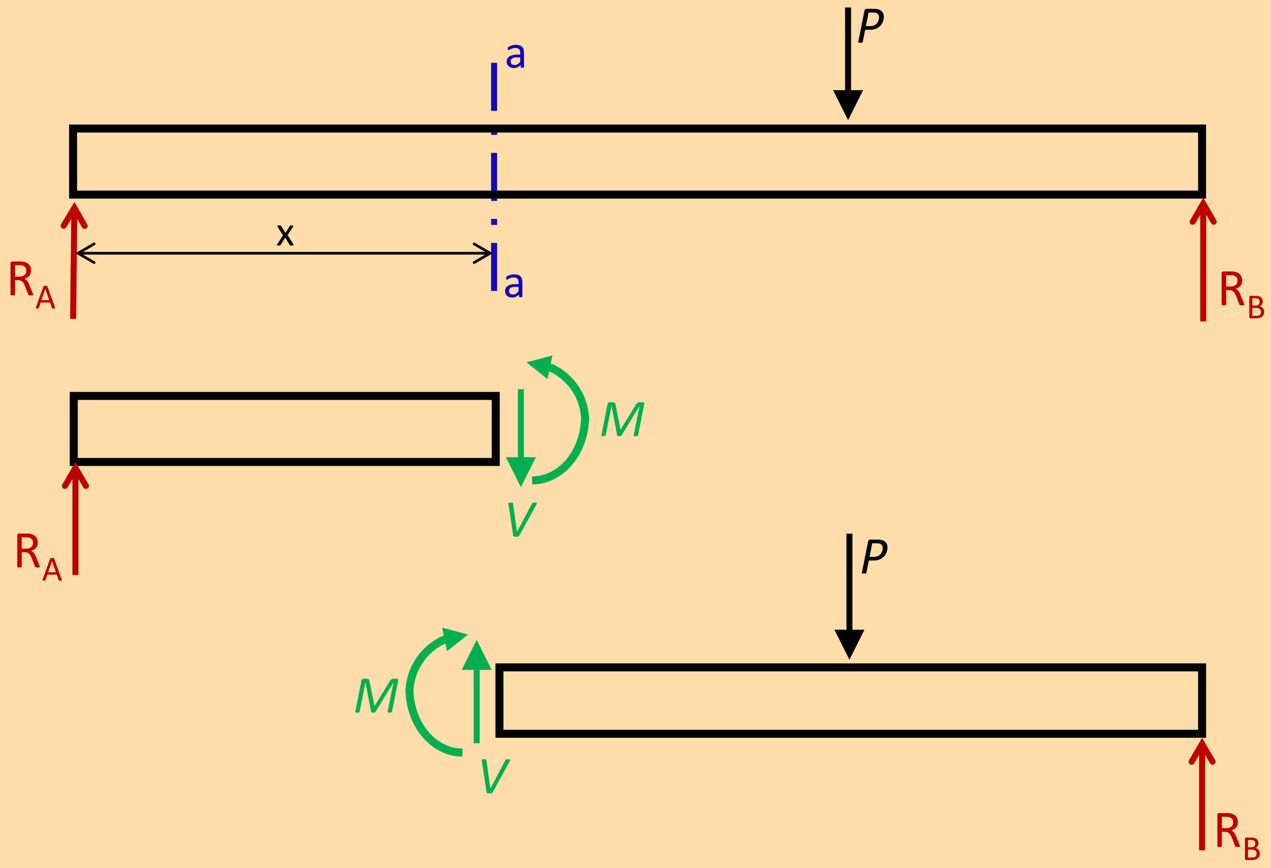
$\Sigma M_A = 0$

$-M_A + 0.5(5)(2)(1/3)(2) + 20(3/5)\,(4) + 15 = 0$

$M_A = 3.3 + 48 + 15$

$M_A = 66.3$ kNm

SHEAR FORCE & BENDING MOMENT DIAGRAM



SHEAR FORCE & BENDING MOMENT DIAGRAM

V = shear force

= the force that tends to separate the member

= balances the reaction R_A

M = bending moment

= the reaction moment at a particular point (section)

= balances the moment, $R_A \cdot x$

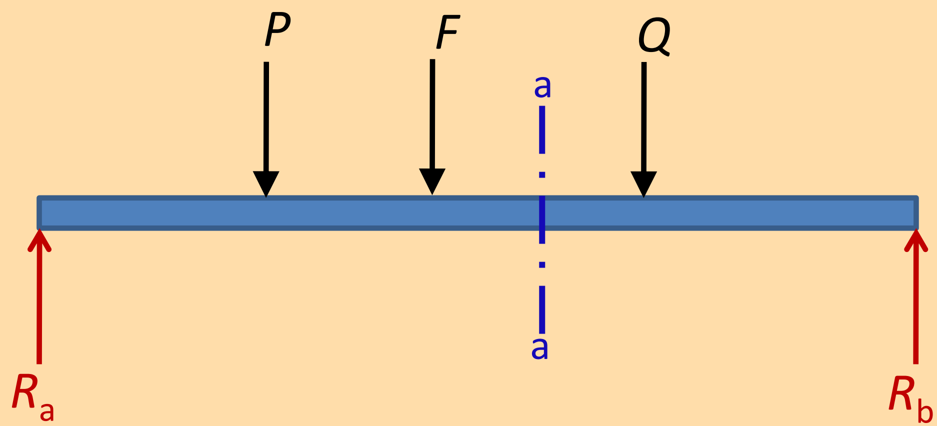
SHEAR FORCE & BENDING MOMENT DIAGRAM

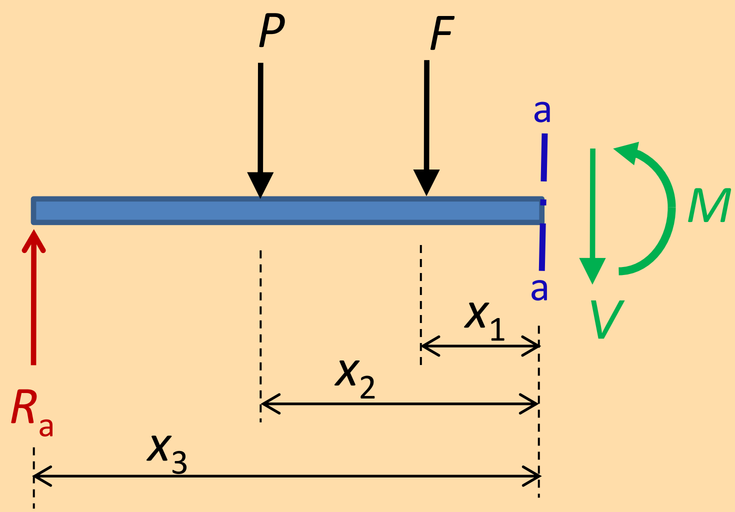
From the equilibrium equations of statics:

$+\uparrow \Sigma F_y = 0;$ $R_A - V = 0$ $\therefore V = R_A$

$+\circlearrowright \Sigma M_{a\text{-}a} = 0;$ $-M + R_A \cdot x = 0$ $\therefore M = R_A \cdot x$

SHEAR FORCE & BENDING MOMENT DIAGRAM



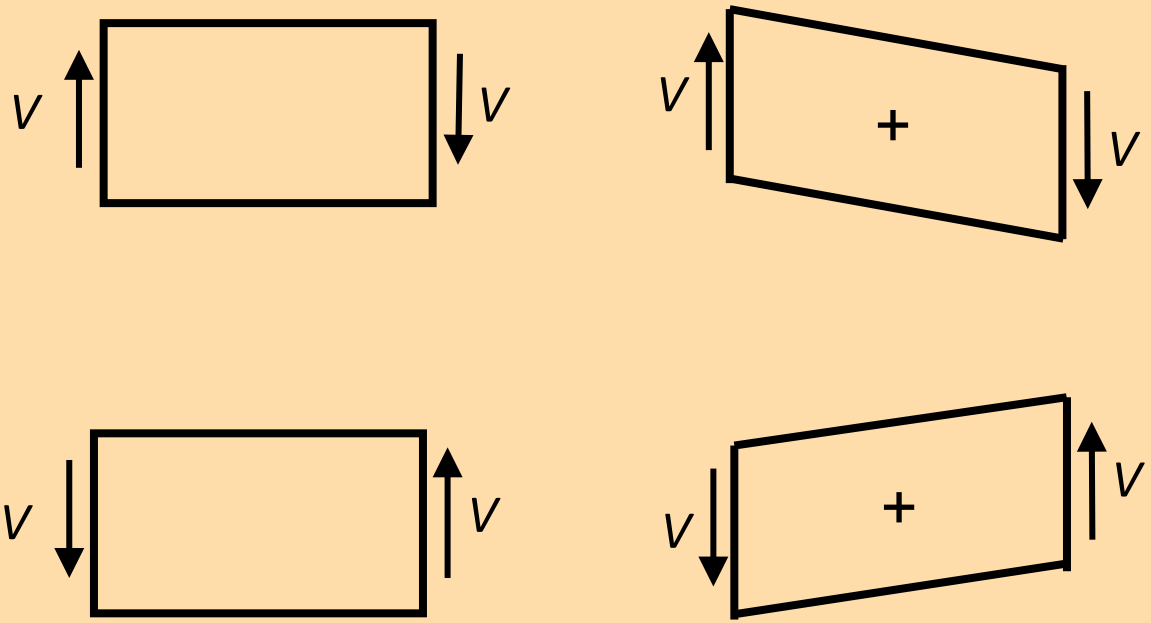


$$\Sigma F_y = 0$$
$$R_a - P - F - V = 0$$
$$V = R_a - P - F$$

$$\Sigma M_{a\text{-}a} = 0$$
$$-M - F \cdot x_1 - P \cdot x_2 + R_a \cdot x_3 = 0$$
$$M = R_a \cdot x_3 - F \cdot x_1 - P \cdot x_2$$

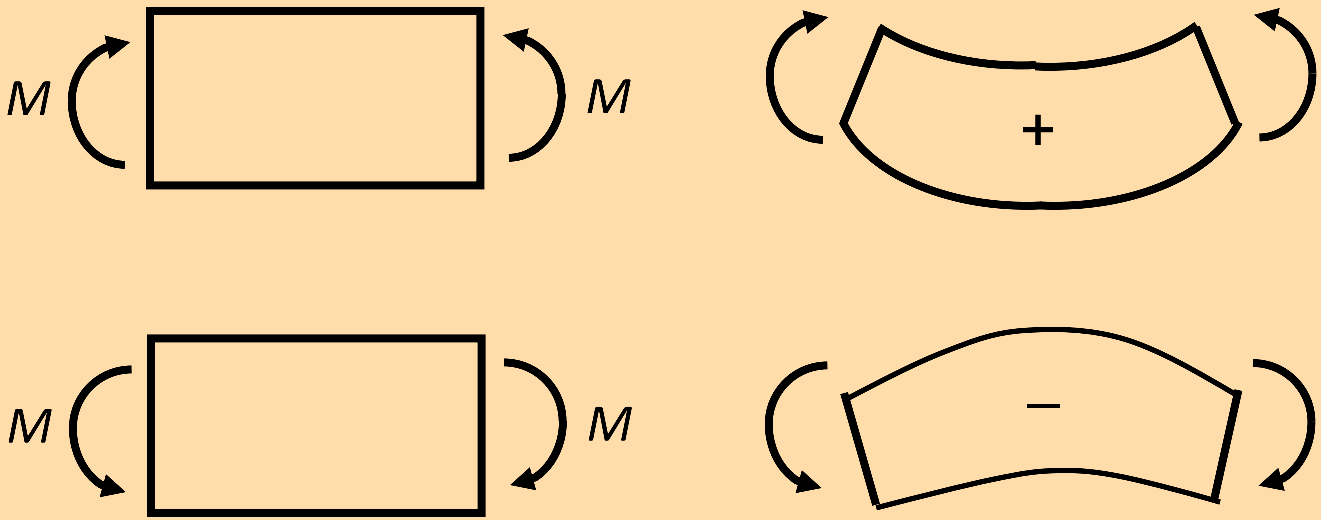
SHEAR FORCE & BENDING MOMENT DIAGRAM

Shape deformation due to shear force:



SHEAR FORCE & BENDING MOMENT DIAGRAM

Shape deformation due to bending moment:



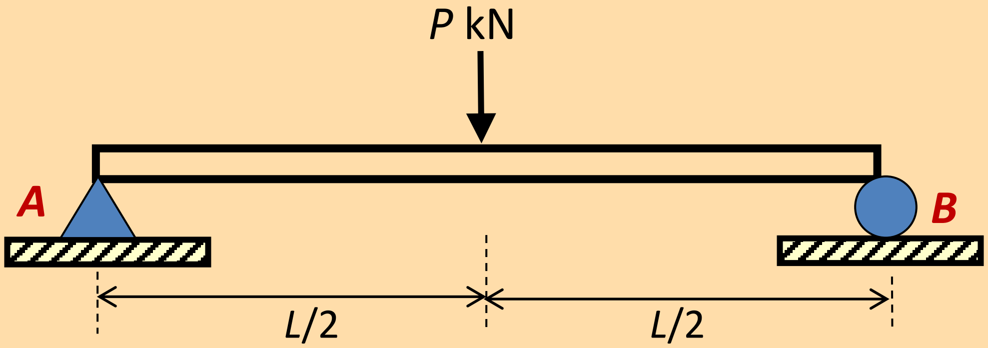
Sign Convention:

- **Positive shear force** diagram drawn **ABOVE** the beam
- **Positive bending moment** diagram drawn **BELOW** the beam

EXAMPLE 4

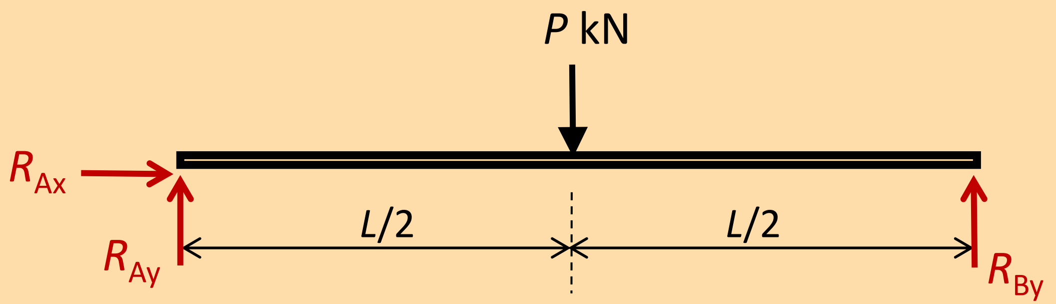
a) Calculate the shear force and bending moment for the beam subjected to a concentrated load as shown in the figure. Then, draw the shear force diagram (SFD) and bending moment diagram (BMD).

b) If P = 20 kN and L = 6 m, draw the SFD and BMD for the beam.



EXAMPLE 4 – Solution

a)



By taking the moment at A:

$\Sigma M_A = 0$

$-R_{By} \times L + P \times L/2 = 0$

$R_{By} = P/2$ kN

$\Sigma F_y = 0$

$R_{Ay} + R_{By} = P$

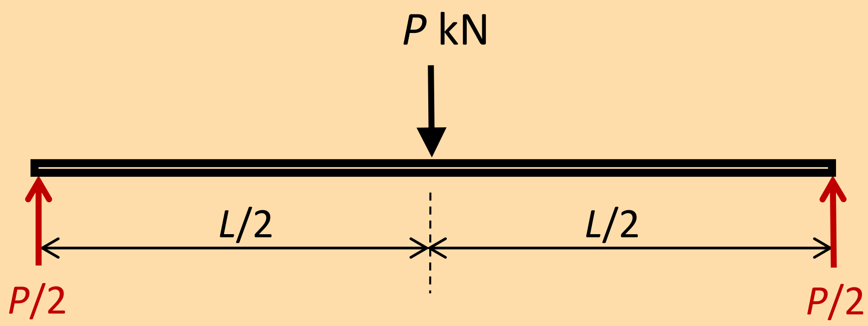
$R_{Ay} = P - P/2$

$R_{Ay} = P/2$ kN

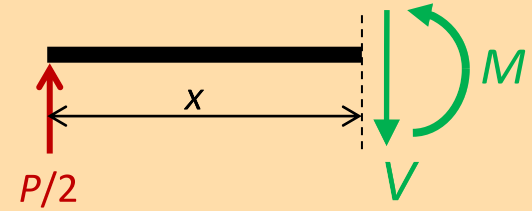
$\Sigma F_x = 0$

$R_{Ax} = 0$

EXAMPLE 4 – Solution



Between $0 < x < L/2$:



$\Sigma F_y = 0,$ $\quad -V + P/2 = 0$

$V = P/2$ kN

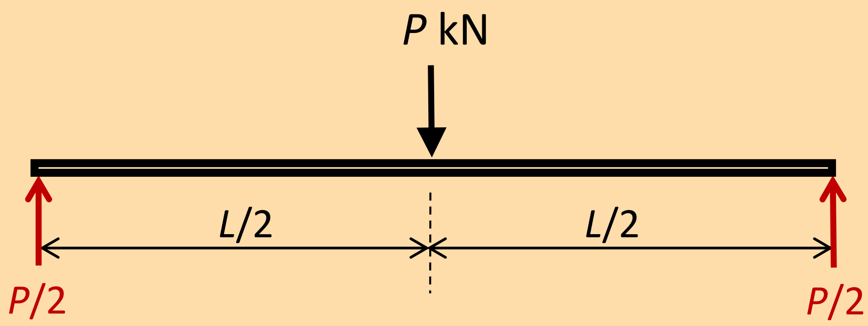
$\Sigma M_{a\text{-}a} = 0,$ $\quad -M + Px/2 = 0$

$M = Px/2$ kNm

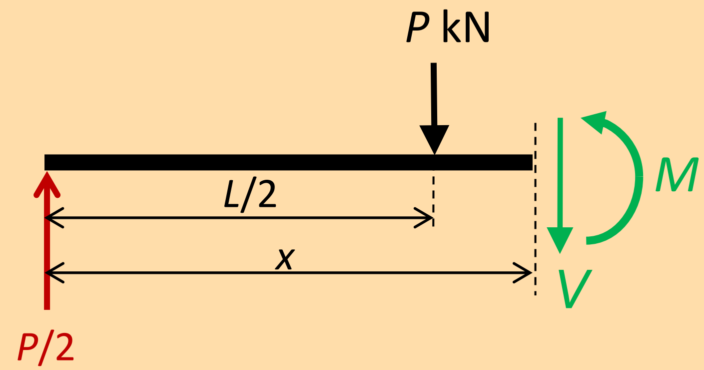
If $x = 0$ m, $\boldsymbol{V = P/2}$ **kN** and $\boldsymbol{M = 0}$ **kNm**

If $x = L/2$ m, $\boldsymbol{V = P/2}$ **kN** and $\boldsymbol{M = PL/4}$ **kNm**

EXAMPLE 4 – Solution



Between $L/2 < x < L$:



$\Sigma F_y = 0,$ $\quad -V + P/2 - P = 0$

$V = -P/2$ kN

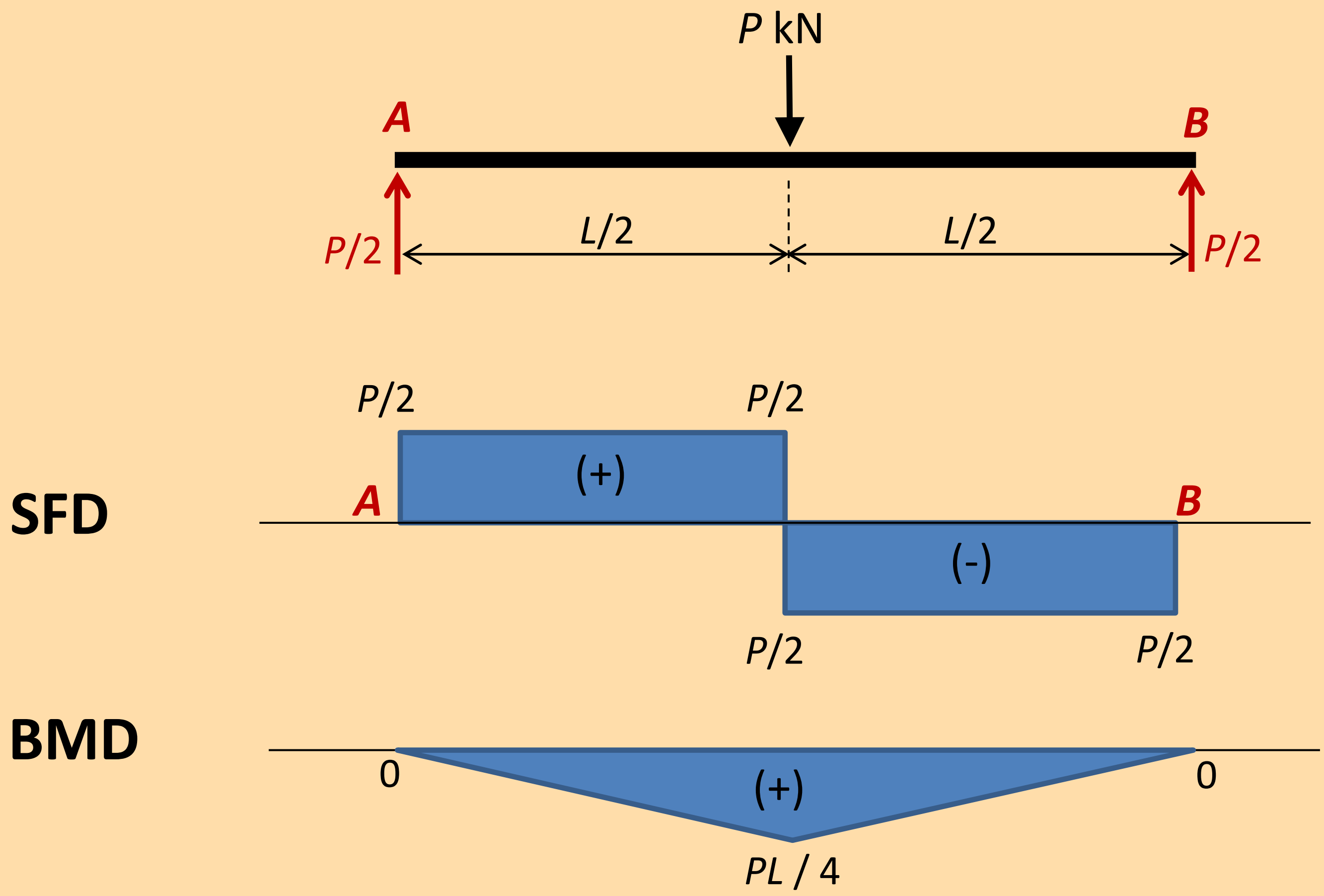
$\Sigma M_{a\text{-}a} = 0,$ $\quad -\mathrm{M} + Px/2 - P(x - L/2) = 0$

$\mathrm{M} = PL/2 - Px/2$ kNm

If $x = L/2$ m, $\boldsymbol{V = -P/2}$ **kN** and $\boldsymbol{M = PL/4}$ **kNm**

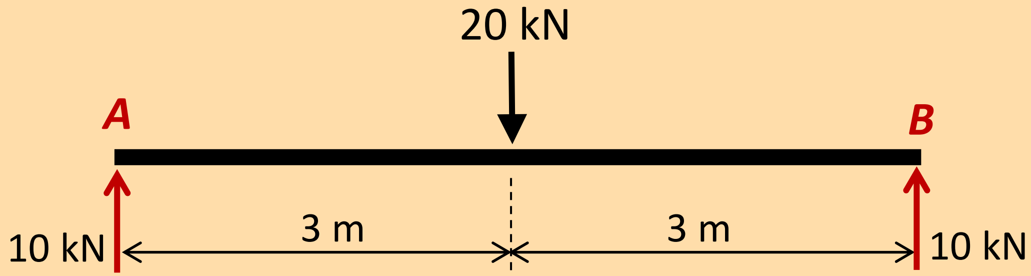
If $x = L$ m, $\boldsymbol{V = -P/2}$ **kN** and $\boldsymbol{M = 0}$ **kNm**

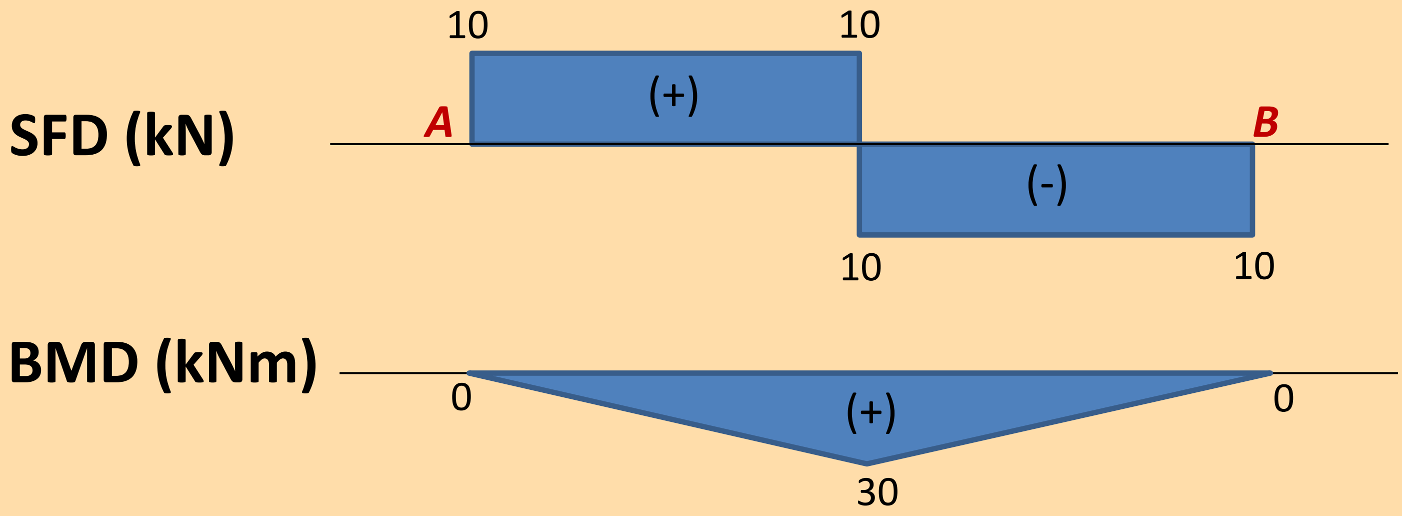
EXAMPLE 4 – Solution



EXAMPLE 4 – Solution

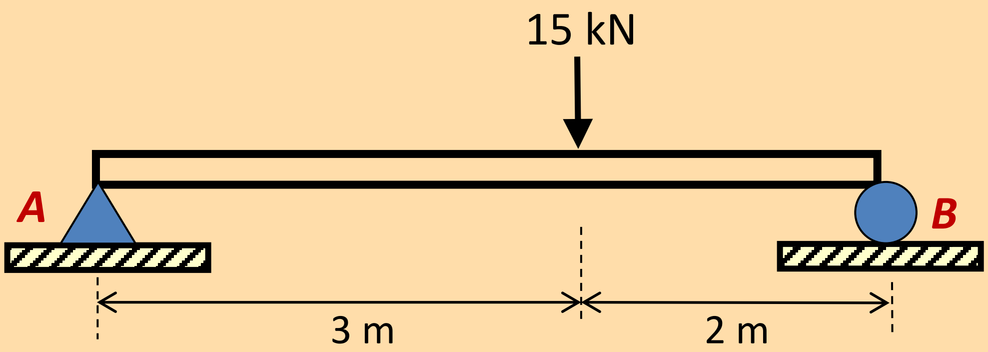
b)



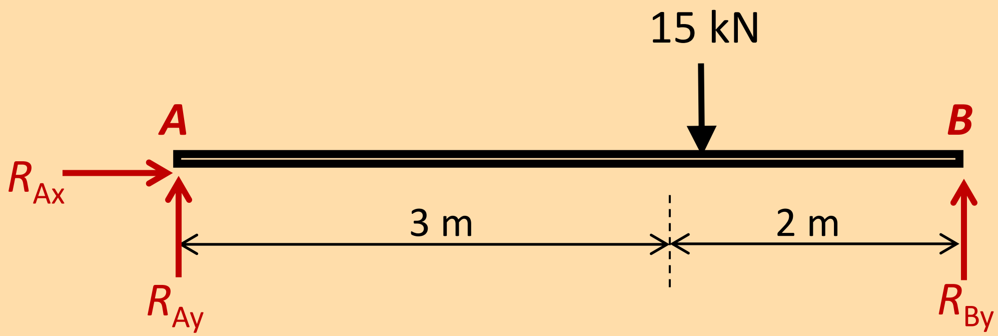


EXAMPLE 5

Calculate the shear force and bending moment for the beam subjected to a concentrated load as shown in the figure, then draw the shear force diagram (SFD) and bending moment diagram (BMD).



EXAMPLE 5 – Solution



By taking the moment at A:

$\Sigma M_A = 0$

$-R_{By} \times 5 + 15 \times 3 = 0$

$R_{By} = 9$ kN

$\Sigma F_y = 0$

$R_{Ay} + R_{By} = 15$

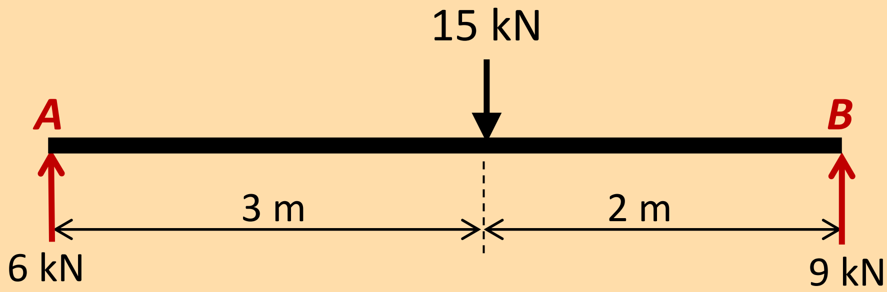
$R_{Ay} = 15 - 9$

$R_{Ay} = 6$ kN

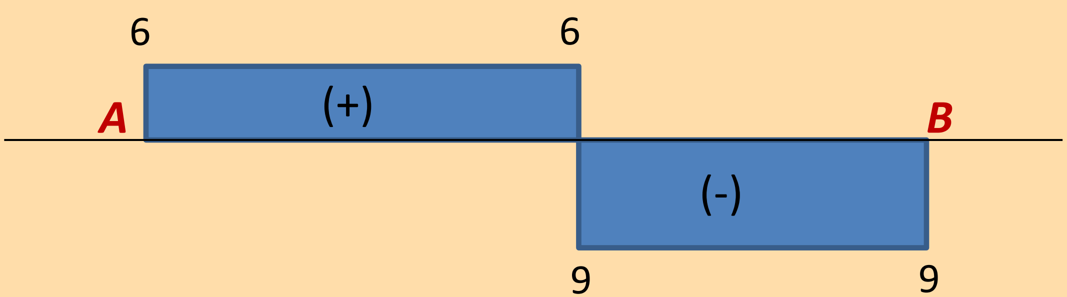
$\Sigma F_x = 0$

$R_{Ax} = 0$

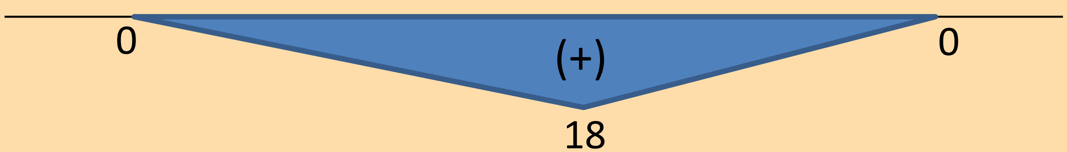
EXAMPLE 5 – Solution



SFD (kN)

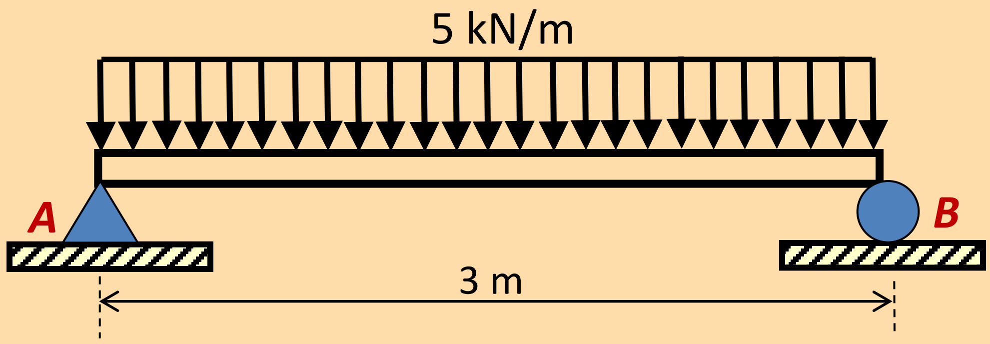


BMD (kNm)

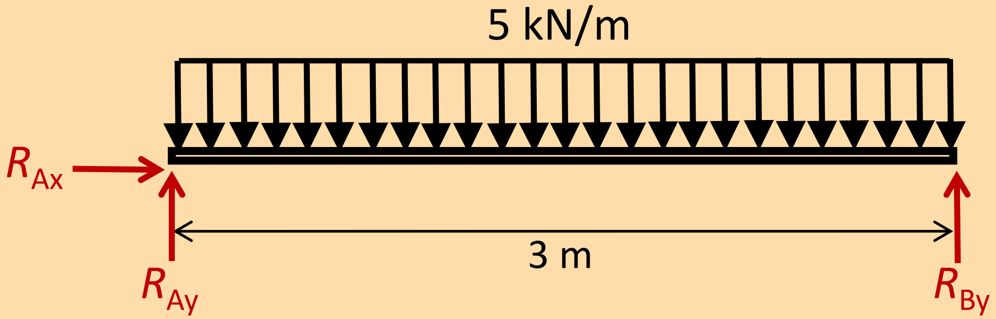


EXAMPLE 6

Calculate the shear force and bending moment for the beam subjected to an uniformly distributed load as shown in the figure, then draw the shear force diagram (SFD) and bending moment diagram (BMD).



EXAMPLE 6 – Solution



By taking the moment at A:

$\Sigma M_A = 0$

$-R_{By} \times 3 + 5 \times 3 \times 3/2 = 0$

$R_{By} = 7.5$ kN

$\Sigma F_y = 0$

$R_{Ay} + R_{By} = 5 \times 3$

$R_{Ay} = 15 - 7.5$

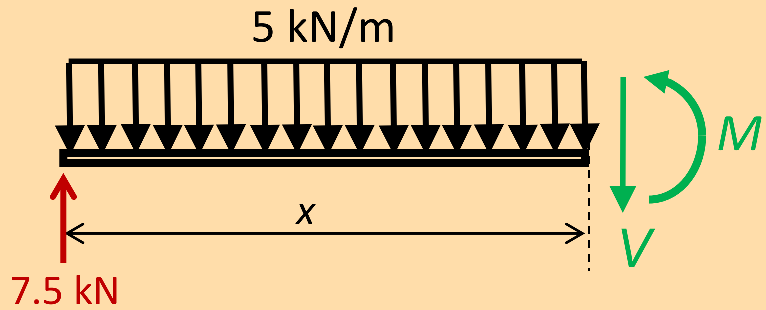
$R_{Ay} = 7.5$ kN

$\Sigma F_x = 0$

$R_{Ax} = 0$

EXAMPLE 6 – Solution

These results for V and M can be checked by noting that $dV/dx = -w$. This is correct, since positive w acts downward. Also, notice that $dM/dx = V$. The maximum moments occurs when $dM/dx = V = 0$.



$\Sigma M_{a\text{-}a} = 0$, $\quad -M + 7.5x - 5x\,(x/2) = 0$

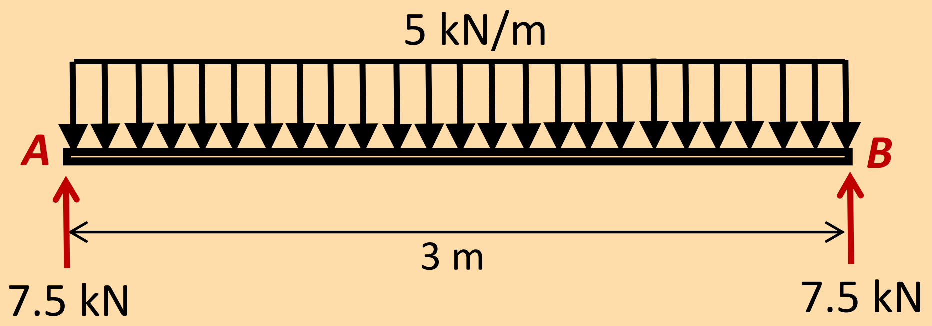
$M = 7.5x - 5x^2/2$

M = maximum when $\dfrac{dM}{dx} = 0$

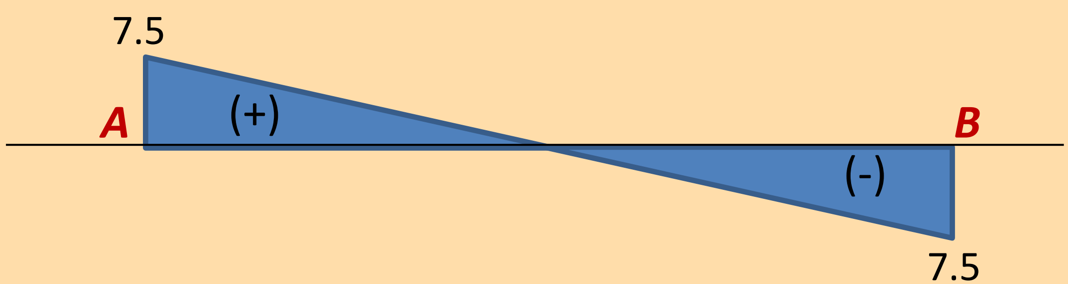
$$\frac{dM}{dx} = 7.5 - 5x = 0 \qquad \therefore\ x = 1.5\ \text{m}$$

Therefore, M_{max} = 5.625 kNm

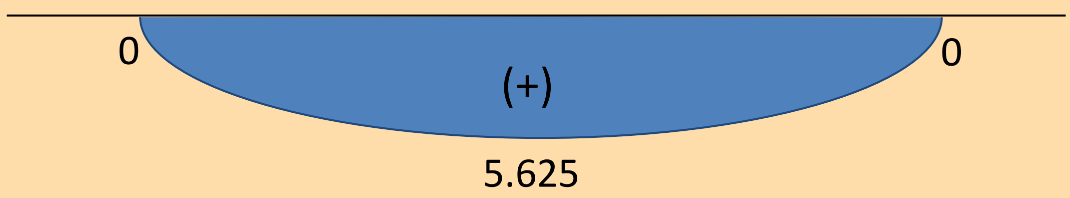
EXAMPLE 6 – Solution



SFD (kN)

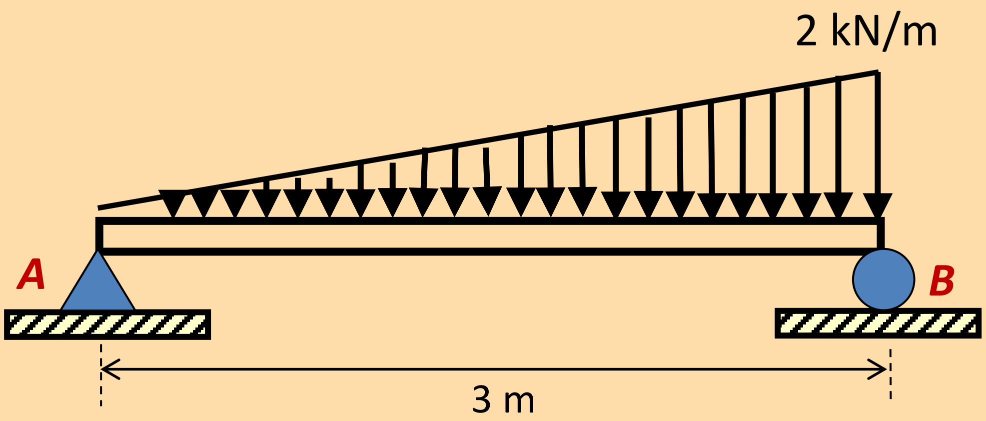


BMD (kNm)

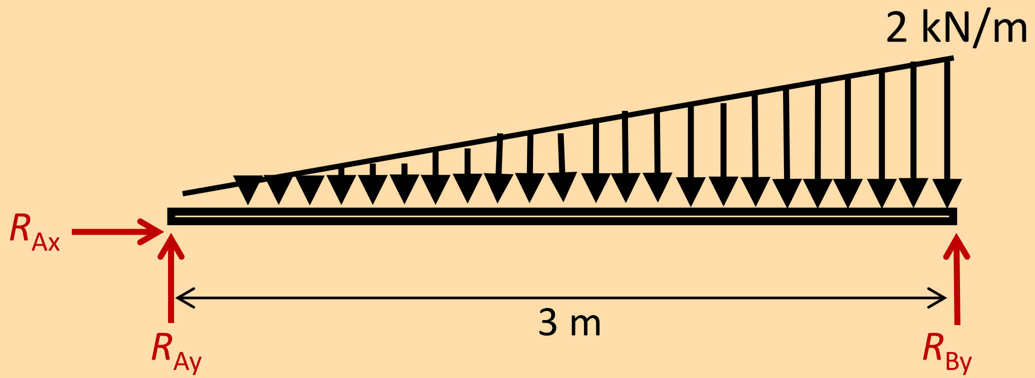


EXAMPLE 7

Calculate the shear force and bending moment for the beam subjected to the loads as shown in the figure, then draw the shear force diagram (SFD) and bending moment diagram (BMD).



EXAMPLE 7 – Solution



By taking the moment at A:

$\Sigma M_A = 0$

$2 \times 3/2 \times 3 \times 2/3 - R_{By} \times 3 = 0$

$R_{By} = 2$ kN

$\Sigma F_y = 0$

$R_{Ay} + R_{By} = 2 \times 3/2$

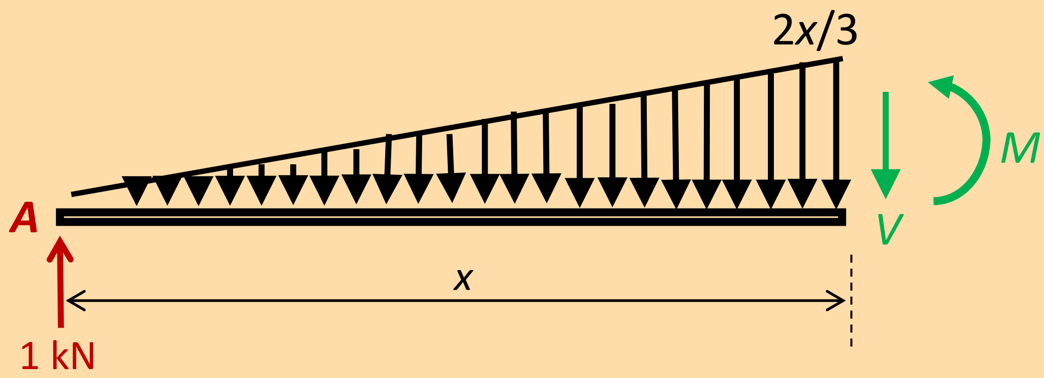
$R_{Ay} = 3 - 2$

$R_{Ay} = 1$ kN

$\Sigma F_x = 0$

$R_{Ax} = 0$

EXAMPLE 7 – Solution



$1 - 2x/3(x)(1/2) - V = 0$

$V = 1 - 2x^2/6$

If $x = 0$, $V = 1$ kN and $x = 3$, $V = -2$ kN

$-M + 1 \times x - 2x/3(x)(1/2)\,(x/3) = 0$

$M = x - x^3/9$

M = maximum when $\frac{dM}{dx} = 0$

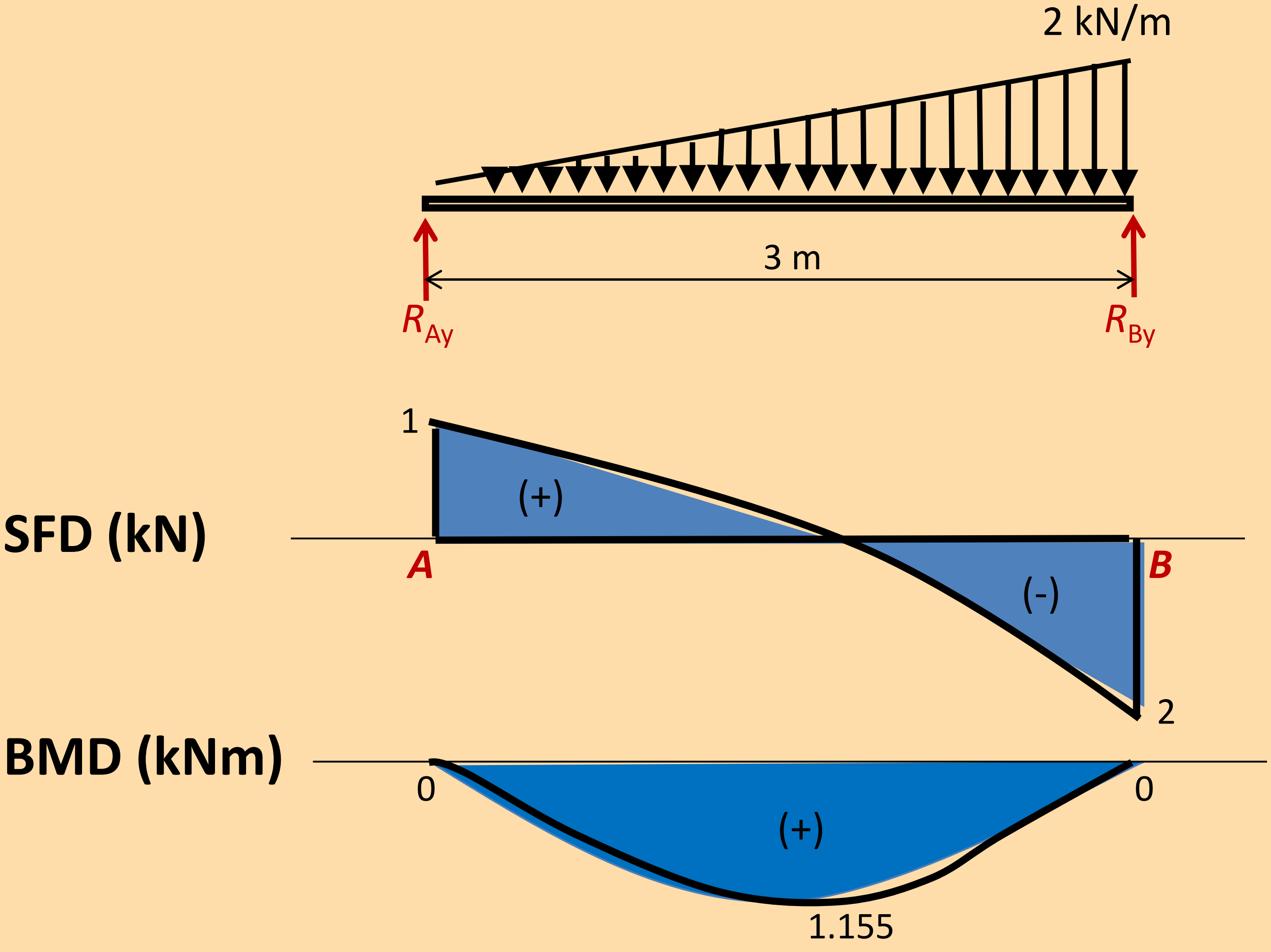
$$\frac{dM}{dx} = 1 - \frac{3x^2}{9} = 0$$

$$x^2 = \frac{9}{3}$$

$$x = \frac{3}{\sqrt{3}} = 1.732m$$

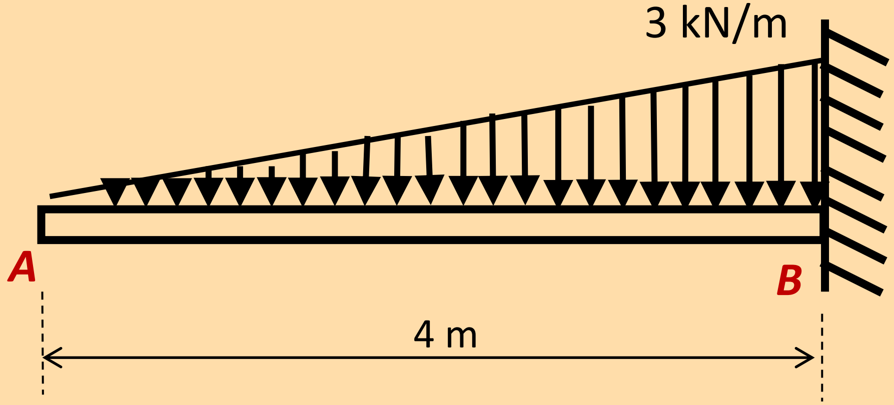
Therefore, M_{max} = 1.155 kNm

EXAMPLE 7 – Solution

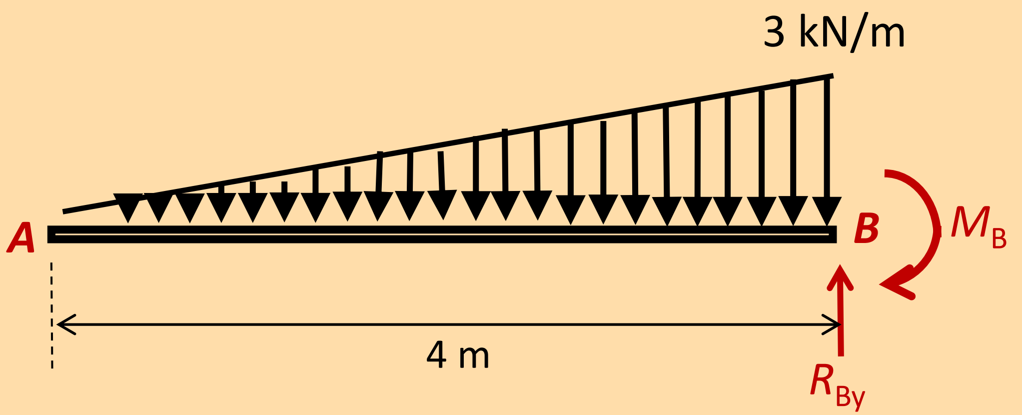


EXAMPLE 8

Calculate the shear force and bending moment for the beam subjected to the loads as shown in the figure, then draw the shear force diagram (SFD) and bending moment diagram (BMD).



EXAMPLE 8 – Solution



By taking the moment at B:

$\Sigma M_B = 0$

$M_B = 3 \times 4/2 \times 4/3$

$M_B = 8$ kNm

$\Sigma F_y = 0$

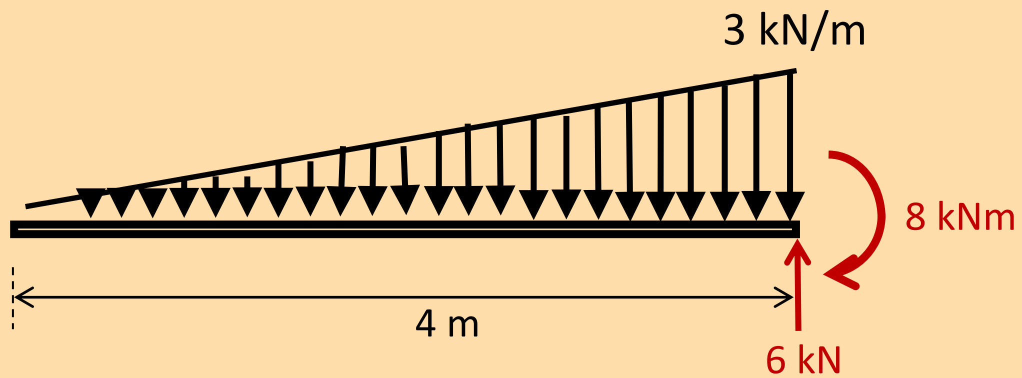
$R_{By} = 3 \times 4/2$

$R_{By} = 6$ kN

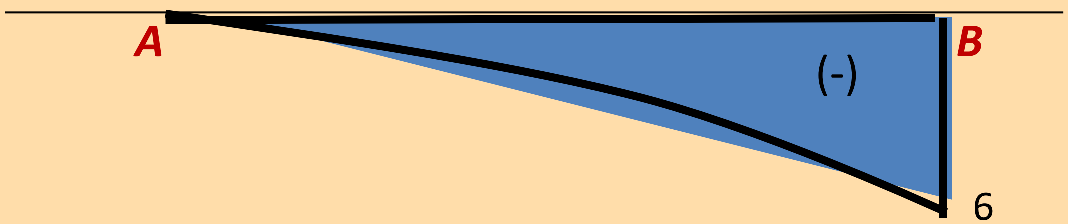
$\Sigma F_x = 0$

$R_{Bx} = 0$

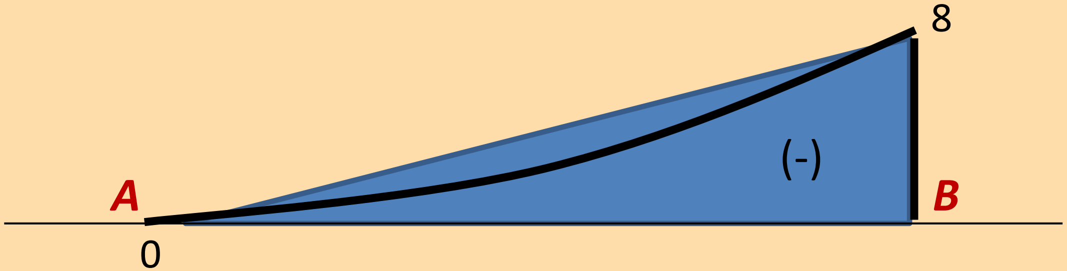
EXAMPLE 8 – Solution



SFD (kN)

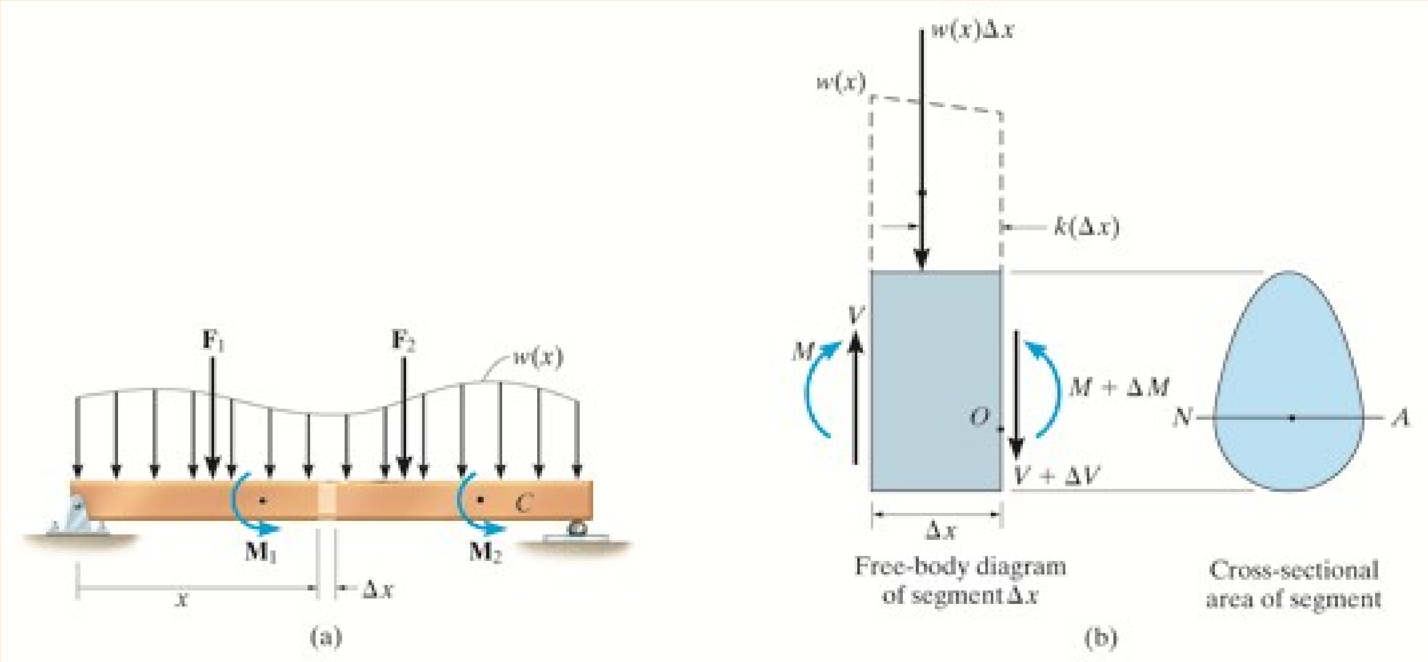


BMD (kNm)



RELATIONSHIP BETWEEN LOAD, SHEAR FORCE & BENDING MOMENT

When a beam is subjected to two or more concentrated or distributed load, the way to calculate and draw the SFD and BMD may not be the same as in the previous situation.



REGION OF DISTRIBUTED LOAD

$$\Sigma F_y = 0; \quad V - w(x)\Delta x - (V + \Delta V) = 0$$

$$\Delta V = w(x)\Delta x$$

$$\Sigma M_0 = 0;$$

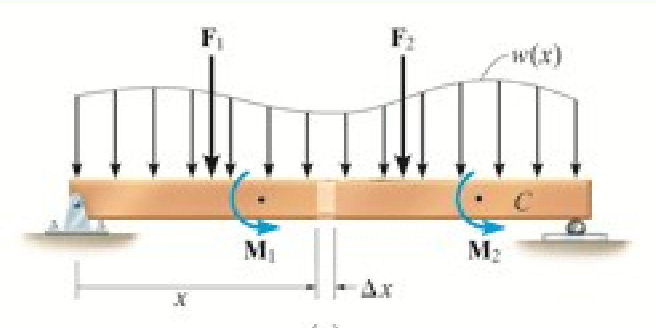
$$-V\,\Delta x - M + w(x)\Delta x[k\,\Delta x] + (M + \Delta M) = 0$$

$$\Delta M = V\,\Delta x - w(x)k\Delta x^2$$

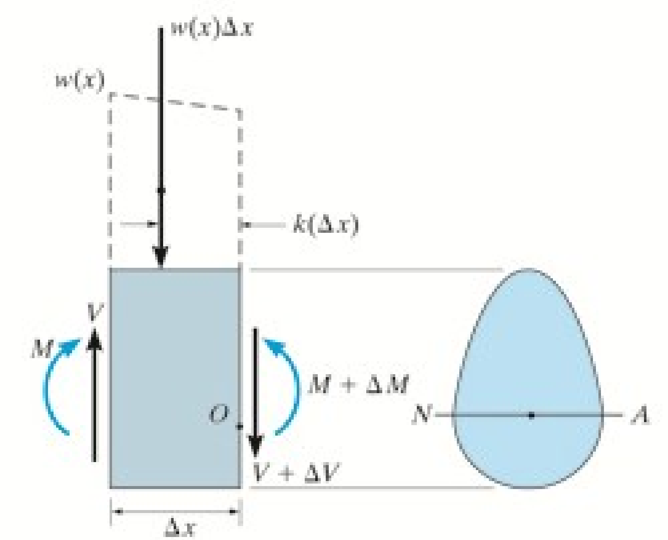
Dividing by Δx and taking the limit as $\Delta x = 0$, the above two equations become:

Slope of the shear diagram at each point $\dfrac{dV}{dx} = -w(x)$ distributed load intensity at each point

Slope of moment diagram at each point $\dfrac{dM}{dx} = V$ Shear at each point



(a)



Free-body diagram of segment Δx

Cross-sectional area of segment

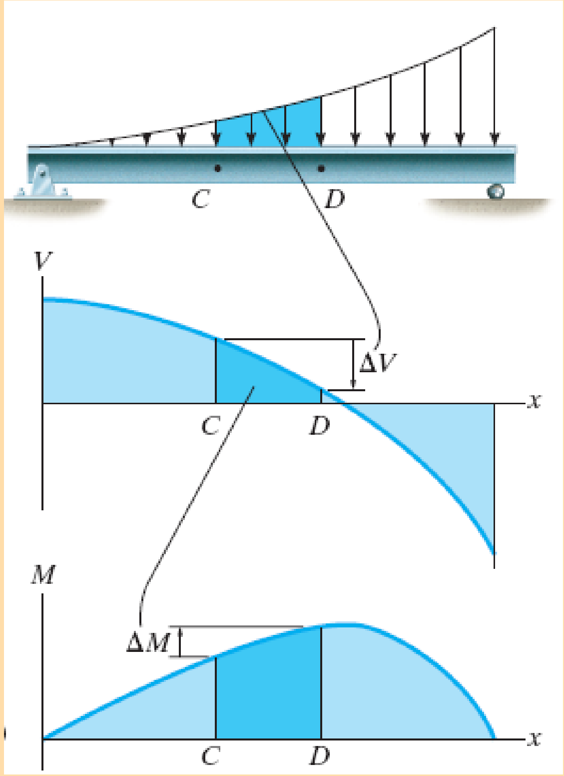
(b)

REGION OF DISTRIBUTED LOAD

- We can integrate these areas between any two points to get change in shear and moment.

Change in shear $\Delta V = -\int w(x)dx$ Area under distributed loading

Change in moment $\Delta M = \int V(x)dx$ Area under shear diagram

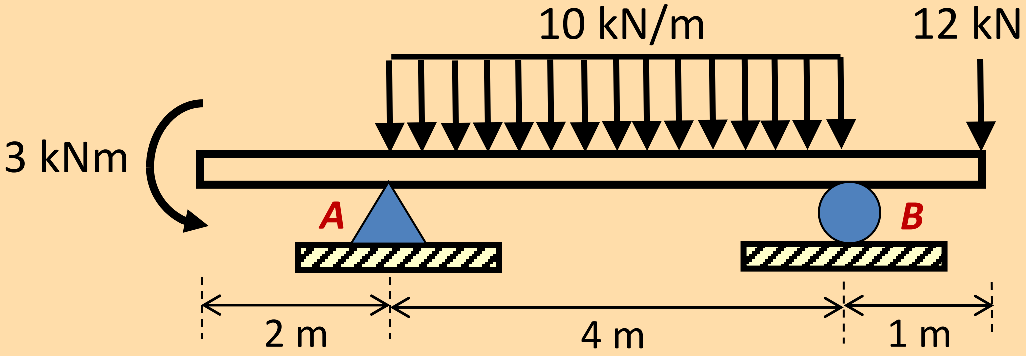


USEFUL TIPS…

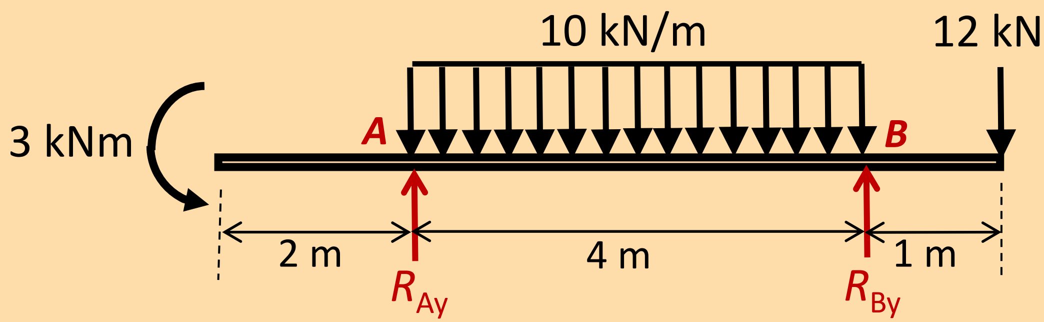
- Slope of bending moment always determined by the shape of shear force lines. The changes in slope (sagging or hogging also depends on the changes in shear force values)
- When shear force intersects BMD axis, there is a maximum moment
- When SF maximum, BM minimum and vice versa
- SFD and BMD always start and end with zero values (unless at the point where there is a moment/couple)
- When a moment/couple acting:
  - Clockwise (↓) (+), Anticlockwise (↑) (-)

EXAMPLE 9

Calculate the shear force and bending moment for the beam subjected to the loads as shown in the figure, then draw the shear force diagram (SFD) and bending moment diagram (BMD).



EXAMPLE 9 – Solution



By taking the moment at A:

$\Sigma M_A = 0$

$-R_{By} \times 4 - 3 + 10 \times 4 \times 4/2 + 12 \times 5 = 0$

$R_{By} = 34.25$ kN

$\Sigma F_y = 0$

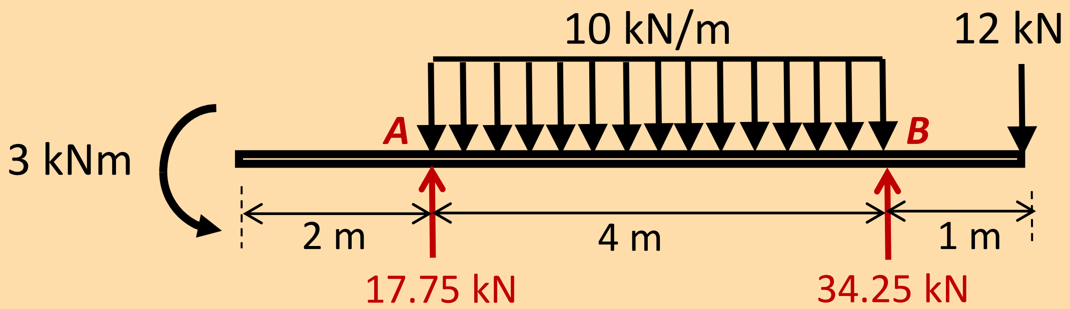
$R_{Ay} + R_{By} = 10 \times 4 + 12$

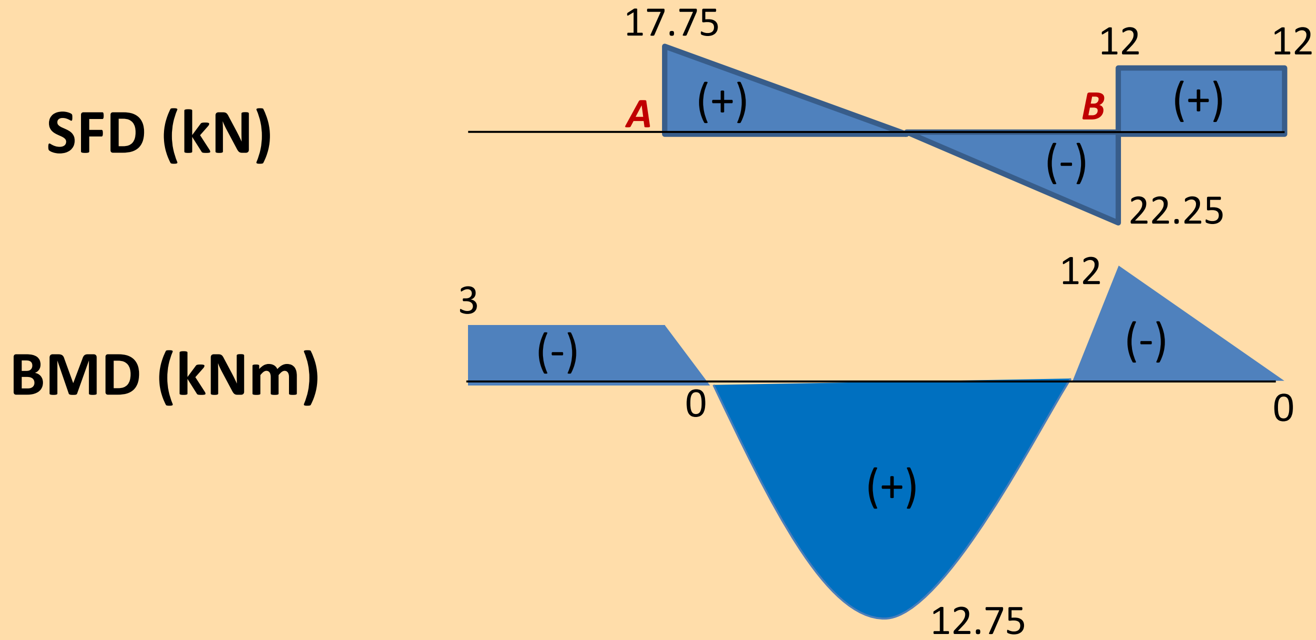
$R_{Ay} = 17.75$ kN

$\Sigma F_x = 0$

$R_{Ax} = 0$

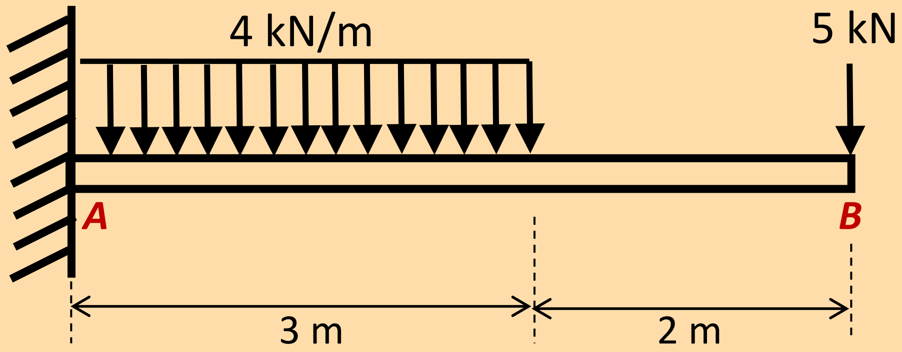
EXAMPLE 9 – Solution



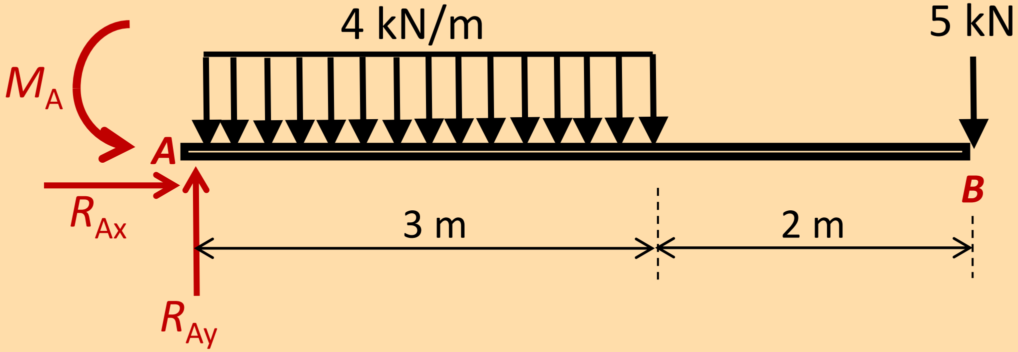


EXAMPLE 10

Calculate the shear force and bending moment for the beam subjected to the loads as shown in the figure, then draw the shear force diagram (SFD) and bending moment diagram (BMD).



EXAMPLE 10 – Solution



By taking the moment at A:

$\Sigma M_A = 0$

$-M_A + 4 \times 3 \times 3/2 + 5 \times 5 = 0$

$M_A = 43$ kNm

$\Sigma F_y = 0$

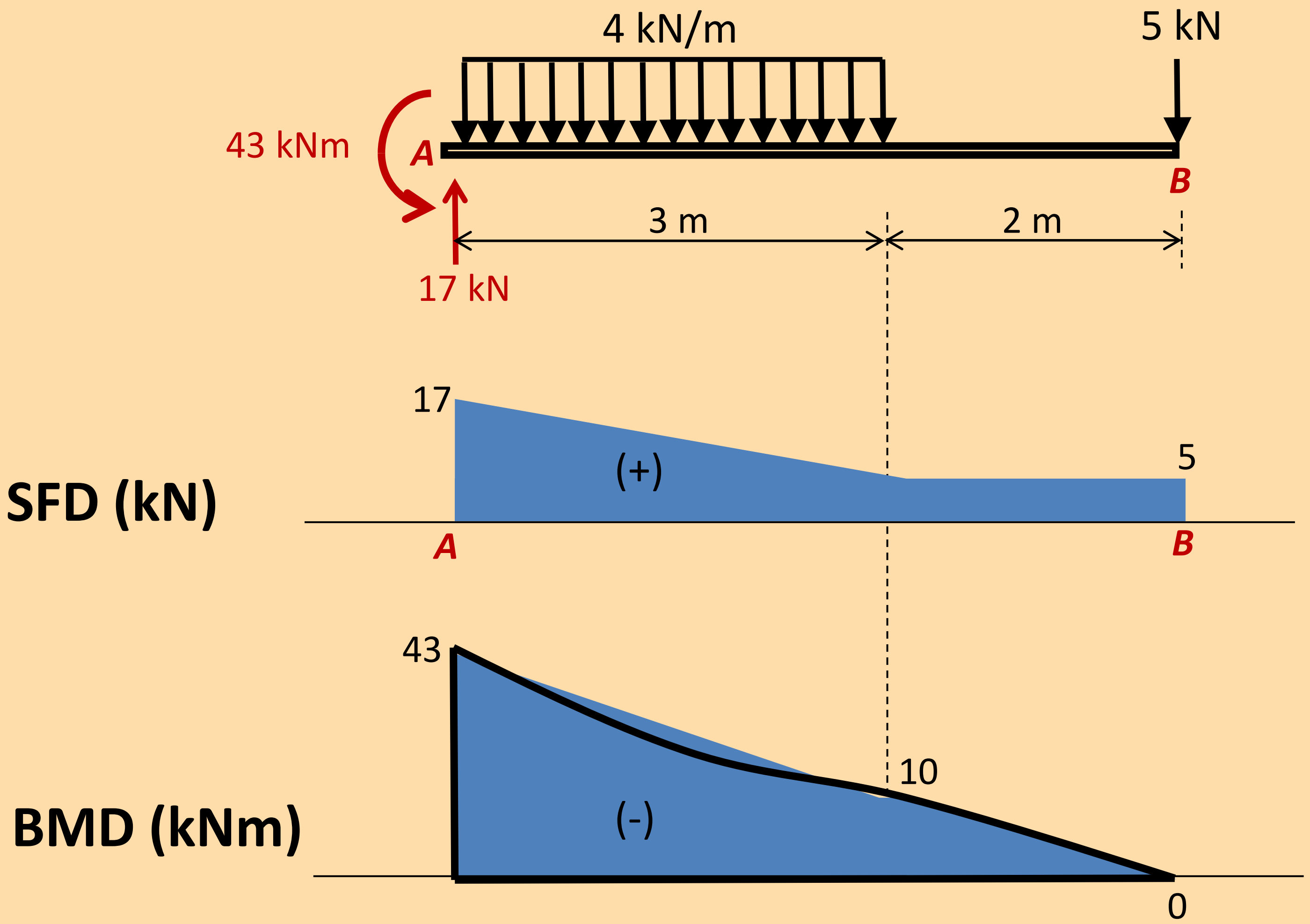
$R_{Ay} = 4 \times 3 + 5$

$R_{Ay} = 17$ kN

$\Sigma F_x = 0$

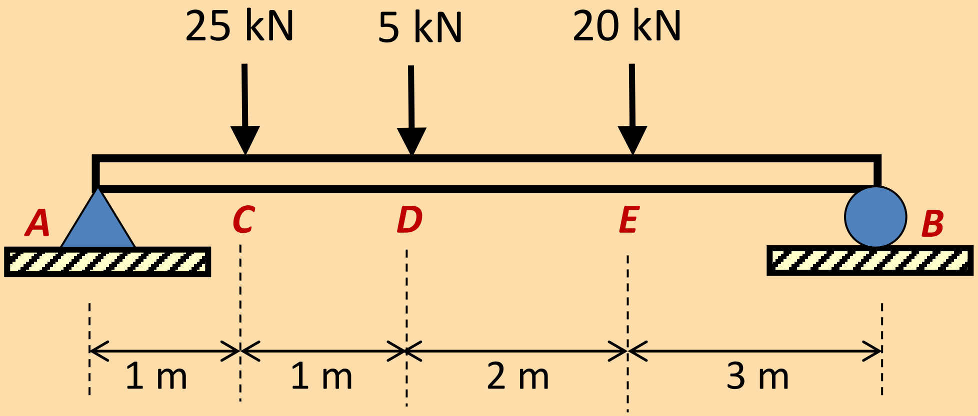
$R_{Ax} = 0$

EXAMPLE 10 – Solution

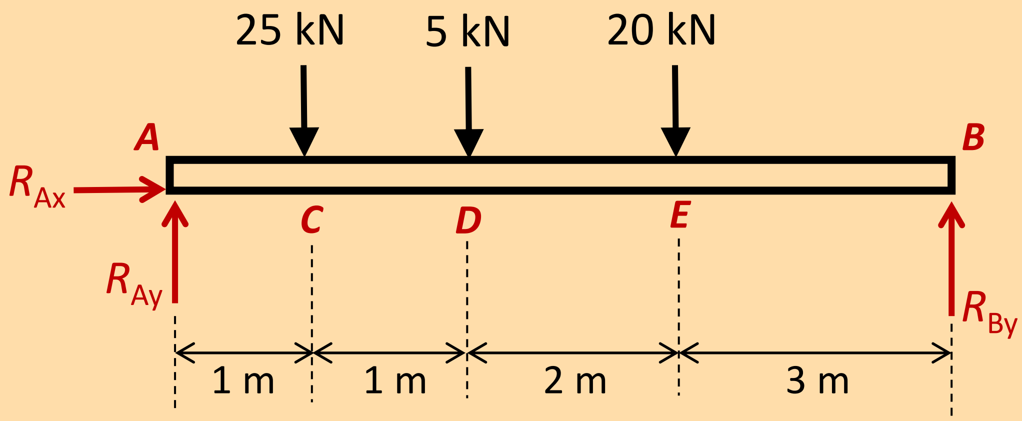


EXAMPLE 11

Calculate the shear force and bending moment for the beam subjected to the loads as shown in the figure, then draw the shear force diagram (SFD) and bending moment diagram (BMD).



EXAMPLE 11 – Solution



By taking the moment at A:

$\Sigma M_A = 0$

$25 \times 1 + 5 \times 2 + 20 \times 4 - R_{By} \times 7 = 0$

$R_{By} = 16.43$ kN

$\Sigma F_y = 0$

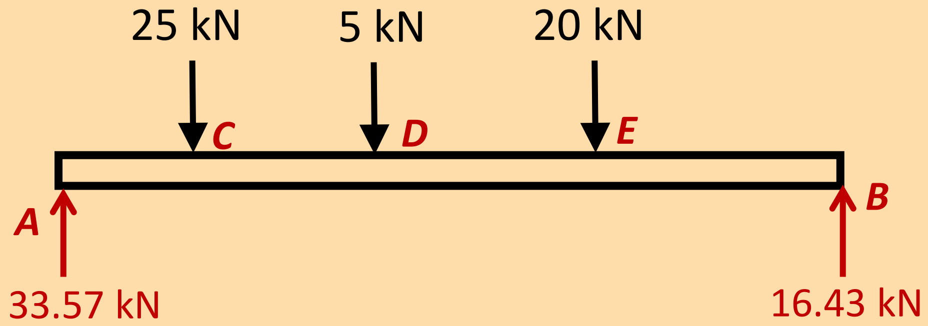
$R_{Ay} + R_{By} = 25 + 5 + 20$

$R_{Ay} = 33.57$ kN

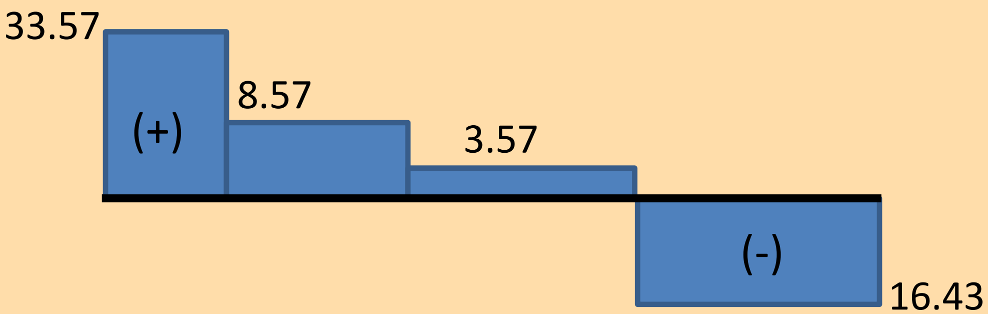
$\Sigma F_x = 0$

$R_{Ax} = 0$

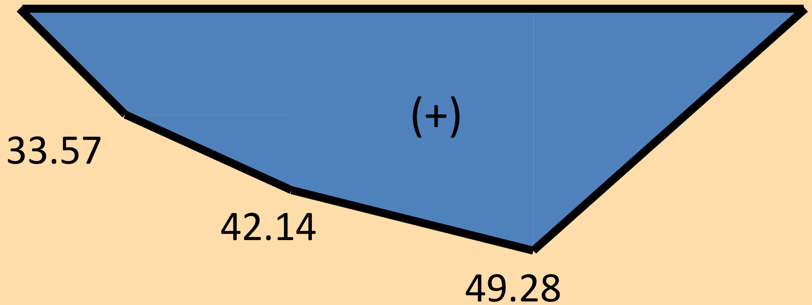
EXAMPLE 11 – Solution



SFD (kN)

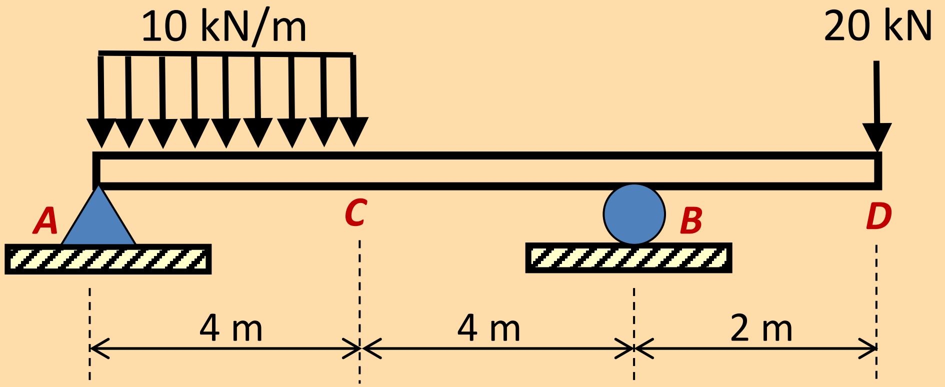


BMD (kNm)

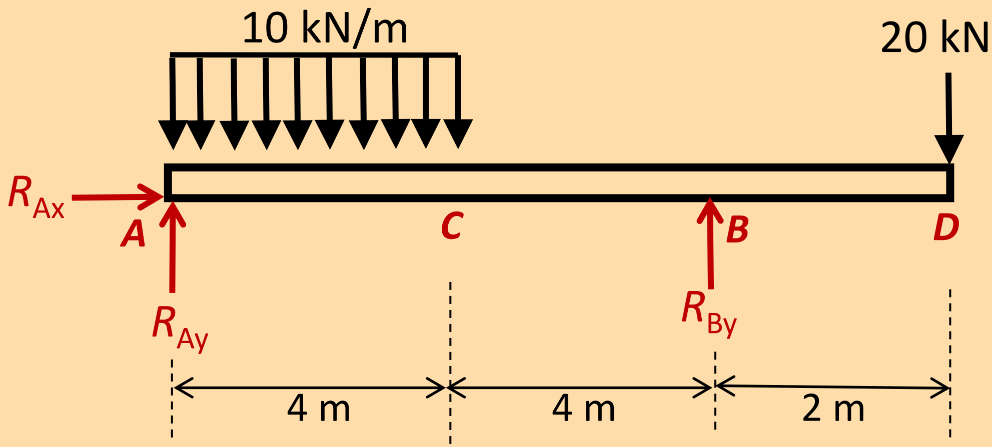


EXAMPLE 12

Calculate the shear force and bending moment for the beam subjected to the loads as shown in the figure, then draw the shear force diagram (SFD) and bending moment diagram (BMD).



EXAMPLE 12 – Solution



By taking the moment at A:

$\Sigma M_A = 0$

$10 \times 4 \times 2 + 20 \times 10 - R_{By} \times 8 = 0$

$R_{By} = 35$ kN

$\Sigma F_y = 0$

$R_{Ay} + R_{By} = 10 \times 4 + 20$

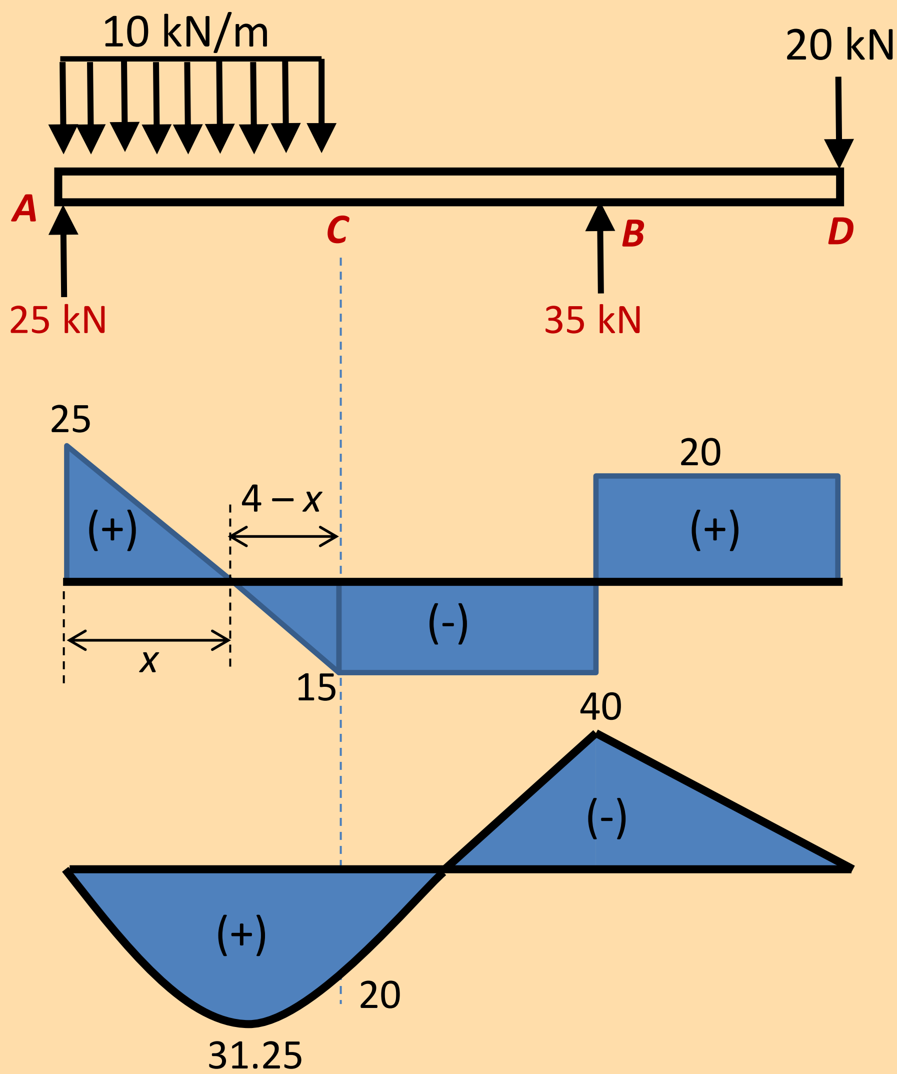
$R_{Ay} = 60 - 35$

$R_{Ay} = 25$ kN

$\Sigma F_x = 0$

$R_{Ax} = 0$

EXAMPLE 12 – Solution



SFD (kN)

BMD (kNm)

$$\frac{x}{25} = \frac{4-x}{15}$$

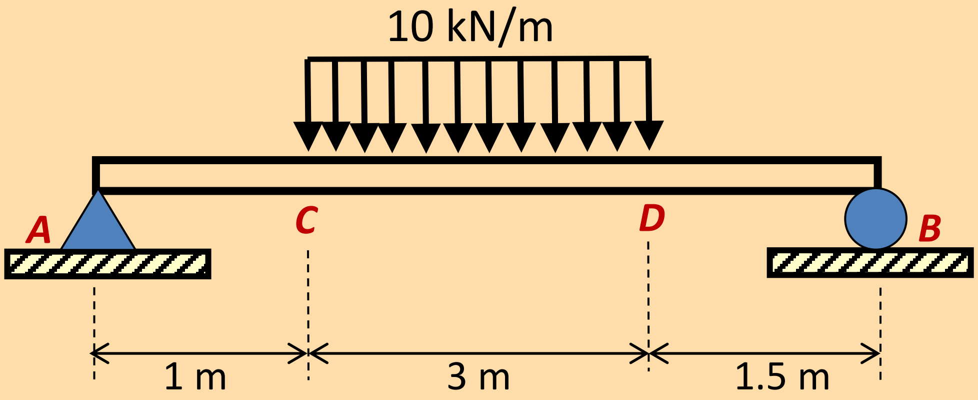
$$15x = 100 - 25x$$

$$40x = 100$$

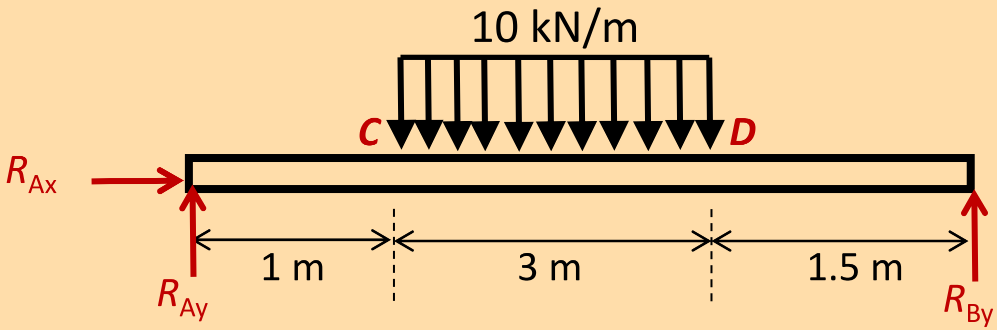
$$x = 2.5$$

EXAMPLE 13

Calculate the shear force and bending moment for the beam subjected to the loads as shown in the figure, then draw the shear force diagram (SFD) and bending moment diagram (BMD).



EXAMPLE 13 – Solution



By taking the moment at A:

$\Sigma M_A = 0$

$10 \times 3 \times 2.5 - R_{By} \times 5.5 = 0$

$R_{By} = 13.64$ kN

$\Sigma F_y = 0$

$R_{Ay} + R_{By} = 10 \times 3$

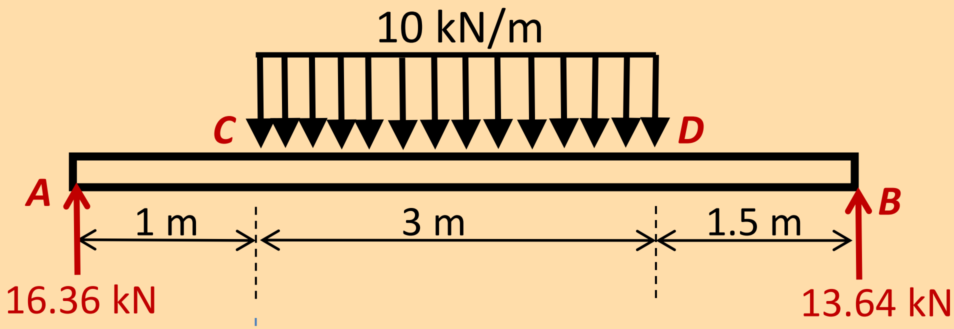
$R_{Ay} = 30 - 13.64$

$R_{Ay} = 16.36$ kN

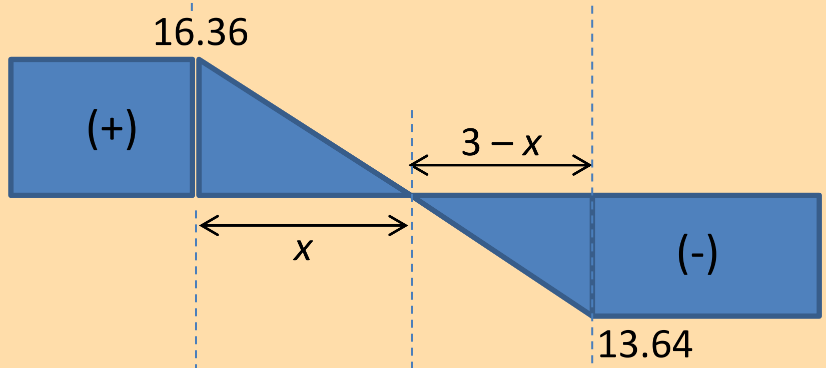
$\Sigma F_x = 0$

$R_{Ax} = 0$

EXAMPLE 13 – Solution



SFD (kN)



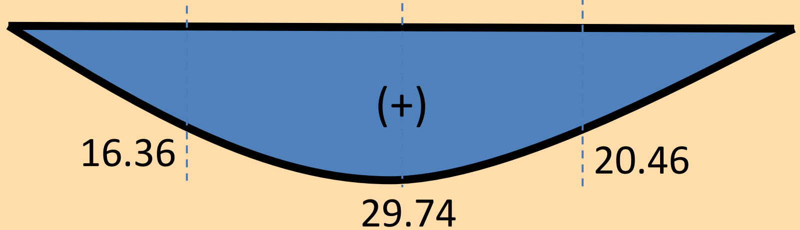
$$\frac{x}{16.36} = \frac{3-x}{13.64}$$

$$13.64x = 49.08 - 16.36x$$

$$30x = 49.08$$

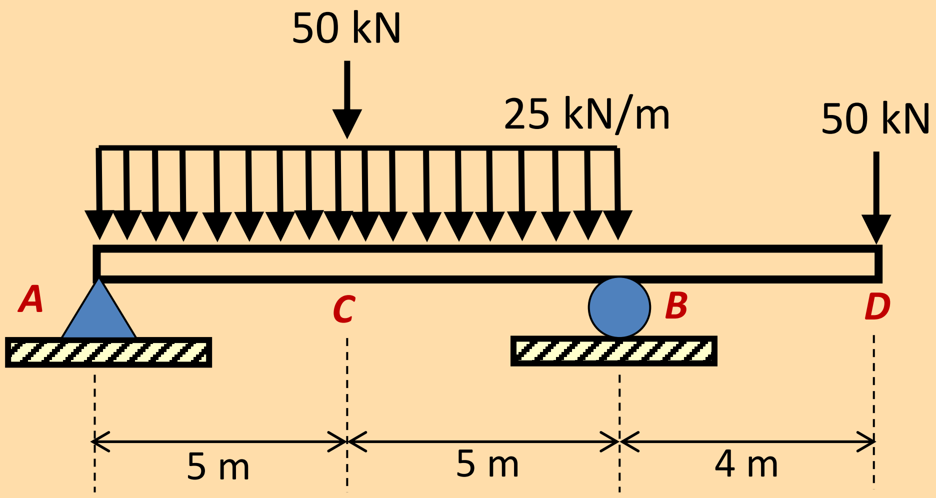
$$x = 1.636$$

BMD (kNm)

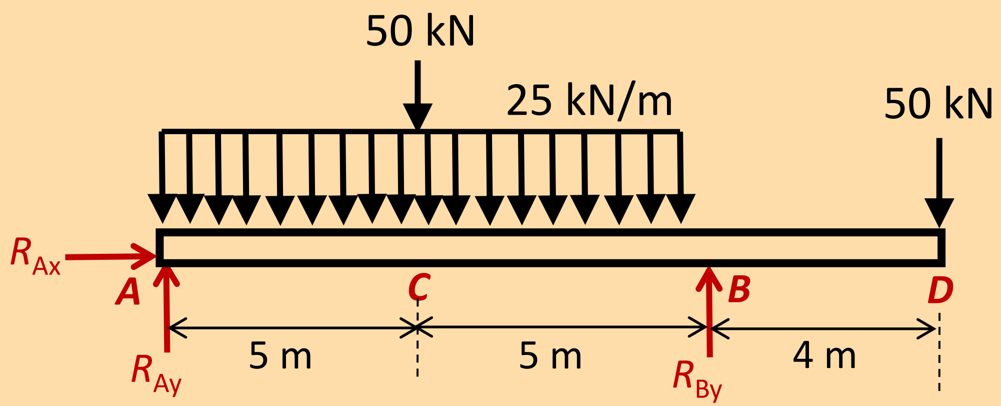


EXAMPLE 14

Calculate the shear force and bending moment for the beam subjected to the loads as shown in the figure, then draw the shear force diagram (SFD) and bending moment diagram (BMD).



EXAMPLE 14 – Solution



By taking the moment at A:

$\Sigma M_A = 0$

$25 \times 10 \times 5 + 50 \times 5 + 50 \times 14 - R_{By} \times 10 = 0$

$R_{By} = 220$ kN

$\Sigma F_y = 0$

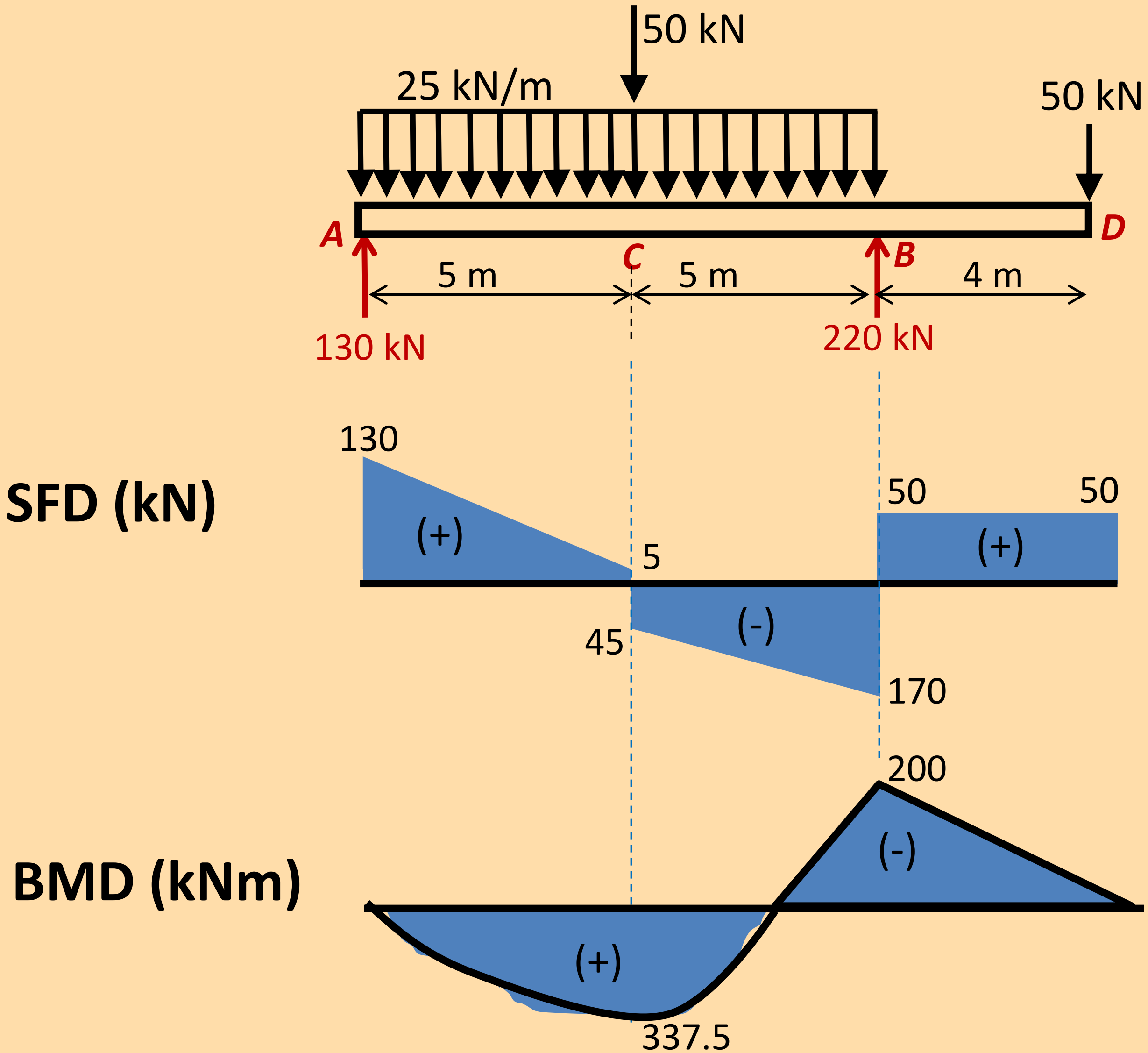
$R_{Ay} + R_{By} = 25 \times 10 + 50 + 50$

$R_{Ay} = 130$ kN

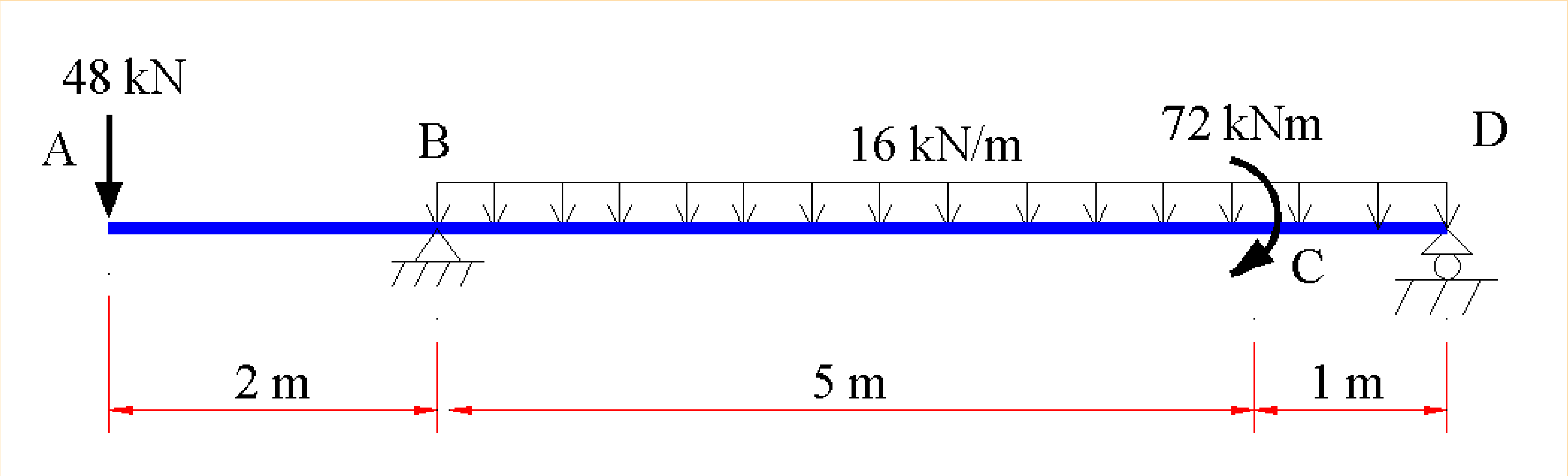
$\Sigma F_x = 0$

$R_{Ax} = 0$

EXAMPLE 14 – Solution



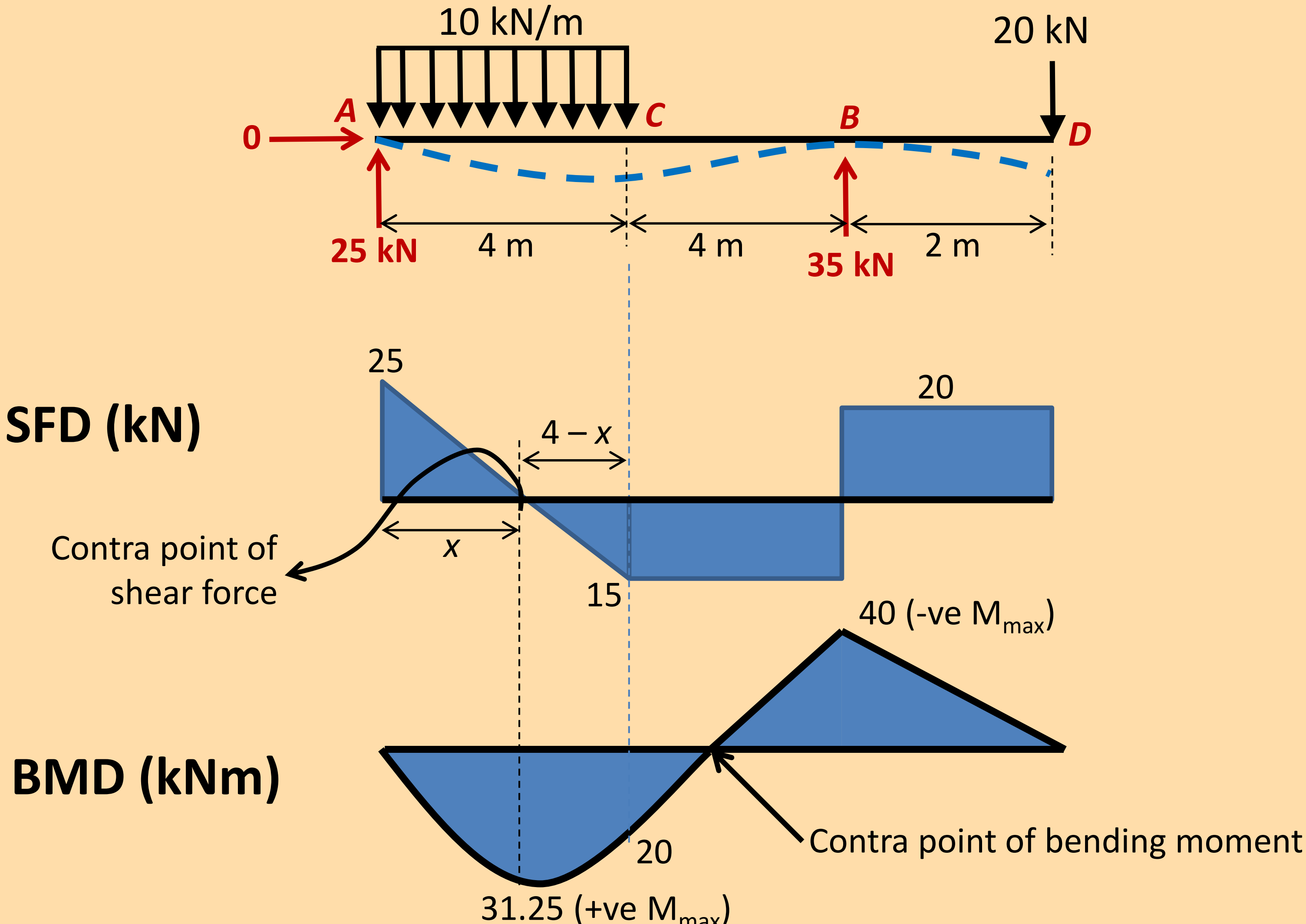
CLASS EXERCISE



CONTRA POINT OF SHEAR FORCE & BENDING MOMENT

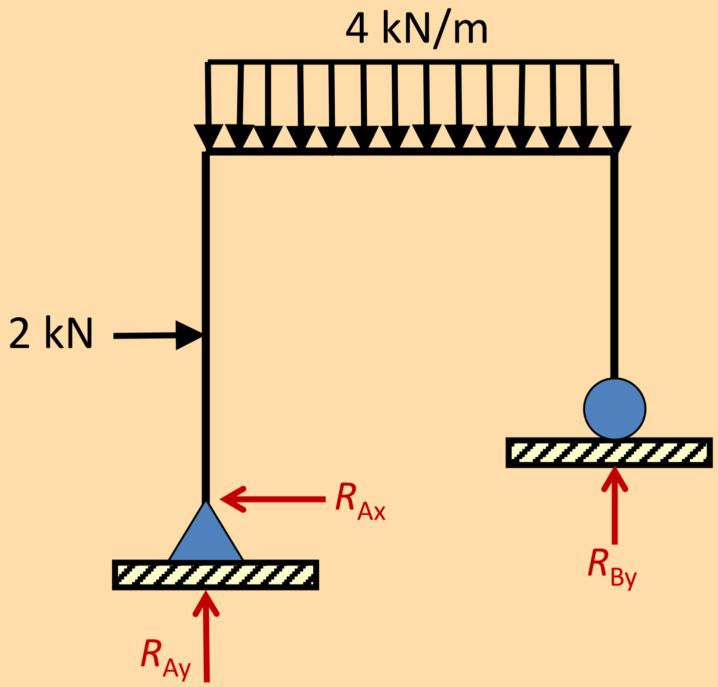
- Contra point is a place where positive shear force/bending moment shifting to the negative region or vice-versa.
- Contra point for shear: ***V = 0***
- Contra point for moment: ***M = 0***
- When shear force is ***zero***, the moment is ***maximum***.
- ***Maximum shear force*** usually occur at the ***support / concentrated load***.

CONTRA POINT OF SHEAR FORCE & BENDING MOMENT



STATICALLY DETERMINATE FRAMES

- For a frame to be statically determinate, the number of unknown (reactions) must be able to solved using the equations of equilibrium.



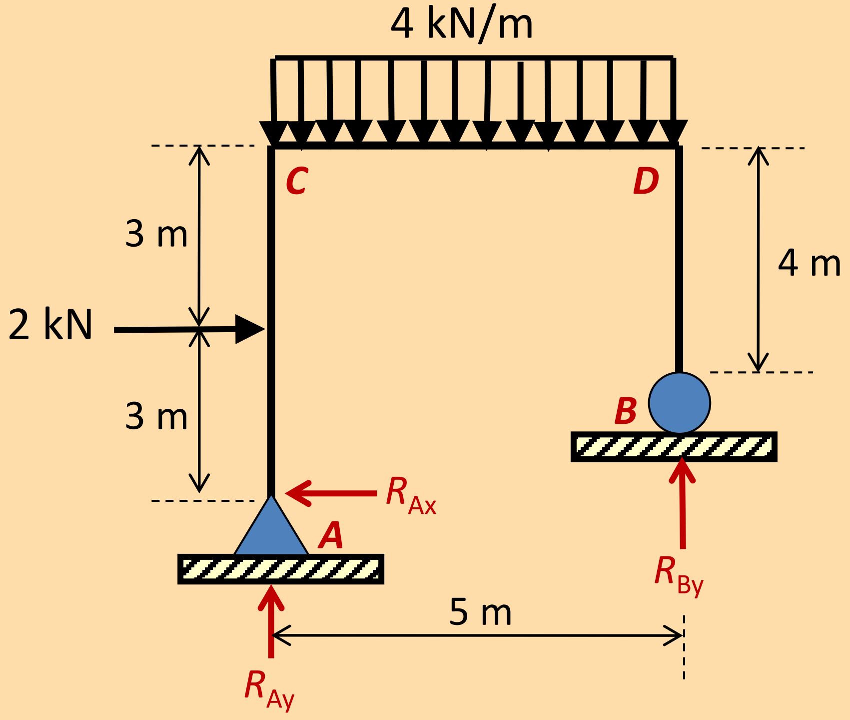
$$\Sigma M_A = 0$$

$$\Sigma F_y = 0$$

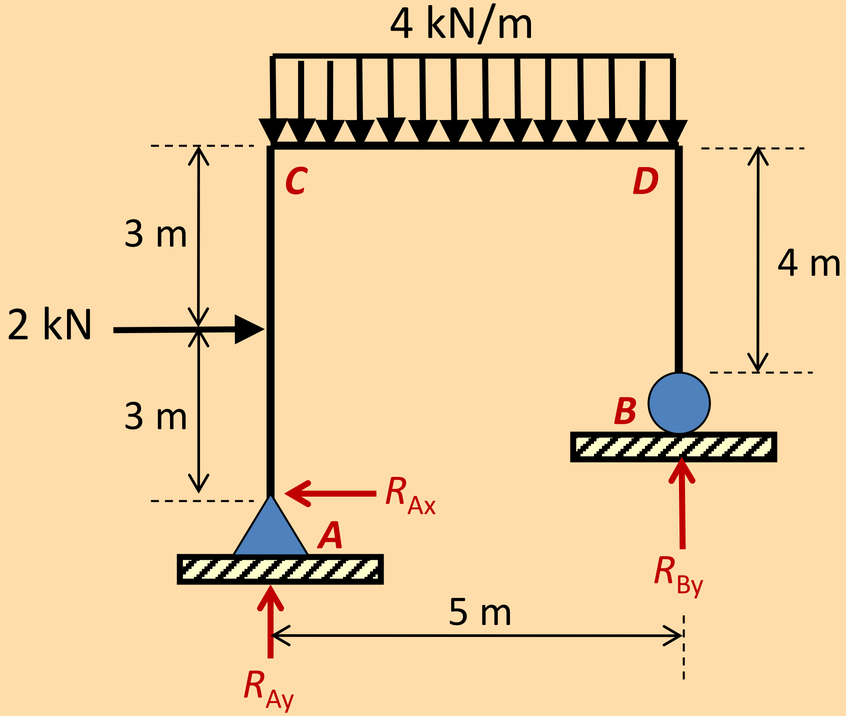
$$\Sigma F_x = 0$$

EXAMPLE 15

Calculate the shear force and bending moment for the frame subjected to the loads as shown in the figure, then draw the shear force diagram (SFD) and bending moment diagram (BMD).



EXAMPLE 15 – Solution



$\Sigma M_A = 0$

$4 \times 5 \times 2.5 + 2 \times 3 - R_{By} \times 5 = 0$

$R_{By} = 11.2$ kN

$\Sigma F_y = 0$

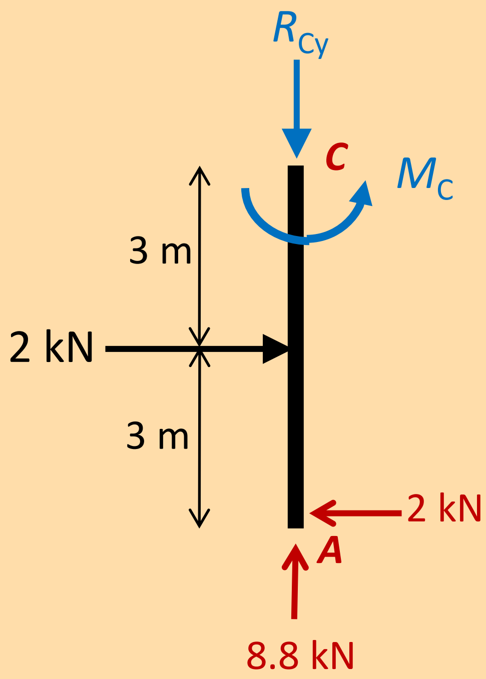
$R_{Ay} + R_{By} = 4 \times 5$

$R_{Ay} = 8.8$ kN

$\Sigma F_x = 0$

$R_{Ax} = 2$ kN

EXAMPLE 15 – Solution



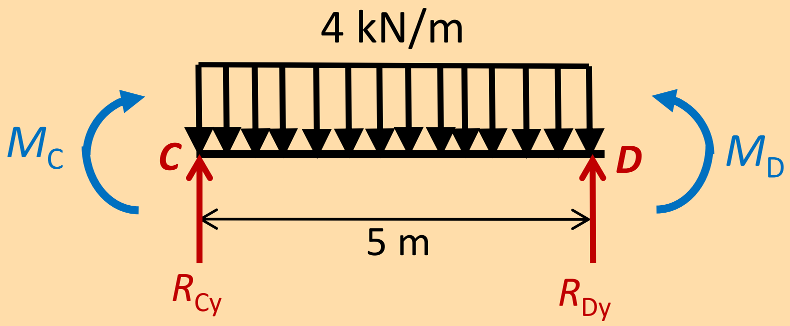
$\Sigma M_A = 0$: $2 \times 3 - M_C = 0$

$\therefore\ M_C = 6$ kNm

$\Sigma F_y = 0$

$\therefore\ R_{cy} = 8.8$ kN

EXAMPLE 15 – Solution



$\Sigma F_y = 0$: $R_{Cy} + R_{Dy} = 4 \times 5$

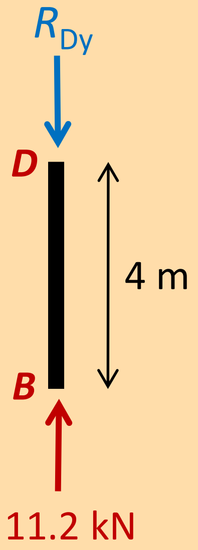
$R_{Dy} = 20 - 8.8 = 11.2$ kN

$\Sigma M_C = 0$:

$M_C + 4 \times 5 \times 2.5 - R_{Dy} \times 5 - M_D = 0$

$M_D = 0$ kNm

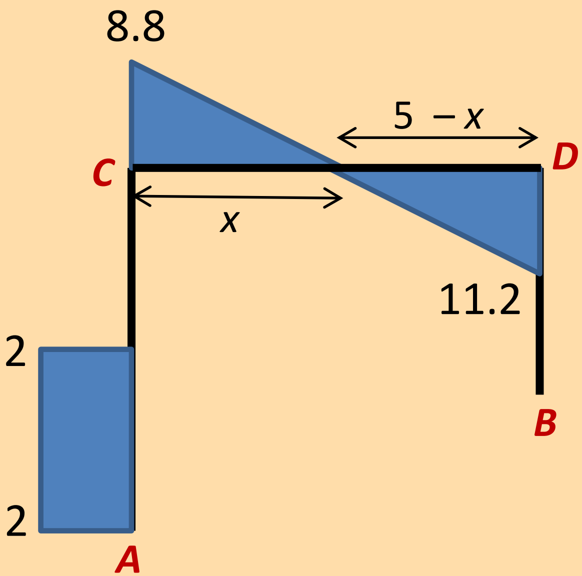
EXAMPLE 15 – Solution



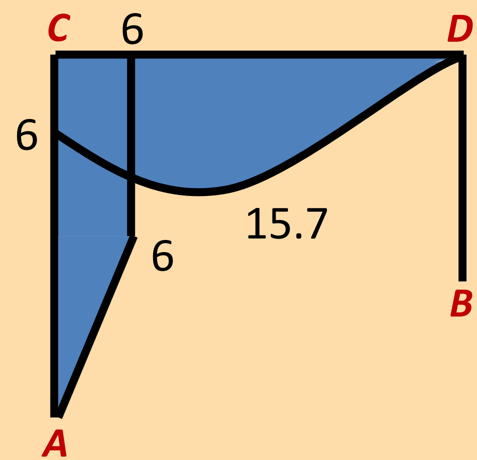
$\Sigma F_y = 0$:

$R_{Dy} = 11.2$ kN

EXAMPLE 15 – Solution



SFD (kN)



BMD (kNm)

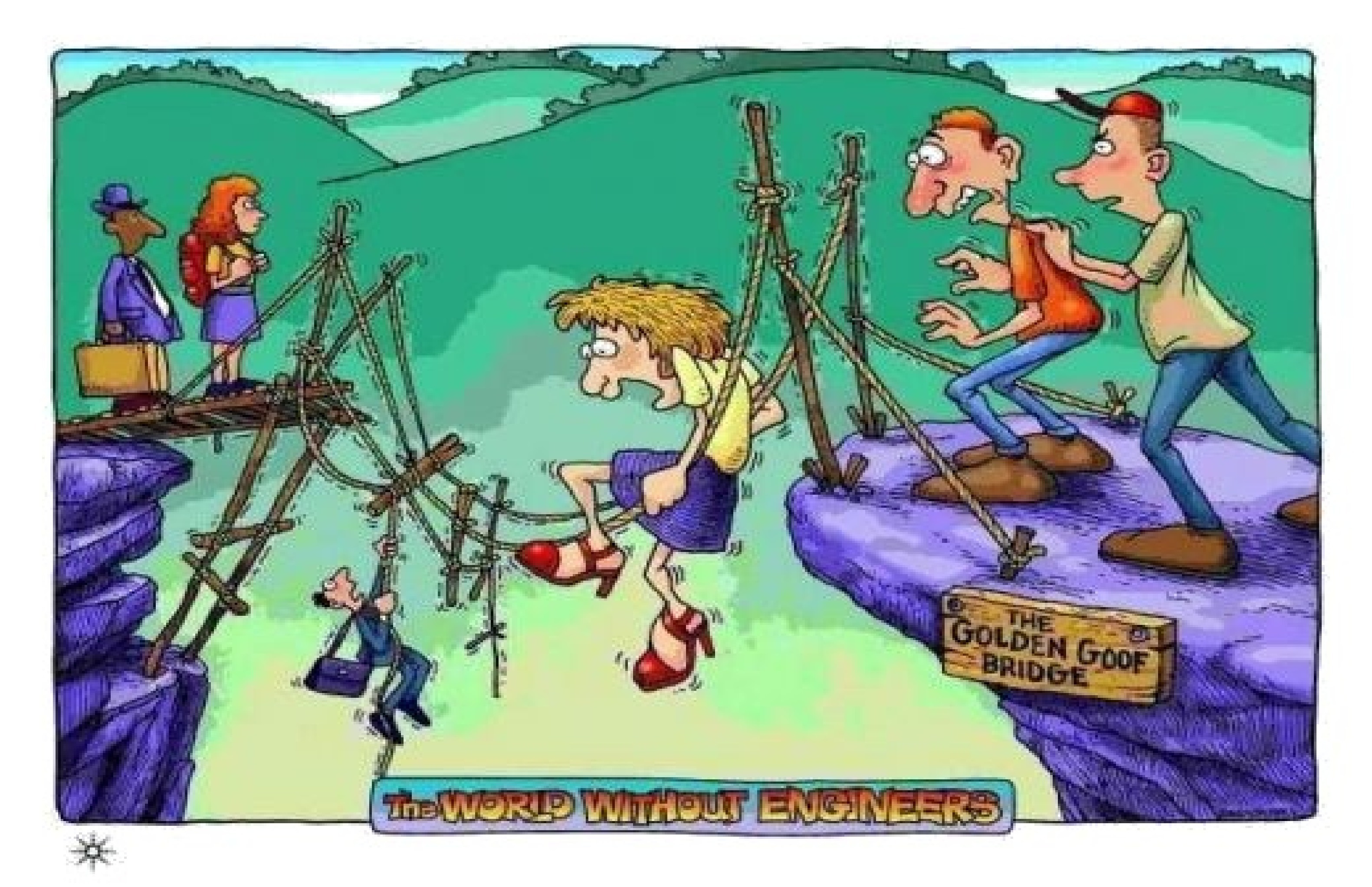
$$\frac{x}{8.8} = \frac{5-x}{11.2}$$

$$11.2x = 44 - 8.8x$$

$$20x = 44$$

$$x = 2.2$$

$M_{max} = 8.8 \times 2.2 \times 0.5 + 6 = 15.7$ kNm


THE GOLDEN GOOF BRIDGE
The WORLD WITHOUT ENGINEERS