## CHAPTER 3. ANALYSIS AND DESIGN OF TWO-WAY SLABS

### 3.1. INTRODUCTION

In reinforced concrete construction, slabs are used to provide flat, useful surfaces. A reinforced concrete slab is a broad, flat plate, usually horizontal, with top and bottom surfaces parallel or nearly so. It may be supported by reinforced concrete beams (and is usually cast monolithically with such beams), by masonry or reinforced concrete walls, by structural steel members, directly by columns, or continuously by the ground.

Slabs may be supported on two opposite sides only, as shown in Figure 3-1a, in which case the structural action of the slab is essentially one-way, the loads being carried by the slab in the direction perpendicular to the supporting beams. There may be beams on all four sides, as shown in Figure 3-1b, so that two-way slab action is obtained. Intermediate beams, as shown in Figure 3-1c, may be provided. If the ratio of length to width of one slab panel is larger than about 2, most of the load is carried in the short direction to the supporting beams and one-way action is obtained in effect, even though supports are provided on all sides.

Concrete slabs in some cases may be carried directly by columns, as shown in Figure 3-1d, without the use of beams or girders. Such slabs are described as flat plates and are commonly used where spans are not large and loads not particularly heavy. Flat slab construction, shown in Figure 3-1e , is also beamless but incorporates a thickened slab region in the vicinity of the column and often employs flared column tops. Both are devices to reduce stresses due to shear and negative bending around the columns. They are referred to as drop panels and column capitals, respectively. Closely related to the flat plate slab is the two-way joist, also known as a grid or waffle slab, shown in Figure 3-1f. To reduce the dead load of solid-slab construction, voids are formed in a rectilinear pattern through use of metal or fiberglass form inserts. A two-way ribbed construction results. Usually inserts are omitted near the columns, so a solid slab is formed to resist moments and shear better in these areas.
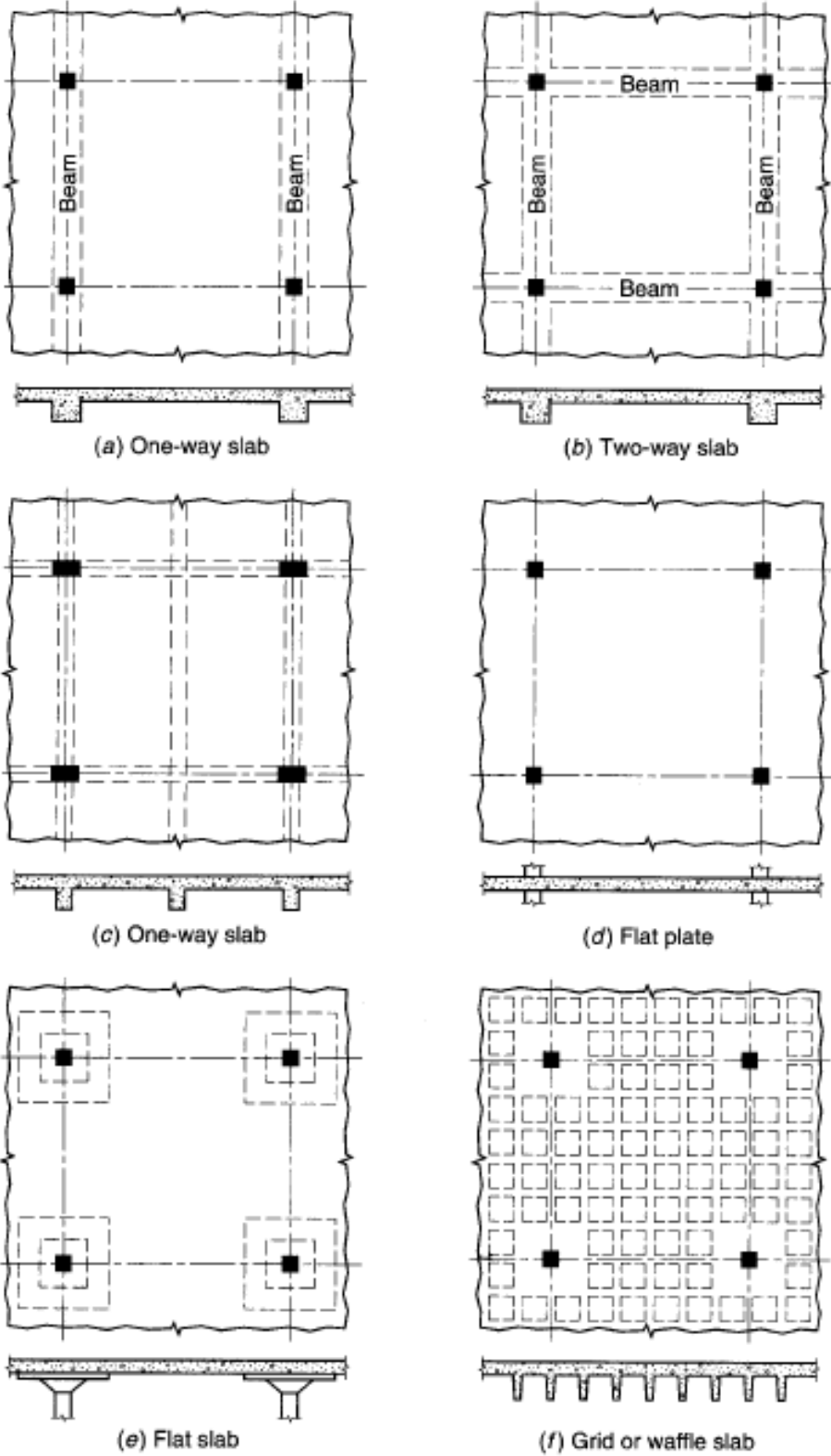
**Figure 3-1 – Types of Structural Slabs**

### 3.2. ANALYSIS AND DESIGN OF TWO WAY SPANNING EDGE SUPPORTED SLABS

#### 3.2.1. BEHAVIOR OF TWO-WAY EDGE-SUPPORTED SLABS

In many cases, rectangular slabs are of such proportions and are supported in such a way that two-way action results. When loaded, such slabs bend into a dished surface rather than a cylindrical one. This means that at any point the slab is curved in both principal directions, and since bending moments are proportional to curvatures, moments also exist in both directions. To resist these moments, the slab must be reinforced in both directions, by at least two layers of bars perpendicular, respectively, to two pairs of edges. The slab must be designed to take a proportionate share of the load in each direction.

The simplest type of two-way slab action is that represented by Figure 3-1b, where the slab, or slab panel, is supported along its four edges by relatively deep, stiff, monolithic concrete beams or by walls or steel girders. If the concrete edge beams are shallow or are omitted altogether, as they are for flat plates and flat slabs, deformation of the floor system along the column lines significantly alters the distribution of moments itself. Two-way systems of this type are considered later in this chapter. The present discussion pertains to the former type, in which edge supports are stiff enough to be considered unyielding.

Such a slab is shown in Figure 3-2 to visualize its flexural performance; it is convenient to think of it as consisting of two sets of parallel strips, in each of the two directions, intersecting each other. Evidently, part of the load is carried by one set and transmitted to one pair of edge supports, and the remainder by the other.
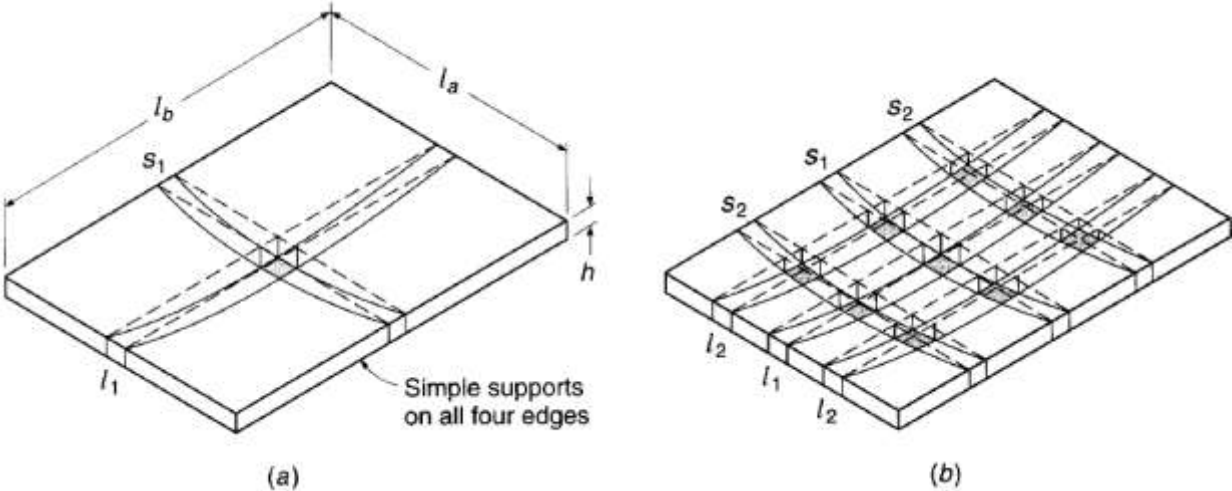


**Figure 3-2 – Two-way slab on simple edge supports: (a) bending of center strips of slab; (b) grid model of slab**

Figure 3-2a shows the two center strips of a rectangular plate with short span $l_a$ and long span $l_b$. If the uniform load is q per square meter of slab, each of the two strips acts approximately as a

simple beam, uniformly loaded by its share of q. because these imaginary strips actually are part of the same monolithic slab; their deflections at the intersection point must be the same. Equating the center deflections of the short and long strips gives

$$\frac{5q_a l_a^4}{384EI} = \frac{5q_b l_b^4}{384EI} \qquad (1)$$

Where $q_a$ is the share of the load q carried in the short direction and $q_b$ is the share of the load q carried in the long direction. Consequently,

$$\frac{q_a}{q_b} = \frac{l_b^4}{l_a^4} \qquad (2)$$

One sees that the larger share of the load is carried in the short direction, the ratio of the two portions of the total load being inversely proportional to the fourth power of the ratio of the spans.

This result is approximate because the actual behavior of a slab is more complex than that of the two intersecting strips. An understanding of the behavior of the slab itself can be gained from Figure 3-2b, which shows a slab model consisting of two sets of three strips each. It is seen that the two central strips $s_1$ and $l_1$ bend in a manner similar to that shown in Figure 3-2a. The outer strips $s_2$ and $l_2$, however, are not only bent but also twisted. Consider, for instance, one of the intersections of $s_2$ and $l_2$. It is seen that at the intersection the exterior edge of strip $l_2$ is at higher elevation than the interior edge, while at the nearby end of strip $l_2$ both edges are at the same elevation; the strip is twisted. This twisting result in torsional stresses and torsional moments that are seen to be most pronounced near the corners. Consequently, the total load on the slab is carried not only by the bending moments in two directions but also by the twisting moments. For this reason, bending moments in elastic slabs are smaller than would be computed for sets of unconnected strips loaded by $q_a$ and $q_b$. For instance, for a simply supported square slab, $q_a = q_b = q/2$. If only bending were present, the maximum moment in each strip would be

$$\frac{(q/2)l^2}{8} = 0.0625ql^2 \qquad (3)$$

The exact theory of bending of elastic plates shows that actually the maximum moment in such a square slab is only $0.048ql^2$, so that in this case the twisting moments relieve the bending moments by about 25 percent.

The largest moments occurs where the curvature is sharpest. Figure 3-2b shows this to be the case at midspan of the short strips $s_1$. Suppose the load is increased until this location is overstressed, so that the steel at the middle of strip $s_1$ is yielding. If the strip were an isolated beam, it would now fail. Considering the slab as a whole, however, one sees that no immediate

failure will occur. The neighboring strips (those parallel as well as those perpendicular to $s_1$), being actually monolithic with it, will take over any additional load that strip $s_1$ can no longer carry until they, in turn, start yielding. This inelastic redistribution will continue until in a rather large area in the central portion of the slab all the steel in both directions is yielding. Only then will the entire slab fail. From this reasoning, which is confirmed by tests, it follows that slabs need not be designed for the absolute maximum moment in each of the two directions (such as $0.048ql^2$ in the example given in the previous paragraph), but only for a smaller average moment in each of the two directions in the central portion of the slab. For instance, one of the several analytical methods in general use permits a square slab to be designed for a moment of $0.036ql^2$. By comparison with the actual elastic maximum moment $0.048ql^2$, it is seen that, owing to inelastic redistribution, a moment reduction of 25 percent is provided.

The largest moment in the slab occurs at midspan of the short strip $s_1$ of Figure 3-2b. It is evident that the curvature, and hence the moment, in the short strip $s_2$ is less than at the corresponding location of strip $s_1$. Consequently, a variation of short-span moment occurs in the long direction of the span. This variation of short-span moment occurs in the long direction of the span. This variation is shown qualitatively in Figure 3-3. The short-span moment diagram in Figure 3-3a is valid along the strip 1-1. Elsewhere, the maximum-moment value is less, as shown. Other moment ordinates are reduced proportionately. Similarly, the long-span moment diagram in Figure 3-3 applies only at the longitudinal centerline of the slab; elsewhere, ordinates are reduced according to the variation shown. These variations in maximum moment across the width and length of a rectangular slab are accounted for in an approximate way in most practical design methods by designing for a reduced moment in the outer quarters of the slab span in each direction.

It should be noted that only slabs with side ratios less than about 2 needs to be treated as two-way slabs. From Equation above, it is seen that for a slab of this proportion, the share of the load carried in the long direction is only on the order of one-sixteenth of that in the short direction. Such a slab acts almost as if it were spanning in the short direction only. Consequently, rectangular slab panels with an aspect ratio of 2 or more may be reinforced for one-way action, with the main steel perpendicular to the long edges.
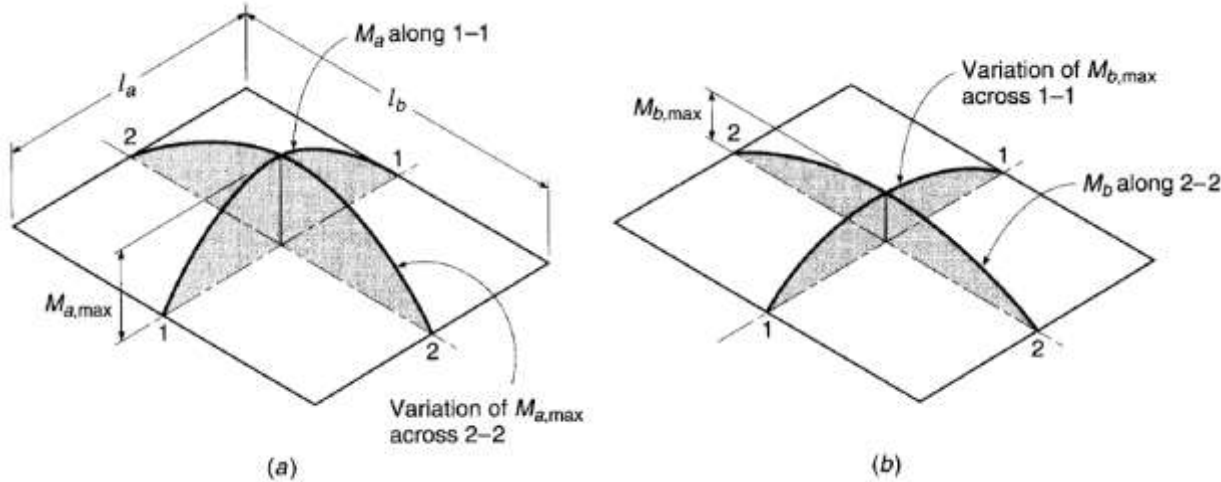
**Figure 3-3 – Moments and moment variations in a uniformly loaded slab with simple supports on four sides**

Consistent with the assumptions of the analysis of two-way edge supported slabs, the main flexural reinforcement is placed in an orthogonal pattern, with reinforcing bars parallel and perpendicular to the supported edges. As the positive steel is placed in two layers, the effective depth d for the upper layer is smaller than that for the lower layer by one bar diameter. Because the moments in the long direction are the smaller ones, it is economical to place the steel in that direction on top of the bars in the short direction. The stacking problem does not exist for negative reinforcement perpendicular to the supporting edge beams except at the corners, where moments are small.

The twisting moments discussed earlier are usually of consequences only at exterior corners of a two-way slab system, where they tend to crack the slab at the bottom along the panel diagonal, and at the top perpendicular to the panel diagonal. Special reinforcement should be provided at exterior corners in both the bottom and top of the slab, for a distance in each direction from the corner in both the bottom and top of the slab, for a distance in each direction from the corner equal to one-fifth the longer span of the corner panel, as shown in Figure 3-4. The reinforcement at the top of the slab should be parallel to the diagonal from the corner, while that at the bottom should be perpendicular to the diagonal.
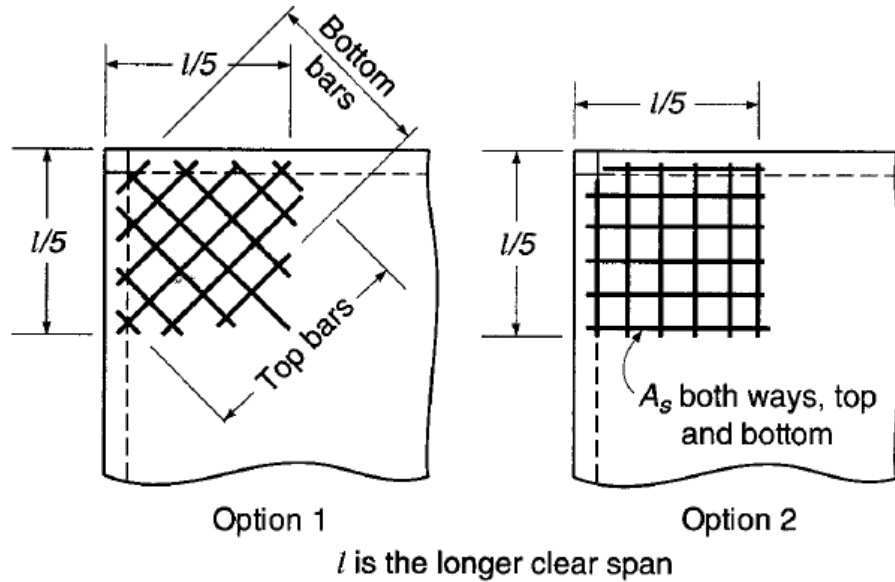
**Figure 3-4 – Special reinforcement at exterior corners of a beam- supported two way slab.**

### 3.2.2. DESIGN OF TWO – WAY EDGE SUPPORTED SLABS USING YIELD-LINE COEEFICIENTS

For rectangular slabs with standard edge conditions and subject to uniformly distributed loads, normally the bending moments are obtained using tabulated coefficients. Such coefficients are provided later in this section.

#### 3.2.2.1. Simply supported slabs

When simply-supported slabs do not have adequate provision to resist torsion at the corners, and to prevent the corners from lifting, the maximum moments per unit width are given by the following equations:

$$M_{sx} = \alpha_{sx} n l_x^2 \tag{4}$$

$$M_{sy} = \alpha_{sy} n l_x^2 \tag{5}$$

Note: Values for $\alpha_{sx}$ and $\alpha_{sy}$ are given in Table 3-1

Where:

$M_{sx}$         Maximum design ultimate moments either over supports or at mid-span on strips of unit width and span $l_x$

$M_{sy}$         Maximum design ultimate moments either over supports or at mid-span on strips of unit width and span $l_y$

$\alpha_{sx}$ and $\alpha_{sy}$    Moment coefficients

$n$           Total design ultimate load per unit area

$l_x$           Length of shorter side

$l_y$                      Length of longer side

The values in Table 3-1are derived from the following equations:

$$\alpha_{sx} = \frac{(l_y/l_x)^4}{8\left(1+(l_y/l_x)^4\right)}$$  (6)

$$\alpha_{sy} = \frac{(l_y/l_x)^2}{8\left(1+(l_y/l_x)^4\right)}$$  (7)

**Table 3-1 - Bending moment coefficients for slabs spanning in two directions at right angles, simply supported on four sides**

| $l_y/l_x$ | 1.0 | 1.1 | 1.2 | 1.3 | 1.4 | 1.5 | 1.75 | 2.0 |
|---|---|---|---|---|---|---|---|---|
| $\alpha_{sx}$ | 0.062 | 0.074 | 0.084 | 0.093 | 0.099 | 0.104 | 0.113 | 0.118 |
| $\alpha_{sy}$ | 0.062 | 0.061 | 0.059 | 0.055 | 0.051 | 0.046 | 0.037 | 0.029 |

### 3.2.2.2. Rectangular panels with restrained edges

In slabs where the corners are prevented from lifting, and provision for torsion is made, the maximum design moments per unit width are given by the following equations.

$$M_{sx} = \beta_{sx} n l_x^2$$  (8)
$$M_{sy} = \beta_{sy} n l_x^2$$  (9)

Note: Values of $\beta_{sx}$ and $\beta_{sy}$ are given in Table 3-2

Where:

$\beta_{sx}$ and $\beta_{sy}$       Moment coefficients

The above equations and the coefficients in Table 3-2 may be derived from the following equations:

$$\beta_y = \left(24 + 2N_d + 1.5N_d^2\right)/1000$$  (10)

$$\gamma = \frac{2}{9}\left[3 - \sqrt{18\frac{l_x}{l_y}\left\{\sqrt{(\beta_y + \beta_1)} + \sqrt{(\beta_y + \beta_1)}\right\}}\right]$$  (11)

$$\sqrt{\gamma} = \sqrt{(\beta_x + \beta_3)} + \sqrt{(\beta_x + \beta_3)}$$  (12)

Where:

$N_d$              Number of discontinuous edges $(0 \le N \le 4)$

$M_{sy}$           Maximum design ultimate moments either over supports or at mid-span on strips of unit width and span $l_y$

$\beta_1$ and $\beta_2$          Hogging moments, per unit width, over the shorter edges divided by $nl_x^2$

$\beta_3$ and $\beta_4$          Hogging moments, per unit width, over the longer edges divided by $nl_x^2$

$l_x$                    Length of shorter side

$l_y$                    Length of longer side

Note: $\beta_1$ and $\beta_2$ take values of $4/3\beta_y$ for continuous edges or zero for discontinuous edges.

$\beta_3$ and $\beta_4$ take values of $4/3\beta_x$ for continuous edges or zero for discontinuous edges.

The conditions in which the equations may be used for continuous slabs only are as follows.

a) The characteristic dead and imposed loads on adjacent panels are approximately the same as on the panel being considered.
b) The span of adjacent panels in the direction perpendicular to the line of the common support is approximately the same as the span of the panel considered in that direction.

**Table 3-2 - Bending moment coefficients for rectangular panels supported on four sides with provision for torsion at corners**

| Type of Panel and moments considered | Short span coefficients, $\beta_{sx}$ | | | | | | | | Long span coefficients, $\beta_{sx}$ for all values of $l_y/l_x$ |
|---|---|---|---|---|---|---|---|---|---|
| | Values of $l_y/l_x$ | | | | | | | | |
| | 1.0 | 1.1 | 1.2 | 1.3 | 1.4 | 1.5 | 1.75 | 2.0 | |
| **Interior panels** Negative moment at continuous edge | 0.031 | 0.037 | 0.042 | 0.046 | 0.050 | 0.053 | 0.059 | 0.063 | 0.032 |
| Positive moment at mid-span | 0.024 | 0.028 | 0.032 | 0.035 | 0.037 | 0.040 | 0.044 | 0.048 | 0.024 |
| **One short edge discontinuous** Negative moment at continuous edge | 0.039 | 0.044 | 0.048 | 0.052 | 0.055 | 0.058 | 0.063 | 0.067 | 0.037 |
| Positive moment at mid-span | 0.029 | 0.033 | 0.036 | 0.039 | 0.041 | 0.043 | 0.047 | 0.050 | 0.028 |
| **One long edge discontinuous** Negative moment at continuous edge | 0.039 | 0.049 | 0.056 | 0.063 | 0.068 | 0.073 | 0.082 | 0.089 | 0.037 |
| Positive moment at mid-span | 0.030 | 0.036 | 0.042 | 0.047 | 0.051 | 0.055 | 0.062 | 0.067 | 0.028 |
| **Two adjacent edges discontinuous** | 0.030 | 0.036 | 0.042 | 0.o047 | 0.051 | 0.055 | 0.062 | 0.067 | 0.028 |
| Negative moment at continuous edge | 0.047 | 0.056 | 0.063 | 0.069 | 0.074 | 0.078 | 0.087 | 0.093 | 0.045 |
| Positive moment at mid-span | 0.036 | 0.042 | 0.047 | 0.051 | 0.055 | 0.059 | 0.065 | 0.070 | 0.034 |
| **Two short edges discontinuous** Negative moment at | 0.046 | 0.050 | 0.054 | 0.057 | 0.060 | 0.062 | 0.067 | 0.070 | - |

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| continuous edge Positive moment at mid-span | 0.034 | 0.038 | 0.040 | 0.043 | 0.045 | 0.047 | 0.050 | 0.053 | 0.034 |
| **Two long edges discontinuous** Negative moment at continuous edge | - | -- | - | - | - | - | - | - | 0.045 |
| Positive moment at mid-span | 0.034 | 0.046 | 0.056 | 0.065 | 0.072 | 0.078 | 0.091 | 0.100 | 0.034 |
| **Three edges discontinuous (one long edge continuous)** Negative moment at continuous edge | 0.057 | 0.065 | 0.071 | 0.076 | 0.081 | 0.084 | 0.092 | 0.098 | - |
| Positive moment at mid-span | 0.043 | 0.048 | 0.053 | 0.057 | 0.060 | 0.063 | 0.069 | 0.074 | 0.044 |
| **Three edges discontinuous (one short edge continuous)** Negative moment at continuous edge | - | - | - | - | - | - | - | - | 0.058 |
| Positive moment at mid-span | 0.042 | 0.054 | 0.063 | 0.071 | 0.078 | 0.084 | 0.096 | 0.105 | 0.044 |
| **Four edges discontinuous** Positive moment at mid-span | 0.055 | 0.065 | 0.074 | 0.081 | 0.087 | 0.092 | 0.103 | 0.111 | 0.056 |
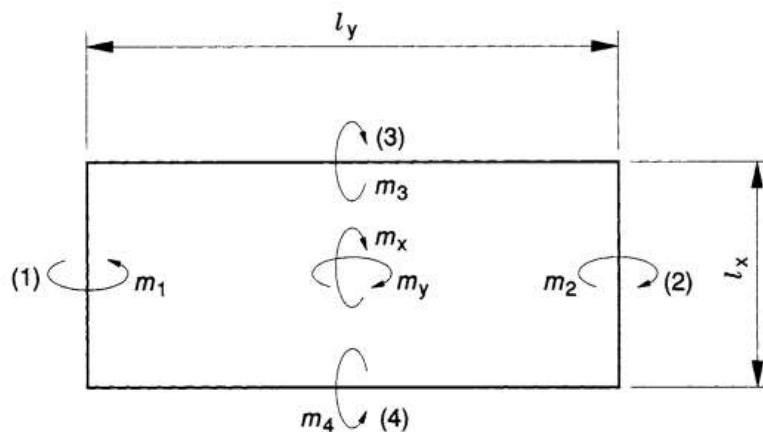


**Figure 3-5 – Explanation of the derivation of the coefficient of Table 3-2**

### 3.2.2.3. Restrained slab with unequal conditions at adjacent panels

$M_{-1}$ and $M_{-2}$ are the support moments for panels 1 and 2, respectively, and $M_{+1}$ and $M_{+2}$ are the span moments for panels 1 and 2, respectively.

In some cases, the bending moments at a common support, obtained by considering the two adjacent panels in isolation, may differ significantly (say by 10%), because of the differing edge condition at the far supports or differing span lengths or loading.

Consider panels 1 and 2 in Figure 3-6. As the support on grid A for panel 1 is discontinuous and support on grid C for panel 2 is continuous, the moments for panels 1 and 2 for the support on grid C for panel 2 is continuous, the moments for panels 1 and 2 for the support on grid B could be significantly different. In these circumstances, the slab may be reinforced throughout for the worst case span and support moments. However, this might be uneconomic in some cases. In such cases, the following distribution procedure may be used:

1) Obtain the support moments for panels 1 and 2 from Table. Treating $M_{-1}$ and $M_{-2}$ as fixed end moments, the moments may be distributed in proportion to the stiffenesses of span $l_x$ in panels 1 and 2. Thus, a revised bending moments $M'_{-B}$ may be obtained for support over B.

2) The span moments in panels 1 and 2 should be recalculated as follows:
$$M'_{+1} = \left(M_{-1} + M_{+1}\right) - M'_{-B} \tag{13}$$
$$M'_{+2} = \left(M_{-2} + M_{-2} + M_{+2}\right) - M'_{-B} - M_{-2} \tag{14}$$
(Note that this assumes that the final moment over C is $M_{-2}$

3) For curtailment of reinforcement, the point of contraflexure may be obtained by assuming a parabolic distribution of moments in each panel.
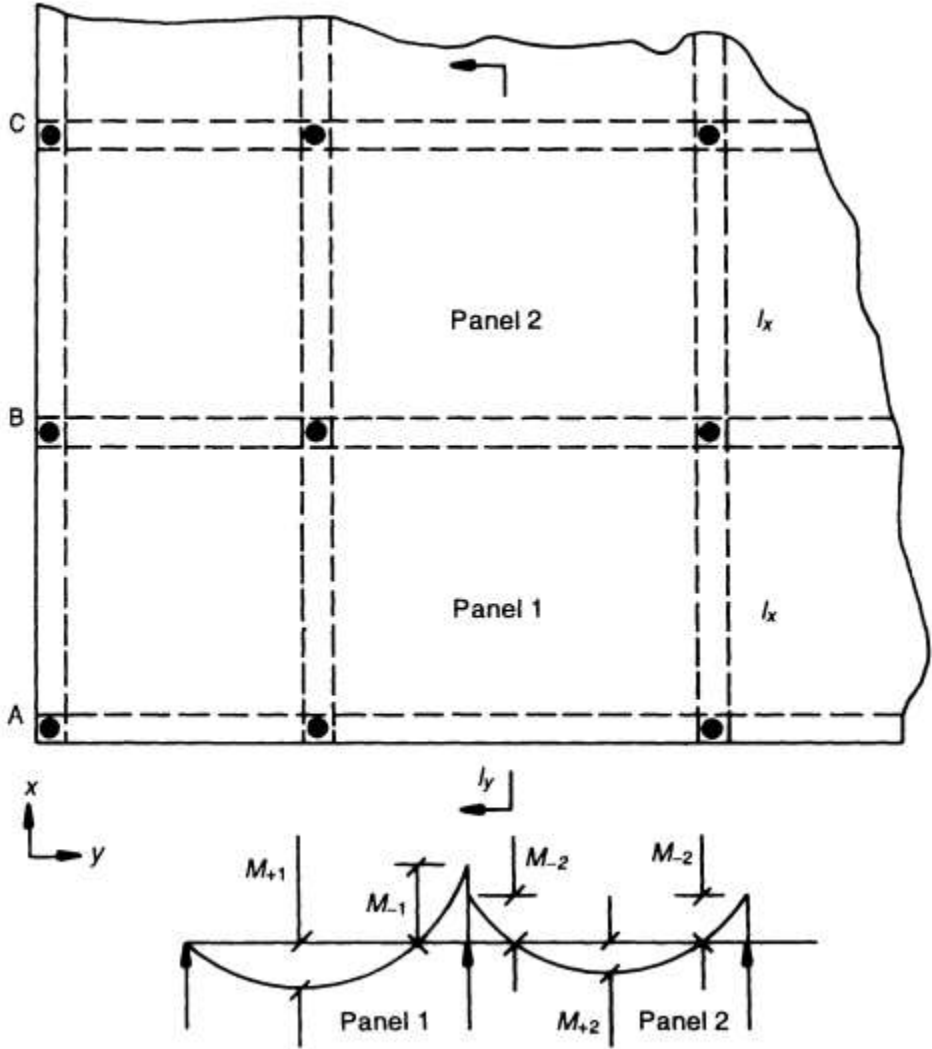
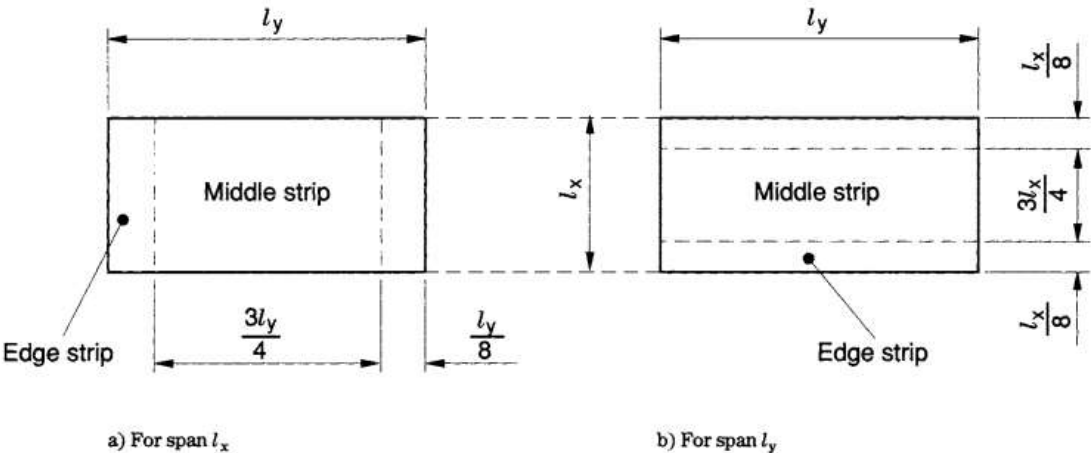**Figure 3-6 – Two-way spanning slabs: unequal edge condition in adjacent panels**



**Figure 3-7 – Division of slab into middle and edge strip**

### 3.2.2.4. Loads on supporting beams

The design loads on beams supporting solid slabs spanning in two directions at right angles and supporting uniformly distributed loads may be assessed from the following equations:

$$\upsilon_{sy} = \beta_{vy} nl_x \tag{15}$$

$$\upsilon_{sx} = \beta_{vx} nl_x \tag{16}$$

Where:

$\upsilon_{sy}$           Design end sear on strips of unit width and span $l_y$ and considered to act over the middle three-quarters of the edge.

$\upsilon_{sx}$           Design end sear on strips of unit width and span $l_x$ and considered to act over the middle three-quarters of the edge.

Where design ultimate support moments are used which differ substantially from those that would be assessed from Table, adjustment of the values given in Table may be necessary. The assumed distribution of the load on a supporting beam is shown in Figure 3-8.
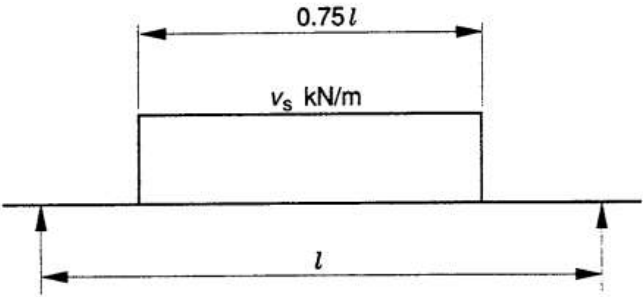


NOTE   $\upsilon_s = \upsilon_{sx}$ when $l = l_y$; $\upsilon_s = \upsilon_{sy}$ when $l = l_x$;

**Figure 3-8 – Distribution of load on a beam supporting a two-way spanning slabs**

**Table 3-3 – Shear force coefficients for uniformly loaded rectangular panels supported on four sides with provision for torsion at corners**

| Type of Panel and location | $\beta_{vx}$ for values of $l_y/l_x$ | | | | | | | | $\beta_{vy}$ |
|---|---|---|---|---|---|---|---|---|---|
| | 1.0 | 1.1 | 1.2 | 1.3 | 1.4 | 1.5 | 1.75 | 2.0 | |
| **Four edges continuous** Continuous edge | 0.33 | 0.36 | 0.39 | 0.41 | 0.43 | 0.45 | 0.48 | 0.50 | 0.33 |
| **One short edge discontinuous** Continuous edge | 0.36 | 0.39 | 0.42 | 0.44 | 0.45 | 0.47 | 0.50 | 0.52 | 0.36 |
| Discontinuous edge | - | - | - | - | - | - | - | - | 0.24 |
| **One long edge discontinuous** Continuous edge | 0.36 | 0.40 | 0.44 | 0.47 | 0.49 | 0.51 | 0.55 | 0.59 | 0.36 |
| Discontinuous edge | 0.24 | 0.27 | 0.29 | 0.31 | 0.32 | 0.34 | 0.36 | 0.38 | - |
| **Two adjacent edges discontinuous** Continuous edge | 0.40 | 0.44 | 0.47 | 0.50 | 0.52 | 0.54 | 0.57 | 0.60 | 0.40 |
| Discontinuous edge | 0.26 | 0.29 | 0.31 | 0.33 | 0.34 | 0.35 | 0.38 | 0.40 | 0.26 |
| **Two short edges discontinuous** Continuous edge | 0.40 | 0.43 | 0.45 | 0.47 | 0.48 | 0.49 | 0.52 | 0.54 | - |
| Discontinuous edge | - | - | - | - | - | - | - | - | 0.26 |
| **Two long edges discontinuous** Continuous edge | - | - | - | - | - | - | - | - | 0.40 |
| Discontinuous edge | 0.26 | 0.30 | 0.33 | 0.36 | 0.38 | 0.40 | 0.44 | 0.47 | - |
| **Three edges discontinuous (one long edge discontinuous)** Continuous edge | 0.45 | 0.48 | 0.51 | 0.53 | 0.55 | 0.57 | 0.60 | 0.63 | - |
| Discontinuous edge | 0.30 | 0.32 | 0.34 | 0.35 | 0.36 | 0.37 | 0.39 | 0.41 | 0.29 |
| **Three edges discontinuous (one short edge discontinuous)** Continuous edge | - | - | - | - | - | - | - | - | 0.45 |
| Discontinuous edge | 0.29 | 0.33 | 0.36 | 0.38 | 0.40 | 0.42 | 0.45 | 0.48 | 0.30 |
| **Four edges discontinuous** Discontinuous edge | 0.33 | 0.36 | 0.39 | 0.41 | 0.43 | 0.45 | 0.48 | 0.50 | 0.33 |

### 3.2.3. DESIGN FOR SHEAR

It is not usual for a slab to contain shear reinforcement; therefore it is only necessary to ensure that the concrete shear stress capacity without shear reinforcement is more than applied shear stress.

### 3.3. FLAT SLABS

Flat slabs design is appropriate for most floor situations and is also suitable for irregular column layouts, curved floor shapes, ramps, etc. The benefits of choosing flat slabs include the following:

- A minimum depth solution leads to least cladding and has advantages in planning height
- Speed of construction
- Flexibility in the plan layout, both in terms of the shape and column layout
- Flat soffit, allowing clean finishes or freedom of layout of services
- The use of economic large spans
- Scope for cutting holes, alterations and repairs
- Scope for much repetition
- Reduction in the need for drop beams (and up-stand beams)
- Scope and space for the use of flying forms
- Good for sound attenuation

### 3.3.1. TYPICAL BEHAVIOR OF A FLAT SLAB

#### 3.3.1.1. Typical bending failure mode

A flat slab spans between column supports without the need for beams. For a regular layout of columns, failure can occur by the formation of hinge lines along the lines of maximum hogging and sagging moments. A complementary set of yield lines can form in the orthogonal direction.
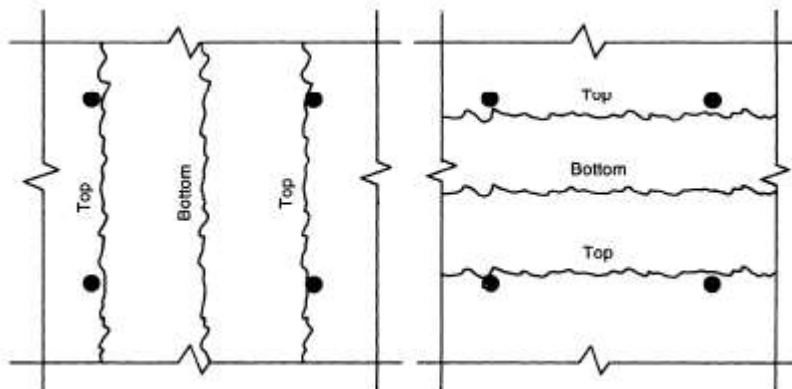


**Figure 3-9 – Possible failure modes of flat slabs**

One misconception of some engineers is to consider a reduced loading when analyzing in a particular direction. The moments applied in each orthogonal direction must each sustain the total loading to maintain equilibrium. There is no sharing of the load by partial resistance in each orthogonal direction.

### 3.3.1.2. Typical deflected shape of an interior panel

The deflected shape of an interior panel of a flat slab on a regular grid of columns under typical in-service conditions is a function of the sum of the deflections in each orthogonal direction as shown in Figure 3-10. Similar deflected shapes will obtain from an irregular grid of columns, but the interaction between adjacent bays may be more complex.
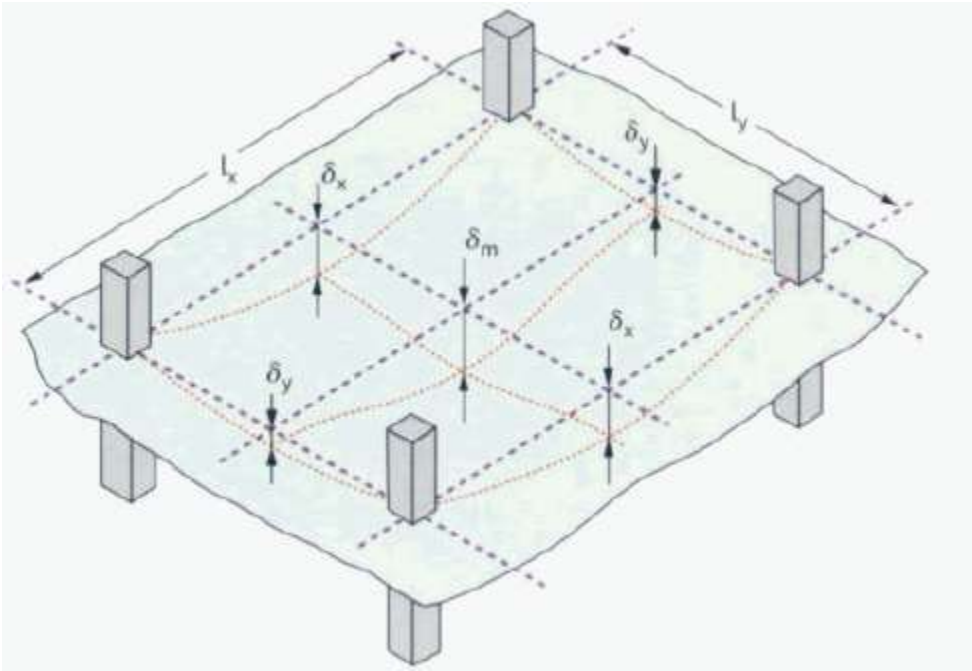


**Figure 3-10 – Typical deflected shape of an interior flat slab panel.**

### 3.3.1.3. Moment contours

The use of finite element methods shows that the distribution of bending moments per unit width is characterized by hogging moments that are sharply peaked in the immediate vicinity of the columns. The magnitude of the hogging moments locally to the column face can be several times that of the sagging moments in the mid-span zones. These moments do occur in practice and the design should take them into account. Redistribution allows a more uniform spread of reinforcement but increases the likelihood of cracking.

A typical distribution of bending stresses for a uniformly distributed load on a flat slab with a regular layout of columns is illustrated in Figure 3-11.
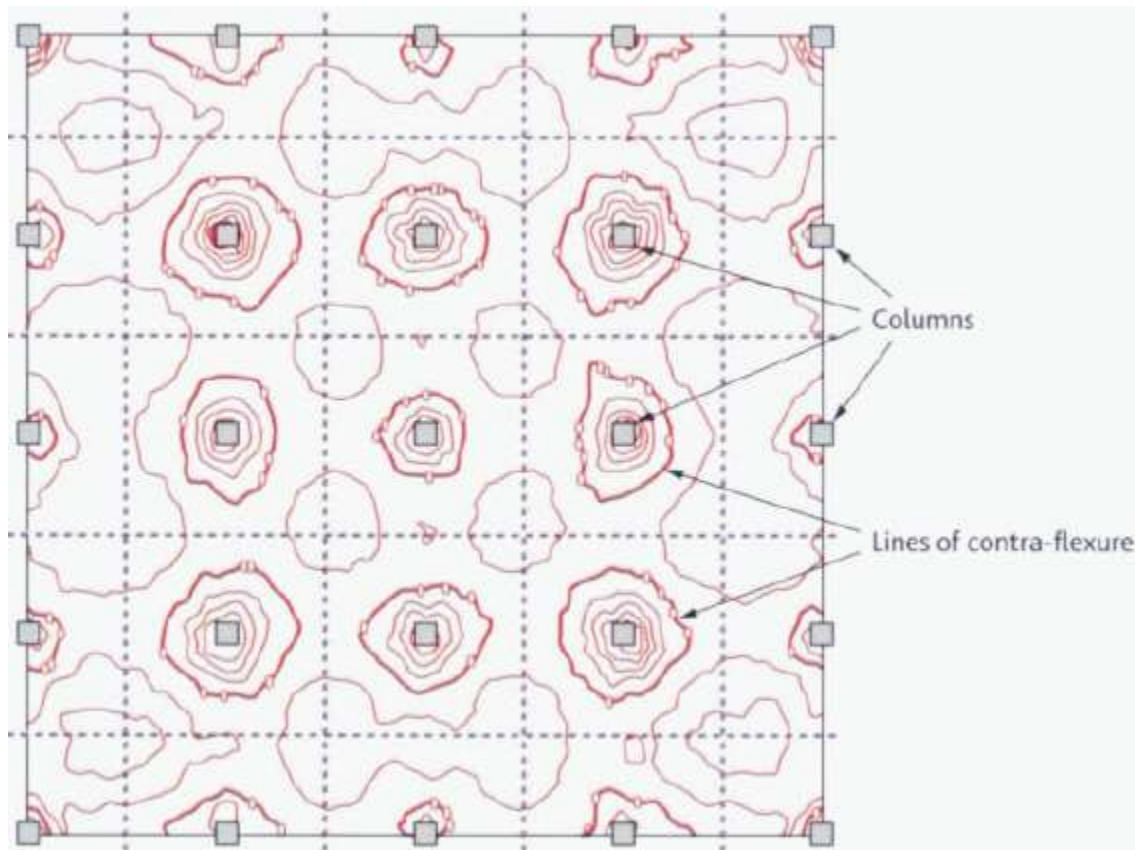
**Figure 3-11 - Typical distribution of bending stress for a flat slab**

### *3.3.1.4. Flexural behavior of a flat slab as the vertical load is increased*
A typical load/deflection curve of a flat slab is shown in Figure 3-12.

As the vertical load on the slab increases, the following changes occur:

- Moments at the supports and mid-span increase elastically until the first cracks occur. These are likely to appear first at the top of the slab close to the column and may occur during construction if the removal of formwork takes place early. Otherwise, this limited cracking may occur under the quasi-permanent combination of actions
- As the loading is increased beyond the characteristic combination of actions, cracking may increase to some way into the span from the column, and cracks may also have started to appear at mid-span. This is unlikely under the frequent or quasi-permanent combination of actions. The cracking increases the non-linear behavior of the slab, although it still behaves elastically as the load increases between the formations of new cracking, and can be modeled elastically-taking account of the tension stiffening of the concrete.
- As the loading is further increased, the reinforcement first starts to yield in the top bars close to the columns and the junction of the slab at edge column starts to behave as a plastic hinge. Apart from this, the slab still behaves elastically as the load increases between the formation of new cracks but with reducing tension stiffening.

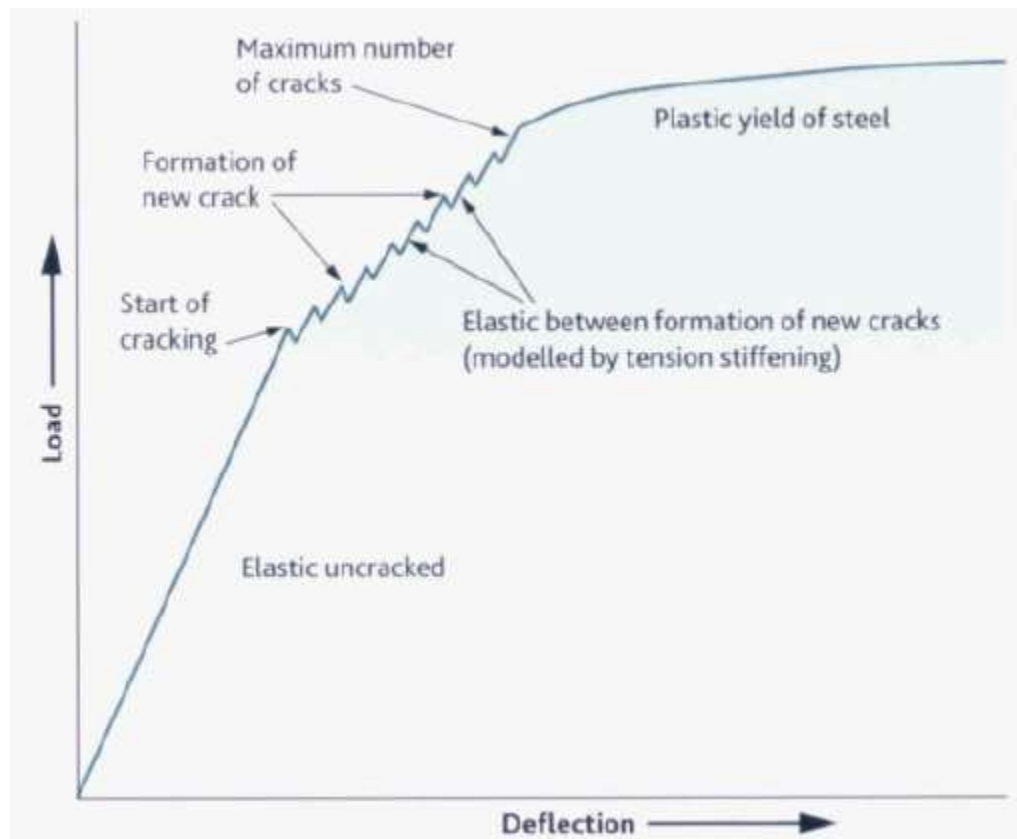- Failure will occur once a failure mechanism is reached



**Figure 3-12 – Typical load/deflection behavior of flat slab**

## 3.3.2. DESIGN OF FLAT SLABS ACCORDING TO ES EN 1992:2014

There are a number of different methods for the design of flat slabs. These include:

- Simplified moment coefficients (based on tests, experience and yield-line methods). This method is suitable for regular layouts of columns where the spans are constant.
- Equivalent frame. This method is suitable for regular layouts of columns, but requires engineering judgment for irregular layouts
- Finite element analysis. This method allows the design of irregular column layouts and can provide the design of reinforcement details. Where the appropriate software is available, it is possible to obtain reasonable assessment of deflections.
- Grillage analysis. This method has similar facilities to finite element models and can also be used for irregular layouts of columns.
- Yield-line methods. These can provide suitable designs for ULS but do not give adequate information for serviceability design.
  However in comparison to ACI Code, the Eurocode does not give detailed procedure for each of the above listed analysis method. Thus, design procedure of flat slabs according to ACI will be discussed and followed throughout this course.

### 3.3.3. DESIGN OF FLAT SLABS USING ACI

ACI Code Section 13.5 allows slabs to be designed by any procedure that satisfies both equilibrium and geometric compatibility, provided that every section has a strength at least equal to the required strength and that serviceability conditions are satisfied. Two procedures for the flexural analysis and design of two-way floor systems are presented in detail in the ACI Code. These are the direct-design method and the equivalent frame design method. These two methods differ primarily in the way in which the slab moments are computed. The calculation of moments in the direct-design method is based on the total statical moment. In this method, the slab is considered panel by panel, and the total moment in each panel is computed in each direction. The statiacal moment then is divided between positive and negative moments, and these are further divided between middle strips and column strips.

The second method is called the equivalent-frame method. Here, the slab is divided into a series of two-dimensional frames (in each direction), and the positive and negative moments are computed via an elastic-frame analysis. Once the positive and negative moments are known, they are divided between middle strips and column strips in exactly the same way as in the direct-design method.

The direct-design method is emphasized in this book because an understanding of the method is essential for understanding the concepts of two-way slab design. In addition, it is an excellent method of checking slab design calculations. In design practice, computer programs based on the principles of the equivalent-frame method often are used.

### 3.3.3.1. Steps in slab design

The steps in the design of a two-way slab are the following:

1. Choose the layout and type of slab to be used. The choice of type of slab is strongly affected by architectural and construction considerations.
2. Choose the slab thickness. Generally, the slab thickness is chosen to prevent excessive deflection in service. Equally important, the slab thickness chosen must be adequate for shear at both interior and exterior columns.
3. Choose the method for computing the design moments. Equivalent-frame methods use an elastic-frame analysis to compute the positive and negative moments in the various panels in the slab. The direct-design method uses coefficients to compute these moments.
4. Calculate the distribution of the moments across the width of the slab. The lateral distribution of moments within a panel depends on the geometry of the slab and the stiffness of the beams (if any). This procedure is the same whether the negative and positive moments are calculated from the direct-design method or from an equivalent-frame method.
5. If there are beams, assign a portion of the column strip moment to the beams.
6. Design reinforcement for the moments from steps 4 and 5. (Note: steps 3 through 6 need to be done for both principal directions.)
7. Check shear strength at a critical section around the columns.

### 3.3.3.2. Beam-to-Slab Stiffness Ratio, $\alpha_f$

Slabs frequently are built with beams spanning from column to column around the perimeter of the building. These beams act to stiffen the edge of the slab and help to reduce the deflections of

the exterior panels of the slab. Very heavily loaded slabs and long-span waffle slabs sometimes have beams joining all of the columns in the structure.

In the ACI Code, the effects of beam stiffness on deflections and the distribution of moments are expressed as a function of $\alpha_f$, defined as the flexural stiffness, $4EI/I$, of the beam divided by the flexural stiffness of a width of slab bounded laterally by the centerlines of the adjacent panels on each side of the beam:

$$\alpha_f = \frac{4E_{cb}I_b/I}{4E_{cs}I_s/I} \tag{17}$$

Because the lengths, $I$, of the beam and slab are equal, this quantity is simplified and expressed in the code as

$$\alpha_f = \frac{E_{cb}I_b}{E_{cs}I_s} \tag{18}$$

where $E_{cb}$ and $E_{cs}$ are the moduli of elasticity of the beam concrete and slab concrete, respectively, and $I_b$ and $I_s$ are the moments of inertia of the uncracked beams and slabs. The sections considered in computing $I_b$ and $I_s$ are shown shaded in **Figure 3-13**. The span perpendicular to the direction being designed is $I_2$.. If there is no beam, $\alpha_f = 0$.
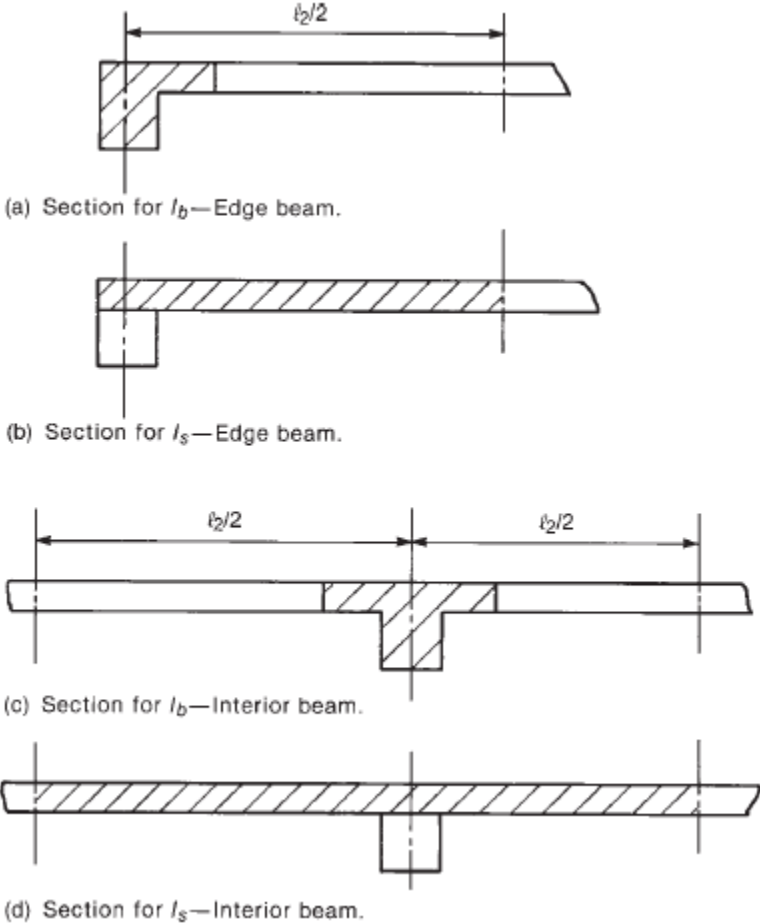
(a) Section for $l_b$—Edge beam.

(b) Section for $l_s$—Edge beam.

(c) Section for $l_b$—Interior beam.

(d) Section for $l_s$—Interior beam.

**Figure 3-13 – Beam and slab sections for calculations of $\alpha_f$**

ACI Section 13.2.4 defines a beam in monolithic or fully composite construction as the beam stem plus a portion of the slab on each side of the beam extending a distance equal to the projection of the beam above or below the slab, whichever is greater, but not greater than four times the slab thickness. This is illustrated in **Figure 3-14**. Once the size of the slab and beam have been chosen, values of $l_b$ and $l_s$ can be computed from first principles.
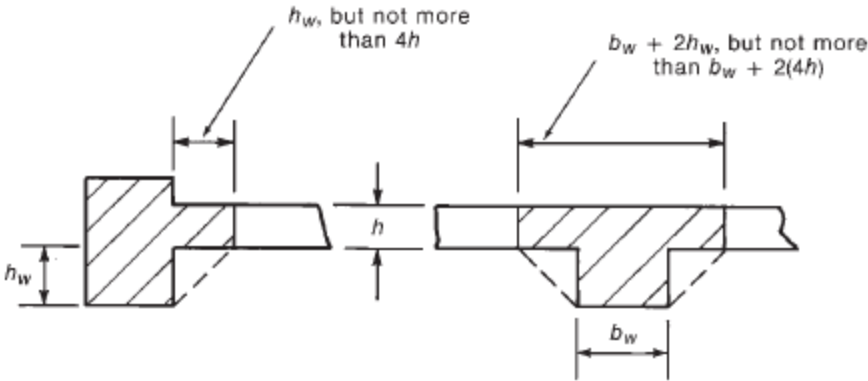
**Figure 3-14 – Cross section of beams as defined in ACI Code Section 13.2.4.**

### 3.3.3.3. The direct-design method

The direct-design method also could have been called "the direct-analysis method", because this method essentially prescribes values for moments in various parts of the slab panel without the need for a structural analysis. The reader should be aware that this design method was introduced in an era when most engineering calculations were made with a slide rule and computer software was not available to do the repetitive calculations required to analyze a continuous-floor slab system. Thus, for continuous slab panels with relatively uniform lengths and subjected to distributed loading, a series of moment coefficients were developed that would lead to safe flexural designs of two-way floor systems.

## ❖ Limitations on the Use of the Direct-Design Method

The direct-design method is easier to use than the equivalent-frame method, but can be applied only to fairly regular multi-panel slabs. The limitations, given in ACI Code Section 13.6.1, include the following:

**1.** There must be a minimum of three continuous spans in each direction. Thus, a nine-panel structure (3 by 3) is the smallest that can be considered. If there are fewer than three panels, the interior negative moments from the direct-design method tend to be too small.

**2.** Rectangular panels must have a long-span/short-span ratio that is not greater than 2. One-way action predominates as the span ratio reaches and exceeds 2.

**3.** Successive span lengths in each direction shall not differ by more than one third of the longer span. This limit is imposed so that certain standard reinforcement cutoff details can be used.

**4.** Columns may be offset from the basic rectangular grid of the building by up to 0.1 times the span parallel to the offset. In a building laid out in this way, the actual column locations are used in determining the spans of the slab to be used in calculating the design moments.

**5.** All loads must be due to gravity only and uniformly distributed over an entire panel. The direct-design method cannot be used for unbraced, laterally loaded frames, foundation mats, or prestressed slabs.

**6.** The service (unfactored) live load shall not exceed two times the service dead load. Strip or checkerboard loadings with large ratios of live load to dead load may lead to moments larger than those assumed in this method of analysis.

**7.** For a panel with beams between supports on all sides, the relative stiffness of the beams in the two perpendicular directions given by $\left(\alpha_{f1}l_2^2\right)/\left(\alpha_{f2}l_1^2\right)$ shall not be less than 0.2 or greater than 5.

Limitations 2 and 7 do not allow use of the direct-design method for slab panels that transmit load as one-way slabs.

## ❖ Distribution of moments within panels –slabs without beams between all supports

**Statical Moment** $M_0$

For design, the slab is considered to be a series of frames in the two directions, as shown in **Figure 3-15**. These frames extend to the middle of the panels on each side of the column lines. In each span of each of the frames, it is necessary to compute the total statical moment, $M_0$. We thus have

$$M_0 = \frac{q_u l_2 l_n^2}{8}$$

(19)

Where

| $q_u =$ | Factored load per unit area |
|---|---|
| $l_2 =$ | Transverse width of the strip |
| $l_n =$ | Clear span between columns |
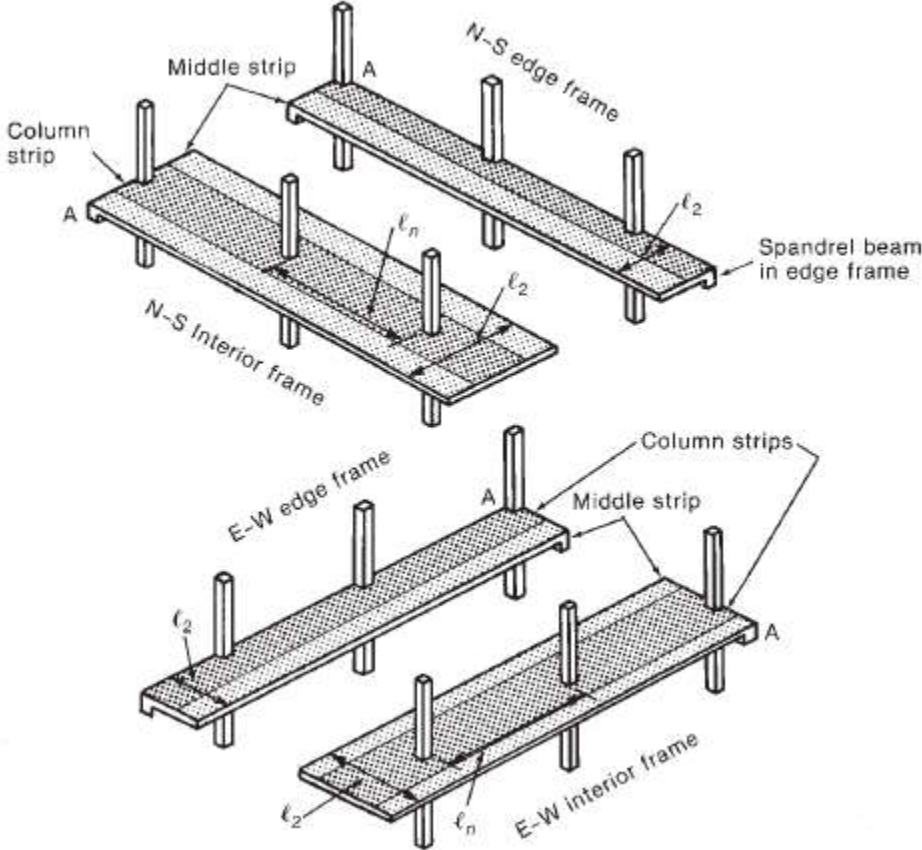


**Figure 3-15 – Division of slab into frame for design**

❖ **Positive and Negative Moments in Panels**

In the direct-design method, the total factored statical moment is divided into positive and negative factored moments according to rules given in ACI Code Section 13.6.3. These are illustrated in **Figure 3-16**. In interior spans, 65 percent of $M_0$ is assigned to the negative moment region and 35 percent to the positive-moment regions. This is approximately the same as for a uniformly loaded, fixed-ended beam, where the negative moment is 67 percent of $M_0$ and the positive moment is 33 percent.

The exterior end of an exterior span has considerably less fixity than the end at the interior support. The division of $M_0$ in an end span into positive- and negative-moment regions is given in **Table 4**. In this table, exterior edge unrestrained" refers to a slab whose exterior edge rests on, but is not attached to, for example, a masonry wall. "Exterior edge fully restrained" refers to a slab whose exterior edge is supported by, and is continuous with, a concrete wall with a flexural stiffness as large or larger than that of the slab.

If the computed negative moments on two sides of an interior support are different, the negative-moment section of the slab is designed for the larger of the two.
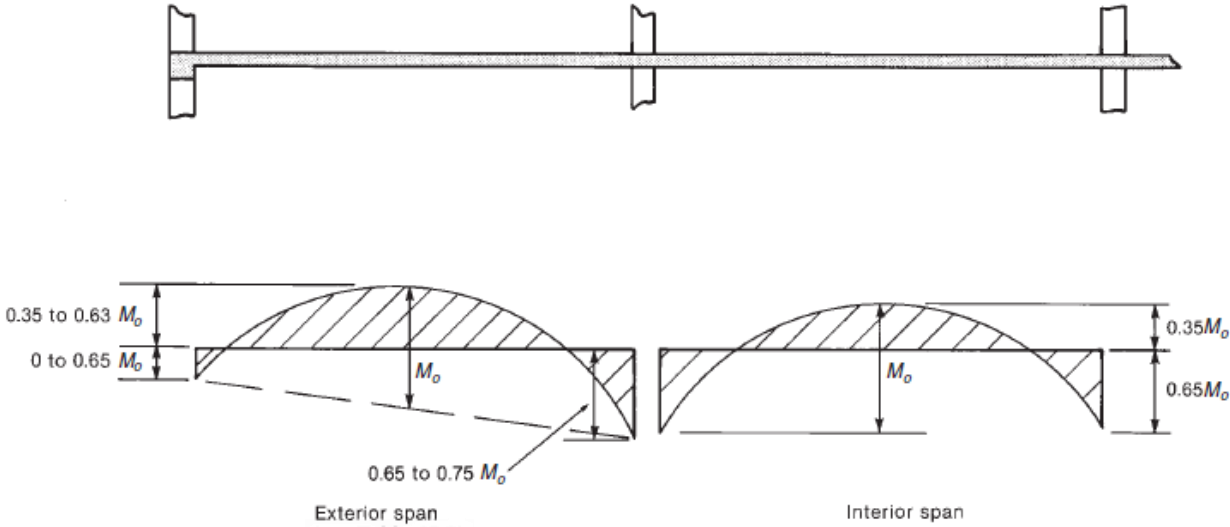


**Figure 3-16 – Assignment of positive- and negative-moment regions**

**Table 4 – Distribution of Total Factored Static Moment, $M_0$, in an Exterior span**

| | (1) Exterior Edge Unrestrained | (2) Slab *with* Beams between *All* Supports | (3) (4) Slab *without* Beams between *Interior* Supports | | (5) Exterior Edge Fully Restrained |
|---|---|---|---|---|---|
| | | | Without Edge Beam | With Edge Beam | |
| Interior Negative Factored Moment | 0.75 | 0.70 | 0.70 | 0.70 | 0.65 |
| Midspan Positive Factored Moment | 0.63 | 0.57 | 0.52 | 0.50 | 0.35 |
| Exterior Negative Factored Moment | 0 | 0.16 | 0.26 | 0.30 | 0.65 |

❖ **Definition of Column Strips and Middle Strips**

As shown in **Figure 3-17**, the moments vary continuously across the width of the slab panel. To aid in steel placement, the design moments are averaged over the width of column strips over the columns and middle strips between the column strips as shown in **Figure 3-17**d. The widths of these strips are defined in ACI Code Sections 13.2.1 and 13.2.2 and are illustrated in **Figure 3-18**. The column strips in both directions extend one fourth of the smaller span, each way from the column line.
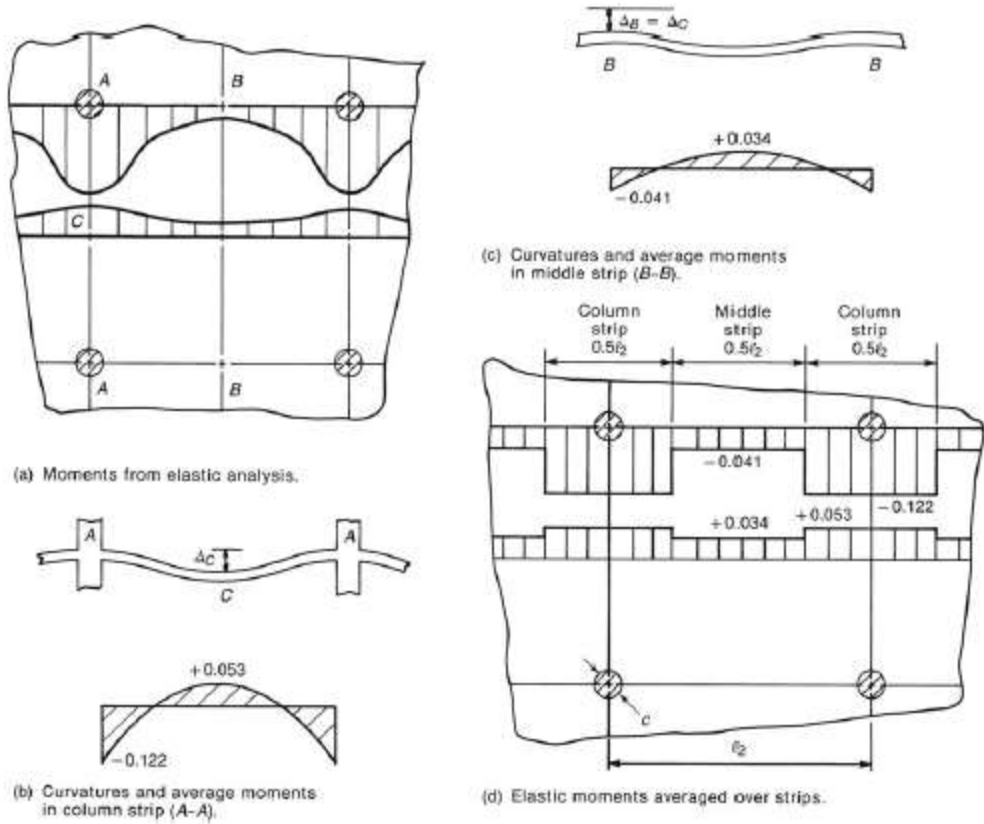
(a) Moments from elastic analysis.

(b) Curvatures and average moments in column strip (A–A).

(c) Curvatures and average moments in middle strip (B–B).

(d) Elastic moments averaged over strips.

**Figure 3-17 – Moments in a slab supported on isolated columns**

❖ **Distribution of Moments between Column Strips and Middle Strips**

ACI Code Section 13.6.4 defines the fraction of the negative and positive moments assigned to the column strips. The remaining amount of negative and positive moment is assigned to the adjacent half-middle strips. The division is a function of $\alpha_{f1} l_2/l_1$, which depends on the aspect ratio of the panel, $l_2/l_1$, and the relative stiffness, $\alpha_{f1}$, of the beams (if any) spanning parallel to and within the column strip.

(a) Short direction of panel.
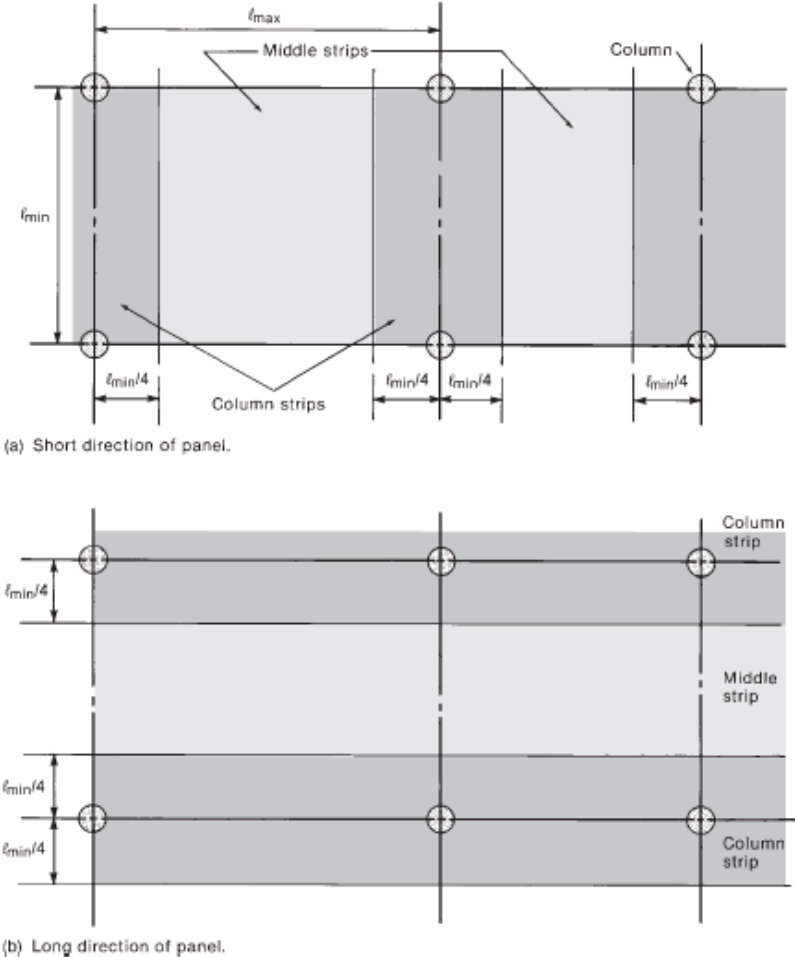
(b) Long direction of panel.

**Figure 3-18 – Definitions of columns and middle strips**

**Table 5** gives the percentage distribution of negative factored moment to the column strip at all interior supports. For floor systems without interior beams, $\alpha_{f1} l_2/l_1$ is taken to be equal to zero, because $\alpha_{f1} = 0$. In this case, 75 percent of the negative moment is distributed to the column strip, and the remaining 25 percent is divided equally between the two adjacent half-middle strips. For cases where a beam is present in the column strip (spanning in the direction of $l_1$ ) and $\alpha_{f1} l_2/l_1 \geq 1.0$, the second row in **Table 5** applies. Some linear interpolation may be required based on the ratio $l_2/l_1$. For cases where $0 \leq \alpha_{f1} l_2/l_1 \leq 1.0$, linear interpolations will be required between the percentages given in the first and second rows of **Table 5**. Similar procedures are used for the distribution of the factored moments at other locations along the span.

**Table 6** gives the percentage distribution of positive factored moment to the column strip at midspan for both interior and exterior spans. For floor systems without interior beams, 60 percent of the positive moment is assigned to the column strip and the remaining 40 percent is divided equally between the adjacent half-middle strips. If a beam is present in the column strip (spanning in the direction of $l_1$), either the percentages in the second row or a linear interpolation between the percentages given in the first and second rows of **Table 6** will apply.

**Table 5 -– Percentage distribution of interior negative factored moment to column strip**

| $\ell_2/\ell_1$ | 0.5 | 1.0 | 2.0 |
|---|---|---|---|
| $(\alpha_{f1}\ell_2/\ell_1) = 0$ | 75 | 75 | 75 |
| $(\alpha_{f1}\ell_2/\ell_1) \geq 1.0$ | 90 | 75 | 45 |

At an exterior edge, the division of the exterior-end factored negative moment distributed to the column and middle strips spanning perpendicular to the edge also depends on the torsional stiffness of the edge beam, calculated as the shear modulus, $G$, times the torsional constant of the edge beam, $C$, divided by the flexural stiffness of the slab spanning *perpendicular* to the edge beam (i.e., *EI* for a slab having a width equal to the length of the edge beam from the center of one span to the center of the other span). Assuming that Poisson's ratio is zero gives $G = E/2$ then this torsional stiffness ratio is defined as

$$\beta_t = \frac{E_{cb}C}{2E_{cs}I_s} \tag{20}$$

where the cross section of the edge beam is as defined in ACI Code Section 13.2.4 . If there are no edge beams, $\beta_t$ can be taken to be equal to zero.

The term C in Eq. (21) refers to the torsional constant of the edge beam. This is roughly equivalent to a polar moment of inertia. It is calculated by subdividing the cross section into rectangles and carrying out the summation where *x* is the shorter side of a rectangle and *y* is the longer side. The subdivision of the cross section of the torsional members is illustrated in **Figure 3-19**. Different combinations of rectangles should be tried to get the *maximum* value of *C*. The maximum value normally is obtained when the wider rectangle is made as long as possible. Thus, the rectangles chosen in **Figure 3-19**b will give a larger value of *C* than will those in **Figure 3-19**a.

$$C = \sum\left[\left(1-0.63\frac{x}{y}\right)\frac{x^3 y}{3}\right] \tag{21}$$

**Table 6 – Percentage distribution of midspan positive factored moment to column strip**

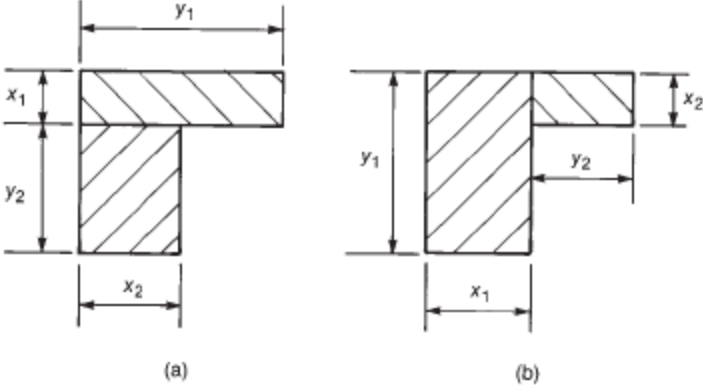| $\ell_2/\ell_1$ | 0.5 | 1.0 | 2.0 |
|---|---|---|---|
| $(\alpha_{f1}\ell_2/\ell_1) = 0$ | 60 | 60 | 60 |
| $(\alpha_{f1}\ell_2/\ell_1) \geq 1.0$ | 90 | 75 | 45 |

**Figure 3-19 – Division of edge members for calculation of torsional constant, C**

**Table 7** gives the percentage distribution of negative factored moment to the column strip at exterior supports. The setup of this table is similar to that used in **Table 6** and **Table 6**, with an addition of two rows to account for presence or absence of an edge beam working in torsion to transfer some of the slab negative moment into the column. When there is no edge beam, all of the negative moment is assigned to the column strip. This is reasonable because there is no torsional edge member to transfer moment from the middle strip all the way back to the column. If a stiff edge beam is present $(\beta_t \geq 2.5)$, **Table 7** gives specific percentages to be assigned to the column strip, depending on the value of $\alpha_{f1}$ and the $l_2/l_1$ ratio, as was done in **Table 6** and **Table 6**. For values of $\beta_t$ between 2.5 and 0.0, and values of $\alpha_{f1} l_2/l_1$ between 1.0 and 0.0, two or three levels of linear interpolation may be required to determine the percentage of negative moment assigned to the column strip.

If a beam is present in the column strip (spanning in the direction of $l_1$), a portion of the column-strip moment is assigned to the beam, as specified in ACI Code Section 13.6.5. If the beam has $\alpha_1 l_2/l_1$ greater than 1.0, 85 percent of the column-strip moment is assigned to the beam and 15 percent to the slab.

**Table 7 – Percentage distribution of Exterior negative factored moment to column strip**

| $\ell_2/\ell_1$ | | 0.5 | 1.0 | 2.0 |
|---|---|---|---|---|
| $(\alpha_{f1}\ell_2/\ell_1) = 0$ | $\beta_t = 0$ | 100 | 100 | 100 |
| | $\beta_t \geq 2.5$ | 75 | 75 | 75 |
| $(\alpha_{f1}\ell_2/\ell_1) \geq 1.0$ | $\beta_t = 0$ | 100 | 100 | 100 |
| | $\beta_t \geq 2.5$ | 90 | 75 | 45 |

❖ **Transfer of Moments to Columns**

**Exterior Columns**

When design is carried out by the direct-design method, ACI Code Section 13.6.3.6 specifies that the moment to be transferred from a slab to an edge column is $0.30M_0$. This moment is used to compute the shear stresses due to moment transfer to the edge column. Although not specifically stated in the ACI Code, this moment can be assumed to be acting at the centroid of the shear perimeter. The exterior negative moment from the direct-design method calculation is divided between the columns above and below the slab in proportion to the column flexural stiffnesses, $4EI/I$ The resulting column moments are used in the design of the columns.

**Interior Columns**

At interior columns, the moment-transfer calculations and the total moment used in the design of the columns above and below the floor are based on an unbalanced moment resulting from an uneven distribution of live load. The unbalanced moment is computed by assuming that the longer span adjacent to the column is loaded with the factored dead load and half the factored live load, while the shorter span carries only the factored dead load. The total unbalanced negative moment at the joint is thus

$$M = 0.65\left[\frac{(qD_u + 0.5q_{Lu})l_2 l_n^2}{8} - \frac{q'_{Du}\, l'_2\left(l'_n\right)^2}{8}\right] \tag{22}$$

Where $q_{Du}$ and $q_{Lu}$ refer to the factored dead and live loads on the longer span and $q'_{Lu}$ refers to the factored dead load on the shorter span adjacent to the column. The values of $l_2$ and $l_n$ refer to the longer of the adjacent spans and the values $l'_2$ and $l'_n$ refer to the shorter span. The factor 0.65 is the fraction of the static moment assigned to the negative moment at an interior support from ACI Code Section 13.6.3.2. A portion of the unbalanced moment is distributed within the slabs, and the rest goes to the columns. It is assumed that most of the moment is transferred to the columns, giving

$$M = 0.07\left[(qD_u + 0.5q_{Lu})l_2 l_n^2 - q'_{Du}\, l'_2\left(l'_n\right)\right] \tag{23}$$

This moment is used to compute the shear stresses due to moment transfer at an interior column. Also, it is distributed between the columns above and below the joint in proportion to their flexural stiffnesses to determine the moments used to design the columns.

### 3.3.3.4. Equivalent frame methods
Equivalent-frame methods are intended for use in analyzing moments in any practical slab-column frame. Their scope is thus wider than the direct-design method. In the direct-design method, the statical moment, $M_0$, is calculated for each slab span. This moment is then divided between positive- and negative regions using arbitrary moment coefficients, which are adjusted to reflect pattern loadings. The transverse distribution of moments to column and middle strips, as defined in the prior section, is the same for both methods.

**Classic Equivalent-Frame Analysis of Slab Systems for Vertical Loads**

The slab is divided into a series of equivalent frames running in the two directions of the building and these frames consist of the slab, any beams that are present, and the columns above and below the slab. For gravity-load analysis, ACI code allows analysis of an entire equivalent frame extending over the height of the building, or each floor can be considered separately with the far ends of the columns being fixed.

❖ **Calculation of Stiffness, Carryover, and Fixed-End Moments**

In the moment-distribution method, it is necessary to compute flexural stiffnesses, K; carry-over factors, COF; distribution factors, DF; and fixed-end moments, FEM, for each of the members in the structure. For a prismatic member fixed at the far end and with negligible axial loads, the flexural stiffness is

$$K = \frac{kEl}{L} \qquad (24)$$

Where k = 4 and the carryover factor is $\pm 0.5$ , the sign depending on the sign convention used for moments. For a prismatic, uniformly loaded beam, the fixed-end moments are $wl^2/12$.

In the equivalent-frame method, the increased stiffness of members within the column-slab joint region is accounted for, as is the variation in cross section at drop panels. As a result, all members have a stiffer section at each end, as shown in **Figure 3-20**b. If the EI used in eq (24) is that at the midspan of the slab strip, k will be greater than 4; similarly, the carryover factor will be greater than 0.5, and the fixed-end moments for a uniform load w will be greater than $wl^2/12$ .
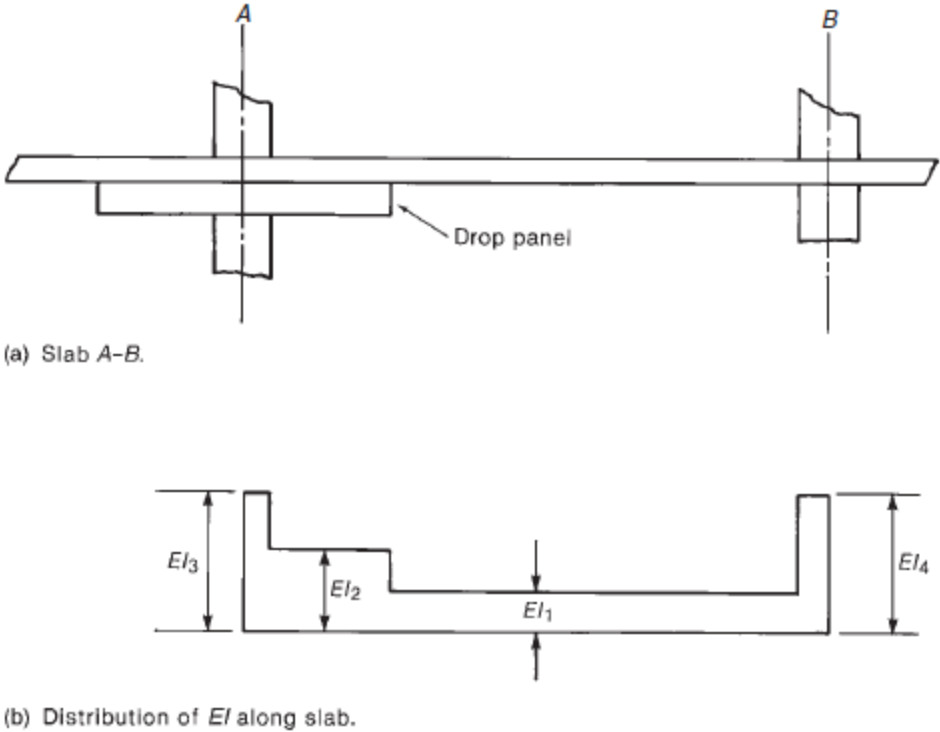


(a) Slab A–B.

(b) Distribution of El along slab.

**Figure 3-20 – Variation in stiffness along a span**

Several methods are available for computing values of k, COF, and FEM. Originally; these were computed by using the column analogy developed by Hardy Cross. Cross observed an analogy between the equations used to compute stresses in an unsymmetrical column loaded with axial loads and moments, and the equations used to compute moments in a fixed-end beam.

❖ **Properties of Slab–Beams**

The horizontal members in the equivalent frame are referred to as *slab-beams.* These consist of either only a slab, or a slab and a drop panel, or a slab with a beam running parallel to the equivalent frame. ACI Code Section 13.7.3 explains how these nonprismatic beams are to be modeled for analysis:

**1.** At points outside of joints or column capitals, the moment of inertia may be based on the gross area of the concrete. Variations in the moment of inertia along the length shall be taken into account. Thus, for the slab with a drop panel shown in **Figure 3-21**a, the moment of inertia at section $A{-}A$ is that for a slab of width $l_2$ (**Figure 3-21**c). At section $B{-}B$ through the drop panel, the moment of inertia is for a slab having the cross section shown in **Figure 3-21**d. Similarly, for a slab system with a beam parallel to $l_1$ as shown in **Figure 3-22**a, the moment of inertia for section $C{-}C$ is that for a slab-and-beam section, as shown in **Figure 3-22**c. Section $D{-}D$ is cut through a beam running perpendicular to the page.

**2.** The moment of inertia of the slab-beams from the center of the column to the face of the column, bracket, or capital (as defined in ACI Code Section 13.1.2 ) shall be taken as the moment of inertia of the slab-beam at the face of the column, bracket, or capital divided by the quantity $\left(1 - c_2/l_2\right)^2$, where $l_2$ is the transverse width of the equivalent frame and $c_2$ is the width of the support parallel to $l_2$.
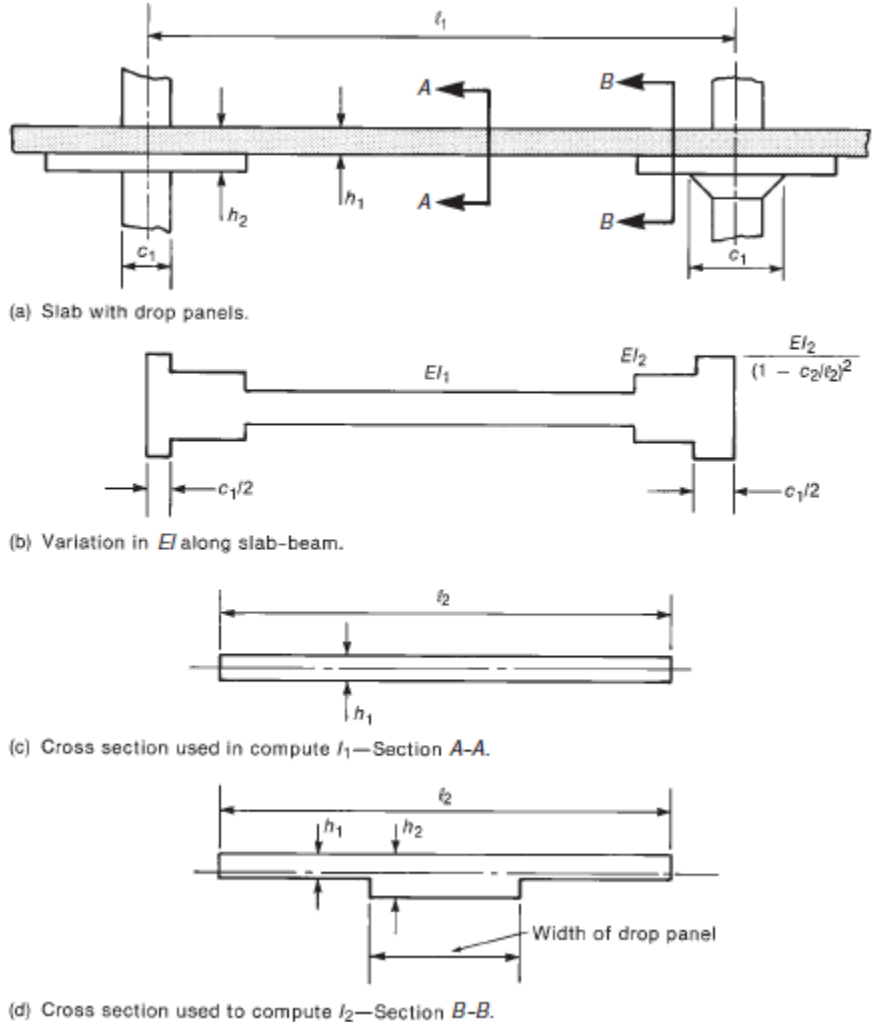
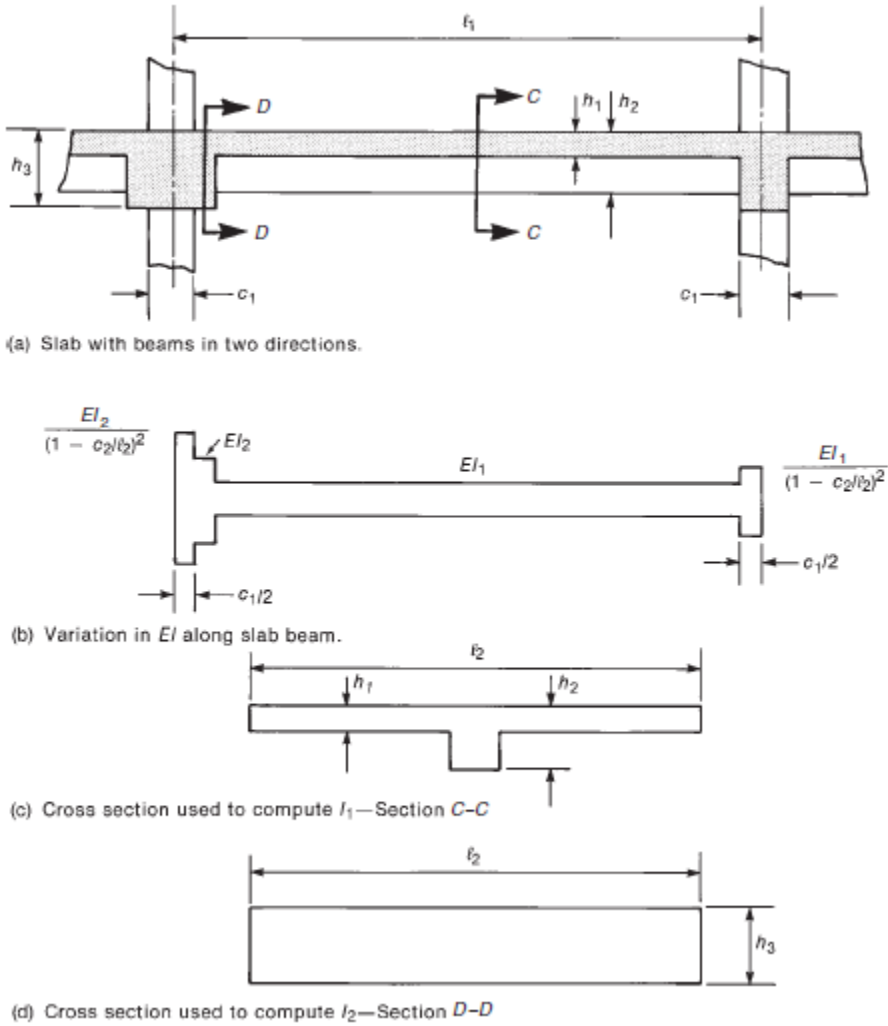**Figure 3-21 – EI values for a slab with a drop panel**

**Figure 3-22 – EI values for a slab and beam**

❖ **Properties of Columns**

In computing the stiffnesses and carryover factors for column, ACI Code Section 13.7.4 states the following:

**1.** The moment of inertia of columns at any cross section outside of the joints or column capitals may be based on the gross area of the concrete, allowing for variations in the actual moment of inertia due to changes in the column cross section along the length of the column.

**2.** The moment of inertia of columns shall be assumed to be infinite within the depth of the slab-beam at a joint. **Figure 3-23** illustrates these points for four common cases.
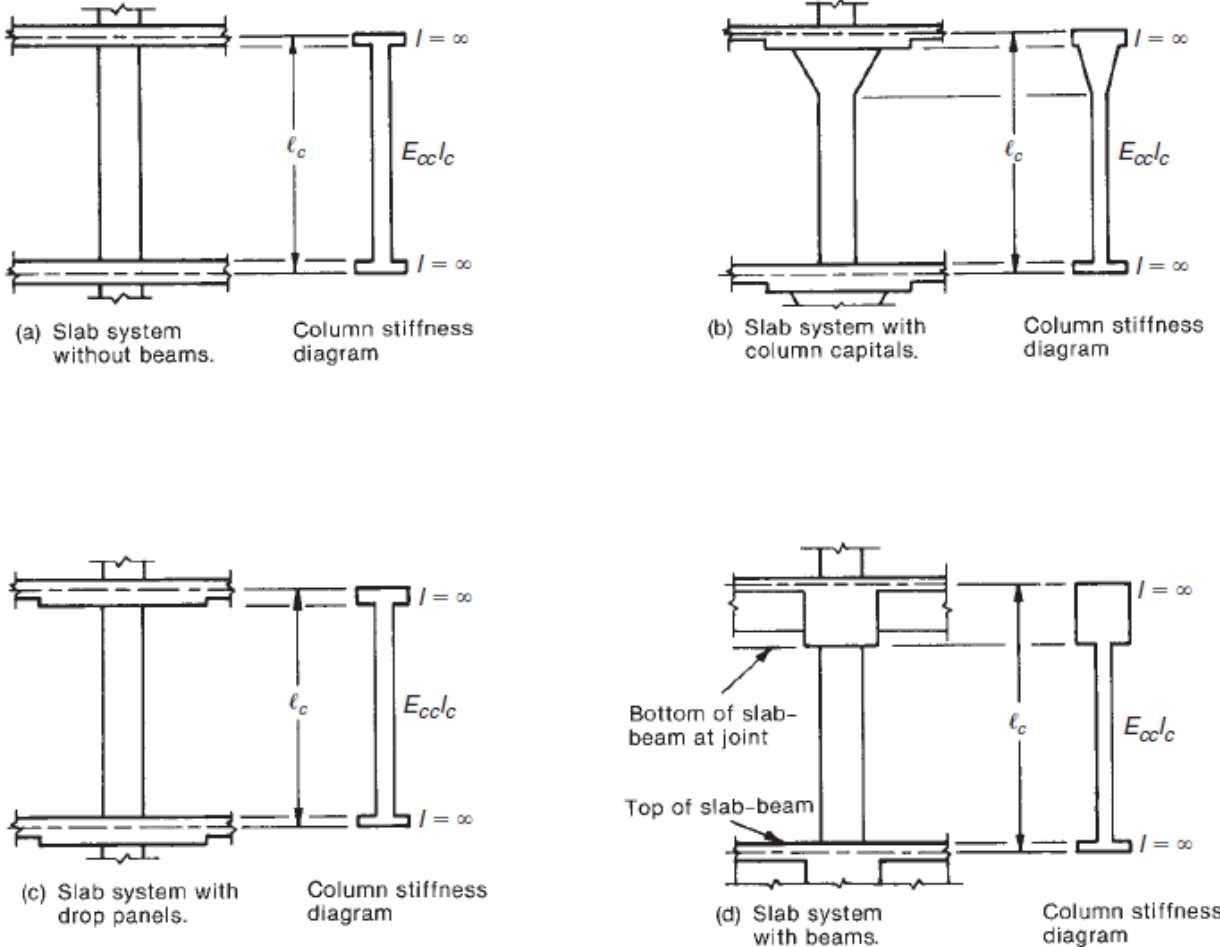
(a) Slab system without beams.    Column stiffness diagram

(b) Slab system with column capitals.    Column stiffness diagram

(c) Slab system with drop panels.    Column stiffness diagram

Bottom of slab–beam at joint

Top of slab–beam

(d) Slab system with beams.    Column stiffness diagram

**Figure 3-23 – Sections for the calculations of column stiffness, $K_c$**

❖ **Torsional Members and Equivalent Columns**

When the beam and column frame shown in **Figure 3-24**a is loaded, the ends of the column and beam undergo equal rotations where they meet at the joint. If the flexural stiffness, $K = M/\theta$, is known for the two members, it is possible to calculate the joint rotations and the end moments in the members. Similarly, in the case shown in **Figure 3-24**b, the ends of the slab and the wall both undergo equal end rotations when the slab is loaded. When a flat plate is connected to a column, as shown in **Figure 3-24**c, the end rotation of the column is equal to the end rotation of the strip of slab $C–D$, which is attached to the column. The rotation at $A$ of strip $A−B$ is greater than the rotation at point $C$, however, because there is less restraint to the rotation of the slab at this point. In effect, the edge of the slab has twisted, as shown in **Figure 3-24**d.

As a result, the *average* rotation of the edge of the slab is greater than the rotation of the end of the column.

To account for this effect in slab analysis, the column is assumed to be attached to the slab-beam by the transverse torsional members $A−C$ and One way of including these members in the

analysis is by the use of the concept of an *equivalent column*, which is a single element consisting of the *columns* above and below the floor and *attached torsional members*, as shown in **Figure 3-24**d. The stiffness of the equivalent column, $K_{ec}$, represents the combined stiffnesses of the columns and attached torsional members:

$$K_{ec} = \frac{M}{average\,rotation\,of\,the\,edge\,beam} \tag{25}$$

The inverse of a stiffness, $1/K$, is called the *flexibility*. The flexibility of the equivalent column, $1/K_{ec}$, is equal to the average rotation of the joint between the "edge beam" and the rest of the slab when a unit moment is transferred from the slab to the equivalent column. This average rotation is the rotation of the end of the columns, $\theta_c$, plus the average twist of the beam, $\theta_{t,avg}$ with both computed for a unit moment:

$$\theta_{ec} = \theta_c + \theta_{t,avg} \tag{26}$$

The value of $\theta_c$ for a unit moment is $1/\sum K_c$ where $\sum K_c$ refers to the sum of the flexural stiffnesses of the columns above and below the slab. Similarly, the value of $\theta_{t,avg}$ for a unit moment is $1/K_t$ where $K_t$ is the torsional stiffness of the attached torsional members. Substituting into Eq.(26) gives

$$\frac{1}{K_{ec}} = \frac{1}{\sum K_c} + \frac{1}{K_t} \tag{27}$$

If the torsional stiffness of the attached torsional members is small, $K_{ec}$ will be much smaller than $\sum K_c$.

(a) Beam and column frame.

(b) Slab and wall frame.

(c) Slab column frame.

Attached torsional member
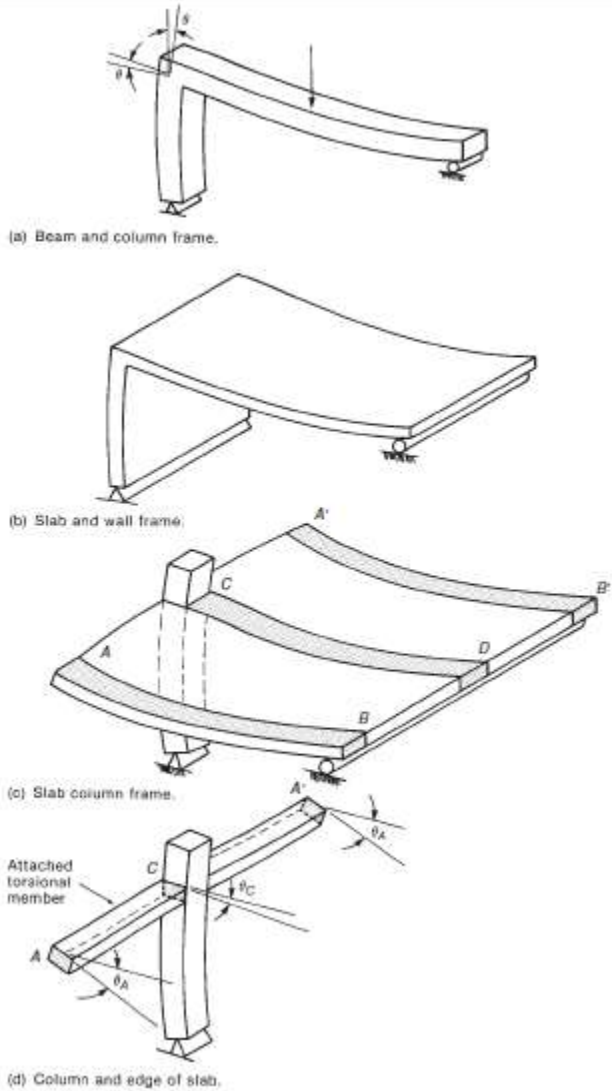
(d) Column and edge of slab.

**Figure 3-24 – Frame action and twisting of edge member**

The derivation of the torsional stiffness of the torsional members (or edge beams) is illustrated in **Figure 3-25**. **Figure 3-25**a shows an equivalent column with attached torsional members that extend halfway to the next column in each direction. A unit torque, $T = 1$ is applied to the equivalent column with half going to each arm. Because the effective stiffness of the torsional members is larger near the column, the moment, $t$, per unit length of the edge beam is arbitrarily assumed to be as shown in **Figure 3-25**b. The height of this diagram at the middle of the column has been chosen to give a total area equal to 1.0, the value of the applied moment.

The applied torques give rise to the twisting-moment diagram shown in **Figure 3-25**c. Because half of the torque is applied to each arm, the maximum twisting moment is ½. The twist angle per unit length of torsional member is shown in **Figure 3-25**d. This is calculated by dividing the twisting moment at any point by $CG$, the product of the torsional constant, $C$ (similar to a polar moment of inertia), and the modulus of rigidity, $G$. The total twist of the end of an arm relative to the column is the summation of the twists per unit length and is equal to the area of the diagram

of twist angle per unit length in **Figure 3-25**d. Because this is a parabolic diagram, the angle of twist at the outer end of the arm is one-third of the height times the length of the diagram:

$$\theta_{t,end} = \frac{1}{3}\frac{\left(1-c_2/l_2\right)^2}{2CG}\left[\frac{l_2}{2}\left(1-\frac{c_2}{l_2}\right)\right] \tag{28}$$

Replacing *G* with *E*/2 gives

$$\theta_{t,end} = \frac{l_2\left(1-c_2/l_2\right)^3}{6CE} \tag{29}$$

This is the rotation of the end of the arm. The rotation required for use in Eq.(27) is the average rotation of the arm, which is assumed to be a third of the end rotation:

$$\theta_{t,avg} = \frac{l_2\left(1-c_2/l_2\right)^3}{18CE} \tag{30}$$

Finally, the torsional stiffness of one arm is calculated as where the moment resisted by one arm is taken as ½ , giving

$$K_t\left(one-arm\right) = \frac{9EC}{l_2\left(1-c_2/l_2\right)^3} \tag{31}$$

ACI Commentary Section R13.7.5 expresses the torsional stiffness of the two arms as

$$K_t = \sum\frac{9E_{cs}C}{l_2\left(1-c_2/l_2\right)^3} \tag{32}$$

Where $l_2$ refers to the transverse spans on each side of the column. For a corner column, there is only one term in the summation.

(a) Column and attached torsional member.

(b) Distribution of torque per unit length along column center line.

(c) Torque diagram.

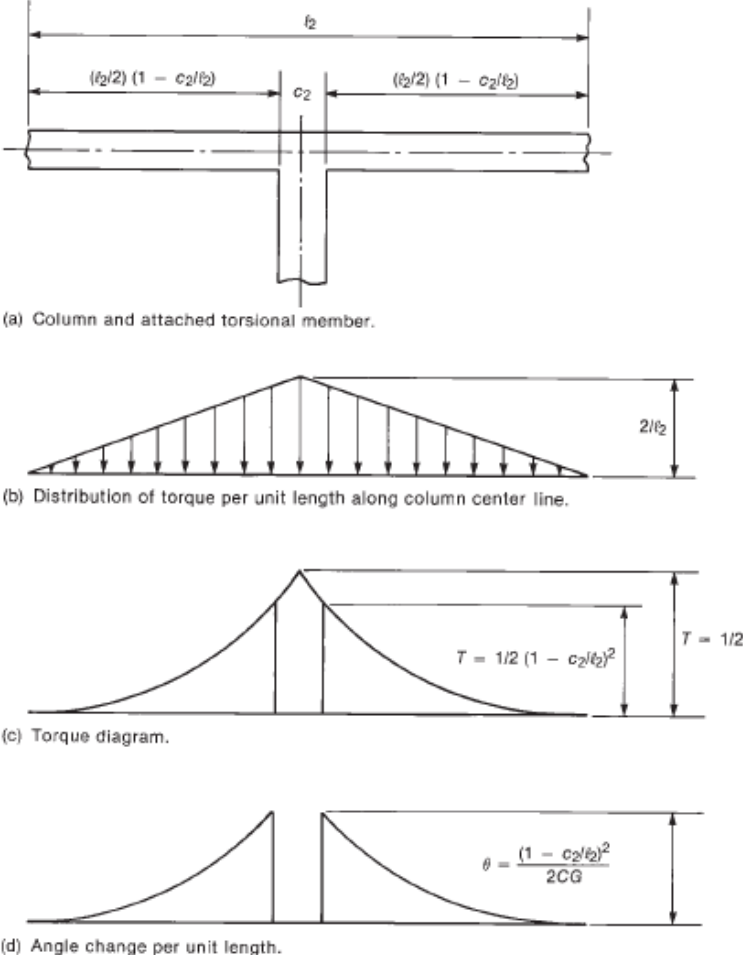(d) Angle change per unit length.

**Figure 3-25 – Calculation of $K_t$**

The cross section of the torsional members is defined in ACI Code Section 13.7.5.1(a) to (c) and is illustrated in **Figure 3-26**. Note that this cross section normally will be different from that used to compute the flexural stiffness of the beam and the beam section used for torsion design (both defined by ACI Code Section 13.2.4). This difference always has been associated with the use of the equivalent-frame method.

The constant C in Eq. (32) is calculated by subdividing the cross section into rectangles and carrying out the summation

$$C = \sum \left[ \left(1 - 0.63 \frac{x}{y}\right) \frac{x^3 y}{3} \right]$$

(33)

where x is the shorter side of a rectangle and y is the longer side.

If a beam parallel $l_1$ to the direction frames into the column, a major fraction of the exterior negative moment is transferred directly to the column without involving the attached torsional member. In such a case, $K_{ec}$ underestimates the stiffness of the column. This is allowed for

empirically by multiplying $K_t$ by the ratio $I_{sb}/I_s$ where $I_{sb}$ is the moment of inertia of the slab and beam together and $I_s$ is the moment of inertia of the slab neglecting the beam stem (ACI Code Section 13.7.5.2).

In a moment-distribution analysis, the frame analysis is carried out for a frame with slab-beams having stiffnesses $K_s$ and with equivalent columns having stiffnesses $K_{ec}$.
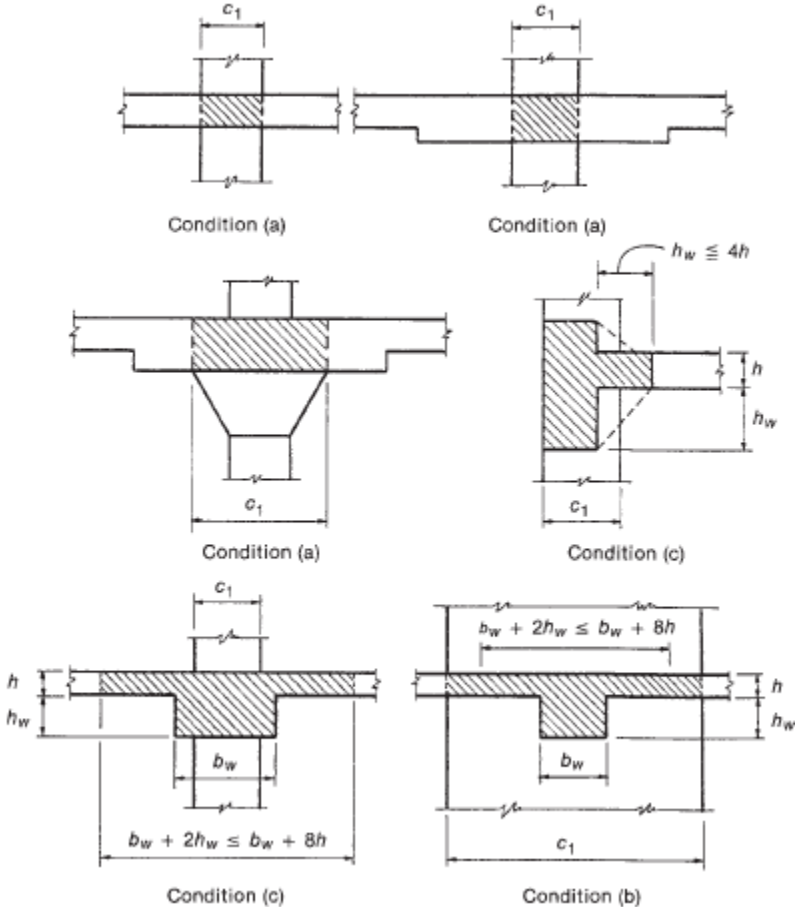


**Figure 3-26 – Torsional members**

### 3.3.4. SHEAR STRENGTH OF TWO-WAY SLABS

A shear failure in a beam results from an inclined crack caused by flexural and shearing stresses. This crack starts at the tensile face of the beam and extends diagonally to the compression zone, as explained in previous sections. In the case of a two-way slab or footing, the two shear=failure mechanisms shown in Figure 3-27 are possible. One-way shear or beam-action shear (Figure 3-27a) involves an inclined crack extending across the entire width of the structure. Two-way shear or punching shear involves a truncated cone or pyramid-shaped surface around the column, as shown schematically in Figure 3-27b. Generally, the punching shear capacity of a slab or

footing will be considerably less than the one-way shear capacity. In design, however, it is necessary to consider both failure mechanisms.
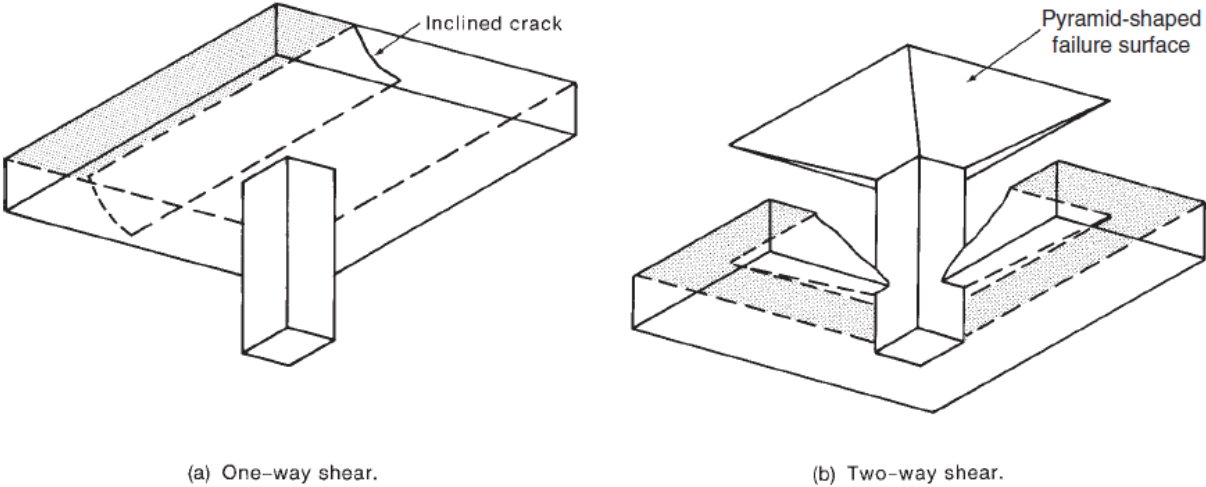


**Figure 3-27 – Shear failure in slabs**

### 3.3.5. PUNCHING SHEAR (CLAUSE 6.4 OF EUROCODE 2)

#### 3.3.5.1. General
The basic control perimeter is set at 2d from the loaded area and has rounded corners as shown in Figure 3-28.
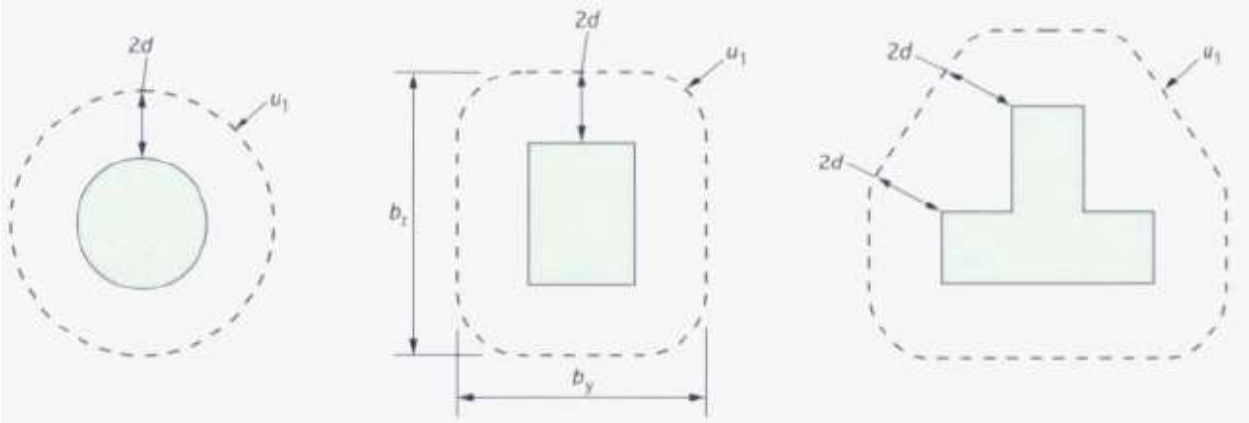


**Figure 3-28 – Typical basic control perimeter around a column**

#### 3.3.5.2. Effective applied shear stress
The effect of slab/column moment transfer on punching shear may be considered as a uniform increase of shear stress at the control perimeter considered. The resulting effective applied shear stress, $v_{Ed}$, is given in Equation (34).

$$v_{Ed} = \beta V_{Ed}/u_l d \tag{34}$$

Where

$V_{Ed}$   = applied shear force

$\beta$   = magnification factor relating to the moment transfer

$u_l$   = length of the control perimeter considered

d   = mean effective depth $(d_x + d_z)/2$

❖ **Simplified approach**

For structures where lateral stability does not depend on frame action and where adjacent spans do not differ by more than 25%, the approximate values for $\beta$ as shown in Figure 3-29 may be used. Otherwise, a more rigorous method of calculating $\beta$ should be adopted as follows:
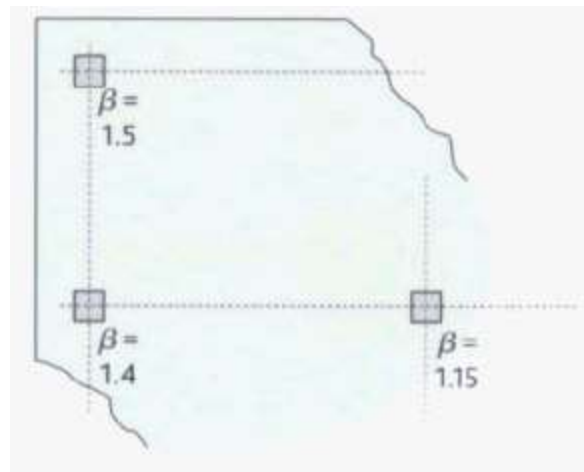


**Figure 3-29 – simplified method for determining the value of $\beta$**

❖ **Internal rectangular columns**

The value of $\beta$ for a rectangular column may be calculated from the approximate Equation (35) (from expression 6.4.3 in Eurocode 2).

$$\beta = 1 + 1.8\sqrt{\left\{\left(M_{Ed,z}/V_{Ed}b_z\right)^2 + \left(M_{Ed,y}/V_{Ed}b_y\right)^2\right\}} \tag{35}$$

Where

$M_{Ed,z}$ and $M_{Ed,y}$   Design applied transfer moments about the z and y axes respectively

$V_{Ed}$   Design applied shear force

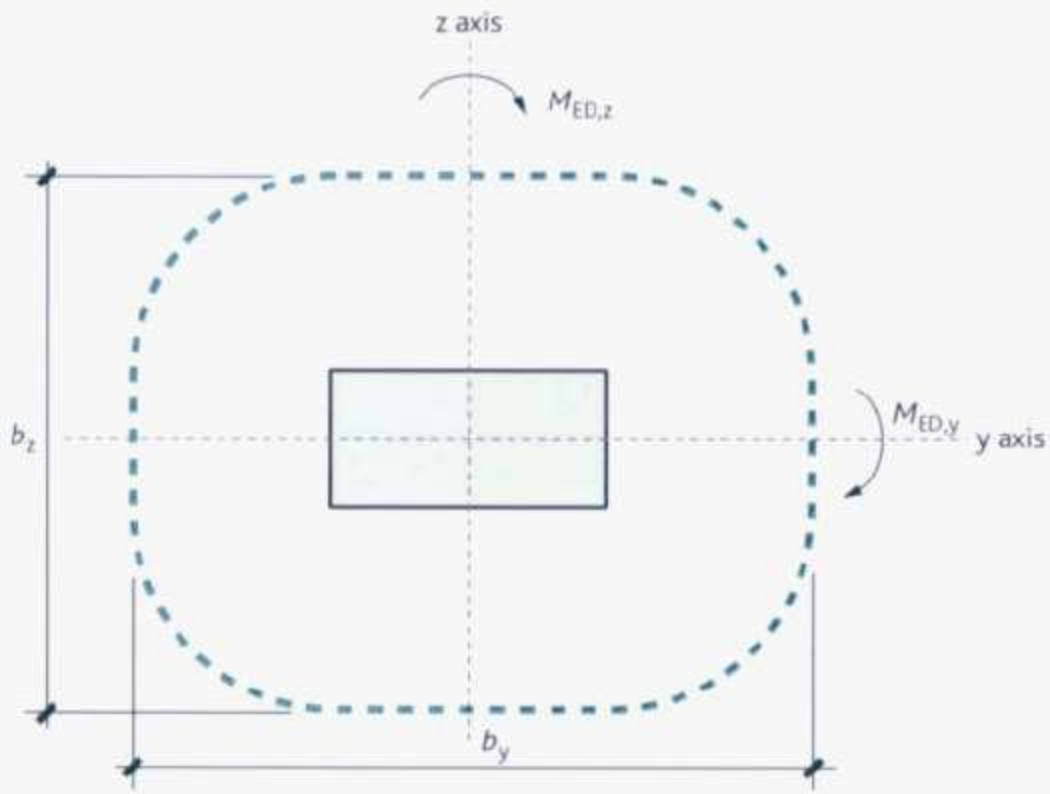$b_z$ and $b_y$   Dimensions of the basic control perimeter

**Figure 3-30 – Effect of applied moments on shear at internal columns**

If more accuracy is required, reference should be made to Clause 6.4.3 of Eurocode 2.

❖ **Edge and corner rectangular columns**

The basic control perimeter for edge and corner columns of flat slabs is shown in Figure 3-31
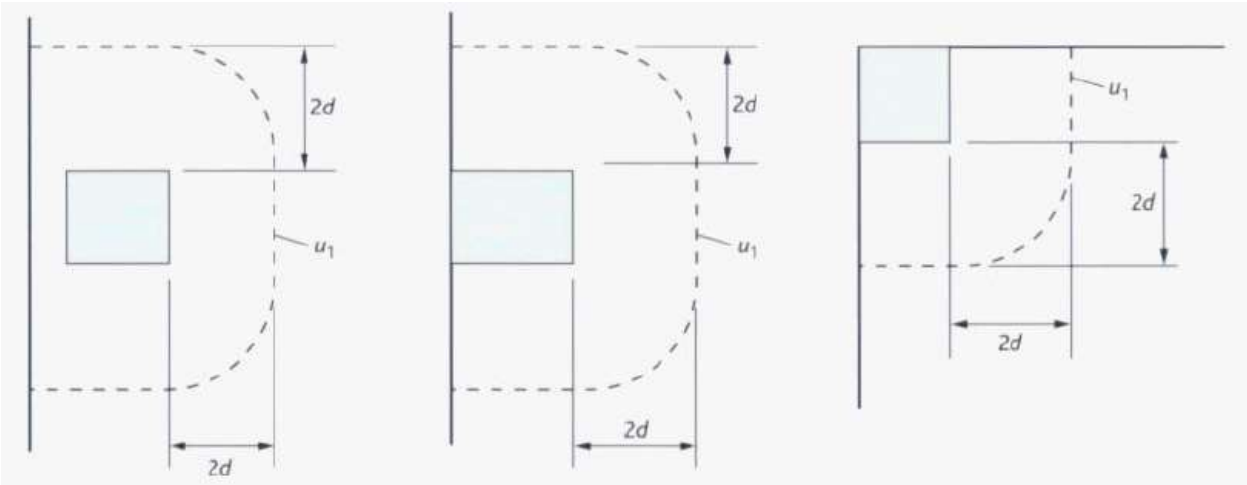


**Figure 3-31 – Basic control perimeter, $u_1$, for edge and corner columns of flat slabs**

In order to assess the effects of moment transfer from the normal uniformly distributed gravity loads on the shear resistance of the slab, a reduced control perimeter, $u_1^*$, is defined as shown in Figure 3-32. This figure also shows the limits for the control perimeter for columns with long or large sections.

For corner and edge columns with bending only about the axis parallel to the edge, Equation may be used.

For corner and edge columns with bending only about the axis parallel to the edge, Equation (36) may be used.
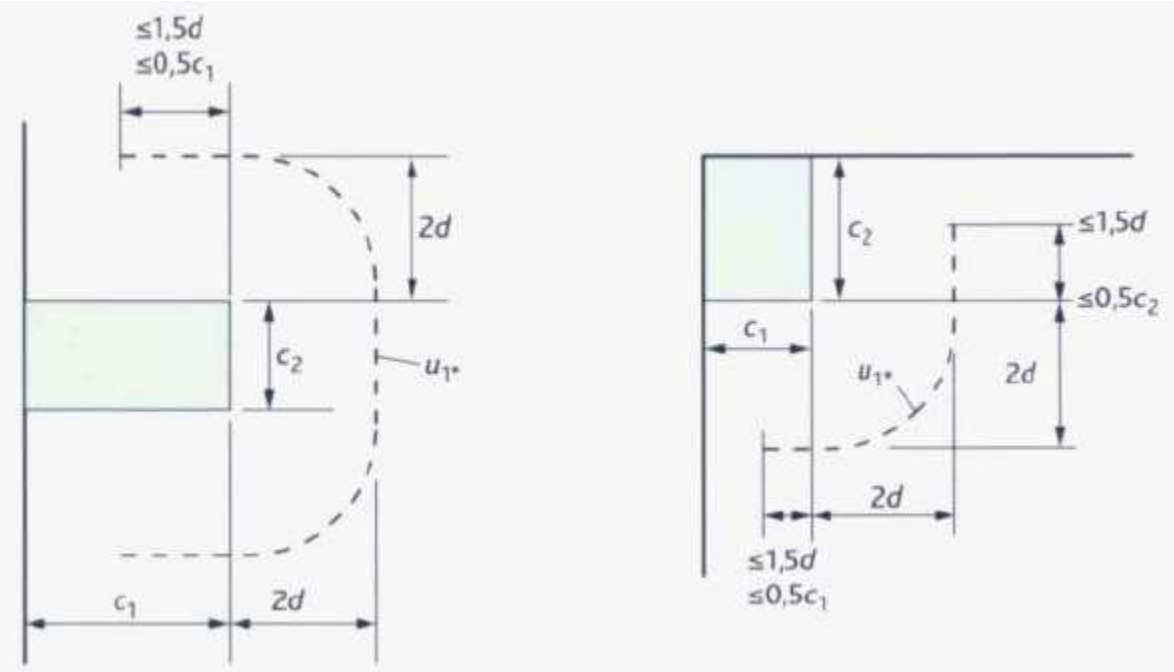
$$\beta = u_1 / u_1^* \tag{36}$$



**Figure 3-32 – Reduced control perimeter, $u_1^*$, to take account of the effects of moment transfer**

For edge columns bent about both axes, Equation (37) may be used.

$$\beta = u_1 / u_1^* + k u_1 e_{par} / W_1 \tag{37}$$

Where:

| $e_{par}$ | Eccentricity parallel to the slab edge resulting from a moment about the axis perpendicular to the slab edge |
| --- | --- |
| $W_1$ | $c_2^2/4 + c_1 c_2 + 4c_1 d + 8d^2 + \pi d c^2$ |

K is determined from the following table:

| $c_1/2c_2$ | $\leq 0.5$ | 1.0 | 2.0 | $\geq 3.0$ |
|---|---|---|---|---|
| k | 0.45 | 0.6 | 0.7 | 0.8 |

❖ **High aspect ratio columns and ends of walls**

Where one or both dimensions of a rectangular column or wall are greater than 3d, the effective perimeter should be calculated in accordance with the rules above as shown in Figure 3-32. The value of $u_1$ in Equations (36) and (37) should be based on values of $c_1$ and $c_2$ which do not exceed 3d.

### 3.3.5.3. Punching shear resistance
❖ **Procedure**

The procedure for checking the punching shear resistance should be as follows:

- Check the effective applied shear stress, $v_{Ed}$, at the basic control perimeter, 2d from the column or column head perimeter (see Figure 3-31 and Equation (38))
- Where the effective applied shear stress shear, $v_{Ed}$, is greater than shear resistance, $V_{Rdc}$, check shear resistance at the column (or column head) perimeter:
  - If the effective applied shear stress, $v_{Ed}$, for that perimeter is greater than the shear resistance, $v_{Rd,max}$ (using Equation (39)), a redesign is necessary (e.g. reduce loads, increase slab depth, introduce column heads or drop panel, reduce span). (it should be noted that calculated for the basic control perimeter.)
  - Otherwise, design punching shear reinforcement: determine the outer perimeter to which it is required (using Equation (40)) and spread the required shear reinforcement evenly over the required area (using Equations (41) and (42)).
- At the design stage, unless the type of shear reinforcement and method of construction is known, it is recommended that the slab is detailed using the traditional loose link arrangement.
- When the maximum shear resistance is greater than the effective applied shear stress at the column perimeter but the shear resistance at the basic control perimeter is less than the effective applied shear stress, shear reinforcement is required.

❖ **Check for punching shear without shear reinforcement (Expression 6.47 of Eurocode 2)**

The punching shear stress resistance, $v_{Rd,c}$, should be calculated for the basic control perimeter using Equation (38). $v_{Rd,c}$ is also limited to that for C50/60 (see Clause 3.1.2 (4) of Eurocode 2 )

$$v_{Rd,c} = 0.12k\left(100\rho_1 f_{ck}\right)^{1/3} \geq 0.035k^{3/2}f_{ck}^{1/2} \tag{38}$$

Where:

$k \qquad = 1 + \sqrt{(200/d)} \le 2.0$

$\rho_1 \qquad = \sqrt{(\rho_{ly} + \rho_{lz})}$

$\rho_{ly}$ and are the mean values of the tension steel in the y and z-directions for a width, b, of slab equal to the column width plus 3d each side. $A_s$ is the area of tensile reinforcement, which extends $\ge (l_{bd} + d)$ beyond the section considered. $l_{bd}$ is the design anchorage length, see Expression 8.4 of Eurocode 2.

❖ **Maximum shear resistance at column perimeter (Clause 6.4.5 (3) of Eurocode 2)**

The value of the maximum shear resistance, $v_{Rd,max}$, should be calculated using Equation (39).

$$v_{Rd,max} = 0.2(1 - f_{ck}/250)f_{ck} \qquad (39)$$

❖ **Position of outer control perimeter (clause 6.4.5 (4) of Eurocode 2)**

The distance from the column face, $r_{out}$, of the control perimeter at which shear reinforcement is not required, $u_{out}$, may be calculated using Equation 14 (see Figure 3-33)

$$r_{out} = \left\{ v_{Ed}/(dv_{Rd,c}) - 2(c_1 + c_2) \right\}/2\pi \qquad (40)$$
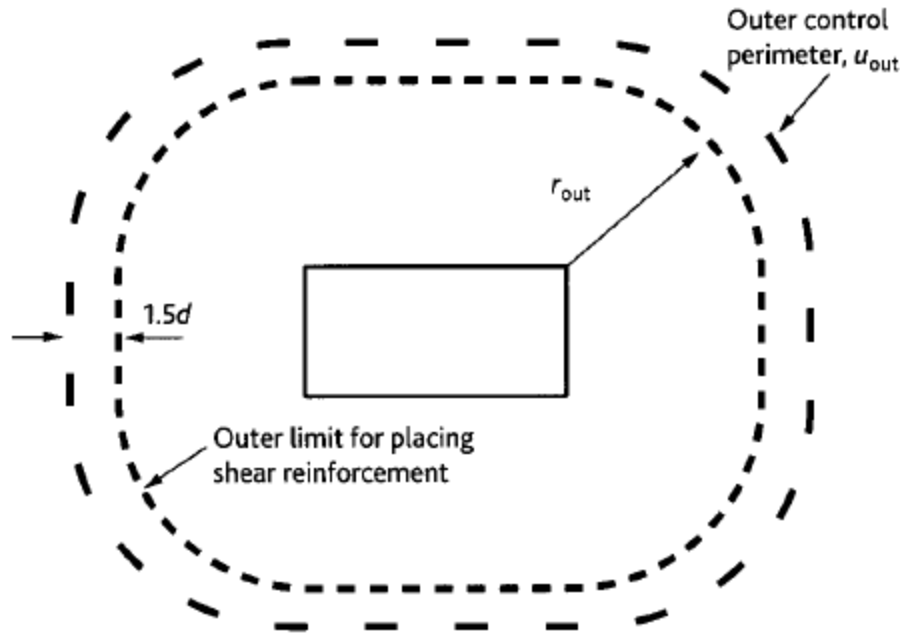
**Figure 3-33 – Outer limit for placing reinforcement**

❖ **Area of shear reinforcement (Expression 6.5.2 of Eurocode 2)**

The punching shear reinforcement required at any perimeter within $(u_{out} - 1.5d)$, using vertical legs, should be calculated using Equation (41).

$$A_{sw}/s_r \geq 2u_1 (v_{Ed} = 0.75v_{Rd,c})/(3f_{ywd,ef}) \qquad (41)$$

Where:

| | |
|---|---|
| $A_{sw}$ | = area of one perimeter of shear reinforcement around the column |
| $u_1$ | = length of the basic control perimeter |
| $s_r$ | = radial spacing of periments of shear reinforcement (mm) |
| $f_{ywd,ef}$ | = effective design strength of the shear reinforcement |
| | $= 250 + 0.25d_{mean} \leq f_{ywd}$ |
| $d_{mean}$ | = mean of the effective depths in the orthogonal directions (mm) |

The minimum area of a vertical leg of shear reinforcement, $A_{sw,min}$, is given by Equation (42).

$$A_{sw,min}/(s_r \times s_t) \geq 0.053\sqrt{(f_{ck}/f_{yk})} \qquad (42)$$

Where

| | |
|---|---|
| $s_r$ | = spacing of shear legs in the radial direction (mm) |
| $s_t$ | = spacing of shear legs in the tangential direction (mm) |

$f_{ck}$ and $f_{yk}$ are in MPa.

The distance between the column face and the nearest shear perimeter taken into account in the design should not exceed d/2.

The actual layout of shear reinforcement will depend on the detailing rules

In situations where the shear reinforcement is not placed at regular orthogonal spacings, a further limit on the length of the outer perimeter is imposed as shown in Figure 3-34.
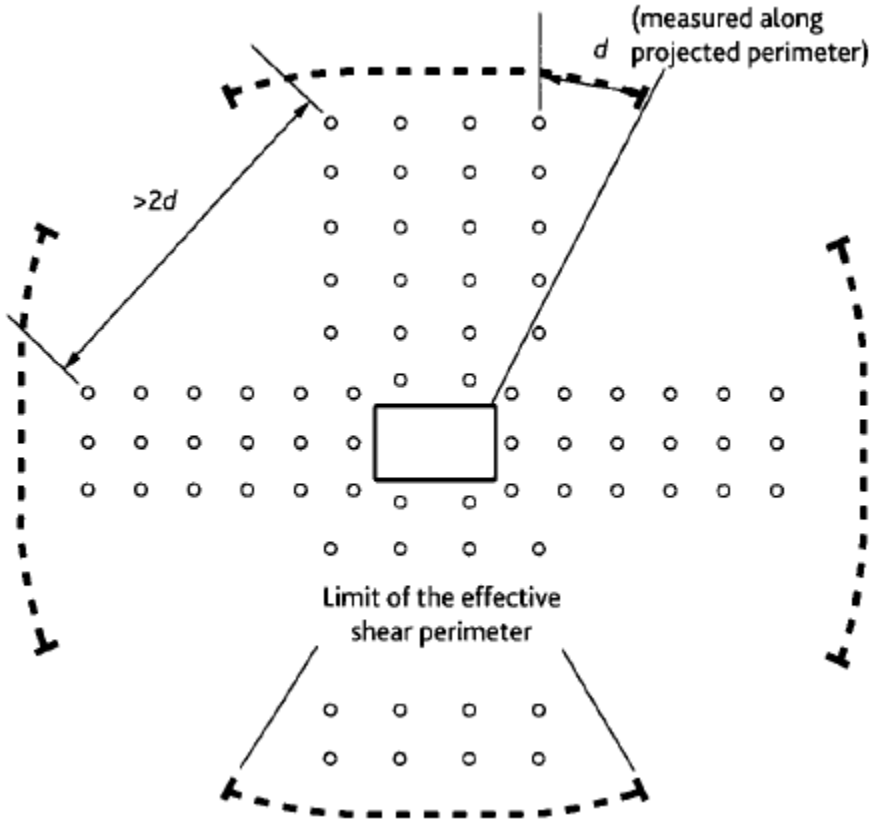


**Figure 3-34 – Limitation o outer perimeter for irregular layout of shear reinforcement**

### 3.3.5.4. Eurocode Rules for spacing and quantity of reinforcement
❖ **Minimum area of principal reinforcement**

Minimum area of principal reinforcement in the main direction is $A_{s,min} = 0.26 f_{ctm} b_t\, d/f_{yk}$ but not less than $0.0013 b_t d$, where $b_t$ is the mean width of the tension zone. For a T-beam with flange in compression, only the width of the web is taken into account in calculating the value of $b_t$.

❖ **Minimum area of secondary reinforcement**

The minimum area of secondary transverse reinforcement is $20\%A_{s,min}$. In areas near supports, transverse reinforcement is not necessary where there is no transverse bending moment.

❖ **Maximum area of reinforcement**

Outside lap locations, the maximum area of tension or compression reinforcement should not exceed $A_{s,max} = 0.04A_c$

❖ **Minimum spacing of reinforcement**

The minimum clear distance between bars should be the greater of:

- Bar diameter
- Aggregate size plus 5 mm
- 20 mm

❖ **Maximum spacing of reinforcement**

For slabs less than 200 mm thick the following maximum spacing rules apply:

- For the principal reinforcement: 3h but not more than 400 mm
- For the secondary reinforcement: 3.5h but not more than 450 mm

The exception is in areas with concentrated loads or areas of maximum moment where the following applies:

- For the principal reinforcement: 2h but not more than 250 mm
- For the secondary reinforcement: 3h but not more than 400 mm
  Where h is the depth of the slab

❖ **Reinforcement in slabs near supports**

In simply supported slabs, half the calculated span reinforcement should continue up to the support.

Where partial fixity occurs along an edge of a slab, but is not taken into account in the analysis, the top reinforcement should be capable of resisting at least 25% of the maximum moment in adjacent span. The reinforcement should extend at least 25% of the maximum moment in the adjacent span, measured from the face of the support. It should be continuous across internal supports and anchored at end supports. At an end support the moment to be resisted may be reduced to 15% of the maximum moment in the adjacent span.

❖ **Corner reinforcement**

If the detailing arrangements at a support are such that lifting of the slab at a corner is restrained, suitable reinforcement should be provided.

❖ **Reinforcement at the free edges**

Along a free (unsupported) edge, a slab normally contains longitudinal and transverse reinforcement, generally arranged as shown in Figure 3-35.

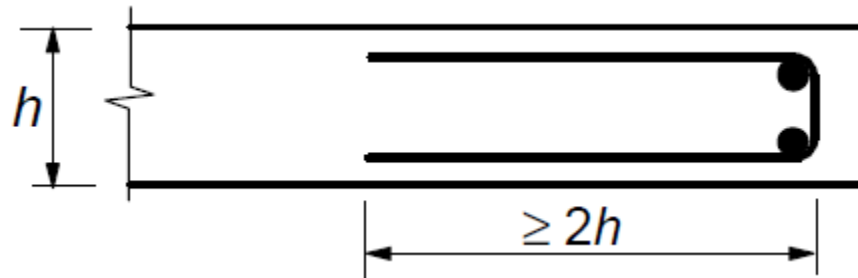The normal reinforcement provided for a slab may act as edge reinforcement.



**Figure 3-35 – Edge reinforcement for a slab**

❖ **Shear reinforcement**

A slab in which shear reinforcement is provided should have a depth of at least 200 mm.

The maximum longitudinal spacing of successive series of links is given by:

$$S_{max} = 0.75d\left(1+\cot\alpha\right) \tag{43}$$

where $\alpha$ is the inclination of the shear reinforcement.

The maximum longitudinal spacing of bent-up bars is given by:

$$S_{max} = d \tag{44}$$

❖ **Rules specific to flat slabs**

**Slab at internal columns**

At internal columns, top reinforcement of area $0.5A_t$ should be placed in a width equal to the sum of 0.125 times the panel width on either side of the column. $A_t$ represents the area of reinforcement required to resist the full negative moment from the sum of the two half panels each side of the column.

Bottom reinforcement ($\geq 2$ bars) in each orthogonal direction should be provided at internal columns and this reinforcement should pass through the column.

**Slab at edge and corner columns**

Reinforcement perpendicular to a free edge required to transmit bending moments from the slab to an edge or corner column should be placed within the effective width $b_e$ shown in Figure 3-36.
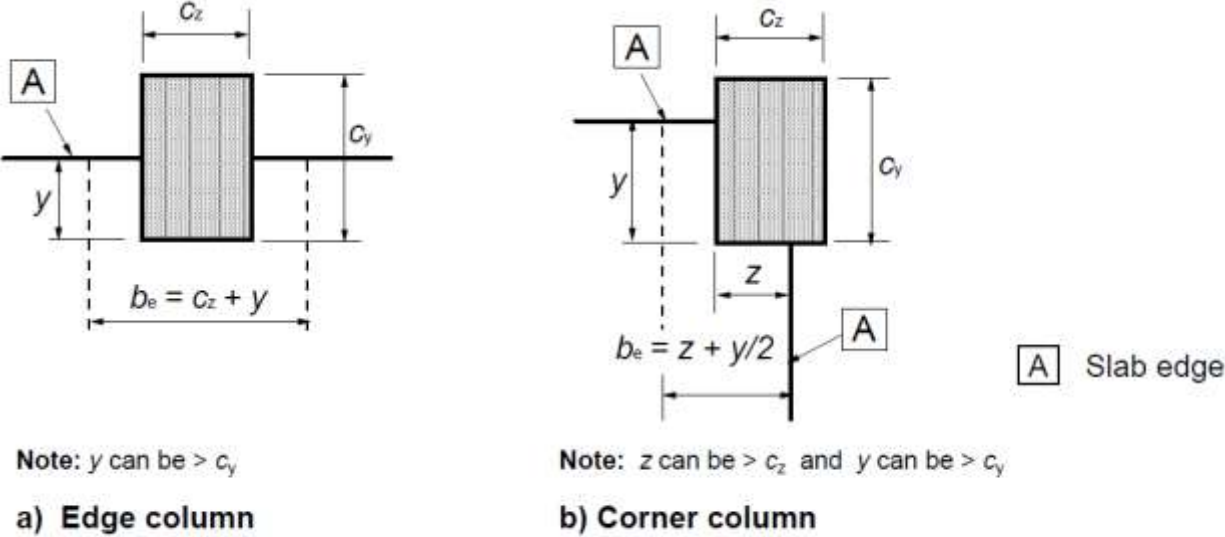


**Note:** $y$ can be > $c_y$

**a) Edge column**

**Note:** $z$ can be > $c_z$ and $y$ can be > $c_y$

**b) Corner column**

$\boxed{A}$ Slab edge

**Note:** $y$ is the distance from the edge of the slab to the innermost face of the column.

**Figure 3-36 – Effective width, $b_e$, of a flat slab**

**Punching shear reinforcement**

–   Where punching shear reinforcement is required it should be placed between the loaded area/column and kd inside the control perimeter at which shear reinforcement is no longer required. If should be provided in at least two perimeters of link legs (see Figure 3-37). The spacing of the link leg perimeters should not exceed 0.75d.

–   The spacing of ink legs around a perimeter should not exceed 1.5d within the first control perimeter (2d from loaded area), and should not exceed 2d for perimeters outside the first control perimeter where that part of the perimeter is assumed to contribute to the shear capacity

–   For bent down bars as arranged in Figure 3-37b one perimeter of link legs may be considered sufficient.
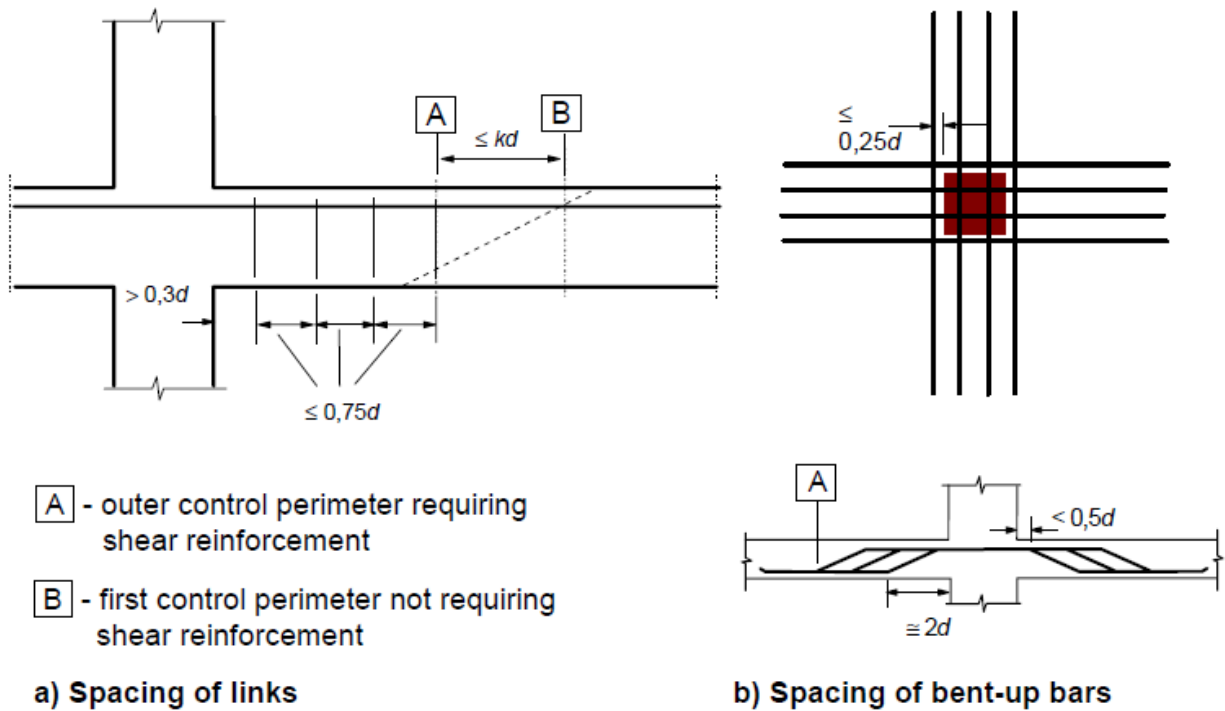
**Figure 3-37 – Punching shear reinforcement**

The recommended value of k is 1.5

–    Where shear reinforcement is required the area of a link leg (or equivalent), $A_{sw,min}$, is given by the following expression

$$A_{sw,min} \cdot (1.5\sin\alpha + \cos\alpha)/(S_r \cdot S_t) \geq 0.08 \cdot \sqrt{(f_{ck})}/f_{yk} \qquad (45)$$

where:

$\alpha$          is the angle between the shear reinforcement and the main steel (i.e. for vertical links $\alpha = 90^0$ and $\sin\alpha = 1$

$S_r$          is the spacing of shear links in the radial direction

$S_t$          is the spacing of shear links in the tangential direction

$f_{ck}$          is in MPa

–    Bent-up bars passing through the loaded area or at a distance not exceeding $0.25d$ from this area may be used as punching shear reinforcement

–    The distance between the face of a support, or the circumstance of a loaded area, and the nearest shear reinforcement taken into account in the design should not exceed d/2. This distance should be taken at the level of the tensile reinforcement; if only a single line of bent-up bars is provided, their slope may be reduced to $30^0$