Chapter 2:

Solving Equations and Inequalities

Unit 2: Vocabulary

1)	equation	
2)	solution (to an equation)	
3)	identity	
4)	contradiction	
5)	inequality	
6)	solution (to an inequality)	
7)	interval	
8)	boundary point	
9)	open interval	
10)	closed interval	
11)	half-open interval	
12)	compound inequality	
13)	conjunction	
14)	disjunction	

Day 1: Solving Equations with Simplification

A-REI.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

Warm-Up Solve for the value of the variable.

1) $4y + 2 = 38$ 2) $87 = 13 - 2x$ 3) $4 - c = 10$

The Solution to an Equation

A solution to an equation is a value or values that make the equation true.

Example Show that 4 is a **solution** to the equation below.
Show that 5 is NOT a **solution**.

$n + 6 = 10$	$n + 6 = 10$

Why is 4 a **solution** to the equation $n + 6 = 10$? ______________________________

____

Why is 5 not a **solution**? ______________________________

____

Exercise Determine if the given number is a **solution** to the equation.

1. $4n + 1 = 17; n = 4$ 2. $2 - 6a = 4; a = 1$

Model Problem #1: Distributive Property and Combining Like Terms

Solve and check: $3(a - 5) + 2a = 15$ Check:

Using the Distributive Property:	Combining Like Terms:

Exercise

Solve and check:

1) $14 - 3c + 7c = 94$

Check:

2) $18 = -6x + 4(2x + 3)$

Check:

Notes:	Model Problem #2A: Distributing a Negative
	$5t - 2(t - 5) = 19$

Notes:	Model Problem #2B: Distributing the -1 $7r - (6r - 5) = 7$

Check for Understanding

Find the solution set: $8b - 4(b - 2) = 24$

Lesson Summary

- Use the distributive property to eliminate parentheses.

  Watch for double negatives. (-)(-) = (+)

- Combine like terms. Like terms contain the same variables, or no variables at all.
- Use the opposite operation to eliminate a term.

Challenge!

Solve for x: $\frac{3}{5}(x+2) = x - 4$

Independent Practice/Homework Solve and check.

1) $5(x + 2) = 20$

2) $-4x - 6x = -20$

3) $30 = 2(10 - y)$

4) $5(3c - 2) + 8 = 43$

5) $-2(p - 8) = 14$

6) $12 = -4(-6x - 3)$

7) $-3(4x + 3) + 4(6x + 1) = 43$

8) $-5 = 6v + 5 - v$

9) $y - (1.4 - y) = 83.6$

10) $3b - (2b - 8) = 38$

11) $7(9 - a) - 3(a - 4) = 3$

12) $2(b - 4) - 4(2b + 1) = 0$

Day 2: Solving Equations with Variables on Both Sides

A-REI.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

Warm-Up

What is the **solution** to an equation? How do we know that 4 is a solution to $3x = 12$?

To solve an equation with variables on both sides, use inverse operations to "collect" variable terms on one side of the equation (and numbers on the other).

NOTE: Equations are often easier to solve when the variable has a positive coefficient.

Keep this in mind when deciding on which side to "collect" variable terms.

Notes:	Model Problem A $7x - 2 = 5x + 6$
Notes:	Model Problem B $2(2x + 1) = 3x$

Check for Understanding

$4a - 3 = 2a + 7$

Write about it: How do you know which way to move the variable term?

What are the steps in solving multi-step equations?	Model Problem C
1) 2) 3)	Solve $4 - 6a + 4a = -1 - 5(7 - 2a)$.

Check for Understanding Solve for x:

$$-5(1 - 5x) + 5(-8x - 2) = -4x - 8x$$

Identities and Contradictions

An **identity** is an equation that is true for all values of the variable. An equation that is an identity has infinitely many solutions. *Look for a number equaling itself.*

Example $2x + 6 = 2(x + 3)$

A **contradiction** is an equation that is not true for any value of the variable. It has no solutions.
Look for a false statement.

Example $x = x + 3$

Exercise Give the solution set to each equation.

$5p - 8 = 1 + 5p - 9$	$3(2v - 1) = 6v - 4$

Give an example of each type of equation. Explain your choice.

a) An equation that is always true

b) An equation that is never true

c) An equation is true only when x = 1

Independent Practice/Homework Solve for the value of the variable.

$5p - 14 = 8p + 4$	$p - 1 = 5p + 3p - 8$
$5n + 34 = -2(1 - 7n)$	$-18 - 6k = 6(1 + 3k)$

$2(4x-3)-8=4+2x$	$3n-5=-8(6+5n)$
$-3(4x+3)+4(6x+1)=43$	$-(1+7x)-6(-7-x)=36$

Give the solution set to each equation.

$4n+6-2n=2(n+3)$	$-v+5+6v=1+5v+3$

A-REI.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

Solve each equation.

1) $-3 + 6a - 3a = -6$

2) $-9 = 5x + 4x$

3) $-0.1x - 0.9 + 0.7x = 0.66$

4) $-3r - 5 = 2(-1 - r)$

5) $-2x + 2(3x + 1) = -3 + 5x$

6) $-21 - 4x = -4(x + 6)$

7) $-2(5k+5)=-6k-14$

8) $-2p-12=-5(-6p-4)$

9) $5x+26=-5+5(x+3)$

10) $-5b+b=-4(3-2b)+4(6b+3)$

11) $2(-2a-1)+3a=-(5a-2)+2a$

12) $-(2-5n)-2(n+4)=n+4+3n-4$

Answers to

1) $\{-1\}$ 2) $\{-1\}$ 3) $\{2.6\}$ 4) $\{-3\}$

5) $\{5\}$ 6) No solution. 7) $\{1\}$ 8) $\{-1\}$

9) No solution. 10) $\{0\}$ 11) $\{2\}$ 12) $\{-10\}$

Day 4: Solving and Representing Inequalities

A-REI.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

<u>Warm-Up</u> Solve for the value of the variable.

$$5(r-1)=2(r-4)-6$$

An <u>inequality</u> is any statement that two quantities are not equal.

<u>Examples</u>

x is greater than 5	$x > 5$
x is less than -4	$x < -4$
x is greater than or equal to 7	$x \geq 7$
x is less than or equal to -2	$x \leq -2$

<u>Note:</u> If an inequality is inverted, we turn it around so that we can read it with the *variable first*.

5 > 3 is the same as saying 3 < 5.
3 > x is the same as saying x < 3.

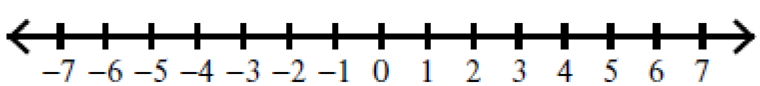
A **solution to an inequality** is any value that makes the inequality true.

<u>Example</u>

1) List 3 solutions and 3 non-solutions to the inequality x < 5.

Solutions: ___________________________ Non-solutions: _____________________________

2) Graph the solution set of x < 5:



3) How many solutions are there to the inequality x < 5? How is this shown on the graph?
____

4) Are all of them integers? If not, name a non-integer solution: ________________

5) Is 5 a solution to the inequality x < 5? Why or why not? How is this shown on the graph?

6) What is the lowest solution in this set? What is the highest? Would your answer change if the inequality were x ≤ 5?

Interval Notation

Besides with a graph, the solutions to an inequality can also be represented in **interval notation**.

An interval is a space between points, called endpoints. **Interval notation** represents a set of numbers using the endpoints and indicates whether the endpoints themselves are included in a set.

An *open interval* does not include the endpoints. An open interval is indicated by parentheses: ( )

A *closed interval* does include the endpoints. A closed interval is indicated by square brackets: [ ]

An interval can also be half-open, including the endpoints on only one side: ( ] or [ )

When there is no endpoint or one or more sides of an interval, we use the symbols ∞ and – ∞.

The symbol ∞ means there is no highest number in the interval. The symbol -∞ means there is no lowest number in the interval.

How to Write a Solution Set in Interval Notation (using the graph)

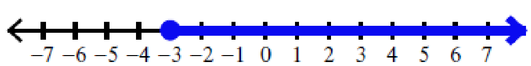
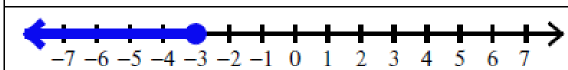
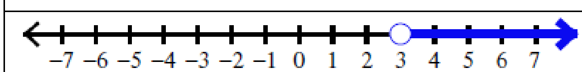
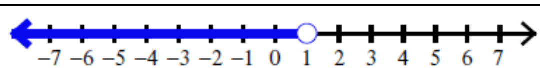
1) Write the lowest and highest value in the solution set (or boundary points) as an ordered pair.

   If it has a left arrow (←), write $-\infty$.
   If it has a right arrow (→), write ∞.

2) Write the correct symbol for the boundary.

   If the interval is closed on that side (•), use [ ].
   If the interval is open on that side (°.) use (, $\pm\infty$) or

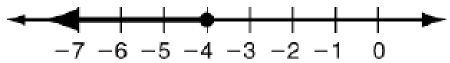
Examples

Graph	*Interval Notation*	*Inequality*
		
		
		
		

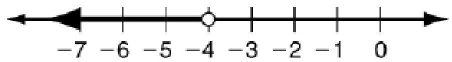
Exercise

Write the inequality indicated by each graph. Then write it in interval notation.

____________________ ____________________



____________________ ____________________



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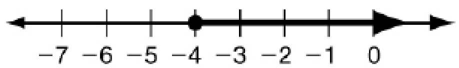


____________________ ____________________

Solving Inequalities

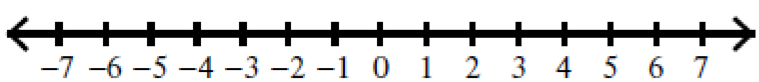
Inequalities are solved just like equations, with a few additional rules to remember.

> Rule #1: Try to collect variables on the left. If this is not possible, the inequality must be inverted before attempting to read it or graph it.

Example 1. $5(2 - r) \geq 3(r - 2)$

Solution Set: ________________________________ Interval Notation: ______________

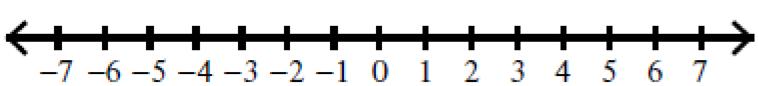
Graph:



Example 2. $-5 \leq -3 + y$

Solution Set: ________________________________ Interval Notation: ______________

Graph:



Rule #2: If you multiply or divide both sides by a negative number, the inequality sign flips.
* note: this is not the same as #1

Example 3. Which value of x is in the solution set of the inequality $-2(x-5) < 4$?

1 0
2 2
3 3
4 5

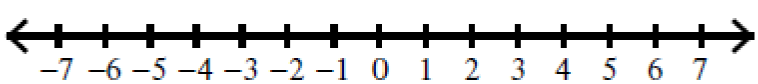
Rule #3: When cross-multiplying a proportion, keep the variable on the same side it starts on. Only negatives on the bottom count for a switch.

Example 4. $4 \leq \frac{k-5}{-2}$

Solution Set: ______________________________ Interval Notation: ____________

Choose a number in the solution set: _______________ and test it below:

Graph:

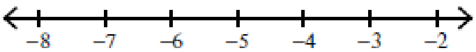


Independent Practice/Homework   Solve for the value of the variable. Express in each notation shown. Graph.

1. $-p+6p \leq 4+6p$

Inequality: ___________

Interval: _____________



2. $5+4x \geq x+8$

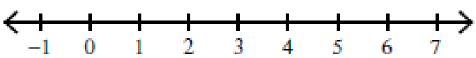
Inequality: ___________

Interval: _____________

3. $4k-4-3k > 13-7k-1+8$

Inequality: ___________

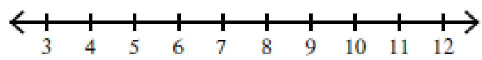
Interval: _____________



4. $r-7 > 9-r$

Inequality: ___________

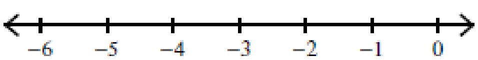
Interval: _____________



5. $6+2x \leq 12+8x-3x$

Inequality: ___________

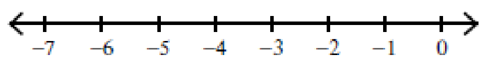
Interval: _____________



6. $-30+5x > 4(6+8x)$

Inequality: ___________

Interval: _____________



7. $7-7(x-7) > -4+5x$

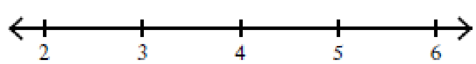
Inequality: ___________

Interval: _____________

8. $-3(2v-5) < -13+v$

Inequality: ___________

Interval: _____________



Day 5: More Practice with Solving Inequalities

A-REI.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

Warm-Up

State the largest negative integer in the solution of this inequality: $-2 < 7x - 2(x - 4)$

Identities and Contradictions

Identities and Contradictions

WORDS	ALGEBRA
Identity	
When solving an inequality, if you get a statement that is always true, the original inequality is an identity, and all real numbers are solutions.	$\begin{array}{r} 1 + x < 7 + x \\ \underline{-x} \quad \underline{-x} \\ 1 < 7 \checkmark \end{array}$
Contradiction	
When solving an inequality, if you get a false statement, the original inequality is a contradiction, and it has no solutions.	$\begin{array}{r} x + 7 < x \\ \underline{-x} \quad \underline{-x} \\ 7 < 0 \times \end{array}$
These properties are also true for inequalities that use the symbols $>$, $\geq$, and $\leq$.	

Model Problems

Solve for x. Tell the solution set.

$x + 5 \geq x + 3$	$2x + 6 \leq 5 + 2x$

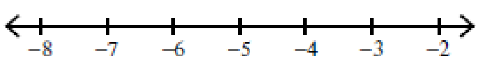
Exercise   Solve for the value of each variable. Tell the solution set.

$2(x-2) \le -2(1-x)$	$4(y+1) < 4y+2$

Independent Practice/Homework   Solve for the value of the variable. Express in each notation shown. Graph.

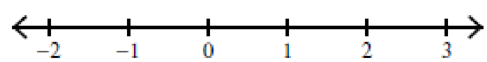
1. $-p+6p \le 4+6p$ Inequality: ___________

   Interval: _____________



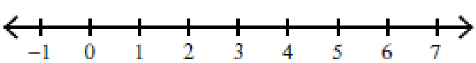
2. $5+4x \ge x+8$ Inequality: ___________

   Interval: _____________



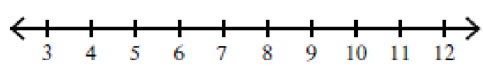
3. $4k-4-3k > 13-7k-1+8$ Inequality: ___________

   Interval: _____________



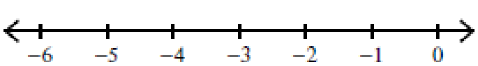
4. $r-7 > 9-r$ Inequality: ___________

   Interval: _____________



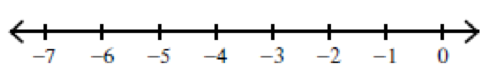
5. $6+2x \le 12+8x-3x$ Inequality: ___________

   Interval: _____________



6. $-30+5x > 4(6+8x)$ Inequality: ___________

   Interval: _____________



7. $7 - 7(x - 7) > -4 + 5x$

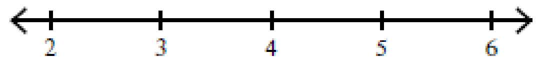
Inequality: ___________

Interval: _____________

8. $-3(2v - 5) < -13 + v$

Inequality: ___________

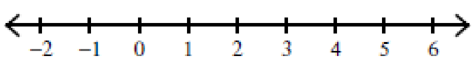
Interval: _____________



9. $-24 \leq 6(5b - 2) - 8(8b - 7)$

Inequality: ___________

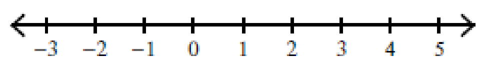
Interval: _____________



10. $7(1 - 5n) - (n + 3) \geq 4$

Inequality: ___________

Interval: _____________



1) $-3n - 4(n - 2) < -7(-2 + n)$

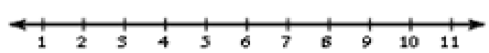
Inequality: _____

Interval: _________

2) $2(-8r - 4) - 1 > -8(1 + 2r)$

Inequality: __________

Interval: ___________



3) $33 + 32v \geq 8v + 3(8v + 7)$

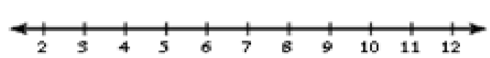
Inequality: __________

Interval: ___________

4) $-4(v - 6) < 3 - 4v$

Inequality: __________

Interval: ___________

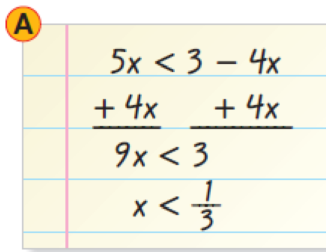


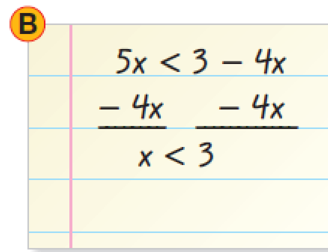
Day 7: Compound Inequalities

A-REI.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

Warm-Up

Two students solved the same inequality. Which solution is incorrect? Explain.



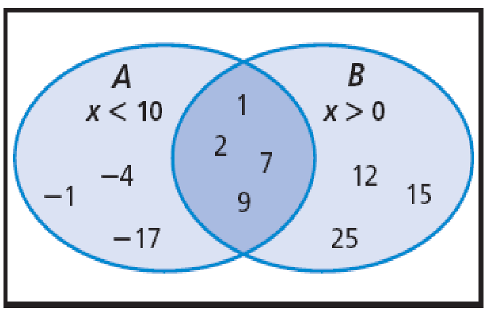


A <u>compound inequality</u> is a statement that combines two simple inequalities using AND or OR.

A statement that combines two inequalities using AND is called a <u>conjunction</u>.
A statement that combines two inequalities using OR is called a <u>disjunction</u>.

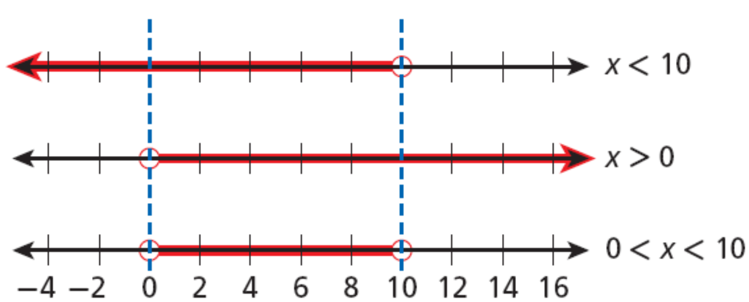
Conjunctions *"AND"*

In this diagram, oval *A* represents some integer solutions of $x < 10$ and oval *B* represents some integer solutions of $x > 0$. The overlapping region represents numbers that belong in **both** ovals. Those numbers are solutions of *both* $x < 10$ *and* $x > 0$.

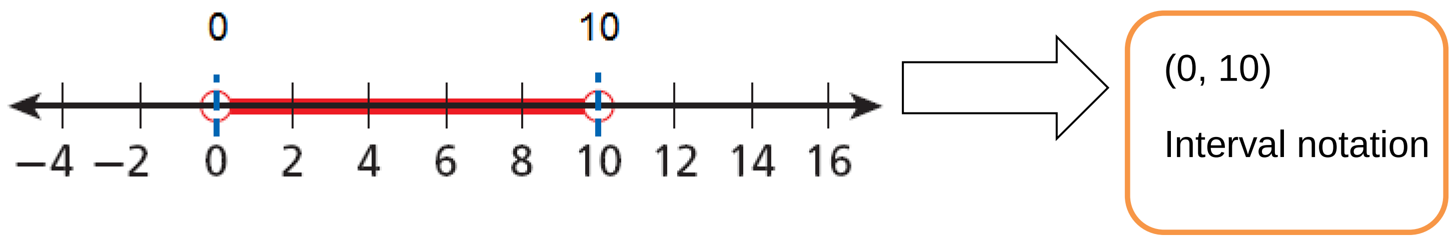


We can ***say*** this solution set in two ways:	We can *write* this solution set in three ways:
1) ______________________________	1) ______________________________
______________________________	2) ______________________________
2) ______________________________	3) ______________________________

You can graph the solutions of a compound inequality involving AND by using the idea of an overlapping region. The overlapping region is called the **<u>intersection</u>** and shows the numbers that are solutions of both inequalities.



We can also write the solution set in **interval notation**:



<u>Model Problems</u>

Consider the inequality $0 < x < 3$.

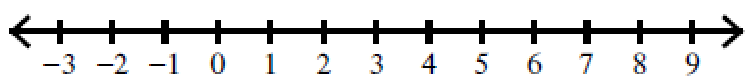
1) Write the inequality as a compound sentence, using the word “and.” ______________________________

2) Write the inequality as a compound sentence, using the symbol $\wedge$. ______________________________

3) Write the inequality in interval notation. ________________

4) Write out the inequality in words.

5) Graph the inequality.



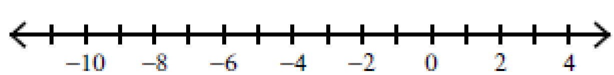
6) How many solutions are there to the inequality? Explain.

7) What are the largest and smallest possible values for x? Explain.

8) If the inequality is changed to $0 \leq x \leq 3$, what are the largest and smallest possible values for x?

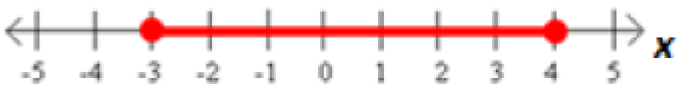
9) Solve and graph the compound inequality:

$$-20 \leq -6m - 2 \leq 58$$



Check for Understanding

1) Write a conjunction that is illustrated by the graph below in three ways:



As a single compound inequality: ________________________________

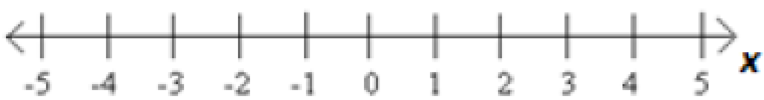
As a compound inequality using “and”: ________________________________

As a compound inequality using $\wedge$: ________________________________

In words: ____

2) Rewrite as a compound sentence and graph the sentence on a number line.

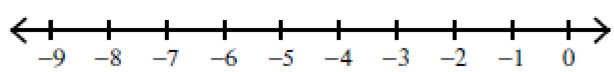
$$1 \leq x \leq 3$$



Compound sentence: ____

3) Solve and graph the solution set:

$$-53 < 9v + 1 < -26$$

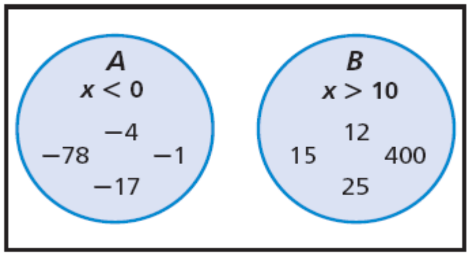


4) When a poll gives a **margin of error**, it states how far an actual measurement may be from an estimated one. For example, if a poll states that 35% of people would vote for a candidate with a margin of error of ±3%, it means that the true measure can be between 32% and 38%.

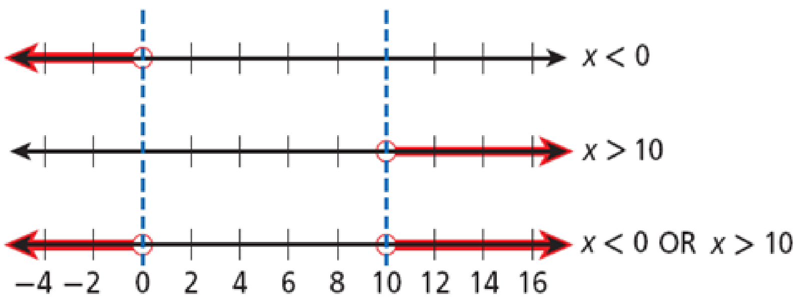
If a poll said that 56% of students in a class like chocolate ice cream with a ±2% margin of error, write an inequality describing what percent of students in that class could like chocolate ice cream.

Disjunctions

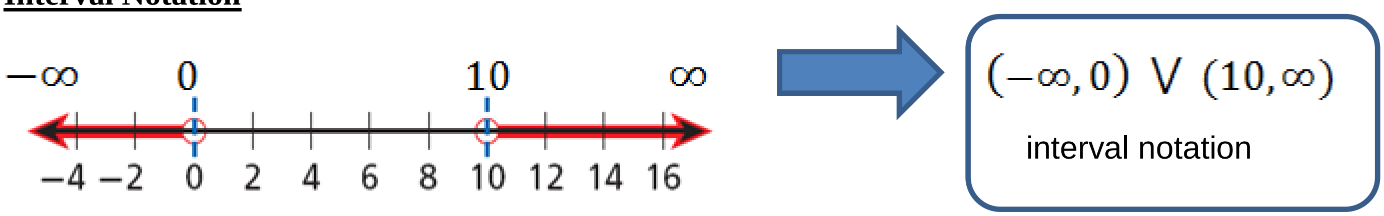
In this diagram, circle *A* represents some integer solutions of $x < 0$, and circle *B* represents some integer solutions of $x > 10$. The combined shaded regions represent numbers that are solutions of *either* $x < 0$ *or* $x > 10$.



You can graph the solutions of a compound inequality involving OR by using the idea of combining regions. The combine regions are called the **union** and show the numbers that are solutions of either inequality.

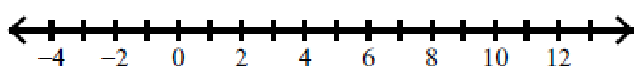


Interval Notation



Solving Disjunctions

$n - 10 \geq 0$ or $-5 + n < -6$

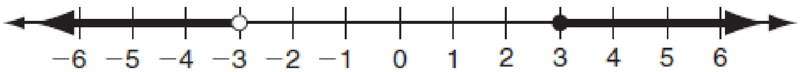


Exercise

1) Name a way the graph of a **conjunction** differs from the graph of a **disjunction**.

2) Do the graphs share any numbers in common? Explain.

3) Write the compound inequality shown by the graph below:



4) Solve and graph the solution set.
Express the solution set in interval notation: $x - 3 < -3$ OR $x - 3 \geq 3$

Interval notation: ______________________________

Summary

Conjunctions AND Statements		Disjunctions OR Statements
$x > 4$ AND $x < 9$ or $4 < x < 9$	← How the compound is written →	$x < 4$ OR $x > 9$
	← How the solution looks →	
values found in *both* inequalities	← Solution →	values found in *either* inequality

Homework 2 – 7

p. 206-208 #16-28 (evens only), 30-35, 46

1) Explain why 4 is a solution to the equation $3x + 4 = 16$.

2) Solve for t: $5-(t+3)=-1+2(t-3)$

3) Solve for x: $6+3x=5(x-1)-3(x-2)$

4) Solve for b:

$$-40 \geq 8b$$

Give one value for b that is:

a) an integer ______________

b) not an integer ____________

5) Solve for t:

$$12 > \frac{t}{-6}$$

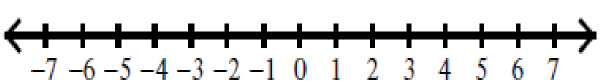
How many values are in the solution set?

6) Solve for x. Express the solution set in interval notation and graph it.

$$2(5-x) < 3x$$

Interval notation: ______________________

Graph:



7) Given the following inequality:

$$12 - 3(x + 1) \geq \frac{1}{2}(3 - 5)$$

a) State the maximum value in the solution set.
b) State the largest integer in the solution set.

8) State the solution set of:

$$d - 2 < \quad d - 4$$

9) State the solution set of:

$$16(s - 2) \leq \quad 4(4s - 5)$$

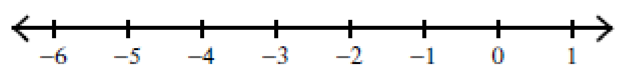
10) Given: $2x + ax + 10 \leq -10$

Determine the smallest integer value of a when x = -4.

11) If $-5x \; < \; 55$, what is the smallest possible integer value of the solution set?

12) Solve and graph the solution set:

$$2 - n > 2 \text{ and } -4n + 1 \leq 21$$



Write the solution set as a single inequality: ______________________________ and in interval notation: ________________

13) Which value of x is in the solution set of the inequality $-4x + 2 > 10$?

1 -2
2 2
3 3
4 -4