CHAPTER 27

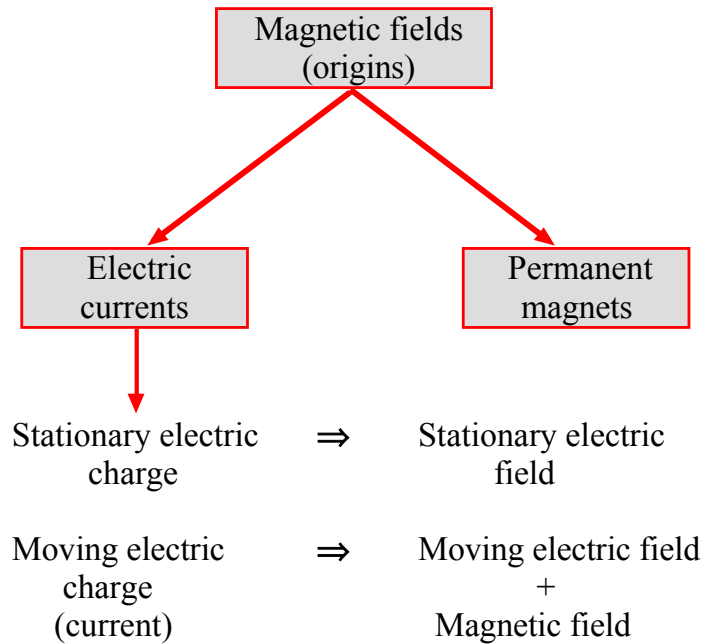***SOURCES OF MAGNETIC FIELD***

- Magnetic field due to a moving charge.

- Magnetic field due to electric currents

    - *Straight wires*
    - *Circular coil*
    - *Solenoid*
    - *Force between two wires*

- Gauss's Law and Ampère's Law

   - *Magnetic field inside a wire*
    - *Toroidal solenoid*

Magnetic fields (origins)

Electric currents

Permanent magnets

| Stationary electric charge | $\Rightarrow$ | Stationary electric field |

| Moving electric charge (current) | $\Rightarrow$ | Moving electric field + Magnetic field |

Much research in the 19th century: magnetism and electricity unified by James Clerk Maxwell (1832-79).

out   in
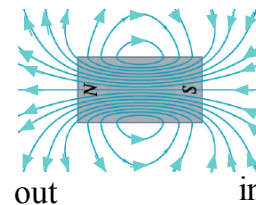
Magnetic poles appear in pairs (N & S), always!

Like poles repel
Unlike poles attract

Magnetic field of a single moving charge:
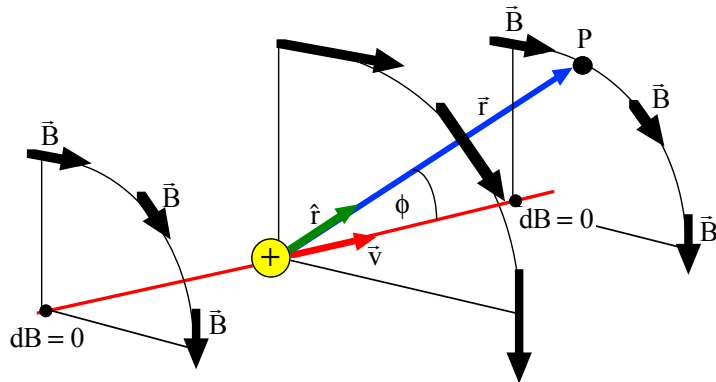
The magnetic field is circular around the direction of motion of the charge. The magnetic field at a point $\vec{r}$ (P), due to a charge q moving with velocity $\vec{v}$ is:
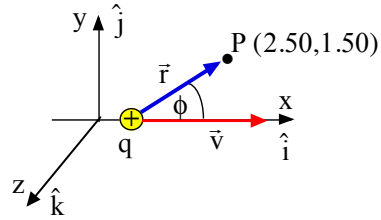
$$\vec{B} = k'\frac{q\vec{v}\times\hat{r}}{|\vec{r}|^2},$$

where $k' = \dfrac{\mu_\circ}{4\pi}$ with $\mu_\circ = 4\pi\times10^{-7}$ T.m/A and $\hat{r} = \dfrac{\vec{r}}{|\vec{r}|}$.

permeability of free space

Question 27.0: A point charge, q = 3.2 μC, is moving along the x-axis with velocity $\vec{v} = 2.50$ m/s. What is the magnetic field produced by this charge at the point $(x,y) = (2.00\ \text{m}, 1.50\ \text{m})$ when it passes the point x = 0.50 m?

We'll solve this problem in two ways.

(a) The field is given

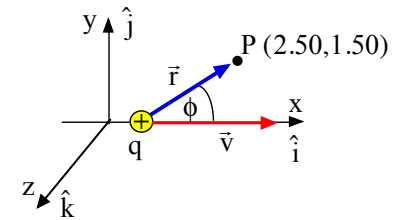by $\vec{B} = \dfrac{\mu_\circ}{4\pi}\dfrac{q\vec{v}\times\hat{r}}{r^2}$,

where $\vec{v} = (2.50 \text{ m/s})\hat{i}$

and $\hat{r} = \dfrac{\vec{r}}{|\vec{r}|} = \dfrac{(2.00\hat{i}+1.50\hat{j})\text{m}}{\sqrt{(2.00\text{m})^2+(1.50\text{m})^2}}$

$\hat{r} = (0.80\hat{i}+0.60\hat{j})\text{m}.$

$\therefore \vec{B} = \dfrac{\mu_\circ}{4\pi}\dfrac{(3.2\times10^{-6}\text{C})(2.50 \text{ m})\hat{i}\times(0.80\hat{i}+0.60\hat{j})\text{m}}{(2.00\text{m})^2+(1.50\text{m})^2}$

$= \left(1\times10^{-7}\right)\dfrac{4.8\times10^{-6}\hat{k}}{6.25} = (7.68\times10^{-14}\text{ T})\hat{k}.$

$\therefore \vec{B} = \dfrac{\mu_\circ}{4\pi}\dfrac{q|\vec{v}||\hat{r}|\sin\phi}{r^2}\hat{k}$

$= \left(1\times10^{-7}\right)\dfrac{(3.2\times10^{-6}\text{C})(2.50 \text{ m/s})(1)(0.60)}{6.25}\hat{k}$
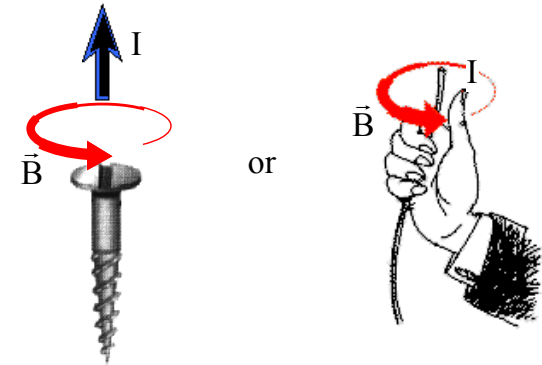
$= (7.68\times10^{-14}\text{ T})\hat{k}.$

(b) We have $\vec{v}\times\hat{r} = |\vec{v}||\hat{r}|\sin\phi\,\hat{k}$, where

$\sin\phi = \dfrac{1.50}{\sqrt{(2.00)^2+(1.50)^2}} = 0.60.$
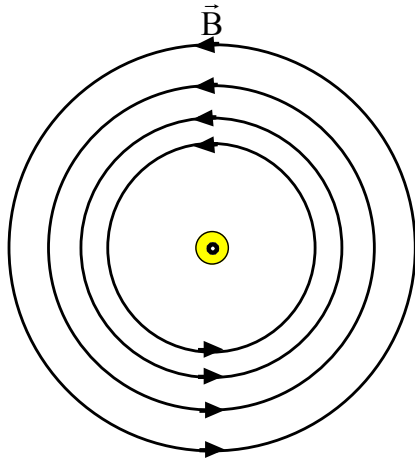
Magnetic field due to a current (Biot-Savart Law):

The magnetic field at a point $\vec{r}$, due to a small element of wire $d\vec{\ell}$ carrying a current I is:

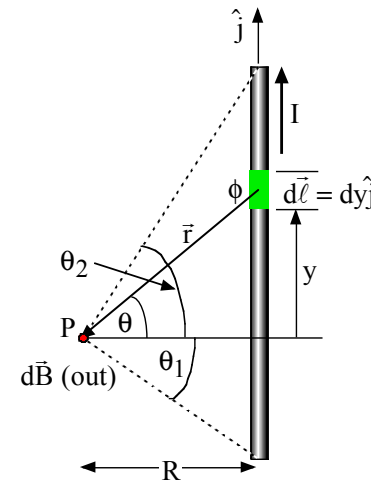$$d\vec{B} = \frac{\mu_\circ}{4\pi} \frac{I \, d\vec{\ell} \times \hat{r}}{|\vec{r}|^2}.$$

For a complete circuit: $\vec{B} = \frac{\mu_\circ}{4\pi} \int \frac{I \, d\vec{\ell} \times \hat{r}}{|\vec{r}|^2}.$

The direction of $\vec{B}$, with respect to the current, is given by the right hand rule:

or

The magnetic field line contours around $d\vec{\ell}$ are circular. The magnetic field shown above is for a current *out* of the page.

Remember, the charges also produce an electric field, but the electric field lines are *radial*.

Magnetic field of a straight wire:

The Biot-Savart law is

$$d\vec{B} = \frac{\mu_\circ}{4\pi} \frac{I d\vec{\ell} \times \hat{r}}{r^2}.$$

Since the contributions to the total magnetic field $\vec{B}$ at P are all parallel (outwards towards us) we need only calculate the magnitude of the field.

Then, we have

$$dB_P = \frac{\mu_\circ}{4\pi} \frac{I dy}{r^2} \sin \phi = \frac{\mu_\circ}{4\pi} \frac{I dy}{r^2} \cos \theta,$$

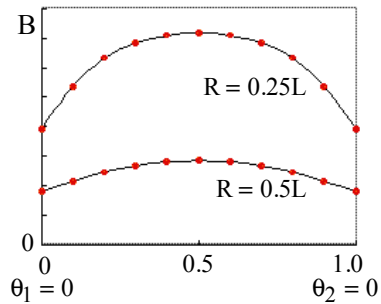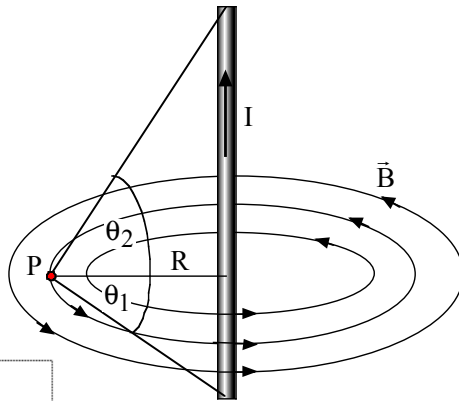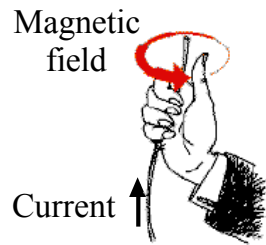where ϕ is the angle between $I d\vec{\ell}$ and $\hat{r}$. But $y = R \tan \theta$, so differentiating we find

$$dy = R \sec^2 \theta . d\theta = R \frac{r^2}{R^2} d\theta = \frac{r^2}{R} d\theta.$$
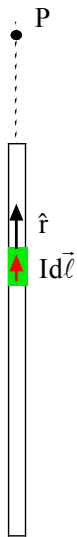
Then, substituting for dy we get

$$dB_P = \frac{\mu_\circ}{4\pi} \frac{I}{r^2} \frac{r^2 d\theta}{R} \cos\theta = \frac{\mu_\circ I}{4\pi R} \cos\theta . d\theta$$

$$\therefore B_P = \frac{\mu_\circ I}{4\pi R} \int_{-\theta_1}^{\theta_2} \cos\theta . d\theta = \frac{\mu_\circ I}{4\pi R} [\sin\theta]_{-\theta_1}^{\theta_2}$$

$$= \frac{\mu_\circ I}{4\pi R} (\sin\theta_2 + \sin\theta_1).$$

Magnetic field

Current

θ_2

θ_1

P

R

I

$\vec{B}$

B

R = 0.25L

R = 0.5L

0

0   0.5   1.0

$\theta_1 = 0$ $\theta_2 = 0$

P

I

Question 27.1: What is the magnetic field at the point P on the axis of the wire?

P

$\hat{r}$

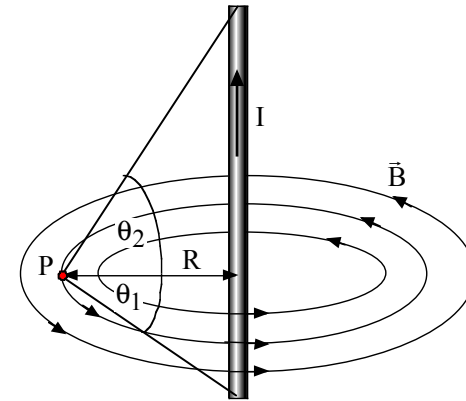$Id\vec{\ell}$

From the Biot-Savart Law:

$$\vec{B} = \frac{\mu_\circ}{4\pi} \int \frac{Id\vec{\ell} \times \hat{r}}{|\vec{r}|^2},$$

But $Id\vec{\ell} \parallel^\ell \hat{r}$ for all elements $d\vec{\ell}$.

$$\therefore Id\vec{\ell} \times \hat{r} = 0$$

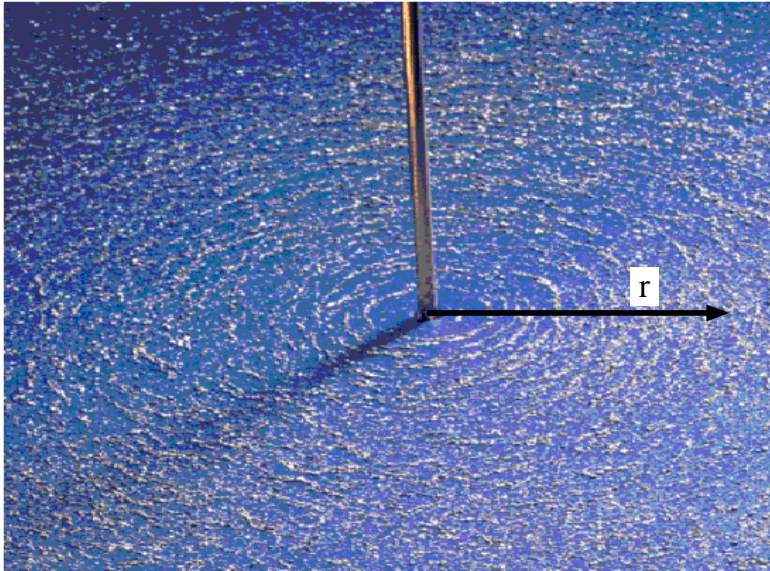$$\therefore |\vec{B}| = 0.$$

Magnetic field of an infinitely long wire:

I

$\vec{B}$

θ_2

P   R

θ_1

For a straight wire $B(R) = \dfrac{\mu_\circ I}{4\pi r}(\sin\theta_1 + \sin\theta_2)$.

As the wire becomes *infinitely* long:

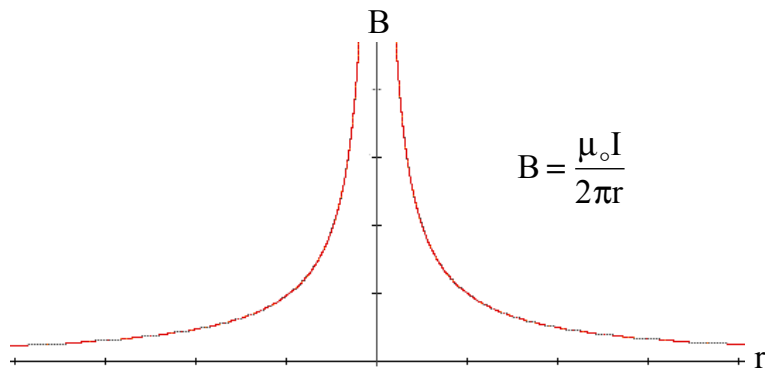$$\theta_1 \to 90° \text{ and } \theta_2 \to 90°.$$
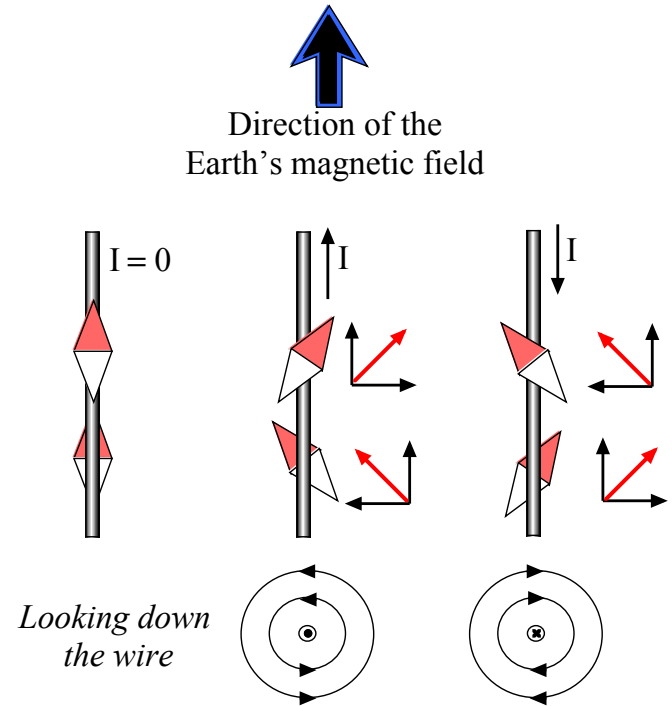
$$\therefore \sin\theta_1 + \sin\theta_2 \to 2,$$

$$\text{so } B(R) = \frac{\mu_\circ I}{2\pi r}$$

Magnetic field lines for a single current carrying wire, indicated by iron filings

$$B = \frac{\mu_\circ I}{2\pi r}$$

Direction of the
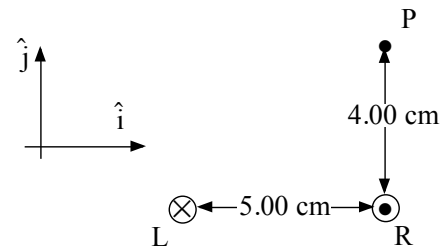Earth's magnetic field

*Looking down
the wire*

Magnetic fields, like electric fields, "add" vectorially. The compass needles point in the direction of the *resultant* of the Earth's magnetic field and the magnetic field produced by the current in the wire.
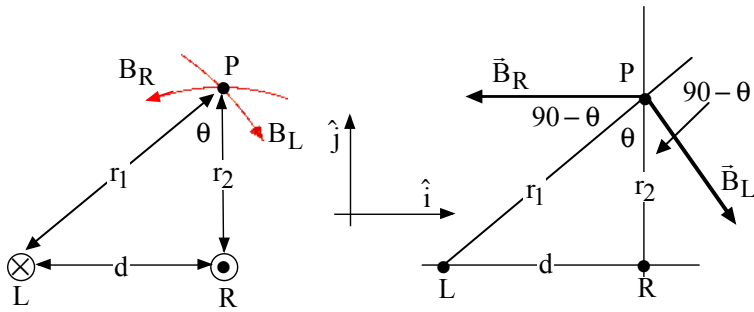
Magnetic field due to two parallel wires:
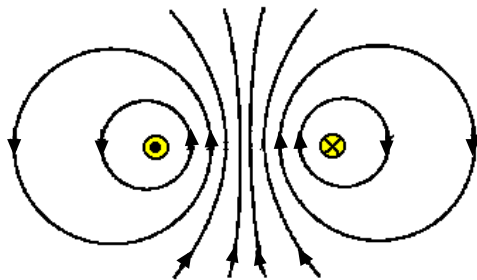
Magnetic field lines add vectorially: shown here is the combined field for two parallel wires carrying a current in opposite directions.

What would be the magnetic field midway between the two wires if the currents are in the same direction?

Question 27.2: Two, very long, parallel wires, L and R, each carry currents $I = 3.00$ A in opposite directions, as shown above. If the wires are spaced 5.00 cm apart, what is the magnetic field vector at a point P a distance 4.00 cm from R, if $\angle LRP = 90°$?

Let the magnetic field at P due to the left hand wire (L) be B_L and the magnetic field at P due to the right hand wire (R) be B_R. We have

$$\theta = \tan^{-1}\left(\frac{5}{4}\right) = 51.3°.$$

Note that, $\vec{B}_L$ is perpendicular to LP and $\vec{B}_R$ is parallel to LR. The resultant magnetic field in the $\hat{i}$ direction is

$$B_i = \left(B_L \sin(90 - \theta)\right) - B_R,$$

and the resultant magnetic field in the $\hat{j}$ direction is

$$B_j = -B_L \cos(90 - \theta).$$

So, the resultant magnetic field is:

$$\vec{B} = B_i\hat{i} + B_j\hat{j}.$$

From earlier, and using the given values,

$$B_L = \frac{\mu_\circ I}{2\pi r_1} = \frac{\mu_\circ I}{2\pi\sqrt{d^2 + r_2^2}}$$

$$= \frac{4\pi\times10^{-7}\times3.00}{2\pi\times0.064} = 9.38\times10^{-6}\,\text{T}$$

and

$$B_R = \frac{\mu_\circ I}{2\pi d} = \frac{4\pi\times10^{-7}\times3.00}{2\pi\times0.04} = 15.0\times10^{-6}\,\text{T}.$$

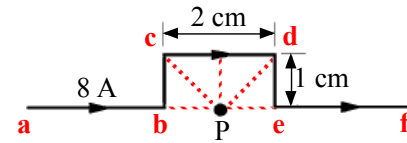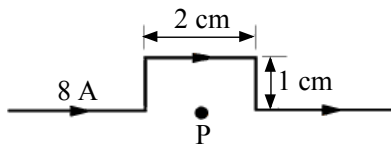$$\therefore B_i = \left(B_L \sin(90 - \theta)\right) - B_R = -9.14\times10^{-6}\,\text{T}$$

and

$$B_j = -B_L \cos(90 - \theta) = -7.32\times10^{-6}\,\text{T}.$$

$$\therefore \vec{B} = \left(-9.14\times10^{-6}\hat{i} - 7.32\times10^{-6}\hat{j}\right)\,\text{T},$$

$$\text{and } |\vec{B}| = 11.7\times10^{-6}\,\text{T}.$$

CHECK: You will find the resultant field at P is parallel to PL. That is a general result for all points P, when RP is perpendicular to LR.

Label the ends of the straight sections **a-f**. Sections **a-b** and **e-f** do not contribute to the magnetic field at P, but **b-c**, **c-d** and **d-e** all produce magnetic fields _into_ the page (so we can simply add them).

Question 27.3: The current in the wire shown below is 8.0 A. Find the magnetic field strength at P.

Field due to **b-c**: $B_1 = \dfrac{\mu_\circ I}{4\pi r}(\sin\theta_1 + \sin\theta_2)$

$$= \frac{4\pi\times10^{-7}\times 8\text{ A}}{4\pi\times 0.01\text{ m}}(\sin 45° + \sin 0) = 5.66\times10^{-5}\text{ T}.$$
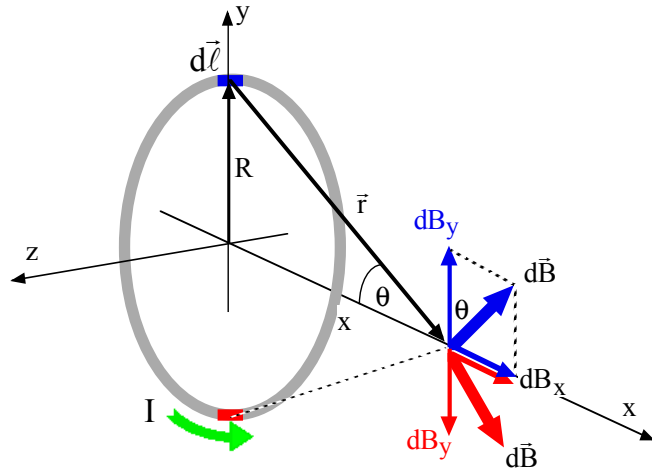
But this is the same as the field due to **d-e**.

Field due to **c-d**: $B_2 = \dfrac{\mu_\circ I}{4\pi r}(\sin\theta_1 + \sin\theta_2)$

$$= \frac{4\pi\times10^{-7}\times 8\text{ A}}{4\pi\times 0.01\text{ m}}(\sin 45° + \sin 45°) = 1.13\times10^{-4}\text{ T}.$$

Therefore total field is:

$$(5.66\times10^{-5} + 1.13\times10^{-4} + 5.66\times10^{-5})\text{ T}$$

$$= 2.26\times10^{-4}\text{ T (into the page)}.$$

Magnetic field on the axis of a circular coil (of radius R):

The current is the same at every point around the coil, so the components of the magnetic field perpendicular to the x-direction (i.e., in the $y - z$ plane) cancel, i.e., the field restricted to the x-direction:

$$\therefore dB_x = dB \sin \theta.$$

But, $dB = |d\vec{B}| = \frac{\mu_\circ}{4\pi} \frac{|Id\vec{\ell} \times \hat{r}|}{r^2} = \frac{\mu_\circ}{4\pi} \frac{Id\ell}{(x^2 + R^2)}$,

where we have substituted $|d\vec{\ell} \times \hat{r}| = d\ell$ since $d\vec{\ell} \perp \hat{r}$.

$$\therefore dB_x = \frac{\mu_\circ}{4\pi} \frac{Id\ell}{(x^2 + R^2)} \sin \theta = \frac{\mu_\circ}{4\pi} \frac{Id\ell}{(x^2 + R^2)} \left(\frac{R}{\sqrt{x^2 + R^2}} \right)$$

$$= \frac{\mu_\circ}{4\pi} \frac{IRd\ell}{(x^2 + R^2)^{3/2}}.$$
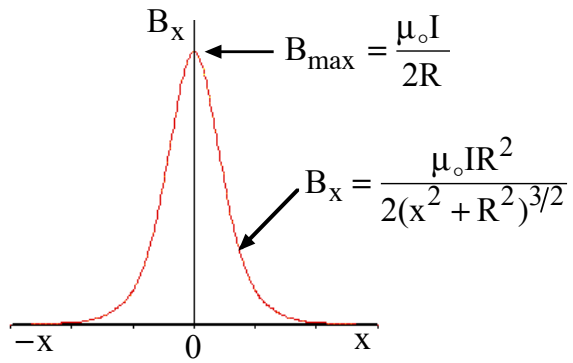
To obtain the total field we integrate around the loop.
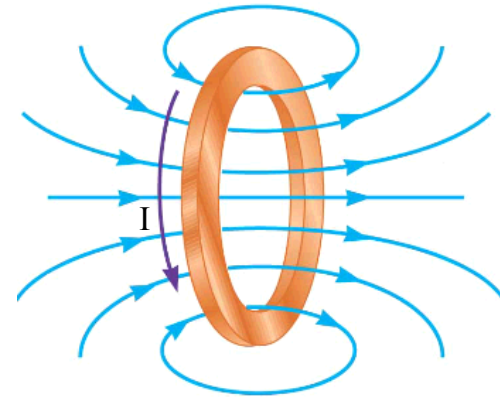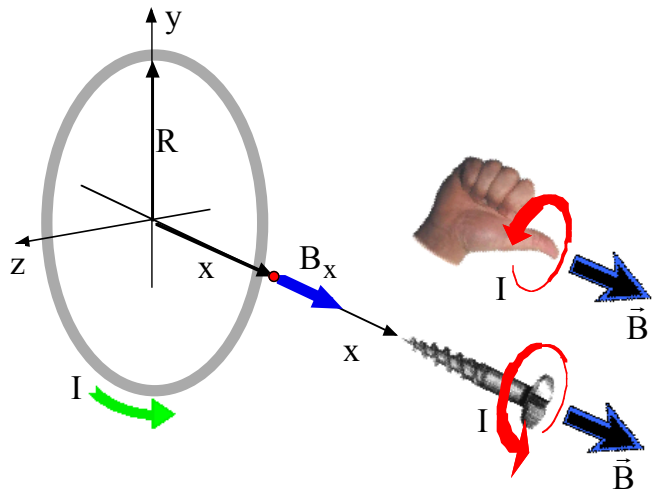
NOTE: x and R are both constant,

$$\therefore B_x = \oint dB_x = \frac{\mu_\circ}{4\pi} \frac{IR}{(x^2 + R^2)^{3/2}} \oint d\ell$$

$$= \frac{\mu_\circ}{4\pi} \frac{IR}{(x^2 + R^2)^{3/2}} 2\pi R = \frac{\mu_\circ}{4\pi} \frac{2\pi IR^2}{(x^2 + R^2)^{3/2}}$$

$$= \frac{\mu_\circ IR^2}{2(x^2 + R^2)^{3/2}},$$

i.e., $\boxed{\vec{B} = B_x \hat{i} = \frac{\mu_\circ IR^2}{2(x^2 + R^2)^{3/2}} \hat{i}.}$

• When $x = 0$: $\quad B_x = \frac{\mu_\circ I}{2R}.$

$$B_x$$

$$B_{max} = \frac{\mu_o I}{2R}$$

$$B_x = \frac{\mu_o I R^2}{2(x^2 + R^2)^{3/2}}$$

$-x$ 0 x

Variation of the magnetic field along the axis of a circular coil. The *direction* of the field is given by the *right hand rule*.

Magnetic field lines in the vicinity of a circular coil carrying a current.

Question 27.4: A closed circuit consists of two semicircles of radii 40.0 cm and 20.0 cm that are connected by straight segments. A current of 3.00 A flows in the circuit in a clockwise direction. Find the magnetic field strength at the point P.
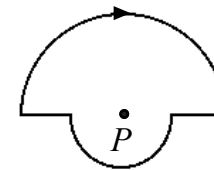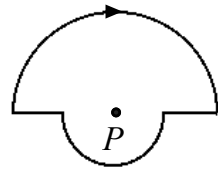
The straight sections do not contribute to the field at P. The net field is due to the two semi-circular arcs. Put $R_1 = 0.40$ m and $R_2 = 0.20$ m.

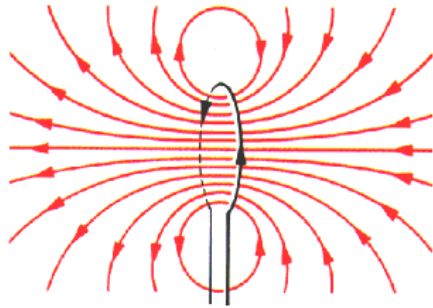$$\therefore \vec{B} = \vec{B}_1 + \vec{B}_2.$$

$$B_1 = \frac{1}{2}\frac{\mu_\circ I}{2R_1} \ (\textit{inwards}) \text{ and } B_2 = \frac{1}{2}\frac{\mu_\circ I}{2R_2} \ (\textit{inwards})$$
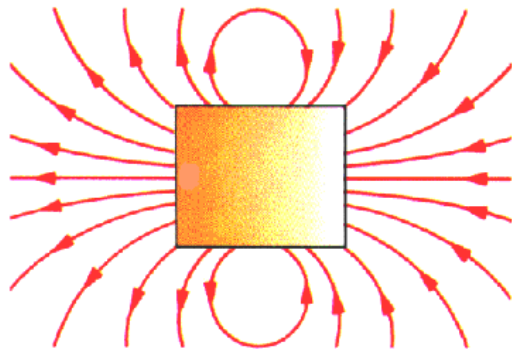
$$\therefore B = B_1 + B_2$$

$$= \left(\frac{1}{2}\right)\frac{\mu_\circ I}{2R_1} + \left(\frac{1}{2}\right)\frac{\mu_\circ I}{2R_2} = \frac{\mu_\circ I}{4}\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

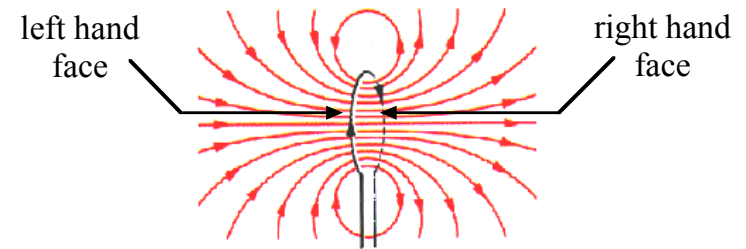$$= \frac{4\pi \times 10^{-7} \times 3 \text{ A}}{4}\left(\frac{1}{0.40 \text{ m}} + \frac{1}{0.20 \text{ m}}\right)$$

$$= 7.07 \times 10^{-6} \text{ T } (\textit{inwards}).$$

Above, magnetic field lines produced by a current carrying coil. Below, magnetic field lines of a permanent magnet.

Notice, they are essentially identical.

left hand face   right hand face

Question 27.5: The magnetic field lines produced by a current carrying coil are very similar to the magnetic field lines of a permanent magnet. Therefore, one face of the coil acts like a NORTH pole and the other acts like a SOUTH pole. Which face of the coil shown here is the north pole?

"equivalent magnets"

| N | S | | N | S |

ATTRACTION

"equivalent magnets"
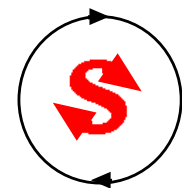
| N | S | | S | N |

REPULSION
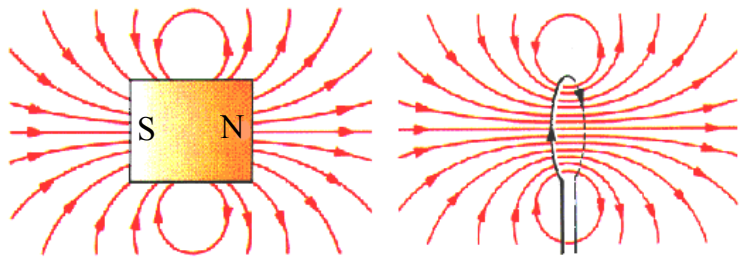
By definition, magnetic field lines travel outward from a NORTH pole. Therefore, the right hand face of this coil acts like a north pole.
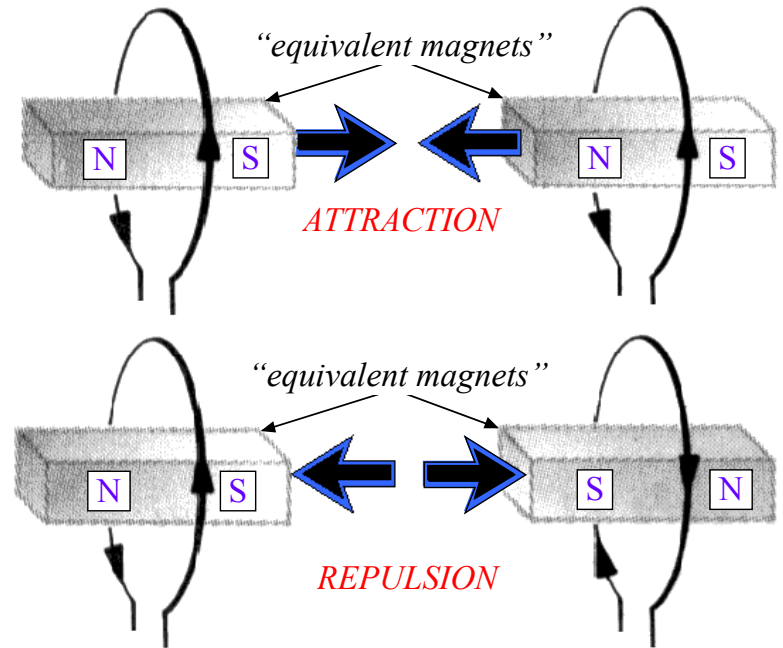
Thus, a current carrying coil has the properties and characteristics of a permanent magnet.
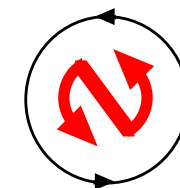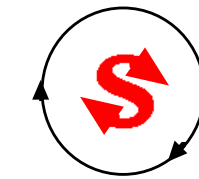
Current CCW.   Current CW

"McDonald rule"
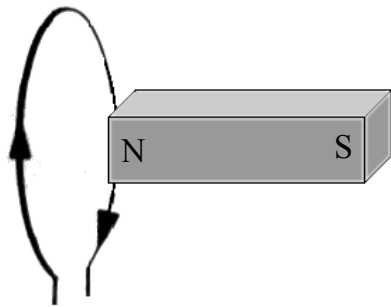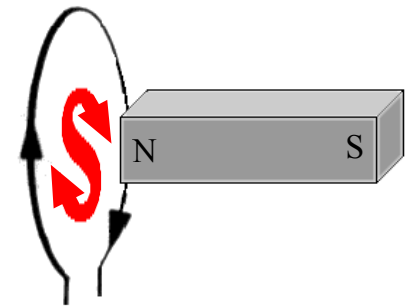
Question 27.6: A current carrying wire and the north end of a magnet are placed close to each other, as shown above. Will there be a mutual atraction or mutual repulsion between the coil and the magnet?

Current CW

From the N-pole end of the magnet, the current in the coil appears clockwise. By the McDonald rule, a cw current means that face of the coil is a S-pole. So, there is a mutual attraction between the coil and magnet!

A **SOLENOID** is a long, tightly wound coil with a constant radius.
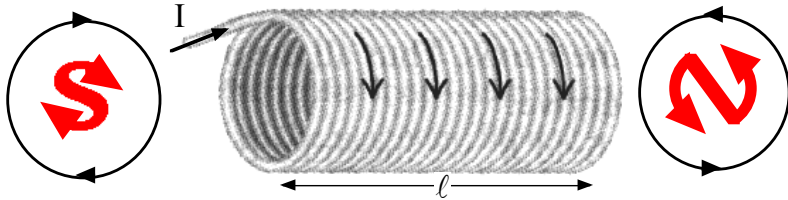
The magnetic field lines within a solenoid are parallel to the axis and the magnetic field strength at the center is:

$$B = \mu_\circ nI$$

where n is the number of turns per unit length ($n = N/\ell$), N is the total number of turns and ℓ is the length.
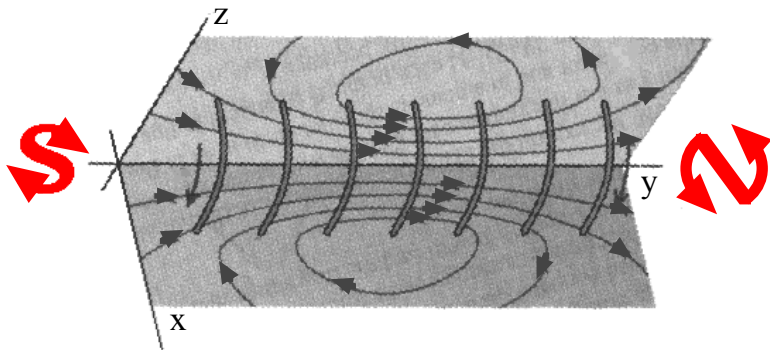
Magnetic field of a solenoid:

Total of N turns

$$n = N/\ell$$

($\ell \gg$ radius)
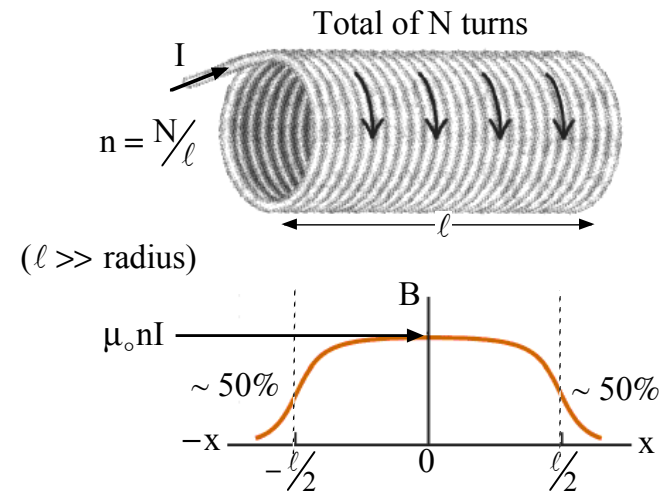
We can increase the strength of the magnetic field by inserting a "core" ... then $\mu_\circ \Rightarrow K_m \mu_\circ \Rightarrow \mu$, i.e, $B = \mu nI$.

Values of K_m vary from 1 to $\sim 10^5$.

Force between two straight wires (Ampère 1820):
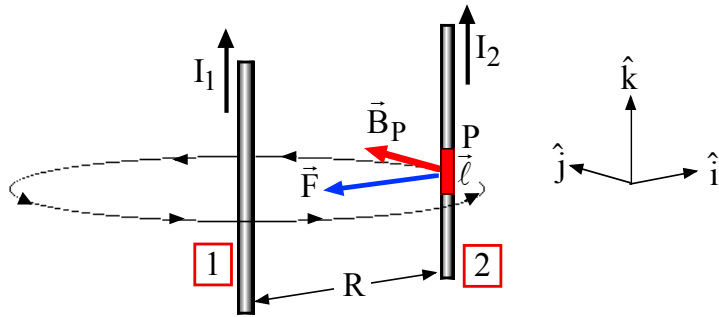
The magnetic field due to wire $\boxed{1}$ at position P is:

$$\vec{B}_P = \frac{\mu_\circ I_1}{2\pi R}\,\hat{j}.$$

Since wire $\boxed{2}$ carries a current, I_2, there is a force:

$$\vec{F} = I_2\vec{\ell}\times\vec{B}_P.$$

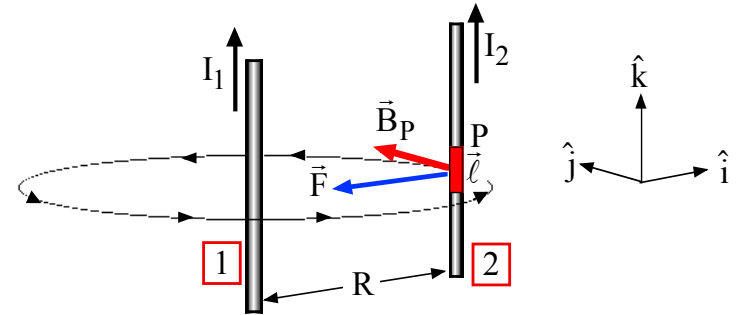acting on the segment $\vec{\ell}$. Since $\vec{\ell}$ is in the $\hat{k}$ direction, $\vec{F}$ is in the $-\hat{i}$ direction, i.e., to the left and attractive. Also, as $\vec{B}_P$ and $\vec{\ell}$ are perpendicular, the magnitude of the force is

$$F = \left|I_2\vec{\ell}\times\vec{B}_P\right| = I_2\ell B_P.$$

Substituting for B_P we find

$$F = I_2\ell B_P = I_2\ell\left(\frac{\mu_\circ I_1}{2\pi R}\right) = \mu_\circ\frac{I_1 I_2\ell}{2\pi R},$$

to the <u>left</u> so the force per unit length is $F/\ell = \mu_\circ\dfrac{I_1 I_2}{2\pi R}$.

A similar analysis indicates that the magnetic field produced by the current in wire $\boxed{2}$ results in a force of equal magnitude towards the <u>right</u> on wire $\boxed{1}$, i.e., in the $+\hat{i}$ direction. Hence, two wires carrying currents in the *same direction* are *attracted* to each other.

<u>Corollary</u>: two wires carrying currents in *opposing directions repel* each other.

Definition of the *AMPÈRE*:

← 1.00 m →

I   I
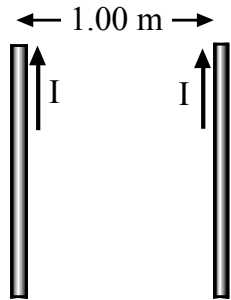
If two parallel wires carry the same current,

i.e., $I_1 = I_2 = I$,

the force per unit length between the wires is:

$$F\big/_\ell = \mu_\circ \frac{I^2}{2\pi R}.$$

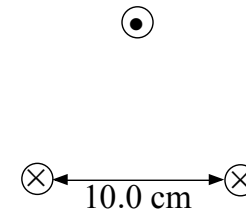Since $\mu_\circ = 4\pi \times 10^{-7}\ \text{T} \cdot \text{m/A}$, a current of one *AMPÈRE* can be defined as:

... the current in two parallel wires of infinite length placed 1.00 m apart that produces a force on each wire of $2.00 \times 10^{-7}\ \text{N}$ *per meter of length.*

i.e., if $I_1 = I_2 = 1$ A and $R = 1.00$ m.

$$\therefore F\big/_\ell = 4\pi \times 10^{-7} \times \frac{1}{2\pi} \Rightarrow 2 \times 10^{-7}\ \text{N/m}$$

⊗ ← 10.0 cm → ⊗

Question 27.7: Three very long straight parallel wires pass through the vertices of an equilateral triangle with sides of length 10.0 cm. If the current in each wire is 15.0 A in the directions shown,

(a) What is the magnetic field at the location of the upper wire due to the currents in the two lower wires?

(b) What force per unit length does the upper wire experience?

(a) Note that $\vec{B}_1$ is perpendicular to the line joining #1 and the upper wire and $\vec{B}_2$ is perpendicular to the line joining #2 and the upper wire. Using the right hand rule their directions are as shown. From the geometry of the problem, the magnetic field vectors are at $30°$ to the x-direction, so the resultant magnetic field is
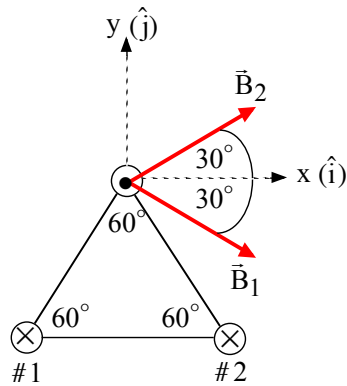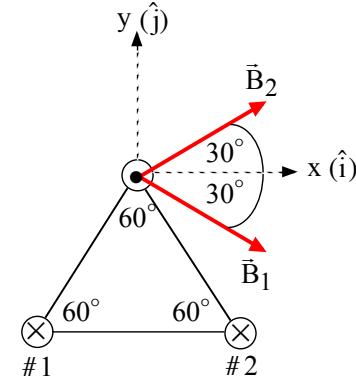
$$\vec{B} = \vec{B}_1 \cos 30° + \vec{B}_2 \cos 30°.$$

But $\left|\vec{B}_1\right| = \left|\vec{B}_2\right| = B$, where $B = \dfrac{\mu_\circ I}{2\pi r} = \dfrac{4\pi \times 10^{-7} \times 15\,A}{2\pi \times 0.10\,m}$

$$= 3.00 \times 10^{-5}\,T.$$

$$\therefore \vec{B} = 2\left(B\cos 30°\right)\hat{i} = 2\left(B\cos 30°\right)\hat{i}$$

$$= 2\left(3.00 \times 10^{-5}\,T \times 0.866\right)\hat{i} = 5.20 \times 10^{-5}\,T\ \hat{i}.$$

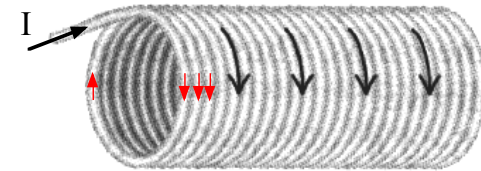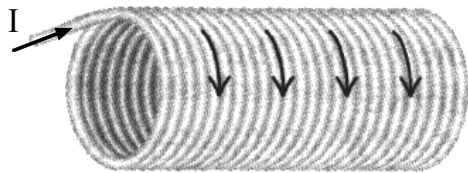(b) The force the lower wires exert on the upper wire is

$$\vec{F} = I\vec{\ell} \times \vec{B} = I\ell\hat{k} \times B\hat{i} = I\ell B\hat{j},$$

i.e., in the y-direction.

$$\therefore \frac{\vec{F}}{\ell} = IB\hat{j} = \left(15.0\,A \times 5.20 \times 10^{-5}\,T\right)\hat{j}$$

$$= \left(7.8 \times 10^{-4}\,N\right)\hat{j}.$$

Question 27.8: Identify all of the forces acting on the windings of a current carrying solenoid ...

The forces acting on the windings of a solenoid are ...

1. Between each turn the currents are parallel so there are attractive forces that tend to reduce the length of the coil.

2. Across a turn the currents are anti-parallel so there are repulsive forces that tend to increase the radius of the coil.

Gauss's Law in magnetism:

Unlike electrical charges, single, isolated magnetic poles - monopoles - do not exist ... *magnetic poles only occur in pairs*. So, the *net magnetic flux* through any closed surface is zero, *always*. Look ...
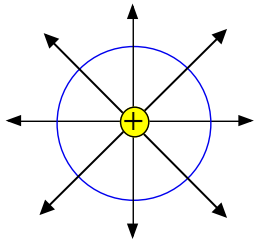
Electric charge with electric field lines; lines start from a +ve charge and end on a −ve charge.
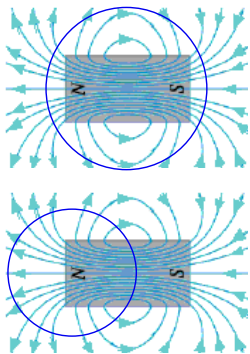
Magnetic poles with magnetic field lines; magnetic field lines are *continuous* so if a line *exits* a Gaussian surface it must also *enter*. So, Gauss's Law tells us ... the *net magnetic flux* through any closed surface is zero, always.

A "sort of equivalent" to Gauss's Law in magnetism is ... *Ampère's Law*:

> *"The line integral of the magnetic field $\vec{B}$ around a closed path is proportional to the algebraic sum of the enclosed currents",*

$$\text{i.e.,} \qquad \oint_c \vec{B} \bullet d\vec{\ell} = \mu_\circ I_{enc}.$$
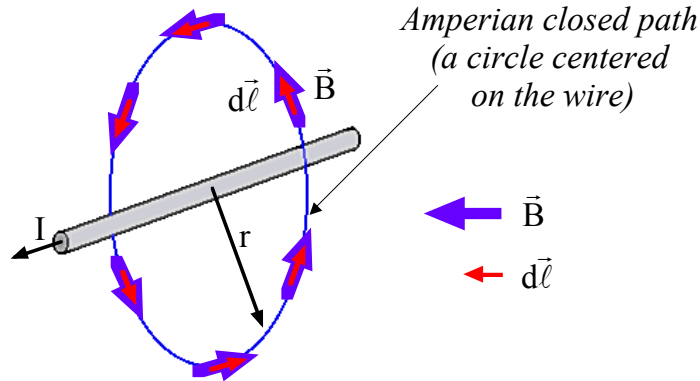
However, Ampère's Law is only useful if ...
• the current is constant, and
• the system has high symmetry.

So it is not so generally applicable as Gauss's Law.

Let's look at some examples.

Example of the use of Ampère's Law:

Amperian closed path
(a circle centered
on the wire)

$d\vec{\ell}$ $\vec{B}$

$\vec{B}$

$d\vec{\ell}$

I   r

Ampere's Law: $\oint_c \vec{B} \bullet d\vec{\ell} = \mu_\circ I_{enc}$.

We choose the closed path to be a circle perpendicular
to the wire and centered on the wire. Then

- $|\vec{B}|$ is constant on the path (by symmetry).

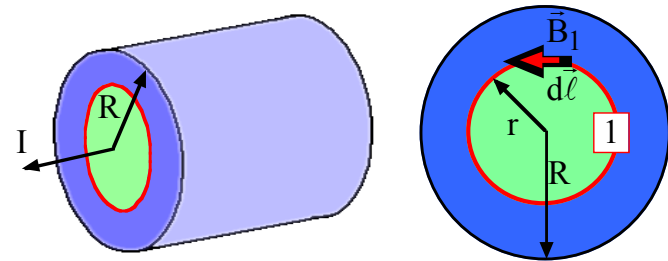- $d\vec{\ell} \parallel \vec{B}$ at all points on the path.

$$\therefore \quad \oint_c \vec{B} \bullet d\vec{\ell} = B\oint_c d\vec{\ell} = B(2\pi r) = \mu_\circ I$$

i.e., $B = \dfrac{\mu_\circ I}{2\pi r}$,

which is the same result we obtained before for a long
straight wire!

Magnetic field inside and outside a current carrying wire:

- *Is there a magnetic field <u>inside</u> a conducting wire?*

1: *<u>Inside</u> the wire (r < R).*

Take a circular Amperian loop $\boxed{1}$ centered on the wire.

By symmetry, $|\vec{B}_1|$ is constant (and parallel to $d\vec{\ell}$) ...

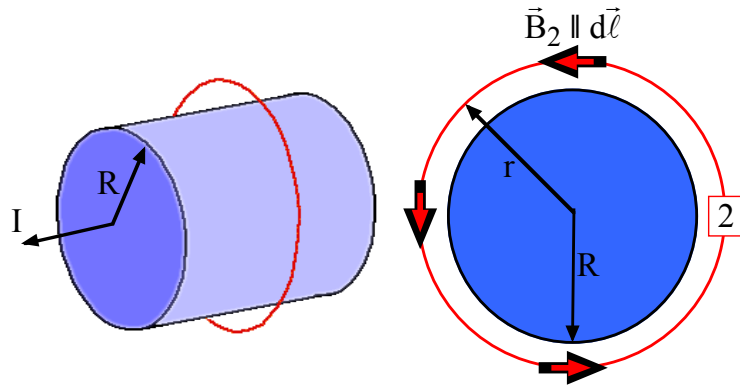$$\therefore \oint_1 \vec{B}_1 \bullet d\vec{\ell} = B_1 \oint_1 d\ell = B_1(2\pi r).$$

The *fraction* of the current enclosed by $\boxed{1}$ is

$$I_{enc} = \frac{\text{Area enclosed by } \boxed{1}}{\text{Total area}} I = \frac{\pi r^2}{\pi R^2} I = \frac{r^2}{R^2} I.$$

$$\therefore B_1(2\pi r) = \mu_\circ I_{enc} = \mu_\circ \frac{r^2}{R^2} I$$

i.e., $B_1 = \dfrac{\mu_\circ r}{2\pi R^2} I \quad (r < R).$

2: *Outside* the wire (r > R).

By symmetry $\left|\vec{B}_2\right|$ is constant (and parallel to $d\vec{\ell}$) ...

$$\therefore \oint_2 \vec{B}_2 \bullet d\vec{\ell} = \oint_2 B_2 d\ell = B_2 \oint_2 d\ell = B_2(2\pi r) = \mu_\circ I_{enc}.$$

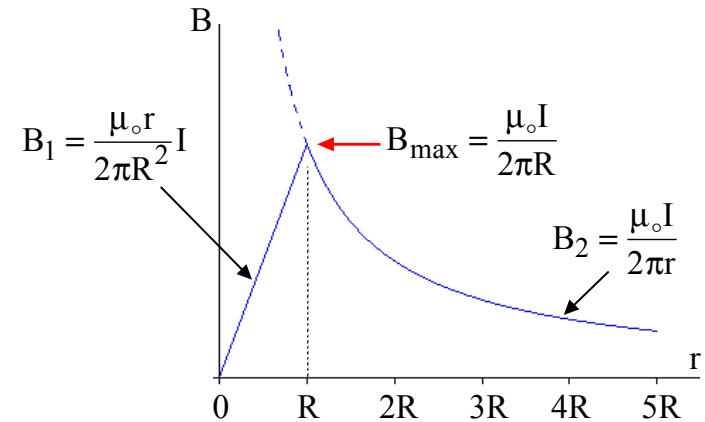The current enclosed by the Amperian loop $\boxed{2}$ is I.

$$\therefore B_2(2\pi r) = \mu_\circ I$$

$$\text{i.e., } B_2 = \frac{\mu_\circ I}{2\pi r} \qquad (r > R).$$

This is just like the field from an infinitely long wire.

Note: it is independent of the radius R.

Here's what the field looks like ...

$$B_1 = \frac{\mu_\circ r}{2\pi R^2} I$$

$$B_{max} = \frac{\mu_\circ I}{2\pi R}$$

$$B_2 = \frac{\mu_\circ I}{2\pi r}$$

The magnetic field inside (r < R) and outside (r > R) a current carrying wire of radius R.

Inside the wire, the field increases linearly from the center to a maximum at the surface of the wire. Outside the wire, the field is the same as the field produced by an infinite wire.
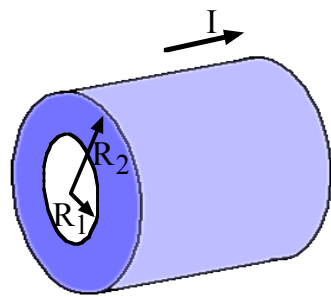
Tube carrying current I

Question 27.9: Is there a magnetic field inside a current carrying hollow tube, i.e., for $r < R_1$? If so, in what direction is it?
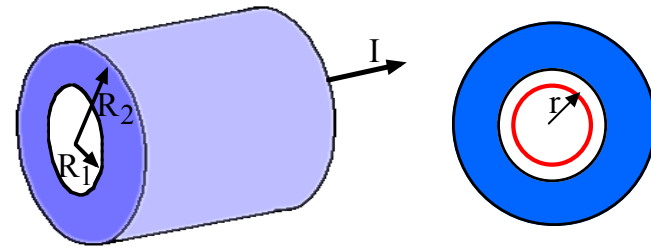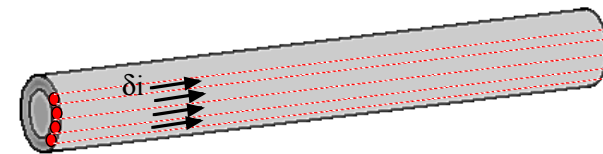

Tube carrying current I

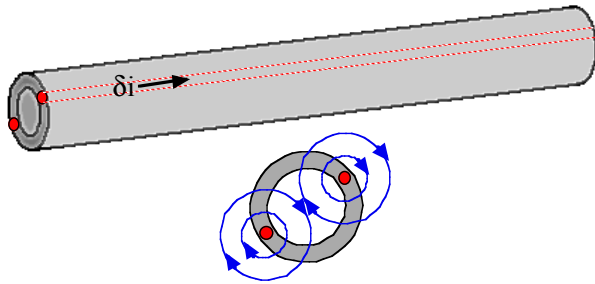Draw a circular Amperian loop radius r (centered on the wire with $r < R_1$). Using Ampère's Law,

$$\oint \vec{B} \bullet d\vec{\ell} = \mu_\circ I_{enc}.$$

But there is no enclosed current so, $\vec{B} \bullet d\vec{\ell} = 0$.

$$\therefore \vec{B} \; cannot \; be \; circular, \; i.e., \; \parallel^\ell \; to \; d\vec{\ell}.$$

Could $\vec{B}$ be perpendicular to $d\vec{\ell}$, i.e., parallel to the axis of the tube? Think of the tube as being made up of individual, small elements each carrying a fraction of the current δi:

Each element acts like a wire carrying part of the total current. It produces a magnetic field that is concentric, i.e., *not down the tube*. But, cancellation from all the diametrically opposite elements produces zero net field inside! So, there is no magnetic field inside the tube at all.

Note that addition of the individual fields <u>outside</u> the tube produce a circular, clockwise set of field lines.
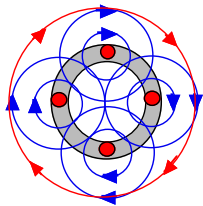
$$B = \frac{\mu_\circ I}{2\pi r} \quad (r > R_2).$$
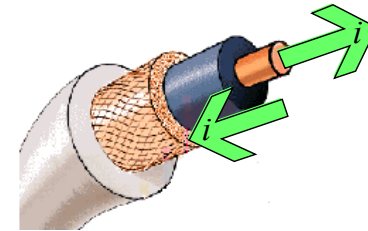
Question 27.10: A coaxial cable is connected so that the instantaneous current to a load flows through the inner central conductor and returns through the outer copper braid, as shown. If you could "see" magnetic field lines, what would they look like *outside* the cable if the instantaneous current in the central conductor is directed towards you?
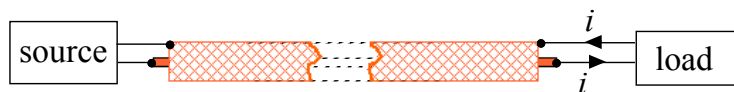
    A: Circular, clockwise.
    B: Circular, counter-clockwise.
    C: Radial, outwards.
    D: Radial, inwards.
    E: There is no field.
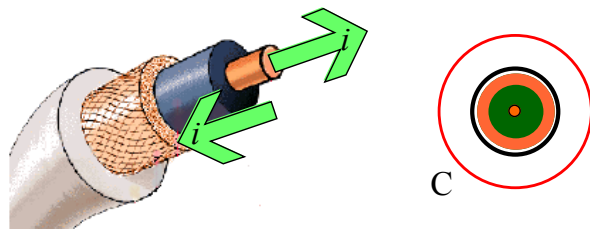
Draw a closed, circular Amperian loop (C) around the coaxial cable. We know

$$\oint_C \vec{B} \bullet d\vec{\ell} = \mu_\circ I_{enc}.$$

But, the net current enclosed $I_{enc} = 0$, so $\vec{B} \bullet d\vec{\ell} = 0$, i.e., $\vec{B}$ cannot be circular. Could $\vec{B}$ be radial? ... _that_ would make $\vec{B} \bullet d\vec{\ell} = 0$. No, because magnetic field lines must be continuous, i.e., loop around. Therefore, the only conclusion is that there is no field outside the coaxial cable.

Connecting coaxial cables this way ensures that there is no external magnetic field even if the current i changes. So, the answer is E.

Question 27.11: A coaxial cable is connected so that the instantaneous current to a load flows through the central conductor and returns through the copper braid as shown. If you could "see" magnetic field lines, what would they look like _between the conductors_ when the instantaneous current in the central conductor is directed towards you?

    A: Circular, clockwise.

    B: Circular, counter-clockwise.

    C: Radial, outwards.
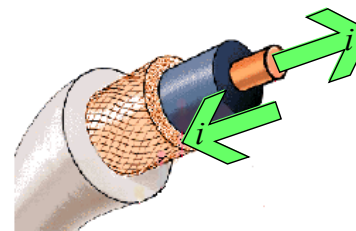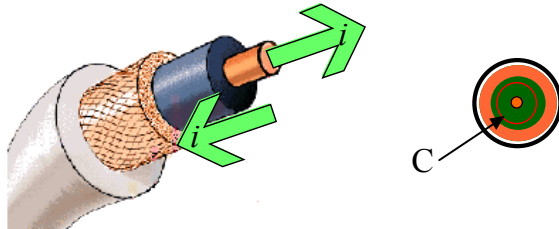
    D: Radial, inwards.

    E: There is no field.

Draw a closed, circular loop (C) with radius r, around the center wire and inside the insulating material. Applying Ampère's Law we get

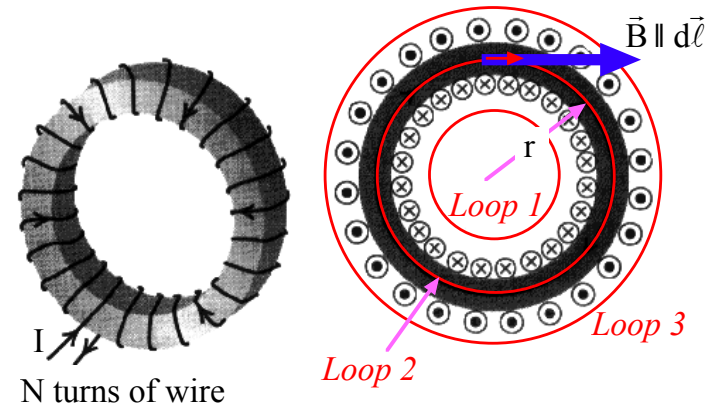$$\oint_C \vec{B} \bullet d\vec{\ell} = \mu_\circ I_{enc}.$$

$$\therefore B(2\pi r) = \mu_\circ i$$

$$\text{i.e., } B(r) = \frac{\mu_\circ i}{2\pi r}.$$

This is the same as for a current carrying wire, and the direction of the field is given by the right hand rule, i.e., counter-clockwise. So, the answer is B.

Note: the outer conductor has absolutely no effect!

Magnetic field of a toroidal solenoid:
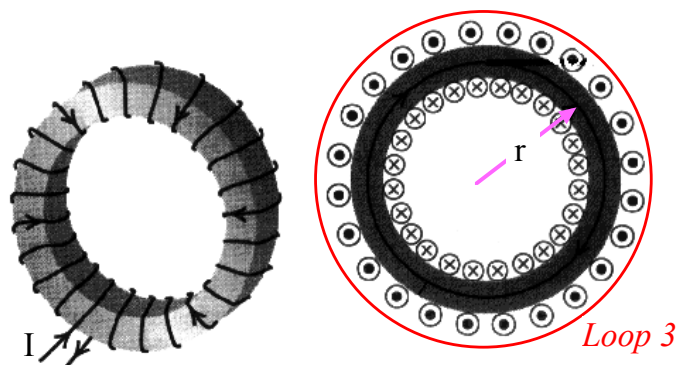

Take three closed (circular) loops ...

   Loop 1: Net current inside *Loop 1* = 0 so $\vec{B} = 0$ everywhere inside the ring.

   Loop 2: By symmetry, $\vec{B}$ is a tangent to the path and $\parallel^\ell$ to $d\vec{\ell}$. Note that $I_{enc} = NI$.

$$\therefore \oint_2 \vec{B} \bullet d\vec{\ell} = 2\pi r B = \mu I_{enc} = \mu NI,$$

where $\mu = K_m \mu_\circ$ and K_m is the relative permeability of the core.

$$\text{i.e., } B = \frac{\mu NI}{2\pi r}.$$

Loop 3

Loop 3: Net current inside *Loop 3* $= 0$ (because of cancellation), so $\vec{B} = 0$ everywhere outside the ring.

Therefore, in an ideal toroidal solenoid, the field is restricted to the *core* of the solenoid and, at radius r, is given by:

$$B = \frac{\mu NI}{2\pi r}.$$

Note that although the field lines are concentric, the field varies across the core; it is largest where r is smallest.

Question 27.12: A tightly wound 1000-turn toroidal solenoid has an inner radius of 1.00 cm and an outer radius of 2.00 cm, and carries a current of 1.50 A. The core of the solenoid is made of soft iron with a relative permeability $K_m = 5500$.

(a) What is the magnetic field strength at a distance of 1.10 cm from the center of the toroid?

(b) What is the magnetic field strength at a distance of 1.50 cm from the center of the toroid?
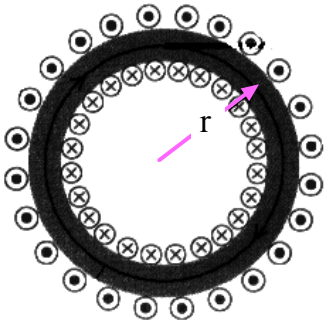
The magnetic field in the *core* of the solenoid at radius r, is given by:

$$B = \frac{\mu NI}{2\pi r}, \text{ where } \mu = 5500\mu_\circ.$$

(a) Substituting the numerical values

$$B(1.10\text{cm}) = \frac{5500 \times (4\pi \times 10^{-7})(1000)(1.50 \text{ A})}{2\pi(1.10 \times 10^{-2} \text{m})}$$
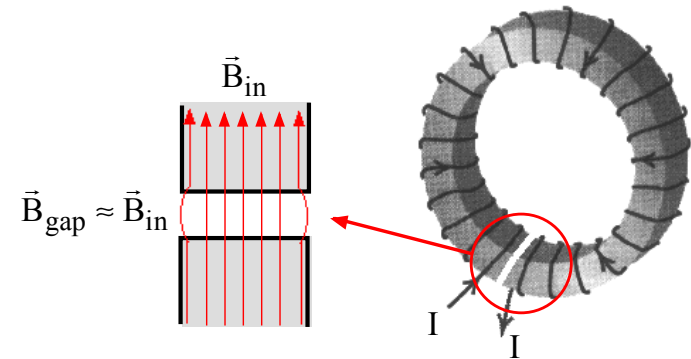
$$= 150\text{T}.$$

(b) Substituting the numerical values

$$B(1.50\text{cm}) = \frac{5500 \times (4\pi \times 10^{-7})(1000)(1.50 \text{ A})}{2\pi(1.50 \times 10^{-2} \text{m})}$$

$$= 110\text{T}.$$

How can we "get" to the field inside the core??

Since magnetic field lines are *continuous* we do not lose much flux (lines/unit area) in a narrow gap. If we wind the coil on a medium with permeability μ, the field in the gap

$$\vec{B}_{gap} \approx \vec{B}_{in} \rightarrow \frac{\mu NI}{2\pi r}$$

and since for some materials $\mu \Rightarrow K_m\mu_\circ \approx 10^5 \mu_\circ$ we can obtain very large magnetic field strengths in the gap.

** This is the basis of an *"electromagnet"* **

Typical laboratory electromagnet

Current carrying coils

S - N

Continuous magnetic
field "circuit"

High permeability
material