CHAPTER 3: BOND GRAPH SYNTHESIS & EQUATION DERIVATION

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PREVIEW QUESTIONS

- What type of mathematical equations are needed?
- How are these equations systematically derived?
- How are the individual constitutive relations of the components connected to generate a mathematical model?

OBJECTIVES & OUTCOMES

Objectives:

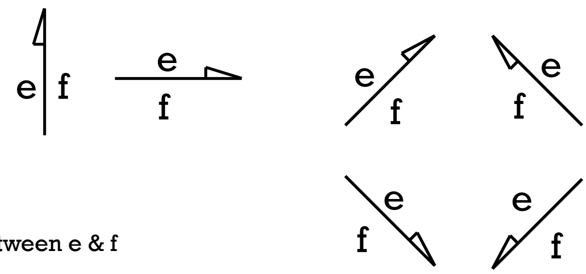
- To effectively use bond graphs to formulate models that facilitate deriving mathematical representations of dynamic systems,
- To be able to systematically derive mathematical representations using bond graphs, and
- To understand the flow of information within a system dynamics model and its relation to mathematical representations.

Outcomes: Upon completion, you should be able to

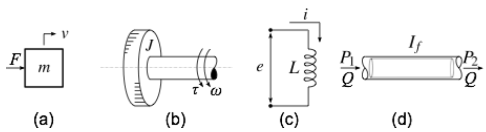
- synthesize bond graph models of mechanical, electrical, and hydraulic systems,
- annotate bond graphs to indicate appropriate power flow and causality, and
- derive mathematical models in the form of differential and algebraic equations using bond graph representations.

RECAP: CHAPTER 2

- Power bond labels

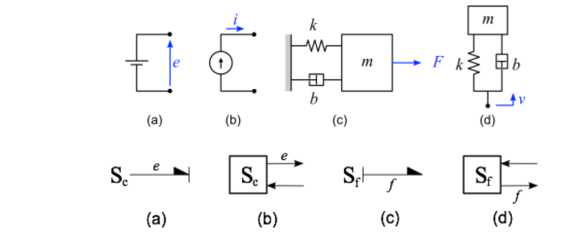


- R-Elements
  - Dissipate Energy
  - Direct algebraic relationship between e & f
- C-Elements
  - Store Potential Energy
  - Derivative Causality
- I-Elements
  - Store Kinetic Energy
  - Derivative Causality

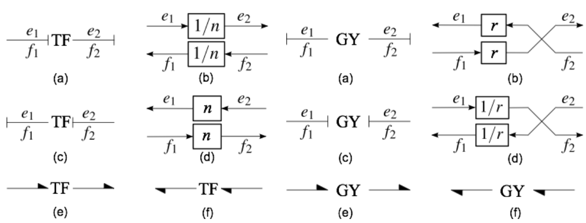


RECAP: CHAPTER 2

- Sources
  - Supply energy



- Transformers
  - Convert energy
  - Power through
- Gyrators
  - Convert energy
  - Power through

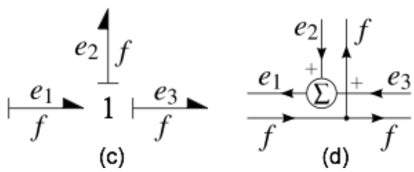


$e_1 = ne_2$
$nf_1 = f_2$

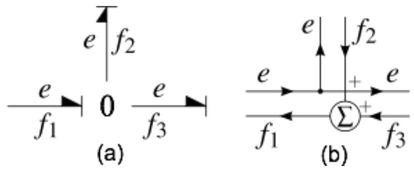
$e_1 = rf_2$
$rf_1 = e_2$

RECAP: CHAPTER 2

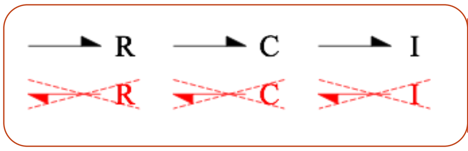
- 1-Junction
  - Common flow
  - Summation of efforts



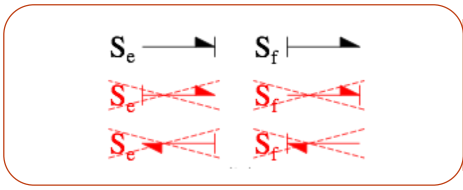
- 0-Junction
  - Common effort
  - Summation of flows



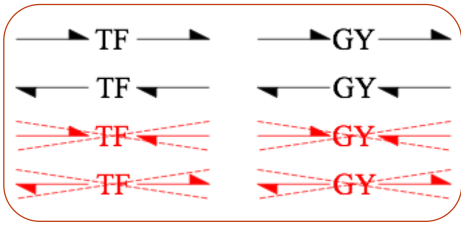
GENERAL GUIDELINES



- Power goes **from the system to** R-, C-, and I-elements



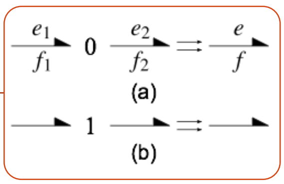
- Sources generally assumed to **supply power to** the system
  - Effort sources specify effort into the system
  - Flow sources specify flow into the system



- 2-ports have a **power through** convention

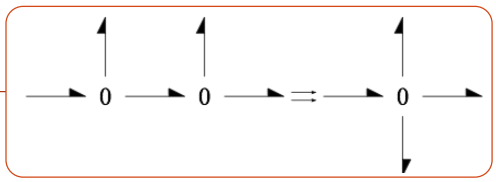
SIMPLIFICATIONS

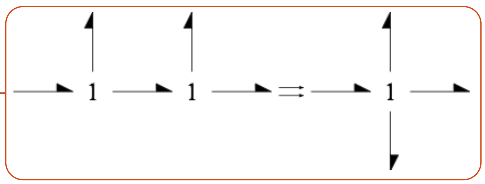
Junctions with two bonds (power in, power out) can be simplified into a single bond



What if power is not showing a power in, power out convention?

Adjacent junctions of the same type are actually the same junction and can be collapsed





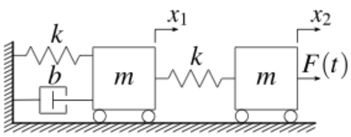
BOND GRAPH SYNTHESIS: MECHANICAL TRANSLATION & ROTATION

1. **Identify distinct velocities (linear/angular)**
2. **Insert the force/torque-generating 1-ports and the energy-conserving 2-ports**
3. **Assign power directions**
4. **Eliminate zero velocity (linear/angular) sources**
5. **Simplify**
6. Assign causality

MECHANICAL TRANSLATION

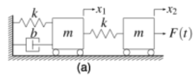
R-Element	Damper or friction
C-Element	Spring
I-Element	Mass
Effort Source	External force
Flow Source	Velocity source or shaker
Transformer	Lever or rocker arm
1-Junction	Common velocity; Sum of forces
0-Junction	Common force; Sum of velocities

Mechanical Translation Example 1



(a)

Figure 3.3



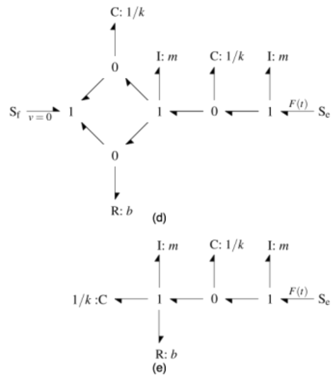
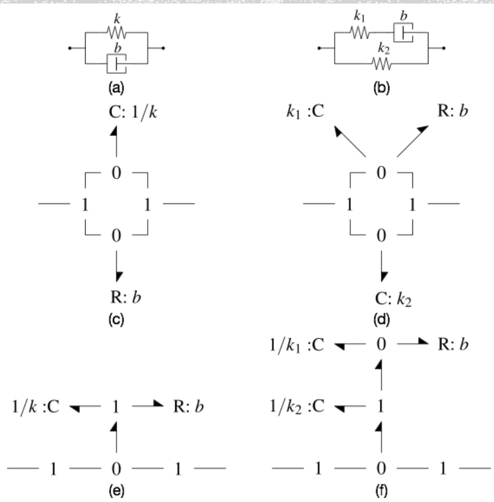
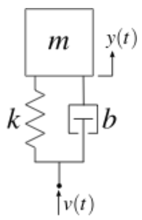


Figure 3.4



MECHANICAL TRANSLATION EQUIVALENCIES

Mechanical Translation Example 2



(b)

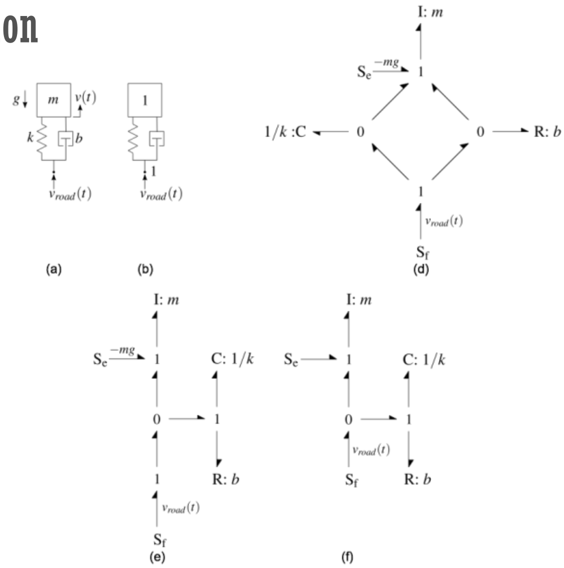
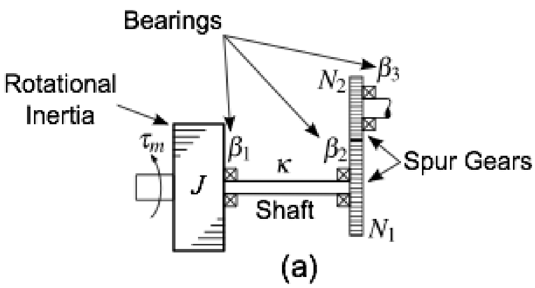


Figure 3.6

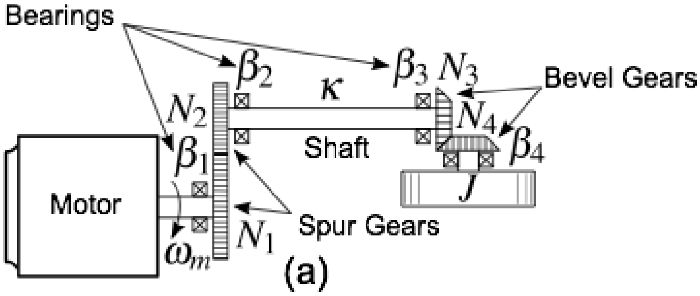
MECHANICAL ROTATION

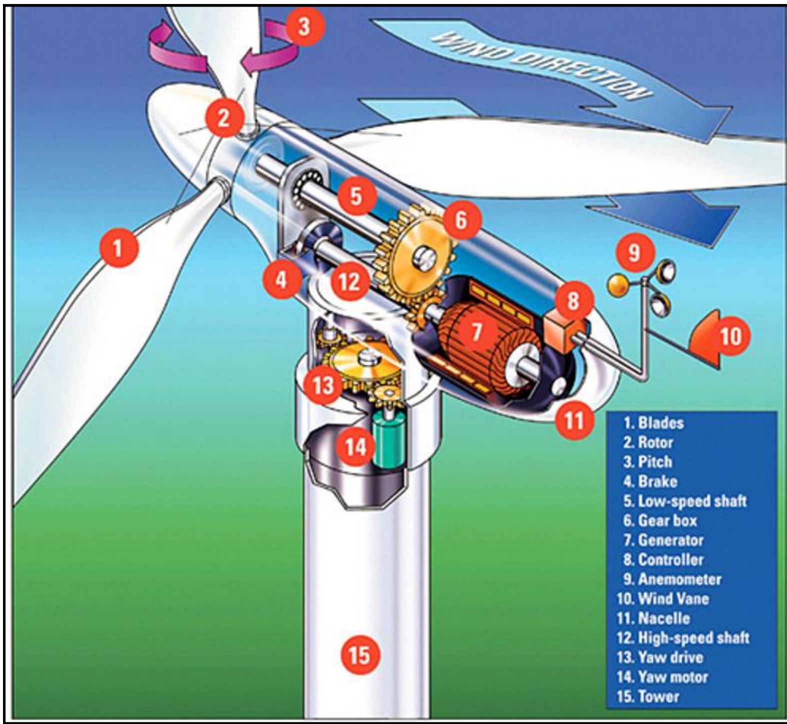
R-Element	Bearing or friction
C-Element	Torsion spring or shaft
I-Element	Rotational inertia
Effort Source	External torque (motor)
Flow Source	Angular velocity source (motor)
Transformer	Gear pair or chain and sprockets
1-Junction	Common angular velocity; Sum of moments (torques)
0-Junction	Common moment (torque); Angular velocity differential

MECHANICAL ROTATION EXAMPLE 1



MECHANICAL ROTATION EXAMPLE 2





CHALLENGE

- Generate a bond graph to predict the response of the system.

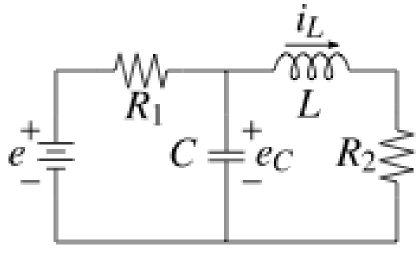
BOND GRAPH SYNTHESIS: ELECTRIC & HYDRAULIC CIRCUITS

1. **Identify distinct voltages/pressures**
2. **Insert 1-port circuit elements and energy-converting 2-ports**
3. **Assign power directions**
4. **Eliminate explicit ground/atmospheric pressure (or reference pressure)**
5. **Simplify**
6. Assign causality

ELECTRICAL CIRCUITS

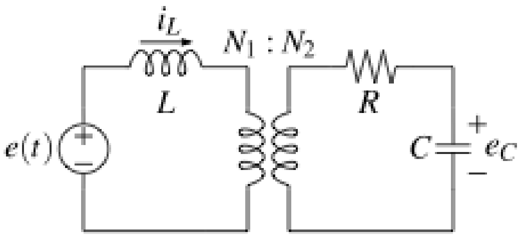
R-Element	Resistor
C-Element	Capacity
I-Element	Inductor
Effort Source	Battery or voltage source
Flow Source	Ideal current source
Transformer	Transformer
1-Junction	Common current; KVL
0-Junction	Common voltage: KCL

ELECTRIC CIRCUIT EXAMPLE 1



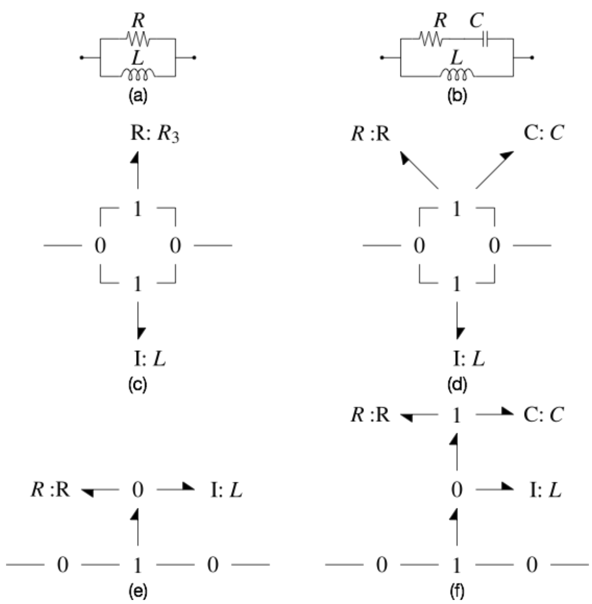
(a)

ELECTRIC CIRCUIT EXAMPLE 2



(a)

ELECTRICAL EQUIVALENCIES



- Electrical elements connected between the same pair of voltages
- Equivalencies can be used to simplify circuit branches connected in parallel
- Circuit elements connected in parallel share a common voltage drop across them

Electric Circuit Example 3

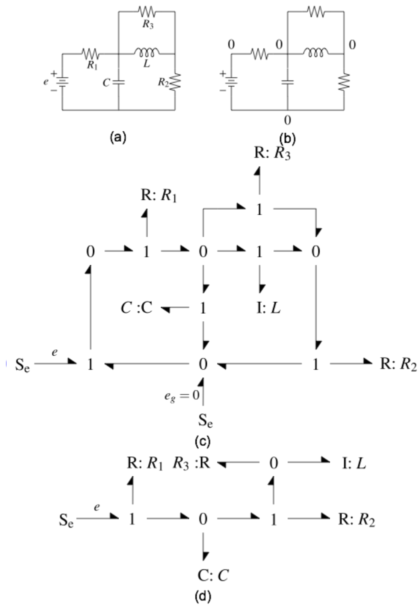


Figure 3.12

HYDRAULIC CIRCUITS

R-Element	Valve or surface roughness
C-Element	Accumulator
I-Element	Slug of fluid
Effort Source	Displacement pump or pressure source
Flow Source	Centrifugal pump or ideal flow source
Transformer	N/A
1-Junction	Common flow; Sum of pressure drops around a loop
0-Junction	Common pressure; Sum of flows into a junction

Hydraulic Circuit Example 1

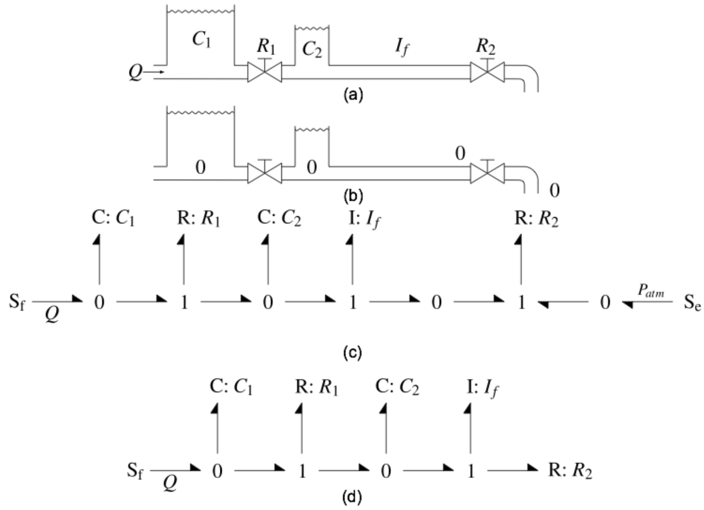


Figure 3.13

Hydraulic Circuit Example 2

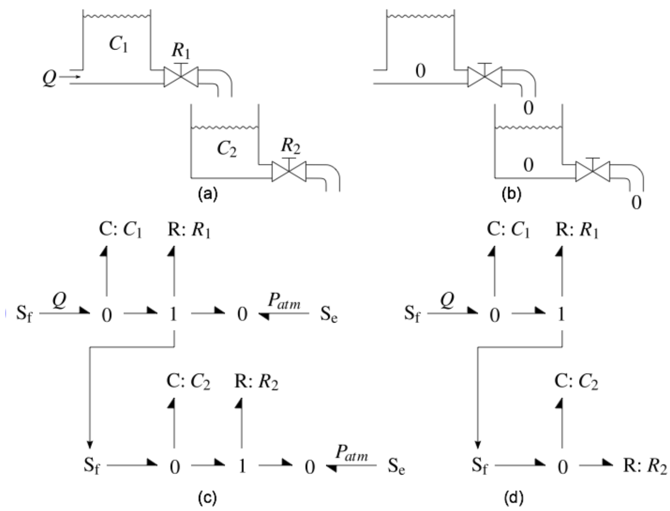
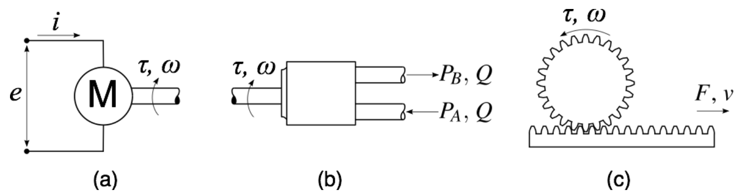


Figure 3.14



MIXED SYSTEMS

- Multiple energy domains that are coupled through transducers
- Procedure
  - Decompose into single energy domain subsystems at the transducers
  - Apply energy specific guidelines to each subsystem
  - Recouple using transducers

A Mixed System Example

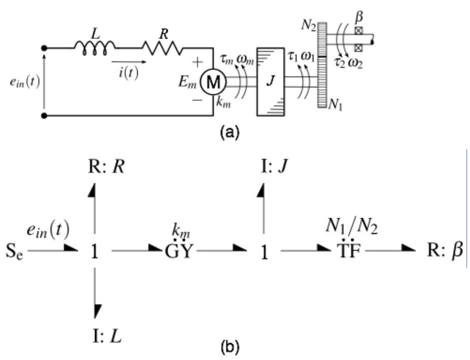
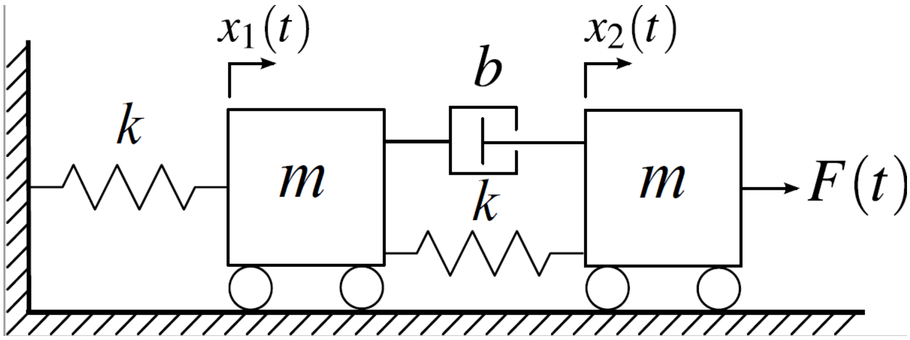


Figure 3.16

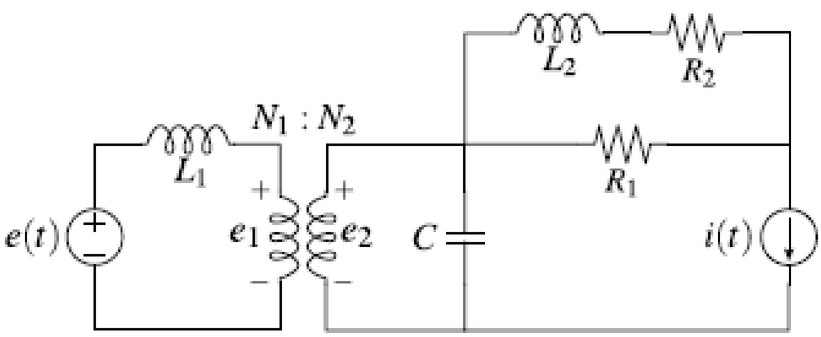
PRACTICE PROBLEM 1

- Synthesize the bond graph for the given system.



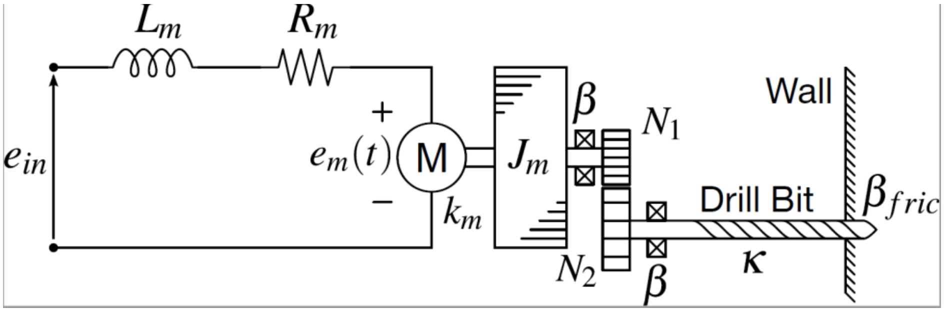
PRACTICE PROBLEM 2

- Synthesize the bond graph for the given system.



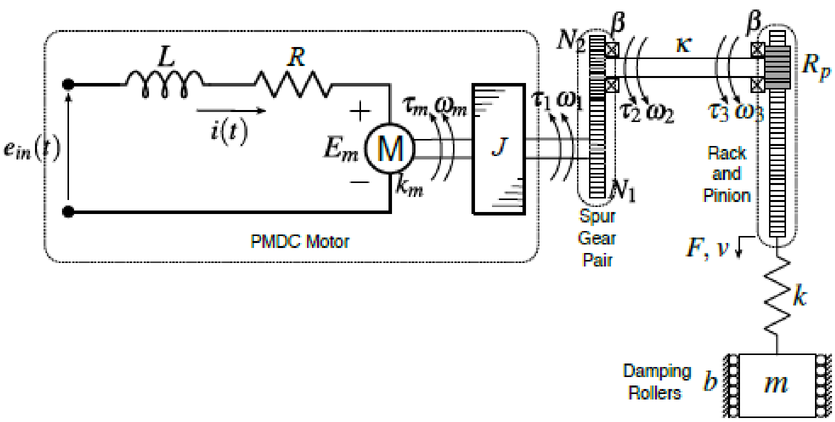
PRACTICE PROBLEM 3

- Synthesize the bond graph for the given system.



PRACTICE PROBLEM 4

- Synthesize the bond graph for the given system.

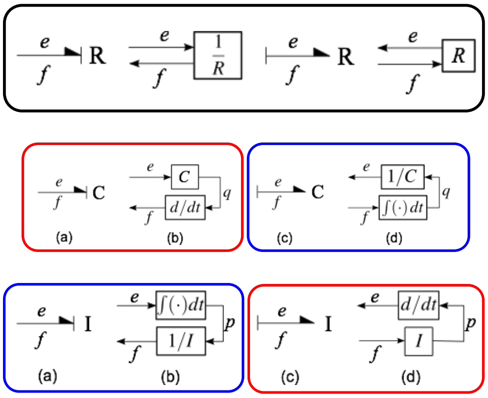


CAUSALITY FOR 1-PORTS

R-Element

C-Element

I-Element



Derivative Causality

Integral Causality

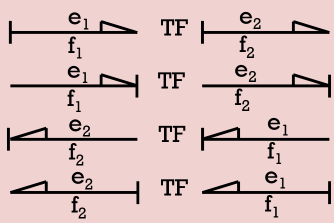
LABELS FOR 1-PORTS AFTER ASSIGNING INTEGRAL CAUSALITY

	Mechanical Translation		Mechanical Rotation		Electric Circuits		Hydraulic Circuits	
	Effort	Flow	Effort	Flow	Effort	Flow	Effort	Flow
C-Elements	kx	$\dot{x}$	$\kappa\Delta\theta$	$\Delta\dot{\theta}$	$\frac{q}{C}$	$\dot{q}$	$\frac{V}{C_f}$	$\dot{V}$
I-Elements	$\dot{p}$	$\frac{p}{m}$	$\dot{h}$	$\frac{h}{J}$	$\dot{\lambda}$	$\frac{\lambda}{L}$	$\dot{\Gamma}$	$\frac{\Gamma}{I_f}$

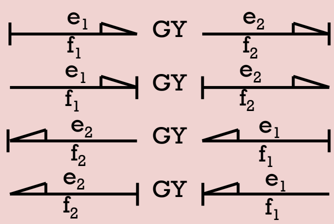
C - Elements	Displacement	Angle	Charge	Volume
I - Elements	Linear Momentum	Angular Momentum	Flux Linkage	Hydraulic Momentum

CAUSALITY FOR 2-PORTS

Transformer



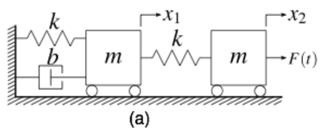
Gyrator



STATE EQUATION DERIVATION

- Synthesize simplified system bond graph
- Assign causality
  - Sources first
  - Then energy-storing elements
  - If unspecified bond remains, select an R-element, assign causality, and propagate
- Label efforts and flows on energy storing elements
- Apply primary conditions
- Apply secondary condition

Mass-Spring-Damper Example



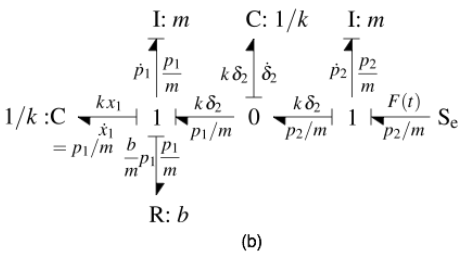
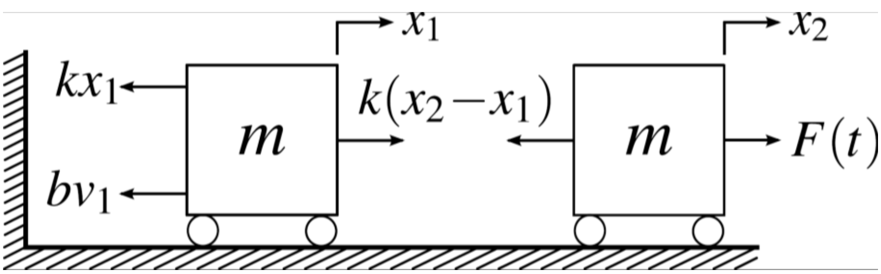


Figure 3.18

FREE BODY DIAGRAM FOR MASS-SPRING-DAMPER EXAMPLE



$\overset{+}{\rightarrow}\Sigma F_x = m\ddot{x}_1 = k(x_2 - x_1) - kx_1 - bv_1$

$\overset{+}{\rightarrow}\Sigma F_x = m\ddot{x}_2 = F(t) - k(x_2 - x_1)$

Mechanical Rotation Example

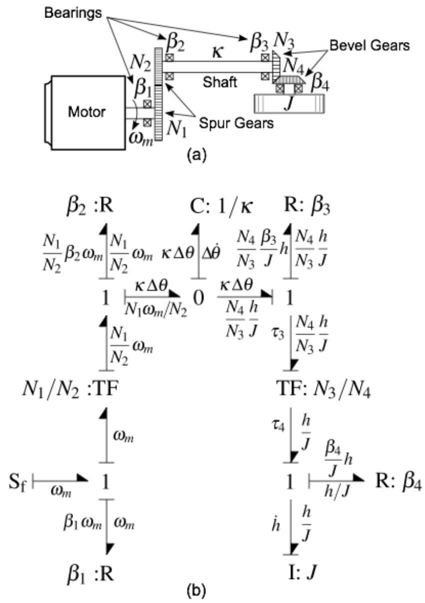


Figure 3.20

Electric Circuit Example

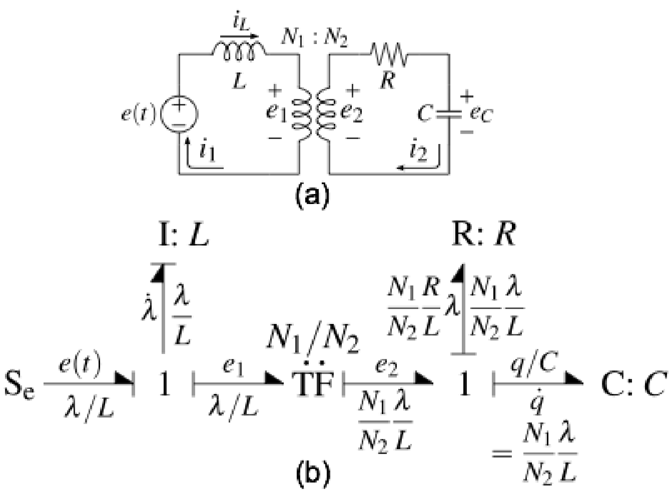


Figure 3.21

Hydraulic Circuit Example

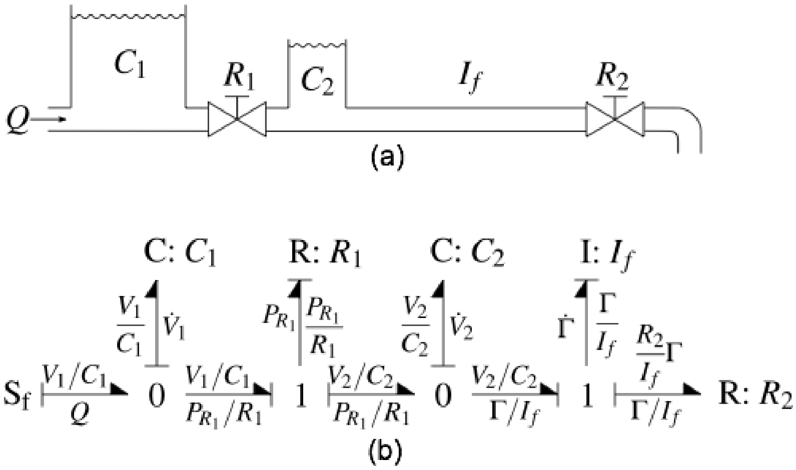


Figure 3.22

Mixed System Example

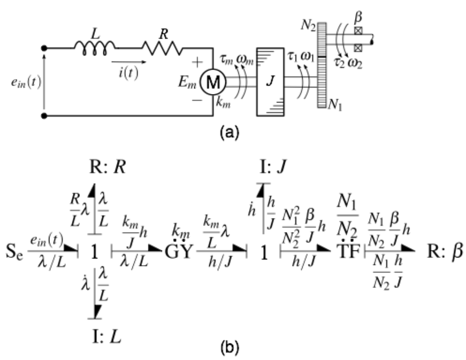


Figure 3.23

Algebraic Loops

- The mass-spring-damper system shown is a model of two railcars being pushed up against a snubber. What if the first railcar was a fully loaded coal car and the second an empty flatbed railcar?

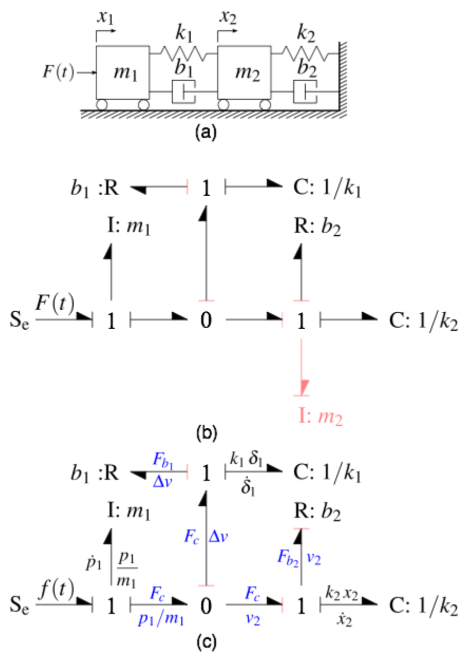
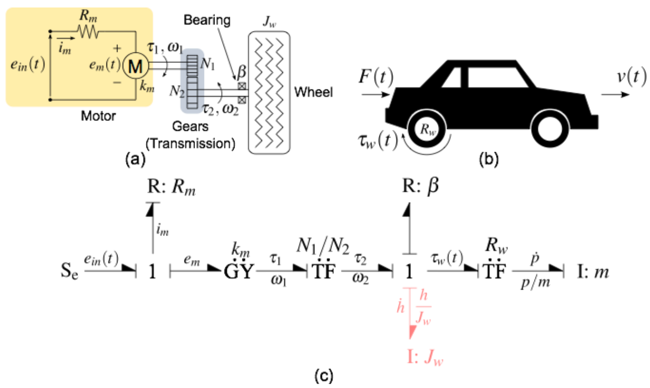
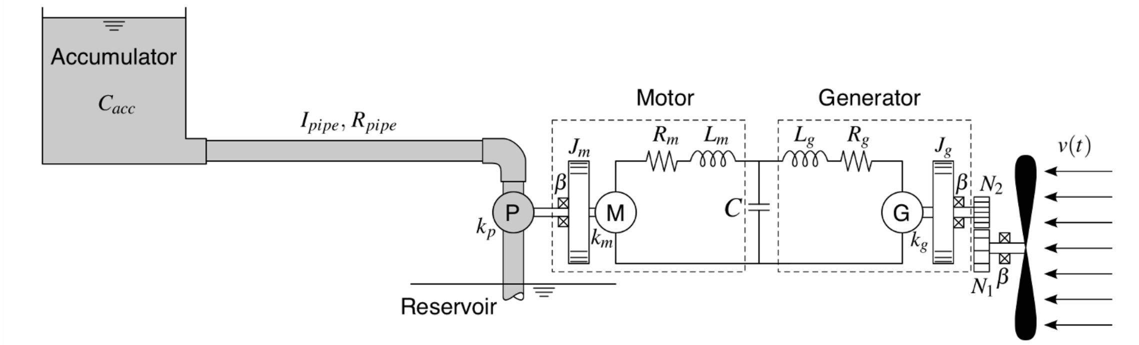


Figure 3.24

DERIVATIVE CAUSALITY

- Energy-storing elements in derivative causality are not dynamically independent, but rather dependent.





CHALLENGE PROBLEM

Synthesize a bond graph and derive the state equations of the following system.

Summary

- As illustrated in Figure 3.1 (a), generally, it is assumed that power flows from the system to energy-storing or dissipating elements.
- Usually, it is assumed that power flows from the source to the system. Moreover, effort sources supply effort as an input and flow sources supply flow inputs (refer to Figure 3.1 (b)).
- Transformers and gyrators have power through convention. As depicted in Figure 3.1 (c), the power goes in one port and out the other.
- Adjacent 0- or 1-junctions can be collapsed into a single junction. Common junction types adjacent to one another are in actuality the same junction and the attached bonds share a common effort or flow (Figure 3.2).
- When synthesizing bond graphs for mechanical systems, we first identify distinct velocities and establish 1-junctions. For each 1-junction we identify elements that are directly associated. For example, inertias are commonly associated with distinct velocities. Then we insert effort-generating 1-ports off of 0-junctions or 2-ports between appropriate pairs of 1-junctions. Next, we eliminate zero-velocity sources and simplify.

Summary Continued

- For circuits (both electric and hydraulic) we first identify distinct potentials (voltages or pressures) and establish 0-junctions. If there are any elements directly associated with these distinct efforts, we place them directly off the associated junction using a bond. We then insert the 1- and 2-ports between pairs of 0-junctions. The 1-ports are placed off of 1-junctions that are inserted between pairs of 0-junctions. Next, we eliminate the ground or reference pressure and simplify.
- Mixed systems can be dissected into subsystems, each of which is of a single energy domain. Each subsystem can be analyzed using the associated guidelines. The subsystems interface at energy-converting transducers which are modeled as either transformers or gyrators. Some examples were provided in Figure 3.15.
- When deriving differential equations from a bond graph one must first assign causality beginning with the sources, then the energy-storying elements, and last, if necessary, the R-elements. At each stage we as- sign the causality to an element and propagate if the causality affects adjacent junctions and/or elements. The process proceeds until all the bonds have an assigned causality. The differential equations result from applying the primary and secondary conditions at the junctions.
- Algebraic loops and derivative causality require extra analysis to derive the differential equations.