

Chapter 3. Concrete Gravity Dams



Dworshak Dam

4/9/2013 oromiyaan ni bilisoomti

3.1 Definition

❑ A concrete gravity dam is a massive concrete structure, roughly **triangular in shape**, and designed so that its **weight ensures structural stability** against the **hydrostatic pressure of the impounded water and other forces that may act on the dam**.

❑ Gravity dams may be classified by plan as **straight gravity dams** and curved gravity dams, depending up on the axis alignment.

❑ In the earlier periods, gravity dams were constructed from masonry. In recent years, however, gravity dams are constructed from concrete.

❑ Gravity dams are

- ✓ permanent structures that require little maintenance
- ✓ constructed to greater heights

A photograph of the Grande Dixence Dam, a massive concrete gravity dam in Switzerland. The dam is a long, grey concrete wall with a series of buttresses, situated in a mountainous valley. The surrounding landscape is steep and covered in green vegetation. The sky is overcast.

The largest concrete gravity Dam

Height 285 m

Length 700 m

Base width 200 m

Volume 6,000,000 m³

Reservoir

Capacity 400,000,000 m³

Catchment area 46 km²

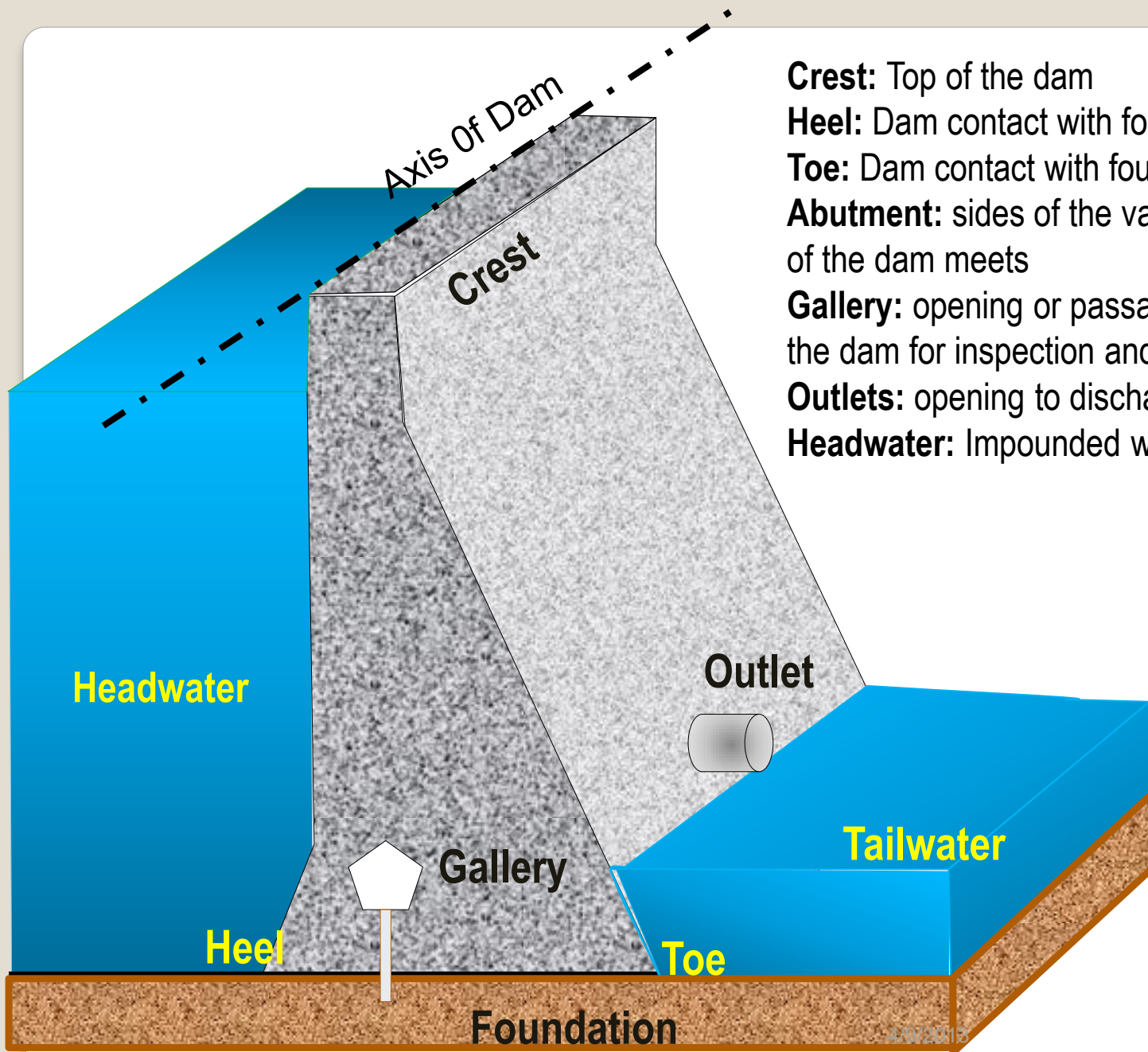
Surface area 4 km²

Max. water depth 284 m

Power station

Installed capacity 2,069 MW

Grande Dixence Dam, Switzerland



Crest: Top of the dam

Heel: Dam contact with foundation on the u/s side

Toe: Dam contact with foundation on the d/s side

Abutment: sides of the valley which the structure of the dam meets

Gallery: opening or passage left in the body of the dam for inspection and drainage purposes

Outlets: opening to discharge water

Headwater: Impounded water



3.2 Types of Concrete Gravity Dams

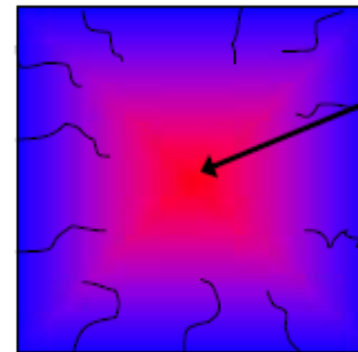
3.2.1 Conventional Concrete Dams (CC Dams)

- ❑ These are dams constructed with Mass concrete

Mass Concrete is any volume of concrete with **dimensions large enough** to require that **measures be taken to cope with generation of heat** from hydration of the cement and attendant volume change to minimizing cracking. (American Concrete Institute ACI)

- ❑ Cement Hydration is a **very exothermic process**, leading to a rise in temperature at the core of very large pours. Expected to reach the maximum within 1 to 3 days after placement.

- ✓ if the temperature rises to 70°C
Delayed ettringite formation (DEF) or
- ✓ if the surface temperature is allowed to deviate greatly from that of the core, i.e., temperature difference between the interior and exterior reaches to 19°C , thermal cracking will develop.

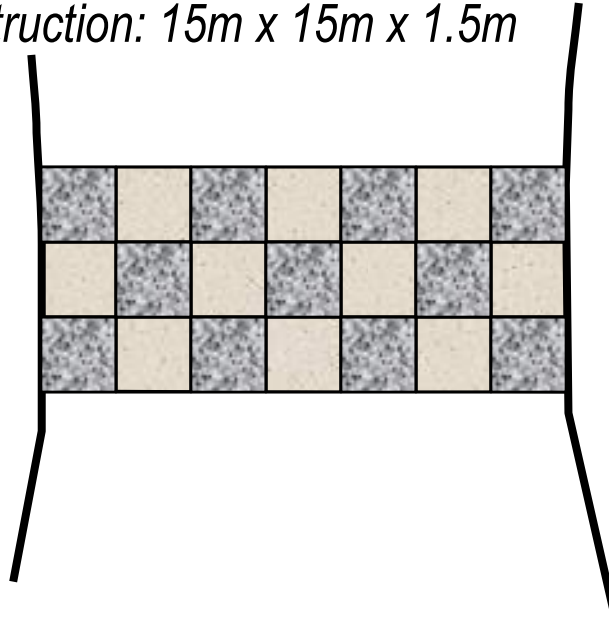


High core temperature during hydration

Cooled surface and thermal cracking

- ❑ Cracks affects **water tightness, durability, internal stresses.**
- ❑ Several methods for controlling cracks due to thermal stresses exist

✓ *Block construction: 15m x 15m x 1.5m*



- ✓ *Mix design that limits heat of hydration*
 - ✓ *reduced cement content*
 - ✓ *using special low heat cement*
 - ✓ *use of pozzolana and other admixtures*
- ✓ *Embedded pipe cooling system*
- ❑ The main disadvantages of CC dams include
 - ✓ They are expensive
 - ✓ They require long construction time

3.2.2 Roller Compacted Concrete Dams(RCC Dams)

❑ Roller-compacted concrete is simply **concrete constructed with the use of earthfill methods**. It was introduced in late 1970s

❑ The **traditional method** of placing, compacting, and consolidating mass concrete is a slow process.

❑ Improvements in earth moving equipments made the construction of earth and rock fill dams speedier and more cost efficient.

❑ According to ACI,

*RCC is a concrete **compacted by roller compaction**. The concrete, in its **unhardened state**, will support a roller while being compacted*

❑ RCC thus differs from conventional concrete in its consistency requirement (zero slump):

✓ *dry enough to support roller*

✓ *wet enough to permit adequate distribution of the binder mortar during mixing and vibration*

Characteristic	RCC	Conventional
Cement (kg/m ³)	<150	150-230
Water to Cement ratio	0.5-0.6	0.5-0.7
90 days strength (MN/m ²)	20-40	18-40
Layer (m)	0.3	1.5-2.5

❑ Advantages of RCC dams include

- ✓ Reduced cost (25 % - 50% less than CC)
 - Lesser cement consumption, less thermal stresses
 - Less formwork
 - Transportation, placement, and compaction is easier.
- ✓ Reduced construction time (1-2yrs)
 - transportation, placement, and compaction is done in highly mechanized way





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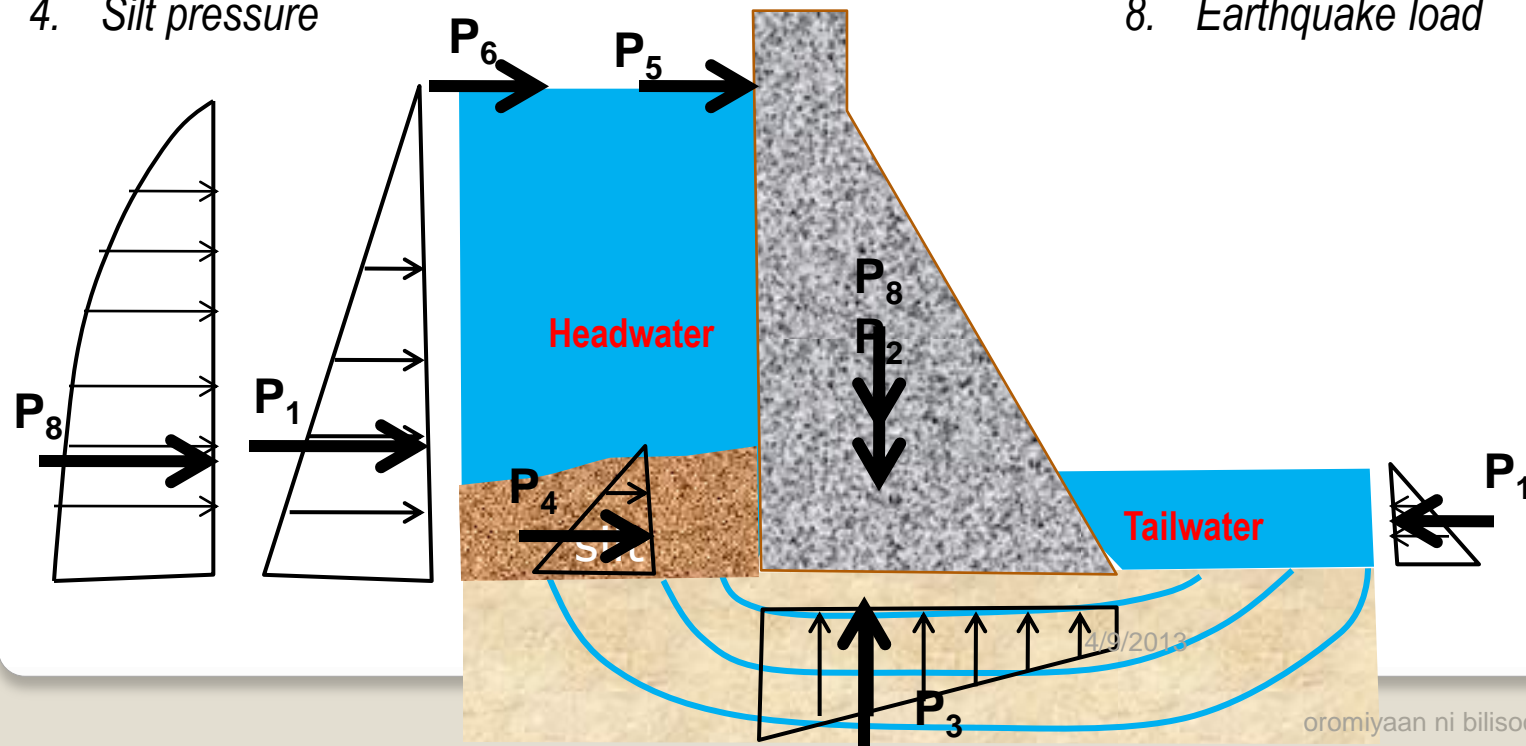
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2.3. Loads on Gravity Dams

❑ The **structural integrity** of a dam must be maintained across the range of circumstances or events likely to arise in service. The design is therefore determined through considerations of corresponding spectrum of loading conditions.

❑ Gravity dams are subjected to the following main loads / forces

1. Water pressure (water load)
2. Weight of the dam
3. Uplift pressure
4. Silt pressure
5. Wave pressure
6. Ice pressure
7. Wind load
8. Earthquake load



❑ Loads can be classified in terms of applicability or relative importance as primary, secondary, and exceptional loads

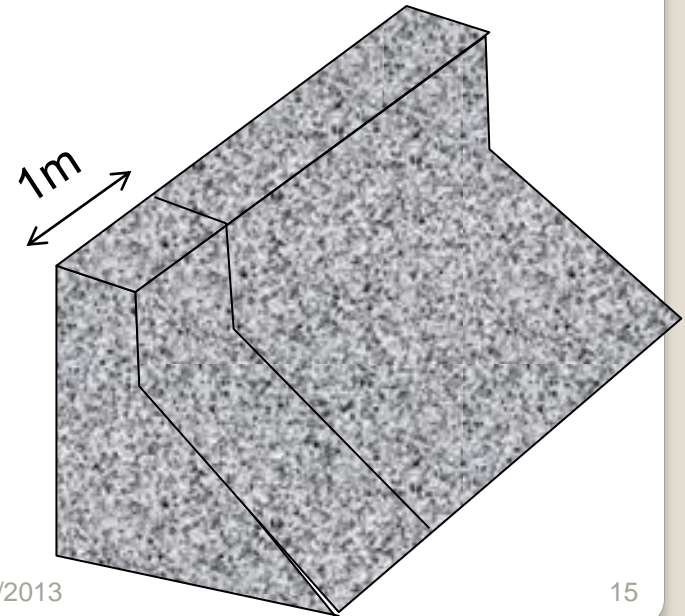
❑ **Primary loads:** are identified as **those of major importance to all dams** irrespective of type. e.g., Water load, self weight, and uplift

❑ **Secondary loads:** are universally applicable although of a lesser magnitude or alternatively **are of major importance to certain types of dam**. e.g. Silt load, wave pressure, thermal pressure

❑ **Exceptional loads:** are so designed on the basis of limited general applicability of occurrence. e.g. Earthquake loads

❑ For convenience in analysis **loads are expressed per metre length of dam**, i.e. they are determined for a two dimensional transverse section with unit width parallel to the dam axis.

❑ It is similarly convenient to account for some loads in terms of resolved horizontal and vertical components, identified by the use of appropriate subscripts, *Ph* and *Pv* respectively.



2.3.1 Primary Loads

2.3.1.1 Water loads

A. Non-overflow section

H – Headwater depth

H' – Tailwater depth

P_H – Horizontal Headwater Pressure Force

P_H' – Horizontal Tailwater Pressure Force

P_V' – Vertical Tailwater Pressure Force

i) U/s vertical face

a) Upstream face: Horizontal force

$$P_H = \frac{1}{2} \gamma_w H^2 \quad \text{The force acts horizontally at } z = \frac{H}{3} \quad \text{from the basis of the dam}$$

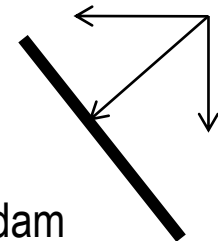
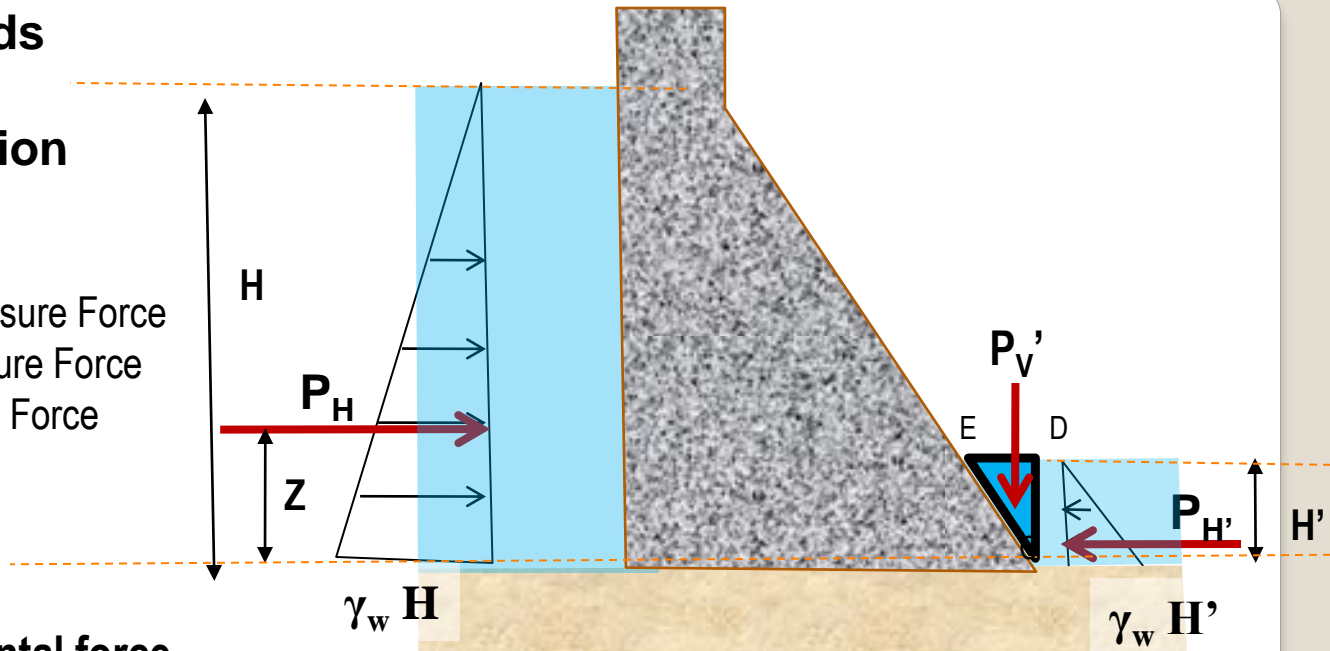
b) Downstream face: Horizontal and vertical forces

Horizontal component

$$P_H' = \frac{1}{2} \gamma_w H'^2 \quad \text{The force acts horizontally at } z' = \frac{H'}{3} \quad \text{from the basis of the dam}$$

Vertical component d/s face

$$P_V' = \gamma_w \text{Area}_{CDE} \quad \text{The force acts vertically at } \frac{\overline{ED}}{3} \quad \text{from the toe of the dam}$$



ii) U/s inclined face

a) Upstream Face:

Horizontal force

$$P_H = \frac{1}{2} \gamma_w H^2$$

It acts horizontally at $z = \frac{H}{3}$
from the basis of the dam

Vertical force

$$P_{V1} = \gamma_w \text{Area}_{FGH}$$

It acts vertically at $\frac{2}{3} \overline{FG'} + \overline{G'C}$ from the toe of the dam

$$P_{V2} = \gamma_w \text{Area}_{HGIJ}$$

It acts vertically at $\frac{1}{2} \overline{FG'} + \overline{G'C}$ from the toe of the dam

c) Downstream face

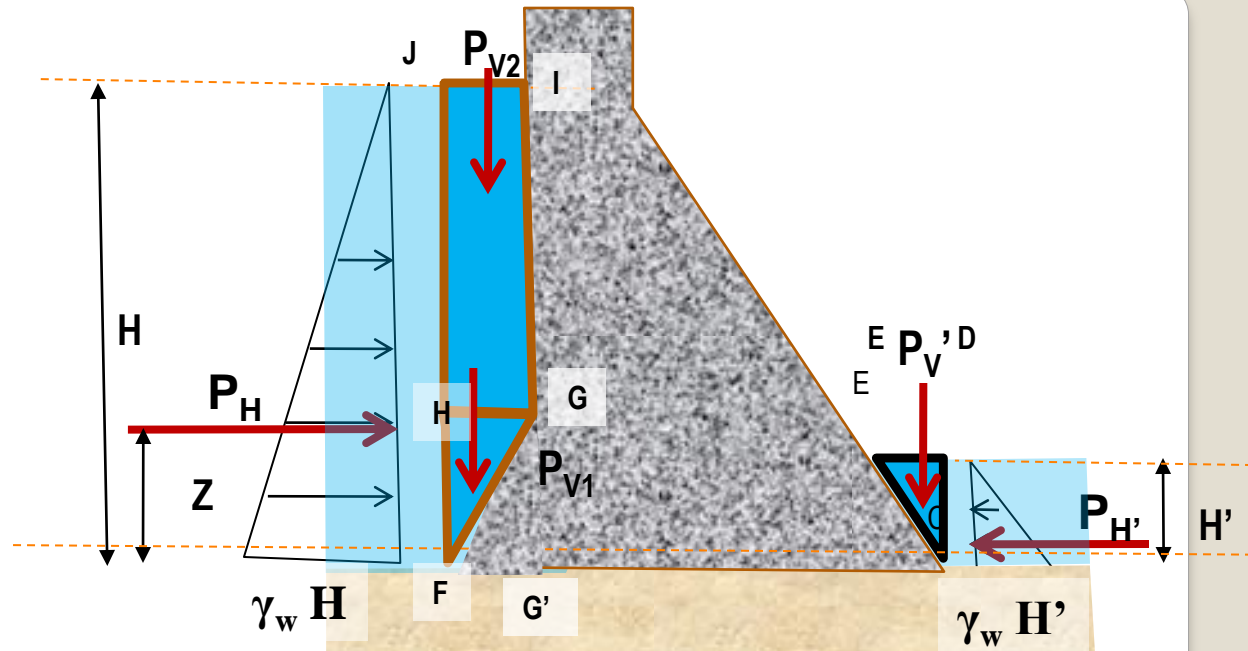
Horizontal force

$$P_{H'} = \frac{1}{2} \gamma_w H'^2$$

It acts vertically at $z' = \frac{H'}{3}$ from the base of the dam

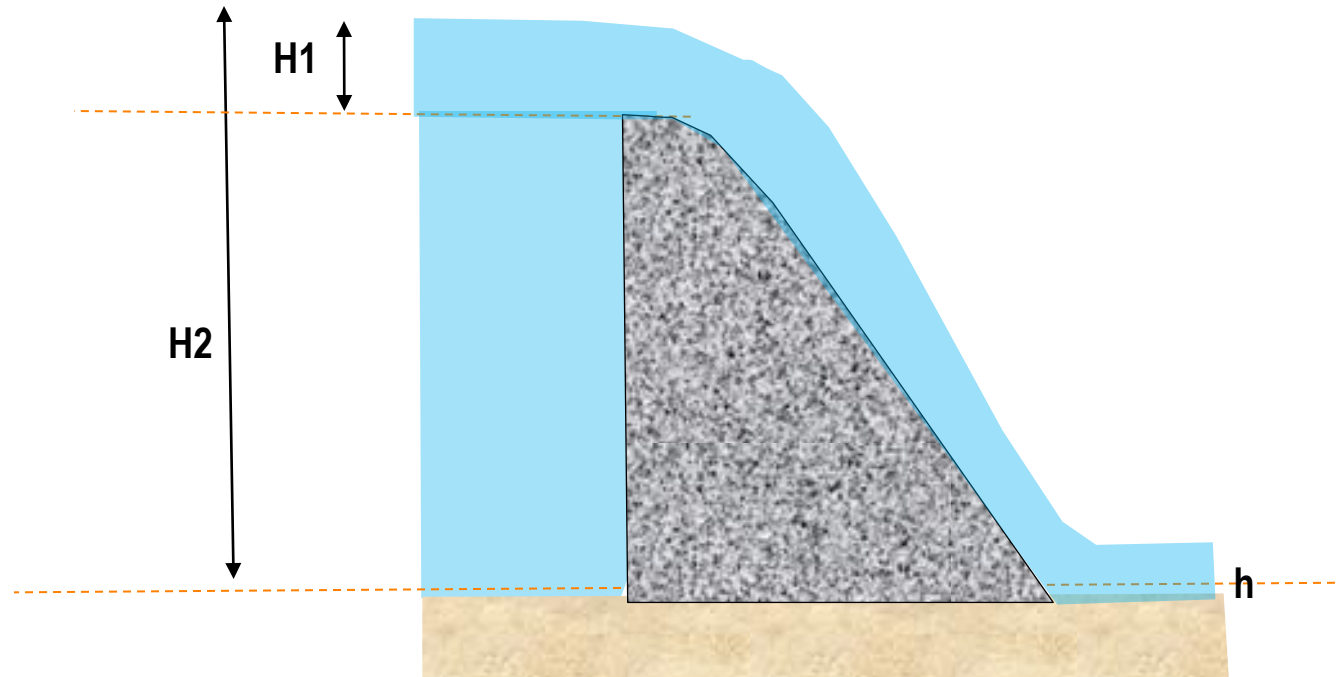
Vertical Force

$P_{V'} = \gamma_w \text{Area}_{CDE}$ it acts vertically at $\frac{\overline{ED}}{3}$ from the toe of the dam

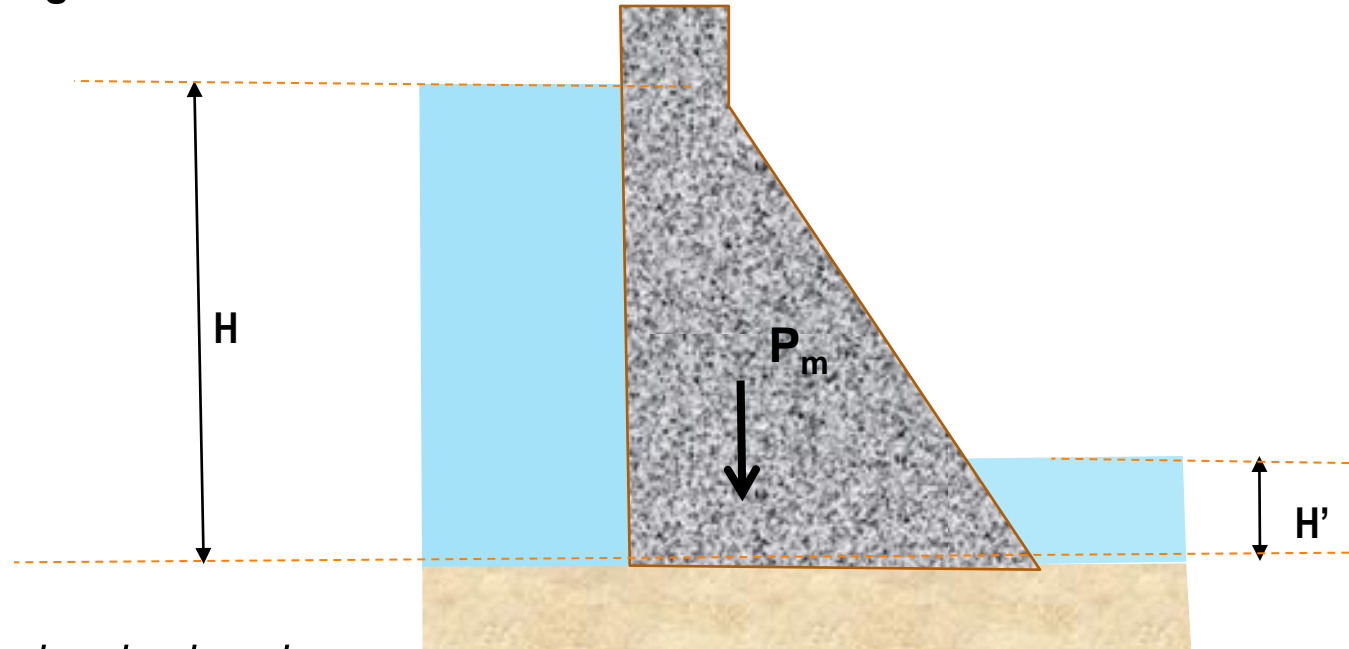


B. Overflow section

Exercise: calculate the water load on an overflow section of a gravity dam



2.3.1.2 Self weight



The weight of the dam is given by

$$P_m = \gamma_c A$$

where

γ_c *is unit weight of concrete*

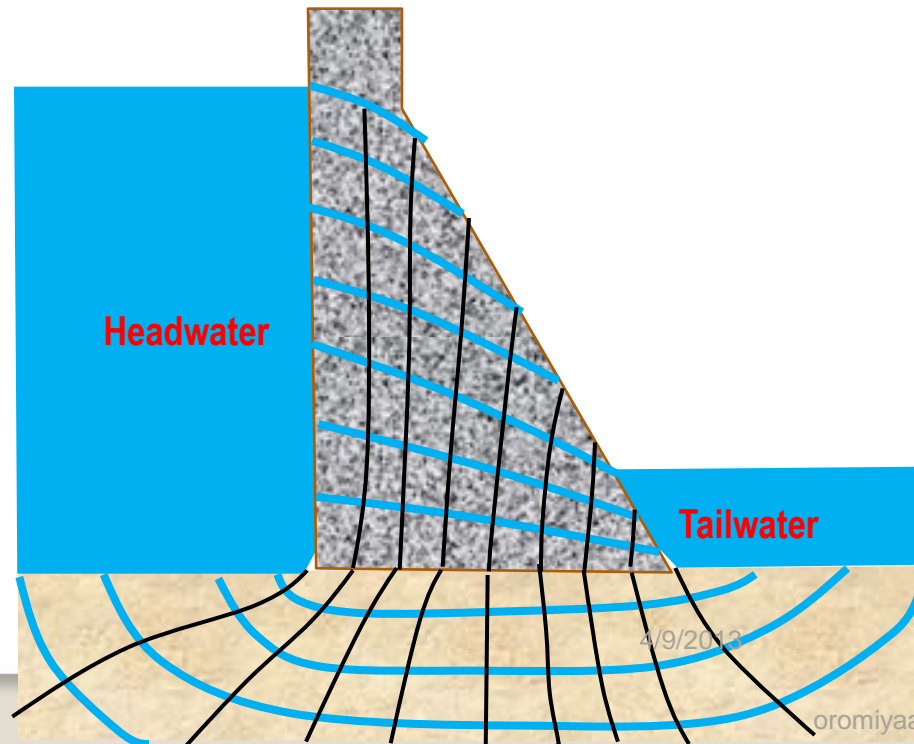
A *is the x-sectional area of the dam*

*The force acts through the **centroid** of the x-sectional area. It will include weight of ancillary structures.*

2.3.1.3 Uplift Force

❑ Both the dam body and foundation material are permeable. Water in the reservoir percolates through the dam body (lift and construction joints) and foundation material

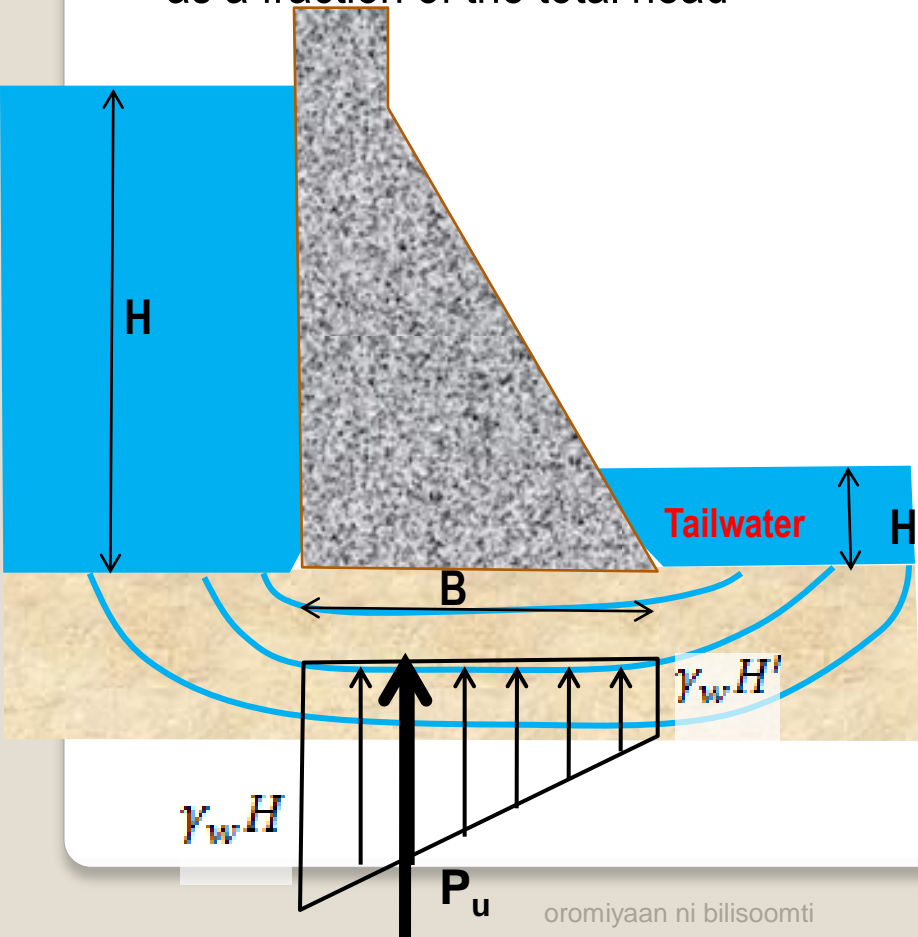
Concrete permeability (cm/s)		Rock permeability (cm/s)		Soil permeability (cm/s)	
Cement		Granite	5×10^{-9}	Gravel	0.01-1
Fresh	2×10^{-4}	Sandstone	1.2×10^{-8}	Sand	10^{-3} to 0.1
Ultimate	6×10^{-11}			Silt	10^{-5} to 10^{-3}
Concrete	10^{-6} to 10^{-8}			Clay	$< 10^{-6}$



❑ The percolating water exerts an uplift pressure within the dam body and at the base of the dam. The uplift in the dam body is small compared to at the base. The uplift force due to the uplift pressure at the base depends on two factors

A) Area factor : the fraction of the actual area of the base over which the uplift pressure is supposed to act. ([Link](#))

B) Intensity factor: the intensity of pressure acting on any point of the base expressed as a fraction of the total head



❑ Uplift at the heel = hydrostatic pressure at the u/s headwater = $\gamma_w H$

❑ Uplift at the heel = hydrostatic pressure at the tailwater = $\gamma_w H'$

❑ Uplift pressure at any intermediate point = linear interpolation

❑ Uplift force = Average pressure intensity \times Area \times Area factor (η)

$$P_u = \frac{1}{2} \gamma_w (H + H') A \eta$$

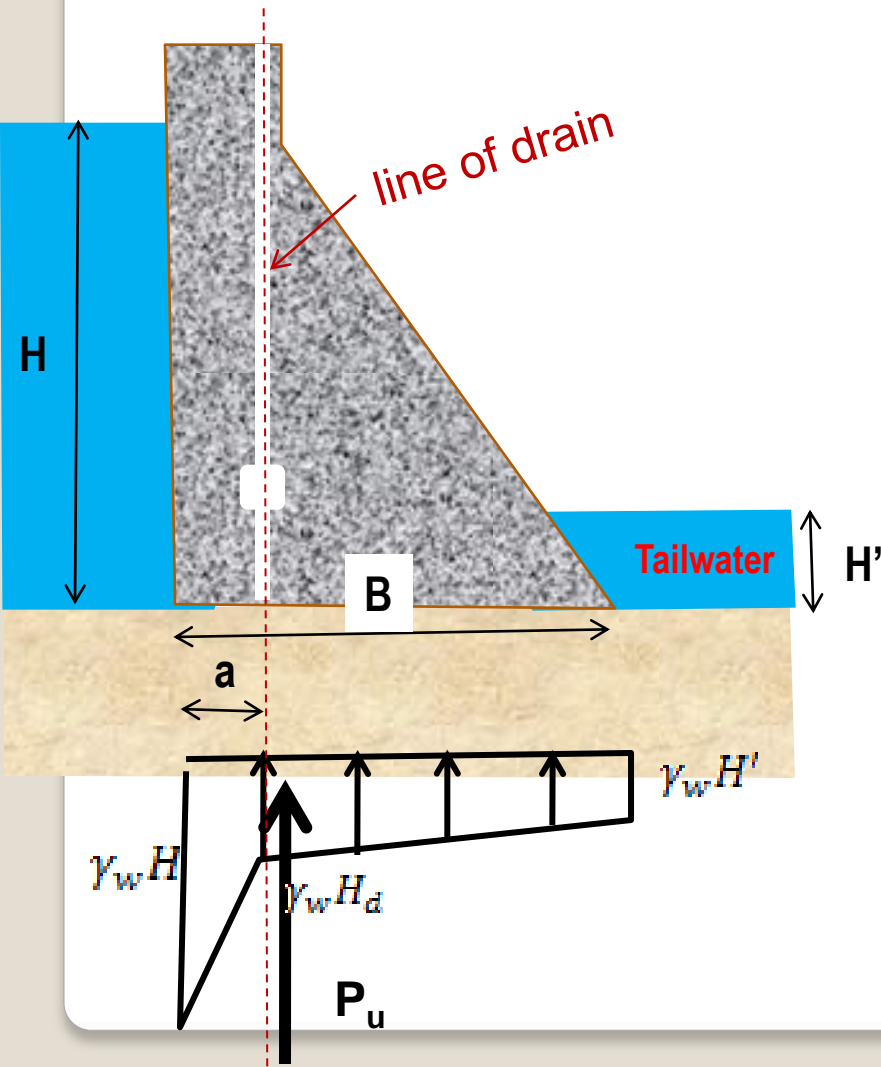
$$\eta = 1; \quad A = B \times 1$$

$$P_u = \frac{1}{2} \gamma_w (H + H') B$$

❑ It acts at $Z = \frac{B}{3} \left(\frac{H + 2H'}{H + H'} \right)$ from the toe

Effects of drains on uplift pressure

❑ To reduce uplift pressure, drains are formed through the body of the dam and also drainage holes are drilled in the foundation rock



❑ Uplift at the heel $= \gamma_w H$

❑ Uplift at the toe $= \gamma_w H'$

❑ Uplift pressure at the line of drain $= \gamma_w H_d$
 H_d – mean effective head at the line of drain

$$H_d = H' + K_d(H - H')$$

K_d is a function of drain geometry

$$K_d = \frac{1}{3} \quad \text{USBR} \qquad K_d = \frac{1}{2} \quad \text{TVA}$$

$$K_d = \frac{1}{4} \text{ to } \frac{1}{2} \quad \text{USACE}$$

❑ The uplift force

$$P_u = \frac{\gamma_w}{2} [H(BK_d + a) + H'(B(2 - K_d) - a)]$$

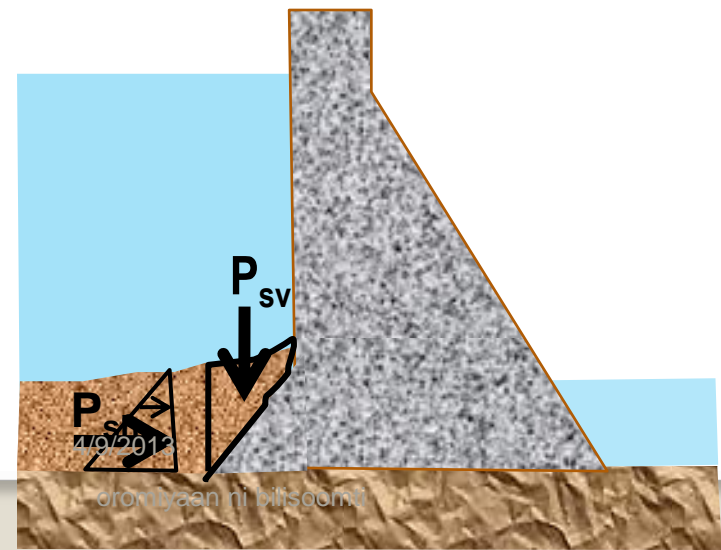
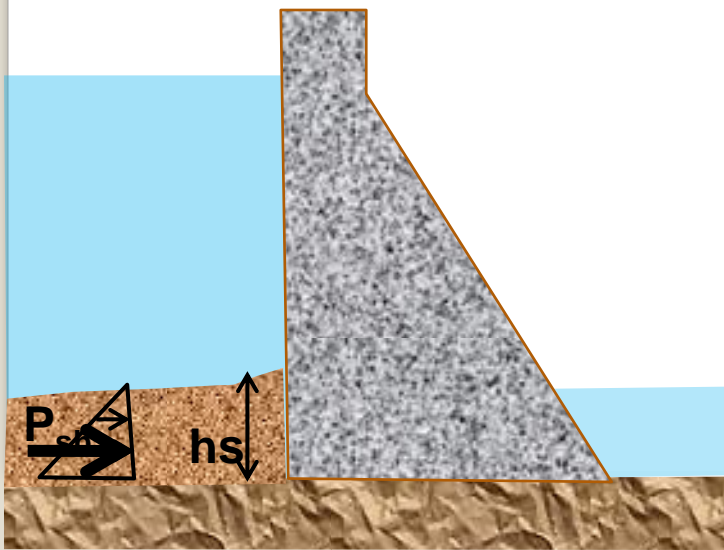
and acts at

2.3.2 Secondary Loads

2.3.2.1 Earth and Silt Load

❑ Gravity dams are sometimes subjected to **earth pressures** on either u/s or d/s face, where the foundation trench is backfilled. Such pressures usually have minor effect on the stability of the structure, and may be ignored in design.

❑ Practically all streams transport silts or fine sediments, particularly during floods. The **gradual accumulation of fine sediments** against the face of the dam generates a resultant horizontal force P_{sh} . The magnitudes of P_{sh} , which is in addition to the water load, is a function of the sediment depth, h_s , the submerged unit weight, γ'_s , and the active lateral pressure coefficient K_s . When the u/s face have flaring, the sediments will generate vertical force P_{sv} .



The **horizontal force** is given by

$$P_{hs} = k_a \gamma'_s \frac{h_s^2}{2}$$

where $\gamma'_s = \gamma_s - \gamma_w$ is the submerged unit weight of the silt sediment

$$k_a = \frac{1 - \sin \phi_s}{1 + \sin \phi_s} \quad \text{is active lateral earth pressure coefficient and}$$

ϕ_s is the angle of shearing resistance

h_s is the height of the silt sediment

The horizontal force act at $z = \frac{h_s}{3}$ above the base of the dam

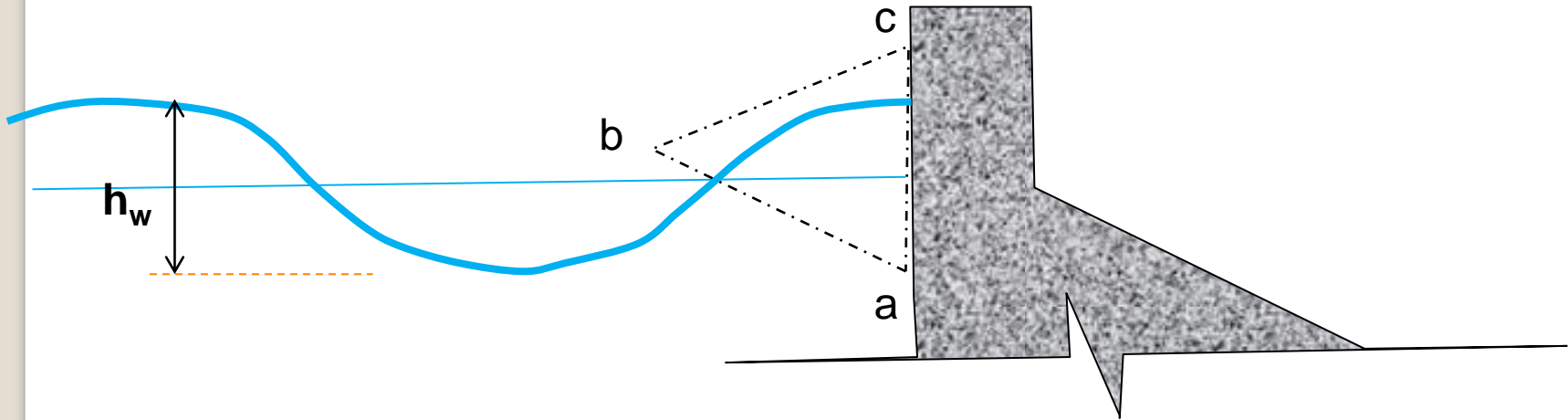
The **vertical force**

$$P_{vs} = \gamma'_s \text{Area}_{ABC}$$

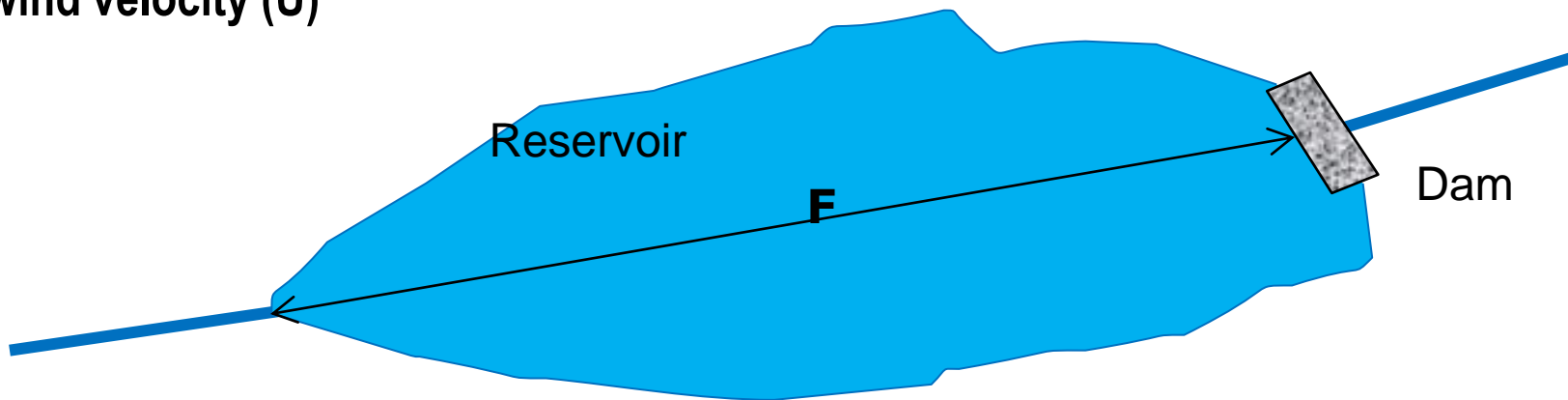
Just after construction of the dam, the depth (h_s) of the silt is zero. It increases gradually with time and finally it becomes equal to the height of the dead storage. It is usual practice to assume the value of h_s is equal to the height of the dead storage above the base.

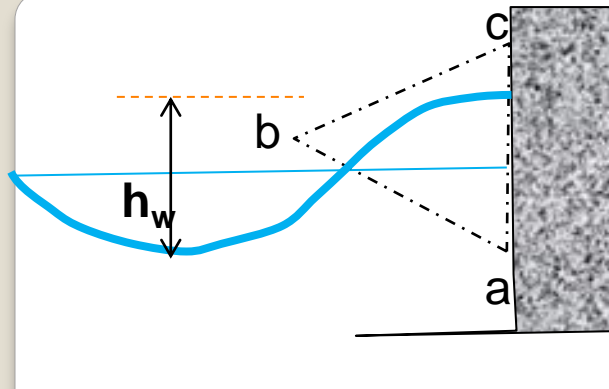
2.3.2.2 Waver pressure

❑ Waves are generated on the surface of the reservoir by the blowing winds, which causes a pressure towards the downstream side



❑ Wave pressure depends on the **wave height (h_w)** which depends on **fetch (F)** and **wind velocity (U)**



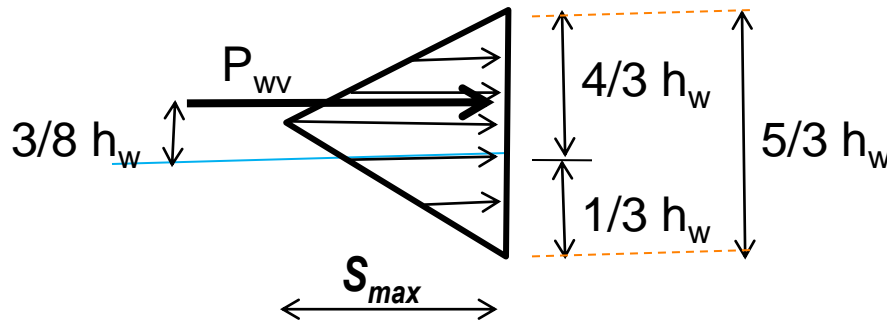


The wave height h_w is given by

$$h_w = 0.032\sqrt{UF} + 0.763 - 0.271\sqrt[4]{F} \quad \text{if } F \leq 32 \text{ km}$$

$$h_w = 0.032\sqrt{UF} \quad \text{if } F \geq 32 \text{ km}$$

h_w - height of wave in m
 U - wind velocity km/hr
 F - Fetch in km



The maximum pressure intensity (S_{max}) due to wave action may be given by

$$S_{max} = 2.4 \gamma_w h_w$$

The pressure distribution may be assumed to be triangular of base $5/3 h_w$. The total force due to wave action

$$P_{wv} = \frac{1}{2} S_{max} \frac{5}{3} h_w$$

$$P_{wv} = 2\gamma_w h_w^2$$

The force acts a distance of $3/8 h_w$ above the reservoir surface

2.3.3 Exceptional loads

2.3.3.1 Earth quake forces

❑ Earthquake **represents the release of built up stress** in the lithosphere. It occurs along fault lines. The released energy propagates in form of **seismic waves** that causes the ground to shake. [Earthquake link](#)



❑ **Ground motions** associated with earthquakes can be **characterized** in terms of *acceleration*. The earthquake ground acceleration are expressed as fraction of gravitational acceleration.

$$\text{ground acceleration} = \alpha g$$

❑ Although the seismic waves propagate in all direction, for design purpose, the accelerations are resolved in horizontal and vertical components.

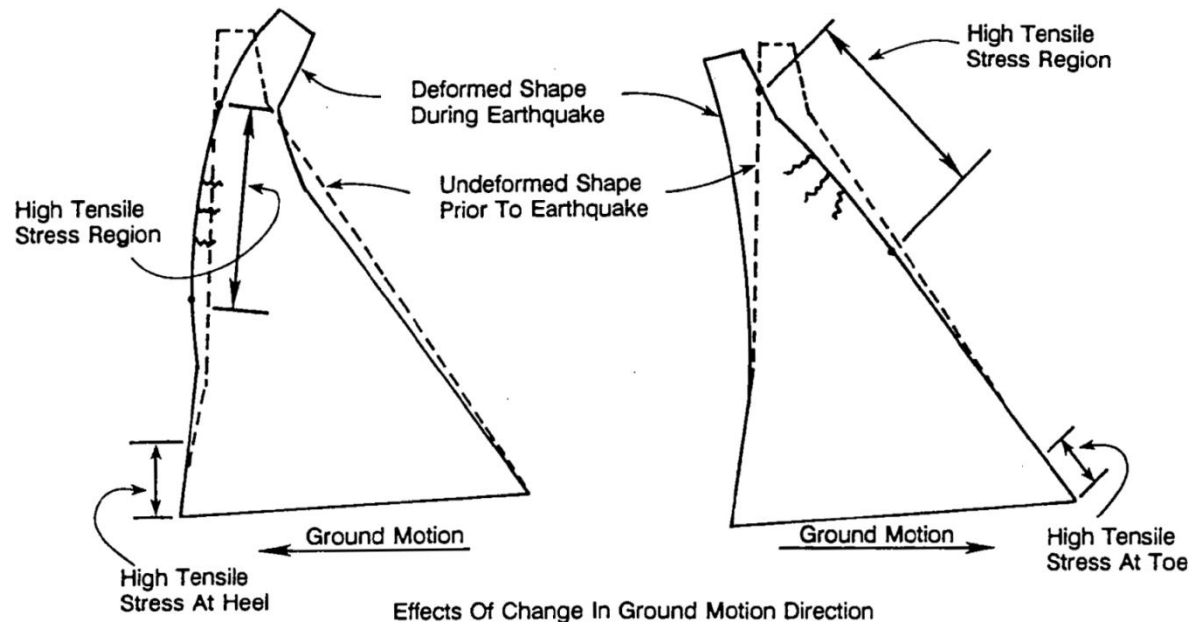
$$\text{Horizontal acceleration} = \alpha_h g$$

$$\text{Vertical acceleration} = \alpha_v g$$



❑ During earth quake , as the ground under a dam moves, the dam must also move with it to avoid rupture. This means that the dam has to resist the inertial force caused by the sudden movement of the earth crest.

✓ Inertial forces always act opposite to the direction earthquake movement.



❑ Two methods for estimating seismic loads exist

✓ **Pseudostatic (seismic coefficient method)**

✓ Dynamic method

❑ Earthquake forces on foundation-dam-reservoir act in two ways

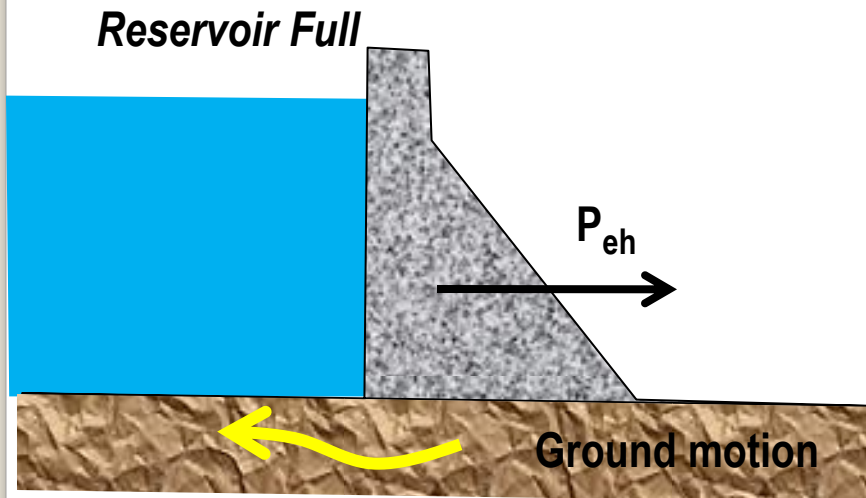
✓ inertial forces / earth quake force on the dam body

✓ increase in water pressure

A. Earthquake forces on the body of the dam

A.1 Effects of horizontal acceleration

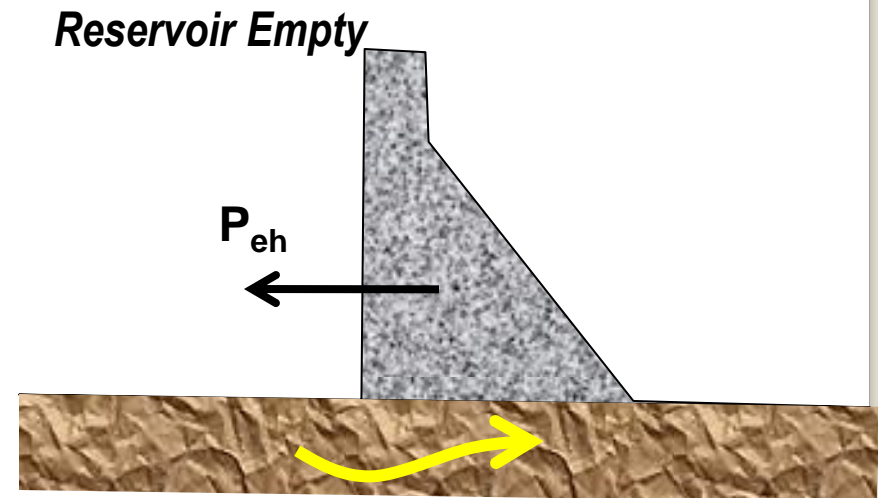
- ☐ The horizontal acceleration can occur in upstream or downstream direction
- ☐ Because a dam is designed for the worst case, the horizontal acceleration is assumed to occur in the direction which would produce the worst combination of the forces.



Earthquake movement: Upstream
Inertial force acts in : Downstream

$$P_{eh} = \frac{W}{g}(g\alpha_h)$$

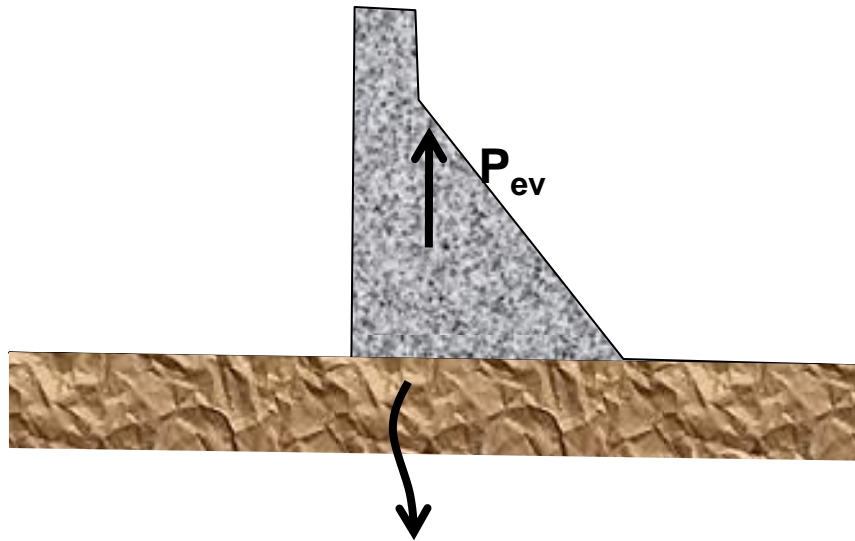
$$P_{eh} = W\alpha_h$$



Earthquake movement: Downstream
Inertial force acts in : Upstream

It acts at the centroid of the dam

A.2 Effects of Vertical Acceleration



Dam is separated from the foundation

$$P_{ev} = \frac{W}{g}(g\alpha_v)$$

$$P_{ev} = W\alpha_v$$

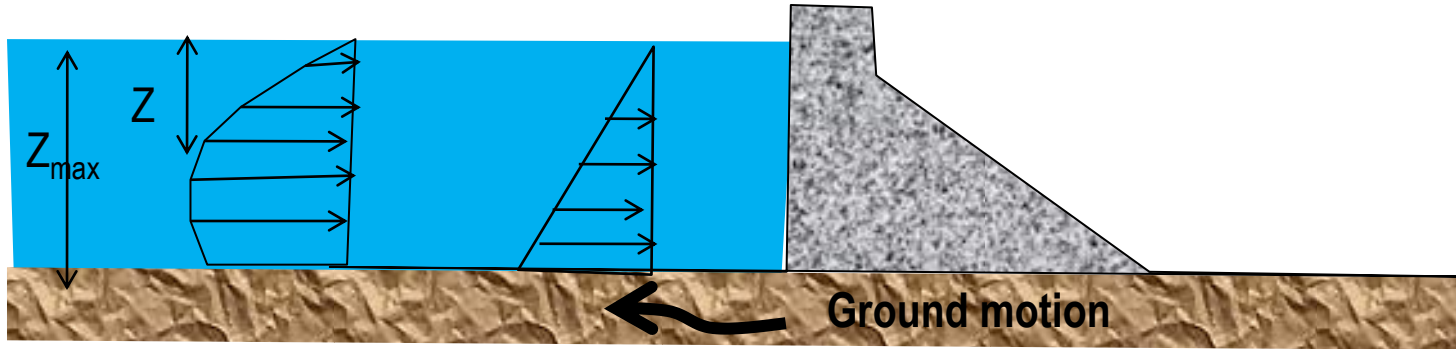
It acts at the centroid of the dam

Its effect is to modify the weight

$$\begin{aligned}\text{Modified weight} &= W - W\alpha_v \\ &= W(1 - \alpha_v)\end{aligned}$$

B. Earthquake forces on the body of water

B.1 Effects of horizontal acceleration



□ Dam and foundation move in upstream direction, they push against the reservoir momentarily increasing the water pressure

An initial estimate of these forces can be obtained using a **parabolic approximation** to the theoretical pressure distribution as analysed in Westergaard (1933). Relative to any elevation at depth Z below the water surface, hydrodynamic pressure p_{ewh} is determined by

$$p_{ewh} = C_e \alpha_h \gamma_w Z_{max} \quad (KNm^{-2})$$

where

Z_{max} is the maximum depth of water at the section of dam considered

C_e is a dimensionless pressure factor, and is a function of Z/Z_{max} and Θ , the angle of inclination of the upstream face to the vertical.

$$C_s = \frac{C_m}{2} \left[\frac{Z}{Z_{max}} \left(2 - \frac{Z}{Z_{max}} \right) + \sqrt{\frac{Z}{Z_{max}} \left(2 - \frac{Z}{Z_{max}} \right)} \right]$$

$$C_m = 0.73 \frac{\theta}{90} \quad \theta, \text{ the angle of inclination of the upstream face to the vertical.}$$

The resultant **hydrodynamic load** is given by

$$P_{ewh} = 0.66 C_s \alpha_h Z \gamma_w \left(\frac{Z}{Z_{max}} \right)^{1/2}$$

And the load acts at **0.4 Z**

B. Effect of vertical acceleration

□ If the dam has an upstream flare / batter the resultant vertical hydrodynamic load, P_{ewv} , effective above an upstream face batter or flare may be accounted for by application of the appropriate seismic coefficient to vertical water load, P_{wv} . It is considered to act through the centroid of area *thus*:

$$P_{ewv} = \pm \alpha_v P_{wv}$$

2.3.4 Load Combinations

❑ All the forces which are discussed in the preceding sections may not act simultaneously on a dam. A concrete dam should be designed with regard to the most rigorous adverse groupings or combination of load which gave a reasonable probability of simultaneous occurrence.

❑ Three **nominated load combinations** are sufficient for almost all circumstances. In ascending order of severity they may be designated as **normal** (sometimes **usual**), **unusual** and **extreme** load combinations, here denoted as NLC, ULC and ELC respectively,

	Load source	Qualification	Load combination		
			NLC	ULC	ELC
Primary	1. Headwater	At Design Flood Level (MWL)		X	
		At Full Reservoir Level (FRL)	X		X
	2. Tail Water	At maximum tail water level	X		X
		Minimum tail water level		X	
	3. Self weight	-	X	X	X
	4. Uplift	Drains functioning	X	X	
		Drains inoperative	X*	X*	X
Secondary	5. Silt	-	X	X	X
Exceptional	Seismic	-			X

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The Indian Standard Criteria (IS: 6512-1972) :

Load Combination	Load
A. – Const Condition	Dam completed but no water in reservoir and no tail water.
B – Normal Operating Condition	Full reservoir elevation, normal dry weather tail water, normal uplift, ice and silt (if applicable).
C – Flood Discharge Condition	Reservoir at maximum flood elevation, all gates open, tail water at flood elevation, normal uplift and silt (if applicable).
D	Combination A with earthquake.
E	Combination B with earthquake.
F	Combination C, but with extreme uplift.
G	Combination E, but with extreme uplift.

2.4. Modes of failure and criteria for structural stability

2.4.1 Modes of failures

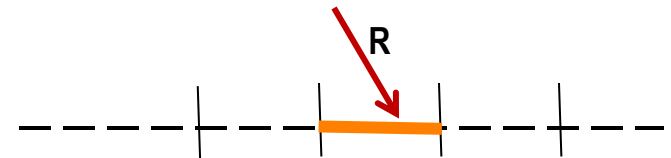
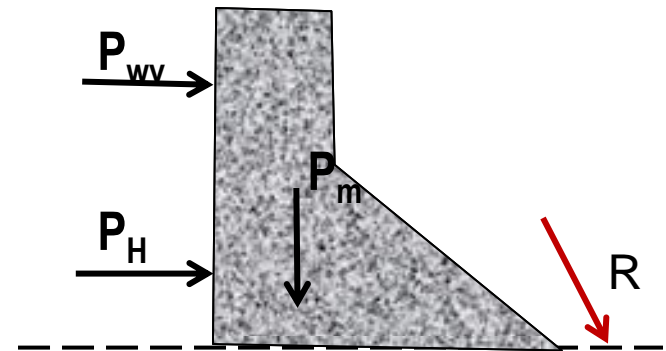
A gravity dam fail in the following ways

- Overturning about the toe
- Sliding or shear failure
- Crushing and cracking

2.4.1.1 Overturning

When the resultant (R) of all the vertical and horizontal forces acting on a dam at any given section passes outside of the toe, the dam will rotate and overturn at the toe.

Generally, a dam is safe against overturning, if the resultant lies within the middle third of the section

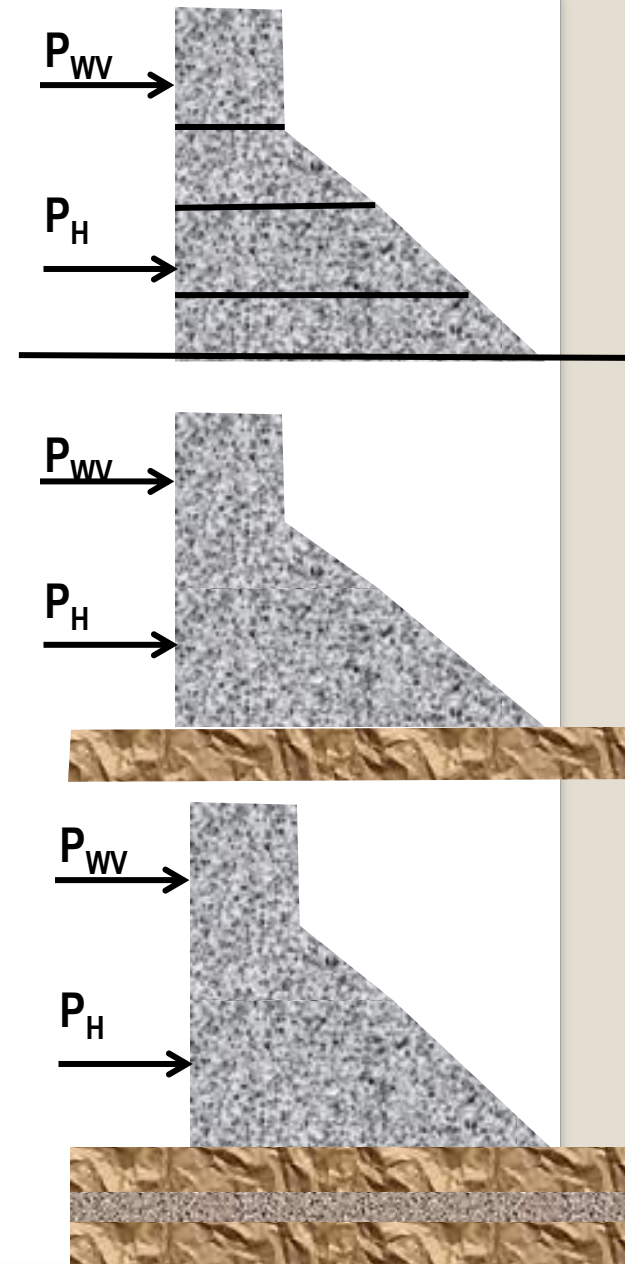


2.4.1.2 Sliding or Shear failure

When the shear stress developed at any potential path/plane due to the applied horizontal and vertical forces exceed the shearing strength of the material along the path or sliding plane, the dam will fail by sliding

Sliding may occur

- a) *At a horizontal lift joint in the dam*
- b) *At the base of the dam i.e. dam – foundation interface*
- c) *Weak joints and seams at joints and strata in the rock*



2.4.1.3 Crushing

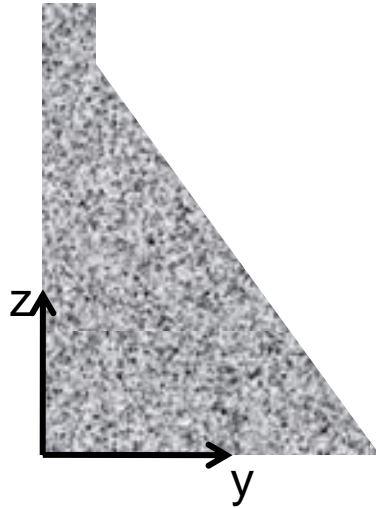
When the stress that are developed at any point in the dam exceed the strength of concrete, the dam may fail by crushing / cracking

- a) Compressive stress exceeding compressive strength of concrete
- b) tensile stress exceeding tensile strength of concrete
- c) When the stress developed at the foundation exceed the bearing capacity of the foundation



2.4.2 Stability Requirements of Gravity Dam

2.4.2.1 Symbols and sign conventions



Symbols

M – moment

$\Sigma H, \Sigma V$ – Horizontal & vertical forces

C – Cohesion

Φ – angle of friction

α – angle of sliding plane

τ – Shear stress

A_s – Area of shearing surface

Stabilizing Forces :

Weight of the dam (Dead Load)

The thrust of the tail water.

Destabilizing Forces :

Head water pressure,

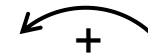
Uplift,

Wave pressure in the reservoir,

Earth and Silt pressure,

Seismic forces,

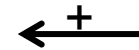
Convention



Restoring Moment



Overturning Moment



+ve Horizontal force



-ve Horizontal force



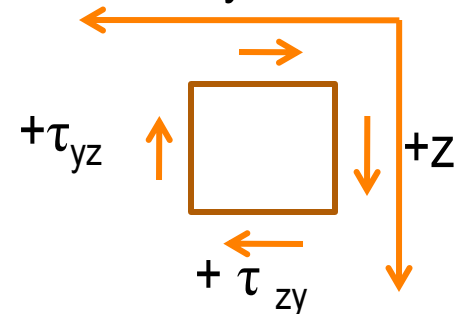
+ +ve Vertical force



- -ve Vertical force

+ve Shear

+y



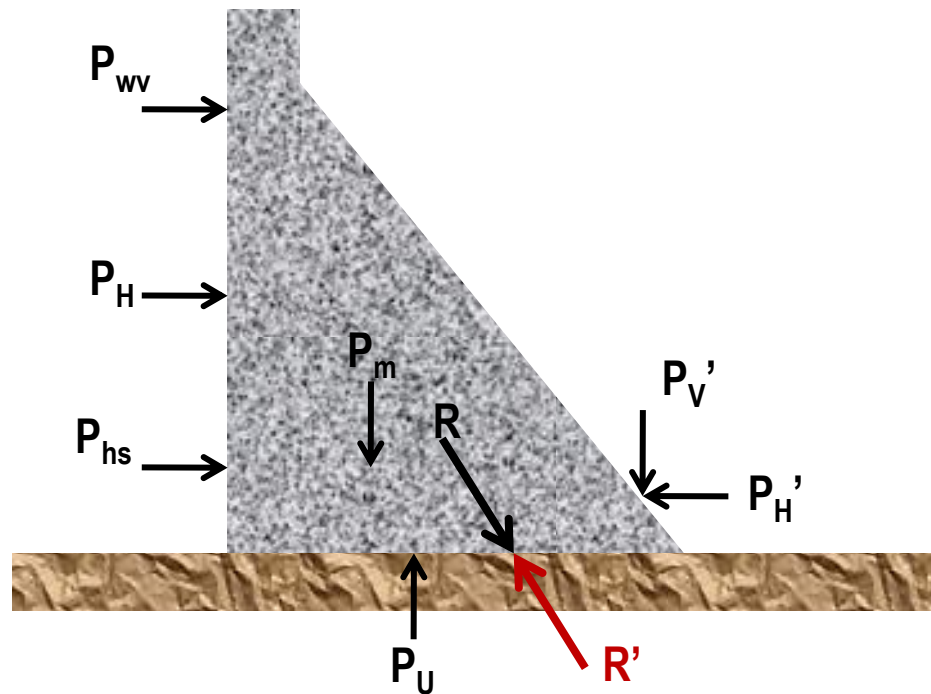
2.4.2.2. Structural Equilibrium

Applied Force = Reactive Force

$\Sigma F_x = 0$ No translational movement

$\Sigma F_y = 0$

$\Sigma M = 0$ No rotational movement



R = Resultant of all loads R' = Foundation Reaction

2.4.2.3 Assumptions in Stability Analysis

1. Concrete used homogeneous, isotropic and elastic
2. Dam consist of a number of vertical cantilivers of unit length. The cantilevers act independently
3. Perfect bond between dam and foundation
4. All loads are transferred by cantilever action by the foundation. No beam action
5. The foundation is strong and unyielding. No movement caused in the foundation due to the imposed loads
6. Small openings, galleries, shafts, do not affect the over all stability.

2.4.2.4 Stability Requirements

❑ A gravity dam must be designed such that it is safe against all possible modes of failures, with adequate factor of safety. A dam may fail

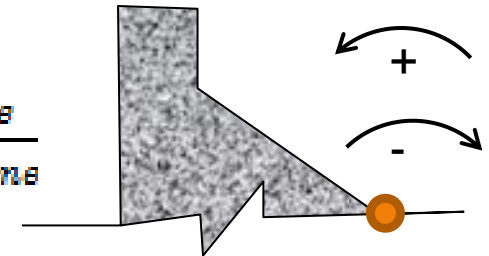
- Overturning
- Sliding and shear
- Crushing

2.4.2.5 Overturning Stability (F_o)

$$F_o = \frac{\sum M_{+ve}}{\sum M_{-ve}}$$
$$= \frac{\text{Sum of restoring moment about the toe of any horizontal plane}}{\text{Sum of overturning moment about the toe of any horizontal plane}}$$

The moments are about the toe of any horizontal plane

$\sum M_{-ve}$ includes the moment generated by uplift.



Criteria

$F_o > 1.25$ Acceptable

$F_o > 1.50$ Desirable

2.4.2.6 Sliding Stability

❑ It is a function of the loading pattern and of the resistant to translational displacement which can be mobilized at any plane. Three methods are available.

- ✓ Sliding Factor (Fss)
- ✓ Shear Friction Factor (FSF)
- ✓ Limit Equilibrium Method (FLE)

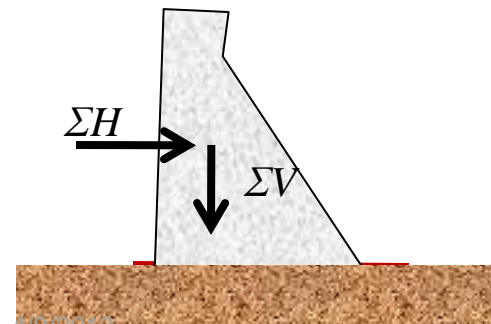
A. Sliding Factor

❑ *Used by dam designers in 1900-1930*

❑ *Resistance to sliding is assumed to be purely frictional with no cohesion*

a) *When the sliding plane is horizontal*

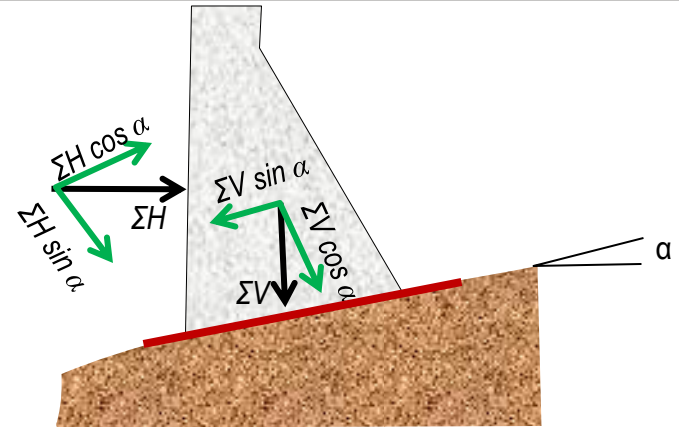
$$F_{ss} = \frac{\sum H}{\sum V}$$



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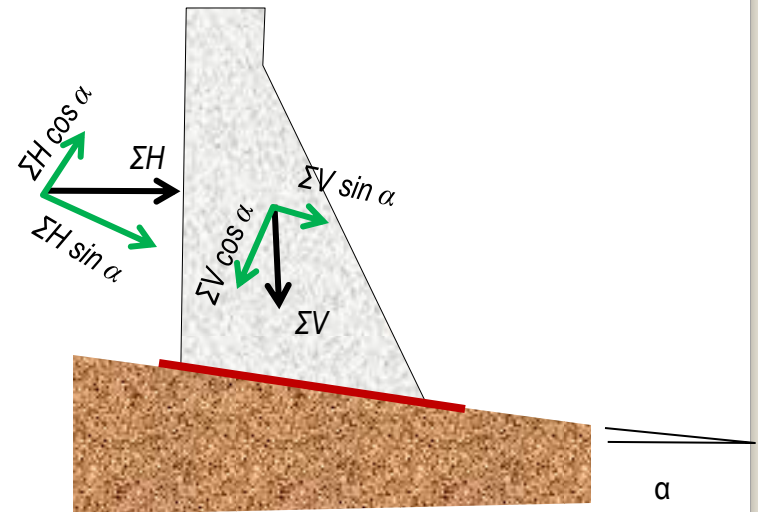
b) When the sliding plane is at an angle +ve α

$$F_{ss} = \frac{\frac{\sum H}{\sum V} - \tan \alpha}{1 + \frac{\sum H}{\sum V} \tan \alpha}$$



c) When the sliding plane is at an angle -ve α

$$F_{ss} = \frac{\frac{\sum H}{\sum V} + \tan \alpha}{1 - \frac{\sum H}{\sum V} \tan \alpha}$$



Criteria

$F_{ss} \leq 0.75$ for NLC

$F_{ss} \leq 0.90$ for ELC

B- Shear Friction Factor (FSF)

❑ Introduced by Henny in 1933

❑ Considers both friction and cohesion for shear resistance

❑ From Mohr –Coulomb

$$\tau_f = C + \sigma_n \tan \phi$$

$$\tau_f A_s = C A_s + \sigma_n A_s \tan \phi$$

$$S_f = C A_s + V_n \tan \phi$$

❑ Shear friction factor is the total resistance to shear and sliding to the horizontal load

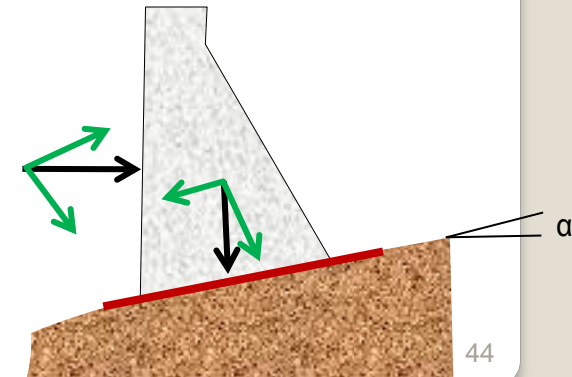
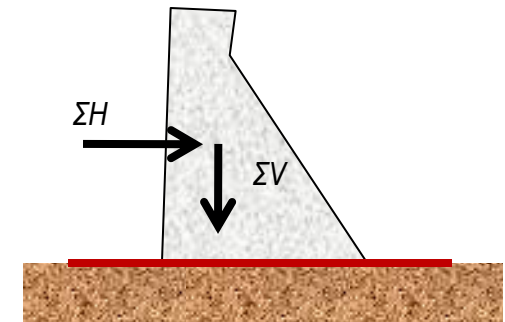
$$F_{SF} = \frac{S}{\Sigma H}$$

a) When the sliding plane is horizontal

$$S = C A_s + \sum V \tan \phi$$

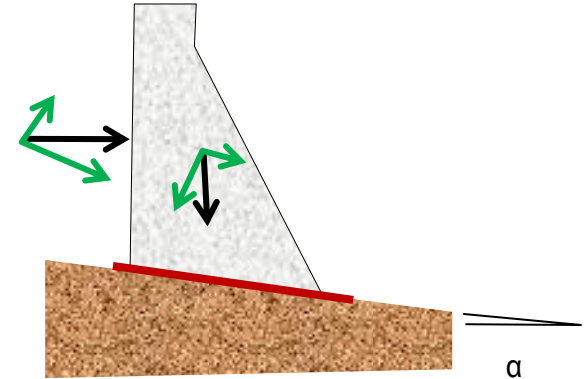
b) When the sliding plane is inclined +ve

$$S = \frac{C A_s}{\cos \alpha (1 - \tan \alpha \tan \phi)} + \sum V \tan(\phi + \alpha)$$



c) When the sliding plane is inclined –ve

$$S = \frac{CA_s}{\cos \alpha (1 + \tan \alpha \tan \phi)} + \sum V \tan(\phi - \alpha)$$



Criteria

	NLC	ULC	ELC
Dam/foundation	3	2	>1

C) Limit equilibrium Factor (FLE)

It follows the conventional soil mechanics logic

$$F_{LE} = \frac{\tau_a}{\tau}$$

τ_a is the shear strength available

τ shear stress developed under the applied loading

τ_a is expressed by mohr-coulomb failure criteria

$$\tau_a = C + \sigma_n \tan \phi$$

a) When the sliding plane is horizontal

$$F_{LE} = \frac{CA_s + V \tan \phi}{\sum H}$$

b) When the sliding plane is inclined at an angle +ve α

$$F_{LE} = \frac{CA_s + (\sum V \cos \alpha + \sum H \sin \alpha) \tan \phi}{\sum H \cos \alpha - \sum V \sin \alpha}$$

c) When the sliding plane is inclined at angle -ve α

$$F_{LE} = \frac{CA_s + (\sum V \cos \alpha - \sum H \sin \alpha) \tan \phi}{\sum H \cos \alpha + \sum V \sin \alpha}$$

Criteria

FLE = 2 NLC

FLE = 1.3 ELC

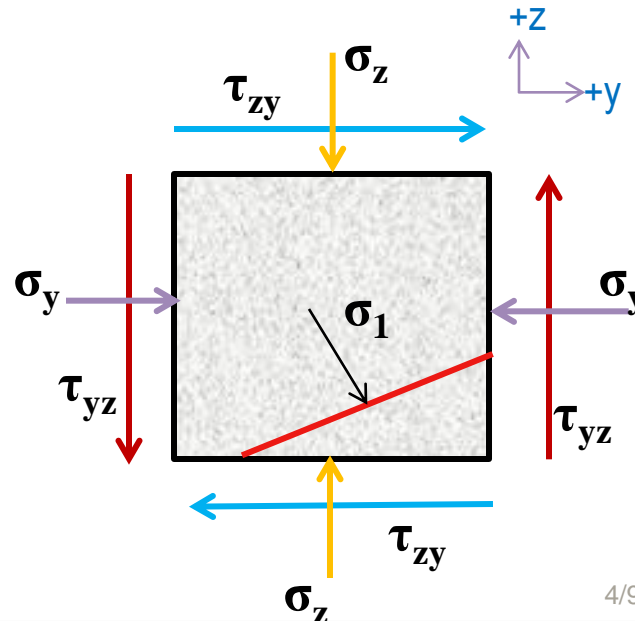
3.4.2.7 Stress Analysis

□ Gravity method is commonly used

- ✓ 2d elastic dam on rigid foundation
- ✓ stress evaluated in the analysis

□ The various stresses that are determined in gravity method are

1. Vertical normal stress, σ_z , on horizontal planes;
2. Horizontal normal stress, σ_y , on vertical planes;
3. Horizontal and vertical shear stresses τ_{zy} & τ_{yz} ;
4. Principal stresses σ_1 & σ_3 ;



A. Vertical Normal Stress

- ❑ Modified beam theory is employed
(i.e., combined Axial Load and Bending Moment)

$$\sigma_z = \frac{\sum V}{A} \pm \frac{\sum M^* y}{I}$$

$\sum M^*$: sum of moment w.r.t centroid of the plane

y : distance from the centroid to point of consideration

I : second moment of area of the plane w.r.t the centroid

$\sum V$: sum of vertical loads excluding uplift

- ❑ For rectangular 2D plane section of unit width

✓ The second moment of area and the eccentricity is given by

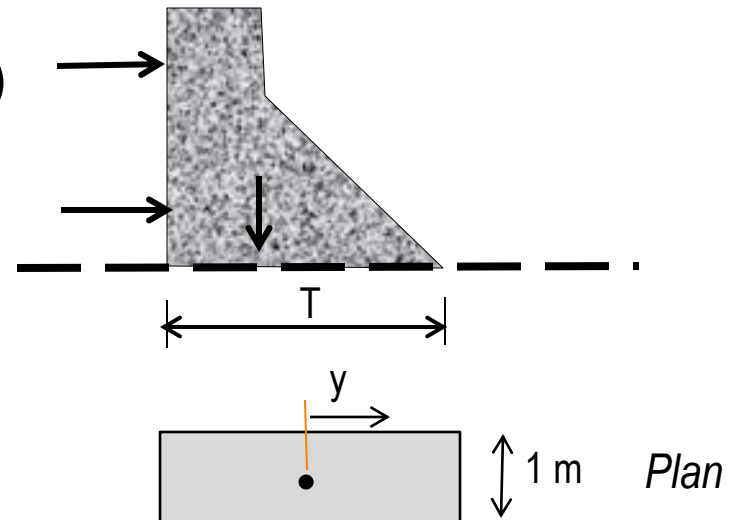
$$I = \frac{T^3}{12} \quad e = \frac{\sum M^*}{\sum V}$$

✓ The vertical normal stress is then given by

$$\sigma_z = \frac{\sum V}{T} \pm \frac{12 \sum V e y}{T^3}$$

✓ The maximum and minimum normal stresses are at $y = T/2$

$$\sigma_z = \frac{\sum V}{T} \left(1 \pm \frac{6e}{T} \right)$$

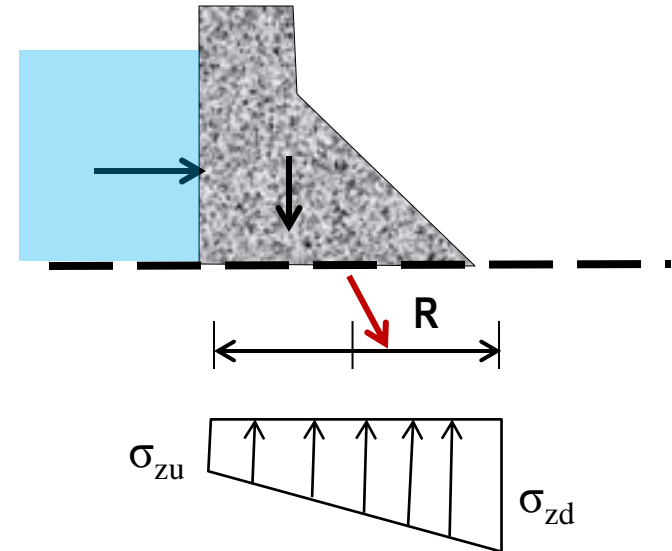


$$\text{Area} = T * 1 = T$$

A.1 For reservoir full condition

U/s face $\sigma_{zu} = \frac{\Sigma V}{T} \left(1 - \frac{6e}{T}\right)$

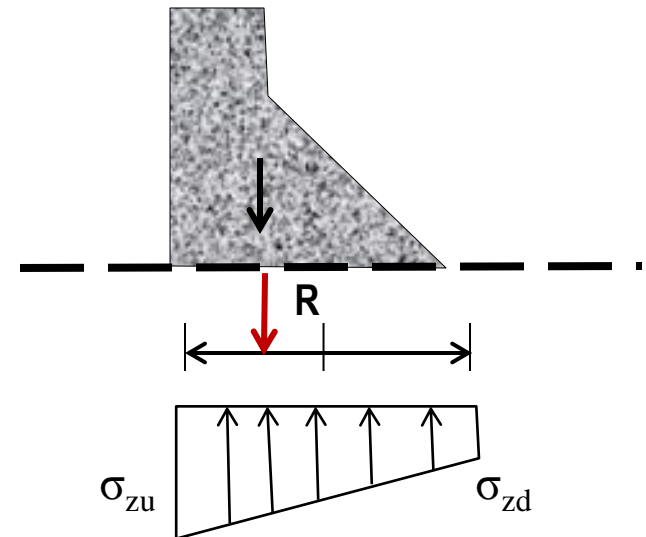
D/s face $\sigma_{zd} = \frac{\Sigma V}{T} \left(1 + \frac{6e}{T}\right)$



A.2 For reservoir empty condition

U/s face $\sigma_{zu} = \frac{\Sigma V}{T} \left(1 + \frac{6e}{T}\right)$

D/s face $\sigma_{zd} = \frac{\Sigma V}{T} \left(1 - \frac{6e}{T}\right)$



For $e > T/6$; -ve stress develop at u/s face, i.e., Tensile stress

B. Shear Stresses

□ Linearly varying normal stress generate numerically equal and complementary

τ_{yz} horizontal shear

τ_{zy} vertical shear

□ Adequate to establish the shear stresses at the boundary

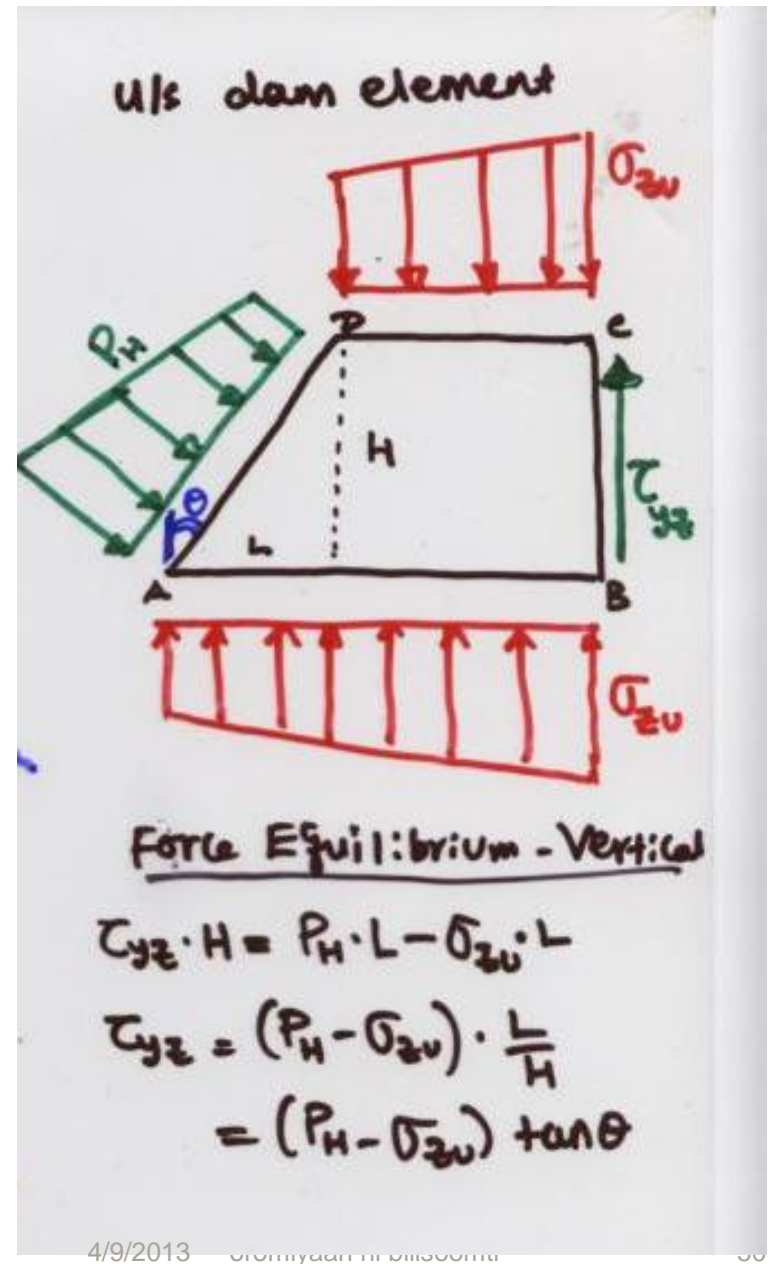
□ Taking a small element on the u/s and d/s faces: [link](#)

U/s face

$$\tau_u = (P_H - \sigma_{zu}) \tan \theta_u$$

D/s face

$$\tau_d = \sigma_{zd} \tan \theta_d$$



C. Horizontal Normal Stress

□ Horizontal normal stress on vertical planes can be determined from consideration of horizontal shear forces.

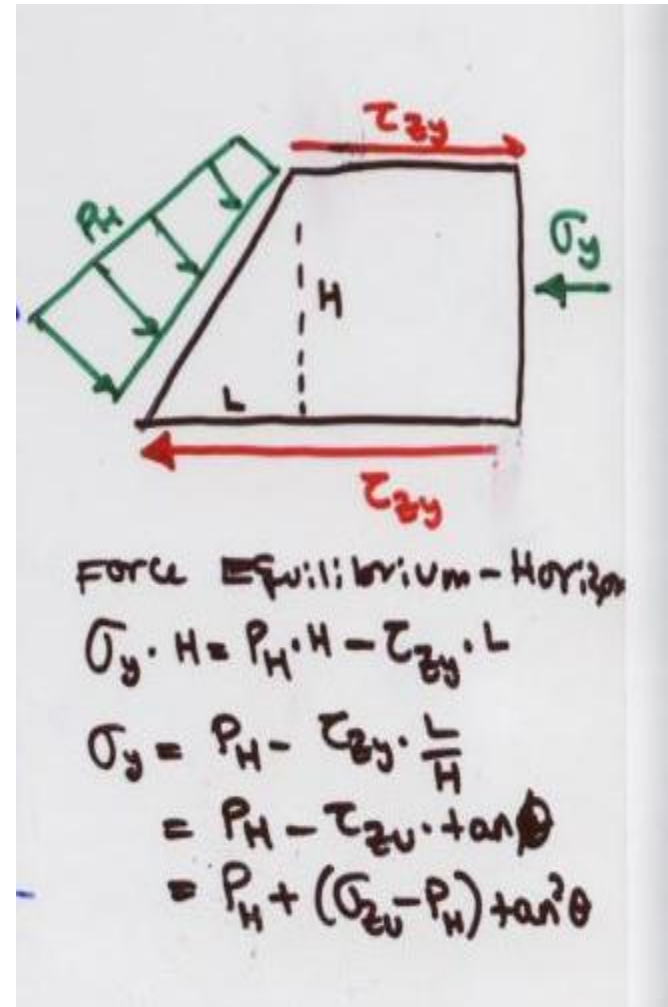
□ Differences in horizontal shear forces is balanced by normal stresses on vertical planes

U/s

$$\sigma_{yu} = P_H + (\sigma_{zu} - P_H) \tan^2 \theta_u$$

D/s

$$\sigma_{yd} = \sigma_{zd} \tan^2 \theta_d$$



D. Principal Stresses

❑ The vertical normal stresses $\sigma_z = \frac{\sum V}{A} \pm \frac{\sum M^* y}{I}$ calculated at the boundaries **are not the maximum stresses** produced anywhere in the dam.

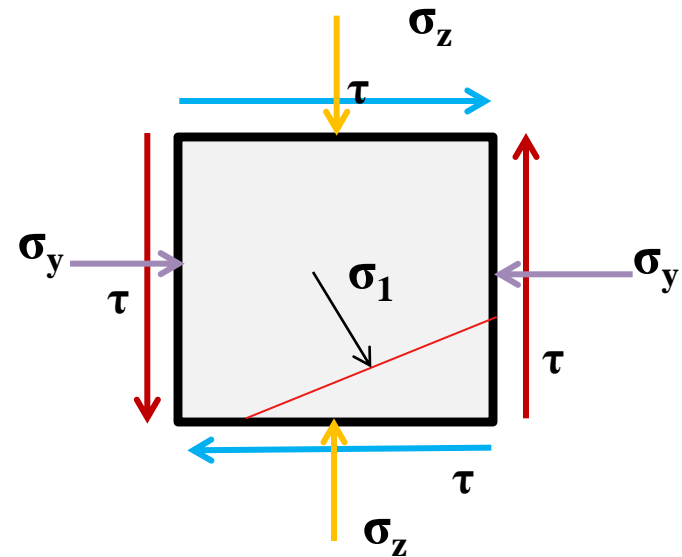
❑ The **maximum normal stress** is the **major principal stress** that will be developed on the **major principal planes**

❑ Given σ_z , σ_y , τ

❑ The major and minor principal stresses are

$$\sigma_1 = \frac{\sigma_z + \sigma_y}{2} + \left[\sqrt{\frac{\sigma_z - \sigma_y}{2}} + \tau \right]^{1/2}$$

$$\sigma_1 = \frac{\sigma_z + \sigma_y}{2} - \left[\sqrt{\frac{\sigma_z - \sigma_y}{2}} + \tau \right]^{1/2}$$



❑ The upstream and downstream faces are places of zero shear, and therefore planes of principal stresses. [Link](#). The boundary values of σ_1 & σ_3 are then determined as follows:

❑ For U/S face

$$\sigma_{1u} = \sigma_{zu}(1 + \tan^2 \theta_u) - P_H \tan^2 \theta_u$$

$$\sigma_{3u} = P_H$$

❑ For D/s face assuming no tail water

$$\sigma_{1d} = \sigma_{zd}(1 + \tan^2 \theta_d)$$

$$\sigma_{3d} = 0$$

❑ Criteria $F_c = \frac{\sigma_c}{\sigma_{max}}$ $\sigma_c =$ allowable compressive strength of concrete
 $\sigma_{max} = \max(\sigma_{1u}, \sigma_{1d})$

	Concrete	Rock
NLC	3	4
ULC	2	2.7
ELC	1	1.3